Angular structure of jet quenching in a hybrid strong/weak coupling model

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4th Heavy Ion Jet Workshop

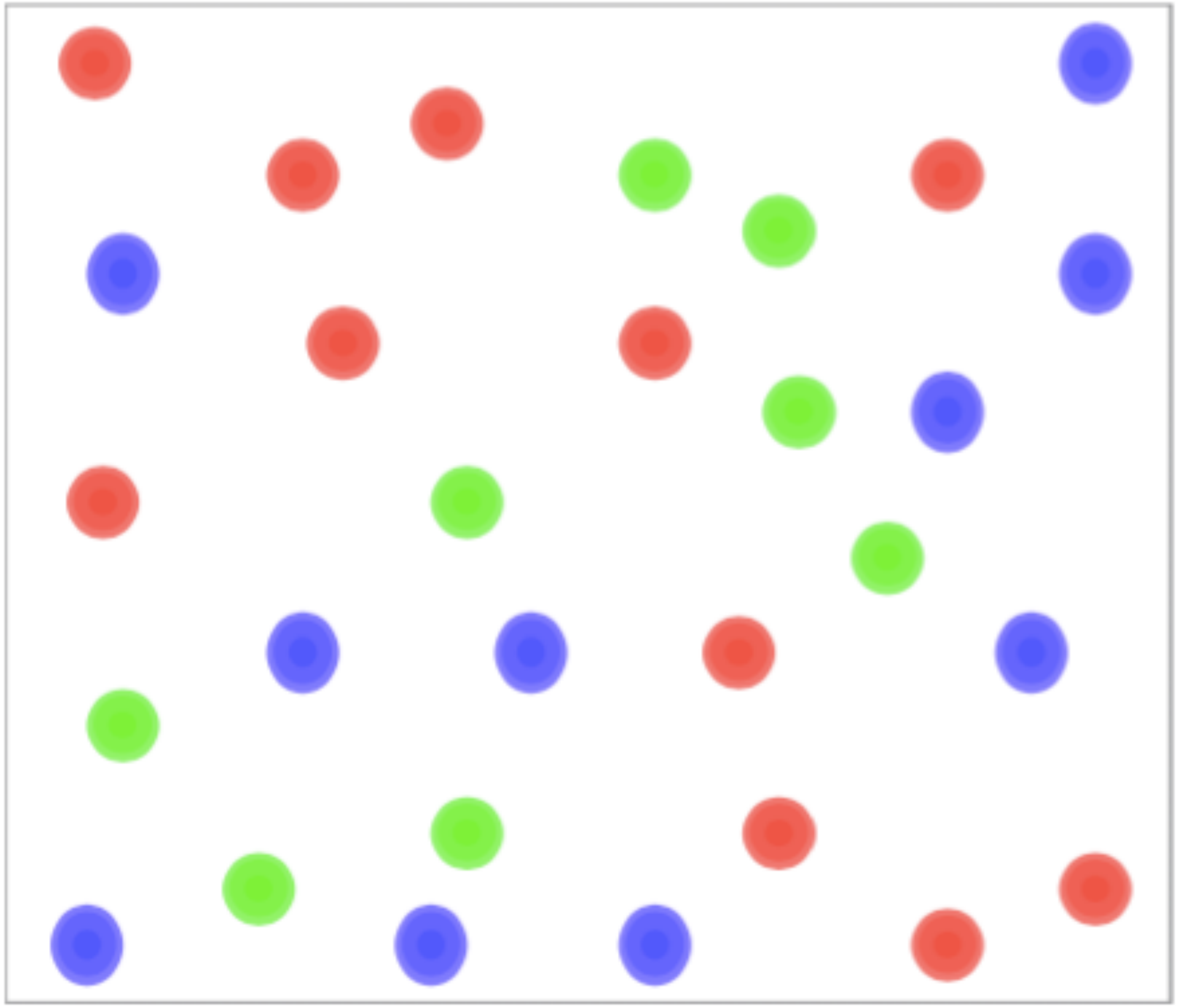
ICCUB

UNIVERSITAT DE BARCELONA

EXCELENCIA MARÍA DE MAEZTU

A Gas of Quarks and Gluons

$T > 10^4\,\mathrm{GeV}$



$$\frac{1}{T} \ll \frac{1}{gT} \ll \frac{1}{g^2T}$$

Inter-particle spacing

Interaction range

Mean free path

A Gas of Quarks and Gluons

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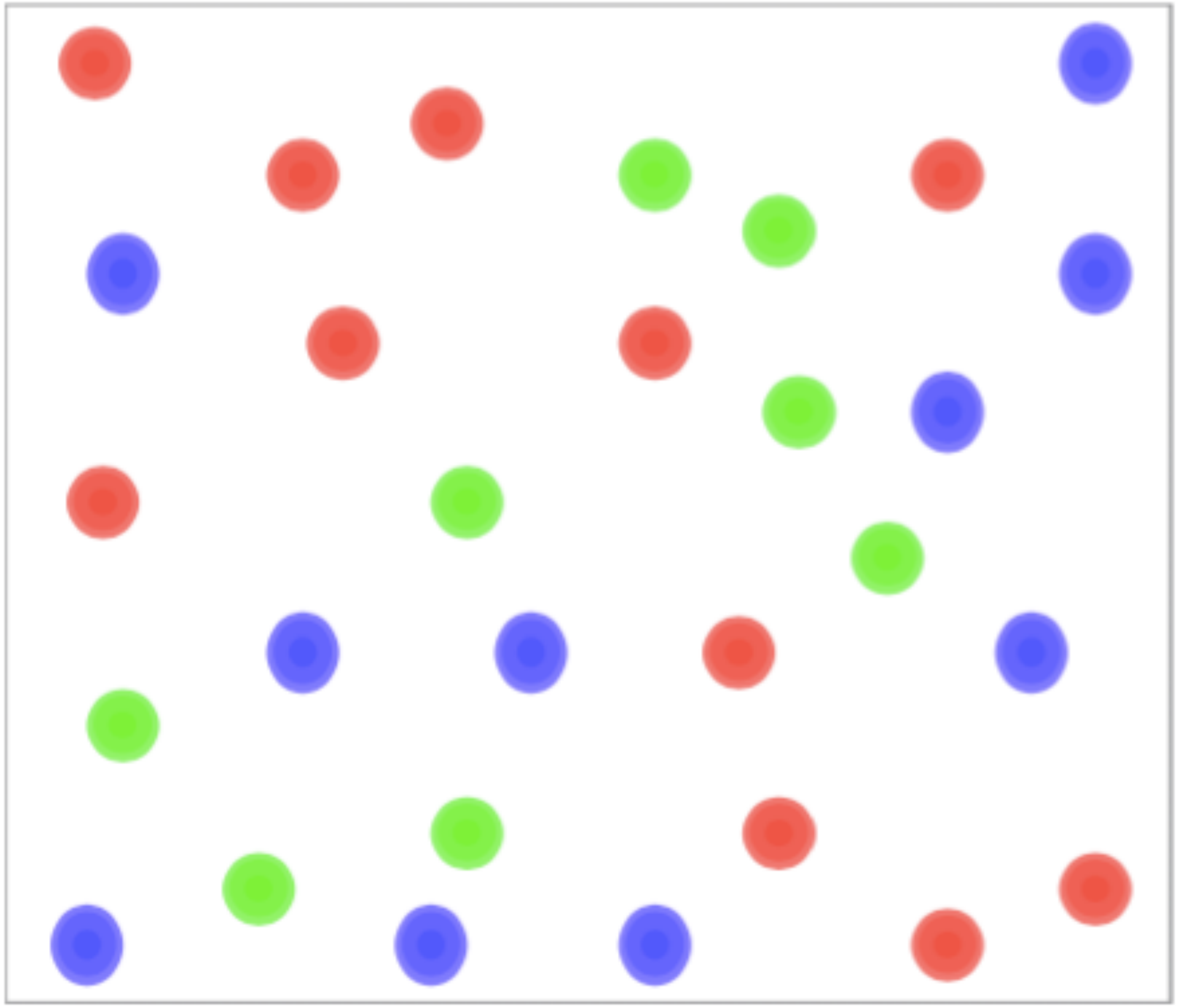


$$\frac{1}{T} \ll \frac{1}{gT} \ll \frac{1}{g^2T}$$

Inter-particle spacing

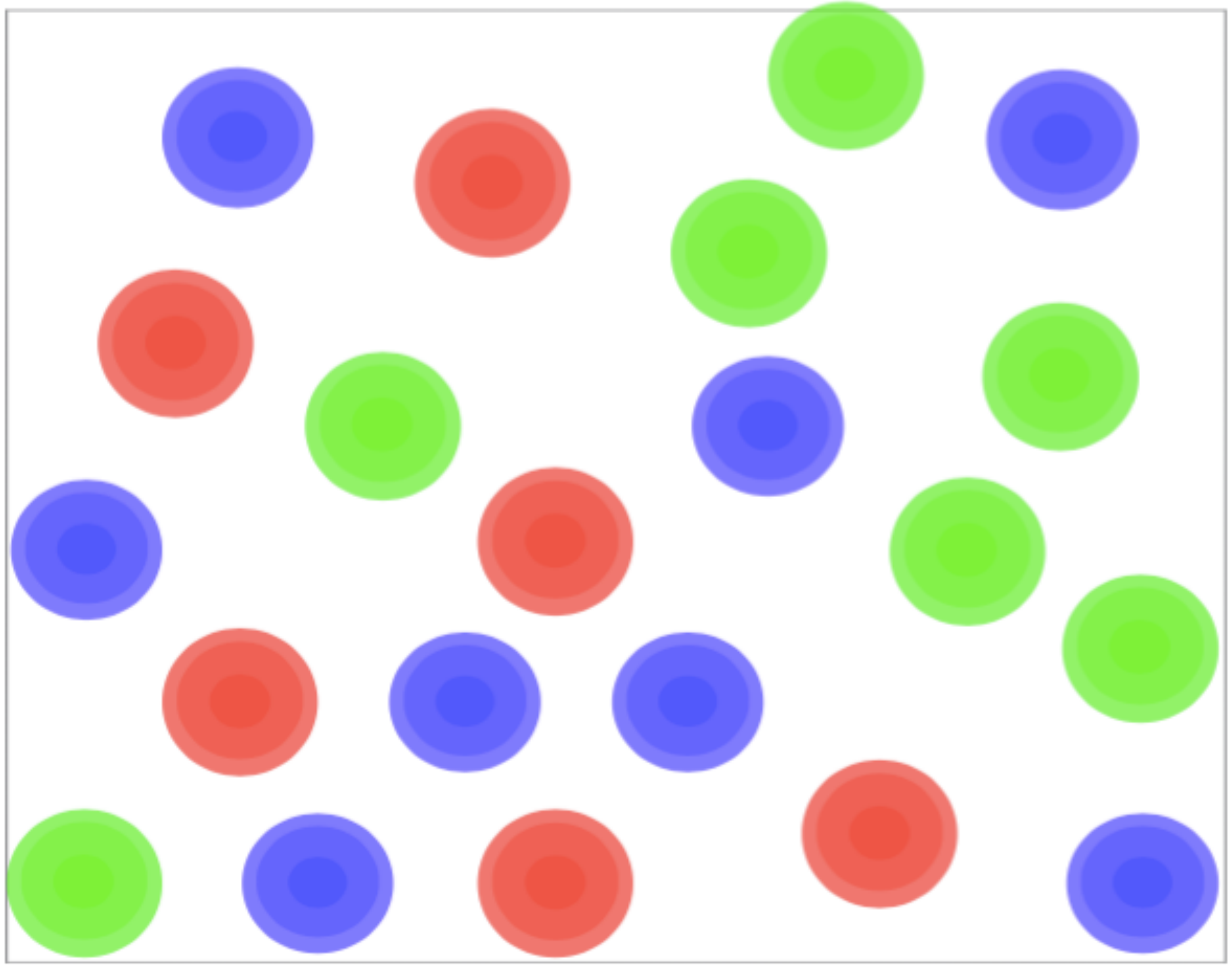
Interaction range

Mean free path

Resummation techniques can bring the validity of perturbative methods to much lower temperatures

What does the plasma look like at different scales?

$T \sim 0.2\,\mathrm{GeV}$



Is it a gas of quarks and gluons?

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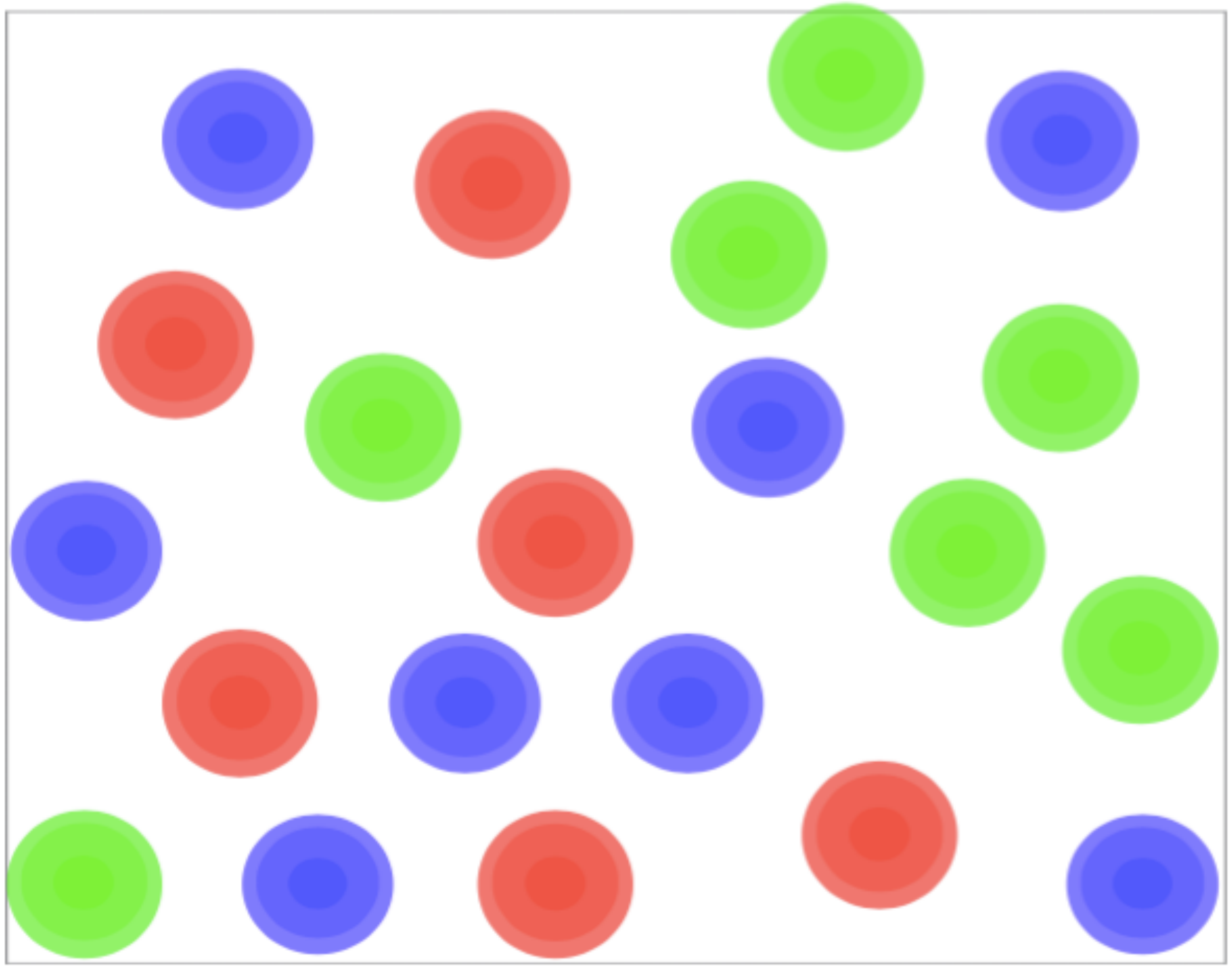

Is it a gas of quarks and gluons?

$$\alpha_s = 0.3 \to g = 2$$

What does the plasma look like at different scales?

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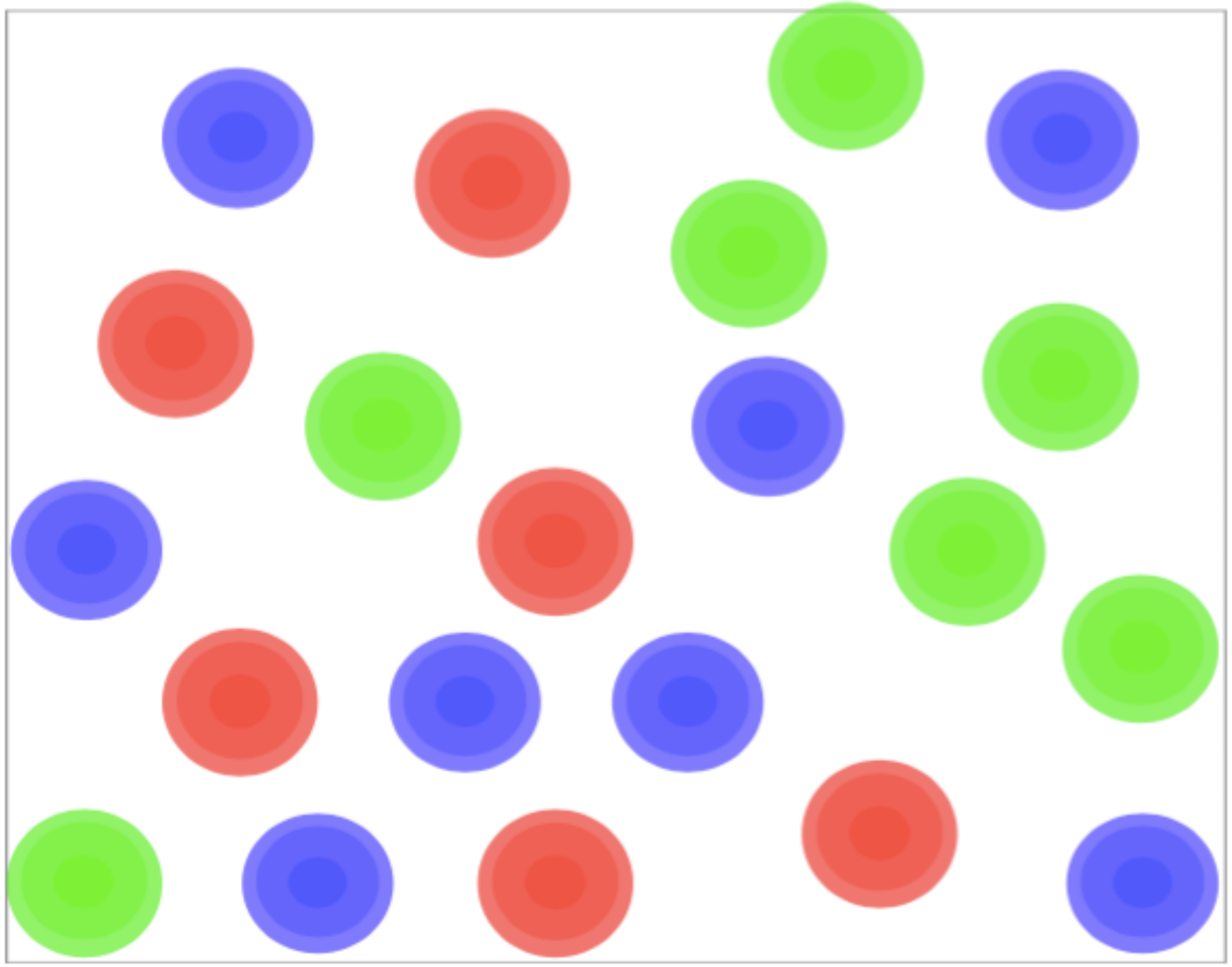

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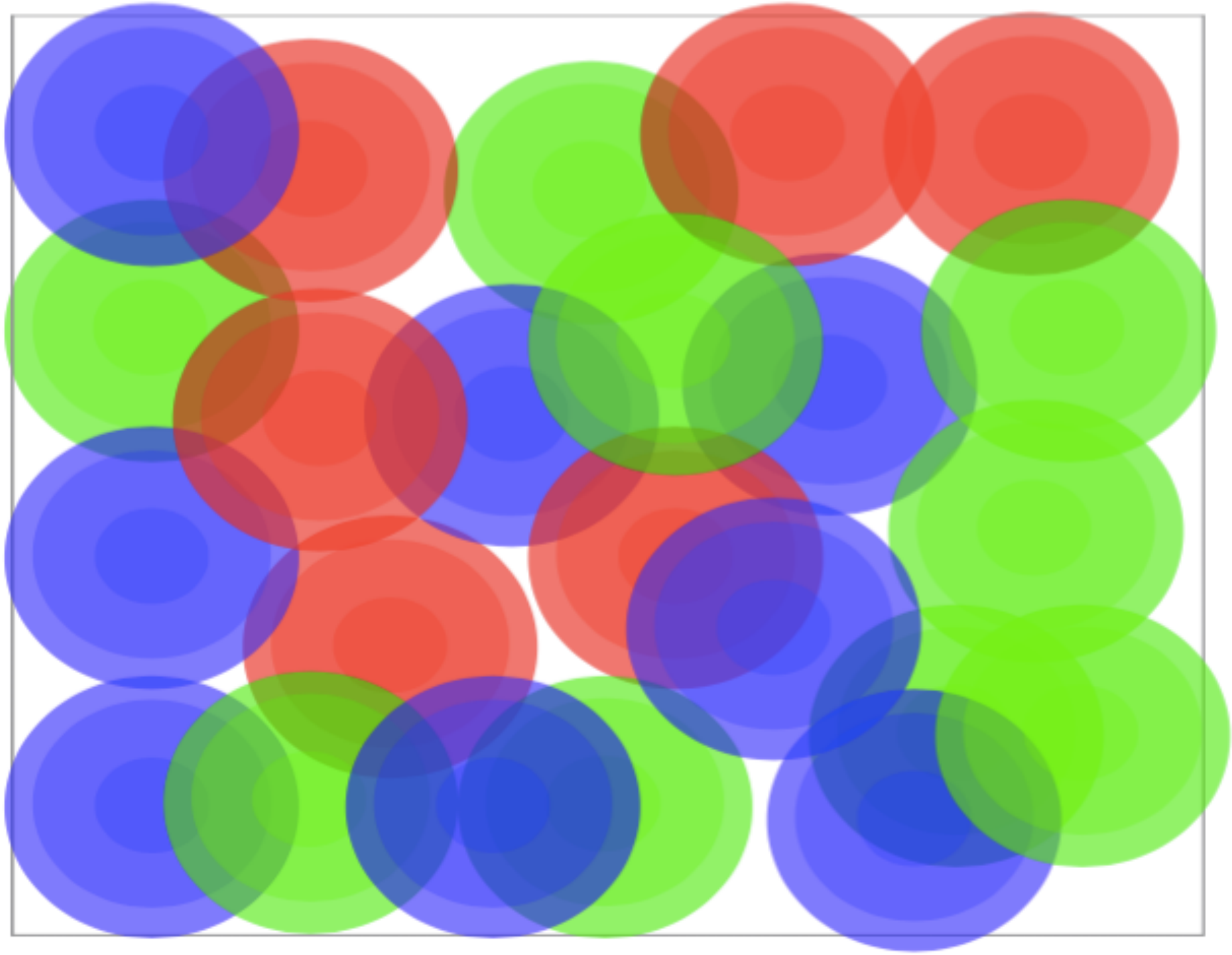

Is it a system with no long lived excitations?

$$\alpha_s = 0.3 \to g = 2$$

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What does the plasma look like at different scales?

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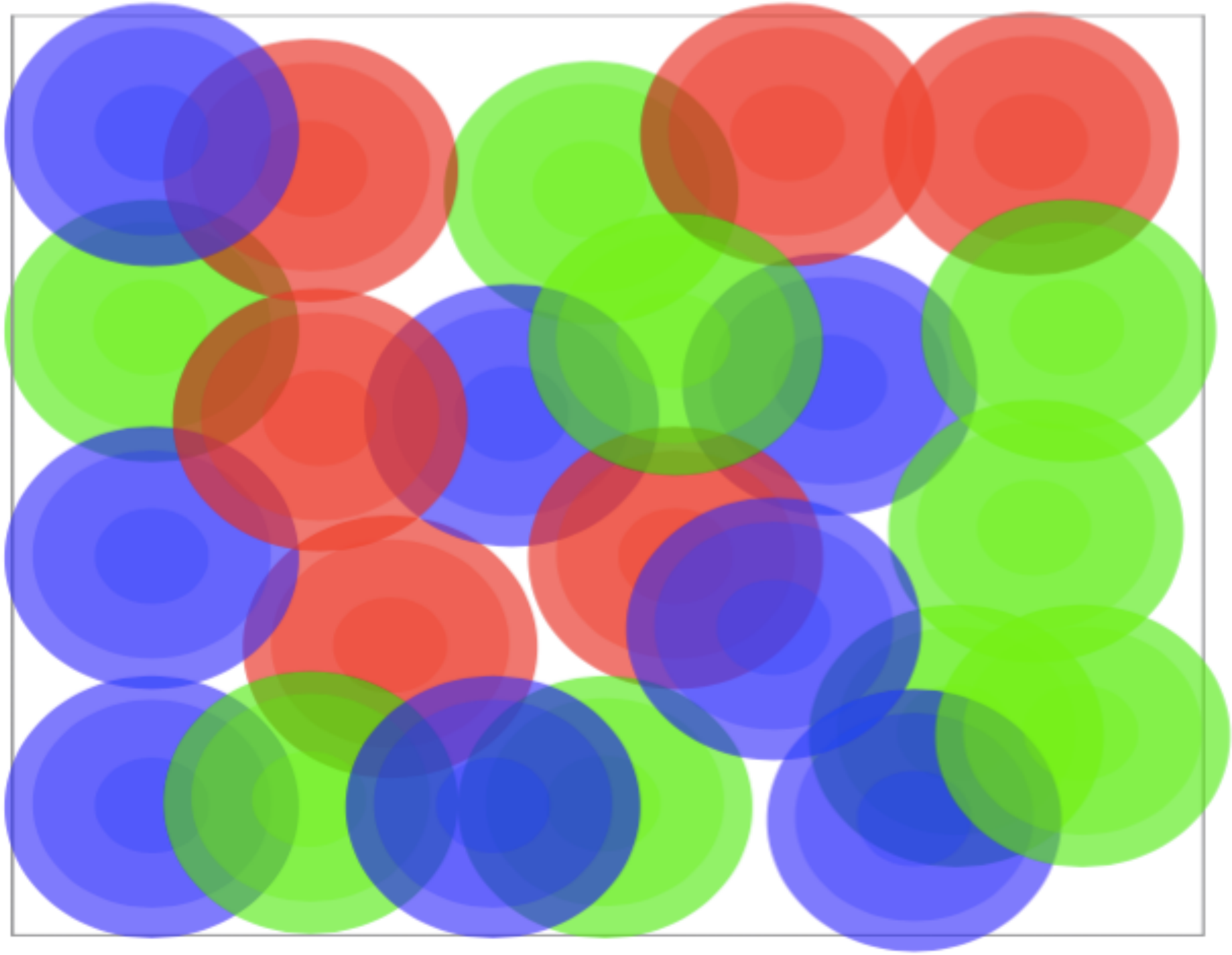

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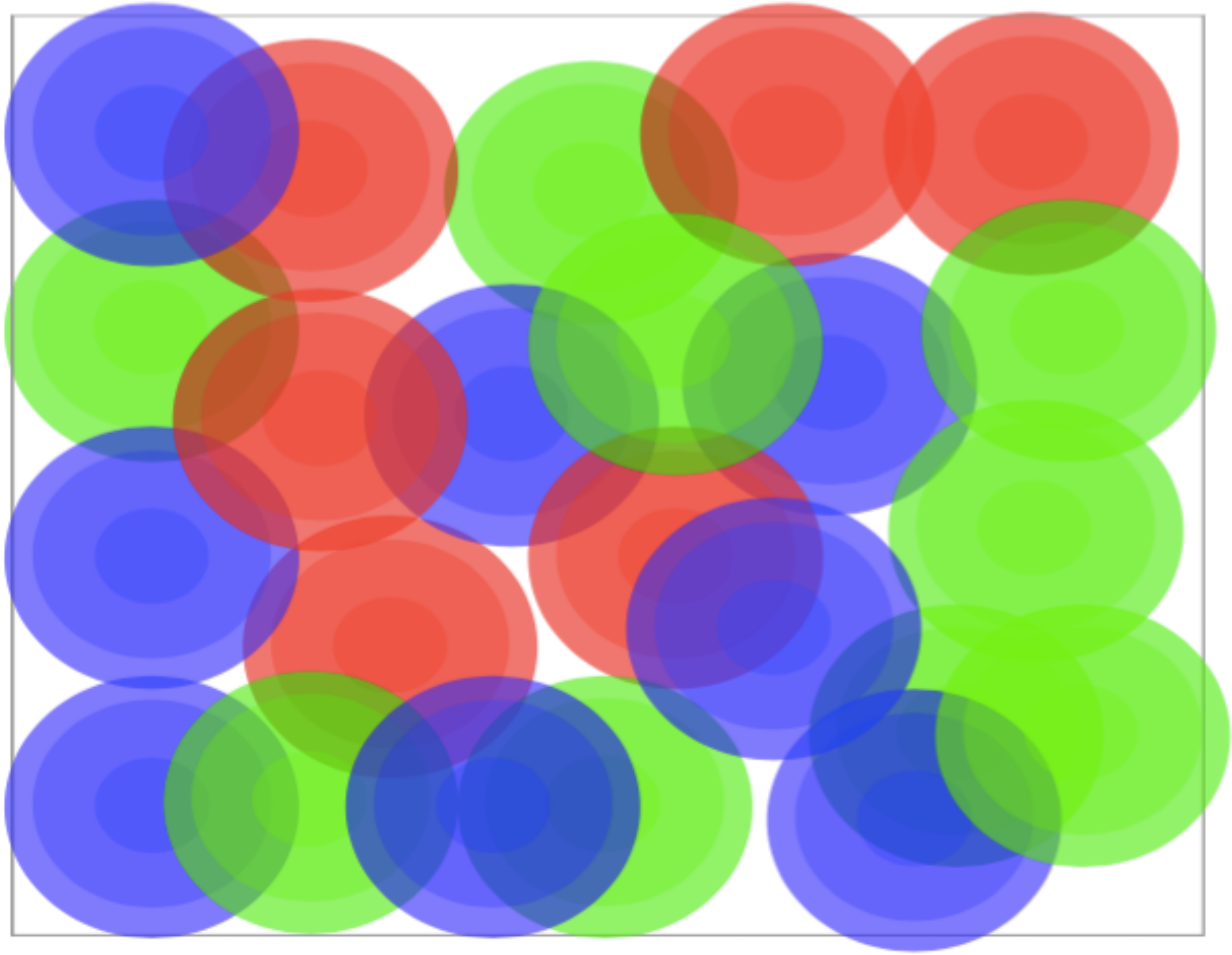


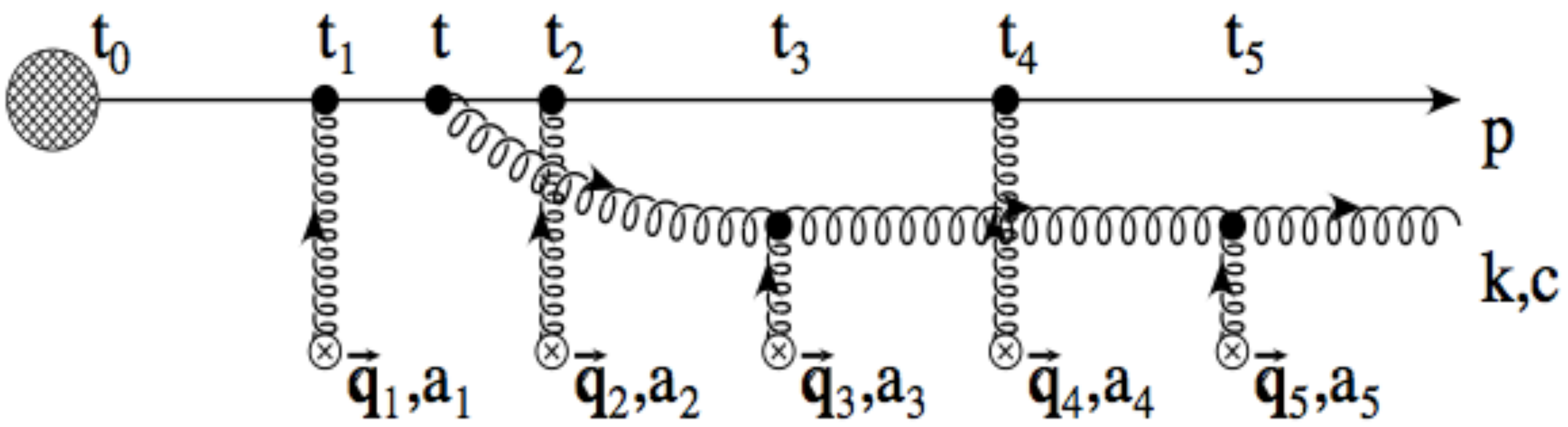
Is it a system with no quasiparticles?

$$\alpha_s = 0.3 \to g = 2$$

$$T \sim gT \sim g^2 T$$

typical momentum
of order T, but will look
different at short distances!

Perturbative Description



BDMPS-Z
(GLV, ASW, AMY, HT…)

Radiative energy loss in a high T plasma (*weak coupling* medium)

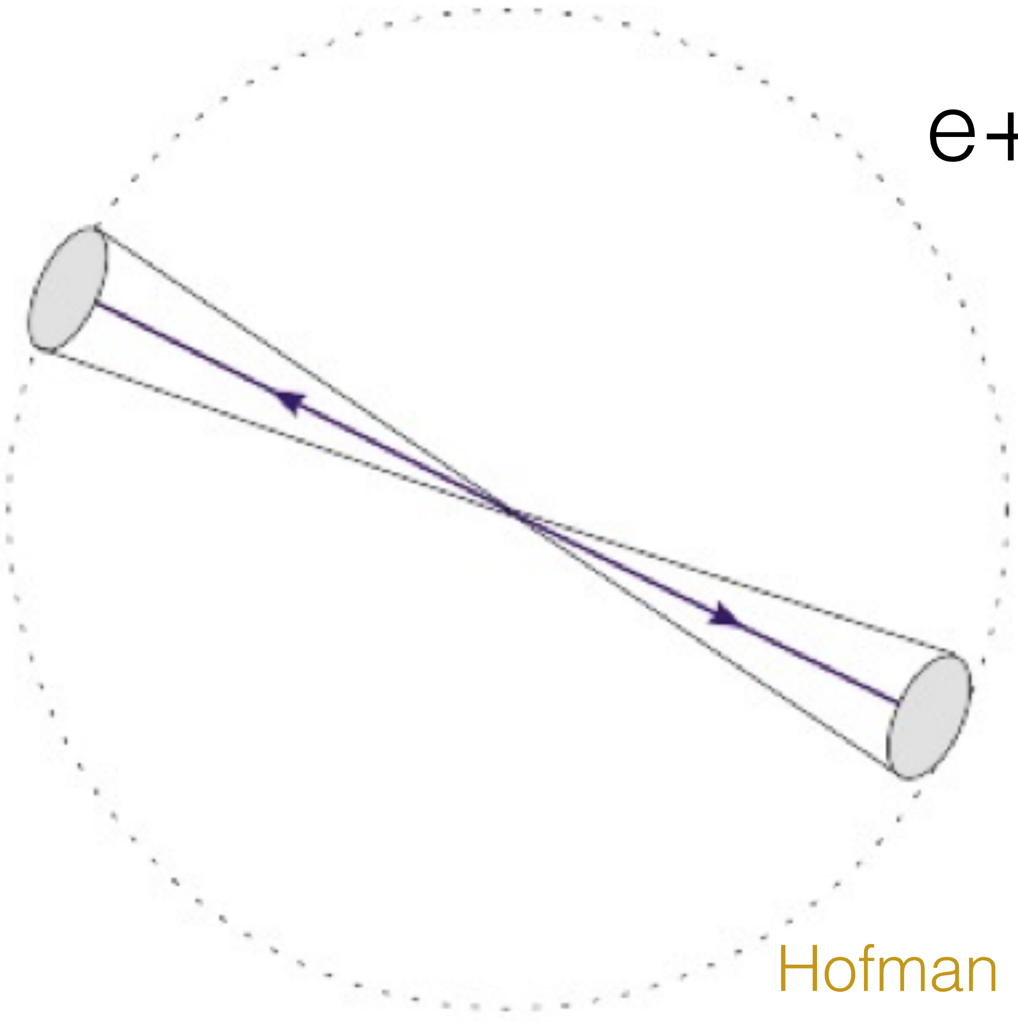
$$\frac{dE}{dx} = \frac{1}{2}\hat{q}L \qquad \hat{q} = \frac{(\text{mean transferred momentum})^2}{\text{length}} \sim \frac{m_D^2}{\lambda_{\text{m.f.p}}}$$

- Stimulated in-medium gluon emission
- Degrades leading parton energy into almost collinear gluons
- Neglects correlations among scatterers $m_D^{-1} \ll \lambda_{m.f.p.}$
- Radiated gluons propagate large distances

Strong Coupling

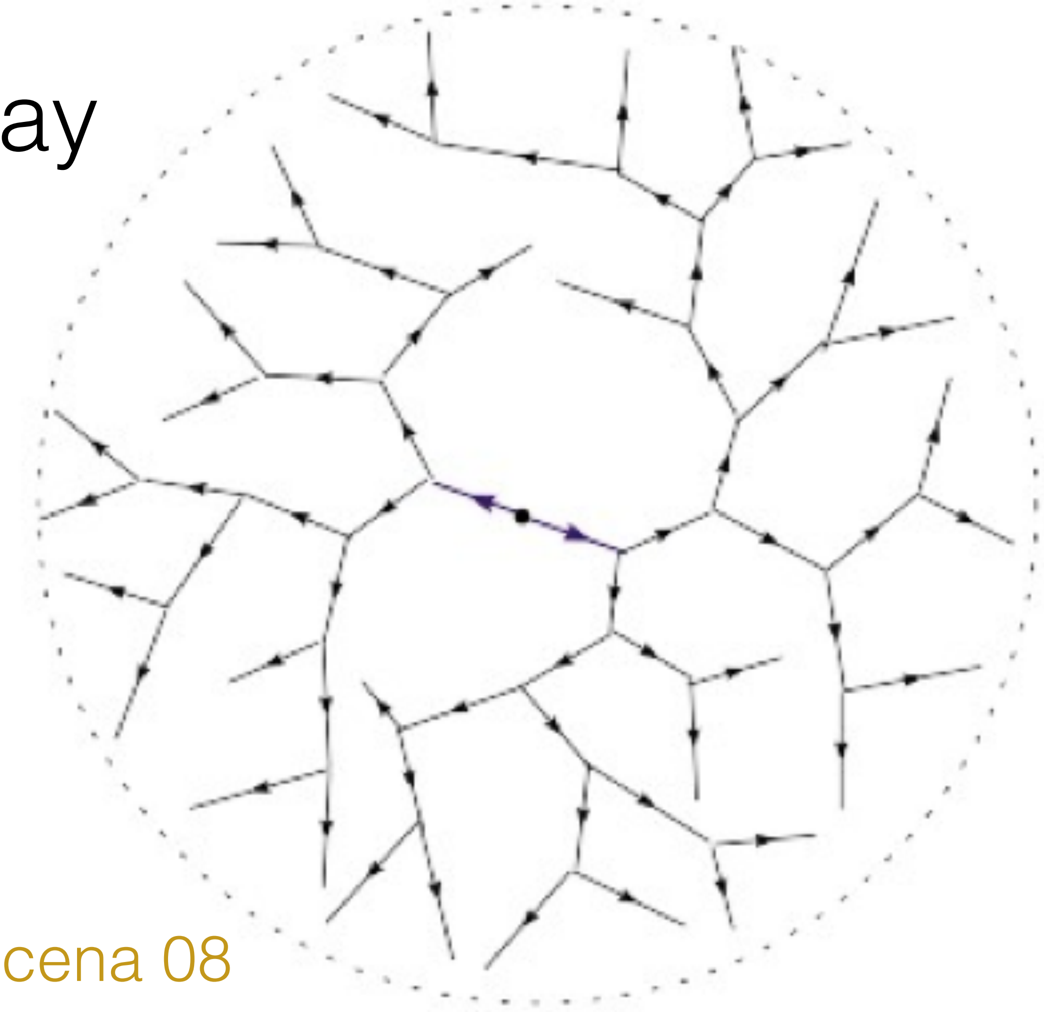
There are no jets in N=4 SYM at strong coupling

e+e- decay



Weak

Hofman and Maldacena 08
Hatta, Iancu, Mueller 08

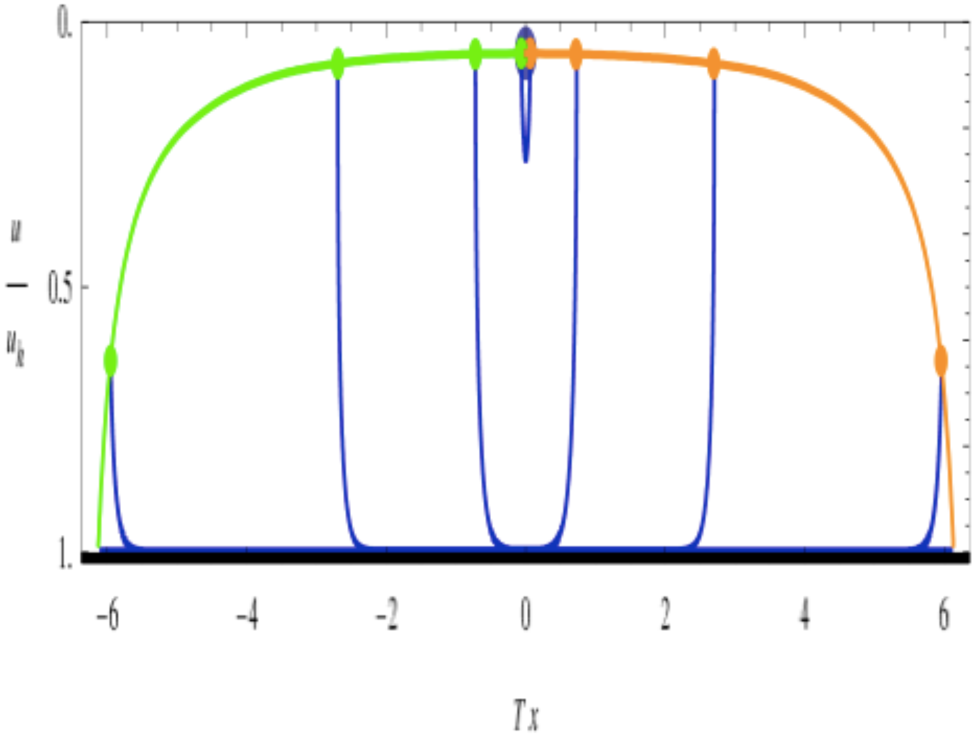


Strong

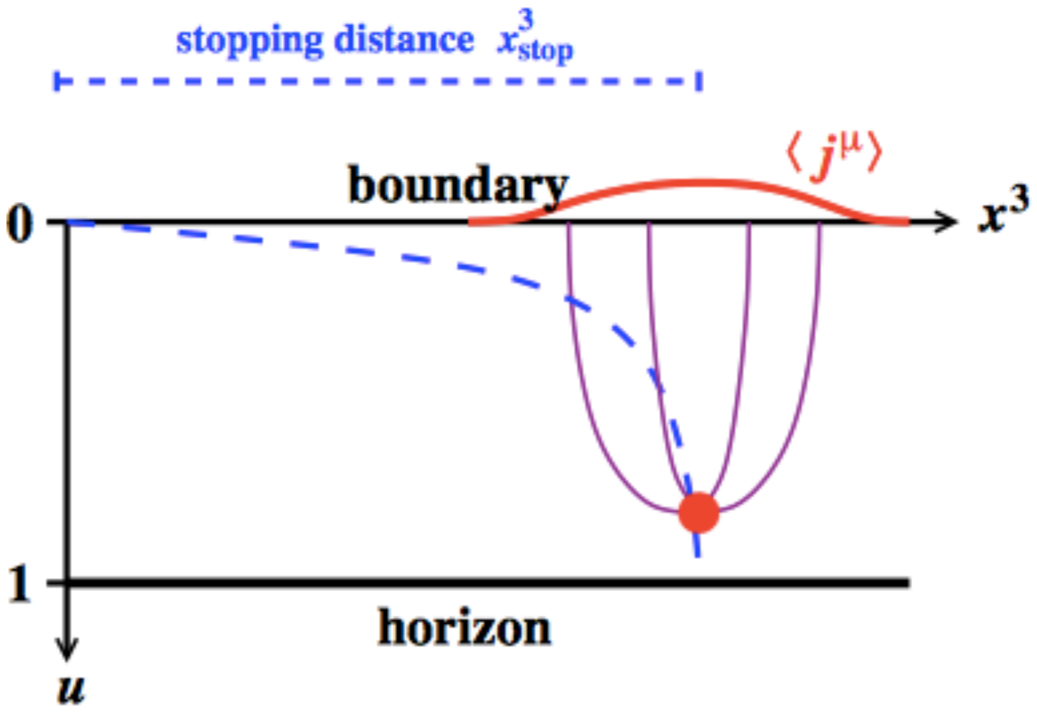
Problem for hard probes

Energetic Excitations

Classical string



Boosted virtual photon



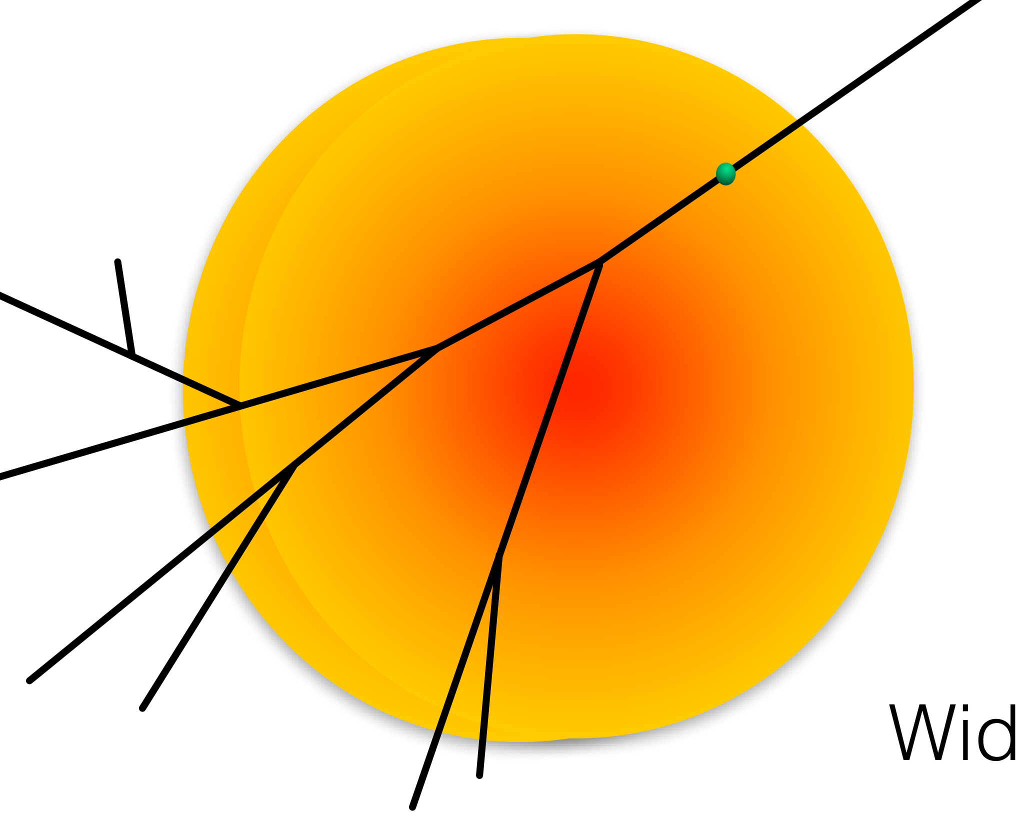
$$x_{\rm stop} = \frac{1}{2\,\kappa_{\rm sc}} \frac{E_{\rm in}^{1/3}}{T^{4/3}}$$

$$\kappa_{\rm sc} \;=\; 1.05\,\lambda^{1/6}$$

$$\kappa_{\rm SC} \propto \lambda^0$$

Chesler, Jensen, Karch, Yaffe 08

Arnold and Vaman 10



A Hybrid Model: Motivation

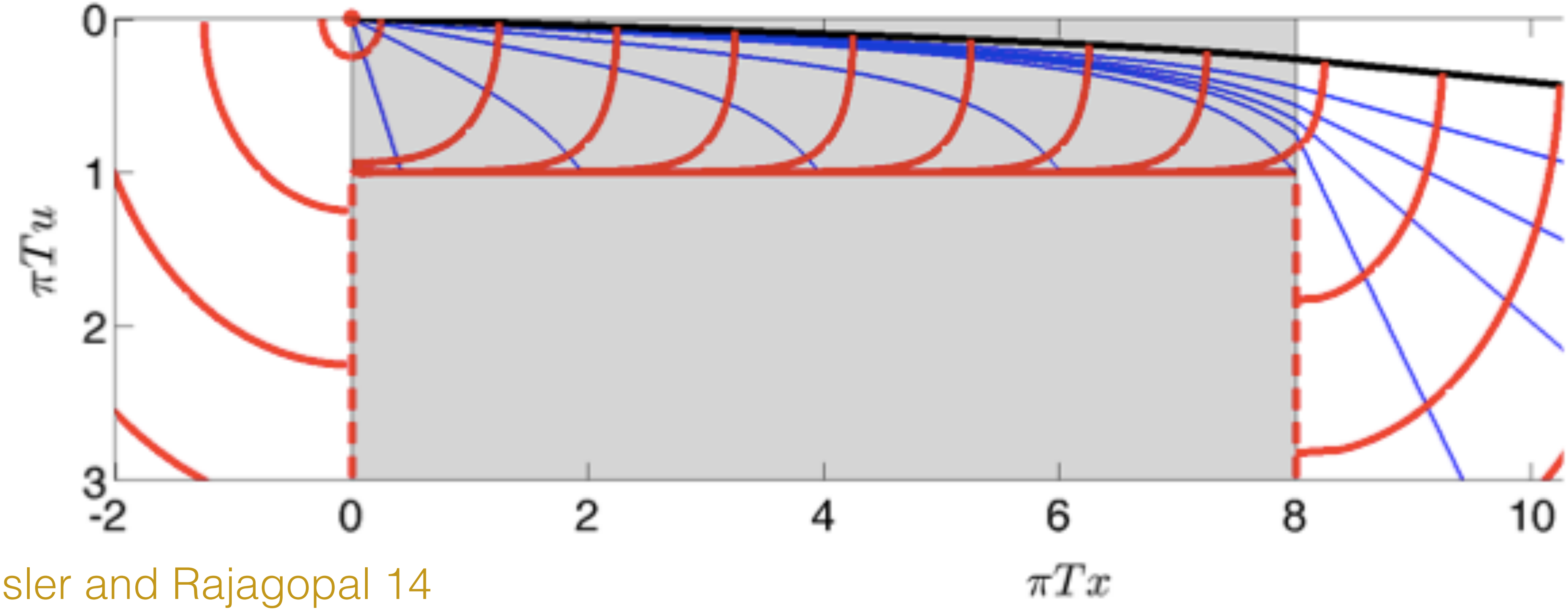
Wide hierarchy of scales in (HE) jet dynamics:

- Production and branching perturbative
- Interaction with QGP non-perturbative

Approached through simple and phenomenological model:

- Vacuum like production and showering
- Differential energy loss rate from holography
- Neglect medium induced modification of splittings (for now)

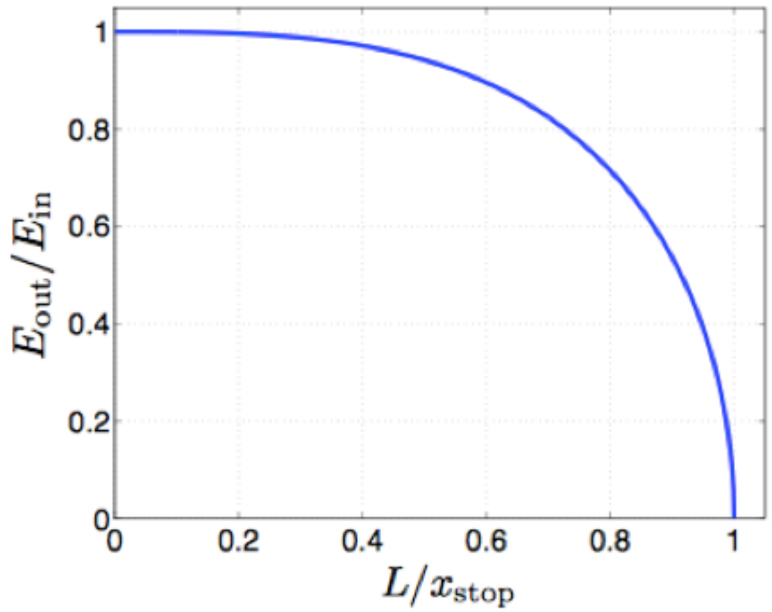
Strongly Coupled Energy Loss



Chesler and Rajagopal 14

$$\frac{1}{E_{\rm in}}\frac{dE}{dx} = -\frac{4}{\pi}\frac{x^2}{x_{\rm stop}^2}\frac{1}{\sqrt{x_{\rm stop}^2 - x^2}}$$

$$x_{\rm stop} = \frac{1}{2\,\kappa_{\rm sc}}\frac{E_{\rm in}^{1/3}}{T^{4/3}}$$



Value of $\boldsymbol{\kappa_{sc}}$ different in different theories

$\lambda \equiv g^2 N_c$

$$\kappa_{sc} \sim \lambda^{1/6}$$

String computations

Gubser et al 08, Chesler et al 08, Ficnar and Gubser 13, Chesler and Rajagopal 14

$$\kappa_{sc} \sim \lambda^{0}$$

U(1) field decays

Hatta, Iancu and Mueller 08, Arnold and Vaman 10

$$\lambda \sim 10 \rightarrow \kappa_{sc} \sim \mathcal{O}(1)$$

We'll use κ_{sc} as our fitting parameter

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$$\kappa_{sc} \sim \lambda^{0}$$

U(1) field decays

Hatta, Iancu and Mueller 08, Arnold and Vaman 10

$$\lambda \sim 10 \to \kappa_{sc} \sim \mathcal{O}(1)$$

expect it to be smaller
in QCD than in N=4 SYM

We'll use κ_{sc} as our fitting parameter

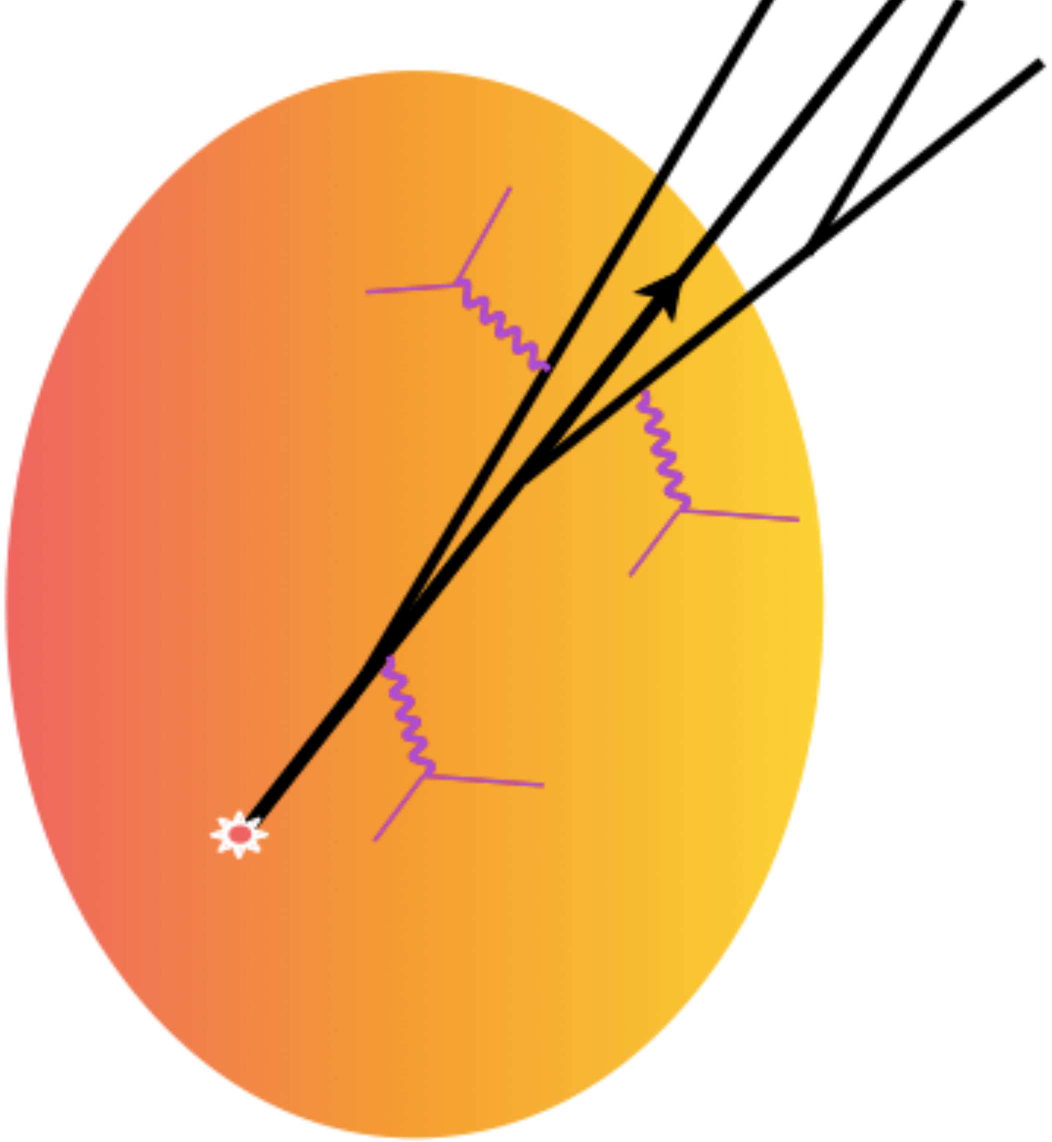
What about gluons?

$$x^{G}_{stop}(E) = x^{Q}_{stop}(E/2)$$

$$\kappa^{G}_{sc} = \kappa^{Q}_{sc}\left(\frac{C_A}{C_F}\right)^{1/3}$$

Chesler et al 08

Monte Carlo Implementation



Jet production and evolution in PYTHIA

Assign spacetime description to parton shower (formation time argument) $\tau_f = \dfrac{2E}{Q^2}$

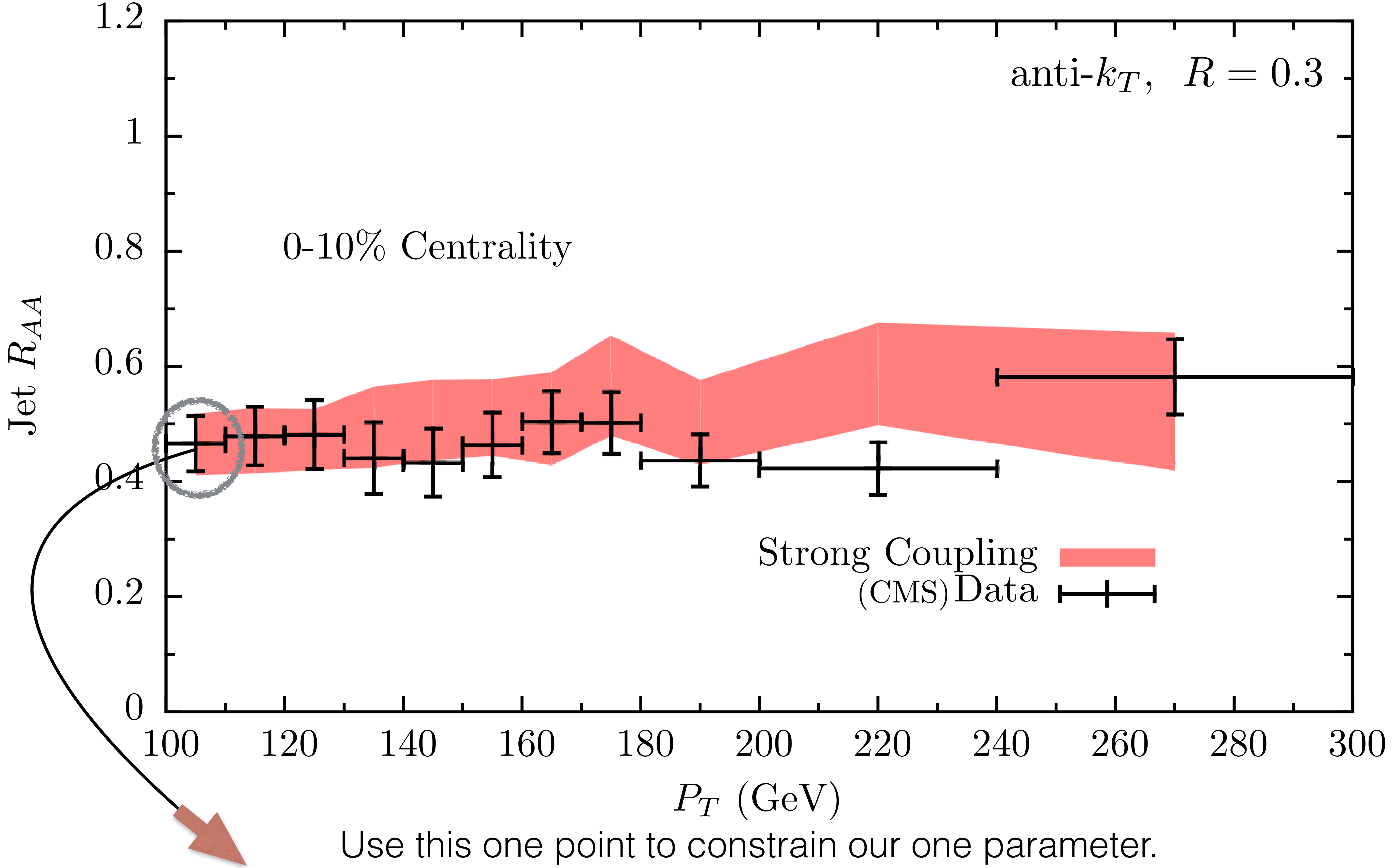
Embed the system into a hydrodynamic background (2+1 hydro code from Heinz and Shen)

Between splittings, partons in the shower interact with QGP, lose energy

Turn off energy loss below a T_c that we vary over $145 < T_c < 170\,\mathrm{MeV}$

Extract jet observables from parton shower

R_{AA}

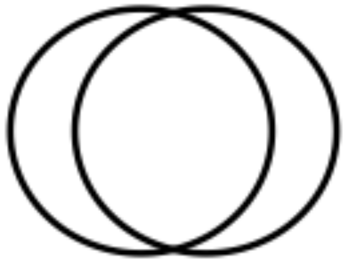


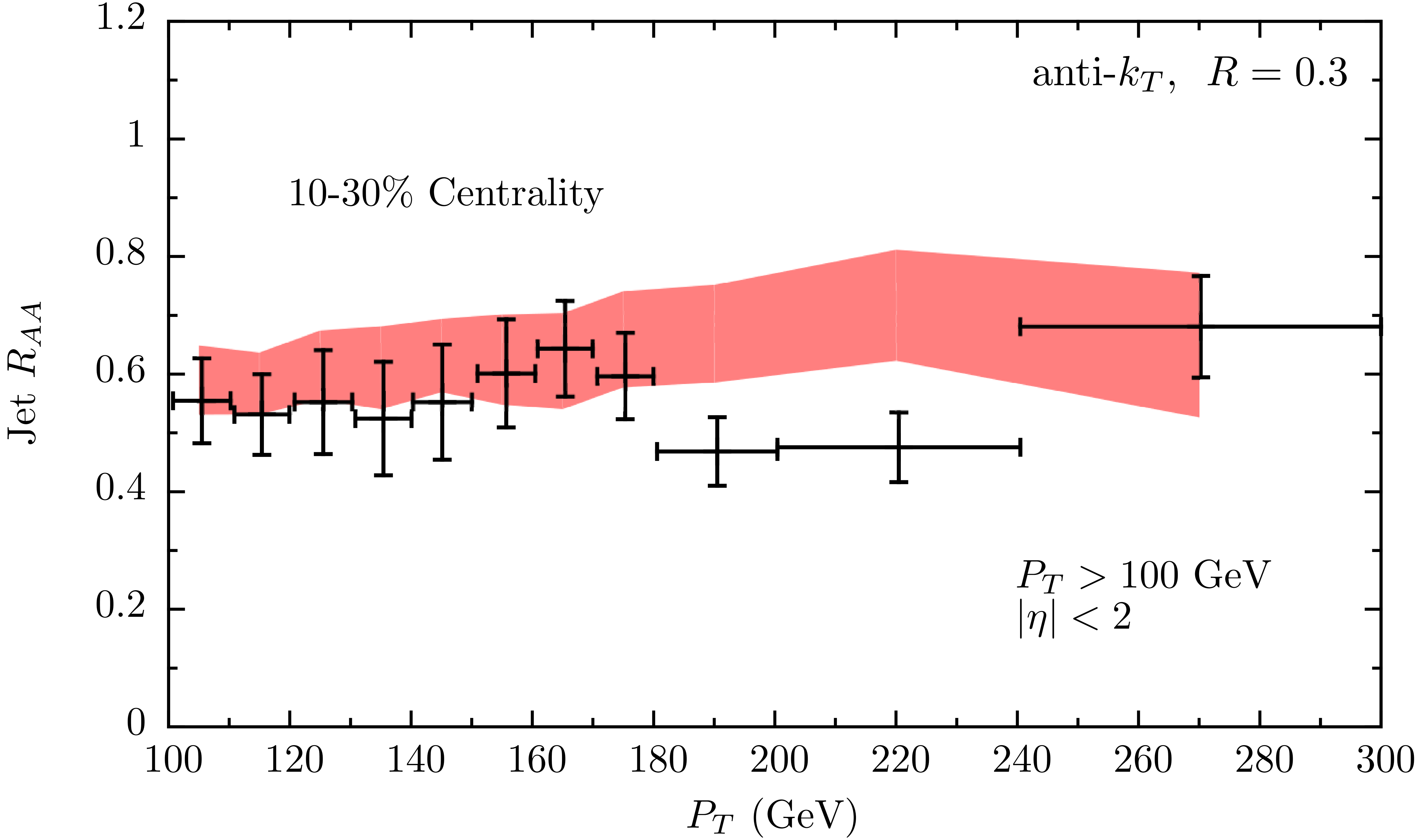
Use this one point to constrain our one parameter.

Bands come from experimental uncertainty on this point

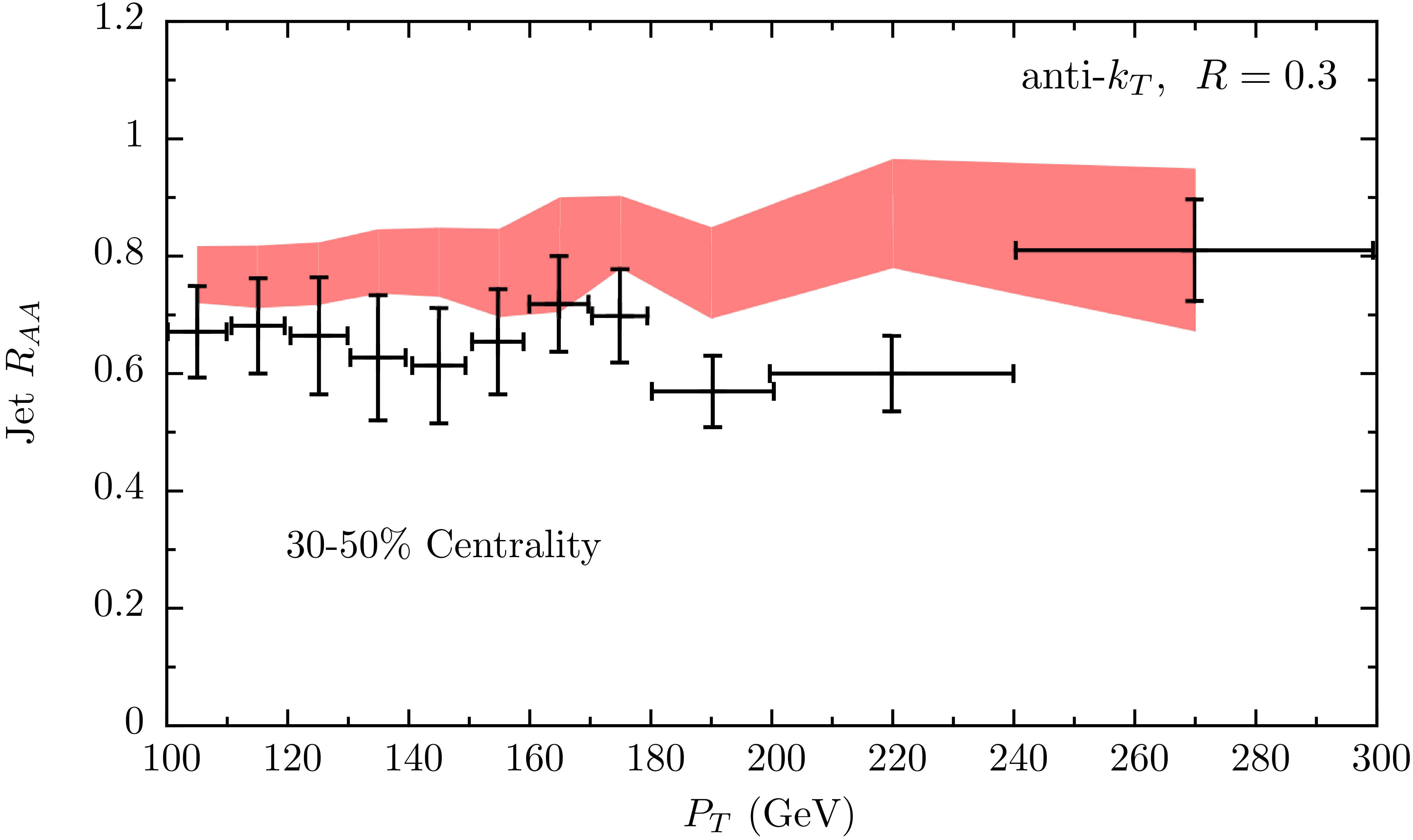
plus varying T_c over $145 < T_c < 170\,\mathrm{MeV}$

R_{AA}

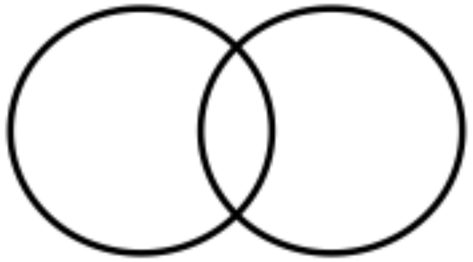


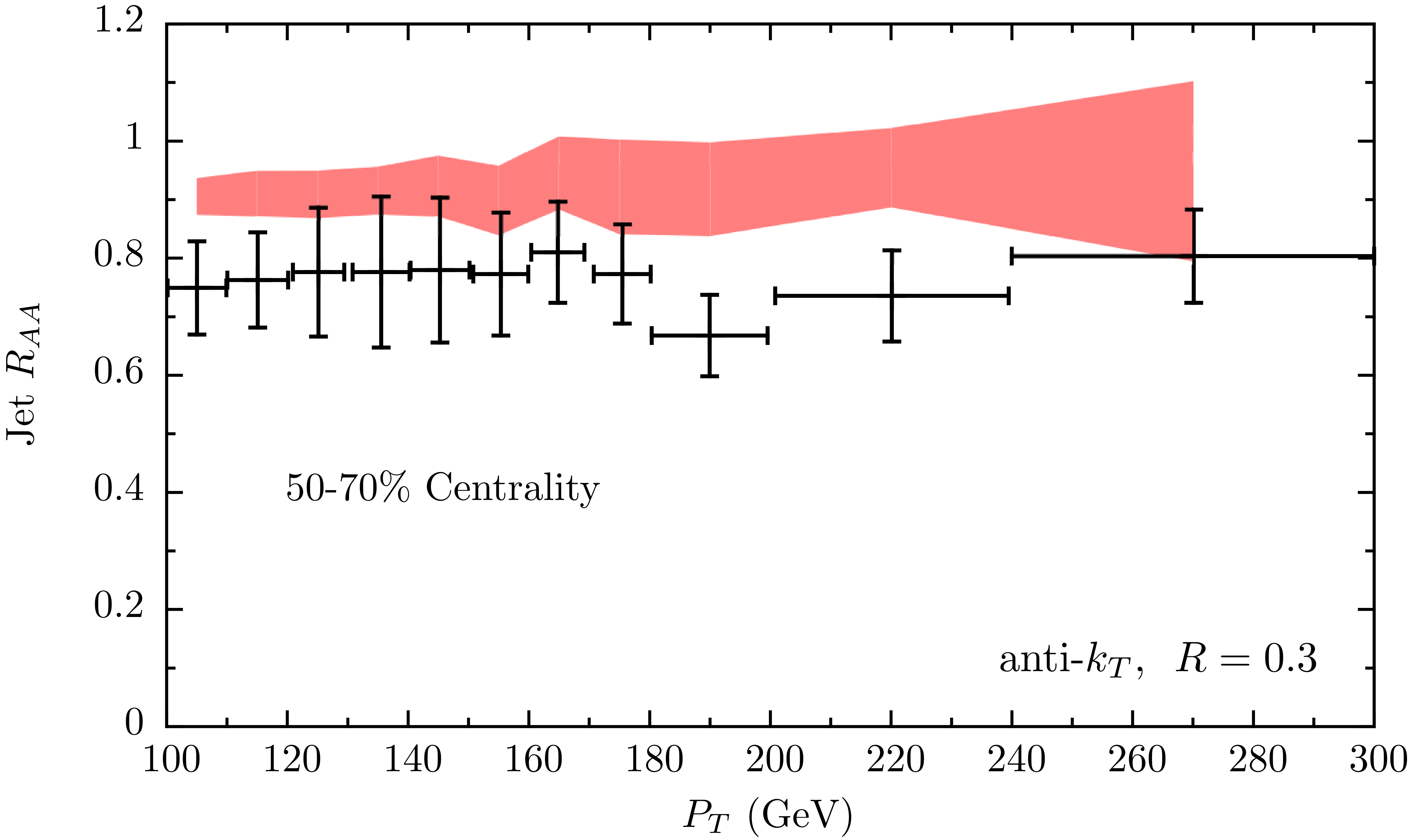

anti-k_T, $R = 0.3$
10-30% Centrality
$P_T > 100$ GeV
$|\eta| < 2$
Jet R_{AA}
P_T (GeV)
0
0.2
0.4
0.6
0.8
1
1.2
100
120
140
160
180
200
220
240
260
280
300

R_{AA}


anti-k_T, $R = 0.3$
30-50% Centrality
Jet R_{AA}
P_T (GeV)
0
0.2
0.4
0.6
0.8
1
1.2
100
120
140
160
180
200
220
240
260
280
300

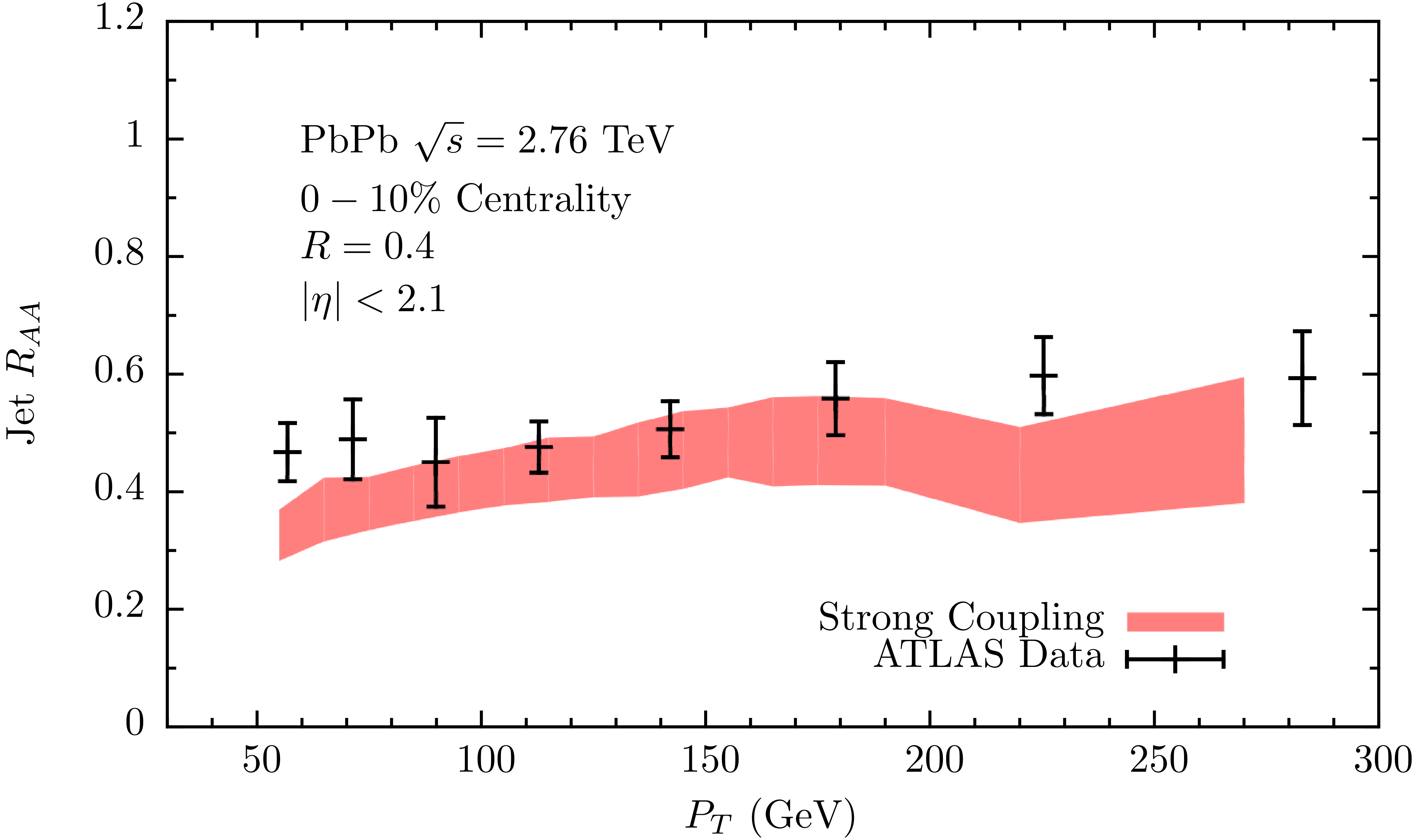
R_{AA}





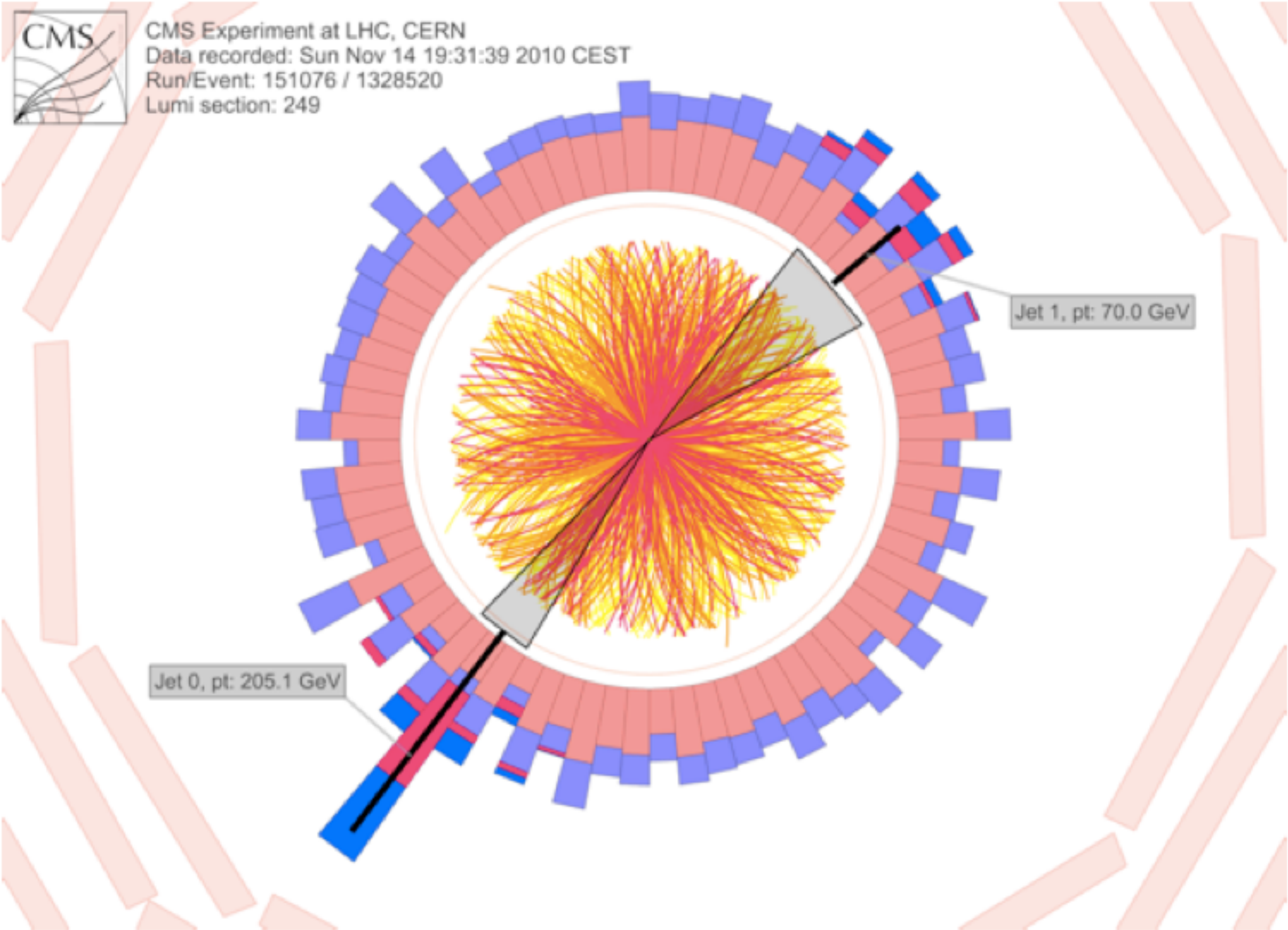
We have only simulated the QGP phase

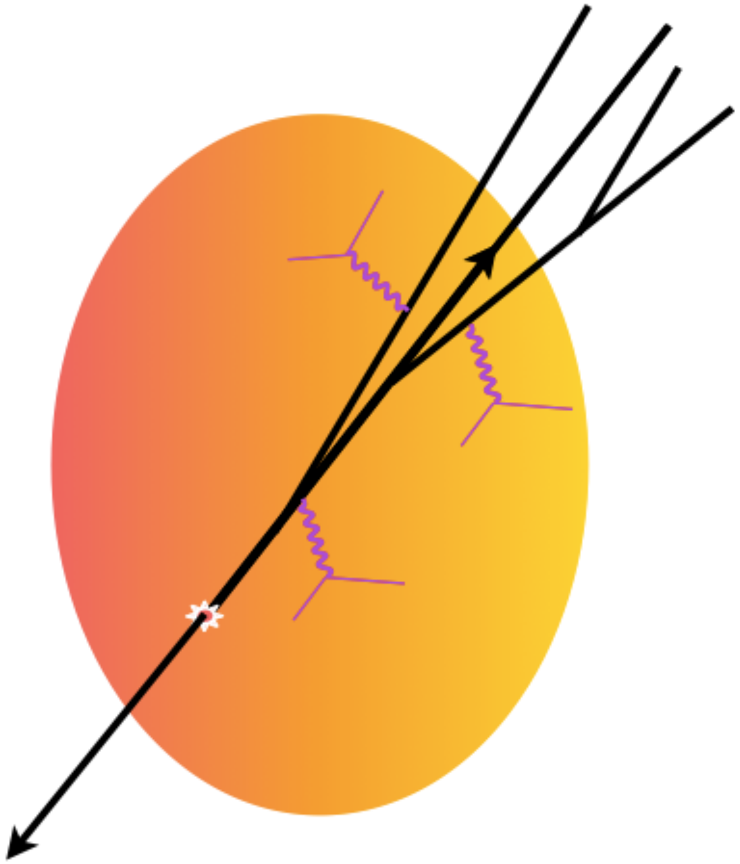
R_{AA}



With current implementation, slightly more quenching for bigger jet radius

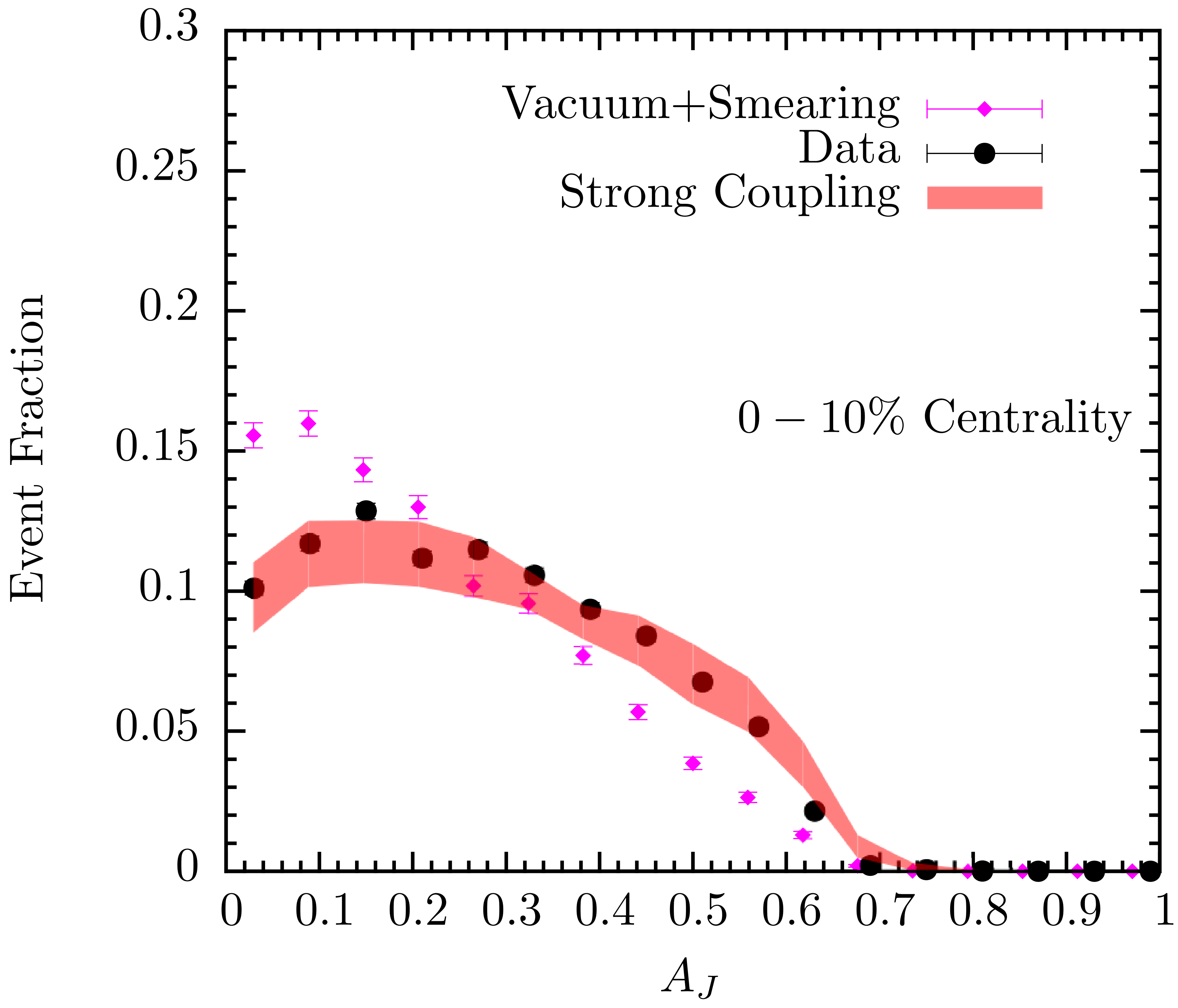
Dijets



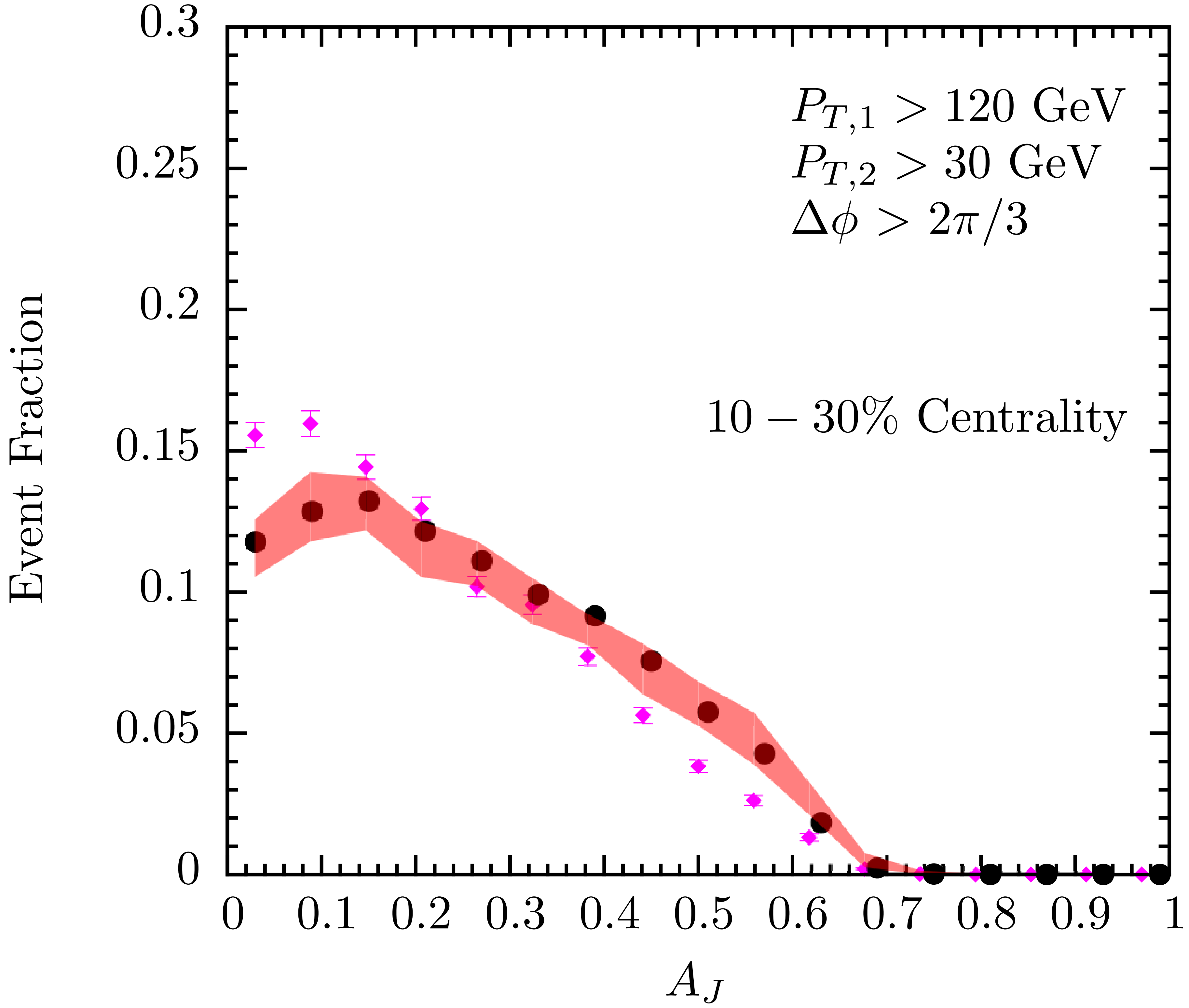


$$A_J = \frac{p_{T_1} - p_{T_2}}{p_{T_1} + p_{T_2}}$$

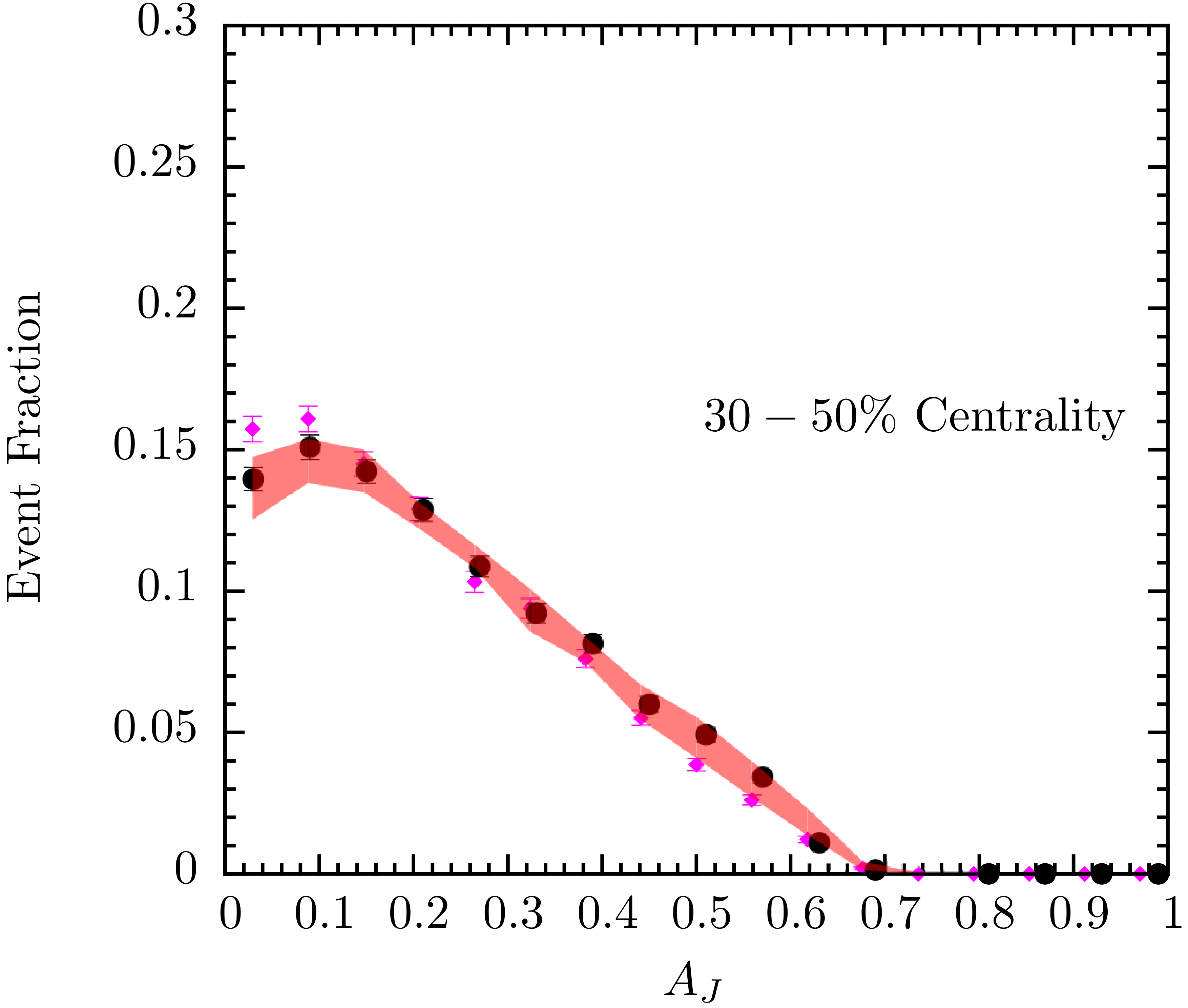
Imbalance


Vacuum+Smearing
Data
Strong Coupling
0 – 10% Centrality
Event Fraction
0
0.05
0.1
0.15
0.2
0.25
0.3
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
A_J

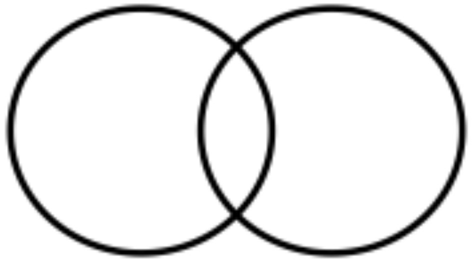
Imbalance

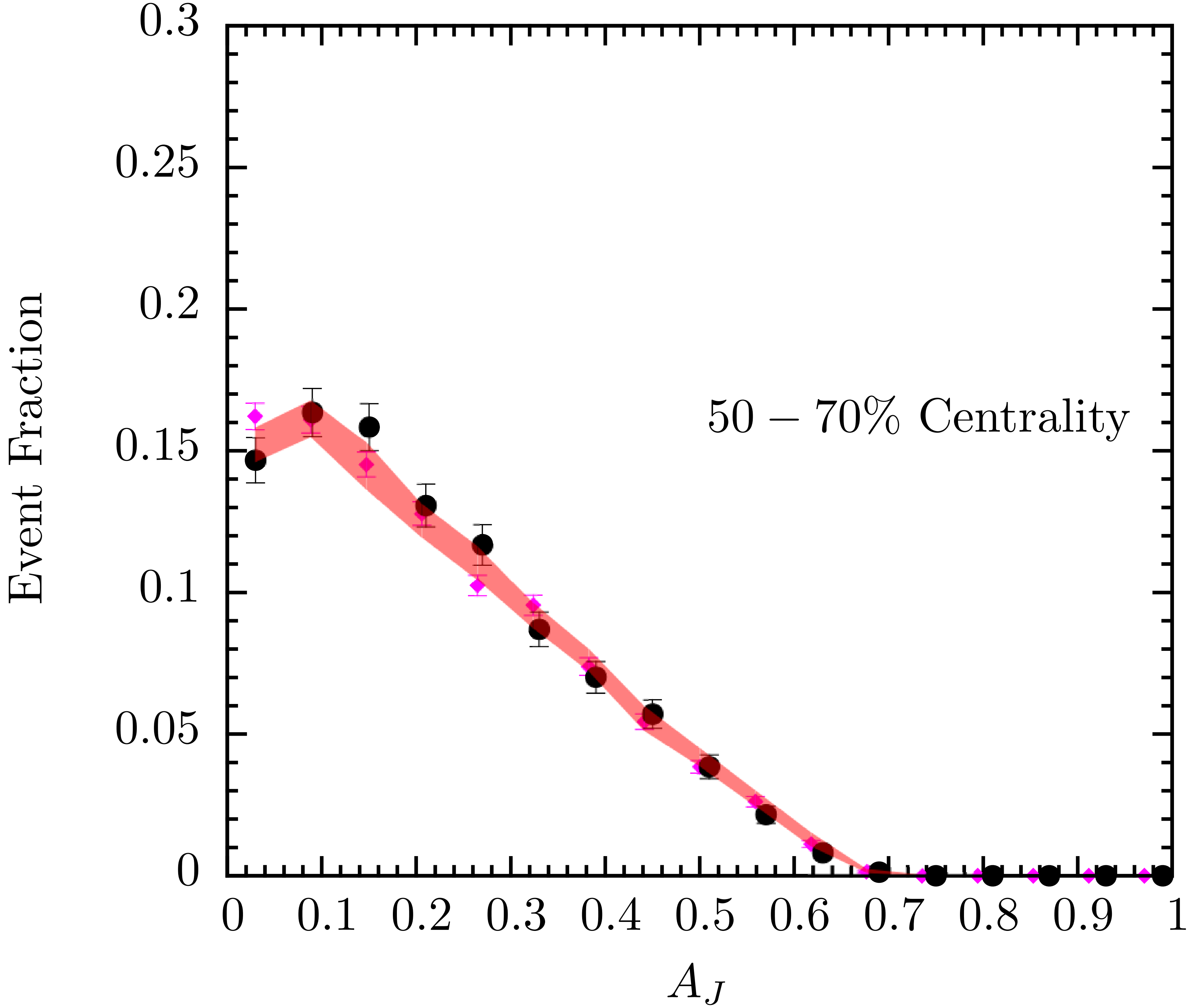

$P_{T,1} > 120$ GeV
$P_{T,2} > 30$ GeV
$\Delta\phi > 2\pi/3$
$10 - 30\%$ Centrality
Event Fraction
0
0.05
0.1
0.15
0.2
0.25
0.3
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
A_J

Imbalance

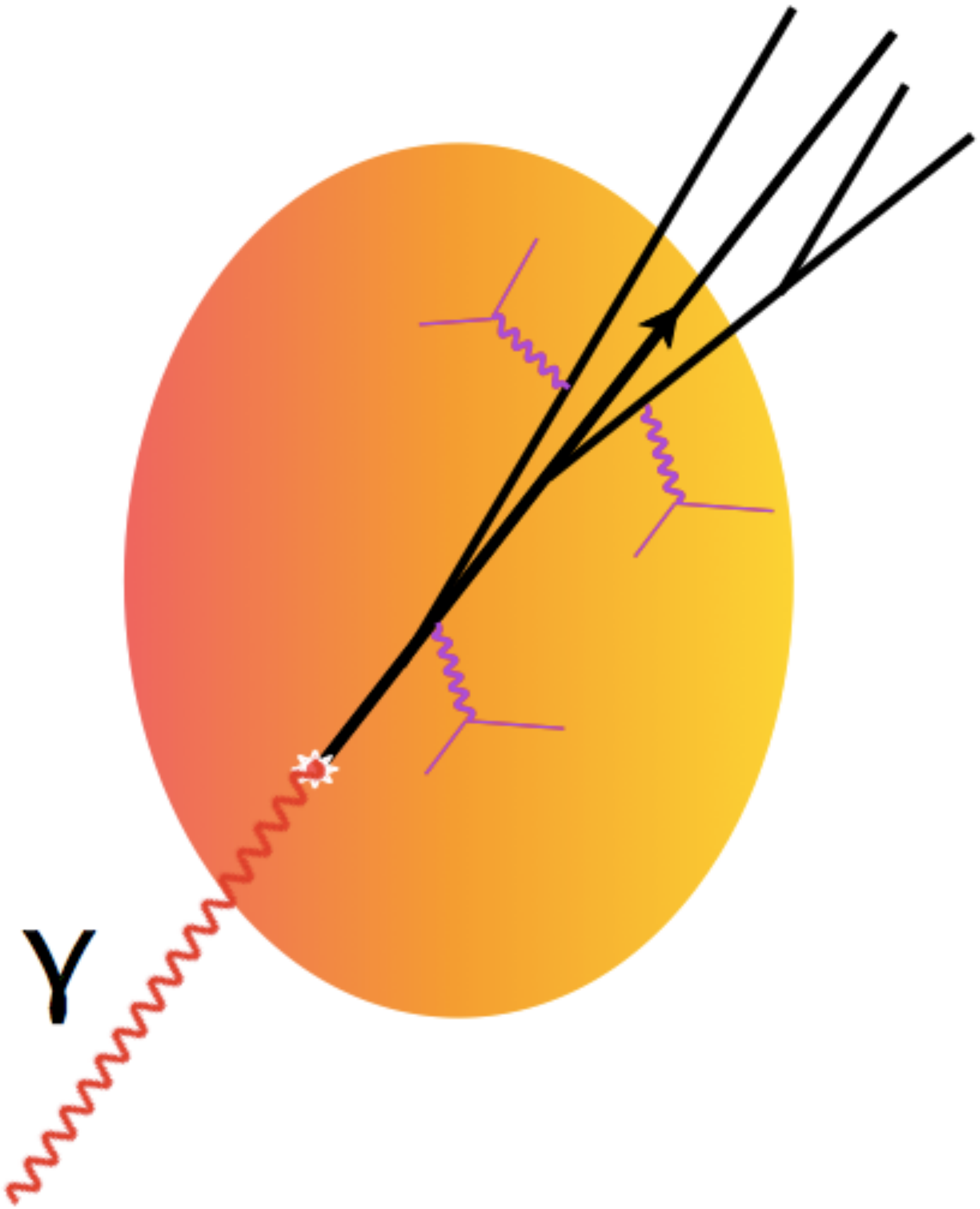

30 – 50% Centrality
Event Fraction
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
A_J

Imbalance




0.3
0.25
0.2
0.15
0.1
0.05
0
Event Fraction
50 − 70% Centrality
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
A_J

Photon Jet



- Photons do not interact with plasma

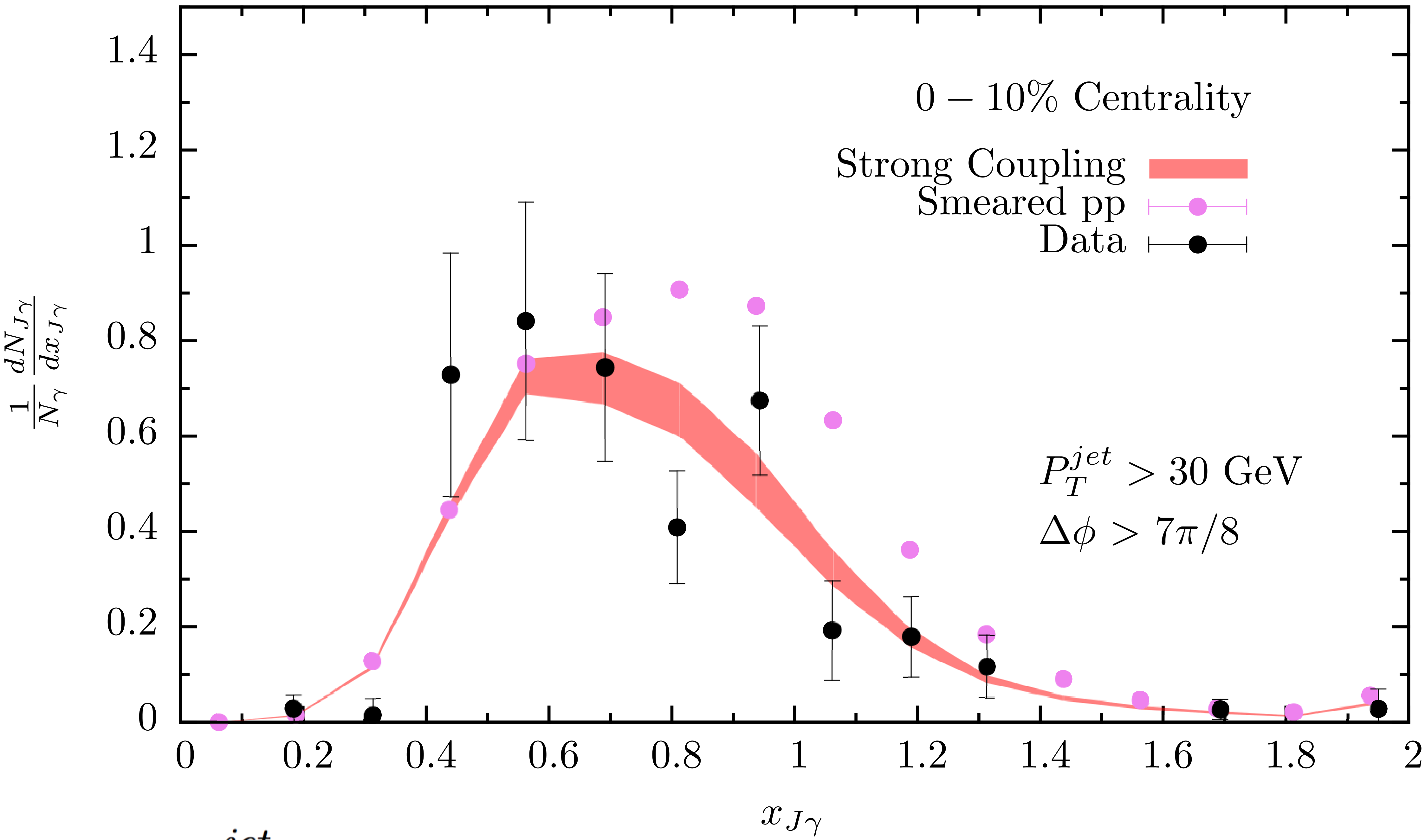
- Look for associated jet

  -Different geometric sampling

  -Different species composition

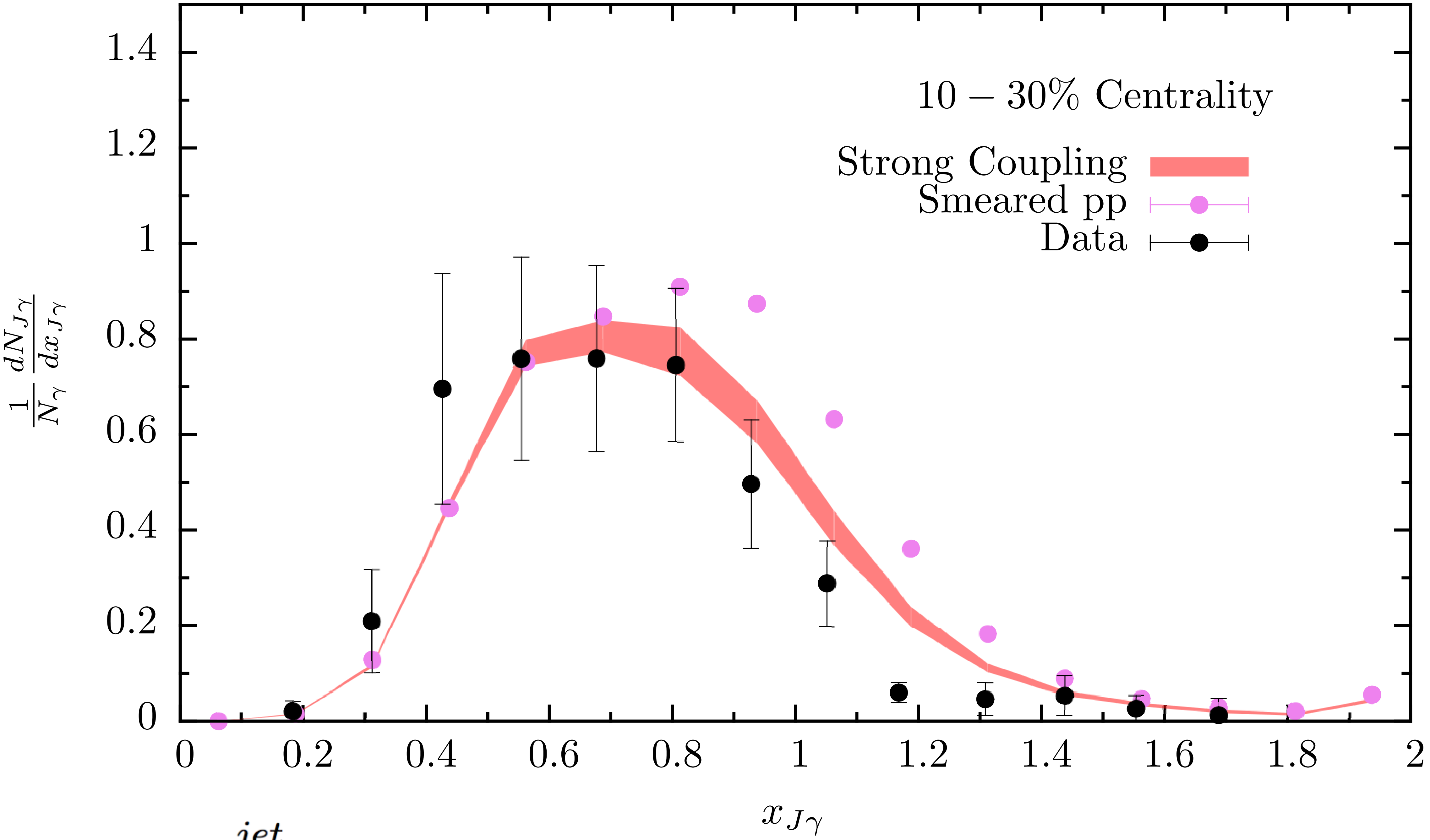
  -E_γ proxy for E_{jet}

Imbalance



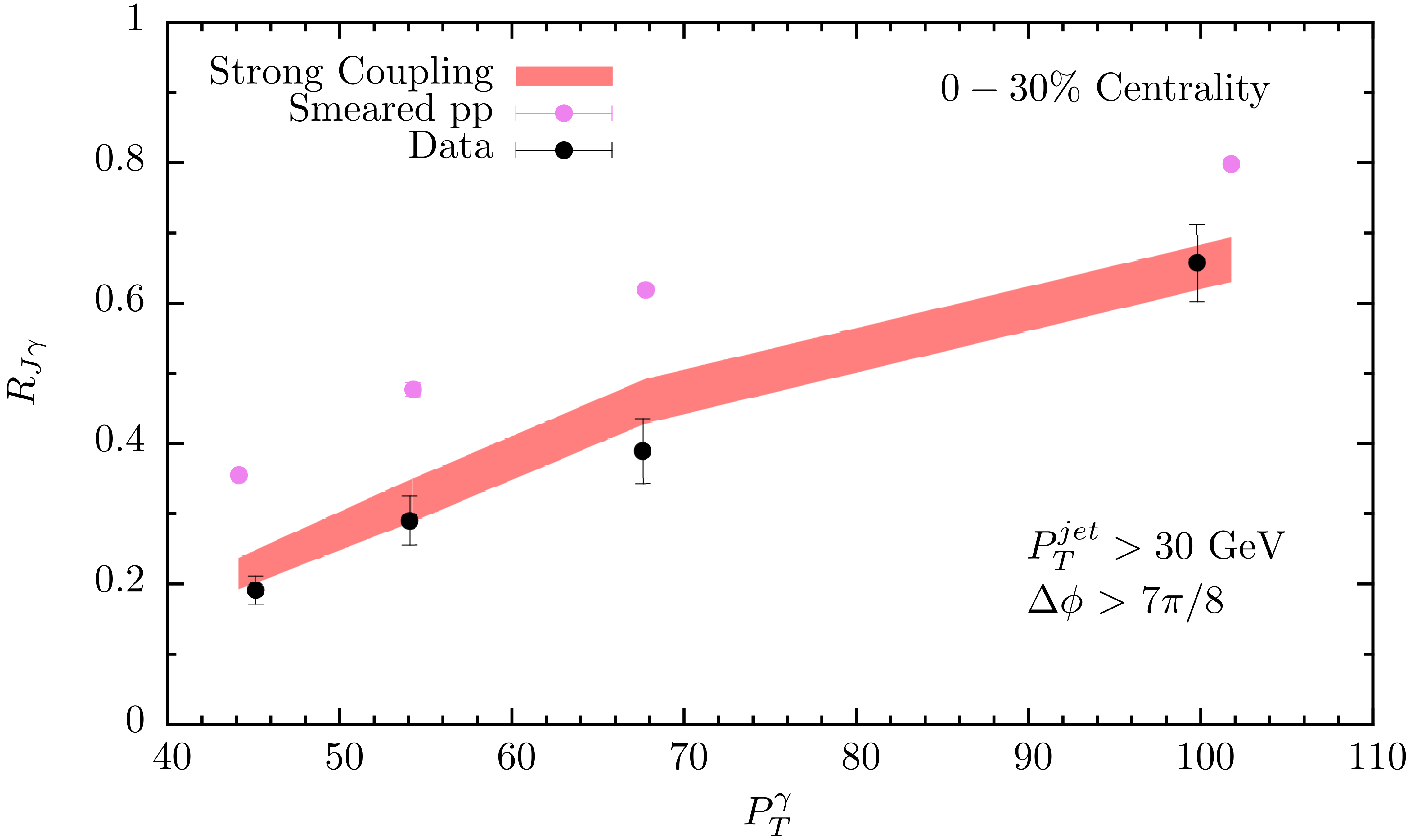
$$x_{J\gamma} = \frac{p_T^{jet}}{p_T^{\gamma}}$$

Imbalance



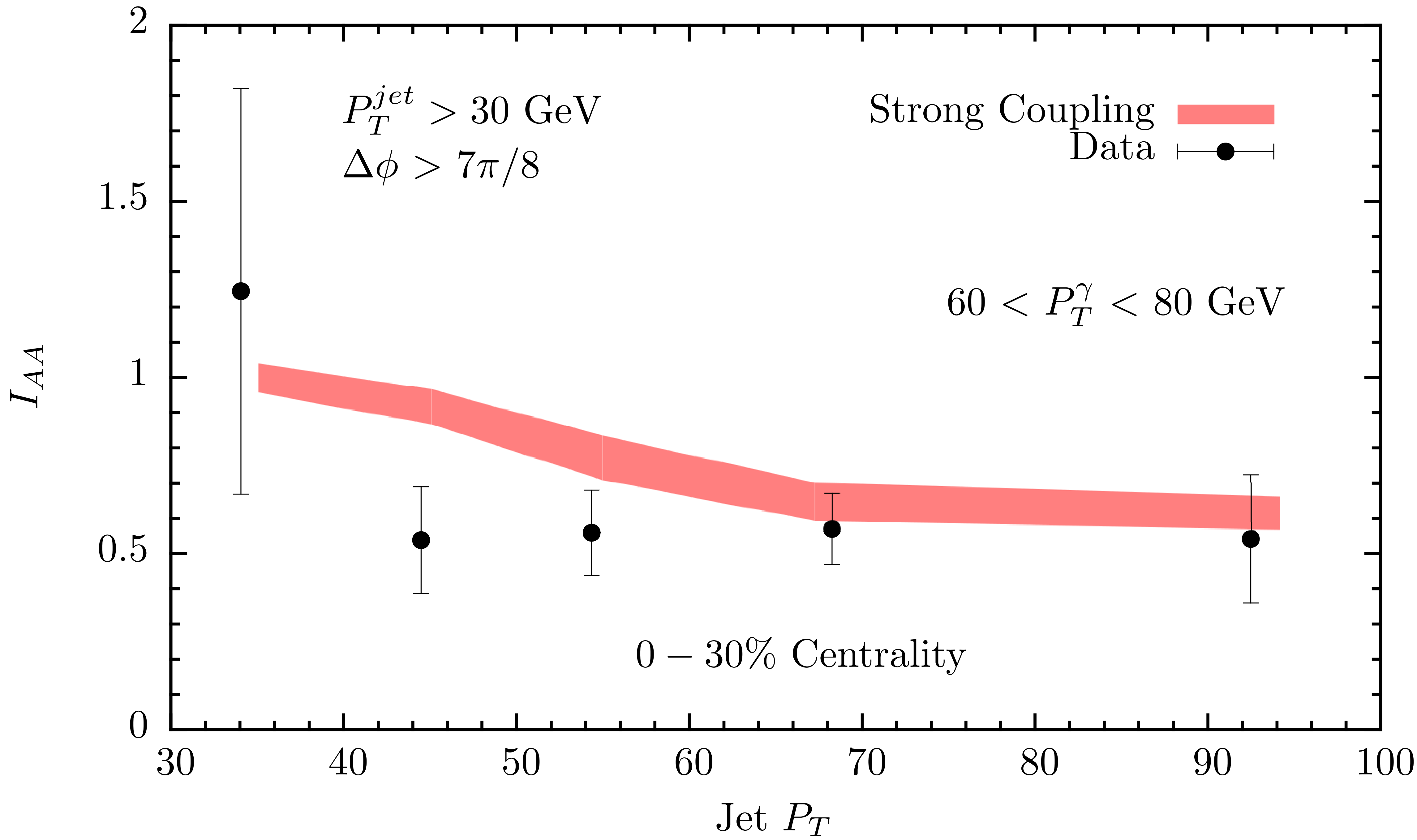
$$x_{J\gamma} = \frac{p_T^{jet}}{p_T^{\gamma}}$$

Jet Suppression



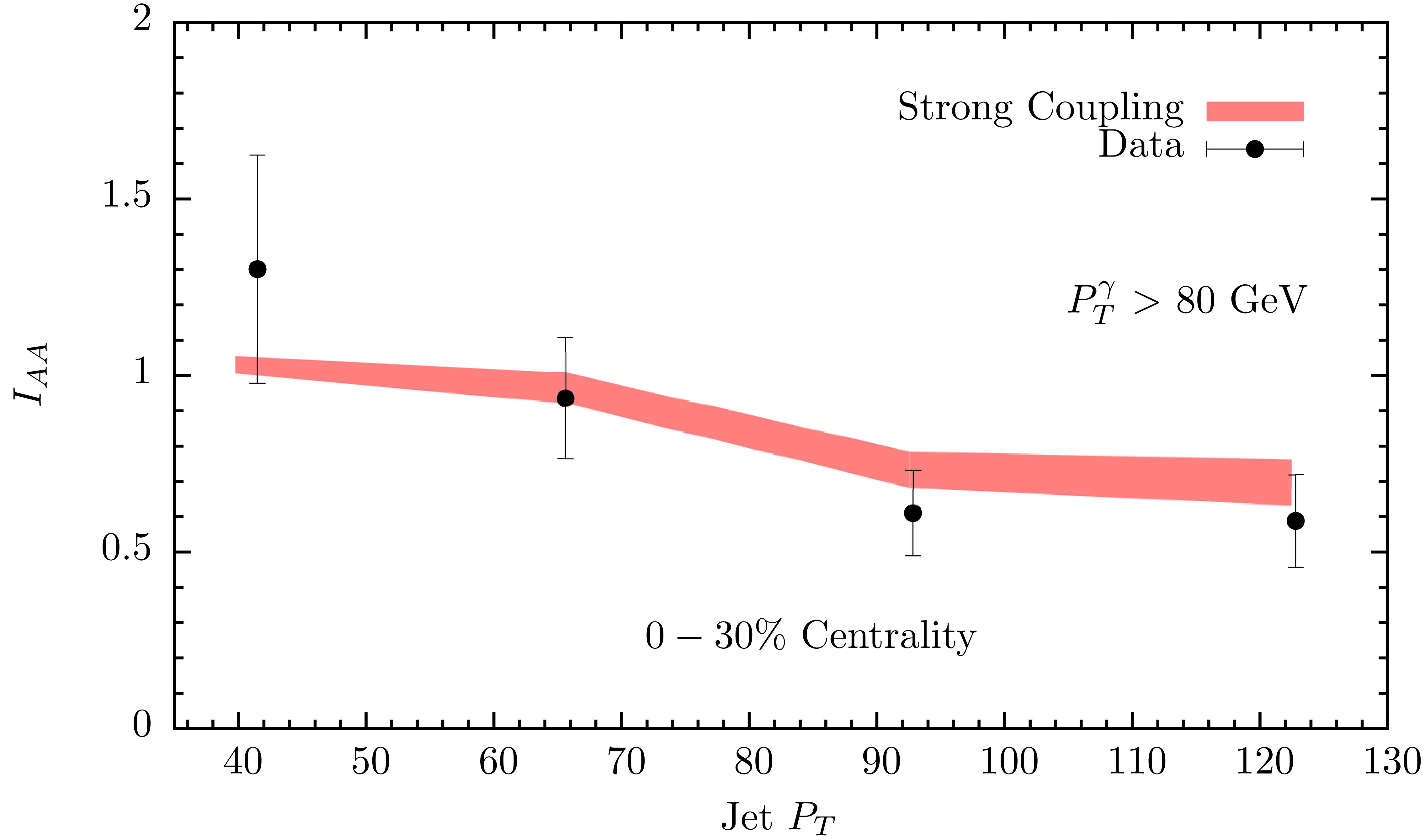
$$R_{J\gamma} = \frac{\text{Number of jets}}{\text{Number of photons}}$$

Spectrum

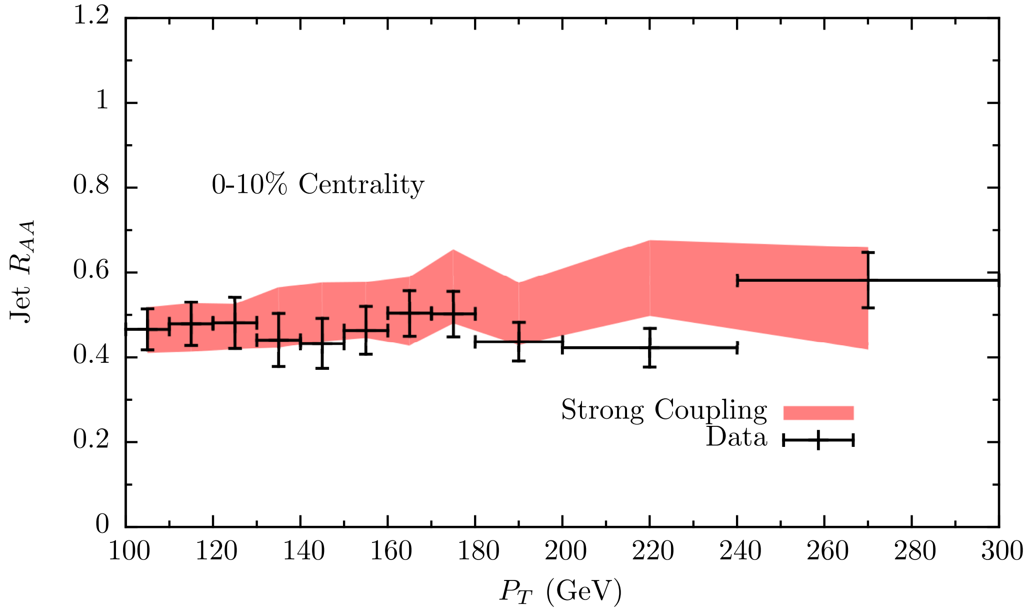


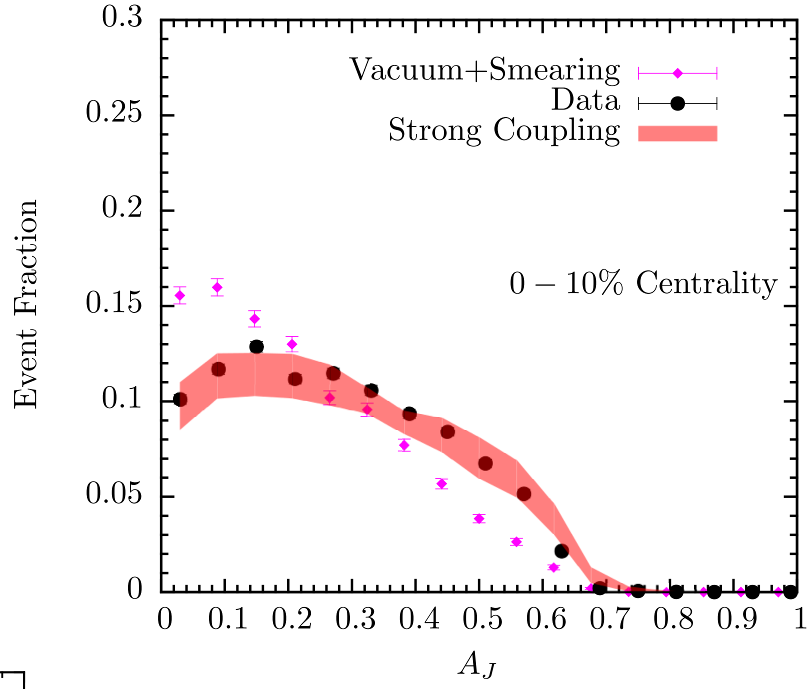
$$I_{AA} = \frac{\text{Number of associated jets in PbPb}}{\text{Number of associated jets in pp}}$$

Spectrum

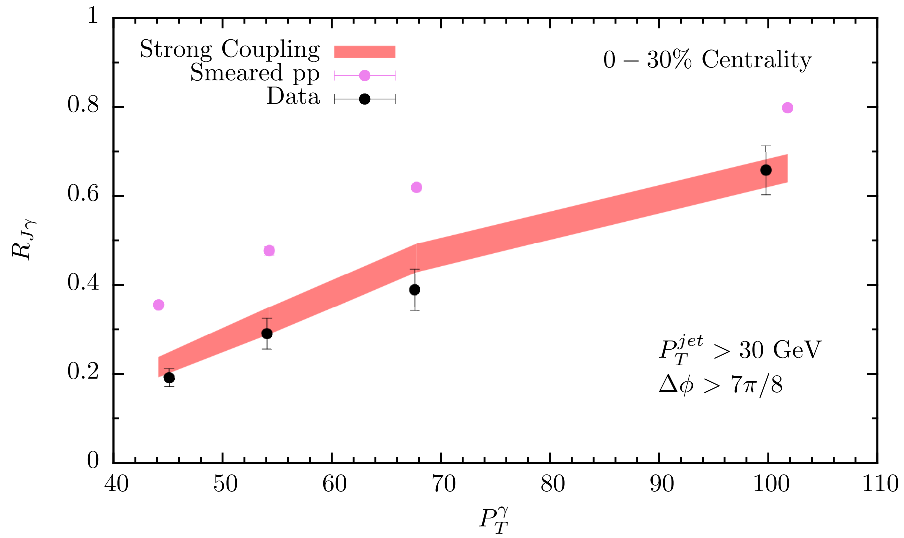


$$I_{AA} = \frac{\text{Number of associated jets in PbPb}}{\text{Number of associated jets in pp}}$$





5 observables
and centrality dependence
all described with
single parameter

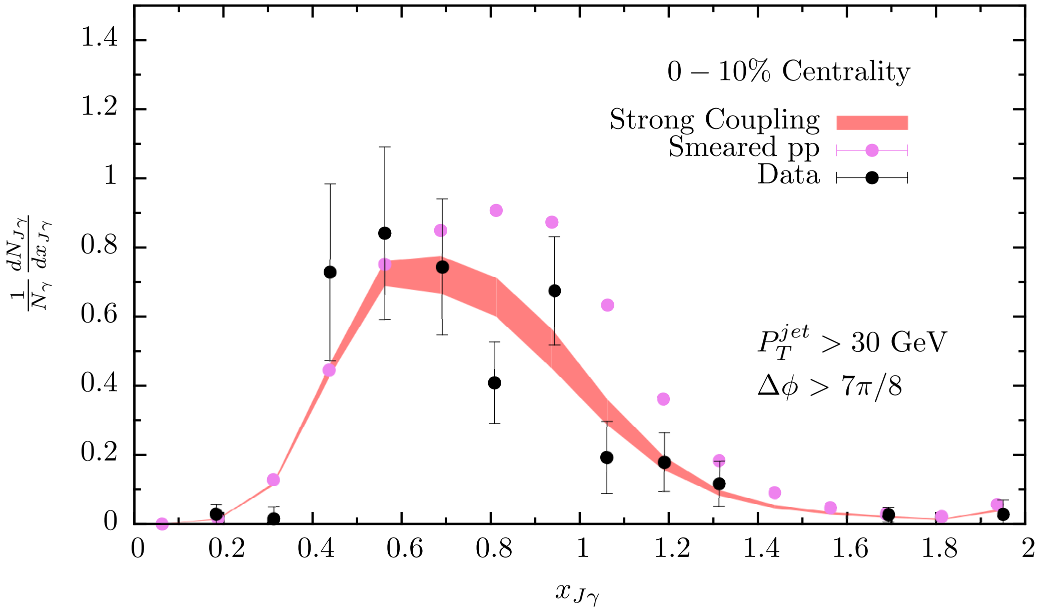


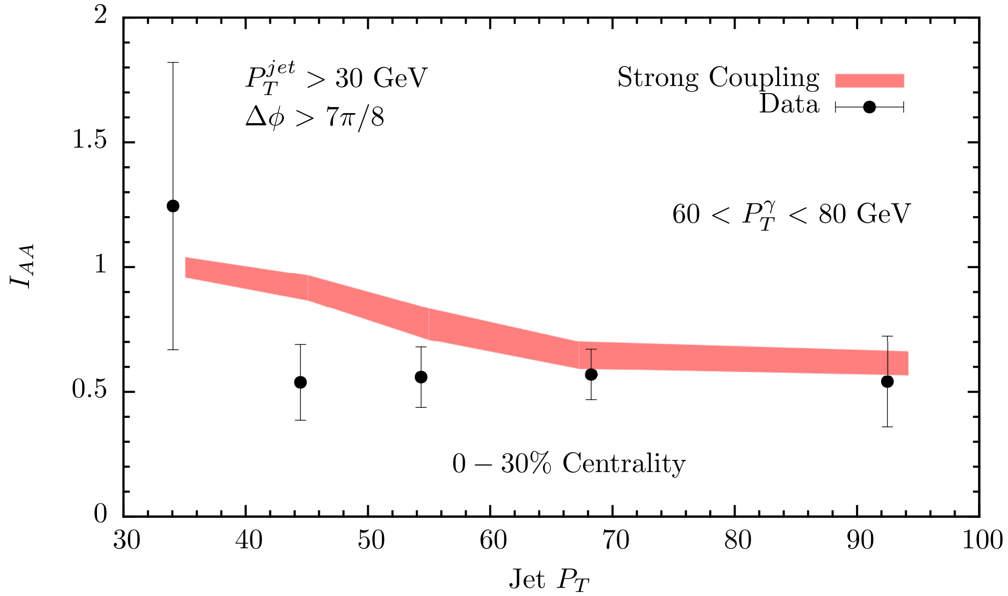
Bands in all plots correspond to

$0.32 < \kappa_{sc} < 0.41$

$\mathcal{O}(1)$ as expected.

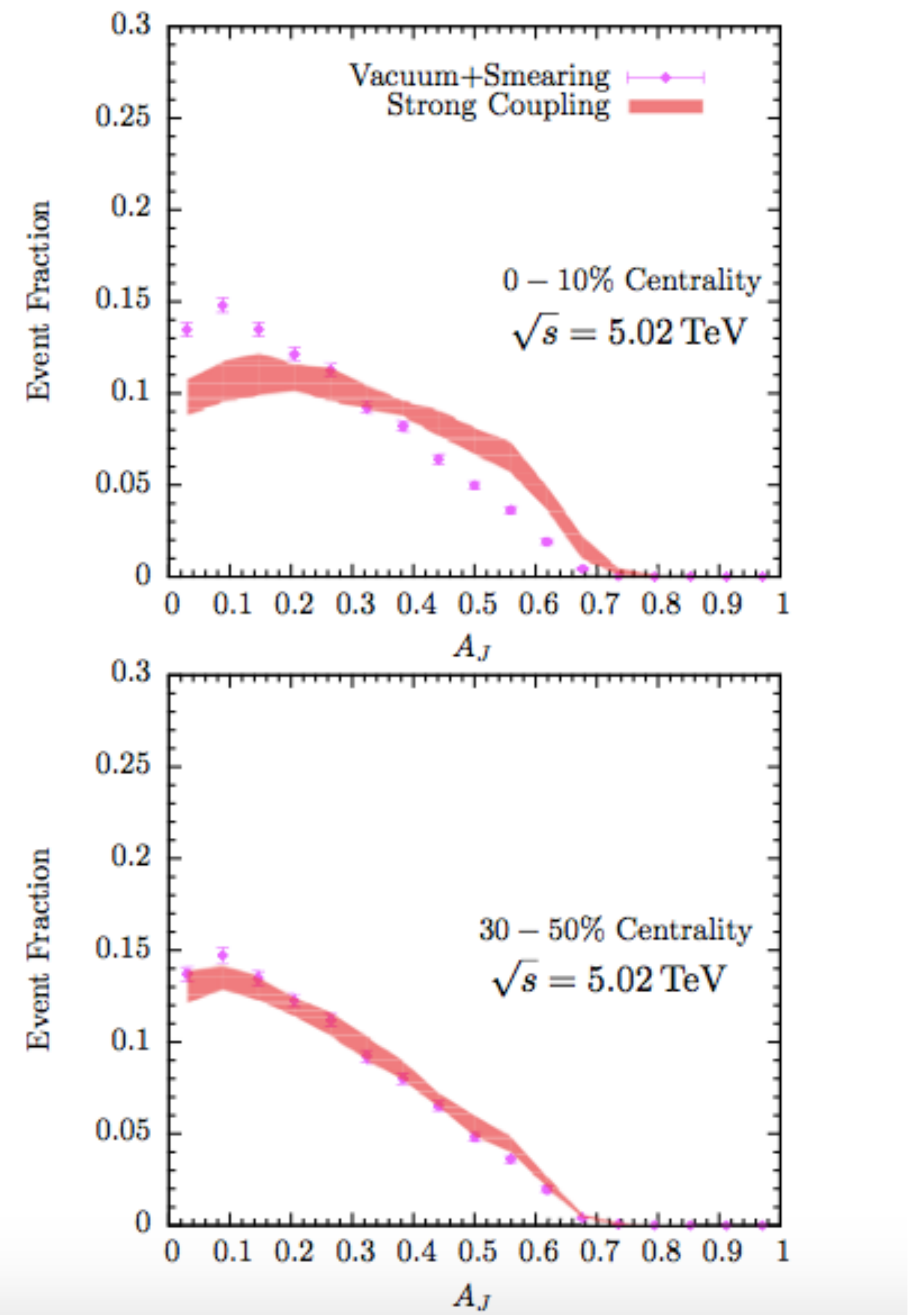
$$x_{stop}^{QCD} \sim (3-4)\, x_{stop}^{\mathcal{N}=4}$$

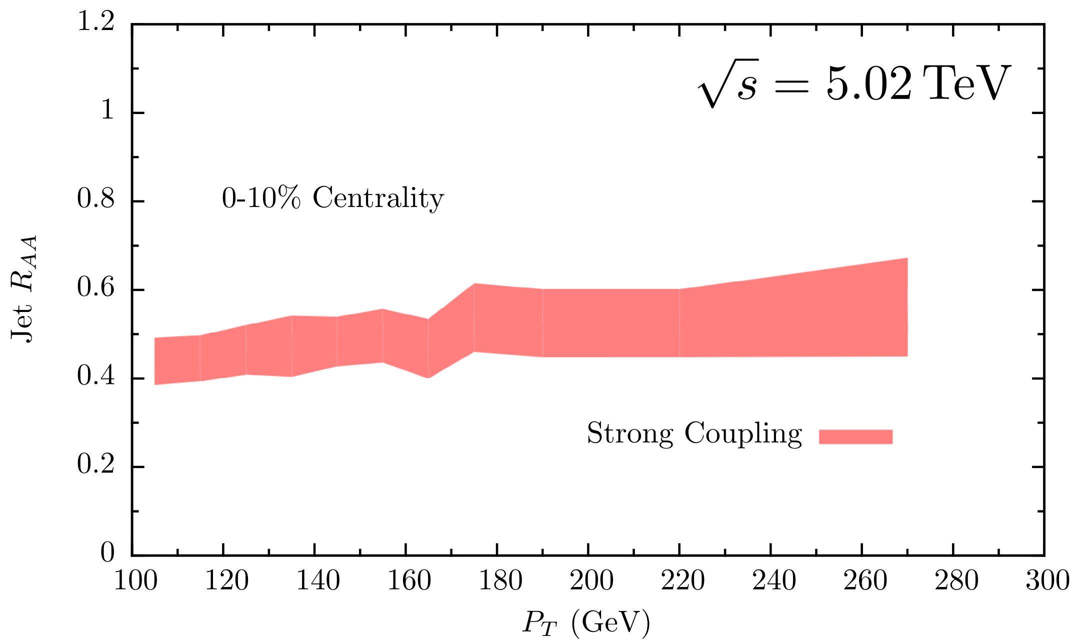


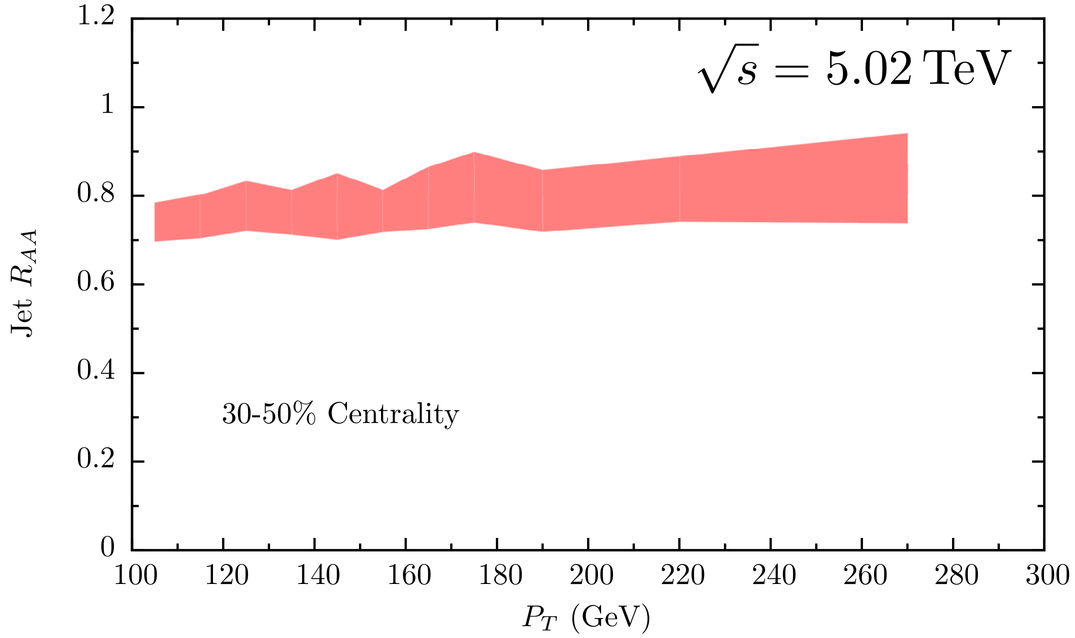


Predictions

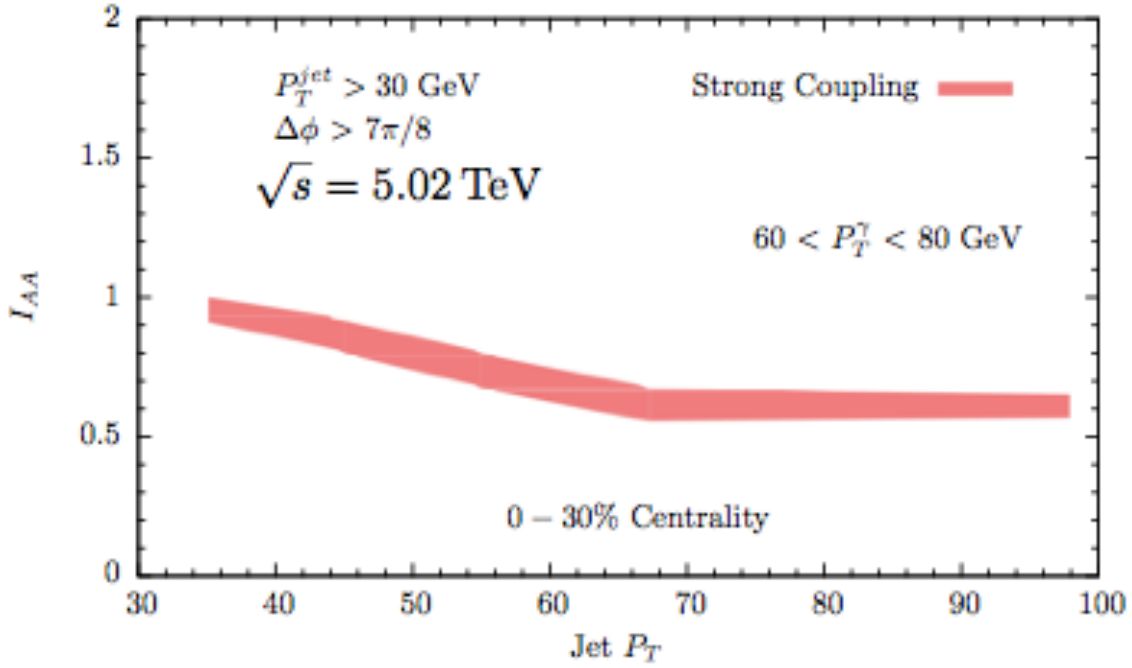
Dijet

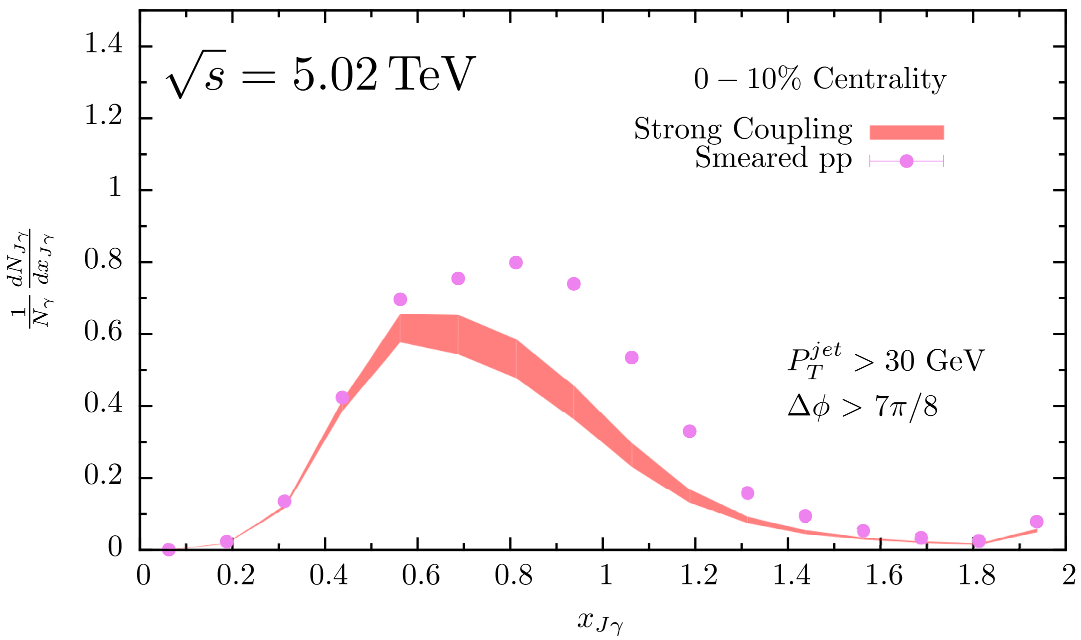

Vacuum+Smearing
Strong Coupling
0 – 10% Centrality
$\sqrt{s} = 5.02$ TeV
Event Fraction
A_J
0.3
0.25
0.2
0.15
0.1
0.05
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
30 – 50% Centrality
$\sqrt{s} = 5.02$ TeV
Event Fraction
A_J

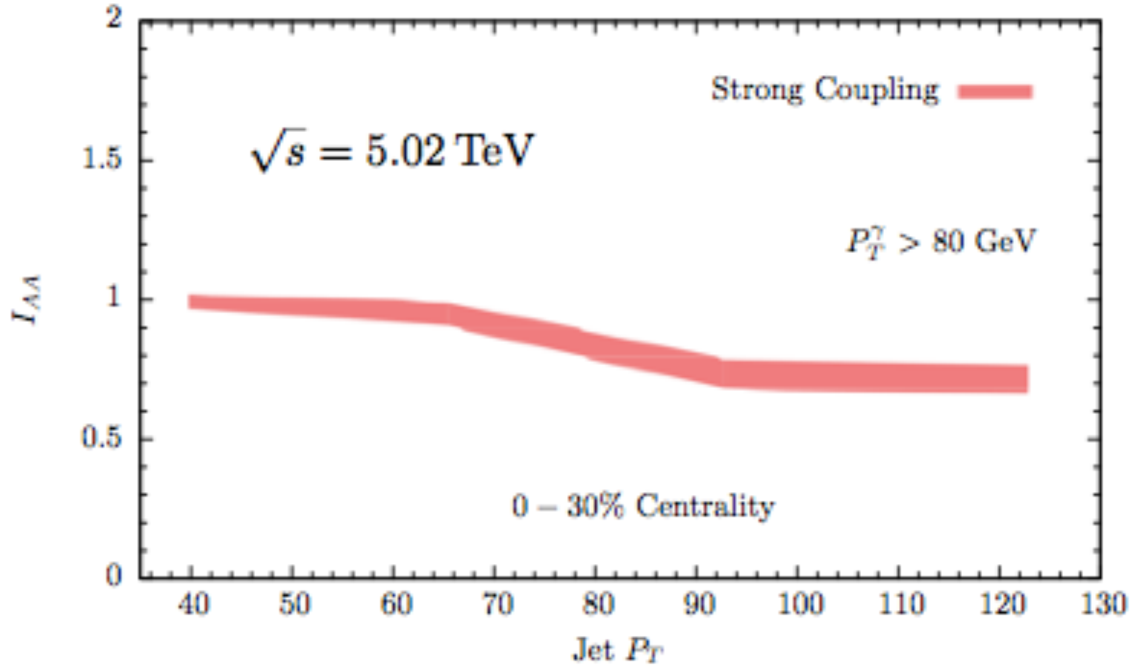

$\sqrt{s} = 5.02$ TeV
0-10% Centrality
Strong Coupling
Jet R_{AA}
P_T (GeV)
1.2
1
0.8
0.6
0.4
0.2
0
100 120 140 160 180 200 220 240 260 280 300

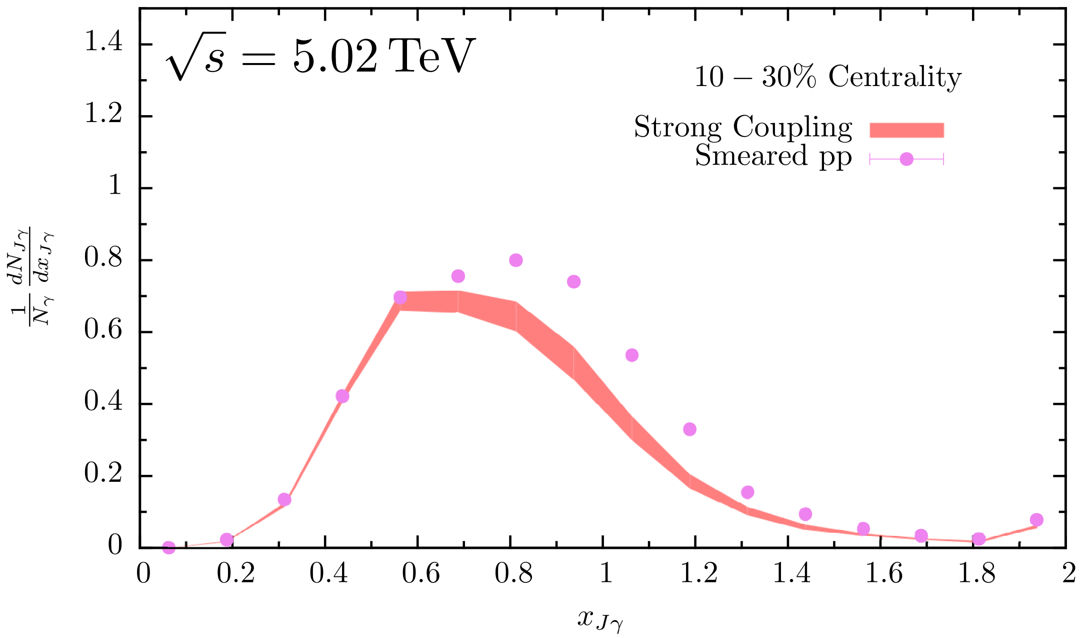

$\sqrt{s} = 5.02$ TeV
30-50% Centrality
Jet R_{AA}
P_T (GeV)
1.2
1
0.8
0.6
0.4
0.2
0
100 120 140 160 180 200 220 240 260 280 300

Photon-Jet

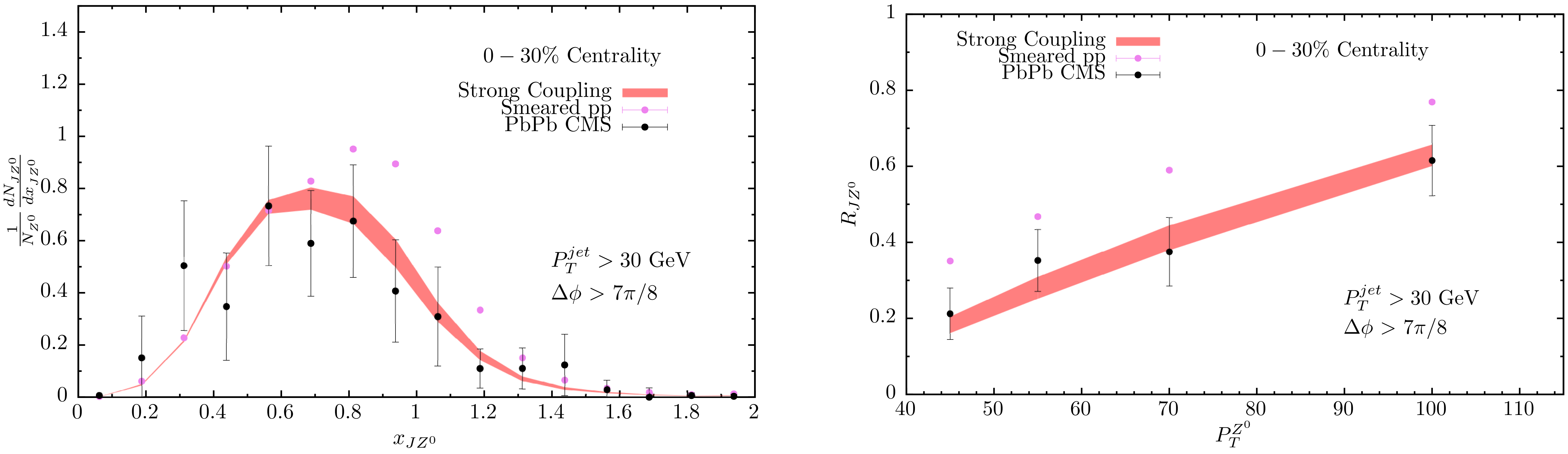

$P_T^{jet} > 30$ GeV
$\Delta\phi > 7\pi/8$
$\sqrt{s} = 5.02$ TeV
Strong Coupling
$60 < P_T^{\gamma} < 80$ GeV
$0 - 30\%$ Centrality
I_{AA}
0
0.5
1
1.5
2
30
40
50
60
70
80
90
100
Jet P_T


$\sqrt{s} = 5.02$ TeV
$0 - 10\%$ Centrality
Strong Coupling
Smeared pp
$P_T^{jet} > 30$ GeV
$\Delta\phi > 7\pi/8$
$\frac{1}{N_\gamma}\frac{dN_{J\gamma}}{dx_{J\gamma}}$
0
0.2
0.4
0.6
0.8
1
1.2
1.4
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
$x_{J\gamma}$


Strong Coupling
$\sqrt{s} = 5.02$ TeV
$P_T^{\gamma} > 80$ GeV
$0 - 30\%$ Centrality
I_{AA}
0
0.5
1
1.5
2
40
50
60
70
80
90
100
110
120
130
Jet P_T

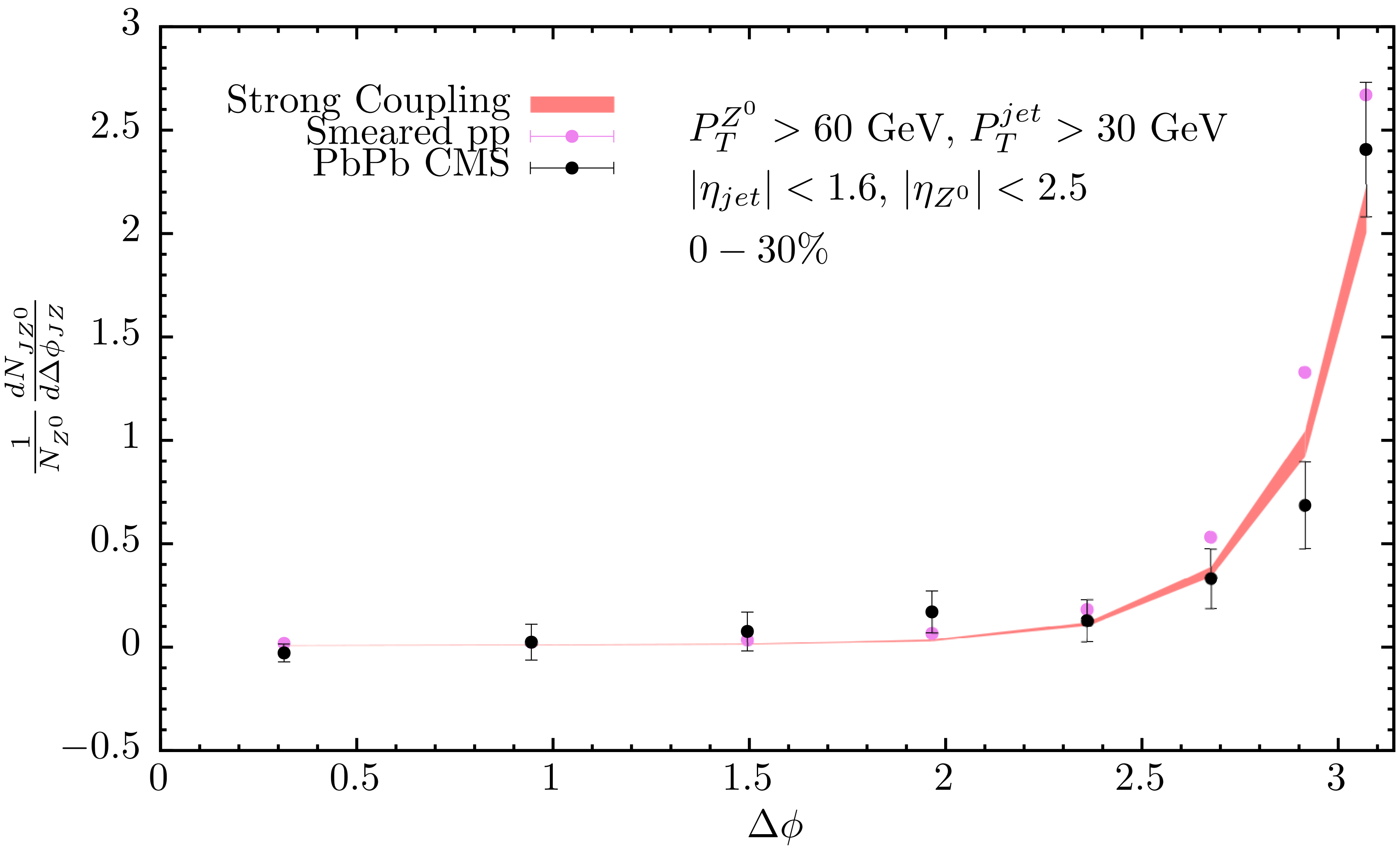

$\sqrt{s} = 5.02$ TeV
$10 - 30\%$ Centrality
Strong Coupling
Smeared pp
$\frac{1}{N_\gamma}\frac{dN_{J\gamma}}{dx_{J\gamma}}$
0
0.2
0.4
0.6
0.8
1
1.2
1.4
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
$x_{J\gamma}$

Z-Jet (5.02 ATeV)



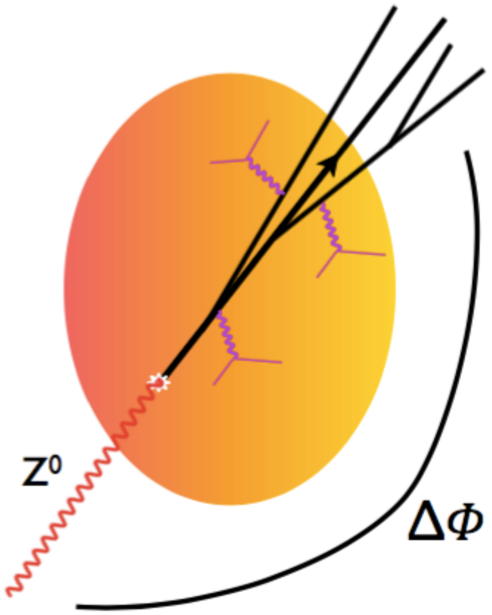
Preliminary CMS data just came out!

Z-Jet Acoplanarity (5.02 ATeV)

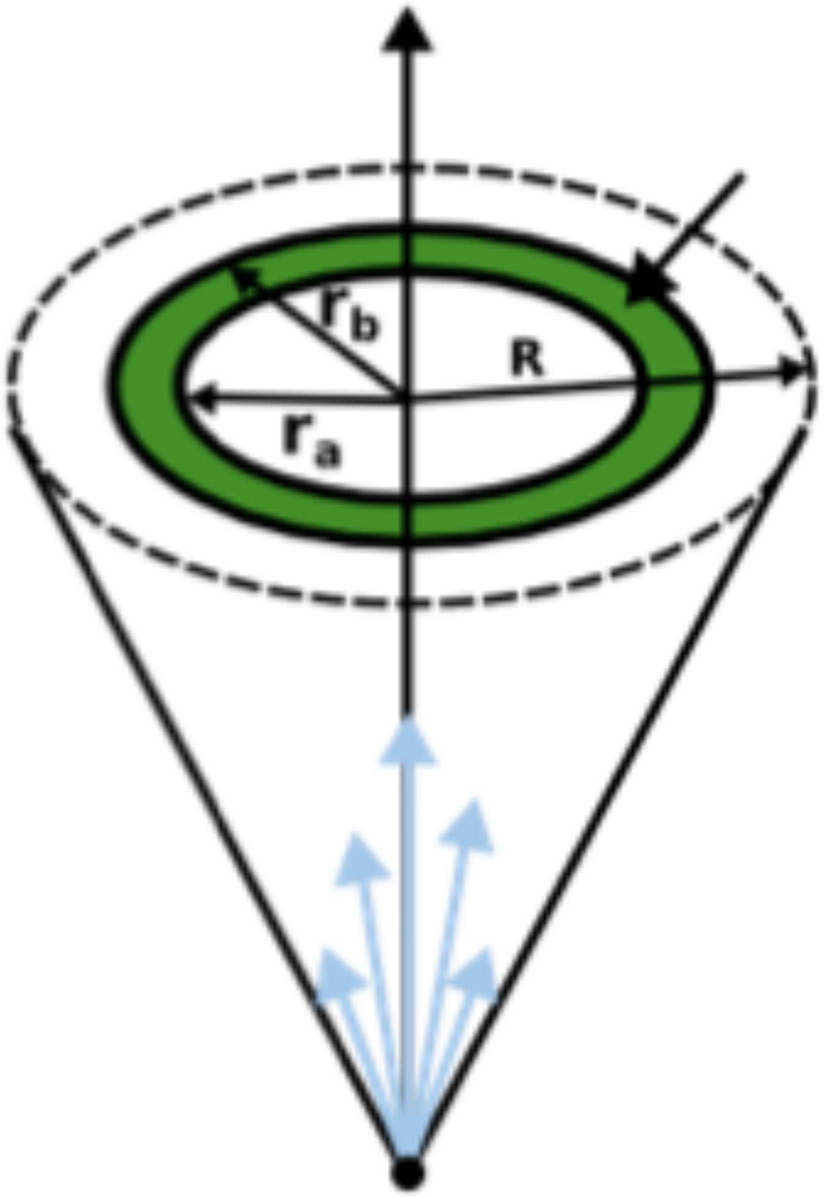


Normalised over the number of Z

Suppression of active jets tends to narrow the distribution



Jet Shapes

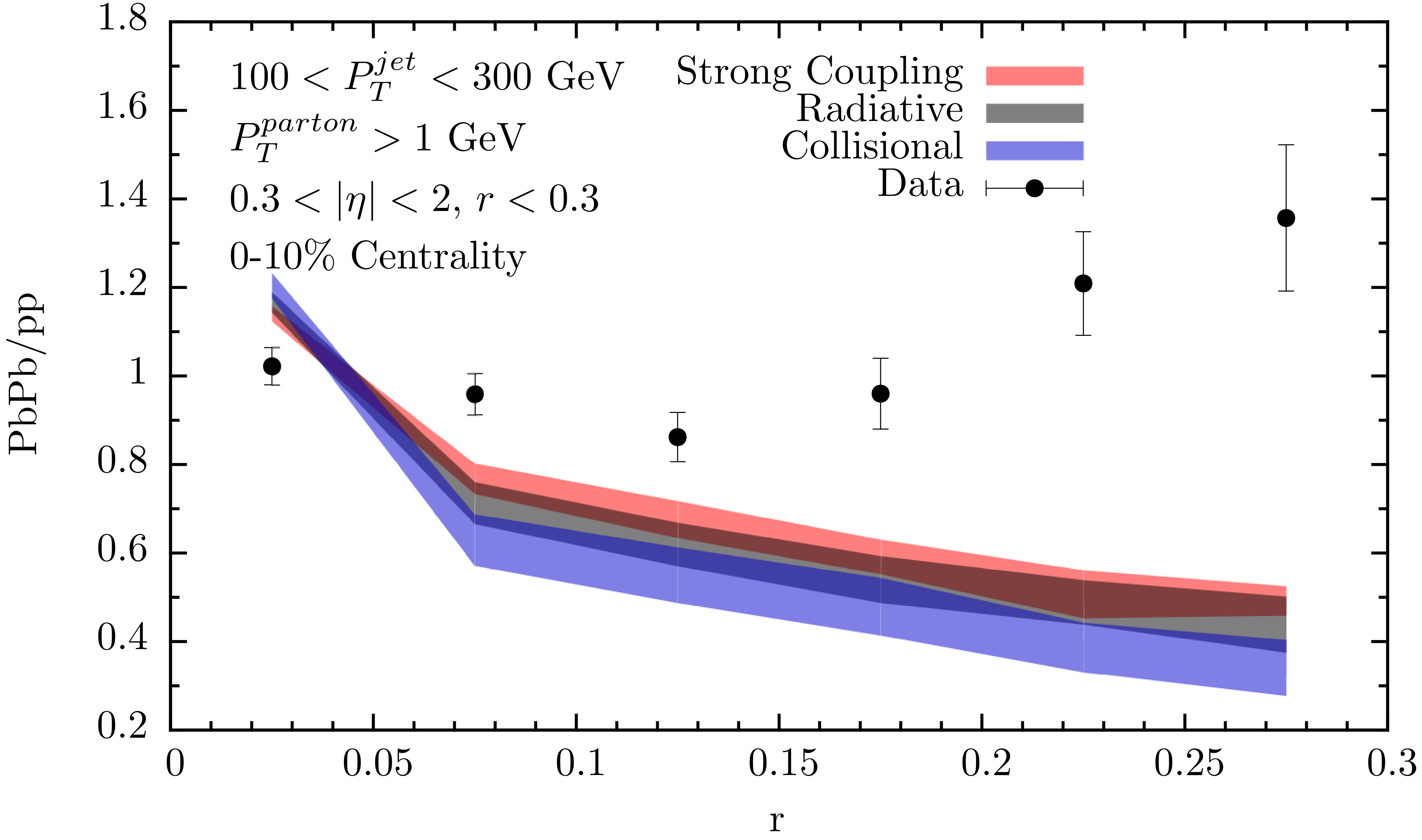


Transverse distribution of energy within the jet

Intra-jet observable robust to hadronization

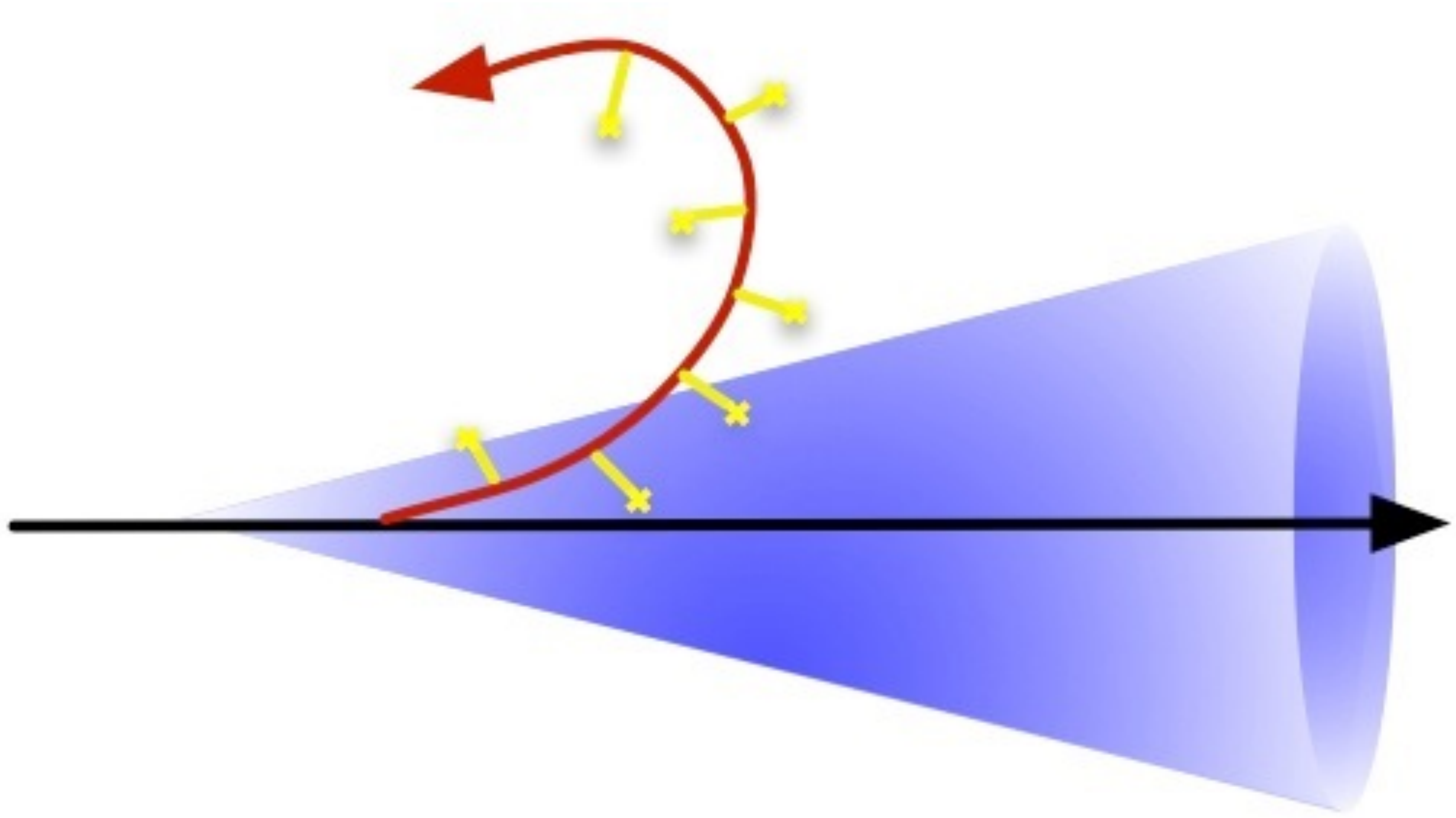
$$\rho(r) = \frac{1}{\Delta r}\frac{1}{N^{\text{jet}}}\sum_{\text{jets}}\frac{p_T(r-\Delta r/2, r+\Delta r/2)}{p_T(0,R)}$$

Jet Shapes



Without broadening, the model as it is
cannot reproduce jet shapes

Broadening



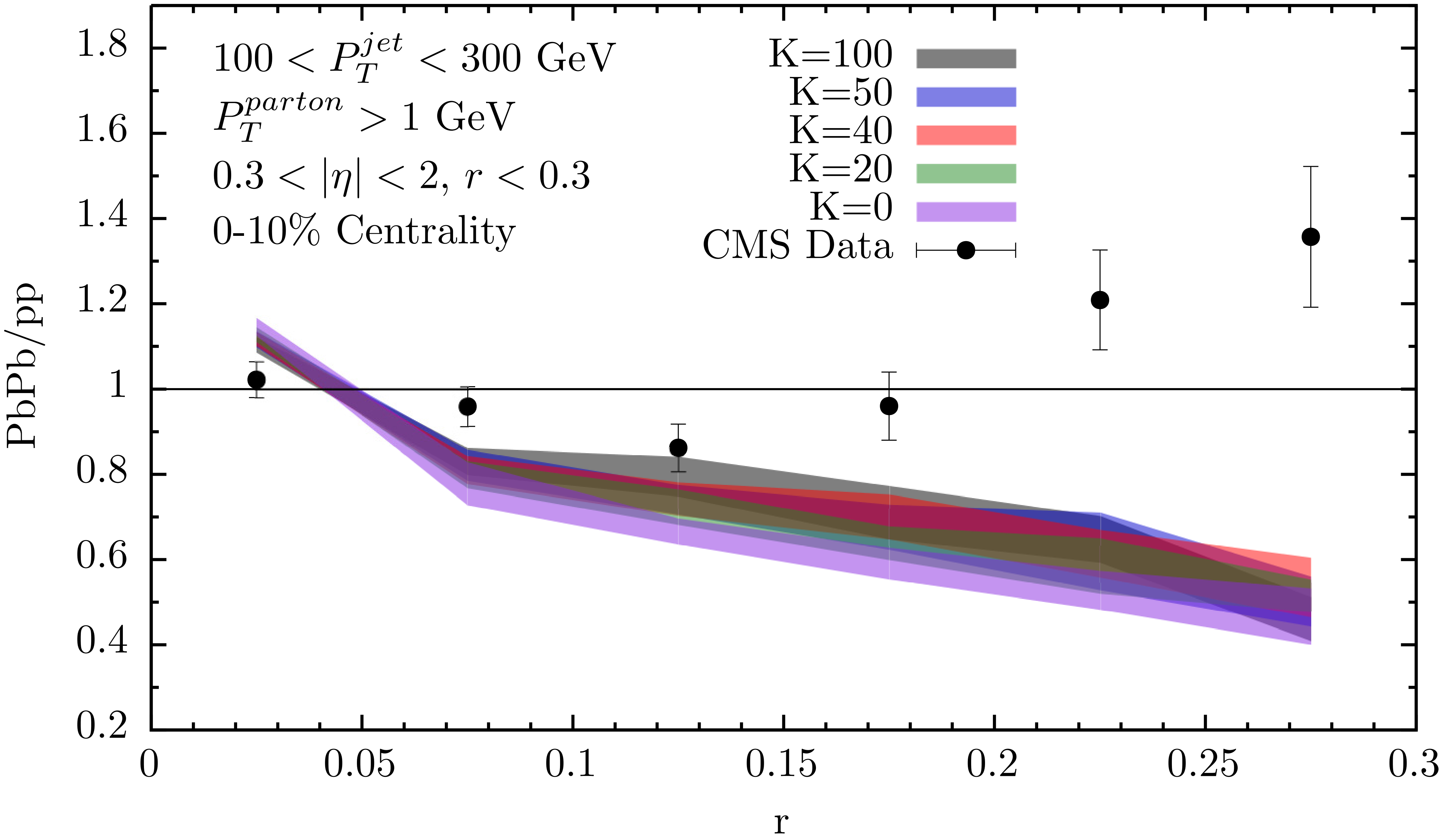
Partons receive transverse kicks according to a gaussian distribution

The width of the gaussian is $(\Delta k_T)^2 = \hat{q}\,dx$

Such mechanism introduces a new parameter $K = \dfrac{\hat{q}}{T^3}$

Transverse kicks can broaden the jet and kick particles out of the jet

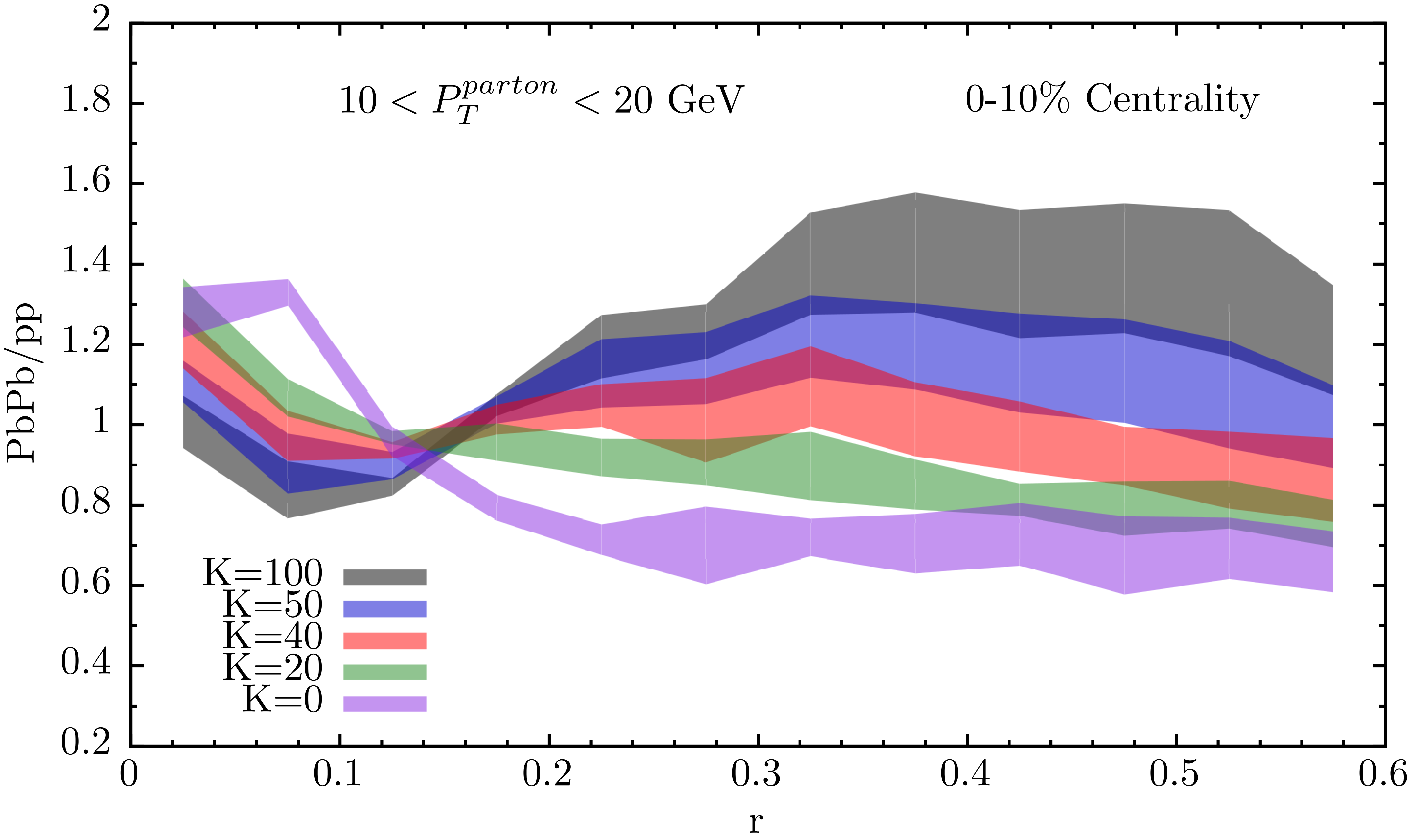
Small sensitivity of standard jet shapes to broadening



Small sensitivity of jet shapes to broadening:

- strong quenching removes soft fragments that appear early
- remaining soft tracks fragment late

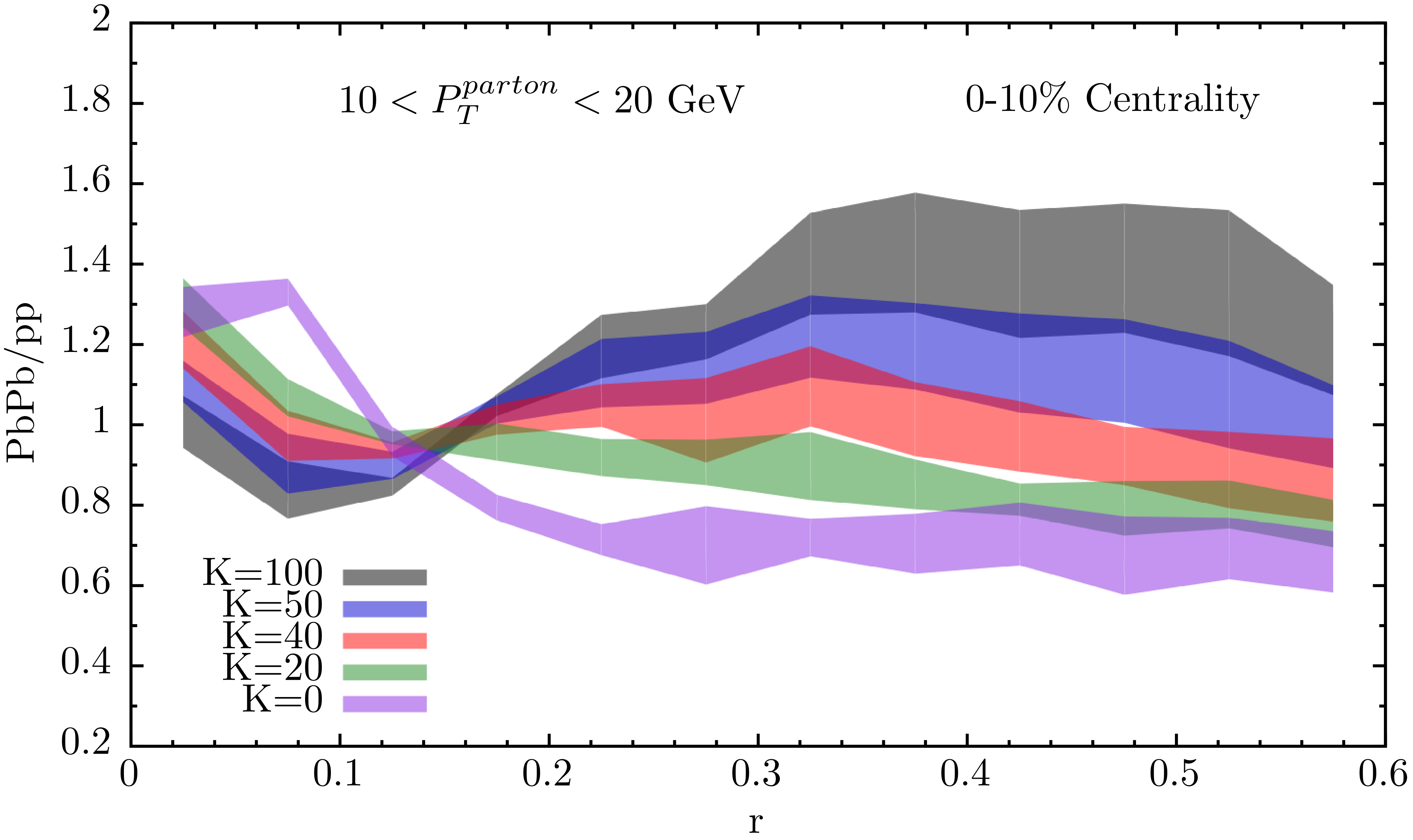
A New Observable, Sensitive to Broadening



Kinematical cuts for partons chosen such that:

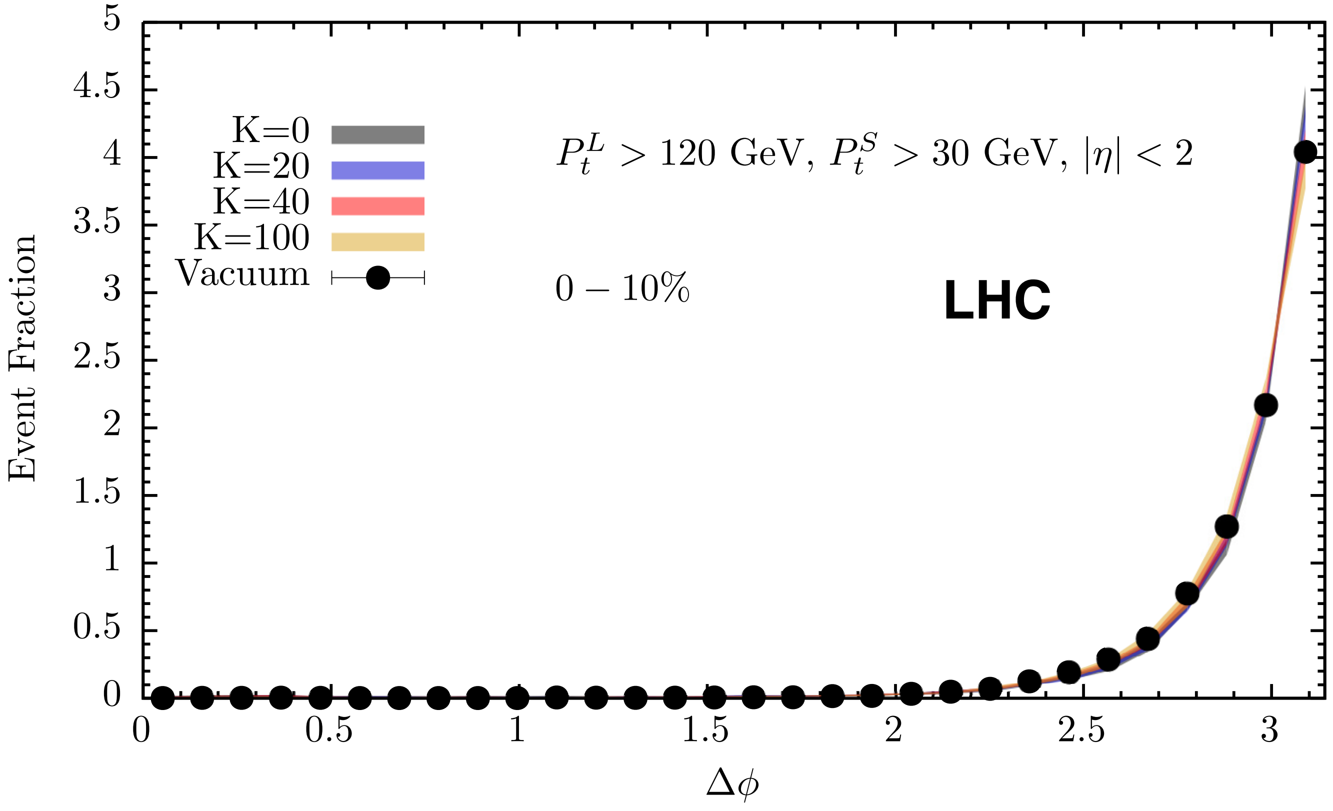
- there is no effect from background (soft tracks)
- we focus on jets without unfragmented cores (hard tracks)

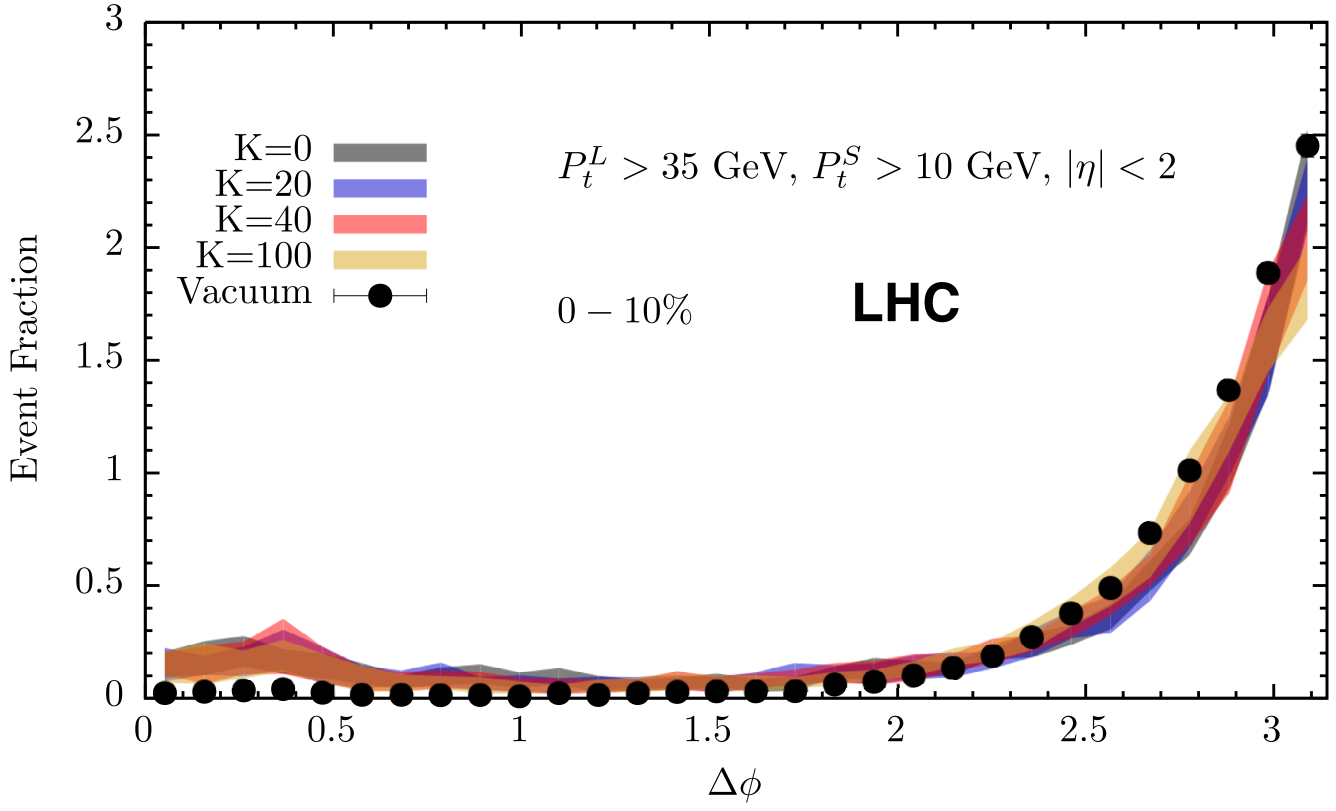
A New Observable, Sensitive to Broadening

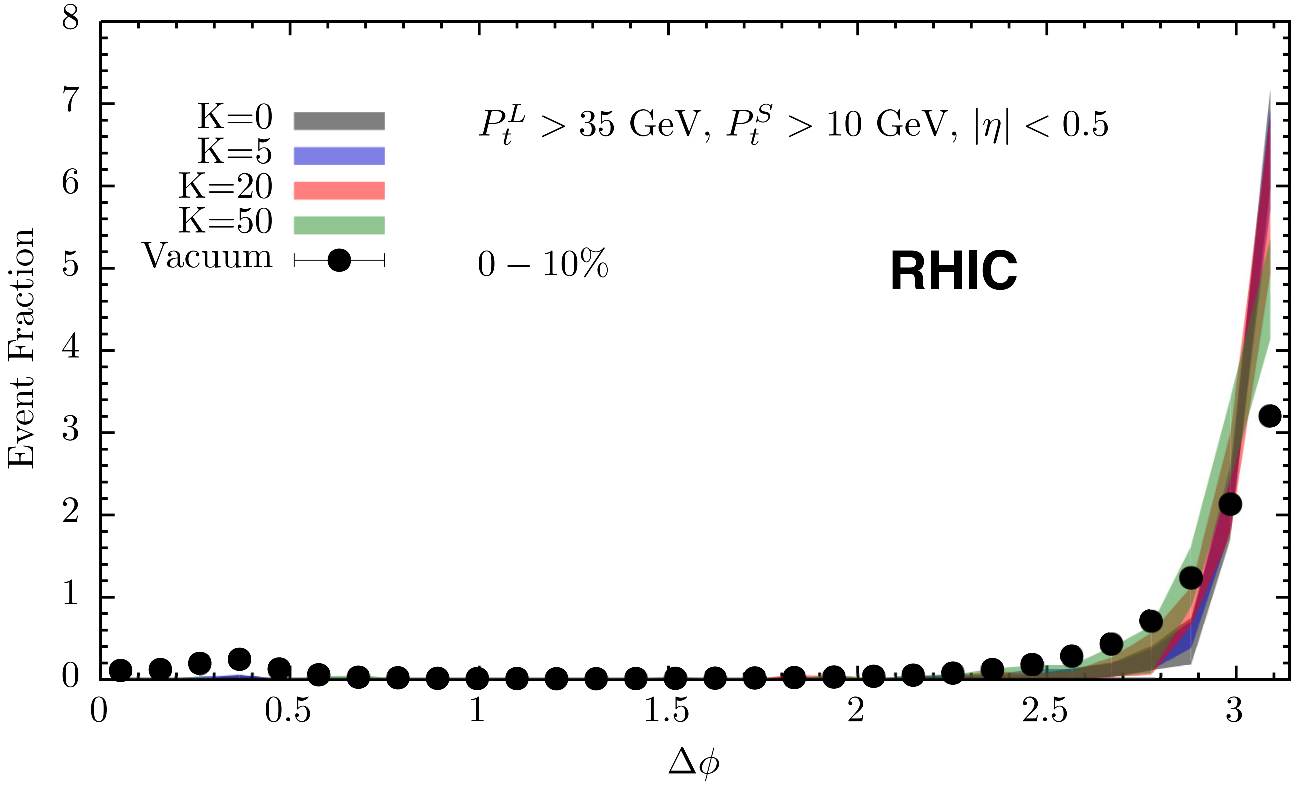


After constraining the Gaussian broadening strength,
the longer term goal will be to look for the rare hard momentum scatterings
given by the short distance quasiparticles in the soup

Dijet Acoplanarities





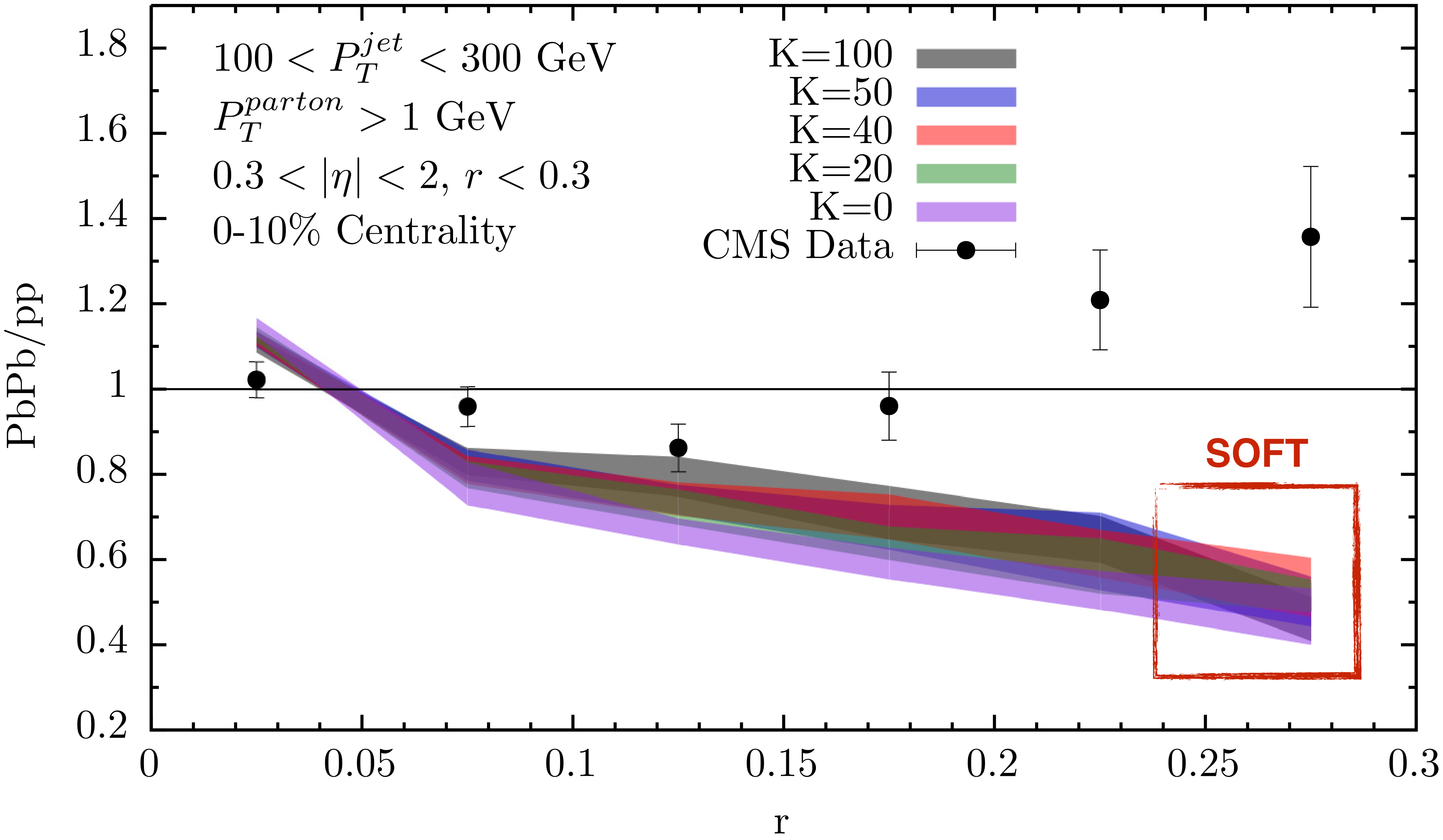


Higher energy jets are narrower: less acoplanar

Energy loss narrows the distributions, while broadening widens them back

Effects strongest for lower energies due to more steeply falling spectrum

Broadening



Large r region dominated by soft tracks,
also sensitive to medium response effects

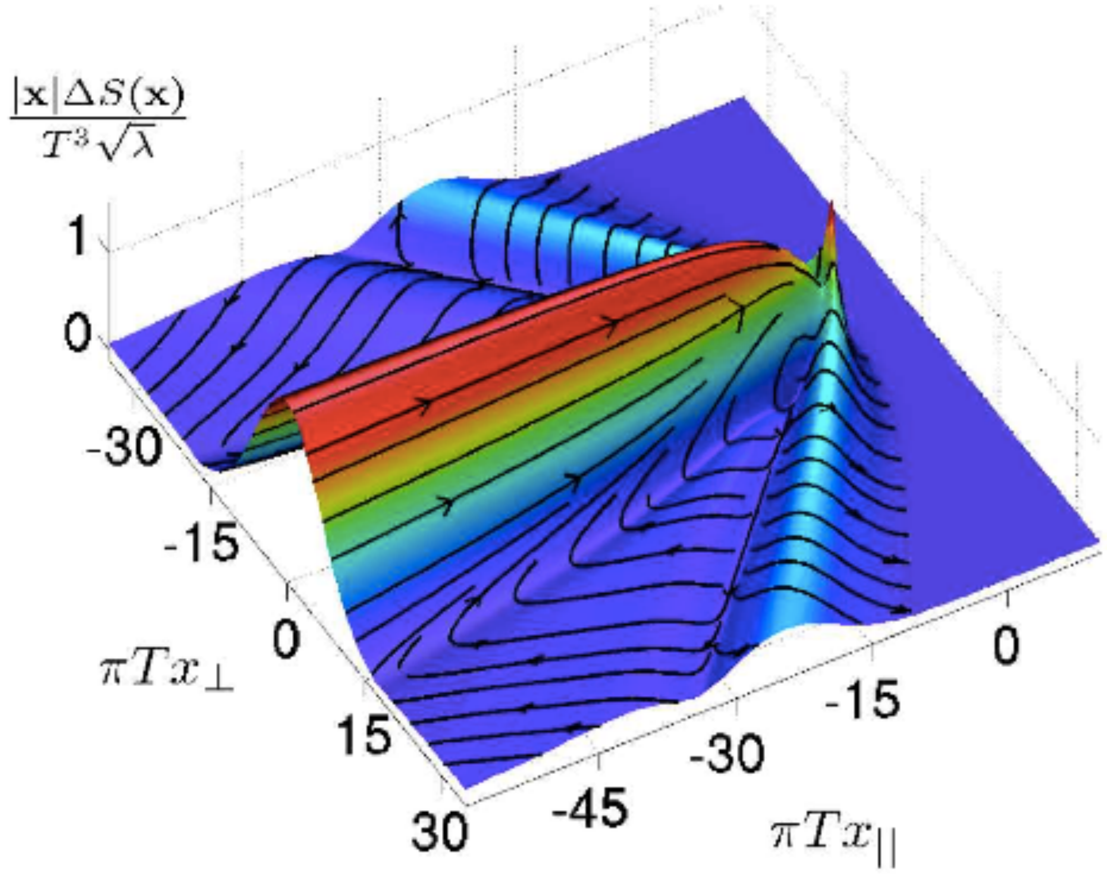
An Estimate of Backreaction

Hydro response to jet passage:

Assumption: small perturbation of hydro

Consequence:

- no details on the perturbation are needed
- distribution fully constrained by **energy-momentum conservation**
- no additional parameters



Chester and Yaffe 0712.0050

An Estimate of Backreaction

Perturbations on top of a Bjorken flow

$$\Delta P^i_\perp = w\tau \int d\eta\, d^2x_\perp\, \delta u^i_\perp$$

$$\Delta P^\eta = 0$$

$$\Delta S = \tau c_s^{-2} s \int d\eta\, d^2x_\perp\, \frac{\delta T}{T}$$

$$c_s^2 = \frac{s}{T}\frac{dT}{ds}$$

Cooper-Frye $$E\frac{dN}{d^3p} = \frac{1}{(2\pi)^3}\int d\sigma^\mu p_\mu f(u^\mu p_\mu)$$

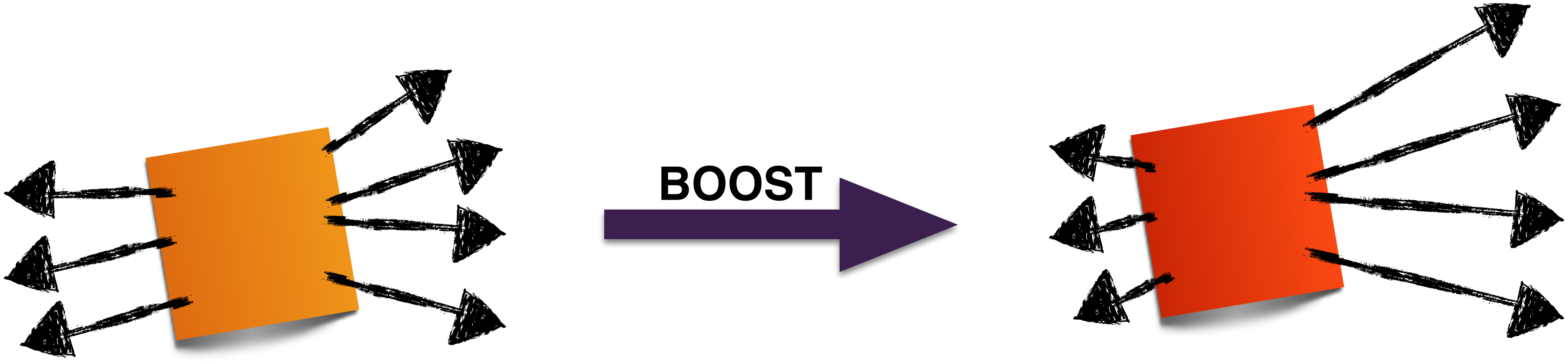
One body distribution

$$E\frac{dN}{d^3p} = \frac{1}{32\pi}\frac{m_T}{T^5}\cosh(y-y_j)e^{-\frac{m_T}{T}\cosh(y-y_j)}$$

$$\left[p_T \Delta P_T \cos(\phi-\phi_j) + \frac{1}{3}m_T \Delta M_T \cosh(y-y_j)\right]$$

An Estimate of Backreaction

One body distribution has negative contributions at large azimuthal separation



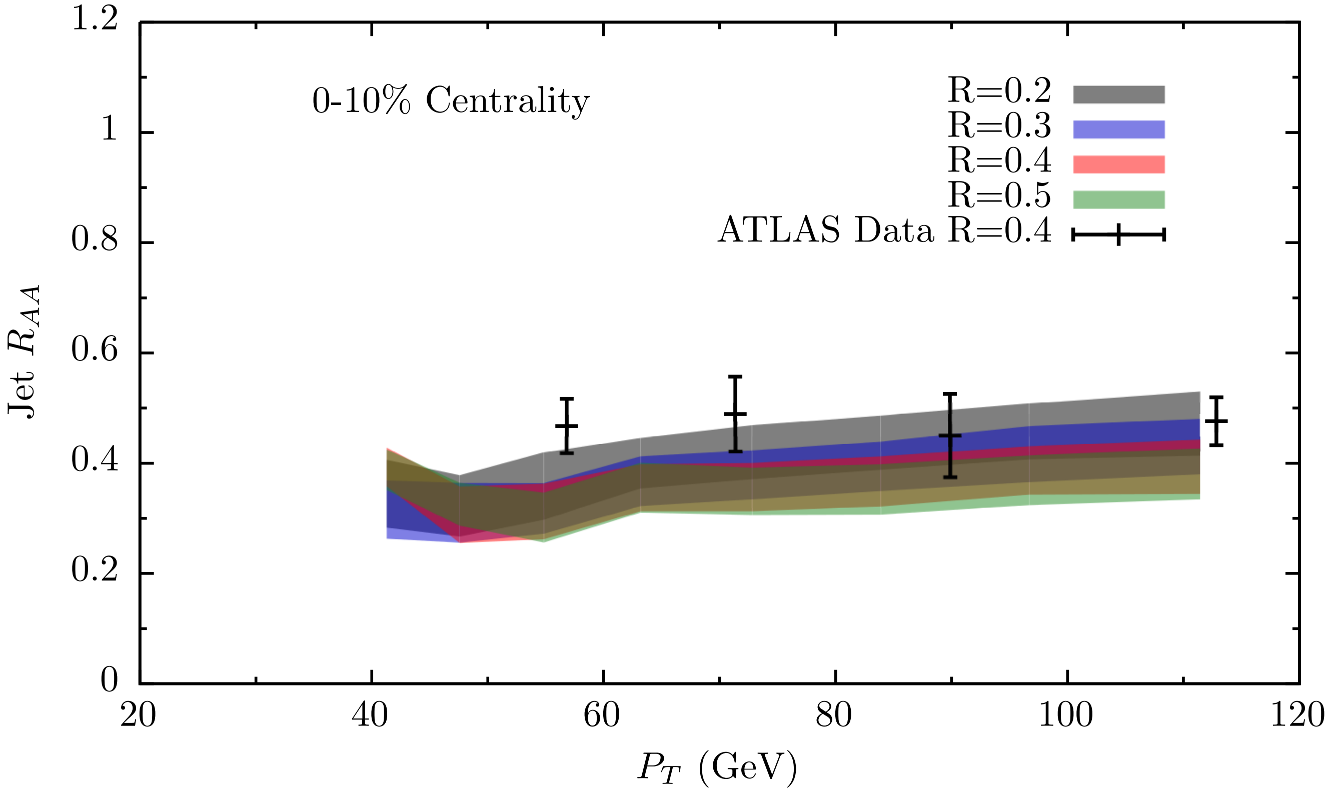
Background diminished w.r.t unperturbed hydro for that region in space

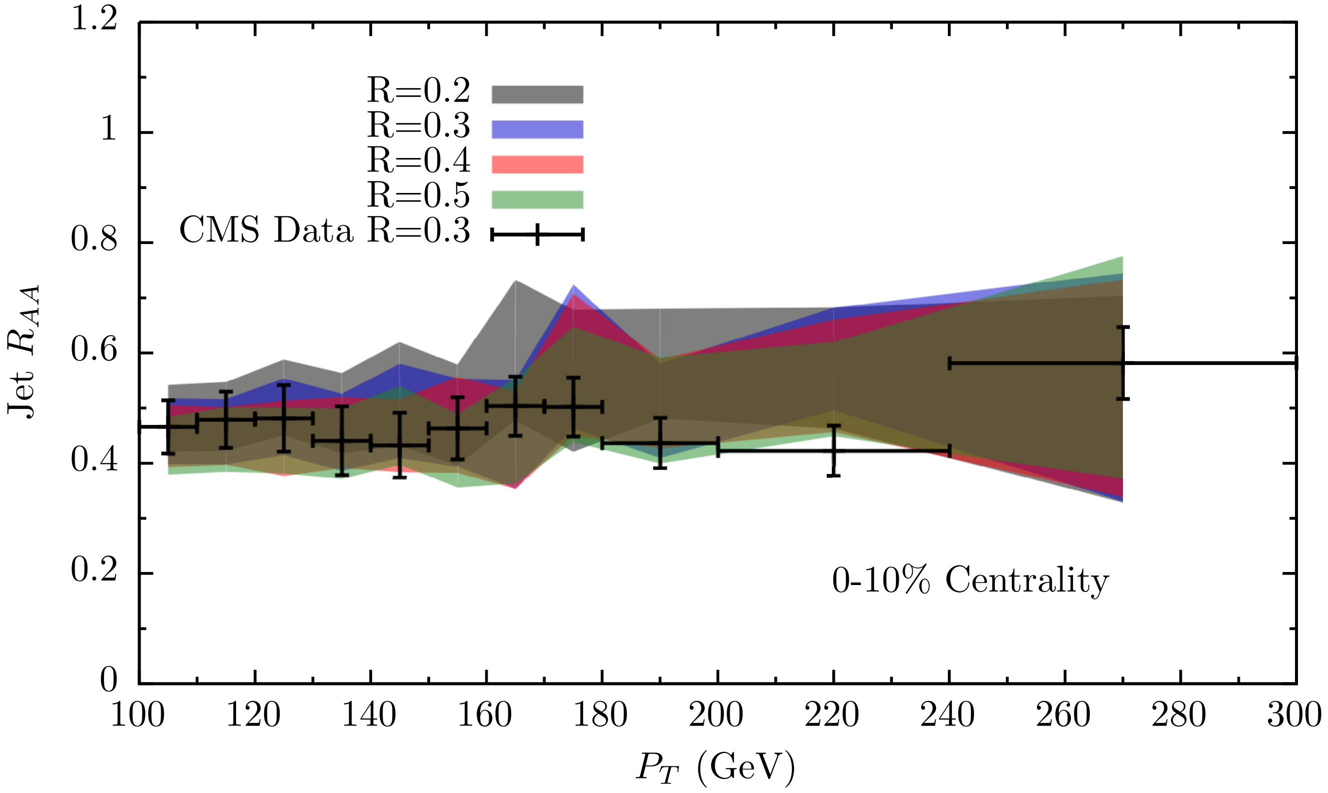
Need to emulate experimental background subtraction

Add background,
embed jets,
subtract background

Event by event, determine the extra particles distribution enforcing energy/momentum conservation via Metropolis algorithm

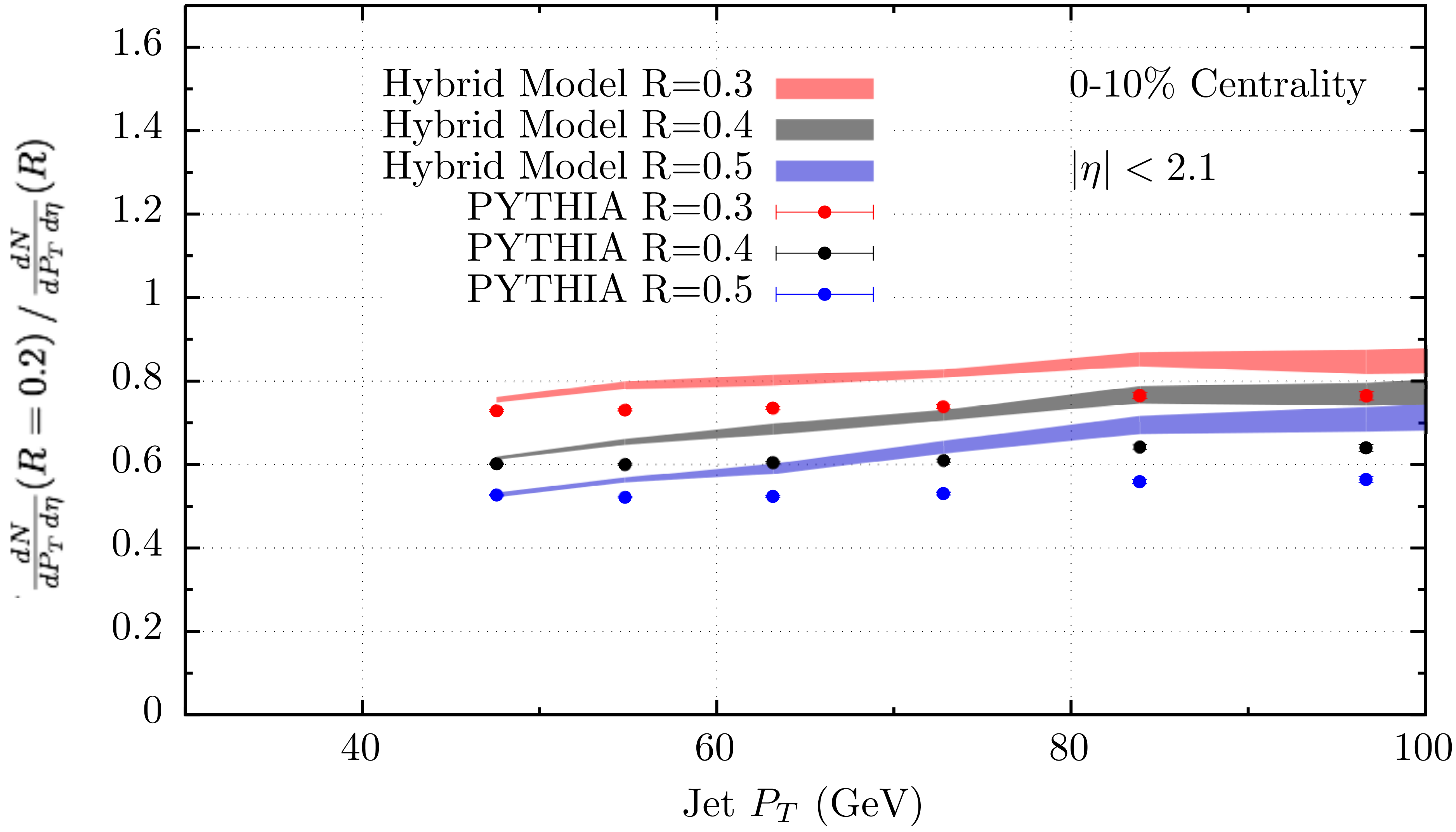
R_{AA} vs R





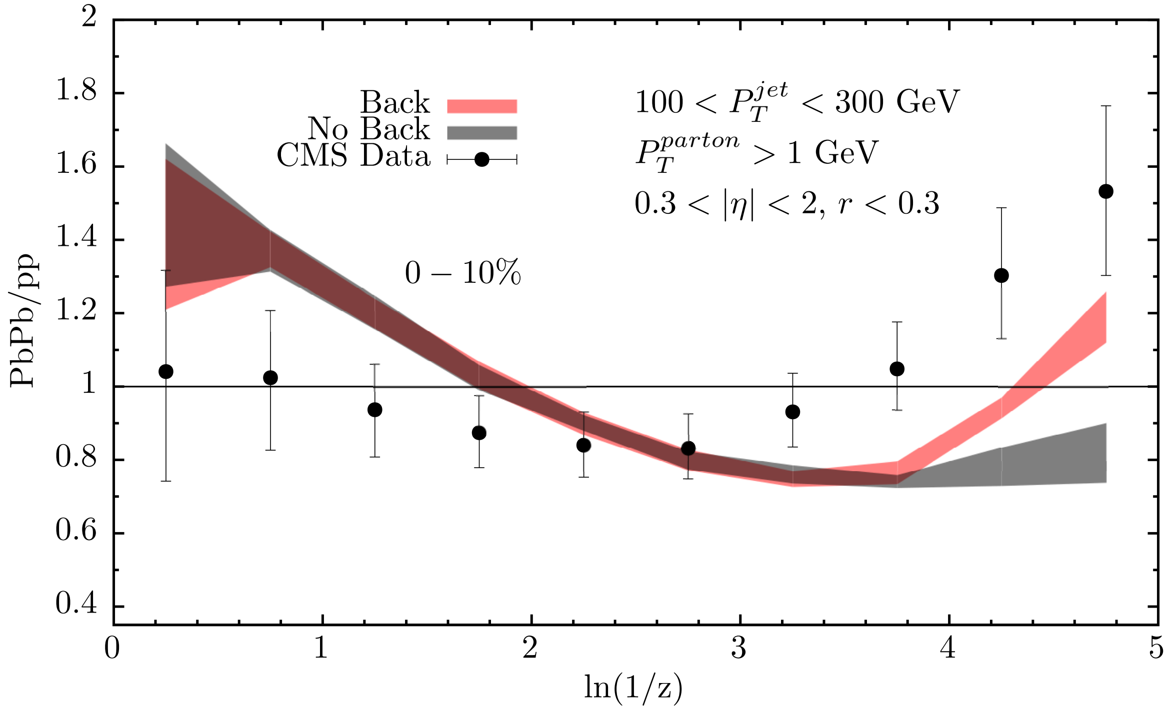
- Had to retune fitting parameter (only at percent level)
- Wider jets are (slightly) more suppressed than narrow ones
- Energy is recovered at wider angles

Jet Spectra Ratios

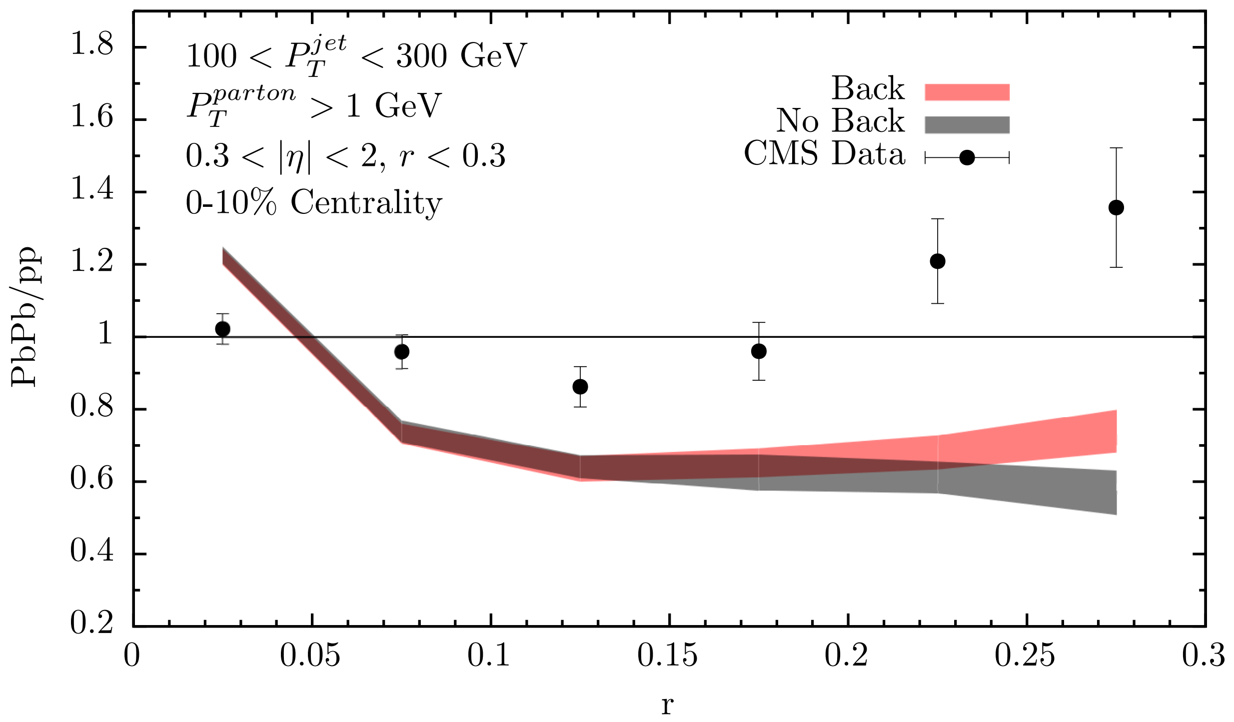


- Higher Pt jets tend to be narrower — increase of ratios with Pt
- Wider jets more suppressed — PbPb ratios always above pp ones
- <#Tracks> increases with Pt — PbPb vs pp separation increases with Pt

Backreaction on Intra-Jet Observables



Fragmentation Functions

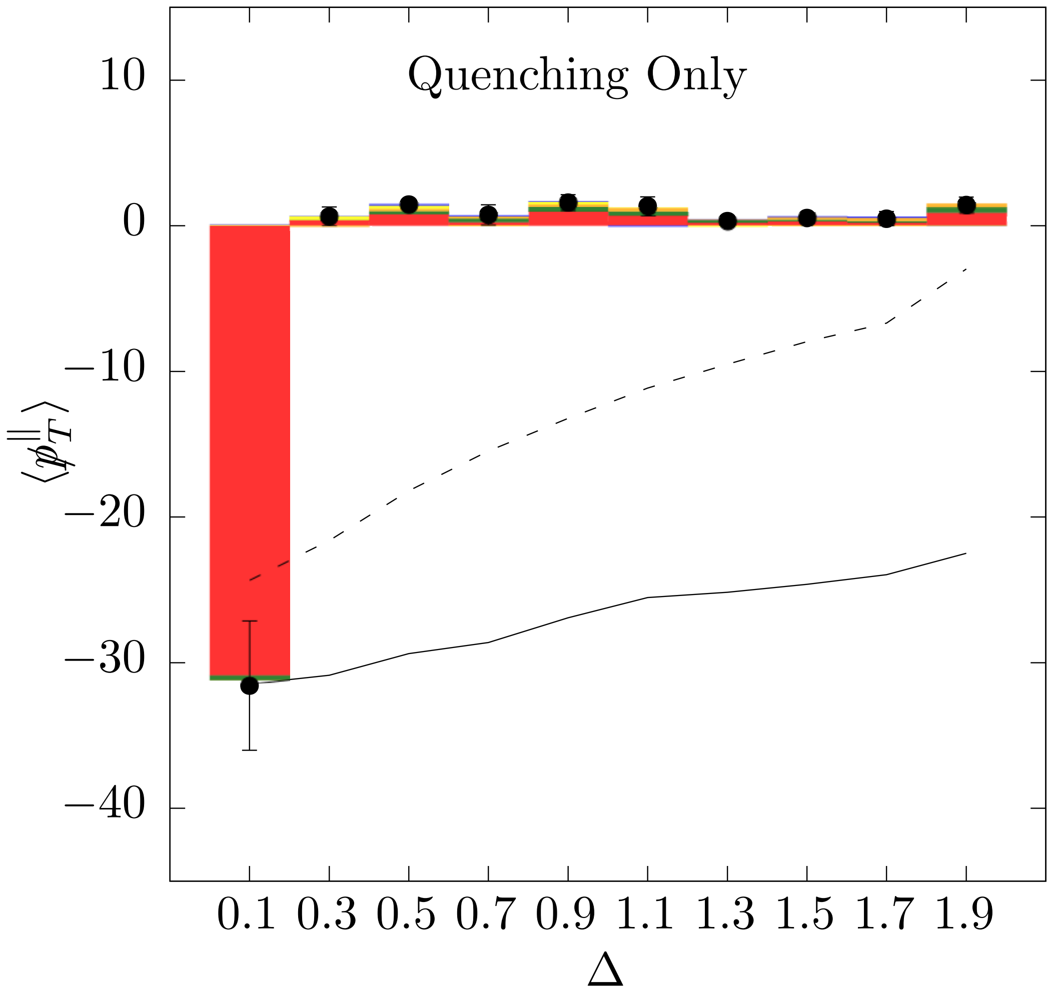


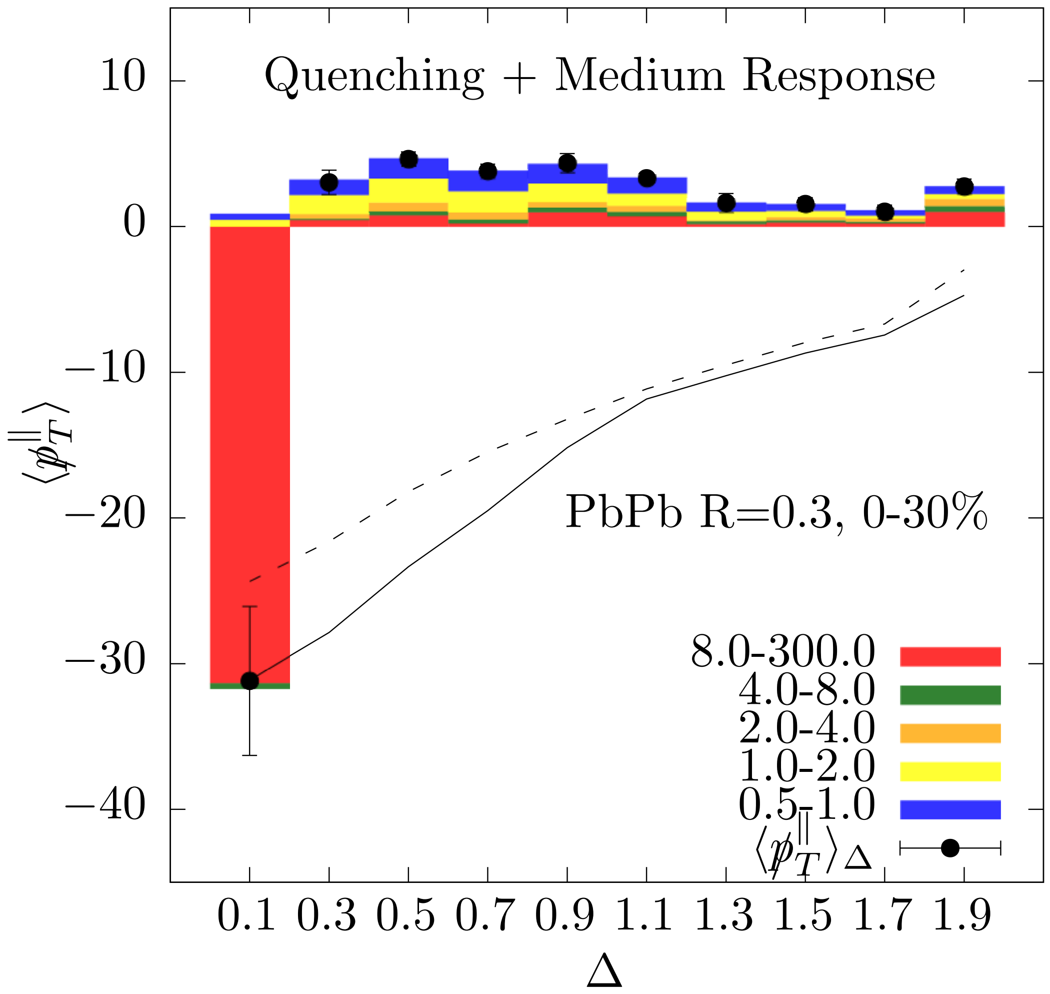
Jet Shapes

- The effect goes in the right direction
- Clearly not enough to explain angular structure

- Oversimplified backreaction?
- Hadronization uncertainties? (medium *and* vacuum)
- Finite resolution effects?

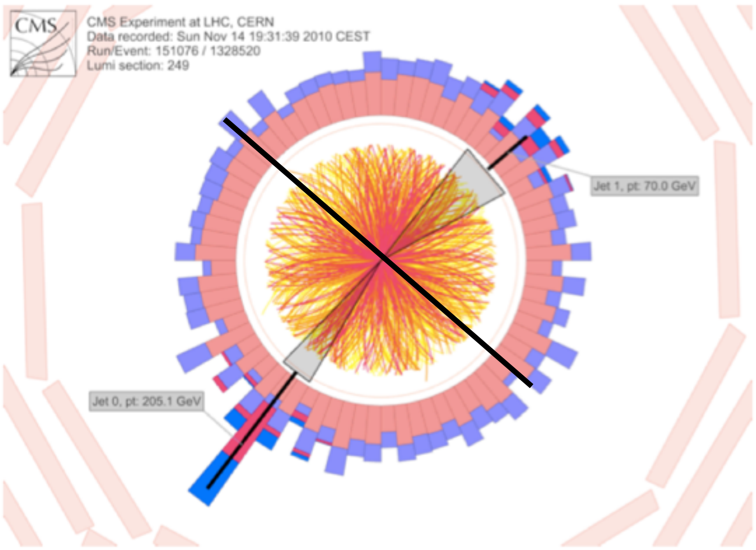
Recovering Lost Energy: Missing Pt





- Energy is recovered at large angles in the form of soft particles

- Adding medium response is essential for a full understanding of jet quenching



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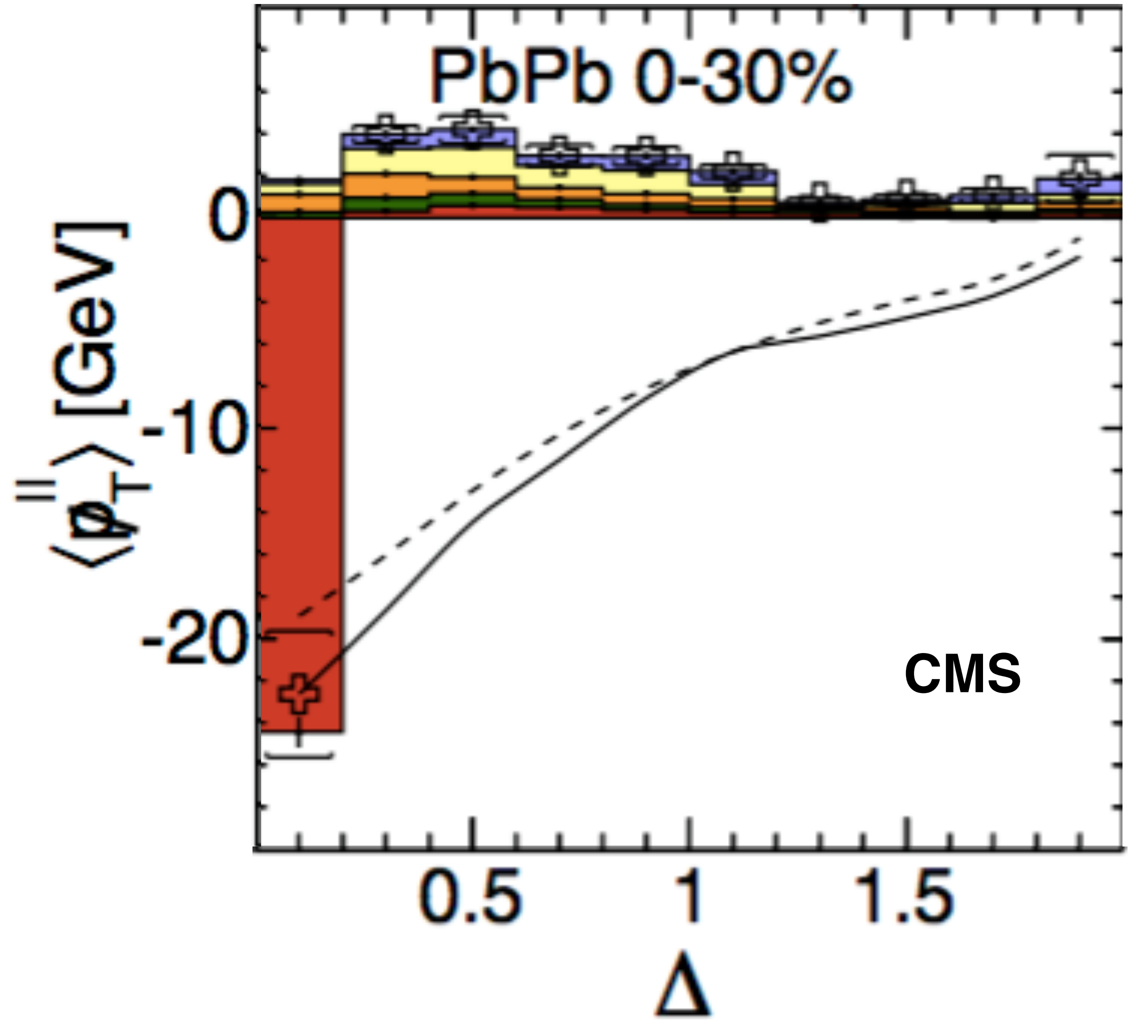

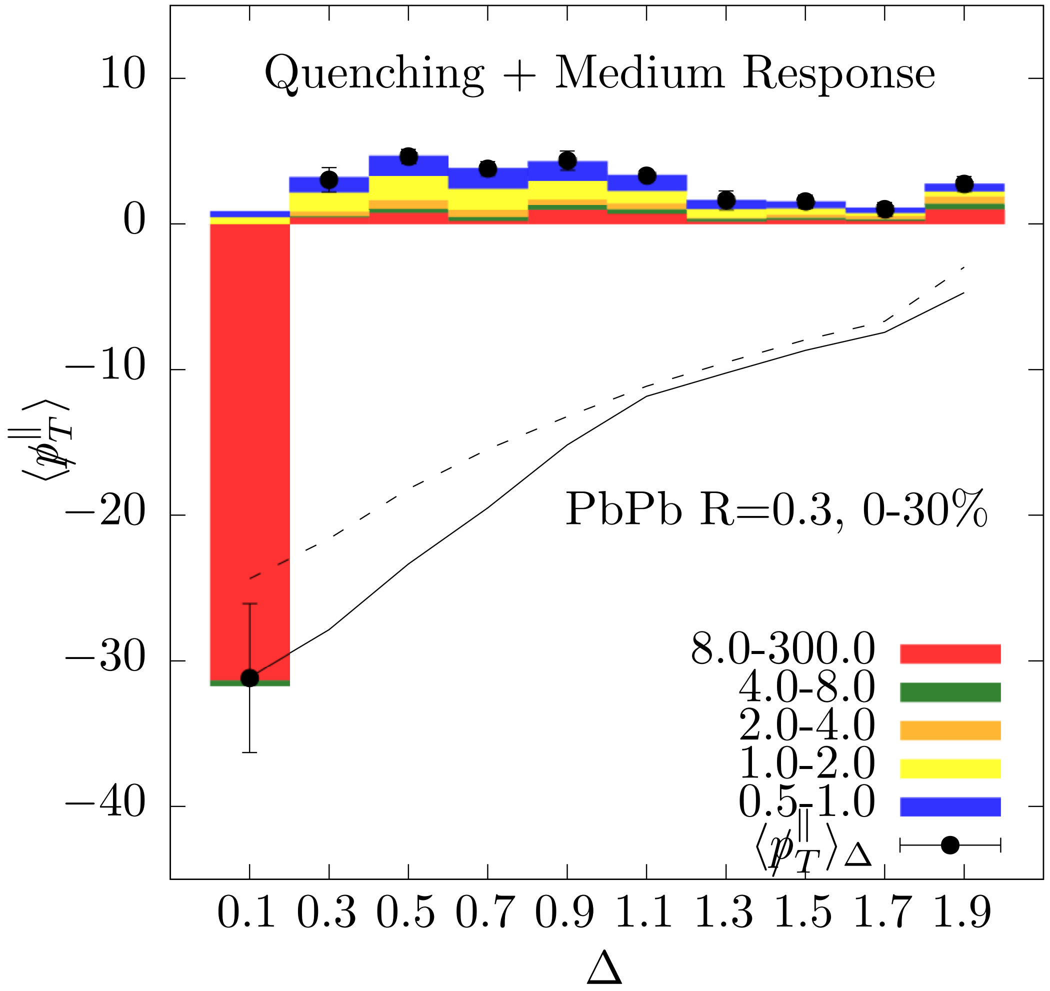

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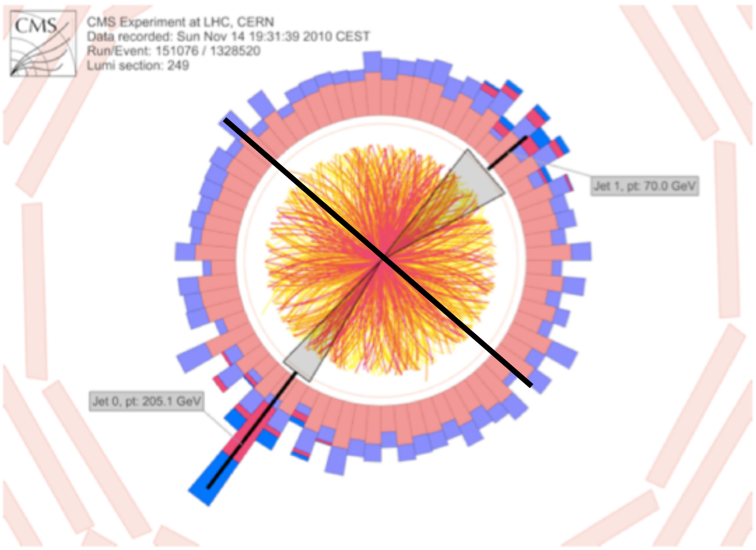

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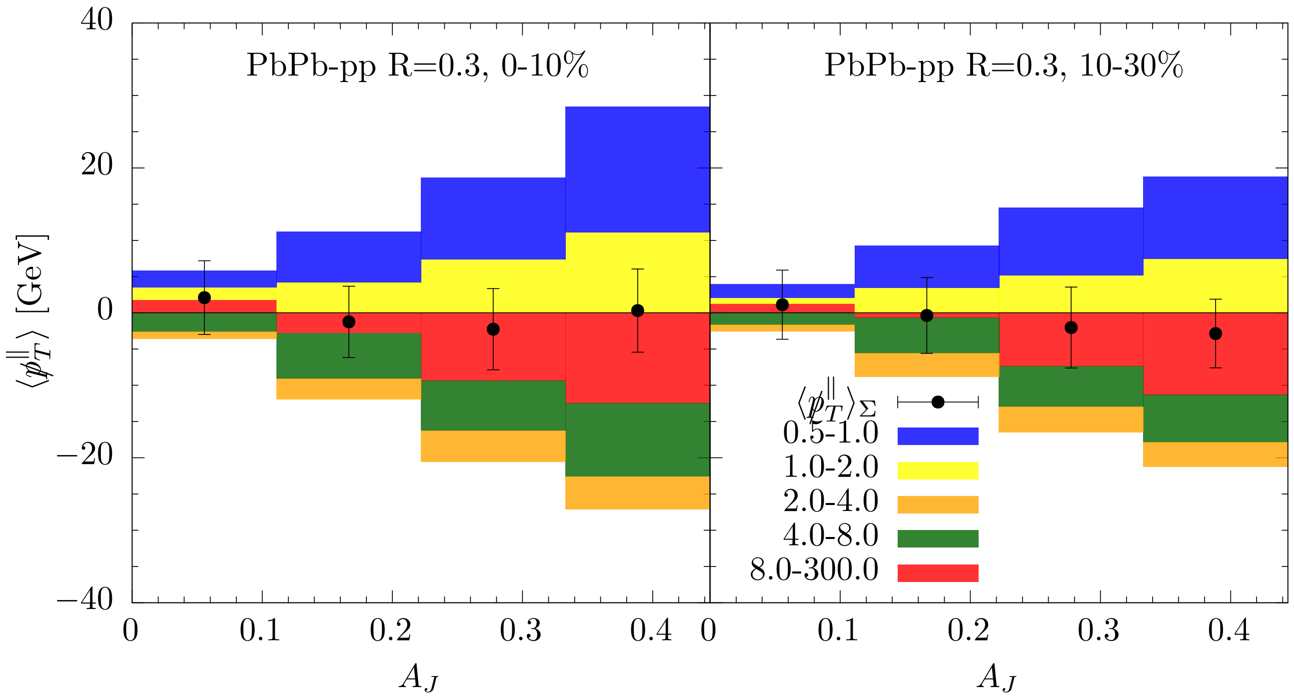
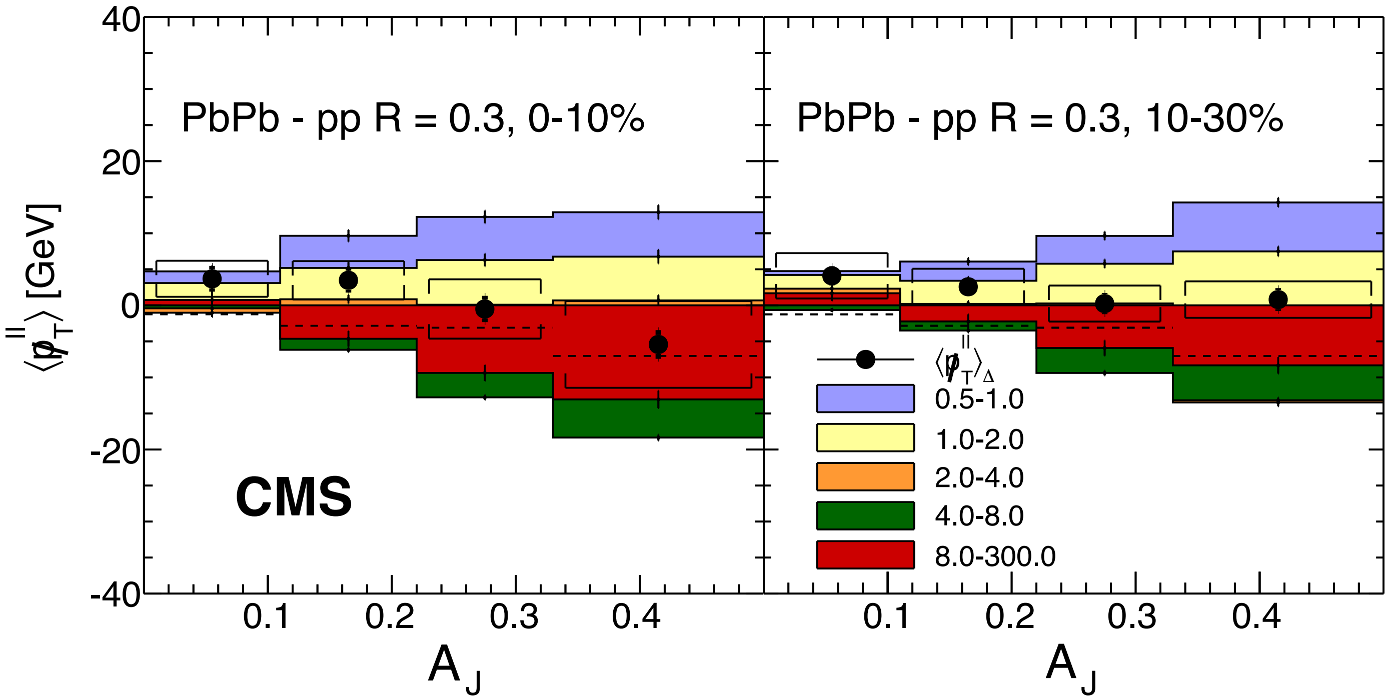
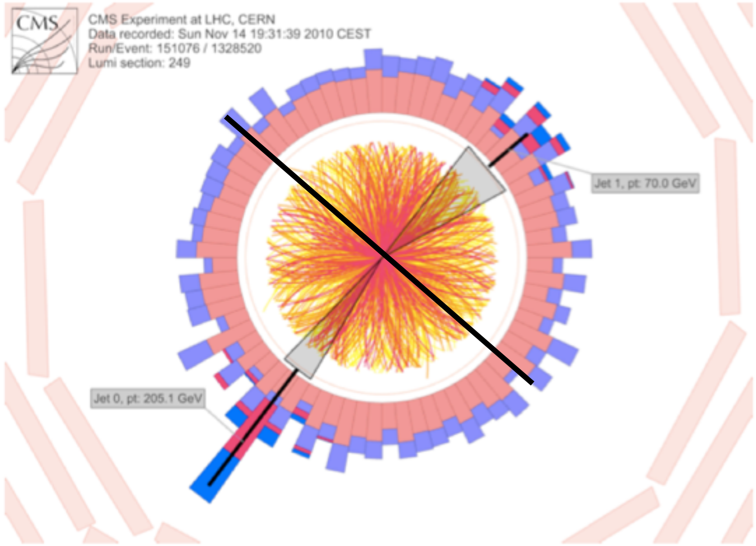


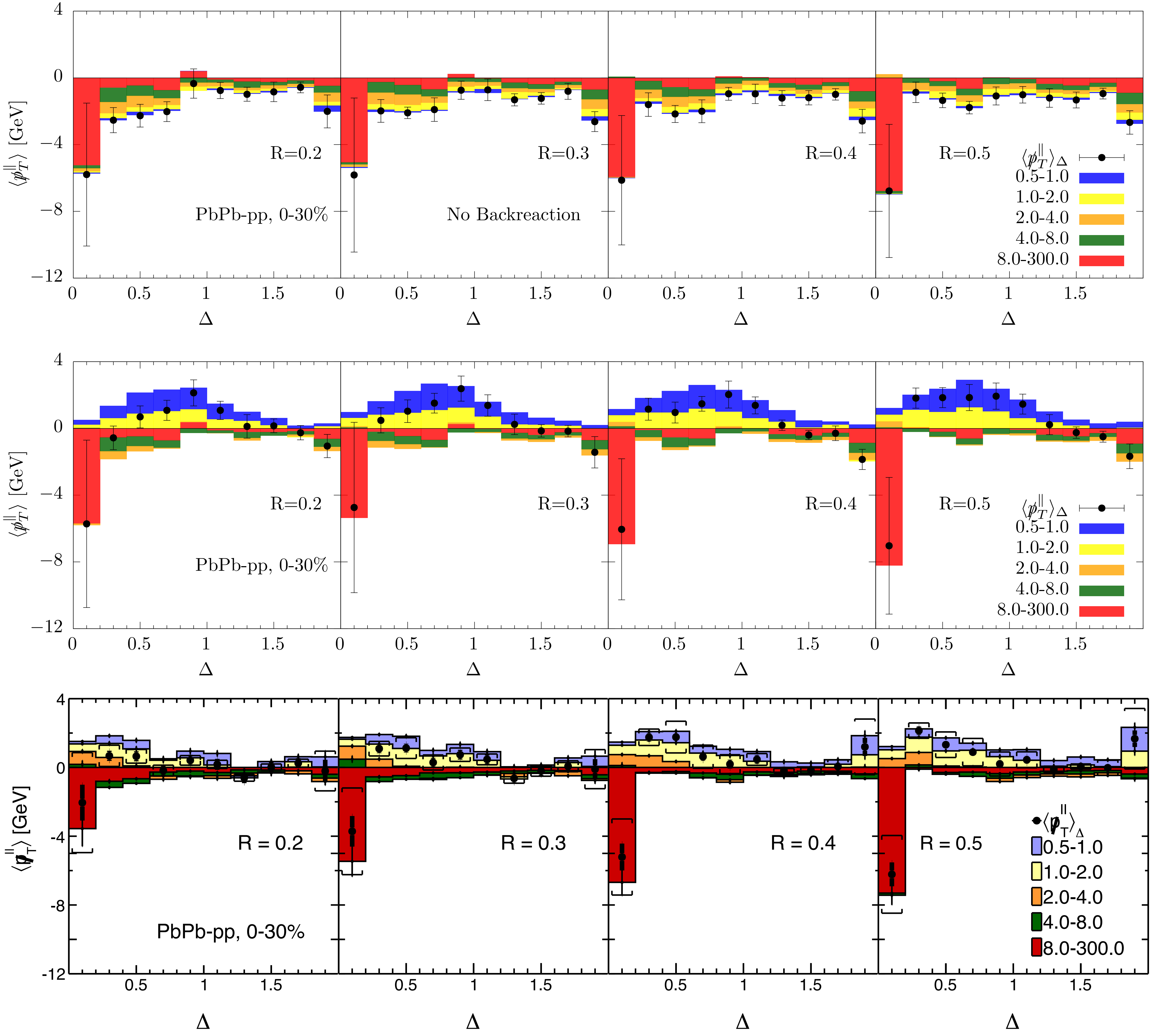


- In PbPb, more asymmetric dijet events are dominated by soft tracks in the subleading jet side

- Discrepancies w.r.t. data in the semi-hard regime motivate improvements to our model



Jet radius dependence of Missing Pt



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- Tension for certain intra-jet observables

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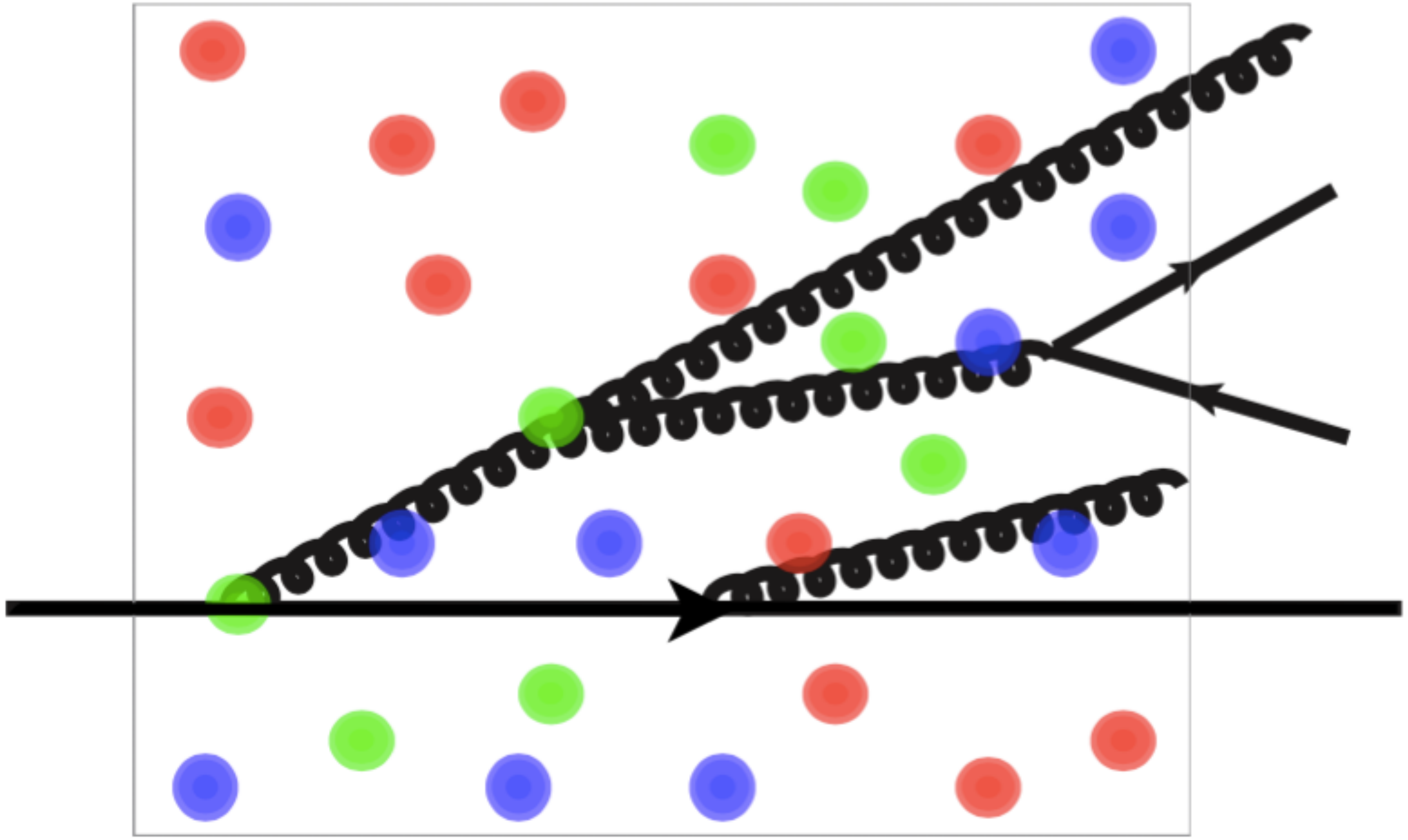
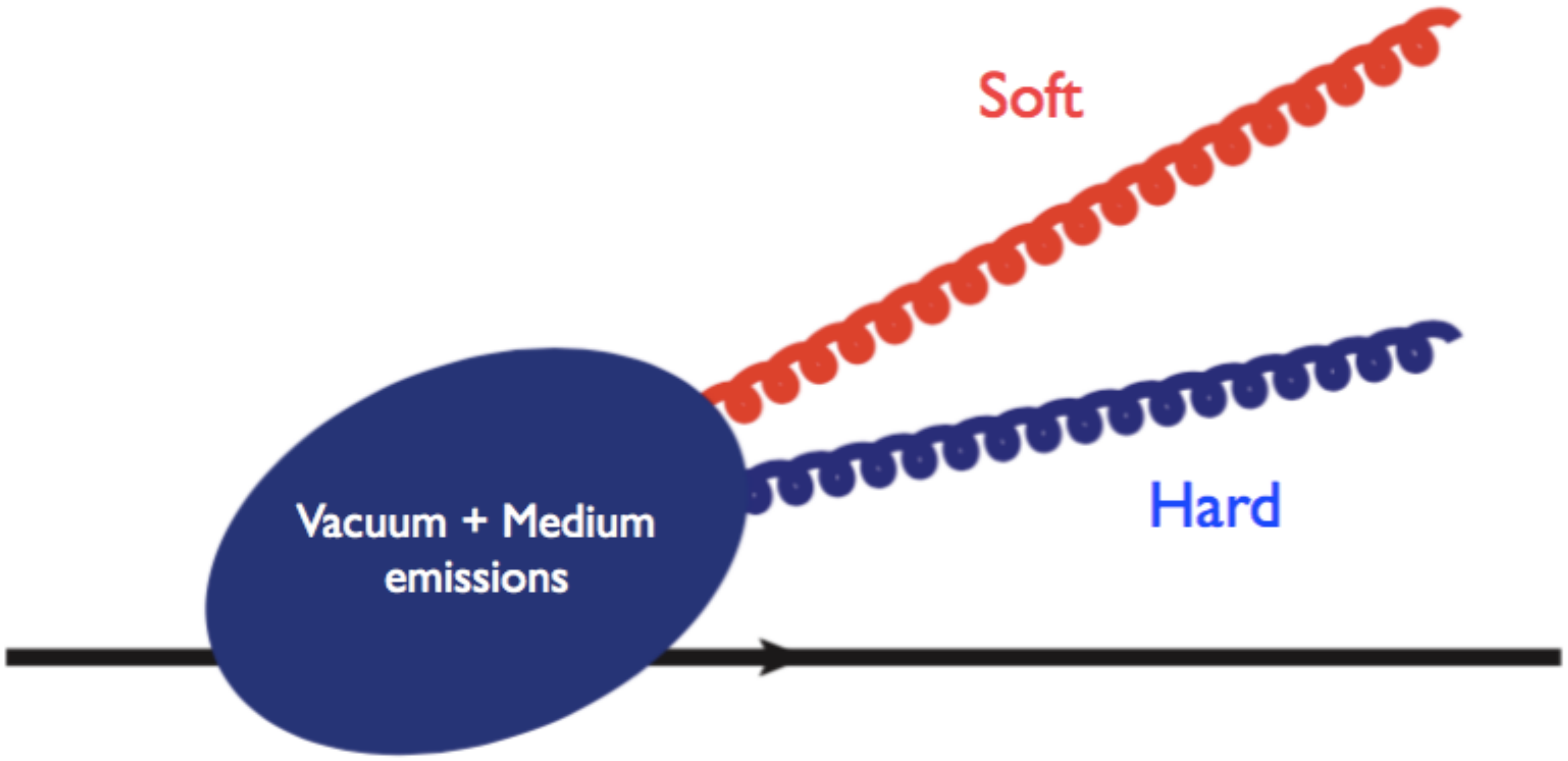
Coherence effects

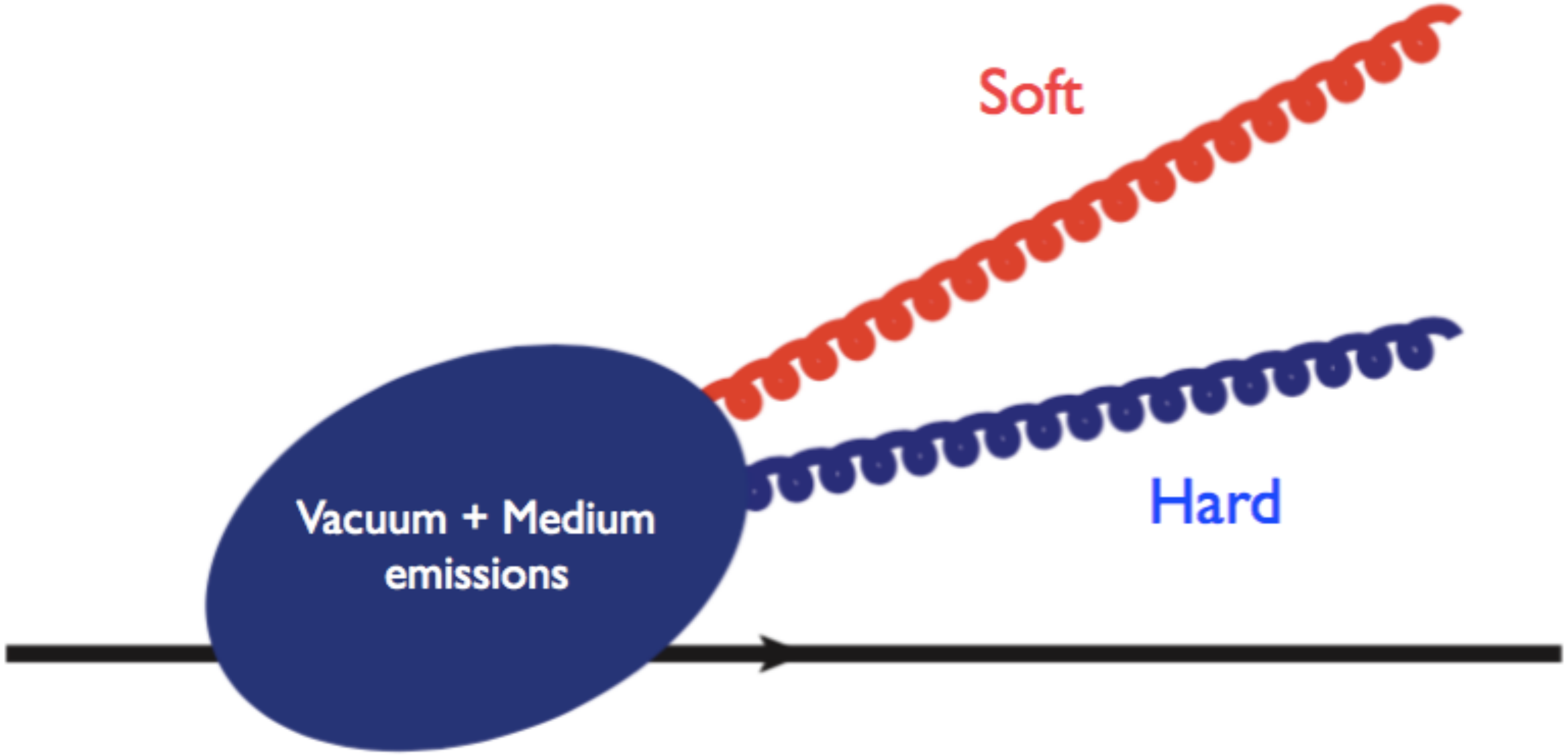


Coherence effects



Compute two gluon inclusive emission off a hard quark

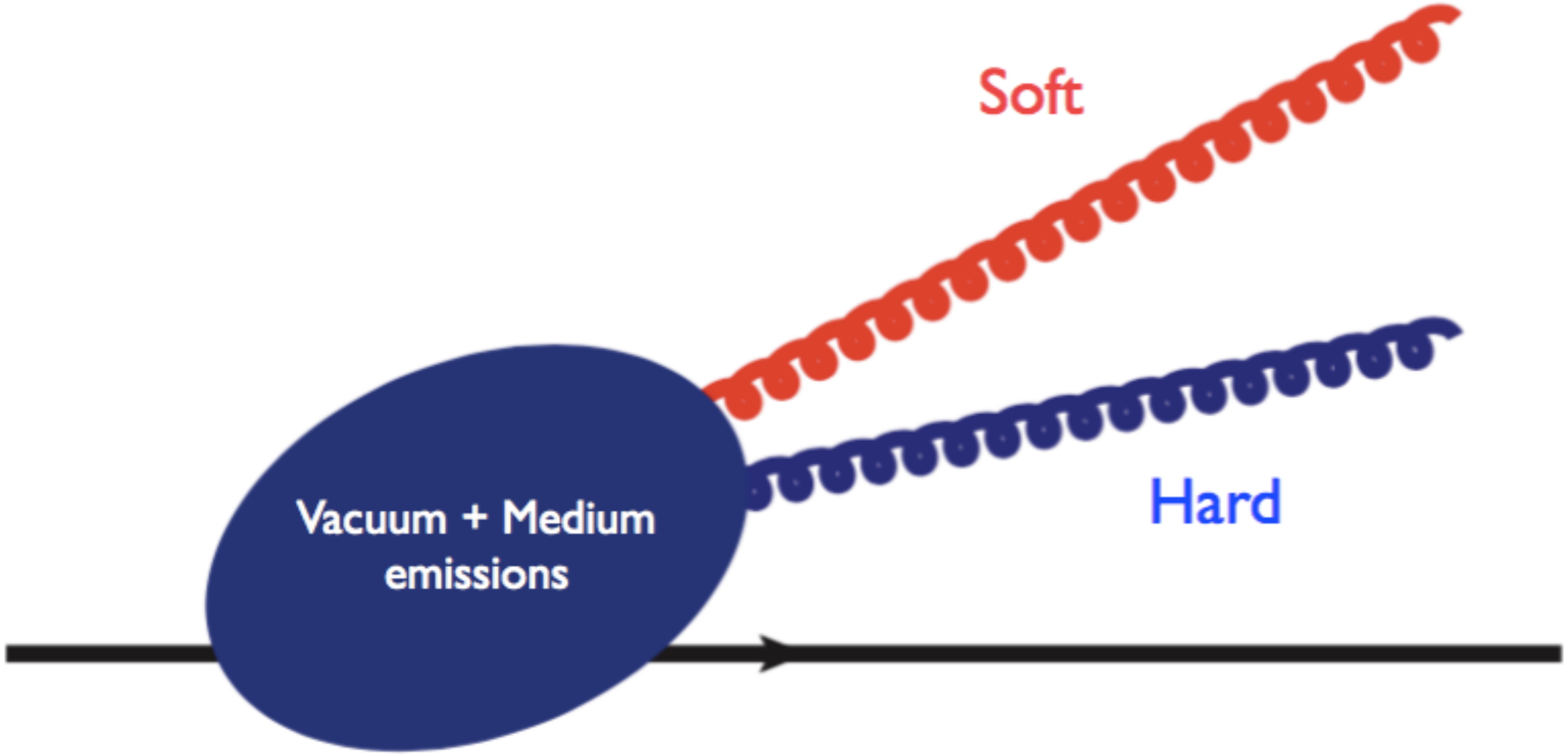
Coherence effects



Compute two gluon inclusive emission off a hard quark

pQCD calculation in N=1 opacity (thin medium)

Coherence effects



Compute two gluon inclusive emission off a hard quark

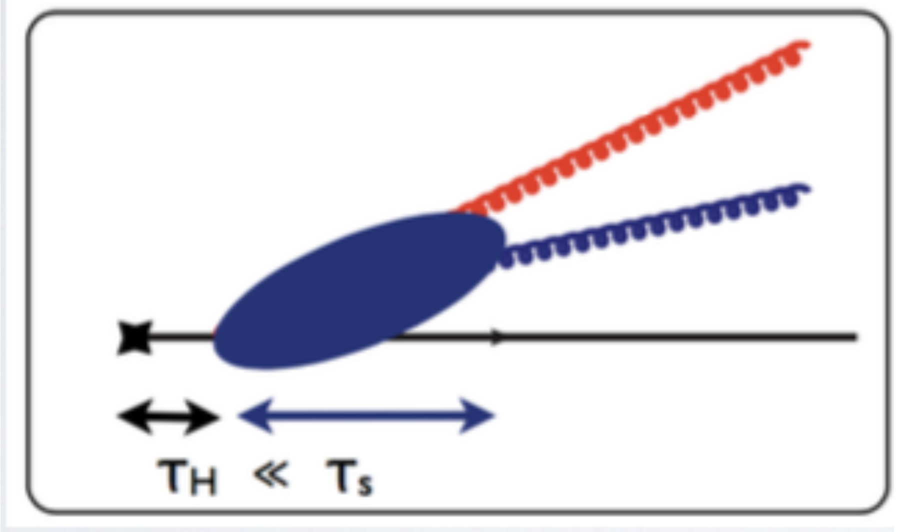
pQCD calculation in N=1 opacity (thin medium)

Provides a full characterisation of interferences
in terms of *formation times*

DP, J. Casalderrey-Solana, K. Tywoniuk 1512.07561

Two Gluon Inclusive Emission

Soft Limit: Ab Initio Antenna



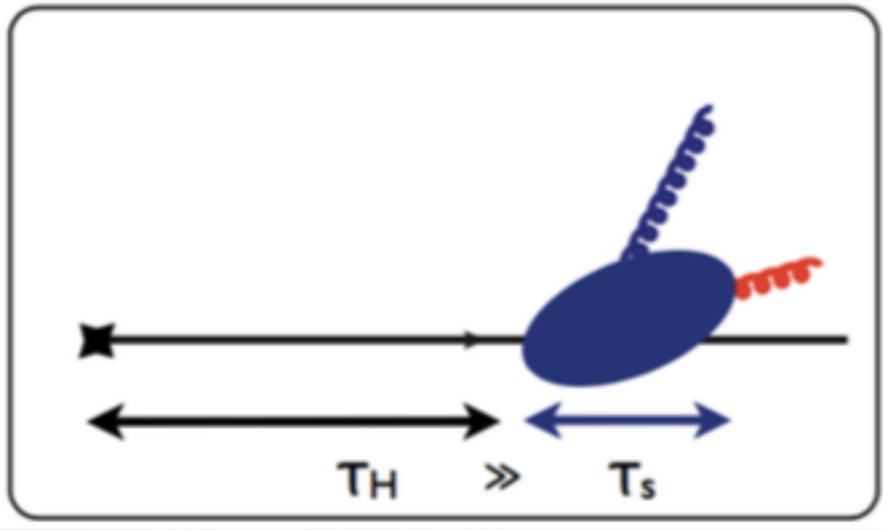
Hard gluon momentum very hard:
decouples from medium scale

Formation time of hard gluon
arbitrarily small

↓

Vacuum Hard
x
Emission off QG Antenna

Collinear Limit: Resolved Antenna



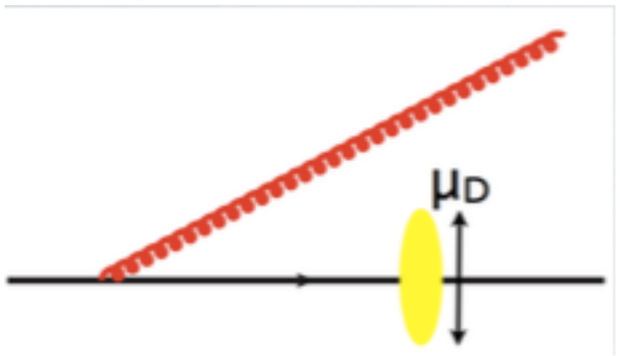
Hard gluon momentum very soft:
decouples due to destructive interferences

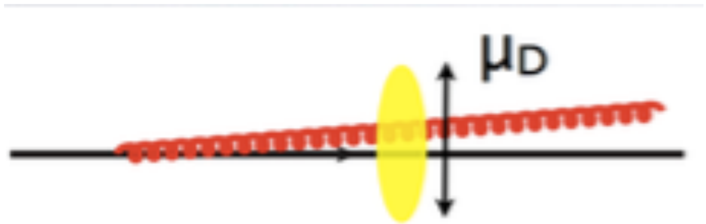
Formation time of hard gluon much longer
than any other time scale

↓

Vacuum Hard
x
Emission. off Resolved Coll. Antenna
(on shell Hard Gluon)

Two Gluon Inclusive Emission





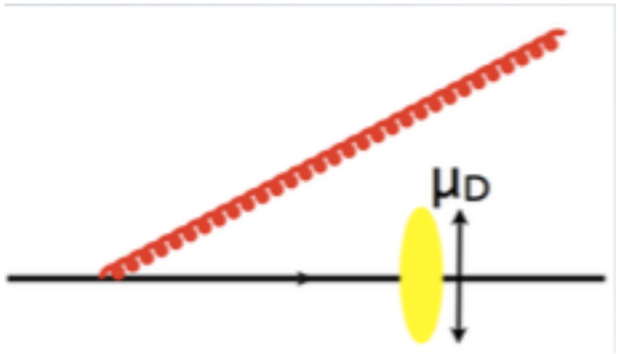
- If the antenna opening angle is larger than the emission angle:

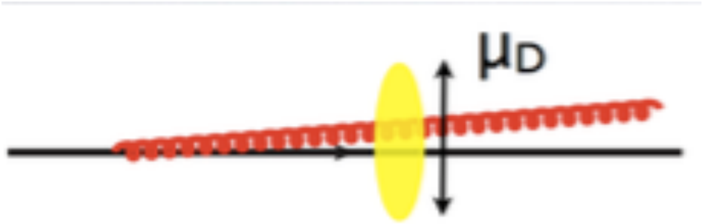
incoherent superposition of emissions off the quark and off the hard gluon

- If the emission angle is larger than the opening angle:

strong interferences

Two Gluon Inclusive Emission





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incoherent superposition of emissions off the quark and off the hard gluon

- If the emission angle is larger than the opening angle:

strong interferences

Take home messages

Partons perceived by the plasma after their formation time

Coherent multipartonic interaction with plasma due to finite resolution power

Conclusions

We have provided a calculation tool for jet quenching

Allows to learn new physics

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- testable against experiments

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- testable against experiments
- successful in a wide range of observables
- predictive

Allows to learn new physics

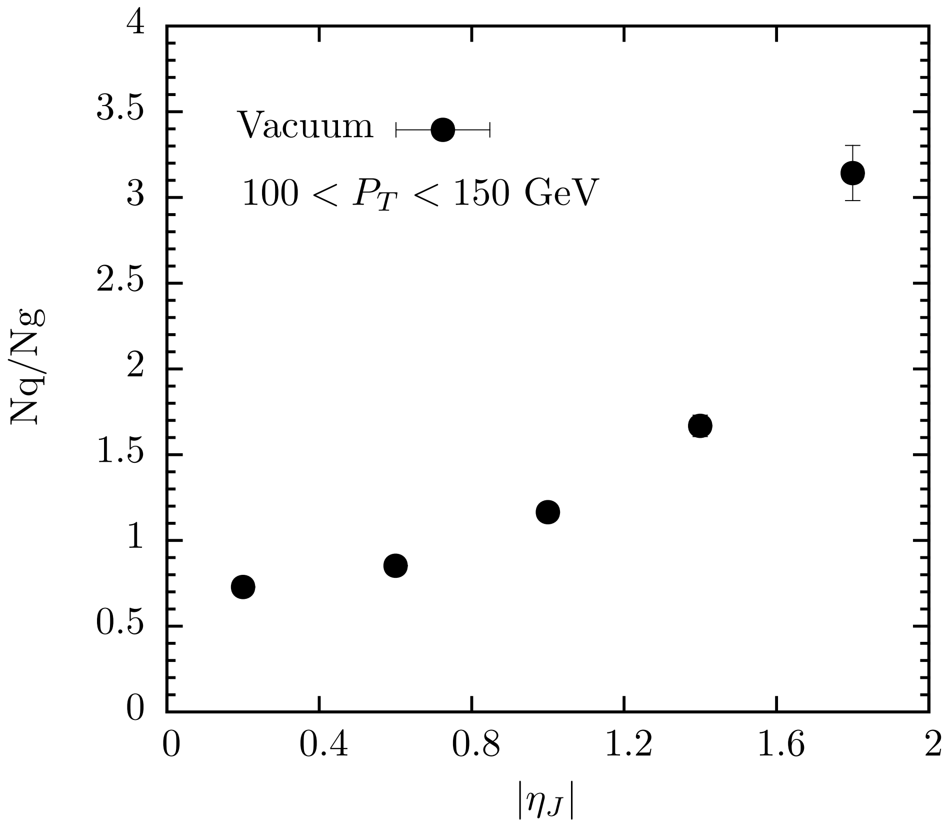
- by testing sensitivity of known observables
- by exploring new observables
- by consistently including relevant effects

Zach will tell us next about the implementation
of one such important effect

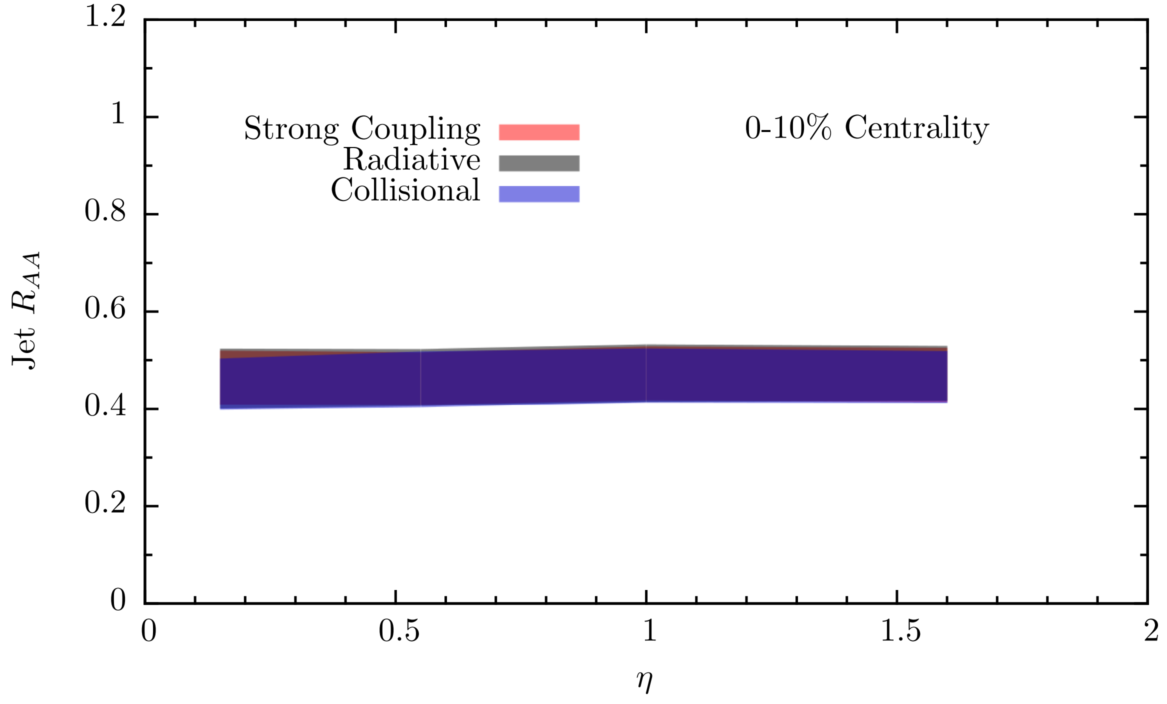
Thank you for your attention!

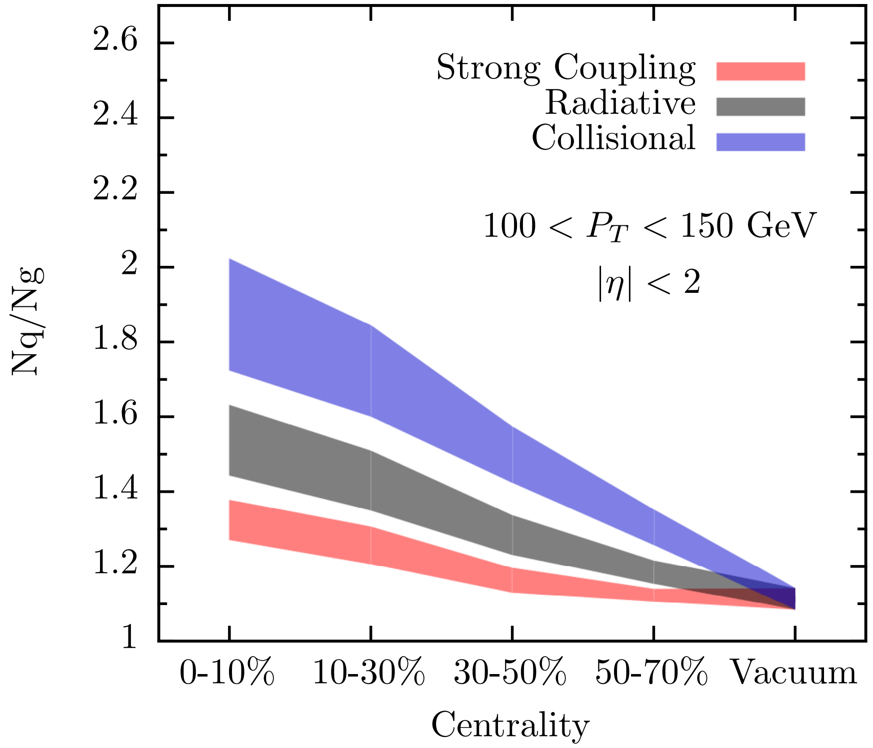
Back-Up Slides

#Quark Jets vs #Gluon Jets

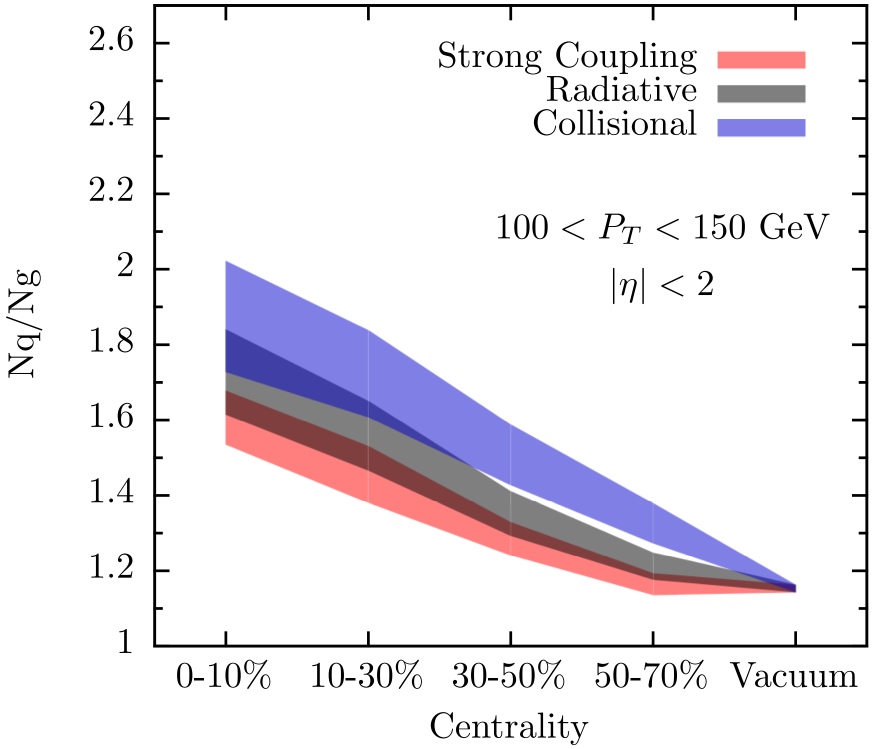


Inclusion of flow effects
preserves boost invariance

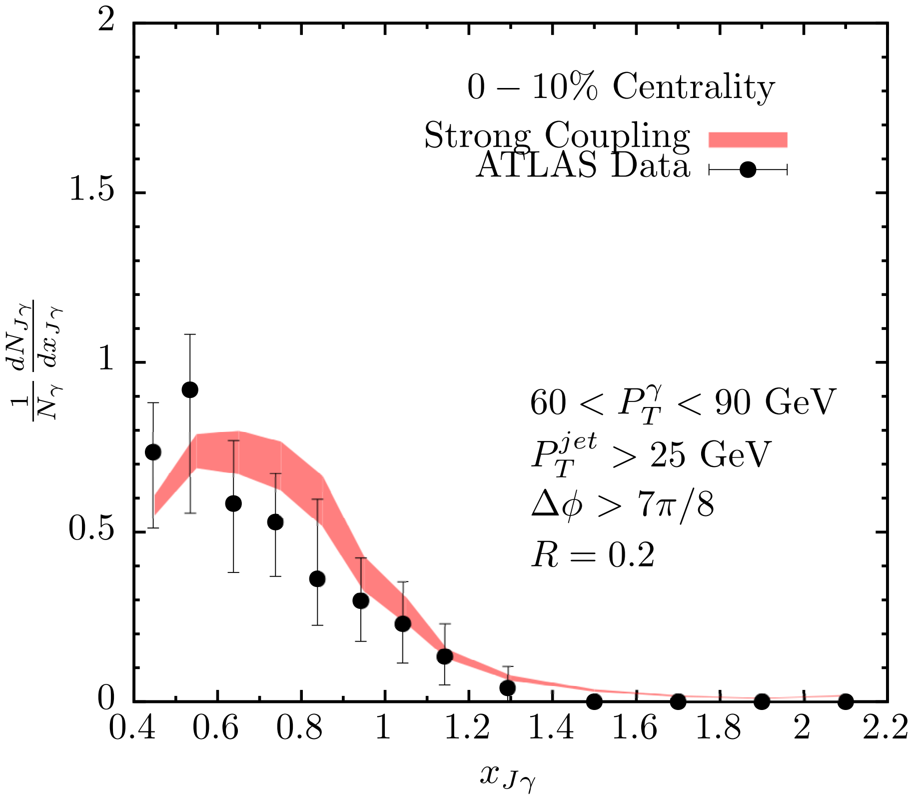


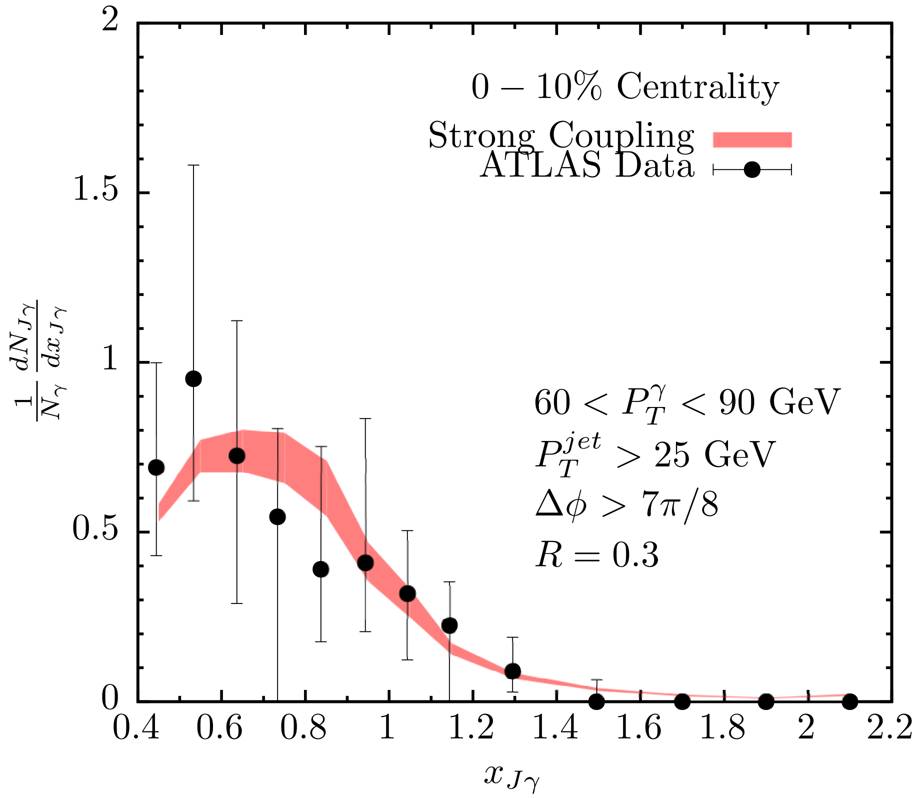


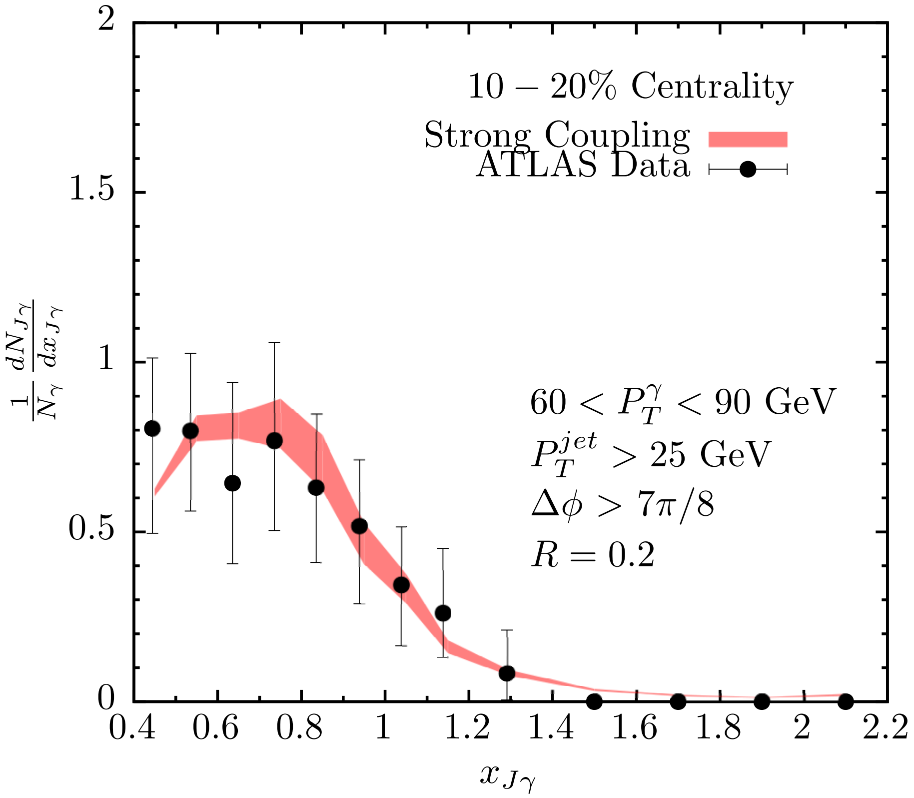
Flow

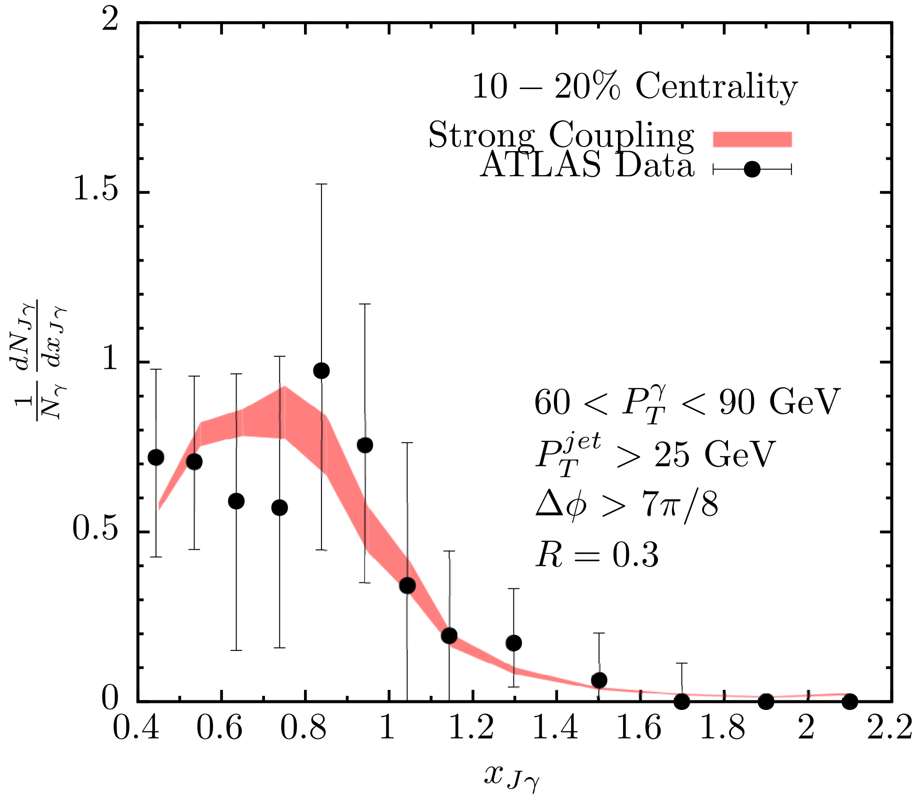


ATLAS Photon Jet Imbalance

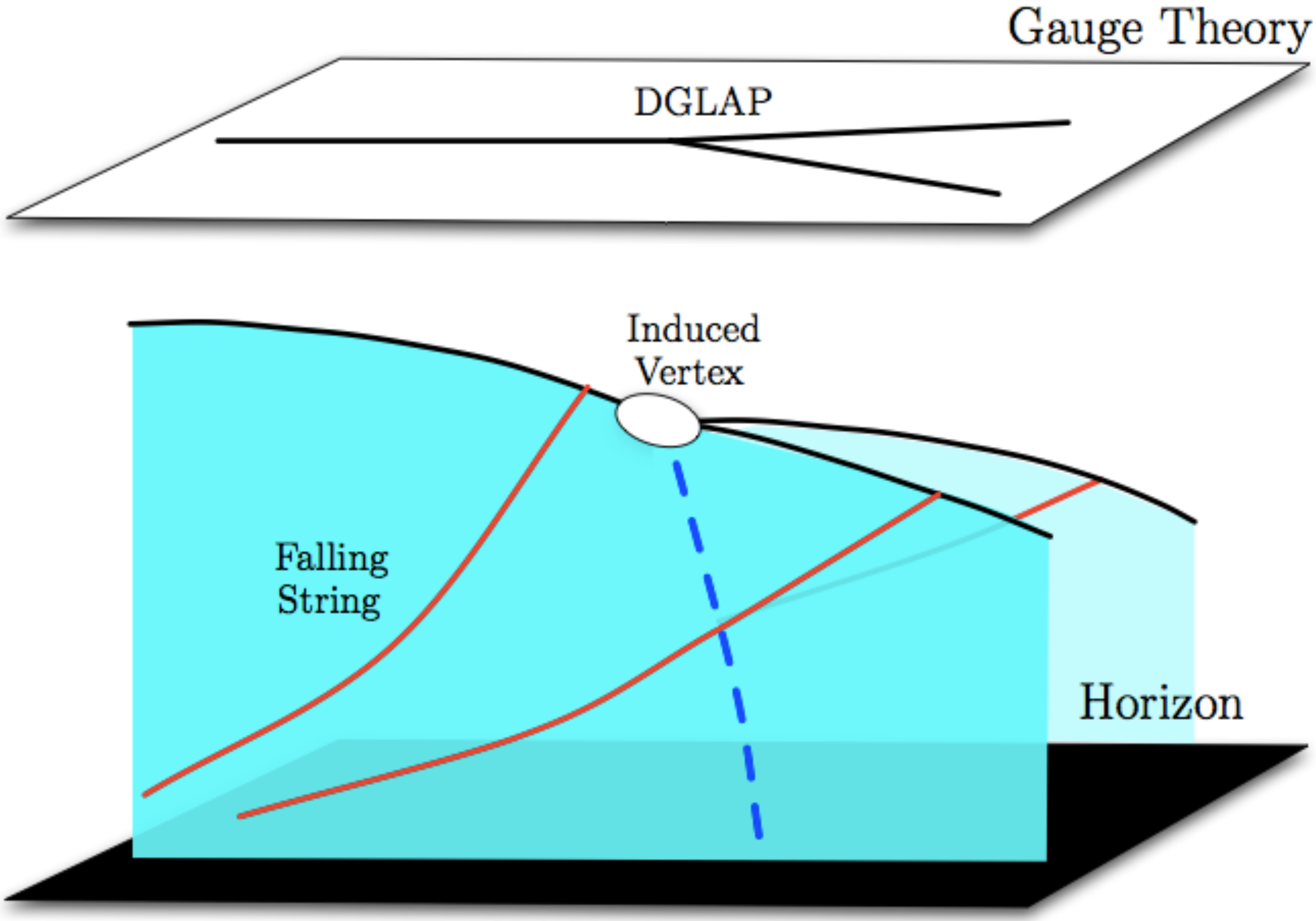




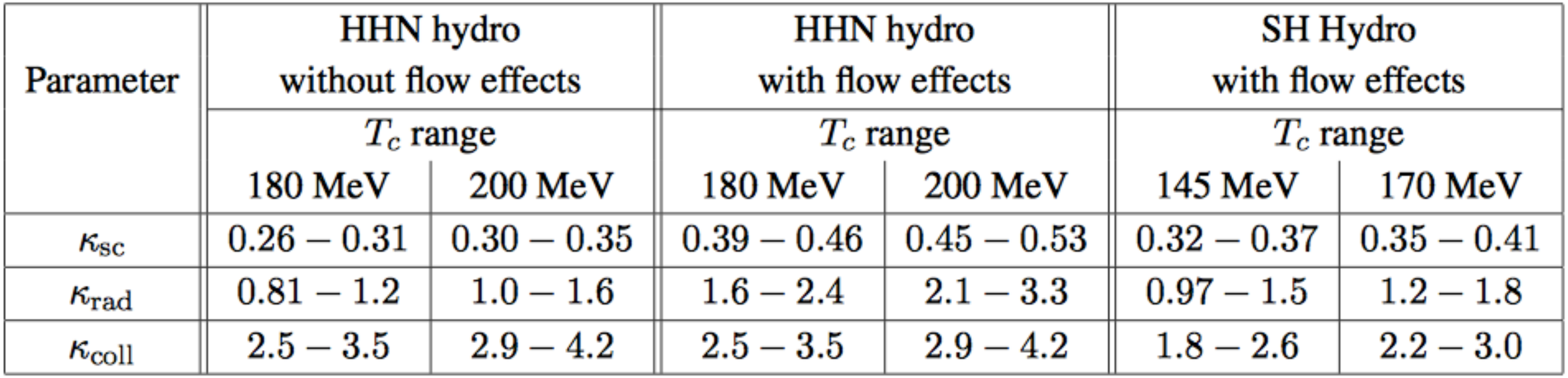




A Heuristic Picture



Parameters



Parameter	HHN hydro without flow effects		HHN hydro with flow effects		SH Hydro with flow effects	
	T_c range		T_c range		T_c range	
	180 MeV	200 MeV	180 MeV	200 MeV	145 MeV	170 MeV
$\kappa_{\rm sc}$	0.26 – 0.31	0.30 – 0.35	0.39 – 0.46	0.45 – 0.53	0.32 – 0.37	0.35 – 0.41
$\kappa_{\rm rad}$	0.81 – 1.2	1.0 – 1.6	1.6 – 2.4	2.1 – 3.3	0.97 – 1.5	1.2 – 1.8
$\kappa_{\rm coll}$	2.5 – 3.5	2.9 – 4.2	2.5 – 3.5	2.9 – 4.2	1.8 – 2.6	2.2 – 3.0

“Radiative”

$${\kappa_{\rm rad}}^{\rm pert} = 2\pi C_F C_A \left(\frac{2N_c + N_f}{6}\right) \alpha_s^3 \log B_{\rm rad} \qquad B_{\rm rad} \approx 1 + 6ET/m_D^2$$

- Large product of coupling times log
- Numerical evaluation reveals some tension
- Large logarithm corrections. Resummation?

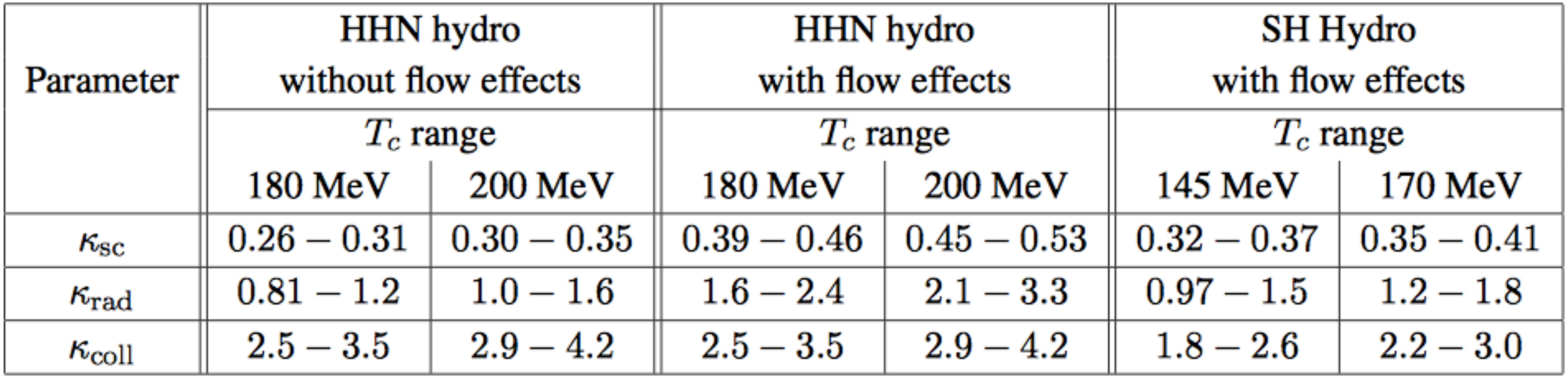
$$g > 1$$

$$m_D > T$$

Casalderrey-Solana and Wang 08, Iancu 14

Blaizot and Mehtar-Tani 14

Parameters



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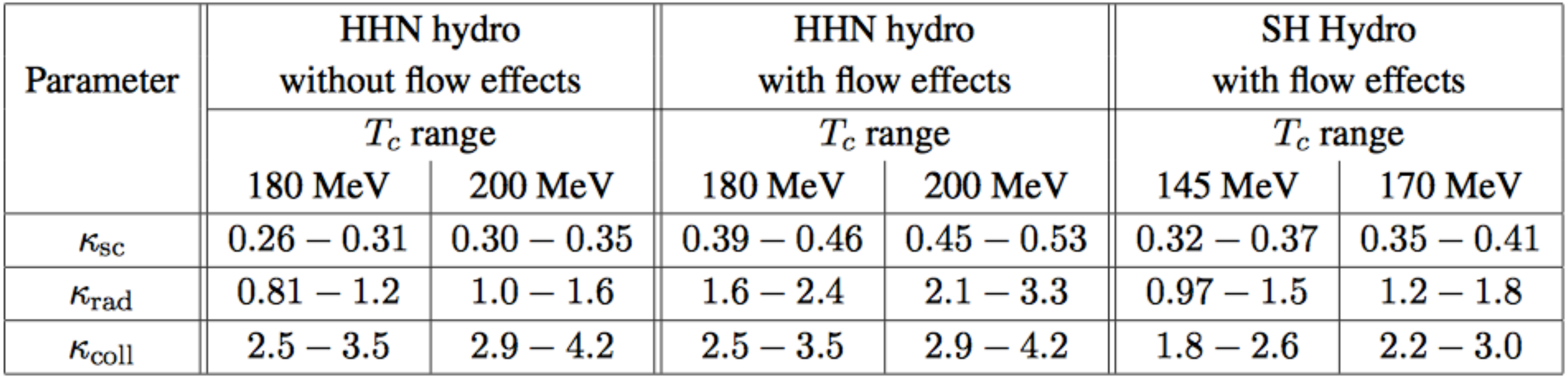
$\kappa_{\rm coll}$	$2.5-3.5$	$2.9-4.2$	$2.5-3.5$	$2.9-4.2$	$1.8-2.6$	$2.2-3.0$

Collisional

$$\kappa_{\rm coll}{}^{\rm pert} = C_F \pi \alpha_s^2 \left(\frac{2N_c + N_f}{6}\right) \log B_{\rm coll} \qquad B_{\rm rad} \approx 1 + 6ET/m_D^2$$

- Large product of coupling times log
- Too large even including log corrections
- Expected to be subdominant

Parameters



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$$x_{\rm stop} = \frac{1}{2\,\kappa_{\rm sc}} \frac{E_{\rm in}^{1/3}}{T^{4/3}}$$

Strong Coupling

Parameter is of order one as expected

$$x_{stop} \sim (3-4) x_{stop}^{\mathcal{N}=4}$$ (via semiclassical strings)

(smaller number of degrees of freedom!)

- All the difference between N=4 and QCD leads to an order one modification of the stopping distance

$\mathcal{N} = 4$ SYM at $T \neq 0$ *vs* QCD at $T > T_c$

$N_c \to \infty, \ \lambda \to \infty$

1101.0618

- Confinement scale and chiral condensate scale play no role above critical temperature
- Regime above T_c in colliders strongly coupled ($\frac{1}{\lambda}$ corrections)
- Different degrees of freedom (how do observables depend on this?)
- $N_c \to \infty$ ($\frac{1}{N_c}$ corrections)
- $0 < N_f \ll N_c$ or $N_f = 0$, but contributions from fundamental representations are important for thermodynamics above T_c
- QCD running of the coupling constant significantly non-conformal just above T_c (but increasingly conformal with higher T)

In Progress: new limit $r = R\,z, \quad z \to 0$

Both gluons can be medium induced $\frac{q}{k_H} = \tilde{q}\,\frac{z}{r}$

New time scales interplay accessible $\frac{\tau_H}{\tau_q} - \frac{\tau_H}{\tau_g} = \mathcal{O}\left(\frac{z}{r}\right)$

Quark: GLV soft + GLV hard + ??? (hard gluon time)

Gluon: $\mathrm{GB}\left[(1-f)(1-\cos(t/\tau_1)) + f(1-\cos(t/\tau_R)) - 2f(1-f)(1-\cos(t/\tau_M))\right]$

$$\tau_H = \frac{2}{R^2 w_H z^2 \theta_S^2}$$

$$1/\tau_R = 1/\tau_g - 1/\tau_q$$

$$1/\tau_M = 1/\tau_R - 1/\tau_1$$

$$f = \frac{\mathbf{k}_H}{2w_H} \cdot \left(\mathbf{k}_H + \frac{\mathbf{k}_S + \mathbf{q}}{1+z}\right)$$

$R \to 0$
$(f \to 0)$

Rich time scales interplay
Intermediate situation between
the studied limits?

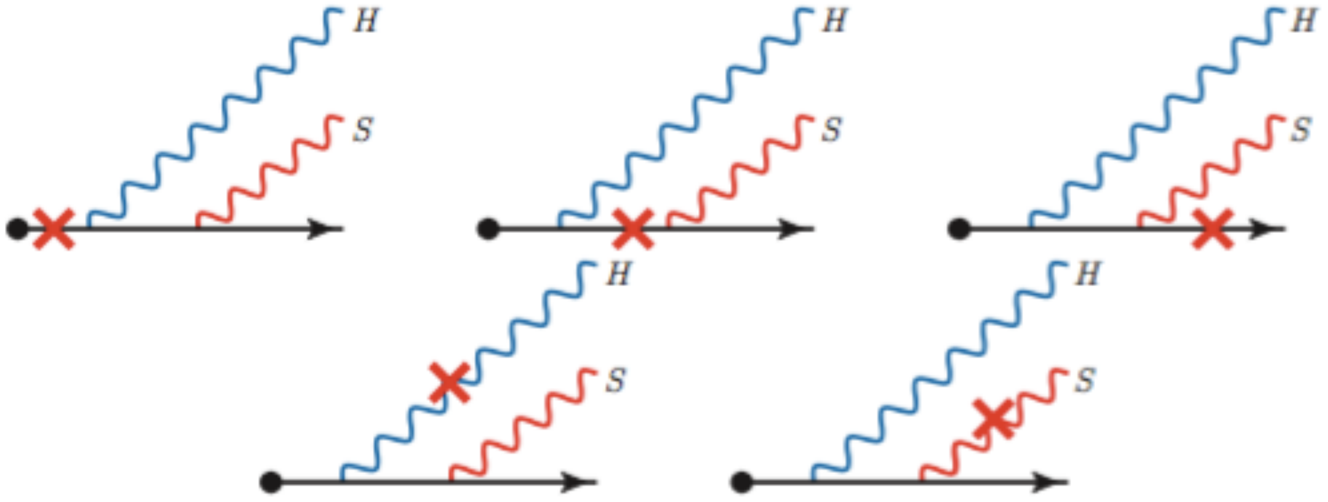
$R \to \infty$
$(f \to 1)$

Collinear Limit
(fully resolved antenna)

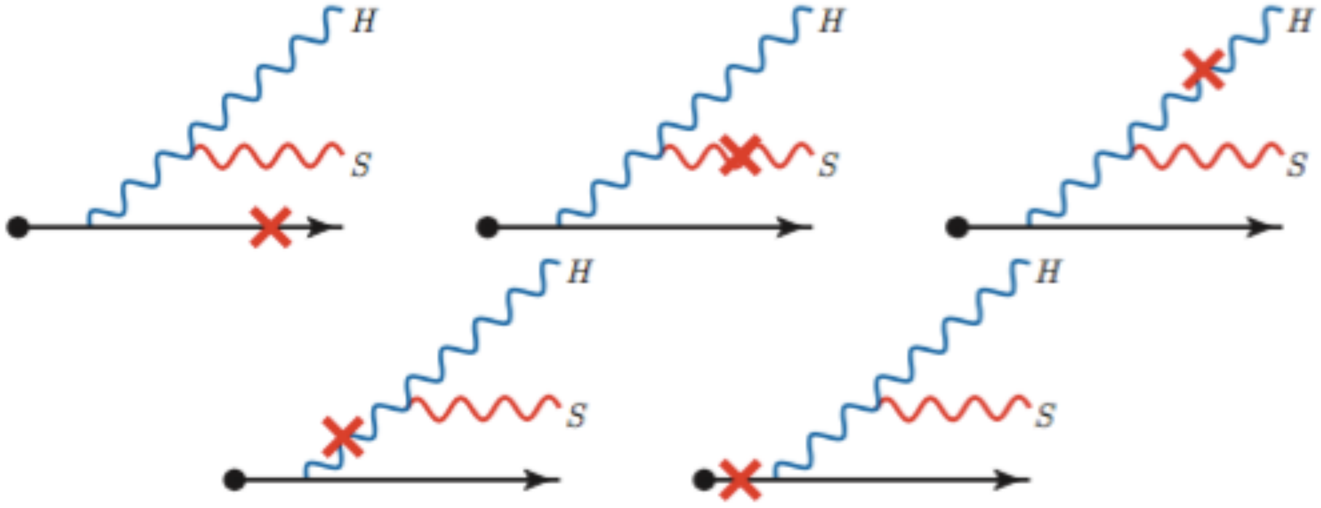
Early Antenna
in the small angle limit

stay tuned…

Diagrams Summary: Real Terms

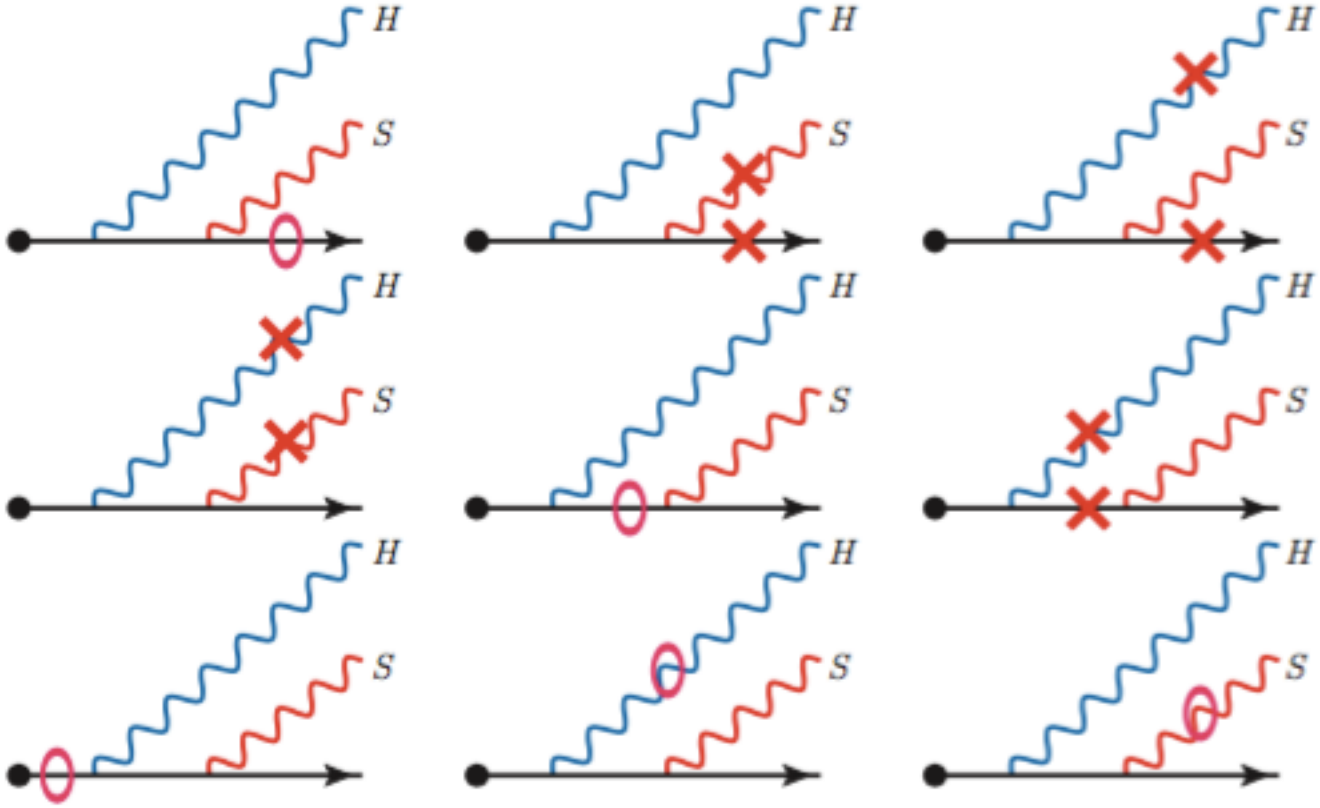


+ interchange Hard and Soft



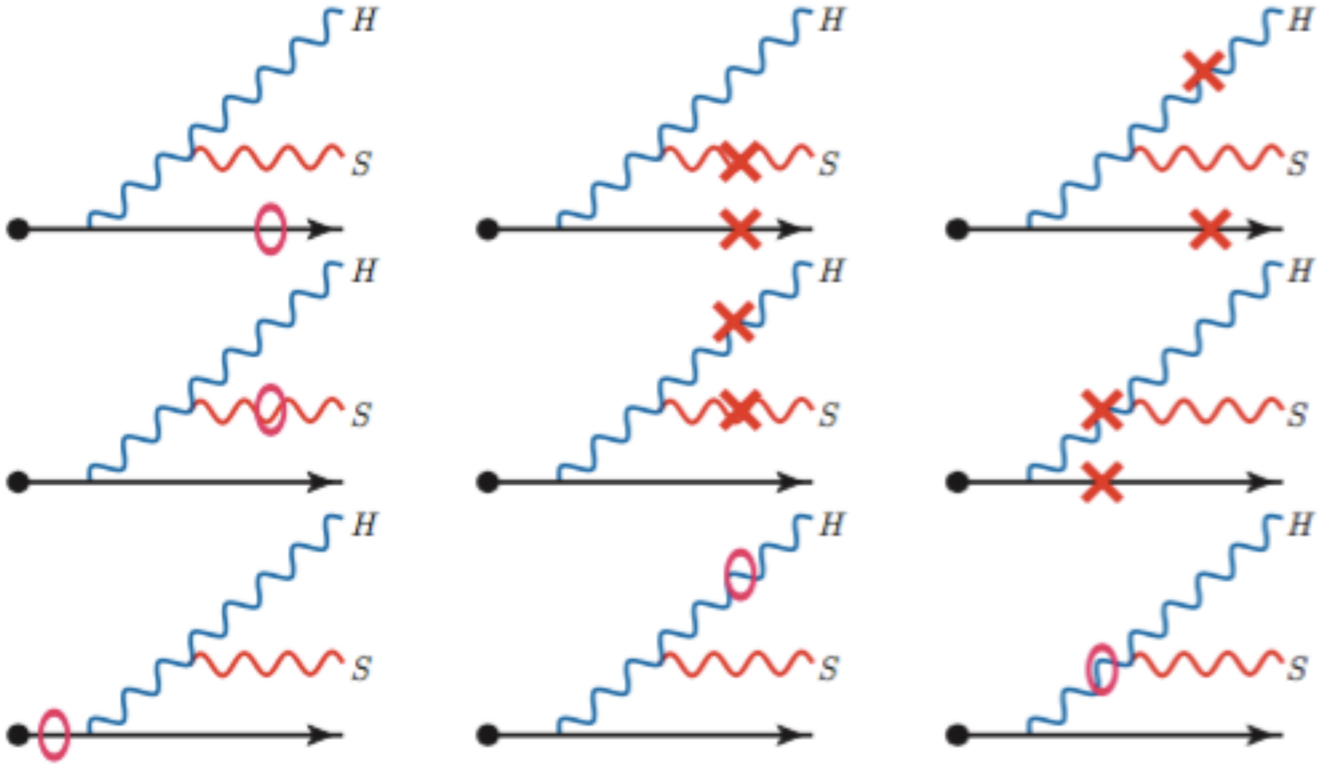
(5.2+5)(5.2+5)=225 terms

Diagrams Summary: Virtual Terms



+ interchange Hard and Soft

(9.2+9).3=81 terms



Analysis of the Induced Rate

$$\langle|\mathcal{M}_{1\mathrm{OP}}|^2\rangle = \langle|\mathcal{M}_{(1)}|^2\rangle + 2\mathrm{Re}\langle\mathcal{M}_{(2)}\mathcal{M}^*_{(0)}\rangle$$

Real contrib. Virtual contrib.

$$\frac{\mathrm{d}^2 N_{1\mathrm{OP}}}{\mathrm{d}\Omega_{k_H}\,\mathrm{d}\Omega_{k_S}} \equiv \frac{1}{\sigma_q^{\mathrm{Born}}}\frac{\mathrm{d}^2\sigma_{1\mathrm{OP}}}{\mathrm{d}\Omega_{k_H}\,\mathrm{d}\Omega_{k_S}} = \frac{1}{2p^+}\langle|\mathcal{M}_{1\mathrm{OP}}|^2\rangle$$

Two gluon emission rate

After averaging over colour, two different terms appear

$$w(x^+;\boldsymbol{q}) = C_F^2 C_A w_Q(x^+;\boldsymbol{q}) + C_F C_A^2 w_G(x^+;\boldsymbol{q})$$

Two gluon emission
off the quark

Hard gluon emission
off the quark which in turn
emits a soft gluon

Full answer can be written as

$$w_I(x^+;\boldsymbol{q}) = \sum_{i=1}^{N_I} \mathcal{P}_I^{(i)}(\boldsymbol{q})\left\{1 - \cos\left[x^+/\tau_I^{(i)}(\boldsymbol{q})\right]\right\}$$

$$I = Q,\, G,\, N_Q = 2 \text{ and } N_G = 19$$

Expansion Parameters

- Note: Study the answer for *any* length of the medium L; expanding prefactors and phases to different order is consistent

Ratio of energies **z**,
by assumption small

$$z = \frac{k_S^+}{k_H^+}$$

In terms of the angles, relative momentum is

$$\boldsymbol{\kappa}_S = k_S^+ \left(\theta_S \, \mathbf{n}_S - \theta_H \, \mathbf{n}_H\right)$$

Motivates introduction of variable **r**

$$r = \frac{\theta_H}{\theta_S}$$

Given that single gluon emission is dominated
by gluons with momentum of the order
of the medium scale, introduce

$$\tilde{q} = \frac{q}{k_S} = \frac{1}{z\theta_S} \frac{q}{k_H^+}$$

This introduces a non-trivial relation with
the hard gluon's momentum

$$\frac{q}{k_H} = \tilde{q} \, \frac{z}{r}$$

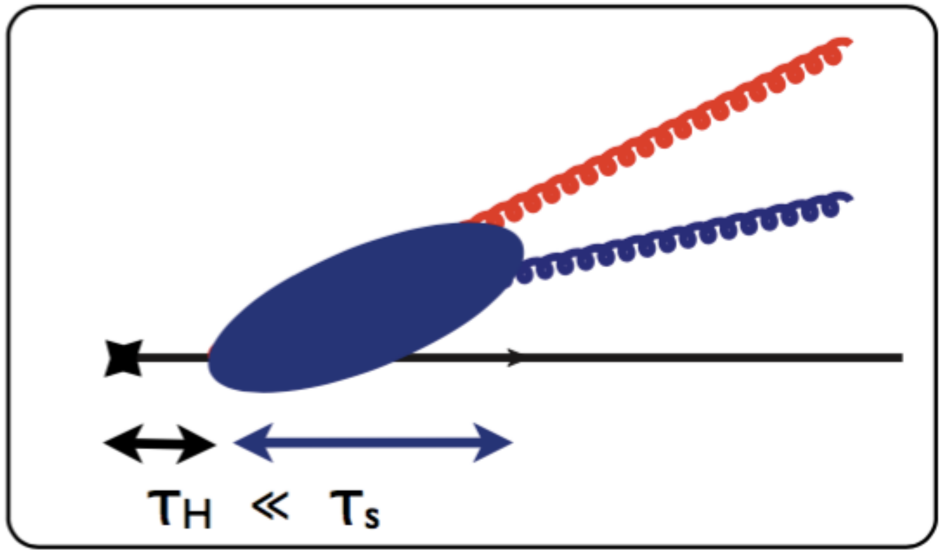
Emission rate in the Soft Limit $z \ll r$

- Hard gluon's momentum gets decoupled from the medium scale: cannot be medium induced

$z \to 0, \quad \text{with} \quad \{r,\, \tilde{q},\, \theta_S,\, k_H^+\}$ fixed.

$\frac{\tau_H}{\tau_S} = \frac{z}{r^2}$ with $\tau_H = 2k_H^+/\boldsymbol{k}_H^2$ and $\tau_S = 2k_S^+/\boldsymbol{k}_S^2$

being the vacuum formation times



Strong ordering of formation times:
hard gluon emitted arbitrarily
close to the hard vertex

Quark: $$w_Q(x^+;\boldsymbol{q}) = \boxed{\frac{4g^2}{\boldsymbol{k}_H^2}} \times \boxed{(-8g^4)\,\frac{\boldsymbol{k}_S\cdot\boldsymbol{q}}{(\boldsymbol{k}_S+\boldsymbol{q})^2\,\boldsymbol{k}_S^2}\left\{1-\cos\left[\frac{(\boldsymbol{k}_S+\boldsymbol{q})^2}{2k_S^+}x^+\right]\right\}}$$

Hard gluon
vacuum emission

Soft gluon induced
N=1 spectrum

Define $\boldsymbol{A}_q = \frac{\boldsymbol{k}_S+\boldsymbol{q}}{(\boldsymbol{k}_S+\boldsymbol{q})^2}, \quad \boldsymbol{B}_q = \frac{\boldsymbol{k}_S}{\boldsymbol{k}_S^2}, \quad \boldsymbol{L}_q = \boldsymbol{A}_q - \boldsymbol{B}_q$ so that $\frac{-\boldsymbol{k}_S\cdot\boldsymbol{q}}{\boldsymbol{k}_S^2(\boldsymbol{k}_S+\boldsymbol{q})^2} = \frac{1}{2}\left(\boldsymbol{L}_q^2+\boldsymbol{A}_q^2-\boldsymbol{B}_q^2\right)$

Emission rate in the Soft Limit $z \ll r$

Gluon:

$$
\begin{aligned}
w_G(x^+;\boldsymbol{q}) = \boxed{\frac{4g^2}{\boldsymbol{k}_H^2}} \times 4g^4 \Bigg\{ & \left(\boldsymbol{L}_g^2 + \boldsymbol{A}_g^2 - \boldsymbol{B}_g^2 - \boldsymbol{A}_q \cdot \boldsymbol{L}_g\right) \left\{1 - \cos\left[\frac{(\boldsymbol{\kappa}_S + \boldsymbol{q})^2}{2k_S^+} x^+\right]\right\} \\
& - \boldsymbol{L}_q \cdot \boldsymbol{A}_g \left\{1 - \cos\left[\frac{(\boldsymbol{k}_S + \boldsymbol{q})^2}{2k_S^+} x^+\right]\right\} \\
& + \boldsymbol{L}_q \cdot \boldsymbol{L}_g \left\{1 - \cos\left[\left(\frac{(\boldsymbol{\kappa}_S + \boldsymbol{q})^2}{2k_S^+} - \frac{(\boldsymbol{k}_S + \boldsymbol{q})^2}{2k_S^+}\right) x^+\right]\right\} \\
& + \mathcal{C}\left(k_H^+, \boldsymbol{k}_H; k_S^+, \boldsymbol{k}_S\right) \sin\left[\frac{\boldsymbol{k}_S^2}{2k_S^+} x^+\right] \sin\left[\frac{\boldsymbol{q} \cdot \boldsymbol{k}_H}{k_H^+} x^+\right] \Bigg\} ,
\end{aligned}
$$

with $\boldsymbol{A}_g = \dfrac{\boldsymbol{\kappa}_S + \boldsymbol{q}}{(\boldsymbol{\kappa}_S + \boldsymbol{q})^2}$, $\boldsymbol{B}_g = \dfrac{\boldsymbol{\kappa}_S}{\boldsymbol{\kappa}_S{}^2}$, $\boldsymbol{L}_g = \boldsymbol{A}_g - \boldsymbol{B}_g$ $\quad$ $\tau_q = \dfrac{2k_S^+}{(\boldsymbol{k}_S + \boldsymbol{q})^2}$, $\tau_g = \dfrac{2k_S^+}{(\boldsymbol{\kappa}_S + \boldsymbol{q})^2}$

and the term with the function $\mathcal{C}\left(k_H^+, \boldsymbol{k}_H; k_S^+, \boldsymbol{k}_S\right) = -\dfrac{1}{4}\dfrac{k_S^+}{k_H^+}\dfrac{\boldsymbol{\kappa}_S \cdot \boldsymbol{k}_H}{\boldsymbol{k}_H^2\, \boldsymbol{k}_S^2\, \boldsymbol{\kappa}_S{}^2}$ vanishes by construction (isotropic medium)

One concludes $\langle|\mathcal{M}_{1\mathrm{OP}}|^2\rangle\Big|_{z\ll r} = \mathcal{P}_{\mathrm{vac}}(\boldsymbol{k}_H) \times \mathcal{P}_{\mathrm{ant}}^{(1)}(\boldsymbol{k}_S)$ with $\mathcal{P}_{\mathrm{vac}}(\boldsymbol{k}_H) = \dfrac{2C_F\, g^2}{\boldsymbol{k}_H^2}$

i.e. the medium interacts with a quark-gluon antenna from the start

Emission rate in the Collinear Limit $(r \to 0)$ $z \ll 1$

$$r \to 0,\ z \to 0, \quad \text{with} \quad \{\tilde{q}, \theta_S, k_H^+\} \text{ fixed}$$

Formation time of hard gluon is parametrically longer than the one of the soft gluon

same as previous limit

Quark: $$w_Q(x^+;\boldsymbol{q}) = \boxed{\frac{4g^2}{\boldsymbol{k}_H^2}} \times \boxed{(-8g^4)\frac{\boldsymbol{k}_S \cdot \boldsymbol{q}}{\boldsymbol{k}_S^2\,(\boldsymbol{k}_S+\boldsymbol{q})^2}\left\{1-\cos\left[\frac{(\boldsymbol{k}_S+\boldsymbol{q})^2}{2k_S^+}x^+\right]\right\}},$$

Gluon: $$w_G(x^+;\boldsymbol{q}) = \boxed{\frac{4g^2}{\boldsymbol{k}_H^2}} \times 4g^2\frac{q^2}{\boldsymbol{k}_S^2\,(\boldsymbol{k}_S+\boldsymbol{q})^2}\left\{1-\cos\left[\boxed{\frac{\boldsymbol{k}_H^2}{2\,k_H^+}}x^+\right]\right\}.$$

$$\tau_H = 2k_H^+/\boldsymbol{k}_H^2$$

new time scale!

The hard gluon is emitted as in vacuum: in this limit the hard gluon momentum is parametrically smaller than the medium scale. Since LPM effect makes the medium induced rate *collinear finite*, it can only come from vacuum dynamics.

If scattering centre is placed before hard gluon formation time, all radiation comes from the quark. If placed after, we also get radiation from the hard gluon with

$$\lim_{r\to 0} \boldsymbol{L}_g^2 = \frac{\boldsymbol{q}^2}{\boldsymbol{k}_S{}^2\,(\boldsymbol{k}_S+\boldsymbol{q})^2}$$ (Gunion-Bertsch; emission off an on-shell gluon) *Why?*

Emission rate in the Collinear Limit $(r \to 0)$ $z \ll 1$

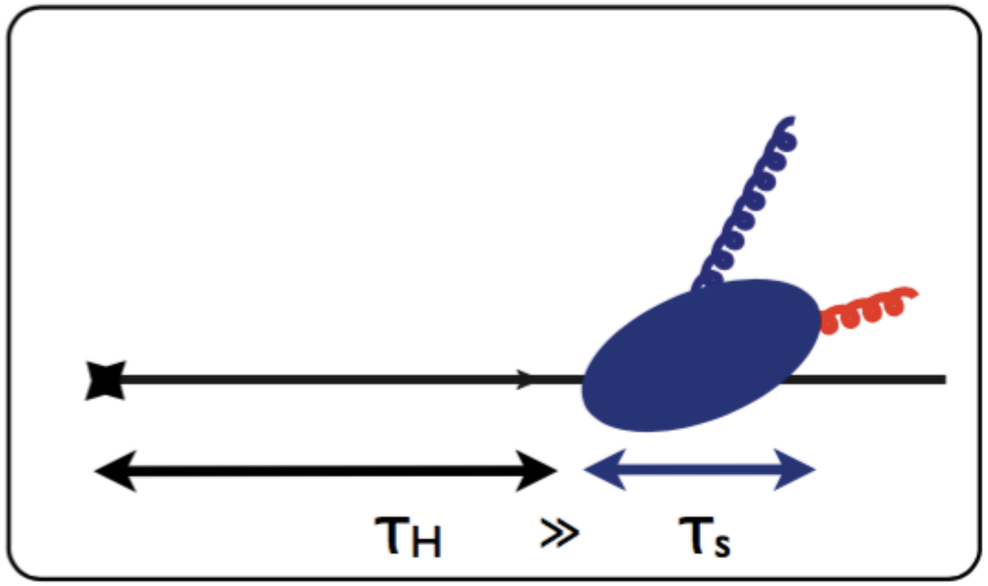
First of all look at the antenna time scales in this limit

$$\frac{\tau_H}{\tau_q} = \mathcal{O}\left(\frac{z}{r^2}\right), \quad \frac{\tau_H}{\tau_g} = \mathcal{O}\left(\frac{z}{r^2}\right), \quad \frac{\tau_H}{\tau_q} - \frac{\tau_H}{\tau_g} = \mathcal{O}\left(\frac{z}{r}\right)$$

The formation of the hard gluon is parametrically longer than the other times scales. The relevant limit for the antenna is $x^+ \to \infty$; so that all phase factors average to zero

By taking the *incoherent* and *small angle* limit of the adjoint part of the *antenna* one recovers the GB spectrum

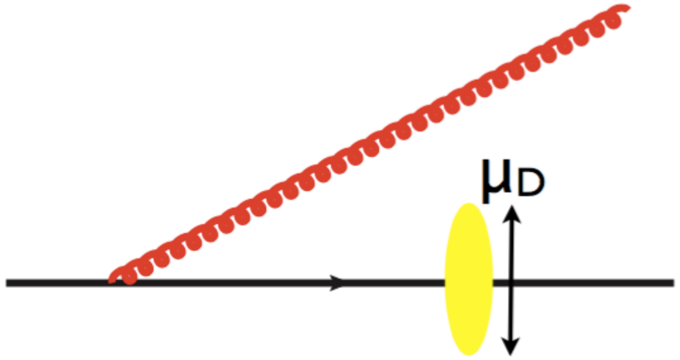
$$\lim_{r \to 0} \left(\boldsymbol{L}_g^2 + (\boldsymbol{A}_g^2 - \boldsymbol{A}_q \cdot \boldsymbol{A}_g) - (\boldsymbol{B}_g^2 - \boldsymbol{B}_q \cdot \boldsymbol{B}_g) \right) = \frac{\boldsymbol{q}^2}{\boldsymbol{k}_S{}^2 \, (\boldsymbol{k}_S + \boldsymbol{q})^2}$$



This means that in this limit, once the antenna is formed it is already *completely resolved*

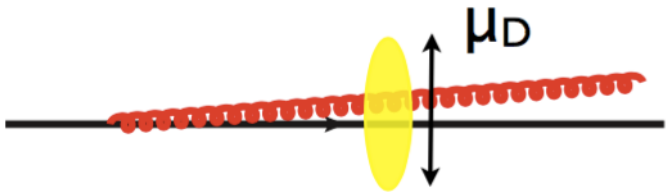
Vacuum emission cancelled by quark interferences. Can only radiate a GB at late times as a stimulated on-shell gluon

Discussion



- If the antenna opening angle is much *larger* than the emission angle, one gets the incoherent superposition of emissions off the quark and off the hard gluon

- If the antenna opening angle is much *smaller* than the emission angle one gets strong interferences



$\boldsymbol{\kappa}_S \approx \boldsymbol{k}_S \sim m_D$ $\quad$ $\boldsymbol{A}_q \approx \boldsymbol{A}_g$, $\boldsymbol{B}_q \approx \boldsymbol{B}_g$ and $\boldsymbol{L}_q \approx \boldsymbol{L}_g$

such that

$$w^{(1)}_{\rm ant}\left(x^+; \boldsymbol{k}_S, k_S^+\right)\Big|_{\theta_H \ll \theta_{\rm med}} = C_F\left[1-\cos\frac{x^+}{\tau_q}\right]\left(\boldsymbol{L}_q^2 + \boldsymbol{A}_q^2 - \boldsymbol{B}_q^2\right) + C_A\left[1-\cos\frac{x^+}{\tau_{\rm res}}\right]\boldsymbol{L}_q^2\,,$$

with the resolution time $\tau_{\rm res}^{-1} \sim m_D\theta_H$

$$\tau_{\rm res}^{-1} = \frac{1}{\tau_q} - \frac{1}{\tau_g} = \frac{2\boldsymbol{q} - \boldsymbol{k}_S - \boldsymbol{\kappa}_S}{2}\,\mathbf{n}$$

Therefore, if at the scattering time the dipole size is $\lambda = \theta_H x^+ \ll \lambda_{\rm res}$ interferences suppress emissions off the hard gluon

$$\boxed{\lambda_{\rm res} = \frac{1}{m_D}}$$

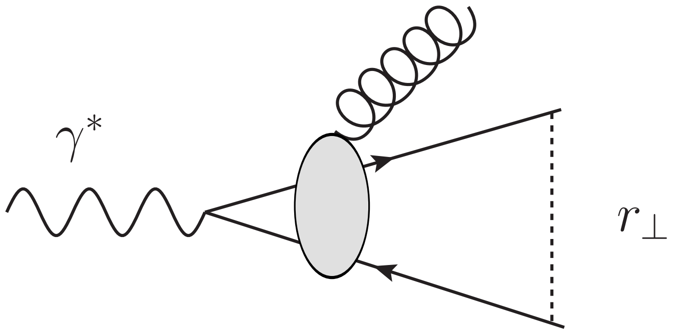
Coherence in vacuum

Heuristic interpretation

Need to think in terms of the *formation time*

Time at which the gluon decorrelates from the quark

$$\tau_f = \frac{w}{k_\perp^2} = \frac{1}{w\theta^2}$$



Transverse size of the gluon is $\lambda_\perp \sim \frac{1}{k_\perp} = \frac{1}{w\theta}$

Size of the antenna when the gluon is being emitted $r_\perp = \theta_{q\bar{q}}\tau_f = \frac{\theta_{q\bar{q}}}{w\theta^2}$

Compare the two:

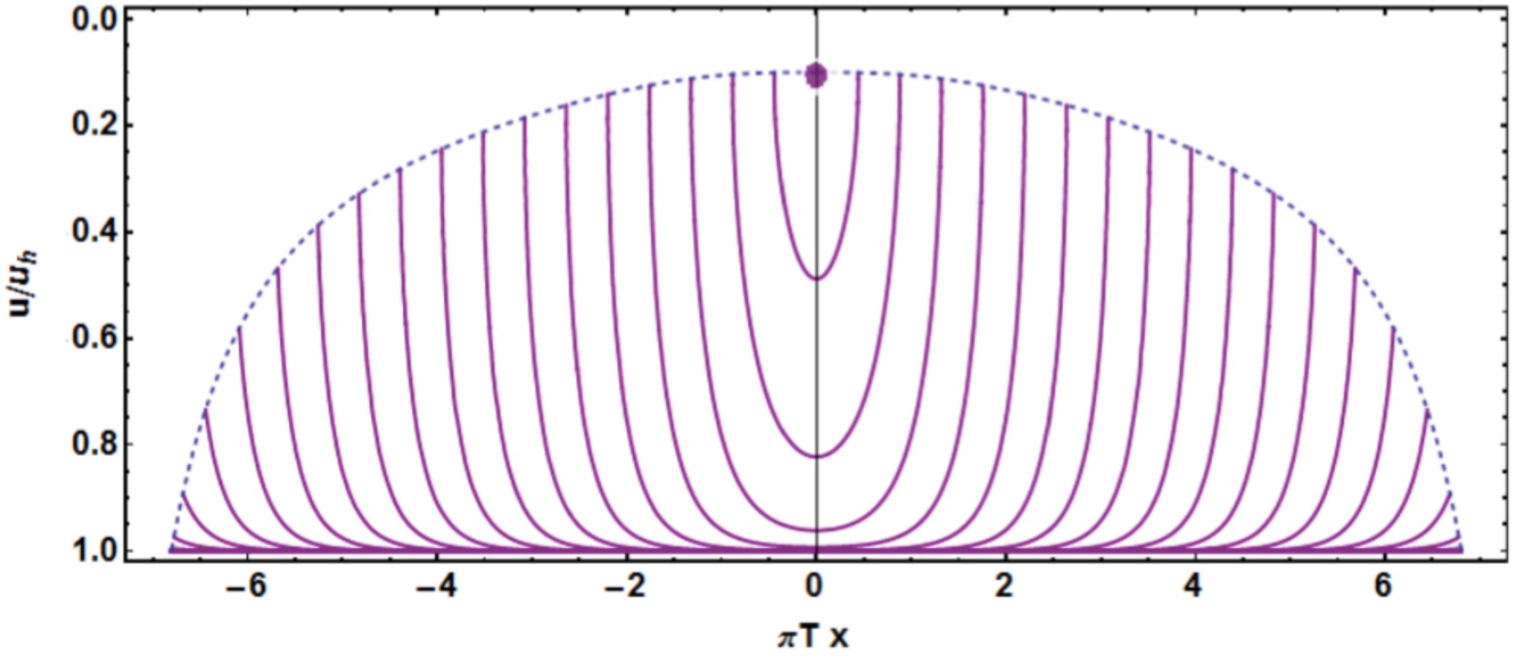
- If $r_\perp < \lambda_\perp$ the gluon cannot resolve the pair: *coherent* No emission (color singlet) $\frac{r_\perp}{\lambda_\perp} < 1 \rightarrow \theta_{q\bar{q}} < \theta_q$

- If $r_\perp > \lambda_\perp$ independent emission by quark and antiquark $\frac{r_\perp}{\lambda_\perp} > 1 \rightarrow \theta_{q\bar{q}} > \theta_q$

Casalderrey-Solana
and Ficnar 1512.00371

Strong Coupling

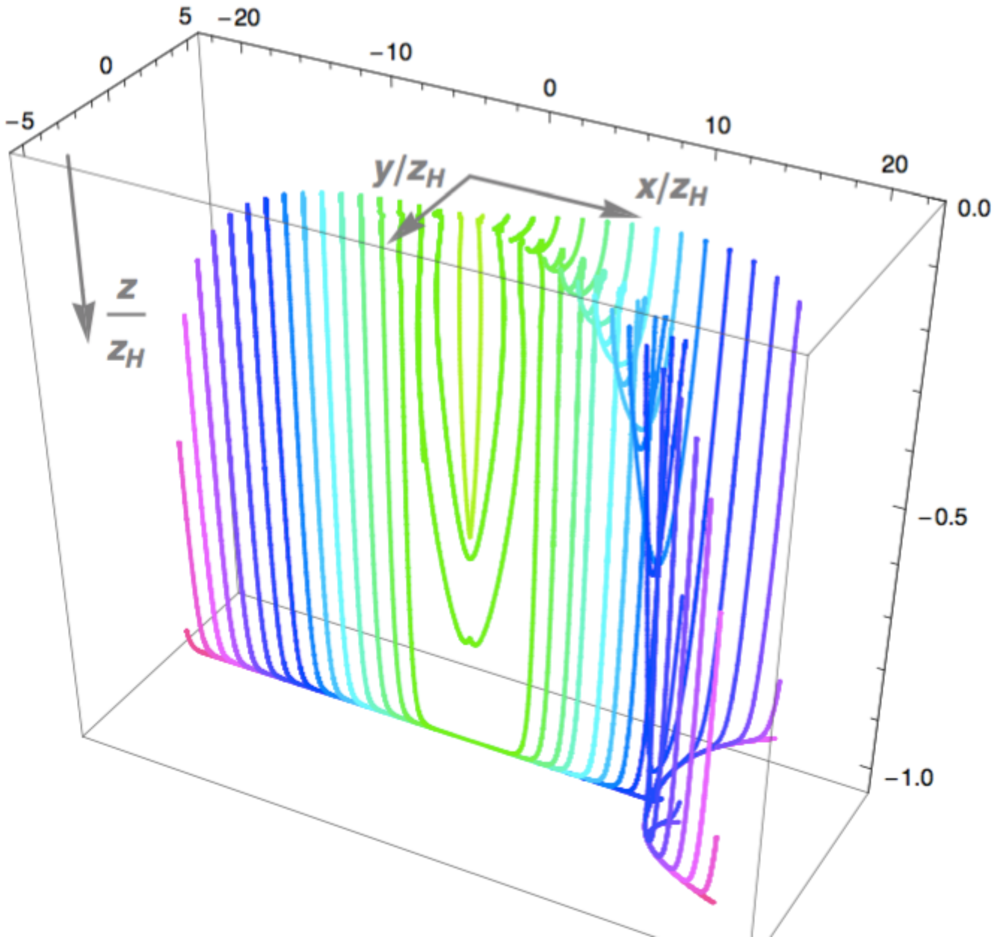
Even though there are no jets at strong coupling, one can use proxies



Light quark-antiquark pair
is dual to a string falling
in AdS(4+1) space

Plasma at T = Black hole with T

Quenching = Falling through
black hole horizon



Introduce a kink on one half of the string:
emulates a quark-gluon system

Find the angle at which the energy loss
saturates: resolution angle at strong coupling

$$\theta_{\rm res} = \frac{2^{4/3}}{\pi}\frac{\Gamma(3/4)^2}{\Gamma(5/4)^2}\left(\frac{E}{\sqrt{\lambda}T}\right)^{-2/3}$$