Phase Structure & Dynamics of QCD
a functional perspective

Hot Quarks 2016

South Padre Island, TX

Nils Strodthoff, LBNL

Deutsche Forschungsgemeinschaft

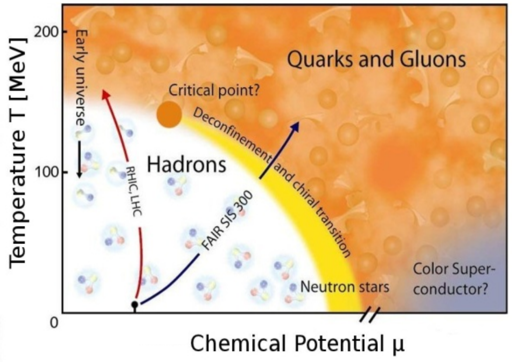
BERKELEY LAB

Fundamental challenges

1. Understanding the **phase structure of QCD** from first principles

    Phase structure at large chemical potentials largely unknown due to **sign problem** in lattice QCD…

    Existence and location of the critical point?



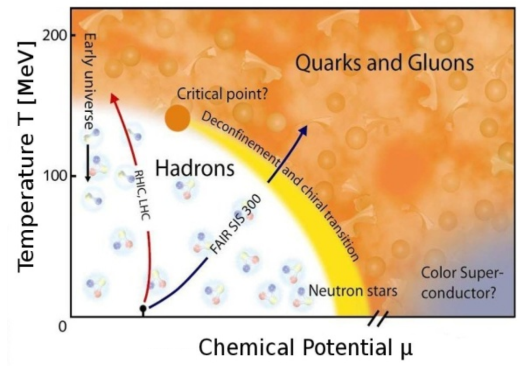
➢ adapted from GSI

Fundamental challenges

1. Understanding the **phase structure of QCD** from first principles

Phase structure at large chemical potentials largely unknown due to **sign problem** in lattice QCD…

Existence and location of the critical point?

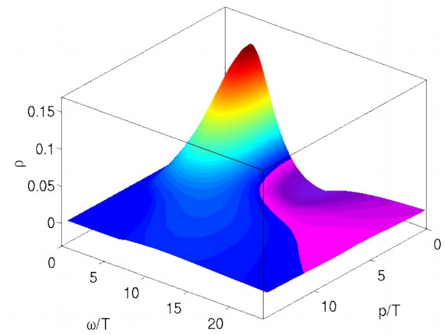


➢ adapted from GSI

2. Understanding the **fundamental properties** of strongly interacting matter from its **microscopic description**

Hadron spectrum
pole masses, decay constants, form factors, scattering amplitudes,...

Realtime observables
elementary spectral functions, transport coefficients...



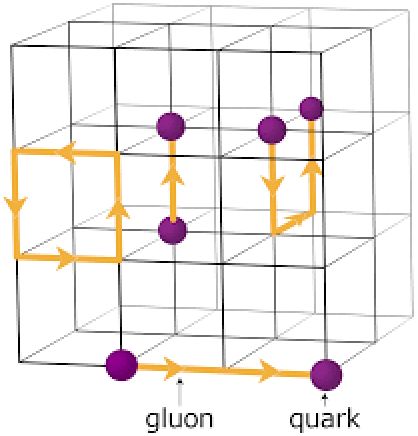
➢ Haas, Fister, Pawlowski
Phys. Rev. D90 091501 (2014)

Difficult to obtain in Euclidean approaches due to analytic continuation

Nonperturbative approaches

Both challenges require **first-principle approaches**:

Lattice QCD

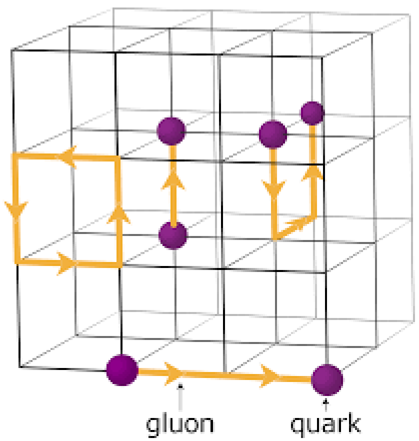


jicfus.jp

Nonperturbative approaches

Both challenges require **first-principle approaches**:

Lattice QCD



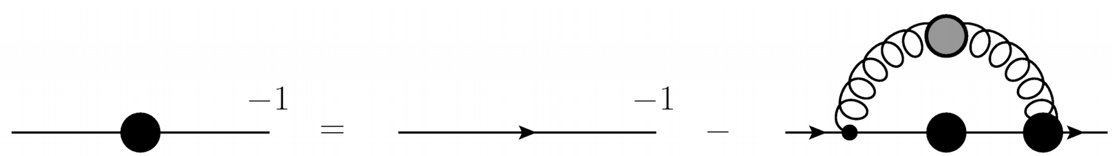
jicfus.jp

Functional approaches

- Dyson-Schwinger equations (DSE)
- n-particle irreducible methods (nPI)
- **Functional Renormalization Group (FRG)**

use relations between off-shell Green's functions

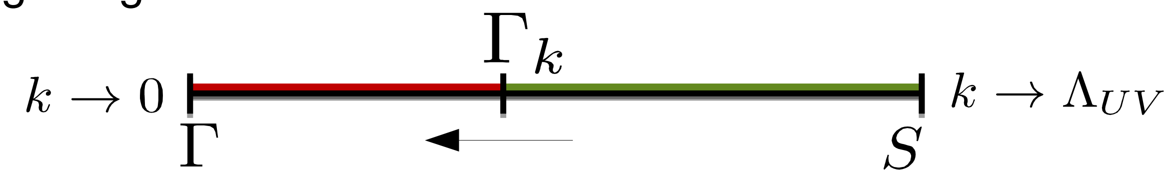
e.g. quark propagator DSE



✓ Complementary to the lattice
✓ No sign problem
✓ Calculation of realtime observables
✓ Effective models incorporated

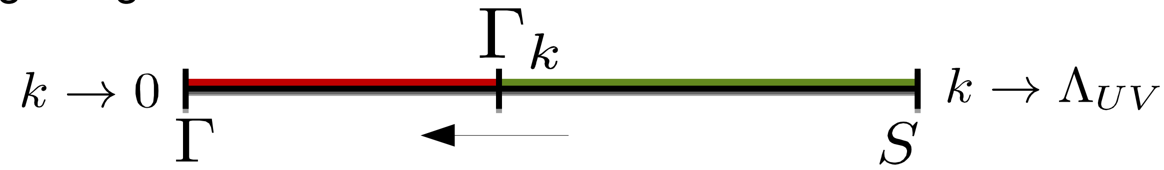
Functional RG for QCD

Spirit of **Wilson RG**: Calculate full quantum effective action by integrating fluctuations with momentum k



Functional RG for QCD

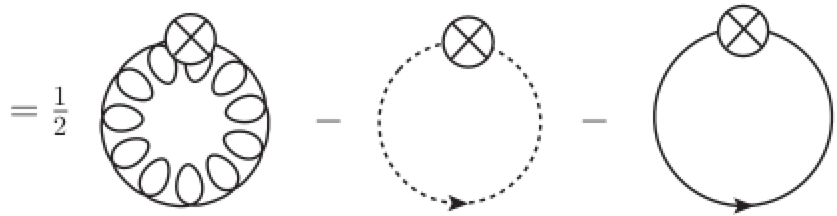
Spirit of **Wilson RG**: Calculate full quantum effective action by integrating fluctuations with momentum k



Functional Renormalization Group (FRG)

Free energy/
Grand potential

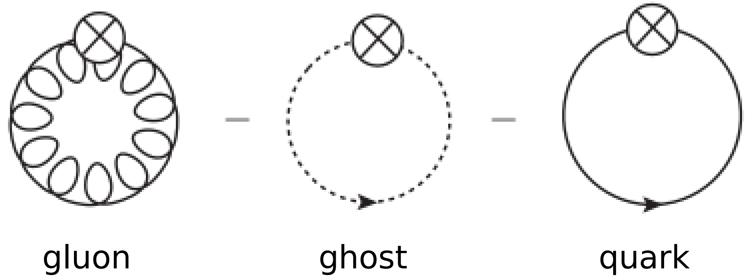
$$\left(\partial_t + \partial_t \Phi_{i,k}[\Phi] \frac{\delta}{\delta \Phi_i}\right) \Gamma_k[\Phi]$$



Dynamical Hadronization

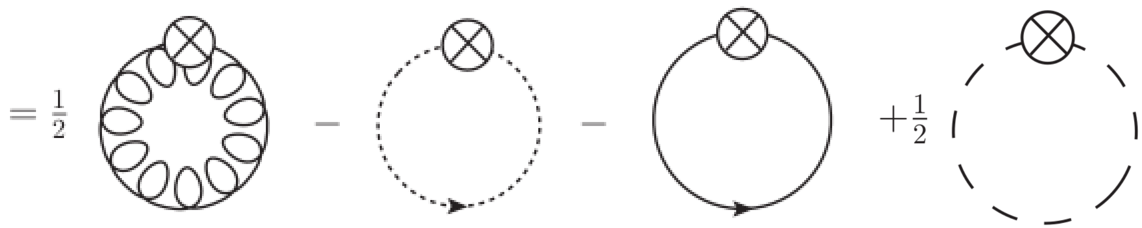
$$\left(\partial_t + \partial_t \Phi_{i,k}[\Phi]\frac{\delta}{\delta \Phi_i}\right)\Gamma_k[\Phi]$$

$= \frac{1}{2}$



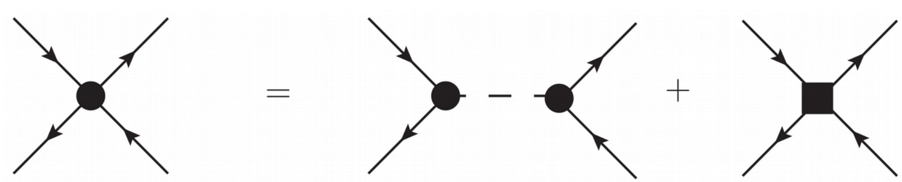
Dynamical Hadronization

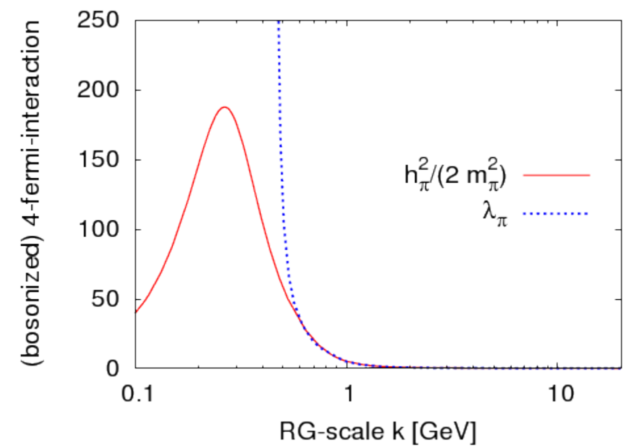
$$\left(\partial_t + \partial_t \Phi_{i,k}[\Phi] \frac{\delta}{\delta \Phi_i}\right) \Gamma_k[\Phi]$$



Dynamical hadronization
Store resonant 4-Fermi structures in terms of effective mesonic interactions

- Gies, Wetterich Phys.Rev. **D65** (2002) 065001





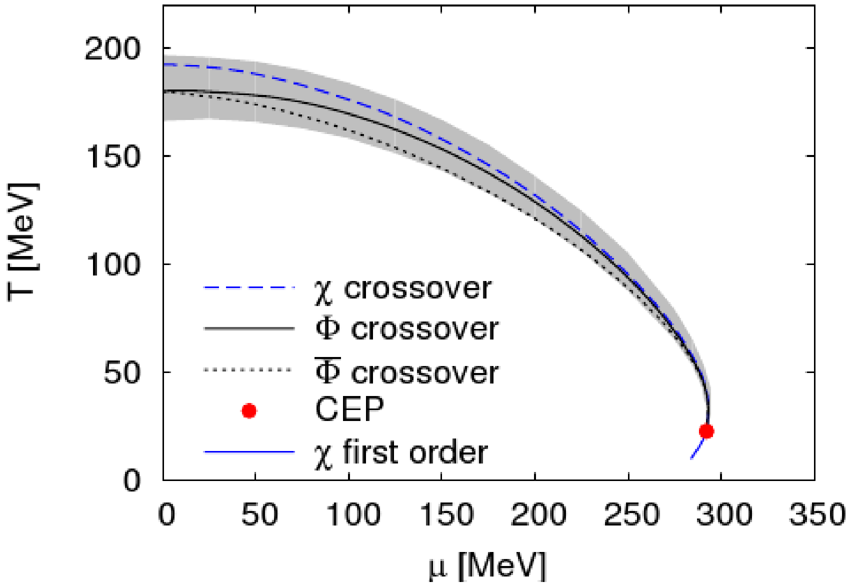
Efficient bookkeeping- no double counting

QCD Phase Structure

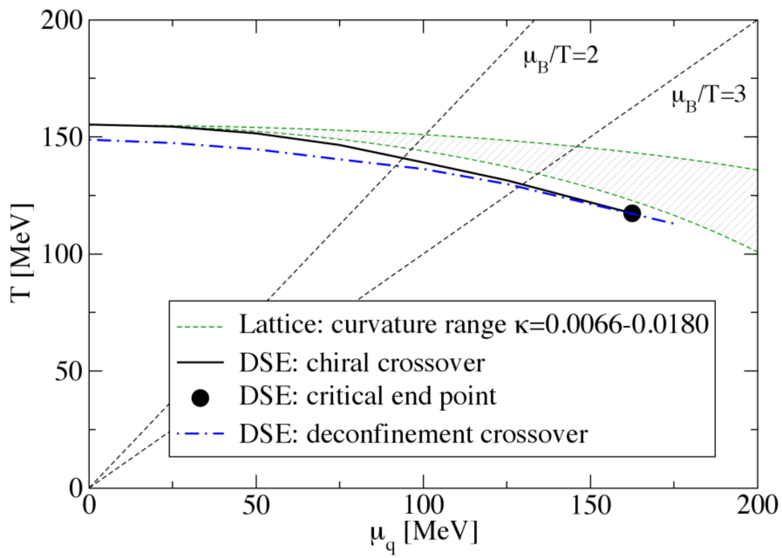
- Mitter, Pawlowski, NSt Phys.Rev. **D91** (2015) 054035
- Cyrol, Fister, Mitter, Pawlowski, NSt Phys.Rev. **D94** (2016) no.5, 054005

Functional methods at T,μ>0

PQM model, Nf=2, FRG



Quark propagator DSE, Nf=2+1

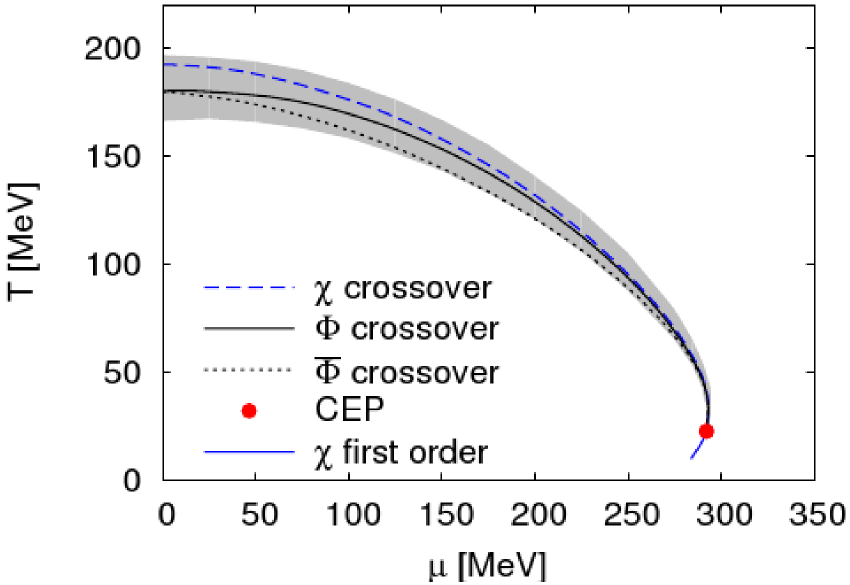


- Herbst, Pawlowski, Schaefer
  Phys.Lett. **B696** (2011)

- Fischer, Luecker, Welzbacher
  Nucl.Phys. **A931** (2014) 774-779

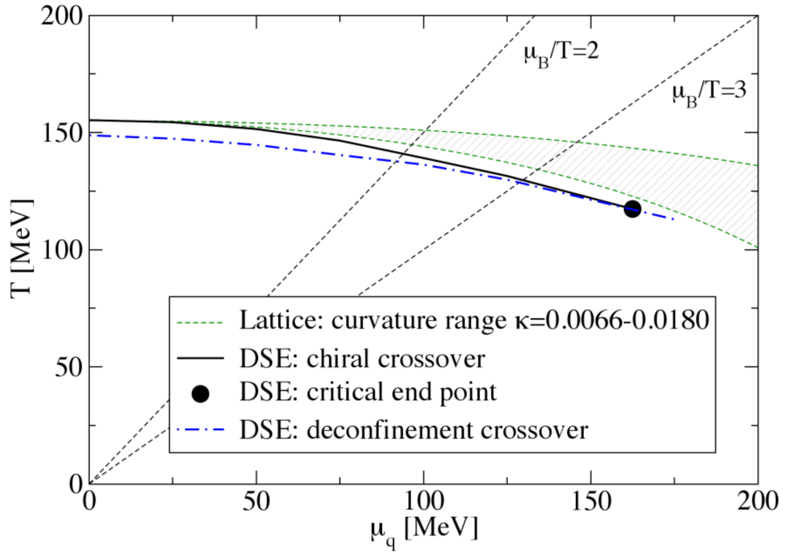
Functional methods at T,μ>0

PQM model, Nf=2, FRG



- Herbst, Pawlowski, Schaefer
  Phys.Lett. **B696** (2011)

Quark propagator DSE, Nf=2+1

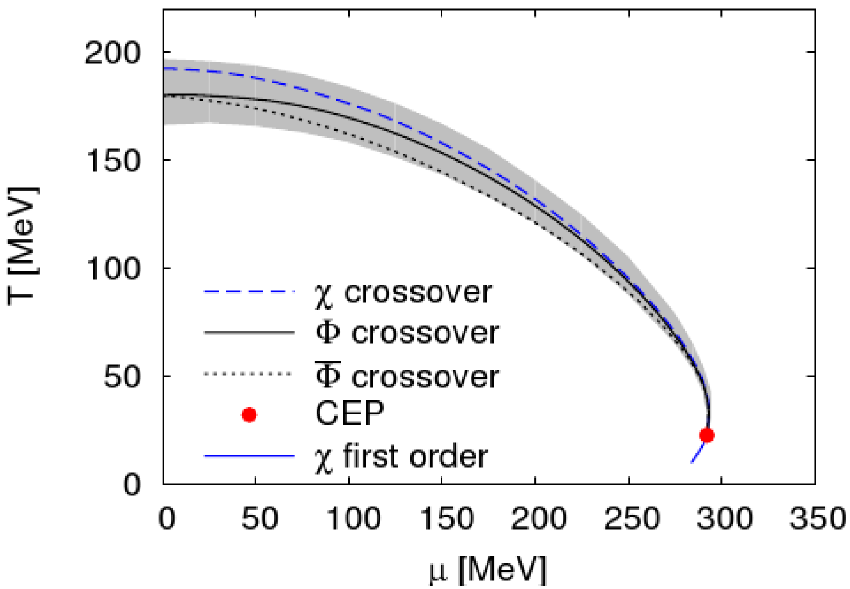


- Fischer, Luecker, Welzbacher
  Nucl.Phys. **A931** (2014) 774-779

But: so far all require additional phenomenological input

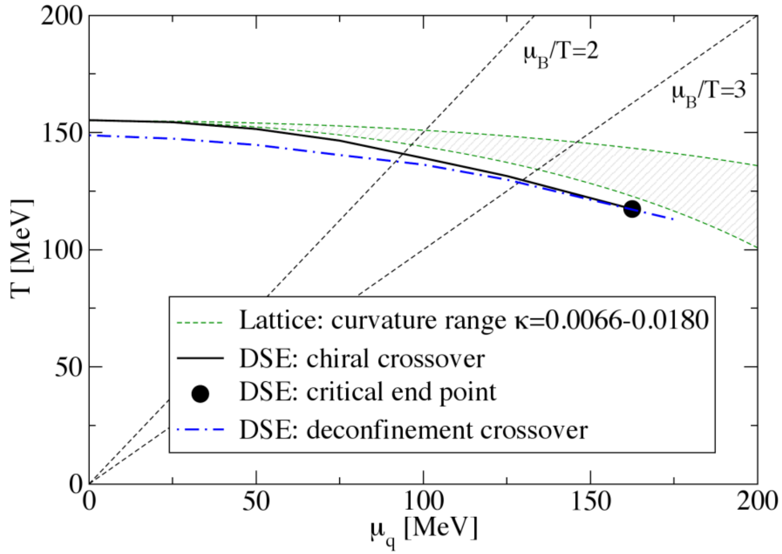
Functional methods at T,μ>0

PQM model, Nf=2, FRG



- Herbst, Pawlowski, Schaefer Phys.Lett. **B696** (2011)

Quark propagator DSE, Nf=2+1



- Fischer, Luecker, Welzbacher Nucl.Phys. **A931** (2014) 774-779

But: so far all require additional phenomenological input

Aim: Quantitative framework for continuum QCD
fundamental parameters of QCD as only input parameters

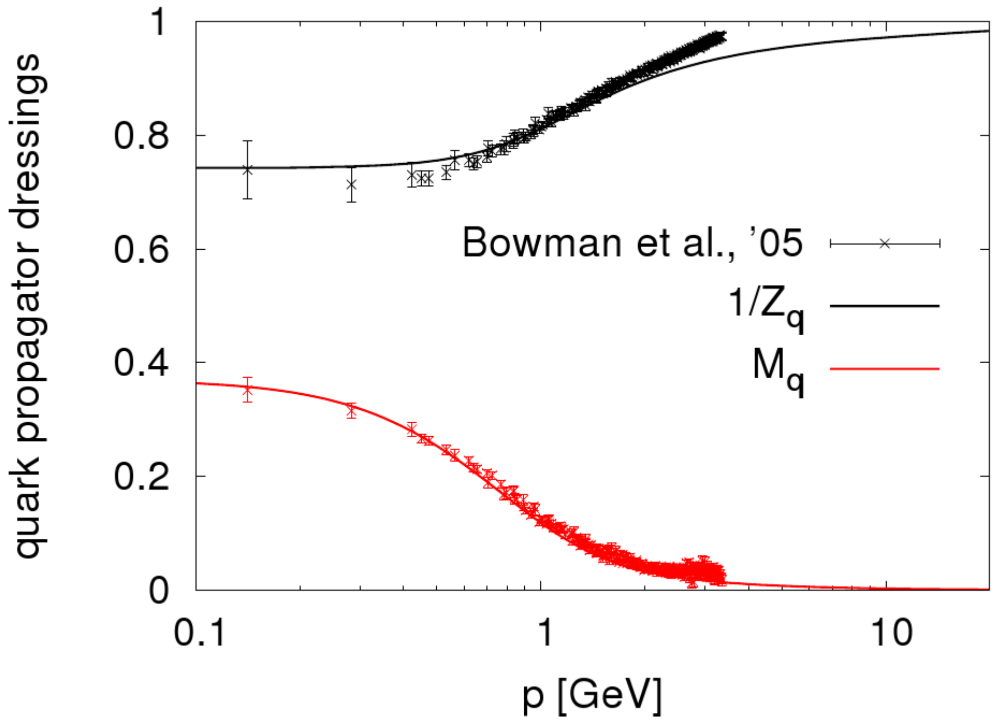
fQCD collaboration
J. Braun, L. Corell, A. K. Cyrol, L. Fister, W. J. Fu, M. Leonhardt, M. Mitter, J. M. Pawlowski, M. Pospiech, F. Rennecke, NSt, N. Wink

Quark propagator (T=0)

Quenched quark propagator

From the full matter system using quenched gluon propagator as only input

$$\Gamma_{\bar{q}q}(p) = Z_q(p)(\mathrm{i}\not{p} + M_q(p))$$



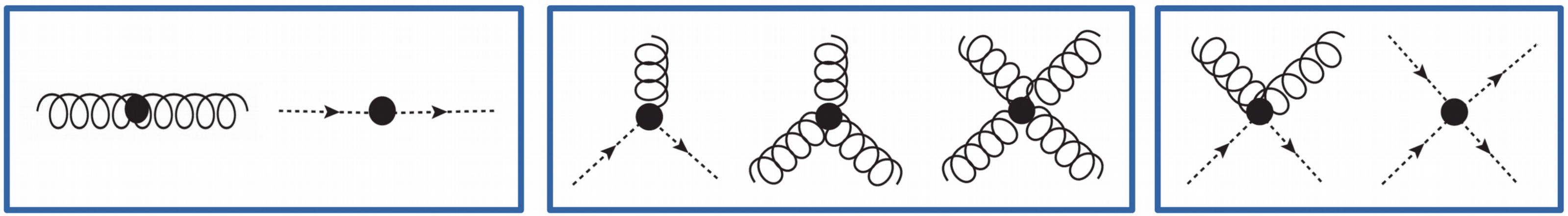
Very good agreement with (quenched) lattice results!

➢ Mitter, Pawlowski, NSt Phys.Rev. **D91** (2015) 054035

Truncation

Vertex expansion: systematic expansion in terms of 1PI vertices

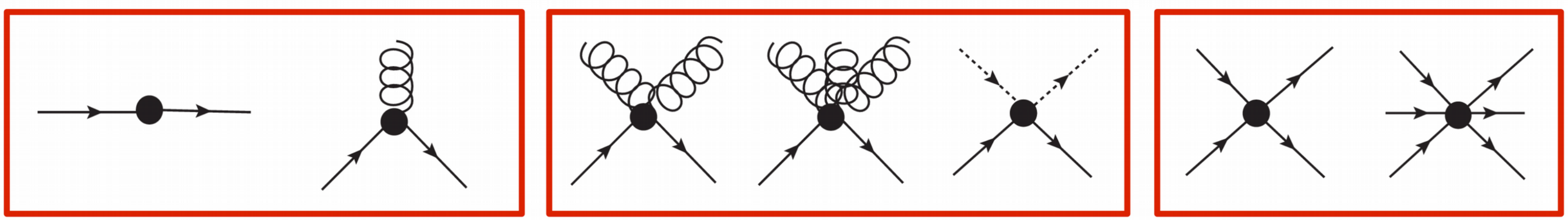
Perturbative relevance counting no longer valid



- full mom. dep.

- classical tensor structure
- mom. dep. (sym. channel)

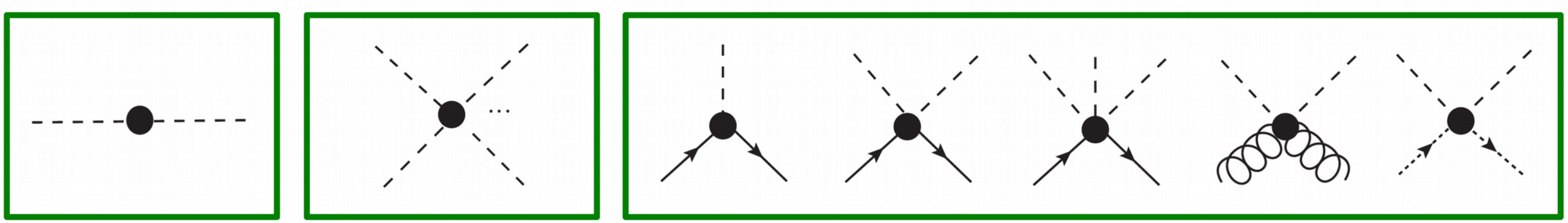
under investigation:
- full tensor structure
- mom. dep. (sym. channel)



- full tensor structure
- full mom. dep.

- partial tensor structure
- mom. dep. (sym. channel)

- full tensor structure
- mom. dep. (single channel)



- full mom. dep.

- via effective potential

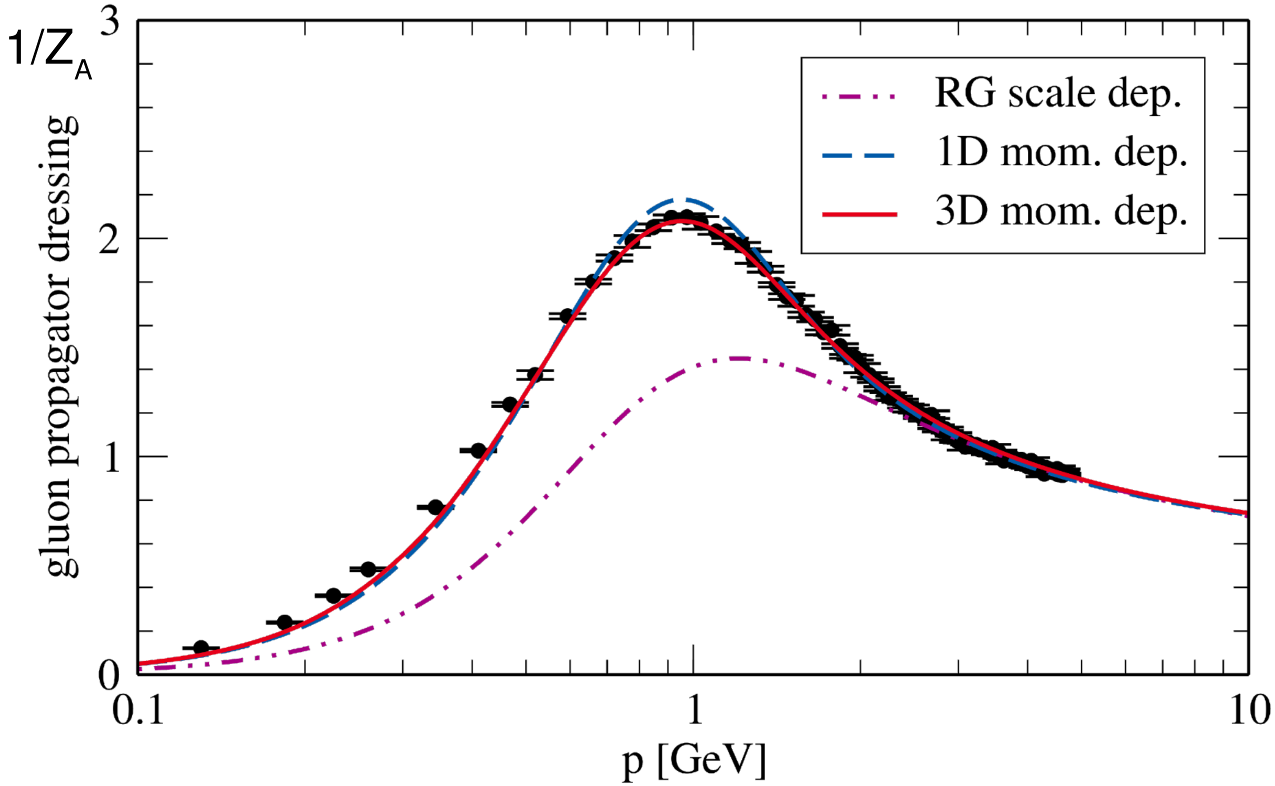
- full tensor structure
- mom. dep. (sym. channel)

Gluon propagator (T=0)

Pure YM gluon propagator

From a self-consistent solution of the transversal 2-,3- and 4-point functions

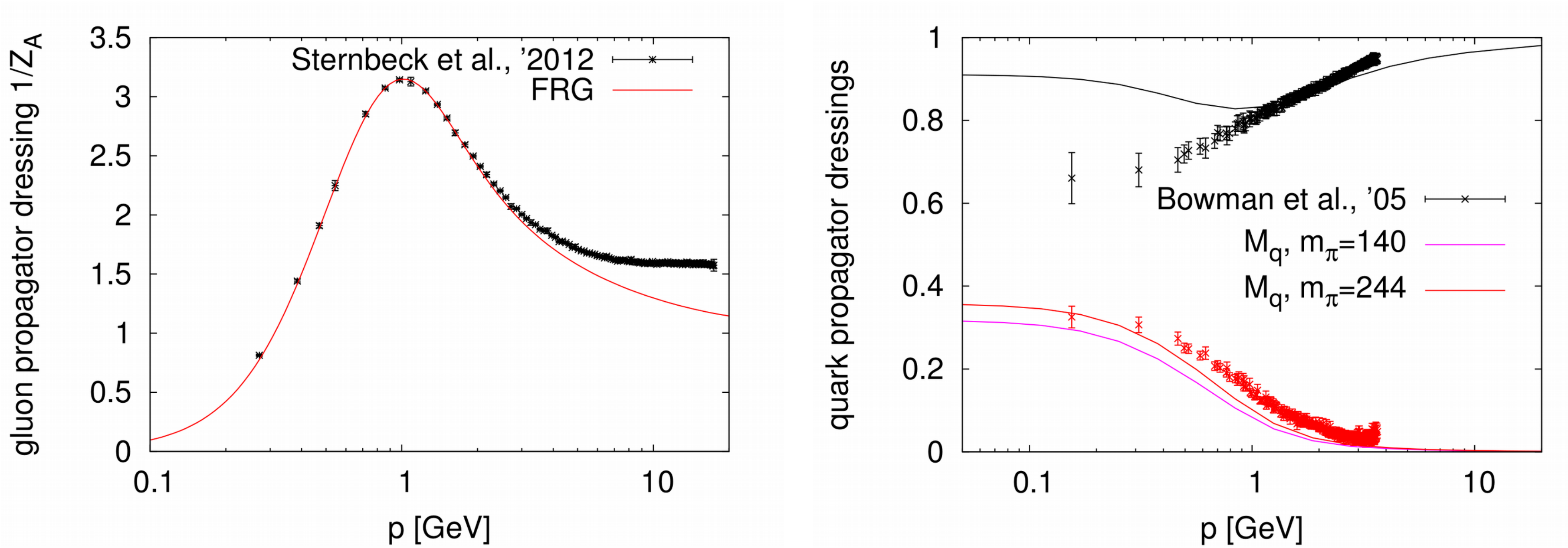
$$\Gamma^{\mu\nu}_{A^2}(p) = Z_A(p)p^2\Pi^{\mu\nu}_T(p)$$



- Cyrol, Fister, Mitter, Pawlowski, NSt Phys.Rev. **D94** (2016) no.5, 054005

Unquenched propagators

Unquenched gluon and quark propagators
From the solution of the coupled matter-glue system



➢ Cyrol, Mitter, Pawlowski, NSt in prep

- ✓ Everything in place for first quantitative results of the full system at finite T and μ
- ✓ Stay tuned for fluctuation observables...

Realtime observables

- Kamikado, NSt, von Smekal, Wambach Eur.Phys.J. **C74** (2014) 2806
- Tripolt, NSt, von Smekal, Wambach Phys.Rev. **D89** (2014) 034010
- Pawlowski, NSt Phys.Rev. **D92** (2015) 9, 094009

- Christiansen, Haas, Pawlowski, NSt PRL **115** (2015) 11, 112002

Spectral Functions

Real-time observables from Euclidean framework

$$\Gamma_R^{(2)}(\omega,\vec{p}) = -\lim_{\epsilon\to 0}\Gamma_E^{(2)}(-\mathrm{i}(\omega+\mathrm{i}\epsilon),\vec{p})$$

$$\rho(\omega,\vec{p}) = \frac{\mathrm{Im}\,\Gamma_R^{(2)}(\omega,\vec{p})}{\mathrm{Im}\,\Gamma_R^{(2)}(\omega,\vec{p})^2+\mathrm{Re}\,\Gamma_R^{(2)}(\omega,\vec{p})^2}$$

requires analytical continuation from Euclidean to Minkowski signature
numerically hard problem

Spectral Functions

Real-time observables from Euclidean framework

$$\Gamma_R^{(2)}(\omega, \vec{p}) = -\lim_{\epsilon\to 0}\Gamma_E^{(2)}(-\mathrm{i}(\omega+\mathrm{i}\epsilon), \vec{p})$$

$$\rho(\omega, \vec{p}) = \frac{\mathrm{Im}\,\Gamma_R^{(2)}(\omega,\vec{p})}{\mathrm{Im}\,\Gamma_R^{(2)}(\omega,\vec{p})^2 + \mathrm{Re}\,\Gamma_R^{(2)}(\omega,\vec{p})^2}$$

requires analytical continuation from Euclidean to Minkowski signature
numerically hard problem

Popular approaches (based on Euclidean data)

- Maximum Entropy Method (MEM)
- Padé Approximants

Spectral Functions

Real-time observables from Euclidean framework

$$\Gamma_R^{(2)}(\omega,\vec{p}) = -\lim_{\epsilon\to 0}\Gamma_E^{(2)}(-\mathrm{i}(\omega+\mathrm{i}\epsilon),\vec{p})$$

$$\rho(\omega,\vec{p}) = \frac{\mathrm{Im}\,\Gamma_R^{(2)}(\omega,\vec{p})}{\mathrm{Im}\,\Gamma_R^{(2)}(\omega,\vec{p})^2+\mathrm{Re}\,\Gamma_R^{(2)}(\omega,\vec{p})^2}$$

requires analytical continuation from Euclidean to Minkowski signature
numerically hard problem

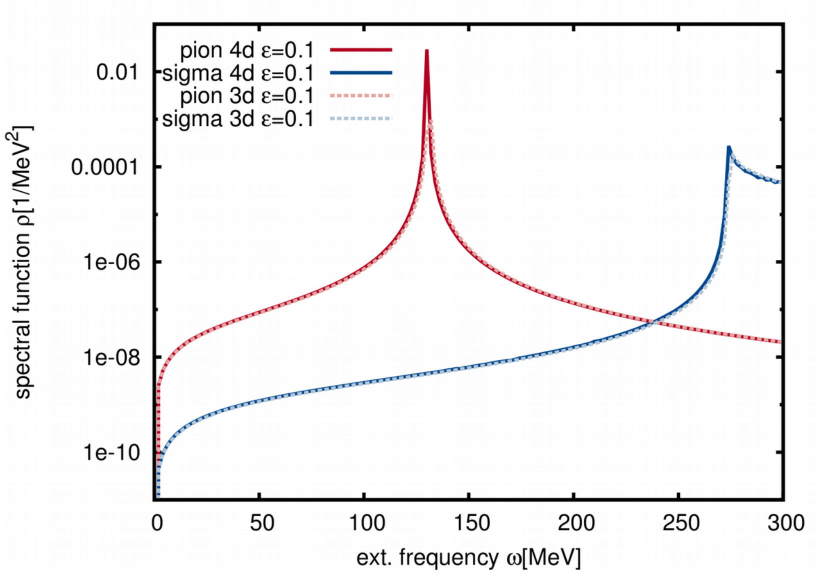
Popular approaches (based on Euclidean data)
- Maximum Entropy Method (MEM)
- Padé Approximants

Alternative: analytic continuation on the level of the functional equation
- Kamikado, NSt, von Smekal, Wambach Eur.Phys.J. **C74** (2014) 2806
- Floerchinger JHEP 1205 (2012) 021
- Strauss, Fischer, Kellermann PRL **109** (2012) 252001

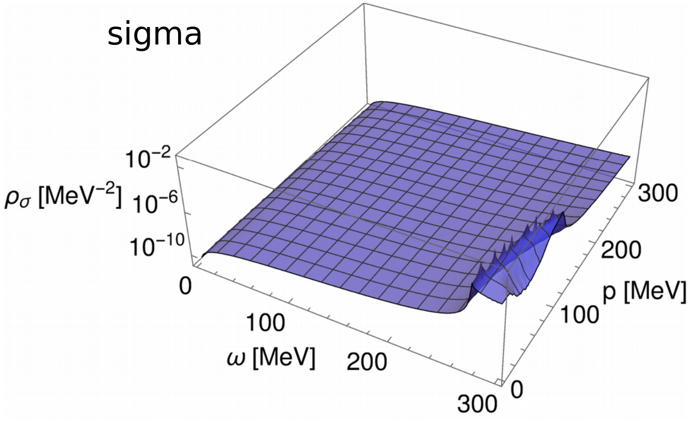
Here Minkowski external momenta appears as external parameters

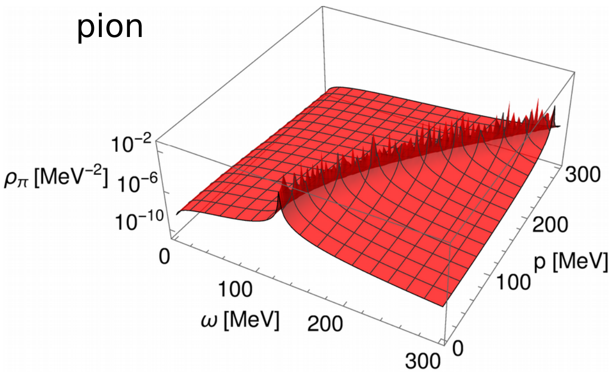
Spectral Functions

O(N) at T=0:



➢ Pawlowski, NSt Phys.Rev. **D92** (2015) 9, 094009





Summary

✔ Directly calculated spectral functions

✔ Tested in scalar and Yukawa models at T,μ>0

✔ Allows the inclusion of full momentum dependence ➢ NSt, in prep

❑ Quark & gluon spectral functions in full QCD

Transport Coefficients

Kubo formula for the shear viscosity

$$\eta = \lim_{\omega \to 0} \frac{1}{20} \frac{\rho_{\pi\pi}(\omega,\vec{0})}{\omega}$$

Require $$\rho_{\pi\pi}(\omega,\vec{p}) = \int_x e^{-\mathrm{i}\omega x_0 + \mathrm{i}\vec{p}\vec{x}} \langle [\pi_{ij}(x), \pi_{ij}(0)] \rangle$$

Transport Coefficients

Kubo formula for the shear viscosity

$$\eta = \lim_{\omega\to 0} \frac{1}{20} \frac{\rho_{\pi\pi}(\omega,\vec{0})}{\omega}$$

Require $\rho_{\pi\pi}(\omega,\vec{p}) = \int_x e^{-\mathrm{i}\omega x_0 + \mathrm{i}\vec{p}\vec{x}} \langle [\pi_{ij}(x), \pi_{ij}(0)] \rangle$

Expansion formula

- Pawlowski Annals Phys. 322 (2007) 2831-2915

$$\langle \pi_{ij}[\hat{A}]\pi_{ij}[\hat{A}] \rangle = \pi_{ij}[G_{A\phi_k}\frac{\delta}{\delta\phi_k} + A]\pi_{ij}[G_{A\phi_k}\frac{\delta}{\delta\phi_k} + A]$$

Transport Coefficients

Kubo formula for the shear viscosity

$$\eta = \lim_{\omega \to 0} \frac{1}{20} \frac{\rho_{\pi\pi}(\omega, \vec{0})}{\omega}$$

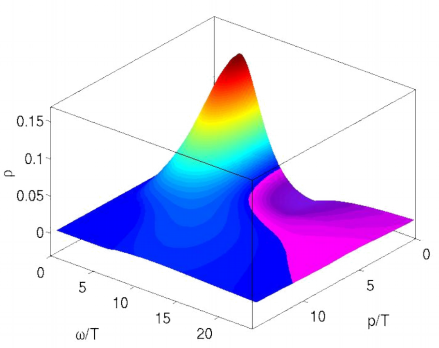
Require $\rho_{\pi\pi}(\omega, \vec{p}) = \int_x e^{-\mathrm{i}\omega x_0 + \mathrm{i}\vec{p}\vec{x}} \langle [\pi_{ij}(x), \pi_{ij}(0)] \rangle$

Expansion formula

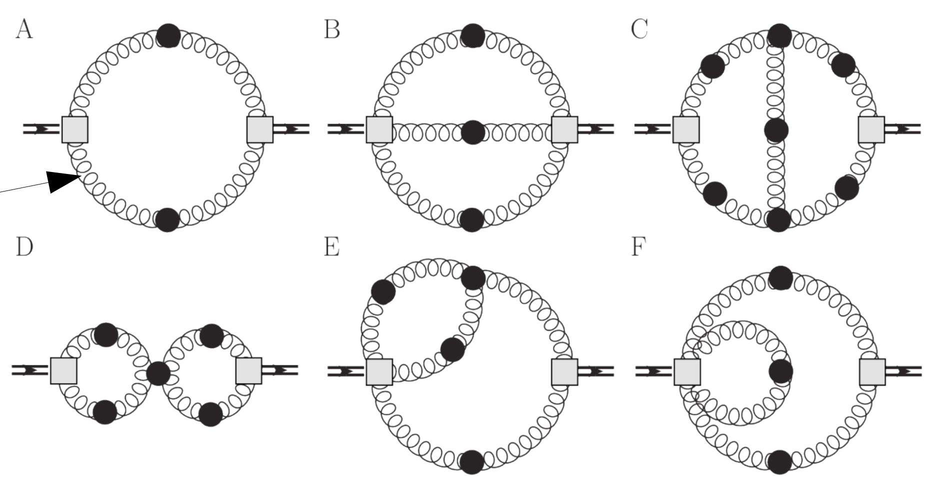
- Pawlowski Annals Phys. 322 (2007) 2831-2915

$$\langle \pi_{ij}[\hat{A}]\pi_{ij}[\hat{A}] \rangle = \pi_{ij}[G_{A\phi_k} \frac{\delta}{\delta\phi_k} + A]\pi_{ij}[G_{A\phi_k} \frac{\delta}{\delta\phi_k} + A]$$

Finite number of diagrams involving **full** propagators/vertices

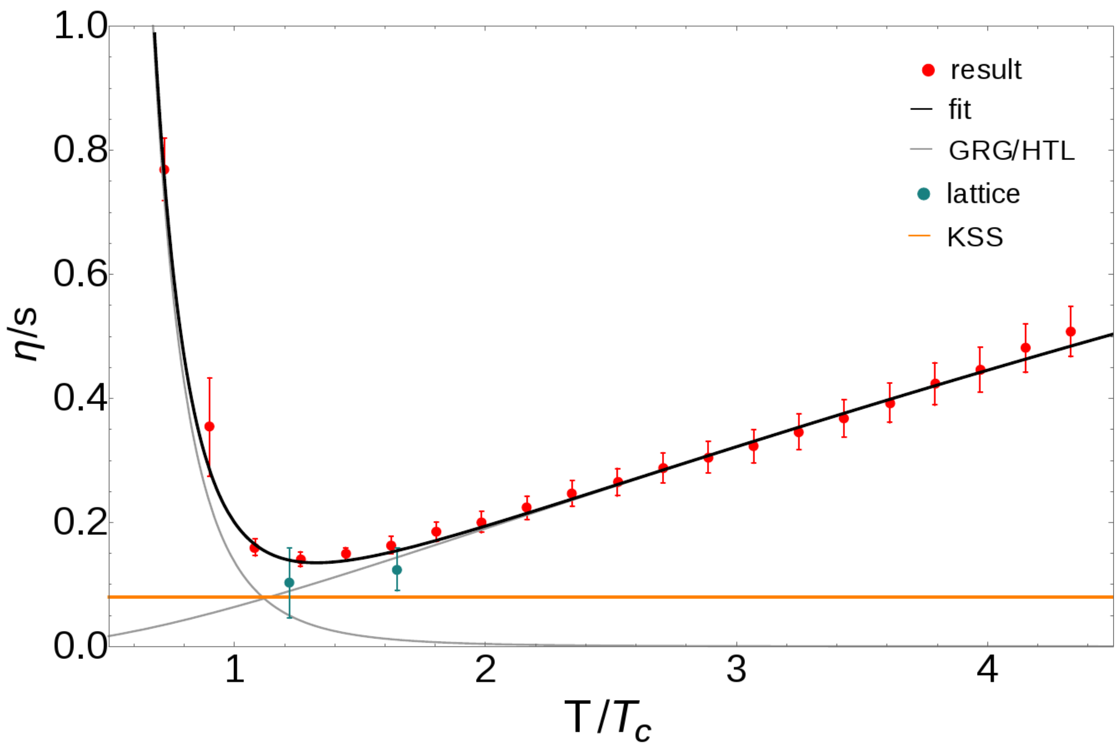


- Haas, Fister, Pawlowski Phys. Rev. D90 091501 (2014)



All diagrams to 2-loop order

η/s in YM theory

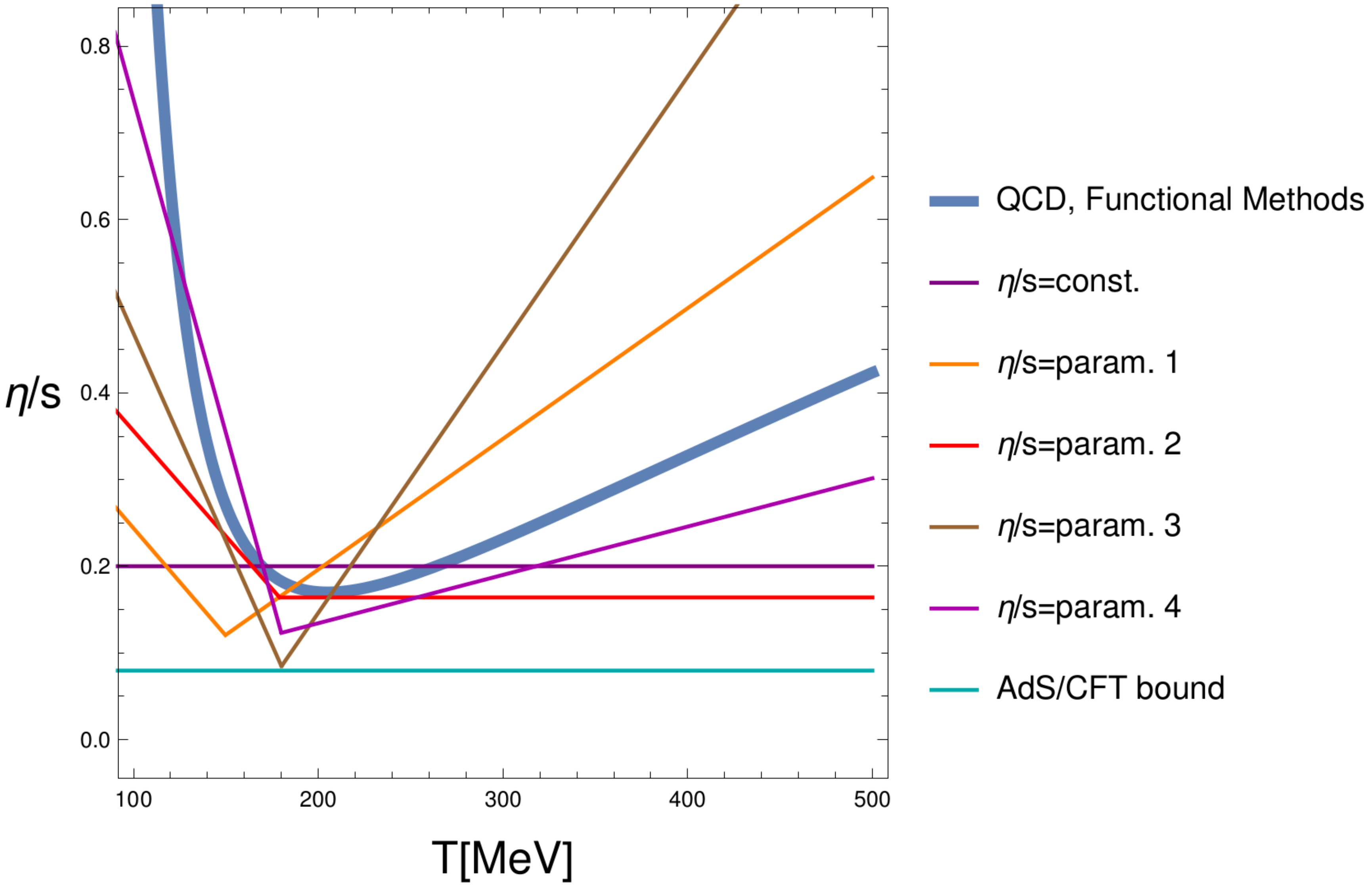


T_{min}=1.26 T_C
Value=0.14

- Christiansen, Haas, Pawlowski, NSt PRL **115** (2015) 11, 112002

Estimate: η/s in QCD

- c.f. Niemi, Eskola, Paatelainen 1505.02677



Summary

- **QCD phase structure**

  towards a quantitative continuum approach to QCD
  - ✓ Quantitative grip on fluctuation physics in the vacuum
  - finite temperature and density
  - Nuclear matter; nuclear binding energy...

- **Realtime observables**
  - **Elementary spectral functions**

    new approach to analytical continuation problem
    - ✓ tested in low energy eff. models (O(N), QM model)
    - vector meson, quark & gluon, charmonium spectral functions

  - **Transport Coefficients**

    from loop expansion involving full propagators and vertices
    - ✓ Global quantitative prediction for η/s in YM theory
    - Full QCD, bulk viscosity, relaxation times

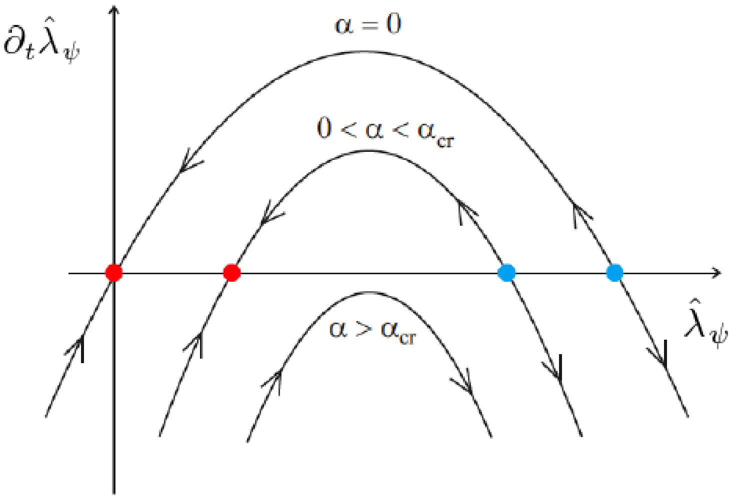
Thank you for your attention!

Backup

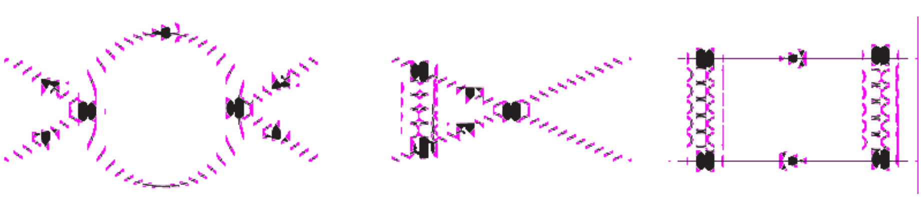
Chiral symmetry breaking

χSB <-> resonance in 4-Fermi int.
(pion pole)

β-function:



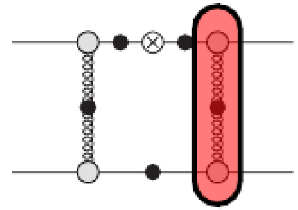
$$k\partial_k \hat{\lambda}_\psi = (d-2)\hat{\lambda}_\psi - a\hat{\lambda}_\psi^2 - b\hat{\lambda}_\psi g^2 - cg^4$$



- Review: Braun J.Phys. **G39** (2012) 033001

Chiral symmetry breaking

χSB <-> resonance in 4-Fermi int.
(pion pole)

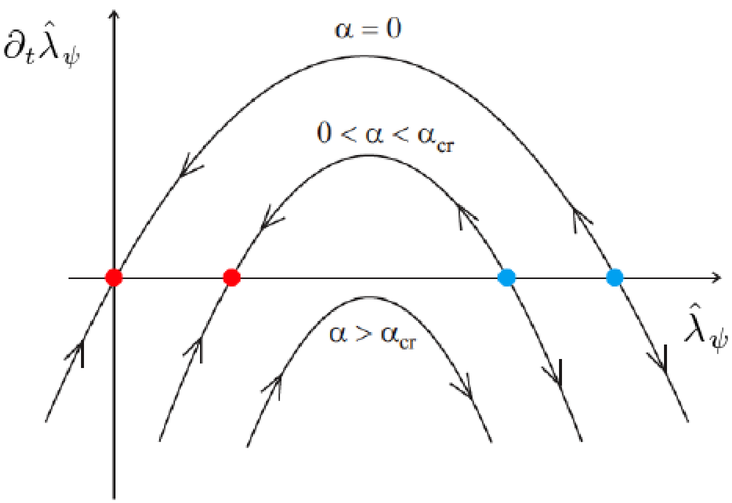


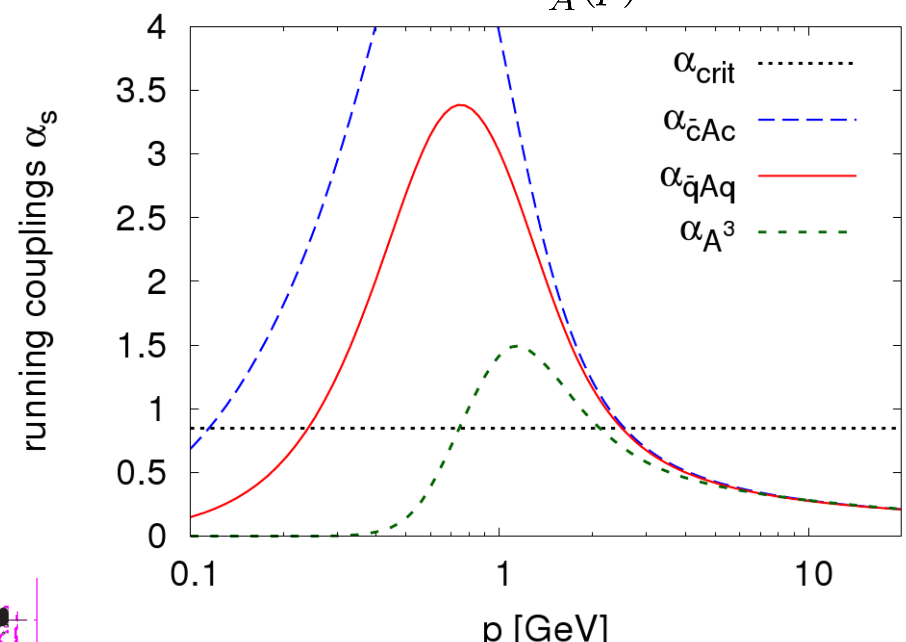
$$\alpha_{\bar{c}Ac}(p) = \frac{Z^2_{\bar{c}Ac}(\bar{p})}{4\pi Z_A(p) Z_c^2(p)}$$

$$\alpha_{\bar{q}Aq}(p) = \frac{Z^2_{\bar{q}Aq}(\bar{p})}{4\pi Z_A(p) Z_q^2(p)}$$

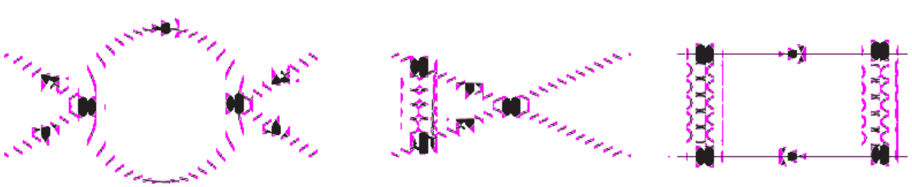
$$\alpha_{A^3}(p) = \frac{Z^2_{A^3}(\bar{p})}{4\pi Z_A^3(p)}$$

β-function:





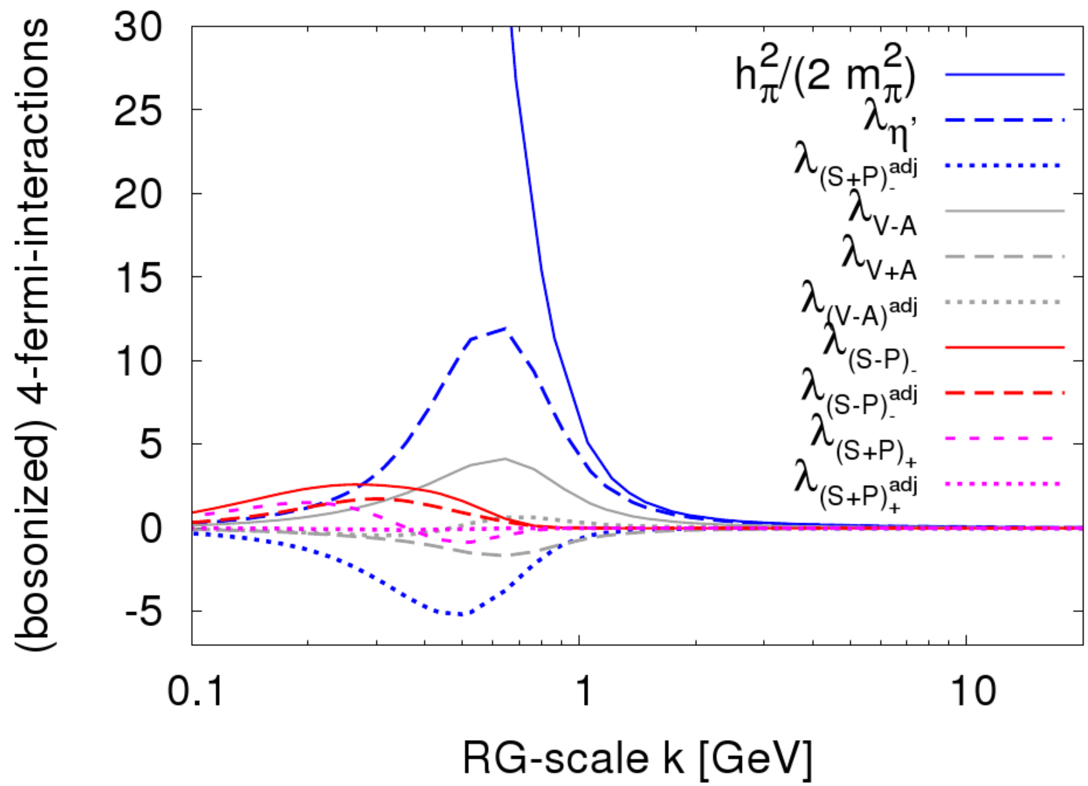
$$k\partial_k \hat{\lambda}_\psi = (d-2)\hat{\lambda}_\psi - a\hat{\lambda}_\psi^2 - b\hat{\lambda}_\psi g^2 - cg^4$$



- Decoupling due to gluon mass gap
- Chiral SB requires to exceed crit. coupling
- area above the critical value decides

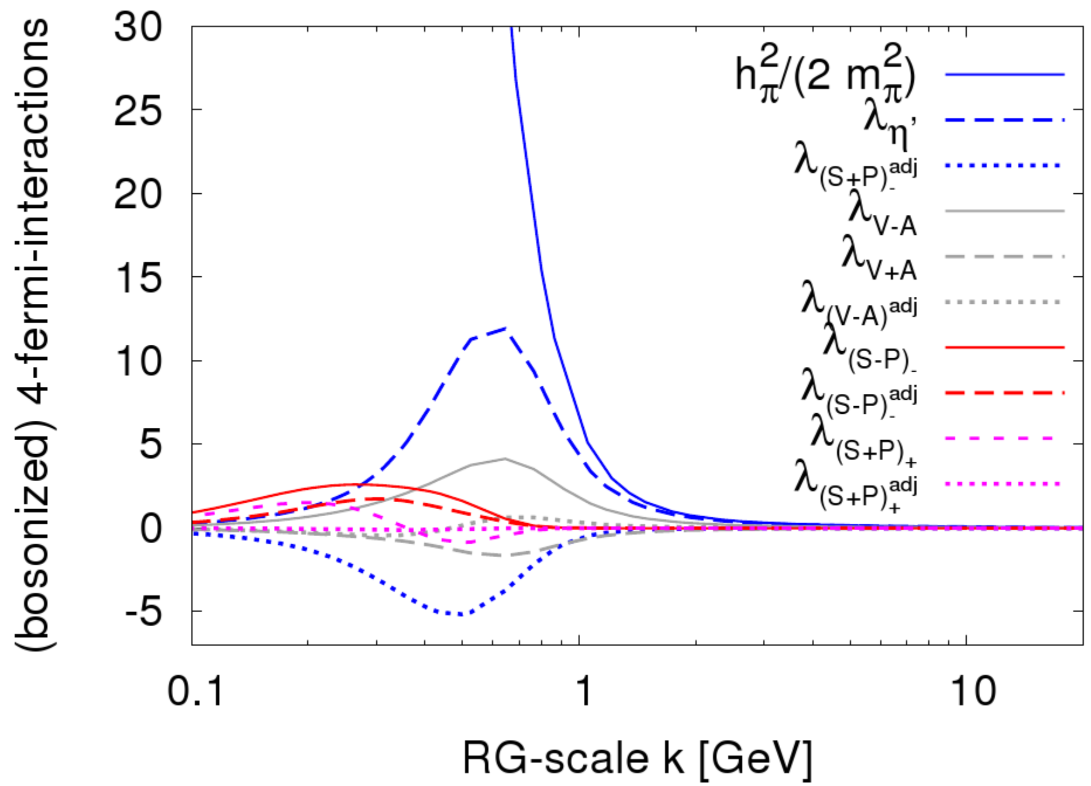
➢ Review: Braun J.Phys. **G39** (2012) 033001

4-Fermi Interactions



(a) Renormalisation group scale dependence of dimensionless four-fermi interactions, see App. B 2 c, and bosonised σ-π channel. Grey: respects chiral symmetry, blue: breaks $U(1)_A$, red: breaks $SU(2)_A$, magenta: breaks $U(2)_A$.

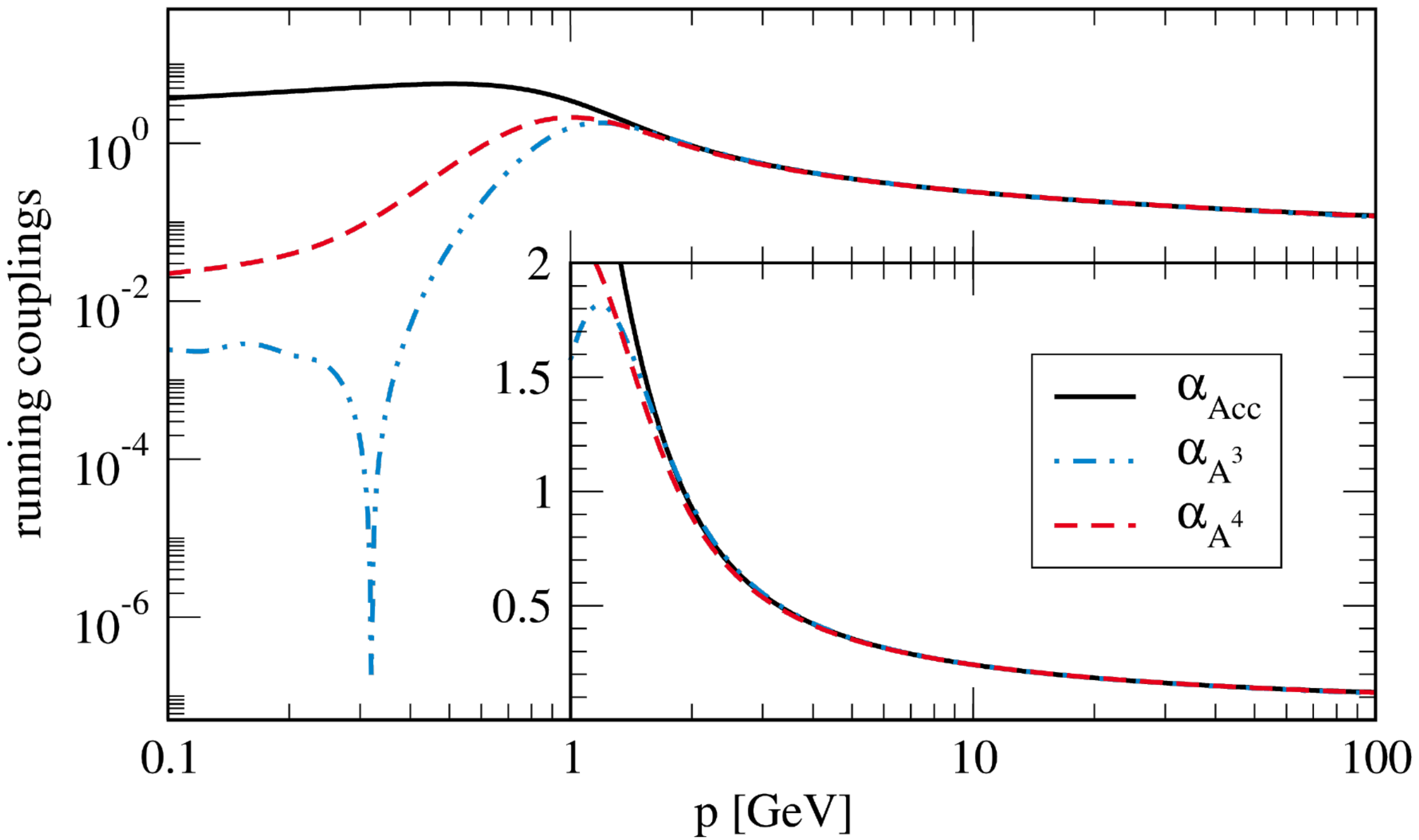
4-Fermi Interactions



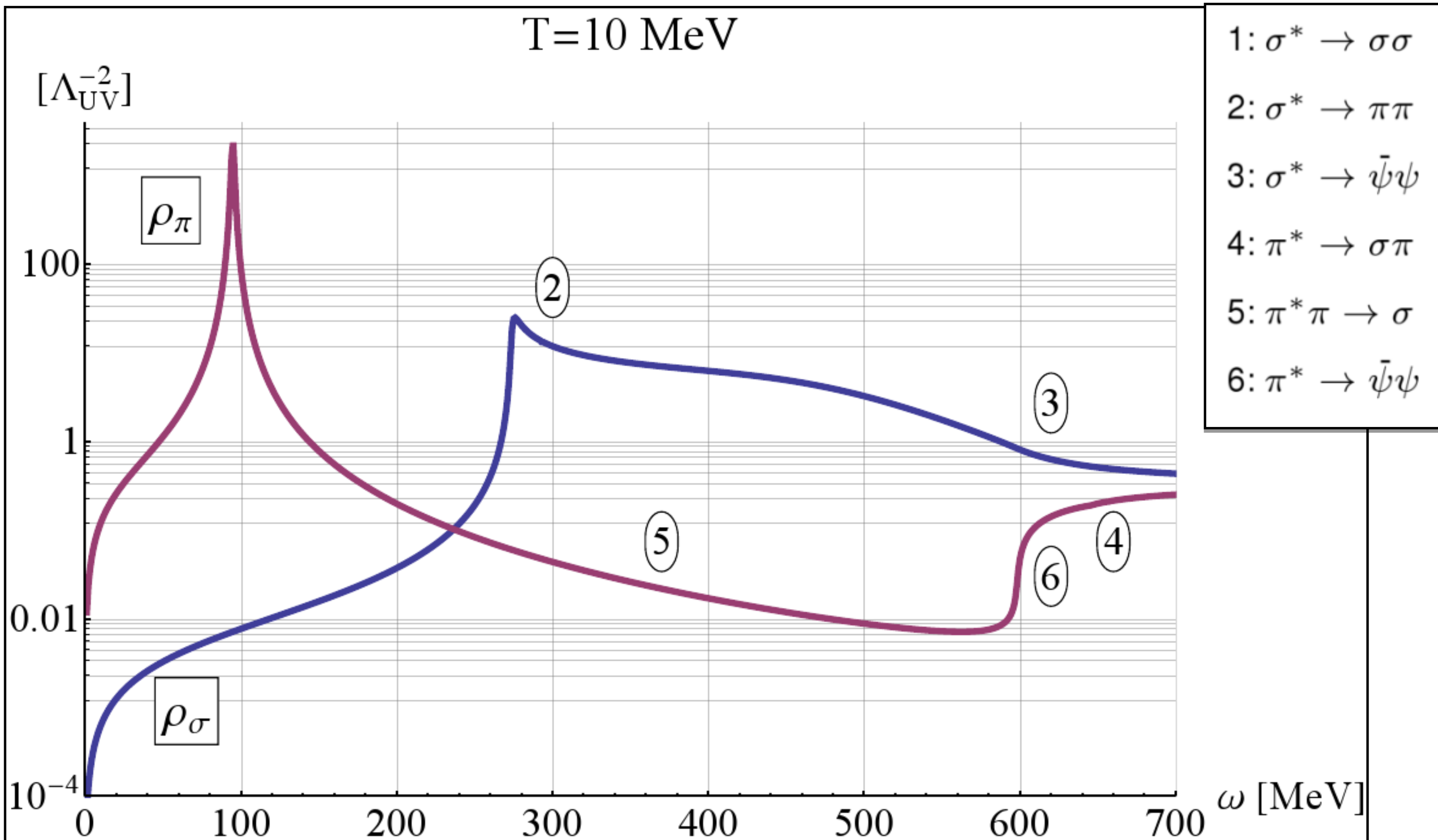
(a) Renormalisation group scale dependence of dimensionless four-fermi interactions, see App. B 2 c, and bosonised σ-π channel. Grey: respects chiral symmetry, blue: breaks $U(1)_A$, red: breaks $SU(2)_A$, magenta: breaks $U(2)_A$.

- Bosonizing the σ-π channel only is sufficient
- In the vacuum: other channels not quantitatively relevant

YM running couplings

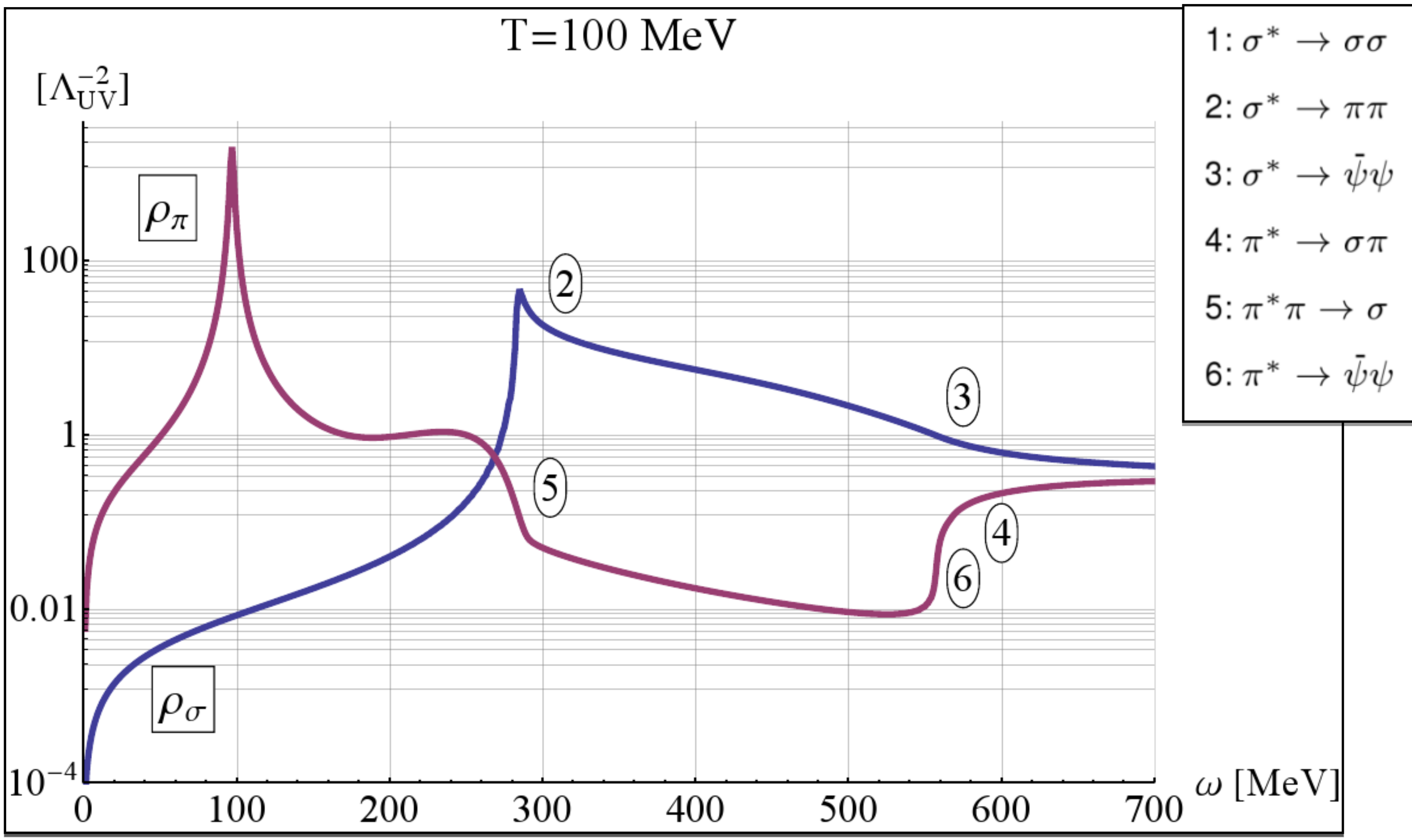


QM Model at T>0



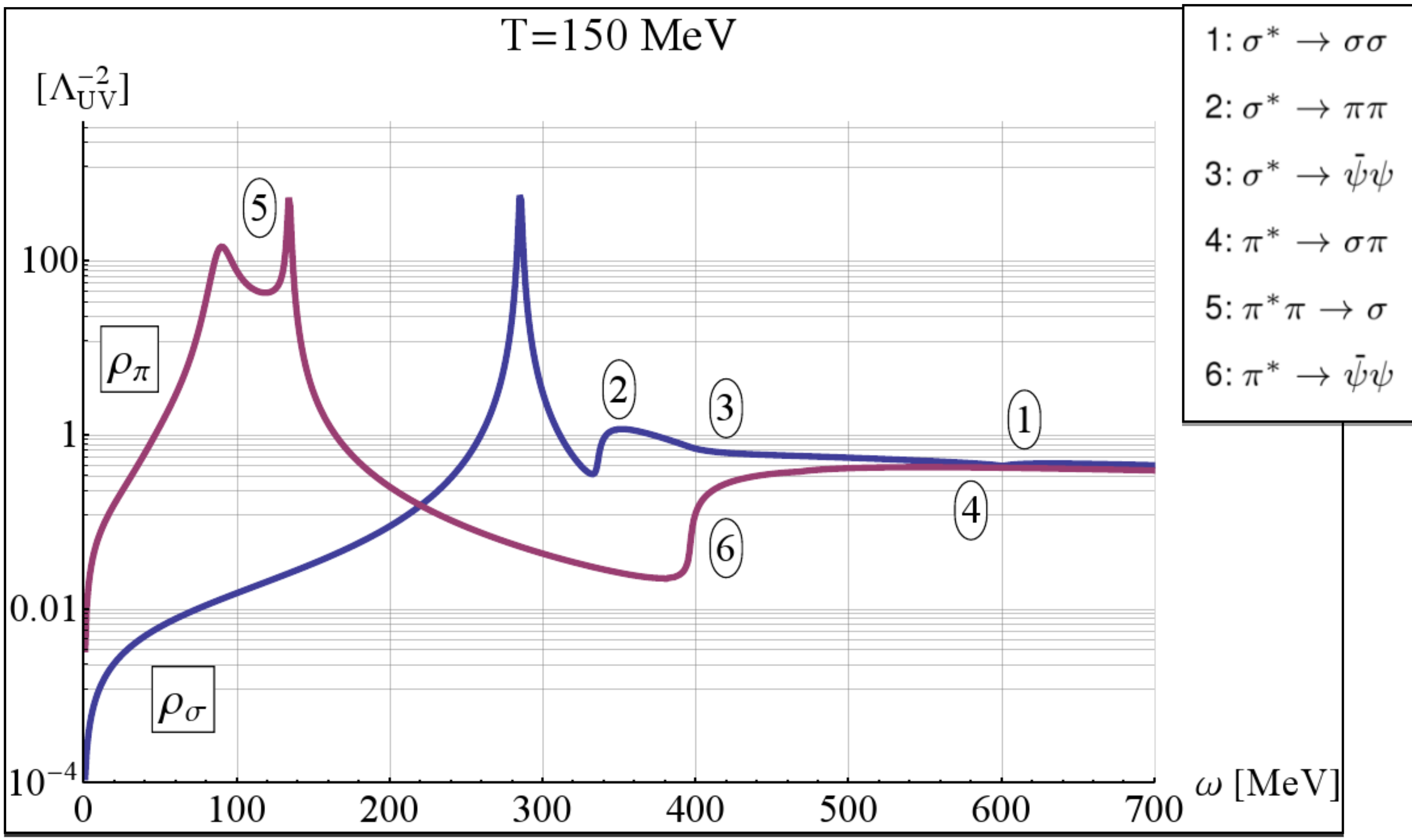
- Tripolt, NSt, von Smekal, Wambach Phys.Rev. **D89** (2014) 034010

QM Model at T>0



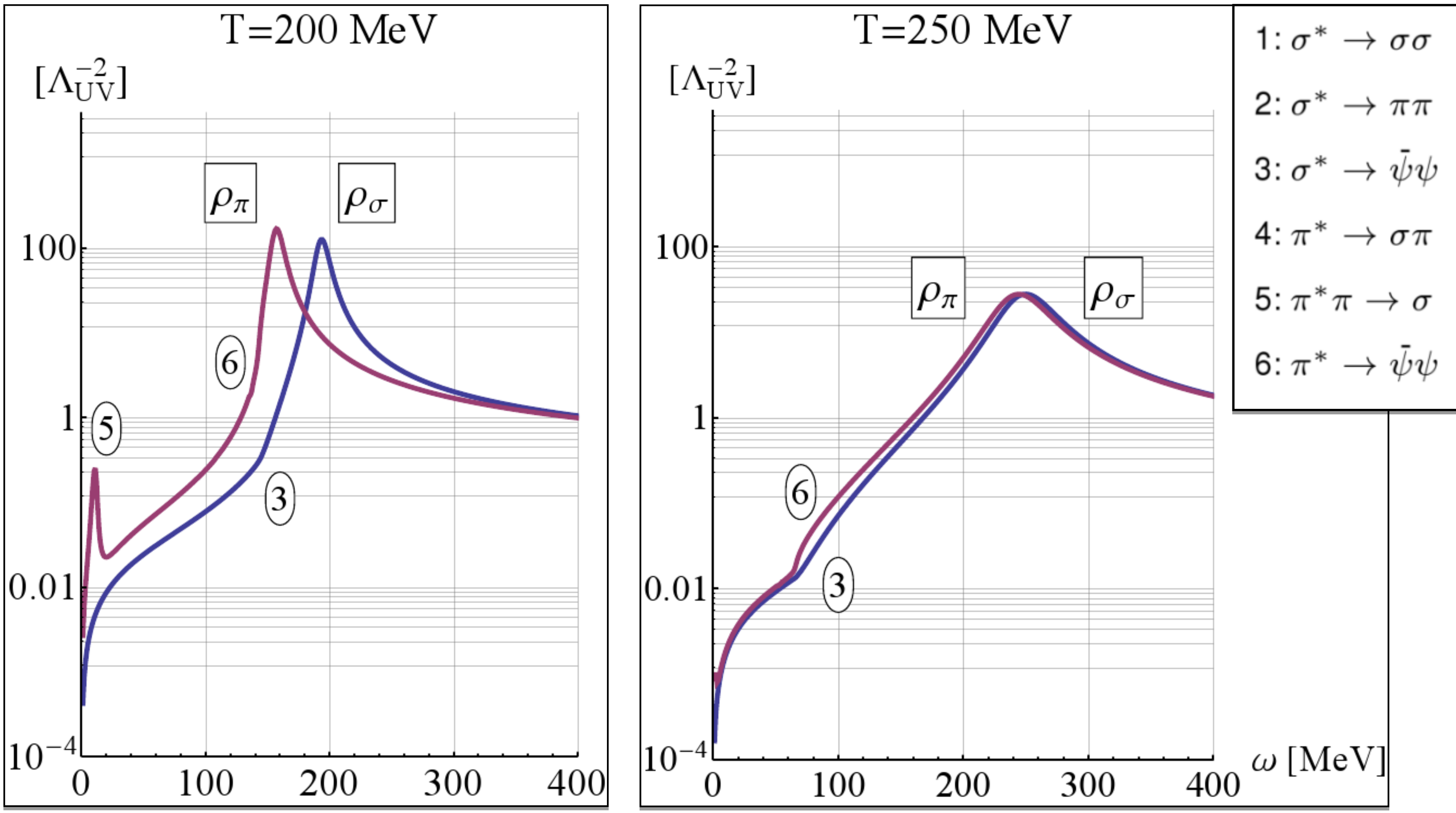
- Tripolt, NSt, von Smekal, Wambach Phys.Rev. **D89** (2014) 034010

QM Model at T>0



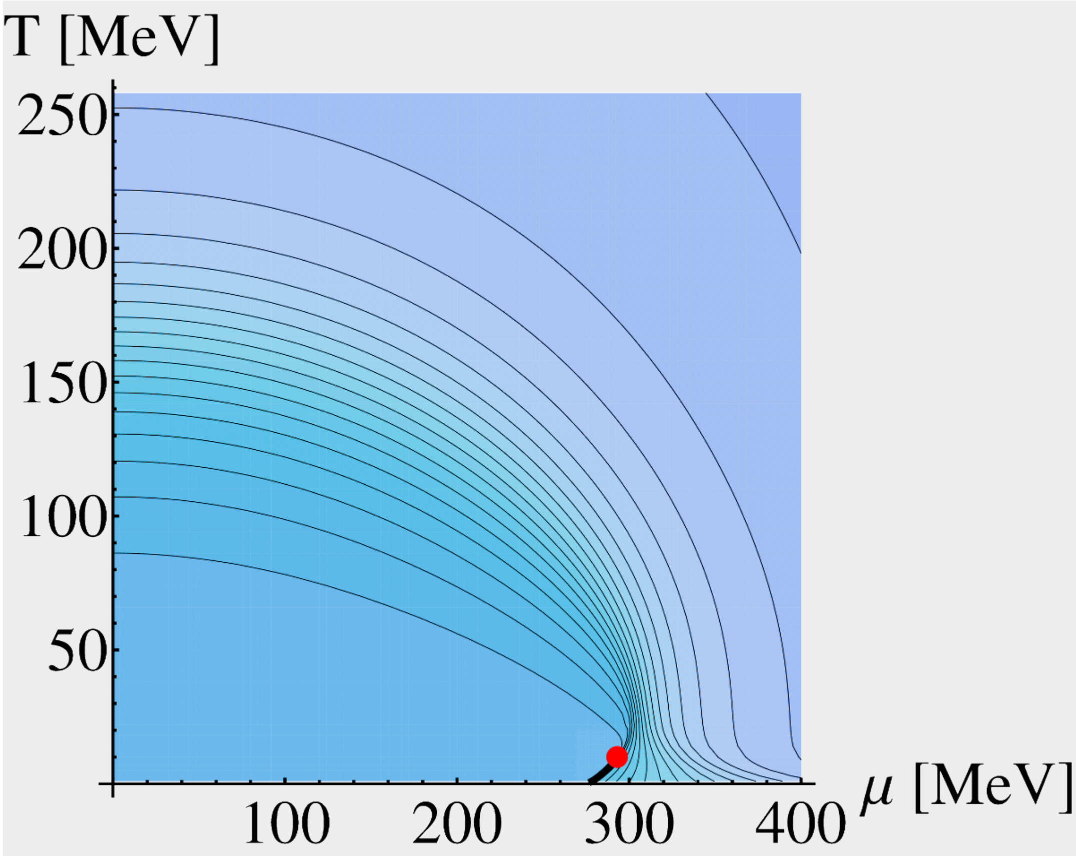
- Tripolt, NSt, von Smekal, Wambach Phys.Rev. **D89** (2014) 034010

QM Model at T>0

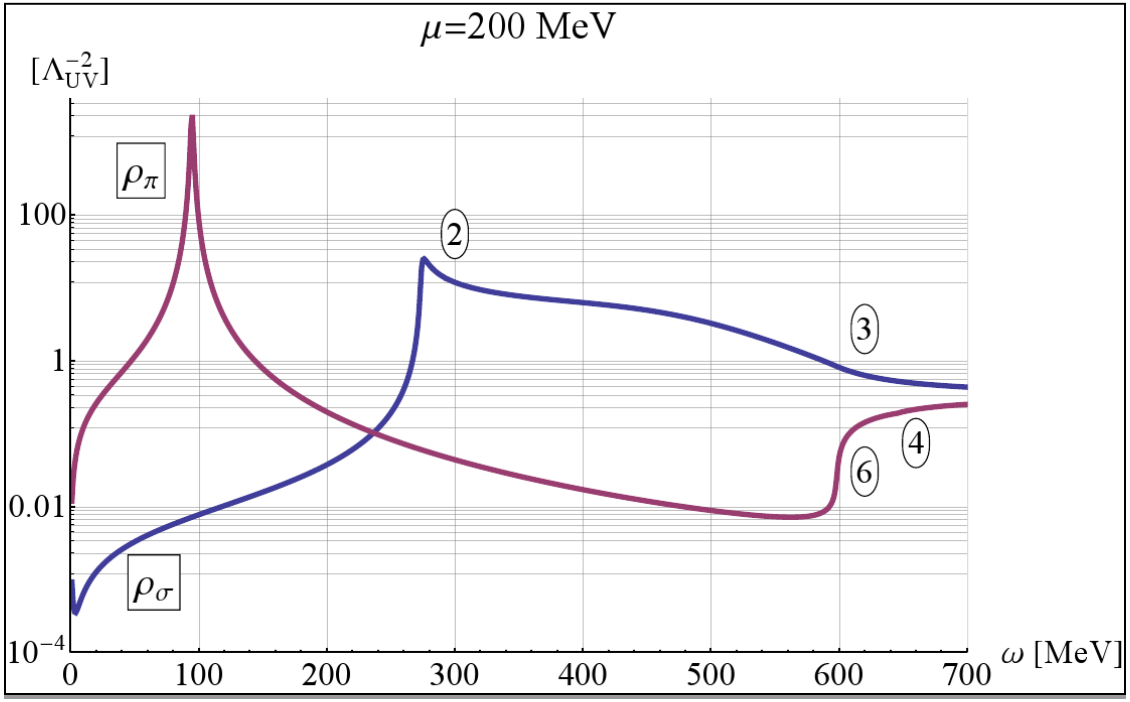


- Tripolt, NSt, von Smekal, Wambach Phys.Rev. **D89** (2014) 034010

QM Model at $\mu > 0$



QM Model at μ > 0



1: $\sigma^* \to \sigma\sigma$

2: $\sigma^* \to \pi\pi$

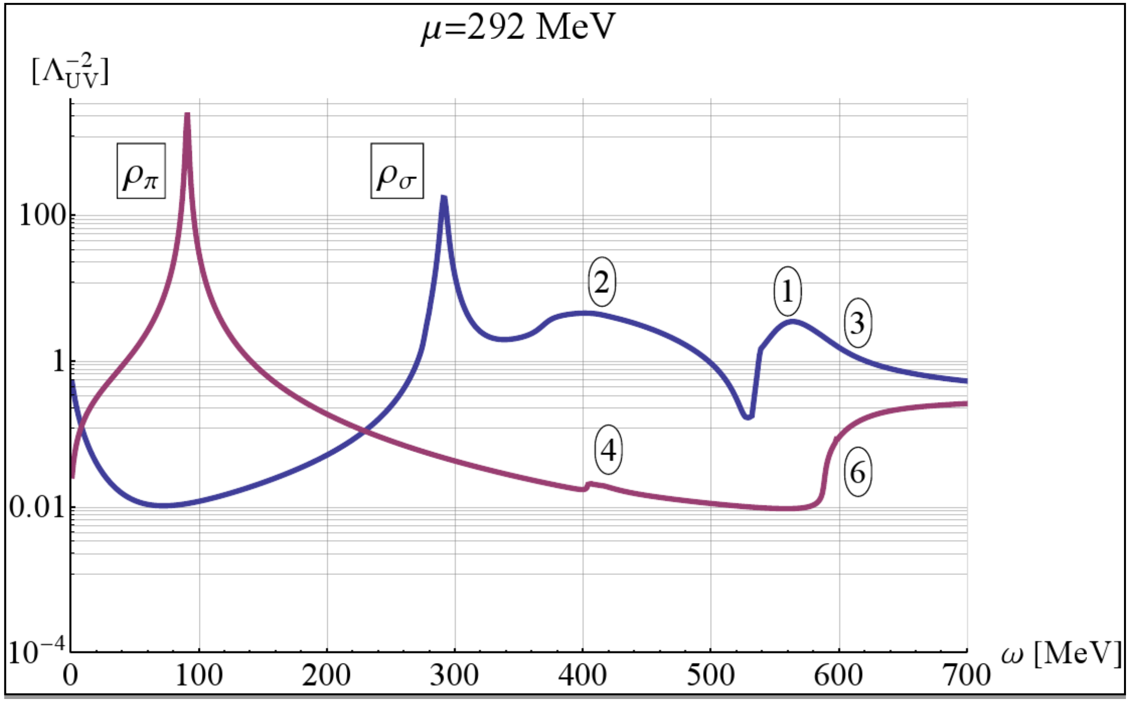
3: $\sigma^* \to \bar{\psi}\psi$

4: $\pi^* \to \sigma\pi$

5: $\pi^*\pi \to \sigma$

6: $\pi^* \to \bar{\psi}\psi$

QM Model at μ > 0



1: $\sigma^* \to \sigma\sigma$

2: $\sigma^* \to \pi\pi$

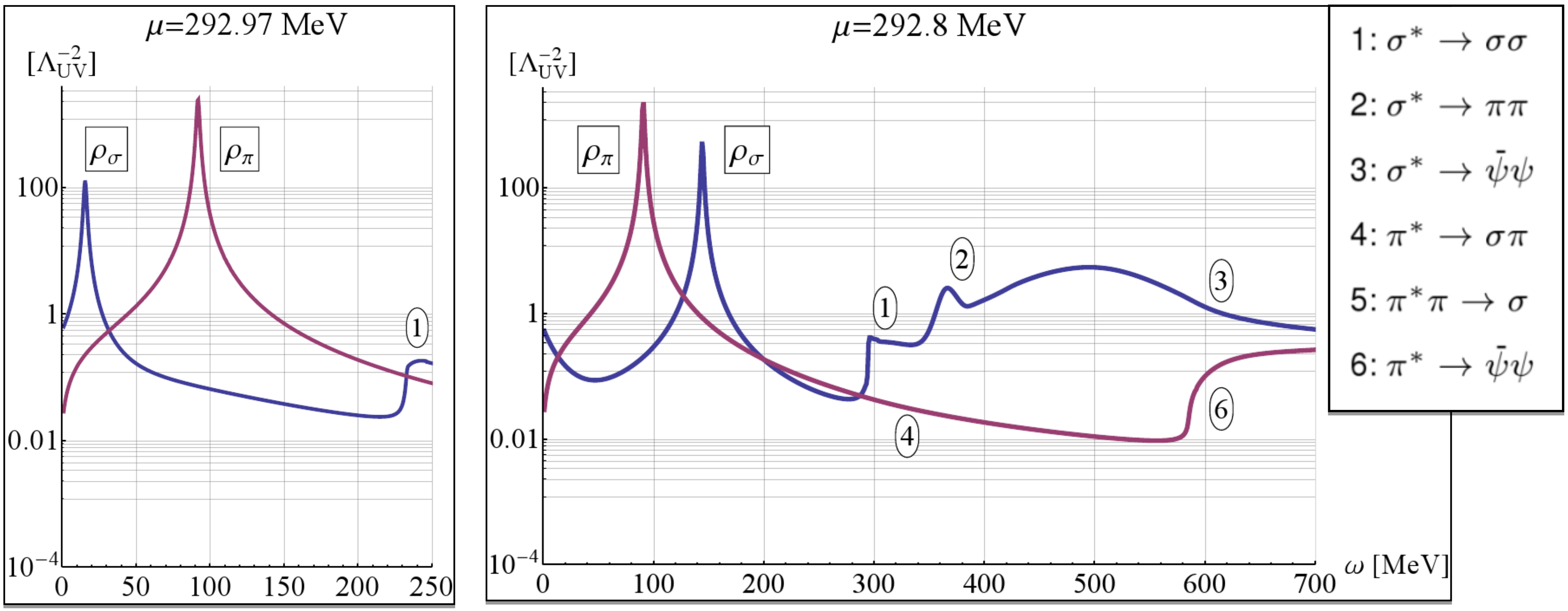
3: $\sigma^* \to \bar{\psi}\psi$

4: $\pi^* \to \sigma\pi$

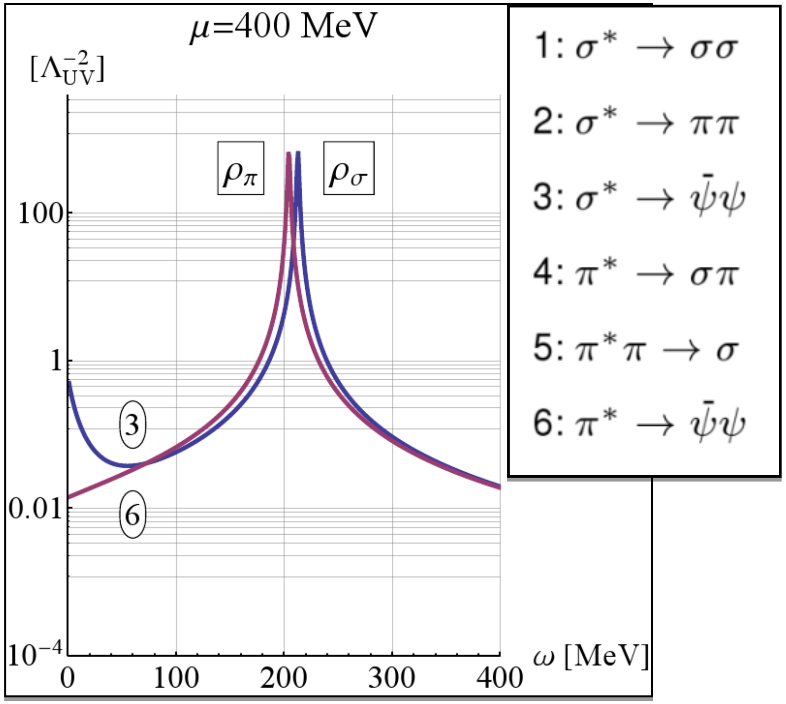
5: $\pi^*\pi \to \sigma$

6: $\pi^* \to \bar{\psi}\psi$

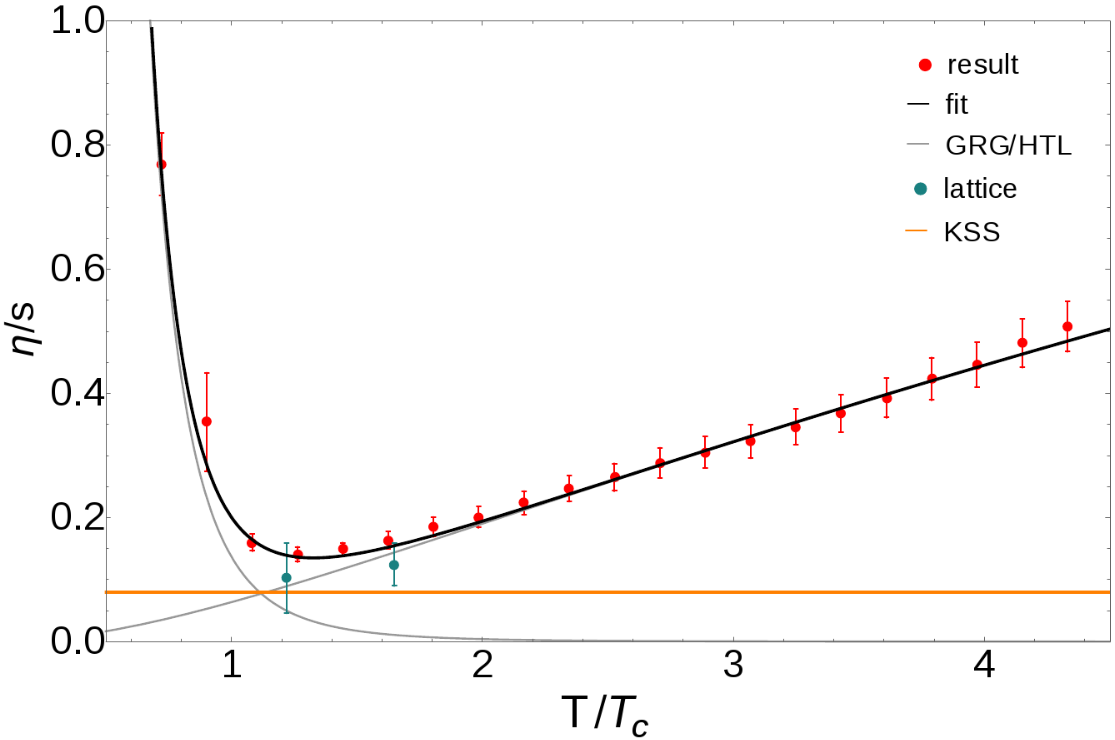
QM Model at μ > 0



QM Model at μ > 0



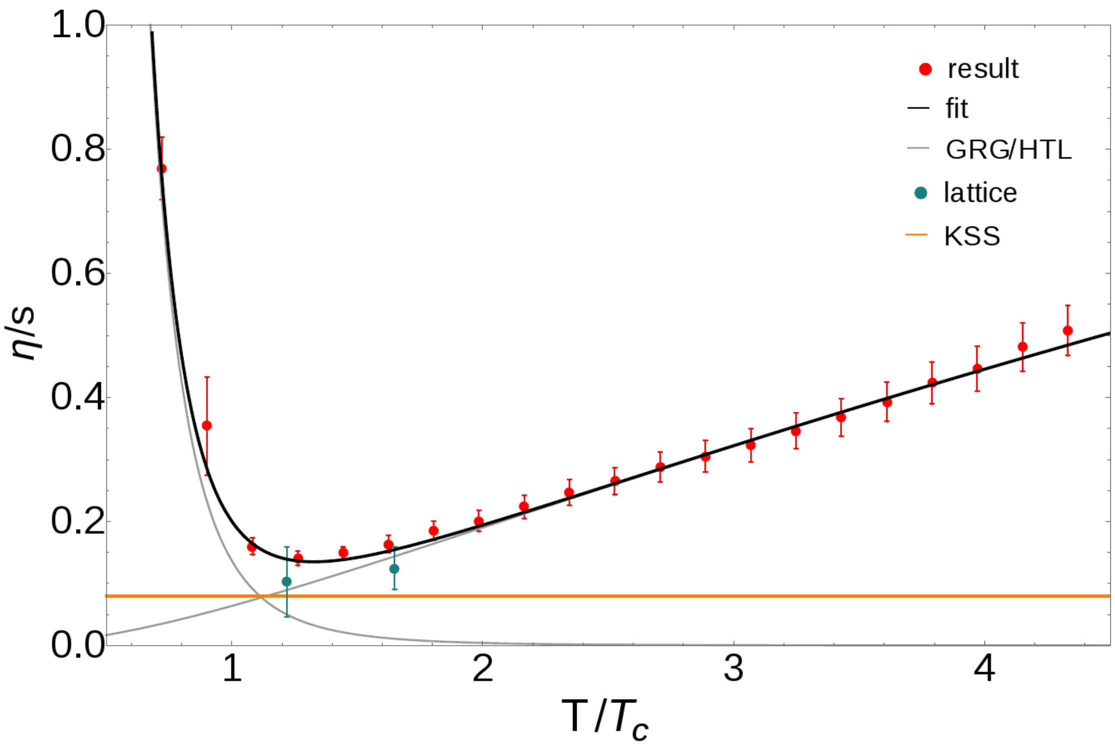
η/s in Yang-Mills Theory



T_{min}=1.26 T_C
Value=0.14

- Christiansen, Haas, Pawlowski, NSt PRL **115** (2015) 11, 112002

η/s in Yang-Mills Theory



T_{min}=1.26 T_C
Value=0.14

- Christiansen, Haas, Pawlowski, NSt PRL **115** (2015) 11, 112002

Direct sum: $$\frac{\eta}{s}(T) = \frac{a}{\alpha_s(cT/T_c)^\gamma} + \frac{b}{(T/T_c)^\delta}$$

$$\gamma = 1.6 \quad a = 0.15 \quad b = 0.14 \quad c = 0.66 \quad \delta = 5.1$$

High T: **consistent with HTL-resummed pert. Theory** (fixing γ) supporting quasiparticle picture

Small T: algebraic decay **glueball resonance gas**

η/s in QCD

From YM to QCD in three simple steps

1. Replace α_s; impose equality at T_C

$$\alpha_s^{N_f=0}\big|_{T_c} = \alpha_s^{N_f=3}\big|_{T_c}$$

2. Genuine quark contributions to η and s
3. Replace GRG by HRG

➢ Demir, Bass PRL **102** (2009) 172302

$$\frac{\eta}{s}(T) = \frac{a}{\alpha_s(cT/T_c)^\gamma} + \frac{b}{(T/T_c)^\delta}$$

$$\gamma = 1.6 \quad a = 4/3 \cdot 0.15 \quad b = 0.16 \quad c = 0.79 \quad \delta = 5$$

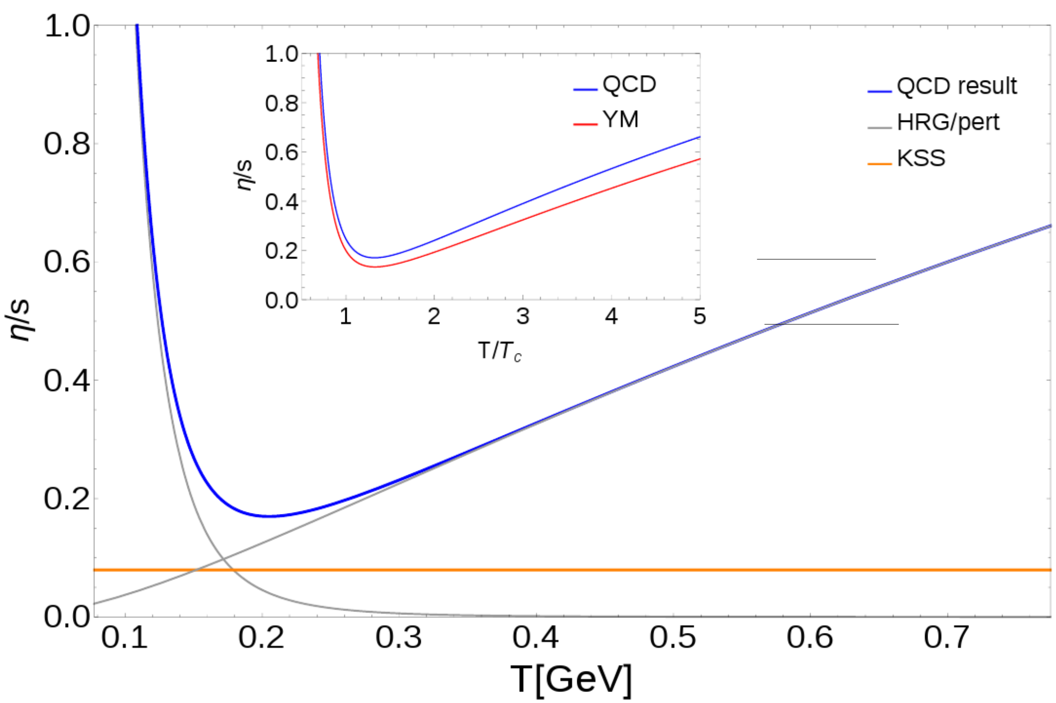
η/s in QCD

From YM to QCD in three simple steps

1. Replace α_s; impose equality at T_C $\quad \alpha_s^{N_f=0}|_{T_c} = \alpha_s^{N_f=3}|_{T_c}$
2. Genuine quark contributions to η and s
3. Replace GRG by HRG ➢ Demir, Bass PRL **102** (2009) 172302

$$\frac{\eta}{s}(T) = \frac{a}{\alpha_s(cT/T_c)^{\gamma}} + \frac{b}{(T/T_c)^{\delta}}$$

$$\gamma = 1.6 \quad a = 4/3 \cdot 0.15 \quad b = 0.16 \quad c = 0.79 \quad \delta = 5$$



T_{min}=1.3 T_C
Value: 0.17

Workflow

...towards 1-click QCD