Fluid dynamics for the anisotropically expanding quark-gluon plasma

Dennis Bazow
The Ohio State University

With:
U. Heinz, M. Martinez, M. Strickland

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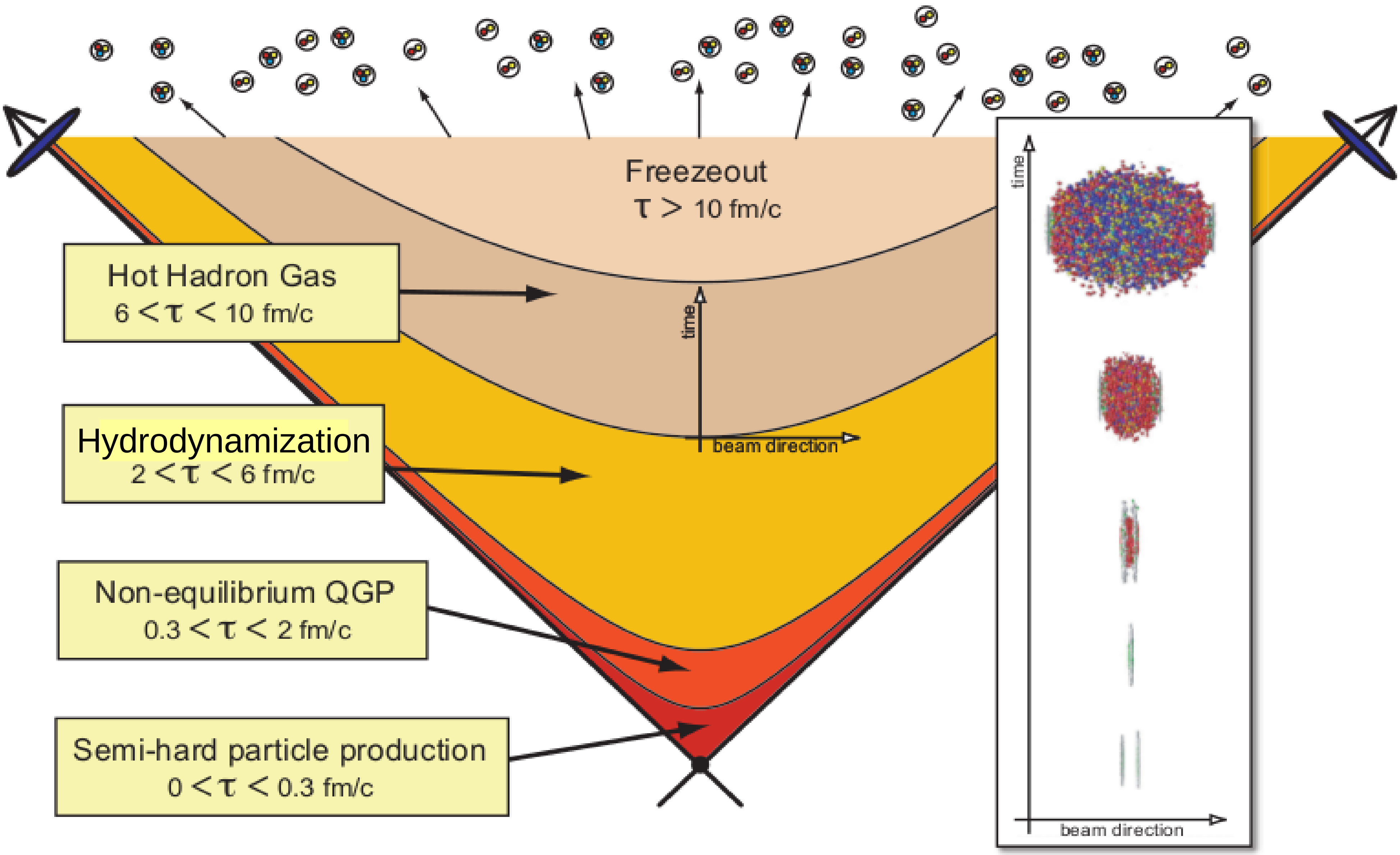
Rough estimate of QGP timescales at LHC



Plot taken from M. Strickland, arxiv:1410.5786

Estimates of early-time momentum anisotropies

Consider Navier-Stokes solution in Bjorken $\pi_{\rm NS} = -\frac{4\eta}{3\tau}$

$$\left(\frac{P_{\rm L}}{\mathcal{P}_\perp}\right)_{\rm NS} = \frac{3\tau T - 16\eta/\mathcal{S}}{3\tau T + 8\eta/\mathcal{S}}$$

RHIC-like initial conditions:

$$T_0 = 400\,{\rm MeV}\,{\rm at}\,\tau_0 = 0.5\,{\rm fm/c} \implies (\mathcal{P}_{\rm L}/\mathcal{P}_\perp)_{\rm NS} \simeq 0.5$$

LHC-like initial conditions:

$$T_0 = 600\,{\rm MeV}\,{\rm at}\,\tau_0 = 0.25\,{\rm fm/c} \implies (\mathcal{P}_{\rm L}/\mathcal{P}_\perp)_{\rm NS} \simeq 0.35$$

Both use $4\pi\eta/\mathcal{S} = 1$

Viscous hydro predicts sizable momentum-space anisotropies

Lets look at hydrodynamics from the Boltzmann equation.

$$p^\mu \partial_\mu f = C[f]$$

Hydrodynamic expansion in from kinetic theory

- linearize around a local equilibrium distribution function

$$y_0 \equiv \frac{u \cdot p}{T} - \frac{\mu}{T}, \qquad y - y_0 \equiv \delta y \ll 1$$

$$f(y) = \underbrace{f_{\text{eq}}(y_0)}_{\equiv f_0} + \underbrace{f_{\text{eq}}(1 - a f_{\text{eq}})\delta y}_{\equiv \delta f} + \mathcal{O}(\delta y^2)$$

Particle momentum-space is approximated at leading-order by a sphere

$$u \cdot p \underset{LRF}{\equiv} \sqrt{m^2 + |\mathbf{p}|^2}$$

Leads to dissipative currents, e.g. shear stress tensor $\pi^{\mu\nu}$

Validity of the distribution function for non-equilibrium systems

For Navier-Stokes

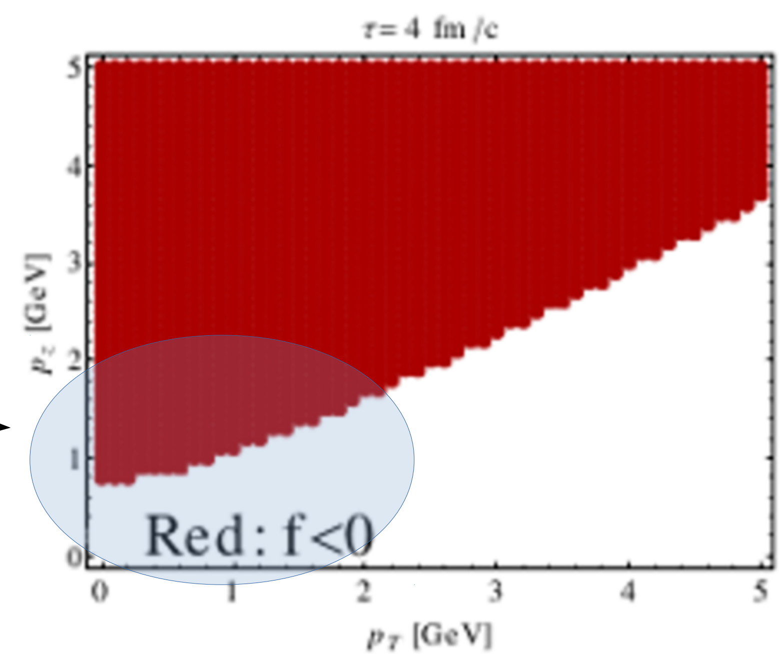
anisotropic in momentum-space

$$f_{NS}^{0+1d} = f_{\rm eq}\left[1 + \frac{\eta}{\mathcal{S}}\left(\frac{p_x^2 + p_y^2 - 2p_z^2}{3\tau T^3}\right)\right]$$

Take $\tau_0 = 0.25\,{\rm fm/c}\,, T_0 = 0.6\,{\rm GeV}\,, 4\pi\eta/\mathcal{S} = 1$

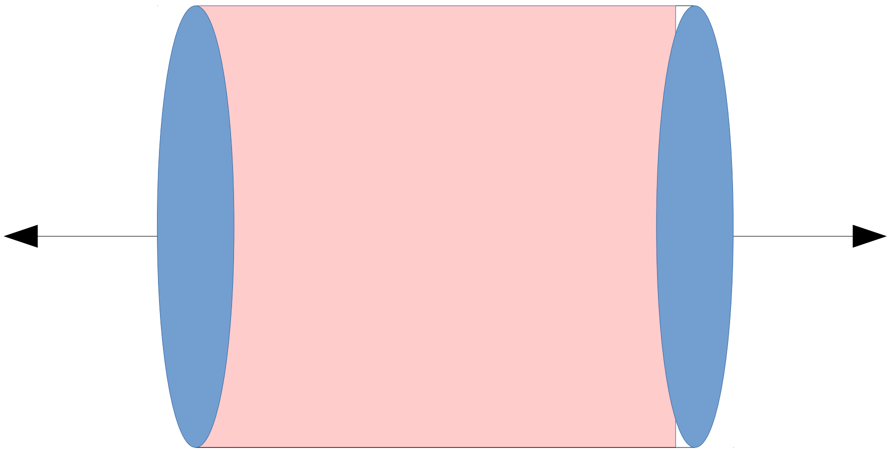
Plot distribution function at time 4 fm/c for IS hydro

f<0 for regions of phase space where hydro is valid



Lets reexamine the problem of a HIC.

Early time QGP



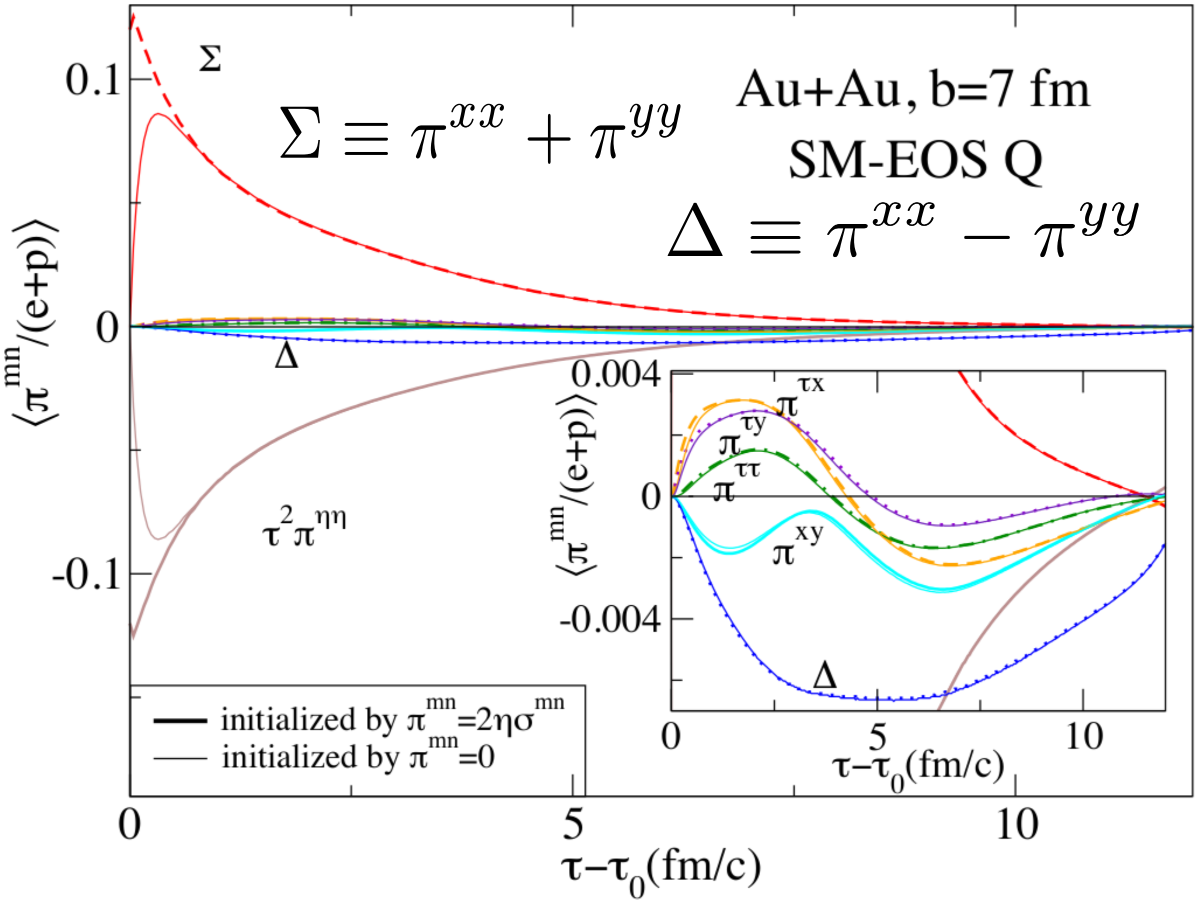
Looks like a tiny one-dimensionally expanding universe

Longitudinal expansion scalar behaves as $1/\tau$

Takes some time to generate significant transverse expansion

Realistic 2+1d HIC simulation

Plot taken from H. Song, arxiv:0908.3656



Σ and $\tau^2\pi^{\eta\eta}$

Are the largest components, related by the traceless condition

$$\Sigma \equiv -\tau^2\pi^{\eta\eta}$$

No need to over complicate things.

Good approximation is to assume spheroidal form in momentum-space at LO. Evolution of these non-hydrodynamic DOF's are treated non-perturbatively

All off-diagonal components are small. Treat them as a perturbation.

Hydrodynamic expansion revisited: a reorganized approach

- Generalized expansion

$$f(x,p) = f_0(x,p)\sum_{\ell,\alpha} a_\alpha(x) P_\alpha^{(\ell)}(p)$$

- f_0 is LO approximation (arbitrary weight factor)
- In order to obtain the most rapid convergence, choose f_0 such that it is as close as possible to the exact solution f
- The choice of f_0 is guided by general insights into the properties of f for the problem at hand

Viscous anisotropic hydrodynamics

- In HIC, rapid longitudinal expansion suggests to use an f_0 distorted along the p_z (beam)-direction with azimuthal momentum-space symmetry
- Expansion around a "local anisotropic equilibrium" momentum-space distribution function with spheroidal symmetry, i.e. Romatschke-Strickland form

In the local rest frame

$$f(x,p) = f_{\rm iso}\left(\beta_a\sqrt{p_\perp^2 + (1+\xi(x))p_z^2}\right) + \delta\tilde{f} \equiv f_{\rm aniso} + \delta\tilde{f}$$

Inverse Temperature-like scale

$\tilde{\Pi}\,,\tilde{V}^\mu\,,\tilde{\pi}^{\mu\nu}$

Lets return to the starting point - the underlying microscopic theory.

$$p^\mu \partial_\mu f = C[f]$$

Effective theory of many-body dynamics of an interacting system

- Hydrodynamics is an effective theory for the evolution of conserved macroscopic quantities
- A full description of the microscopic dynamics must include an infinite set of nonhydrodynamic modes

Hydrodynamic modes satisfy the dispersion relation

$$\lim_{\mathbf{k}\to 0} \omega_n(\mathbf{k}) = 0$$

Dominate the system at large spatial and temporal length scales

Must include at least the first nonhydrodynamic mode due to causality

Nonhydrodynamic modes

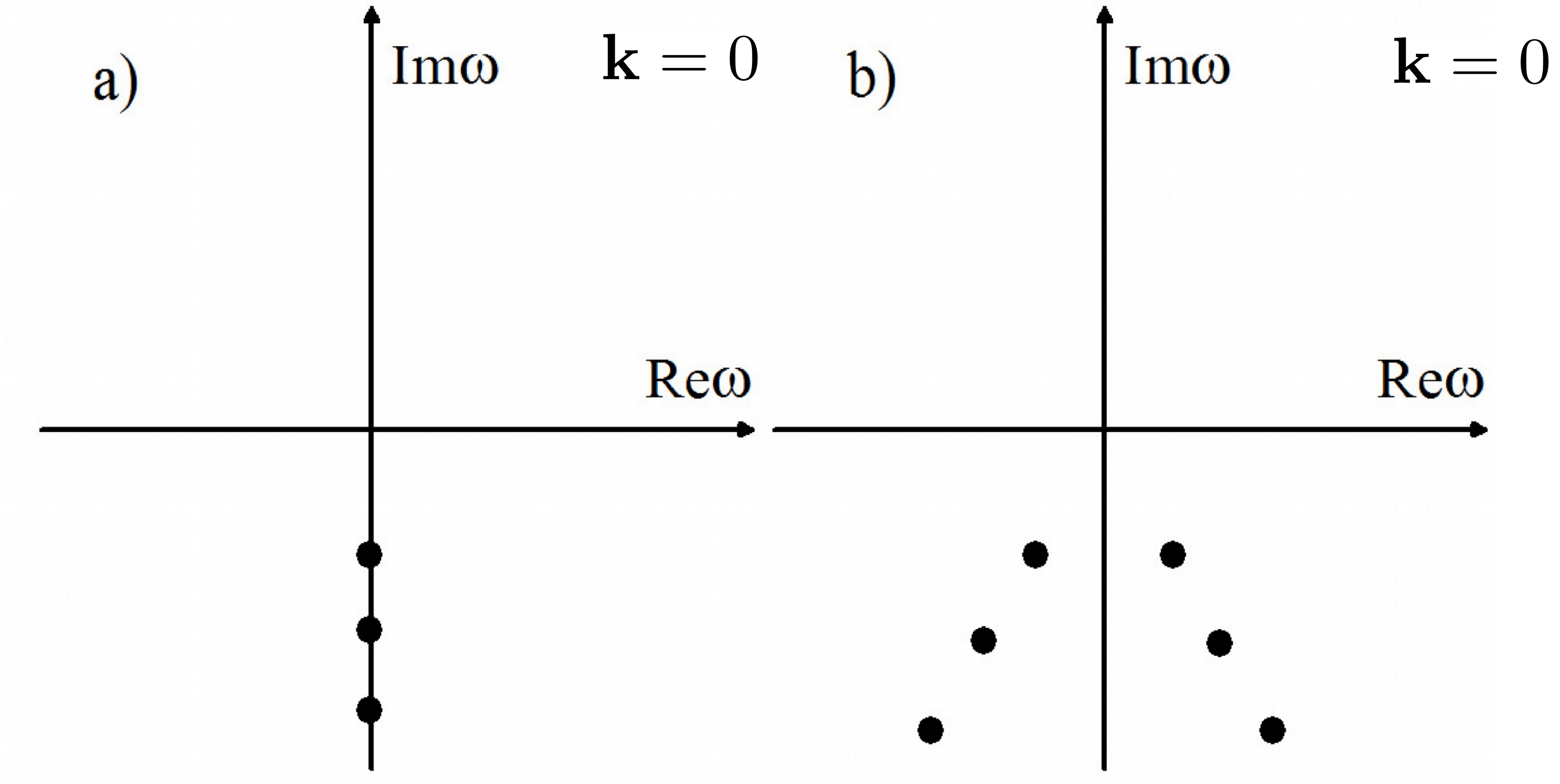
$$\lim_{\mathbf{k}\to 0} \omega_n(\mathbf{k}) \neq 0$$

Inclusion of higher n, extends the theory to describe higher frequencies

Systematic coarse gaining procedure leads to effective theories with varying DOF and nonhydrodynamic modes.

Eigenmodes of the fluid dynamic equations

Plot taken from Noronha&Denicol, arxiv:1104.2415



Weakly coupled theories based on the BE.
Denicol&Noronha&Niemi&Rischke, arxiv:1102.4780

Strongly coupled theories based on the Ads/CFT duality.
Starinets, arxiv:hep-th/0207133

Eigenmodes of the fluid dynamic equations

Generic conclusions

Strongly coupled plasmas: shear stress tensor relaxes in an oscillatory manner

Weakly coupled plasmas: shear stress tensor relaxes towards equilibrium without oscillating (exponentially damped)

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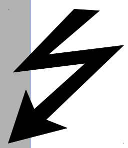

Main assumption

BE was expanded around an isotropic local equilibrium distribution

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arxiv:1102.4780

Goal

- Relax assumption of expansion of BE around an isotropic local equilibrium distribution
- What happens to the nonhydrodynamic modes? (Do they still lie on the negative imaginary axis of the complex frequency plane?)

Main ingredients

$$f_{\mathrm{a},\mathbf{k}} = f_{\mathrm{eq}}(\beta_{\mathrm{a}}\sqrt{m^2 + k_\perp^2 + (1+\xi)k_z^2})$$

$$\beta_{\mathrm{a}} \equiv 1/\Lambda$$

Related to the energy density at point x whose evolution is controlled by energy conservation, varies on hydrodynamic time scales.

Evolution of anisotropy parameter, like that of all other dissipative flows, is not constrained by conservation laws and thus happens on microscopic time scales.

$$\xi \sim \pi^{\mu\nu} \sim \mathrm{R}_\pi^{-1}$$

$$\mathrm{R}_\pi^{-1} \equiv |\pi^{\mu\nu}|/\mathcal{P}_0$$

- Adapt method of Denicol&Niemi&Molnar&Rischke, arxiv:1202.4551 to anisotropic systems

$\xi \sim \mathcal{O}(1)$ in our power counting scheme in terms of modified inverse Reynolds number and Knudsen number

$$\mathrm{Kn} \equiv \frac{\ell_{\mathrm{micro}}}{L_{\mathrm{macro}}}$$

- Consider only shear effects

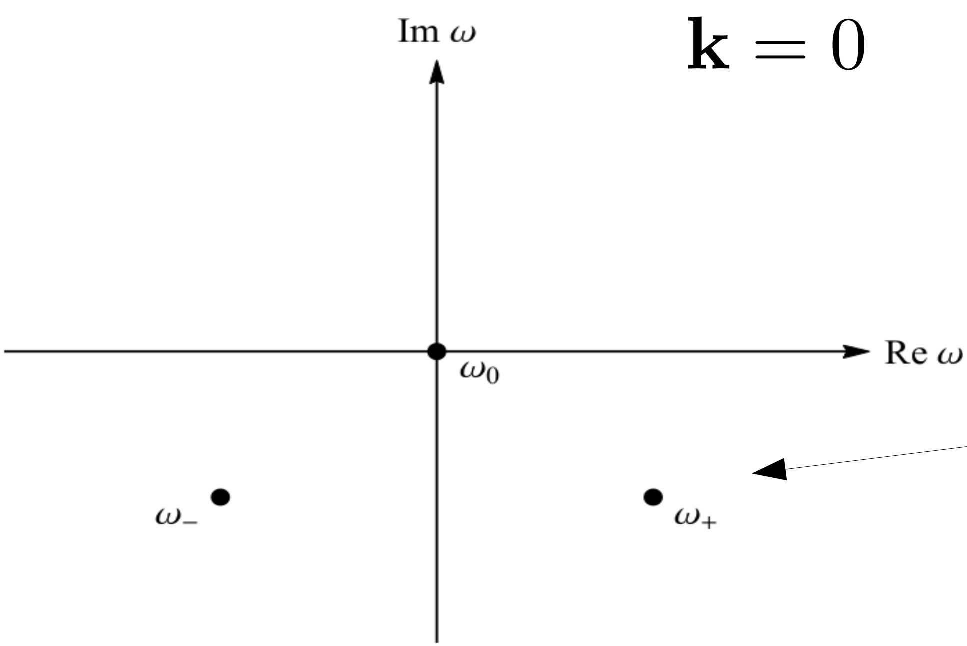
For example, $\mathrm{Kn} = \tau_\pi \theta$

Resulting equations of motion

$$(\tau_\pi^{(n)})^{\mu\nu}_{\alpha\beta}\dot{\tilde{\pi}}^{\alpha\beta} + (\Omega_{n0}^{(2)})^{\mu\nu}_{\alpha\beta}\tilde{\pi}^{\alpha\beta} + (\tau_\xi^{(n)})^{\mu\nu}\dot{\xi} + (\lambda_z^{(n)})^{\mu\nu\lambda}\dot{z}_\lambda$$
$$+(\mathcal{D}_2^{(n)})^{\mu\nu} + (\mathcal{D}_1^{(n)})^{\mu\nu}$$
$$= (\eta_\theta^{(2,n)})^{\mu\nu}\theta + (\eta_\sigma^{(2,n)})^{\mu\nu\alpha\beta}\sigma_{\alpha\beta} + (\eta_\omega^{(2,n)})^{\mu\nu\alpha\beta}\omega_{\alpha\beta}$$
$$+(\mathcal{J}^{(n)})^{\mu\nu} + (\mathcal{K}^{(n)})^{\mu\nu}$$

$$(\mathcal{D}_2^{(n)})^{\mu\nu} \supset \ddot{\xi}$$

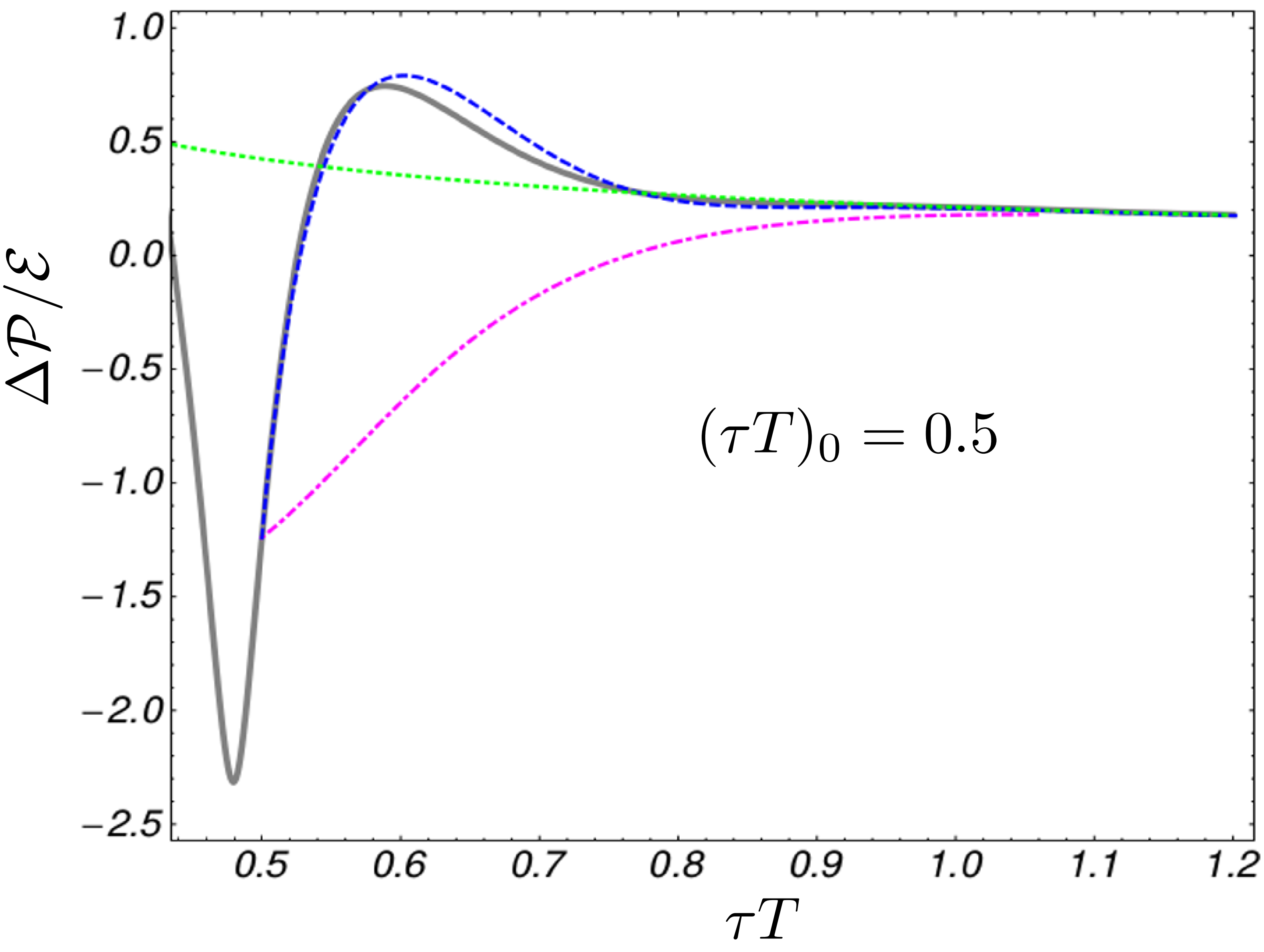
Second order time derivatives of ξ identify this anisotropy parameter as a quasi-normal mode of the non-equilibrium dynamics which undergoes transient oscillations that are damped on a microscopic time scale $(\tau_\xi^{(n)})^{\mu\nu}$



Transient oscillations from the BE

What can we learn about these transient oscillations?

- We do not have any estimates for their magnitudes yet
- So, lets see what happens in strongly coupled systems
- Heller&Janik&Spalinski&Witaszczyk, arxiv:1409.5087
  - studied the effect of including second-order time derivatives on the transition between pre-equilibrium dynamics at early and hydrodynamic behavior at later stages of a nuclear collision



Gray: numerical solution based on AdS/CFT;
Green: Navier-Stokes;
Red: Isreal-Stewart;
Blue: Inclusion of 2nd-order time derivatives

$$\Delta\mathcal{P} \equiv \mathcal{P}_\perp - \mathcal{P}_\mathrm{L}$$

Conclusions and outlook

- Viscous anisotropic hydrodynamics is a more efficient way to solve relativistic fluid dynamics for HIC
  - Especially at early times
- Expansion around an anisotropic distribution function leads to transient oscillations of the slowest nonhydrodynamic modes in the Boltzmann equation
  - These oscillations (present in both weakly and strongly coupled theories) might play a role in the early-time dynamics
  - Needed when larger Knudsen numbers are important

Backup

Strategy

- Adopt method of Denicol&Niemi&Molnar&Rischke, arxiv:1202.4551 to anisotropic systems
- Replace BE with an infinite hierarchy of moment equations.

$$\tilde{\rho}_r^{\mu_1\cdots\mu_\ell}(x) \equiv \int dK\,(u\cdot k)^r k^{\langle\mu_1}\cdots k^{\mu_\ell\rangle}\delta\tilde{f}_{\mathbf{k}}$$

See Molnar&Niemi&Rischke, arxiv:1602.00573 for a better expansion basis

- Now solving BE is equivalent to solving the dynamical equations for $\tilde{\rho}_r^{\mu\nu}$ (with $\ell = 2$ only for shear effects) by applying

$$\Delta^{\mu\mu}_{\alpha\beta}(u\cdot\partial)\,\mathrm{to}\,\tilde{\rho}_r^{\mu\nu}$$

Results in

$$\Delta^{\mu\nu}_{\alpha\beta}\dot{\tilde{\rho}}_r^{\alpha\beta} \equiv +\sum_{n=0}^{N_2}(\mathcal{A}^{(2)}_{rn})^{\mu\nu}_{\alpha\beta} = \mathcal{L}_r^{\mu\nu}\dot{\xi} + \mathcal{M}_r^{\mu\nu\lambda}\dot{z}_\lambda + (\alpha^{(2)}_{\sigma r})^{\mu\nu\lambda\rho}\sigma_{\lambda\rho} + \cdots$$

Main ingredient

- Asymptotic “Navier-Stokes” limit involves time derivatives of the anisotropy parameter

$$\rho_i^{\mu\nu} \simeq \Omega_{i0}^{(2)} \pi^{\mu\nu}$$

← Expansion around isotropic local equilibrium state

$$\tilde{\rho}_i^{\mu\nu} \simeq \Omega_{i0}^{(2)} \tilde{\pi}^{\mu\nu} + \hat{\ell}_i^{\mu\nu} \dot{\xi} + \hat{m}_i^{\mu\nu\lambda} \dot{z}_\lambda + \cdots$$

Plug back into the equation $\dot{\tilde{\rho}}_r^{\mu\nu}$

Results in second-order comoving time derivatives of the anisotropy parameter

Time derivative of LO distribution

$$\delta \dot{f} \equiv -\dot{f}_0 - (u{\cdot}p)^{-1}[p{\cdot}\nabla(f_0 + \delta f) - C[f]]$$

$$\dot{f}_0 \supset \dot{\beta}_0\,,\dot{u}^\mu$$

Replace with conservation laws in the form

$$\dot{\beta}_0 \equiv (J_{10}/D_{20})[(\mathcal{E}_0 + \mathcal{P}_0)\theta - \pi^{\mu\nu}\sigma_{\mu\nu}]$$

$$\dot{u}^\mu \equiv [\nabla^\mu \mathcal{P}_0 - \Delta^\mu_\alpha \partial_\beta \pi^{\alpha\beta}]/(\mathcal{E}_0 + \mathcal{P}_0)$$

Only spatial (in the LRF) gradients of the local thermodynamic fields remain

$$\delta \dot{\tilde{f}} \equiv -\dot{f}_{\rm aniso} - (u{\cdot}p)^{-1}[p{\cdot}\nabla(f_{\rm aniso} + \delta \tilde{f}) - C[f]]$$

$$\dot{\tilde{f}}_{\rm a} \supset \dot{\beta}_{\rm a}\,,\dot{u}^\mu\,,\dot{\xi}\,,\dot{z}^\mu$$

$\dot{\beta}_{\rm a}\,,\dot{u}^\mu$ Can be replaced with the conservation laws (similar expressions as above).

Nonhydrodynamic modes, no conservation laws to replace in terms of spatial gradients

$$\beta_{\rm a} \equiv \Lambda^{-1}$$

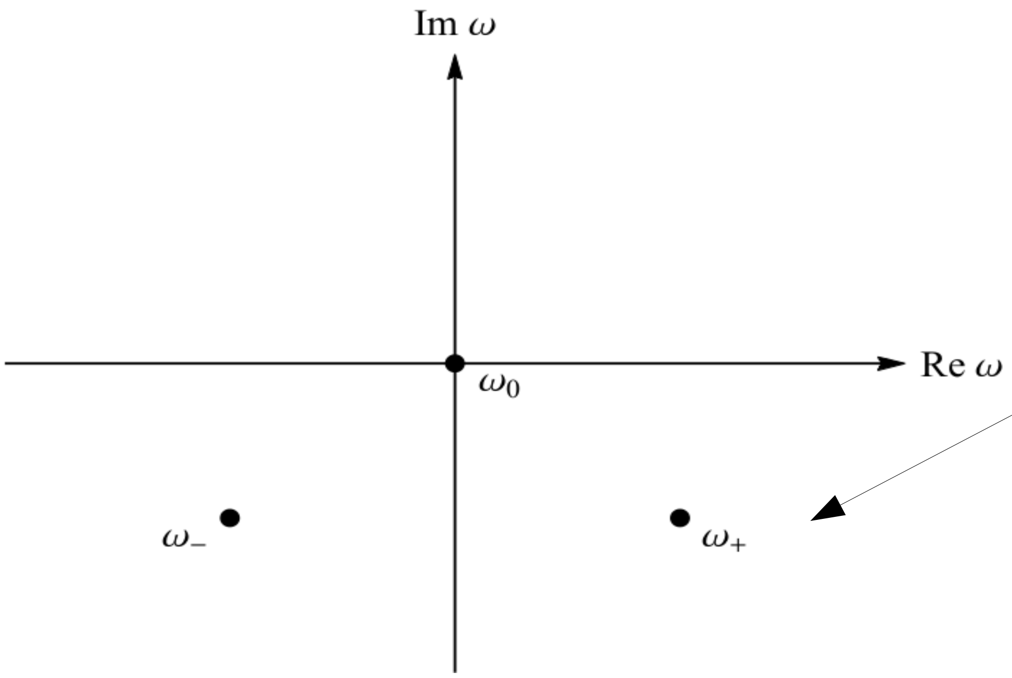
Eigenmode analysis

- Linearize equations of motion by considering a linear perturbation around a static background

$$\xi = \xi_0 + e^{i\omega t - ikz}\delta\xi \qquad \tilde{\pi}^{\mu\nu} = \tilde{\pi}_0^{\mu\nu} + e^{i\omega t - ikz}\delta\tilde{\pi}^{\mu\nu}$$

$$u^\mu = u_0^\mu + e^{i\omega t - ikz}\delta u^\mu \qquad \delta u^\mu = (0, 0, \delta u^y(t,z), 0)$$

Flow in the y-direction with a gradient in the z-direction



Transient oscillations from the BE

Shear channel eigenmodes in the zero wave number limit.

How well does viscous anisotropic hydro work?

Evolution equation for ξ

No kinetic definition for ξ

- For now, what's the easiest thing to do?
- Assume relaxation time approx. $C \equiv -\frac{u \cdot p}{\tau_{\rm eq}}(f - f_{\rm eq})$
- High-energy limit, we ignore chemical potential;

$\tilde{V}^\mu \equiv 0$

$\partial_\mu J^\mu$ no longer couples to dissipative currents. Use it.

↑

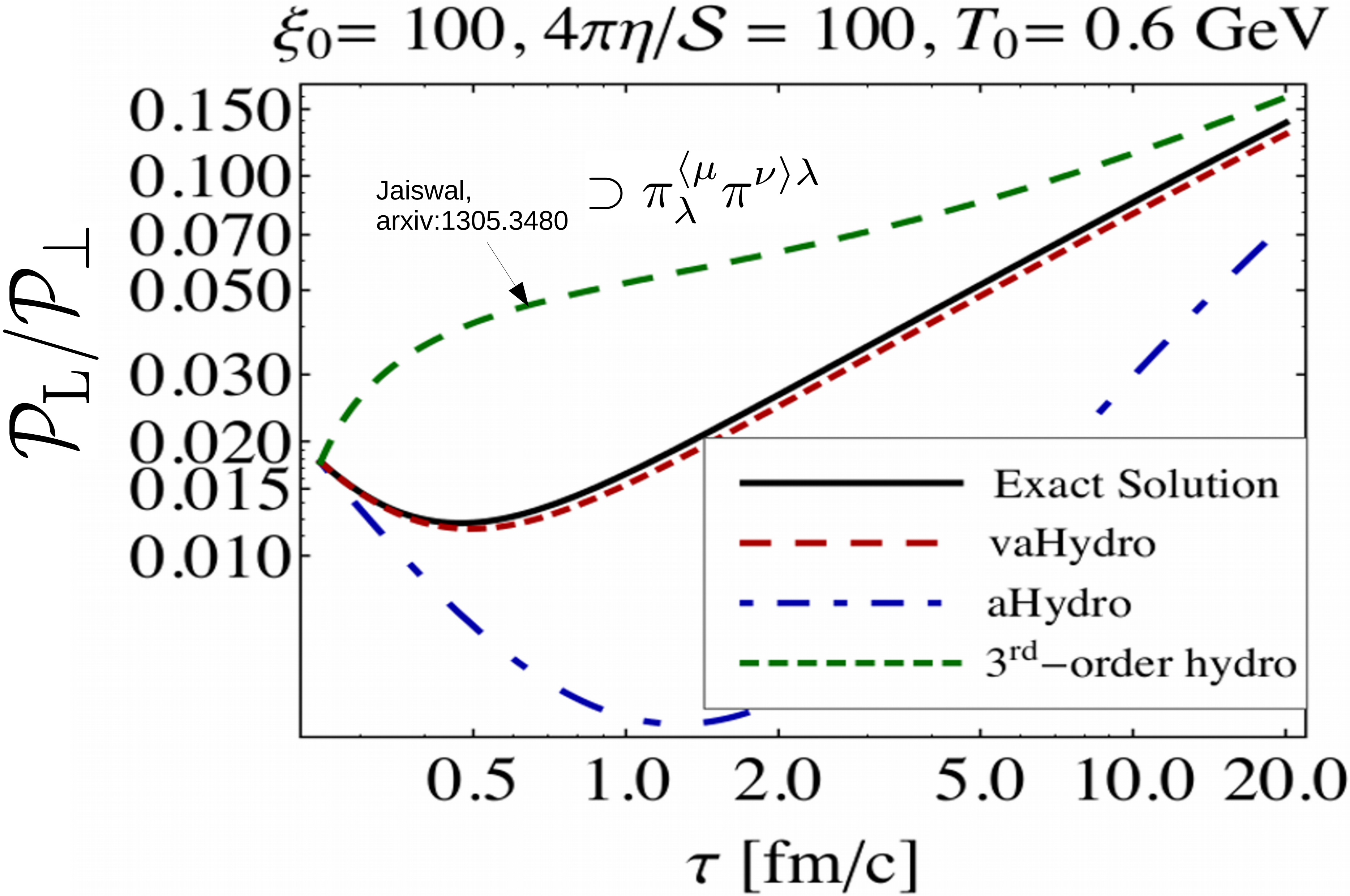
Caveat: no way to conserve particle number

$$\partial_\mu J^\mu = 0 \rightarrow \partial_\mu J^\mu = \mathcal{C}$$

← Non-vanishing source term.

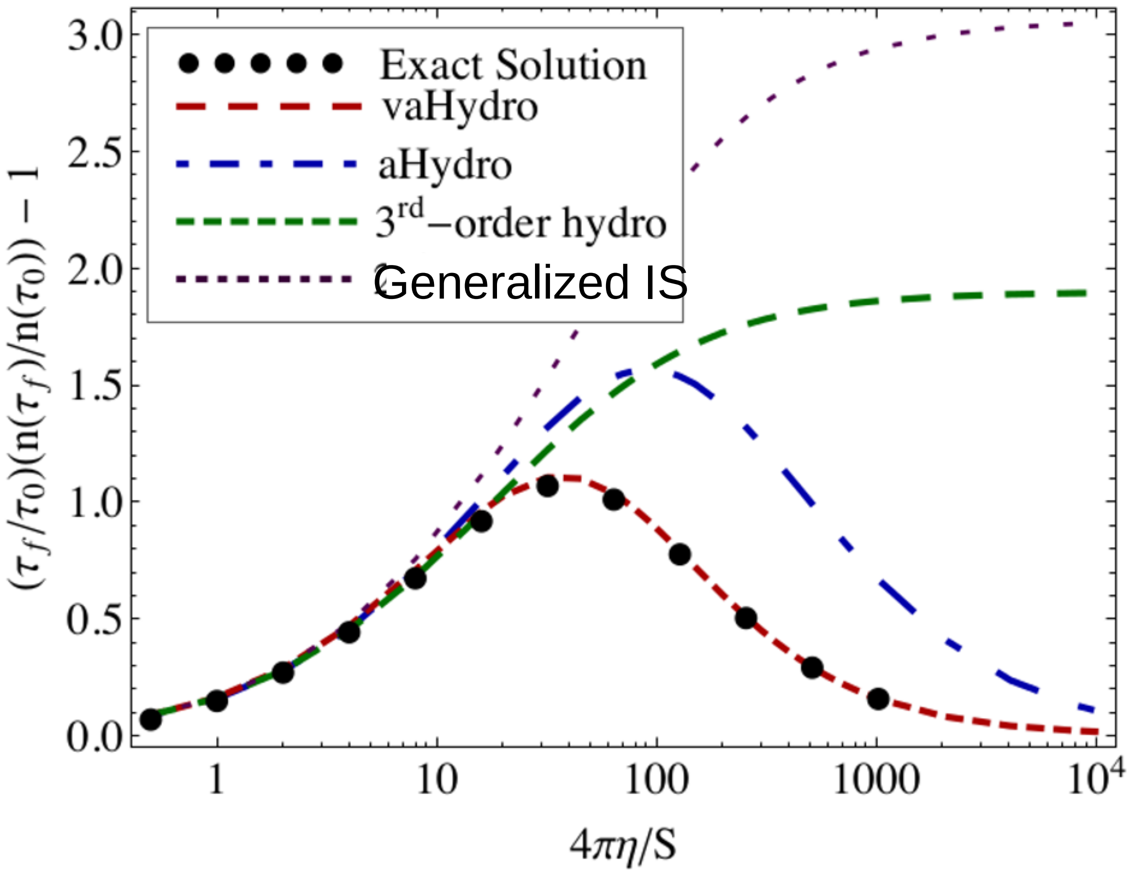
$$\mathcal{C} \equiv \int dP \, C[f]$$

$$\frac{\dot{\xi}}{1+\xi} + 6\frac{\dot{\Lambda}}{\Lambda} - 2\theta = 2\frac{1}{\tau_{\rm eq}}(1 - \sqrt{1+\xi}\mathcal{R}^{3/4}(\xi))$$



DB&Heinz&Strickland, arxiv:1311.6720

Entropy (particle) production



Generalized Isreal-Stewart hydrodynamics, Denicol&Koide&Rischke, arxiv:1004.5013

DB&Heinz&Strickland, arxiv:1311.6720