Initial conditions for hydrodynamics from weakly coupled pre-equilibrium evolution

Aleksas Mazeliauskas

September 17, 2016
Hot Quarks, South Padre Island

A.M., S. Schlichting, J.-F. Paquet and D. Teaney, (work in progress)

L. Keegan, A. Kurkela, A.M. and D. Teaney, JHEP 1608, 171 (2016) (perturbations)

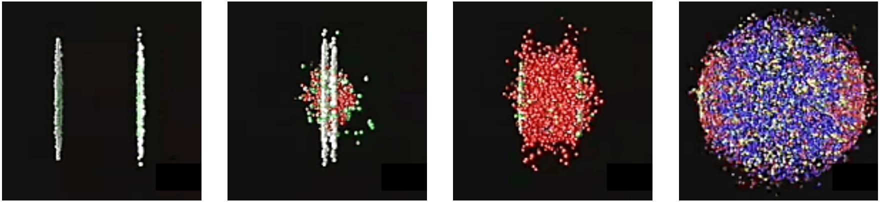
A. Kurkela, Y. Zhu, Phys. Rev. Lett. 115, 182301 (2015) (uniform background)

Stony Brook University

Motivation

Heavy ion collisions in a nutshell

pre-equilibrium



$\tau = 0$ $\qquad$ $\tau \sim 1\,\mathrm{fm}/c$ $\qquad$ $\tau \sim 10\,\mathrm{fm}/c$

colliding nuclei

microscopic
initial conditions
CGC, EKRT

QGP expansion
hydrodynamics

hadronization
UrMQD

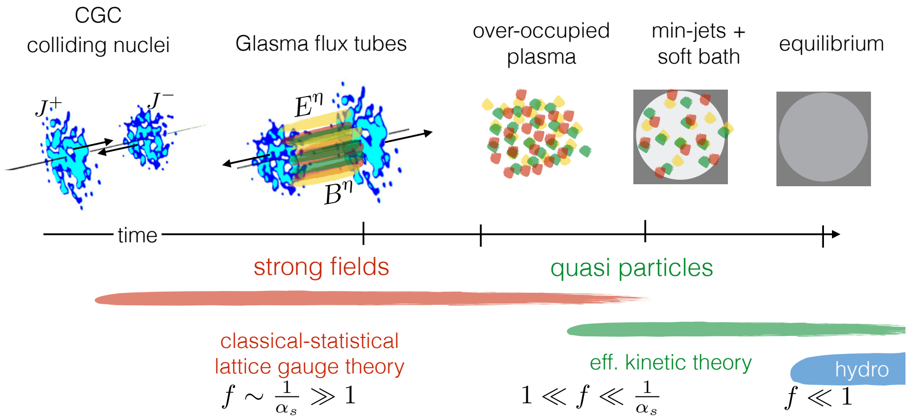
Goal: to understand transition from initial conditions to hydro expansion

Weak coupling picture of equilibration

$\alpha_s \ll 1$ — coupling constant

f — occupation number

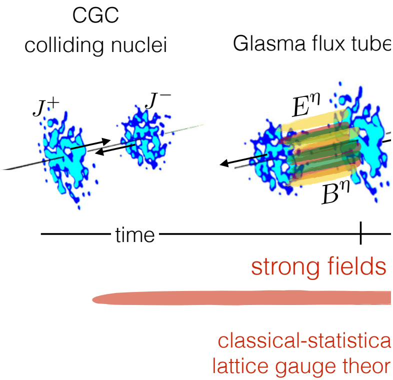
Sören Schlichting, Initial Stages 2016



Effective kinetic theory—bridge to hydrodynamics.

Weak coupling picture of equilibration

$\alpha_s \approx 0.1 - 0.3$ — coupling constant
f — occupation number



$f \sim \frac{1}{\alpha_s} \gg 1$ $\quad 1 \ll f \ll \frac{1}{\alpha_s}$ $\quad f \ll 1$

Effective kinetic theory—bridge to hydrodynamics.

Weak coupling picture of equilibration

$\alpha_s \ll 1$ — coupling constant
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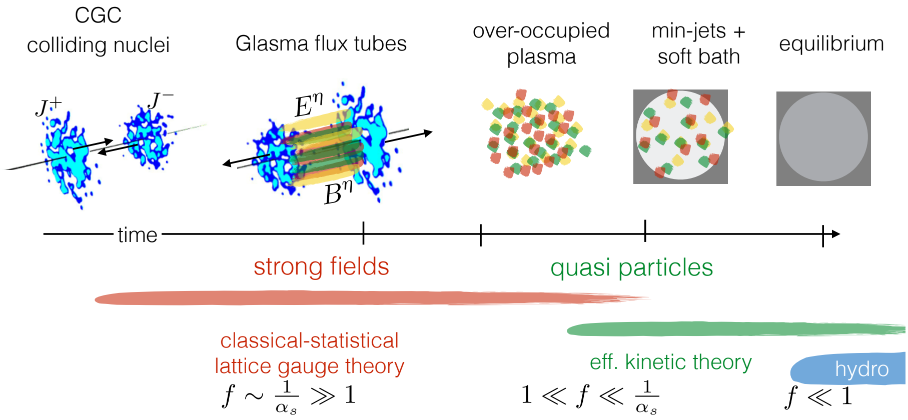
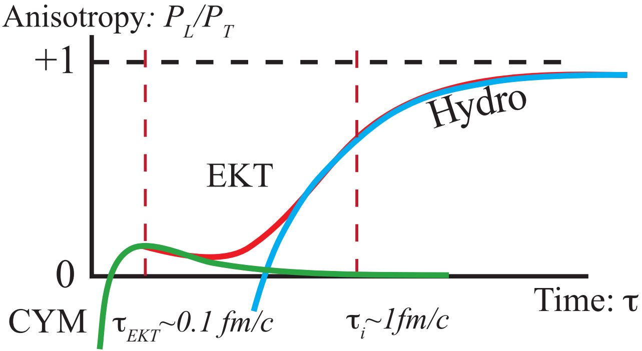


Effective kinetic theory—bridge to hydrodynamics.

Pre-equilibrium evolution in kinetic theory

From kinetic theory to hydro

CYM — classical Yang-Mills evolution
EKT — effective kinetic theory



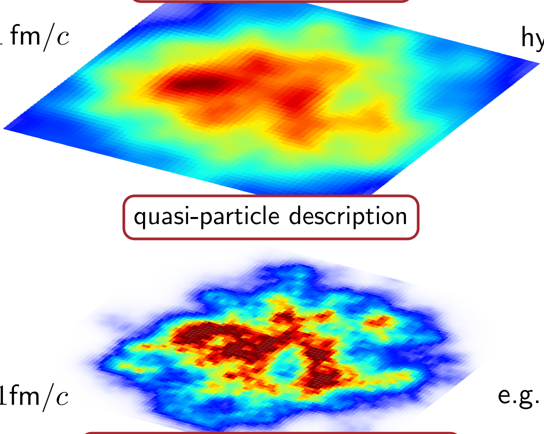
Use kinetic theory for a smooth approach to hydrodynamics.

Pre-equilibrium evolution

close to equilibrium,
onset of hydrodynamics

$\tau_{\text{init}} \sim 1\,\text{fm}/c$

hydro



τ

quasi-particle description

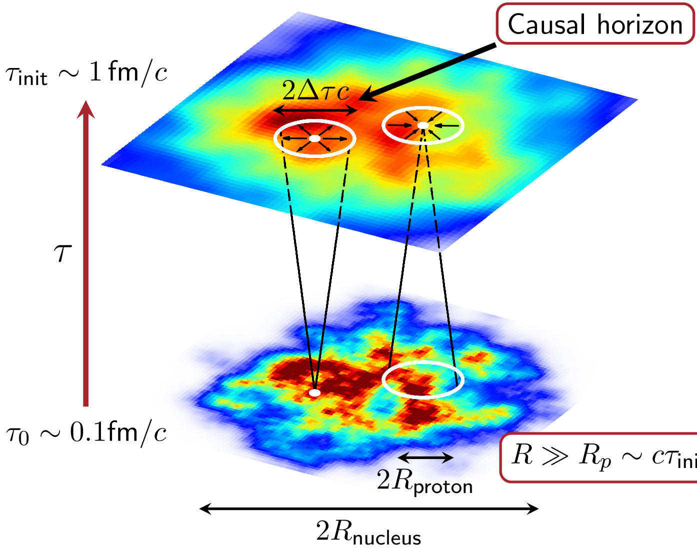
kinetic theory

$\tau_0 \sim 0.1\text{fm}/c$

e.g. CGC

far from equilibrium initial state,
strong QCD fields, CGC, CYM

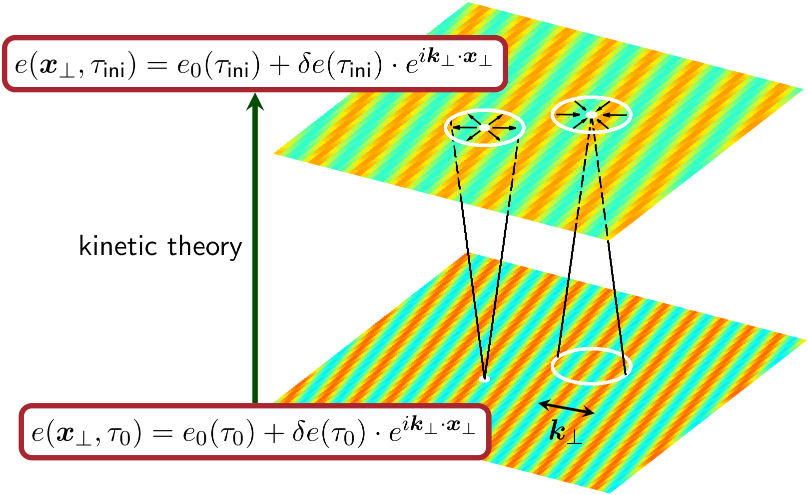
Pre-equilibrium evolution



$R \gg R_p \sim c\tau_{\text{init}} \sim l_{\text{mfp}}$

Linear perturbations in transverse plane

Expand around local energy density

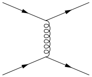


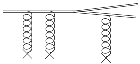
Kinetic theory for QGP

Bottom-up thermalization Baier, Mueller, Schiff, and Son (2001)
Effective kinetic theory Arnold, Moore, Yaffe (2003)

- Boltzmann equation for $f(\boldsymbol{p})$ – hard gluon distribution function.
- Spatial gradients, e.g. Bjorken expansion.
- Particle collisions:
  - elastic 2 to 2 scatterings between hard gluons
  - medium induced 1 to 2 colinear splittings
- Leading order accurate at weak coupling*.

$$\partial_\tau f + \frac{\boldsymbol{p}}{|p|}\cdot\nabla f - \underbrace{\frac{p_z}{\tau}\partial_{p_z} f}_{\text{Bjorken expansion}} = -\underbrace{\mathcal{C}_{2\leftrightarrow 2}[f]} - \underbrace{\mathcal{C}_{1\leftrightarrow 2}[f]},$$





LO acccurate in α_s, *here $\alpha_s \approx 0.3$.

Linear perturbations in transverse plane

Gluon distribution function at initial time

$$f(\tau, \boldsymbol{p}, \boldsymbol{x}_\perp) = \underbrace{\bar{f}_{\boldsymbol{p}}}_{\text{uniform background}} + \underbrace{\delta f_{\boldsymbol{k}_\perp, \boldsymbol{p}} e^{i\boldsymbol{k}_\perp \cdot \boldsymbol{x}_\perp}}_{\text{transverse perturbations}} .$$

Linearize Boltzmann equation in perturbations

$$(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z})\bar{f}_{\boldsymbol{p}} = -\mathcal{C}[\bar{f}] \qquad \text{background}$$

$$(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z} + \frac{i\boldsymbol{p}_\perp \cdot \boldsymbol{k}_\perp}{p})\delta f_{\boldsymbol{k}_\perp, \boldsymbol{p}} = -\delta\mathcal{C}[\bar{f}, \delta f] \qquad \text{perturbation}$$

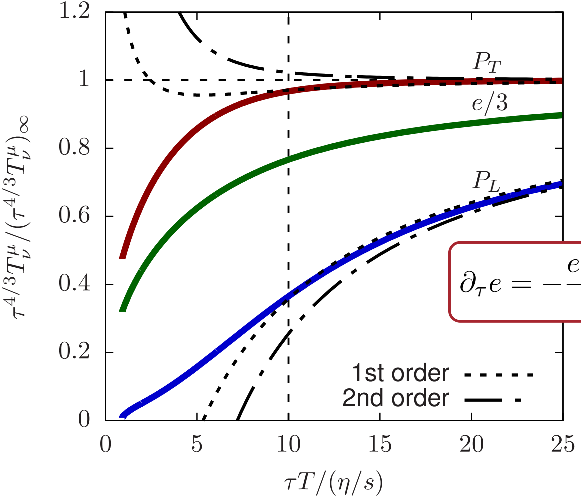
Study evolution of energy δe and momentum g^x perturbations.

$$\delta e(\tau, k) \equiv \delta T^{00} = \nu_g \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} p^0 \delta f, \tag{1}$$

$$g^x(\tau, k) \equiv \delta T^{0x} = \nu_g \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} p^x \delta f, \tag{2}$$

Background approach to hydro

Hydro constitutive equation: $T^{zz}(e) = P_L(e) = \frac{1}{3}e - \frac{4}{3}\frac{\eta}{\tau} - \frac{8}{9}\frac{\tau_\pi \eta - \lambda_1}{\tau^2}.$



$\eta/s = 0.63$

$\tau_\pi = 5.1\eta/sT$

$\lambda_1 = 0.8\eta\tau_\pi$

$$\boxed{\partial_\tau e = -\frac{e + P_L}{\tau}}$$

Background approach to hydro

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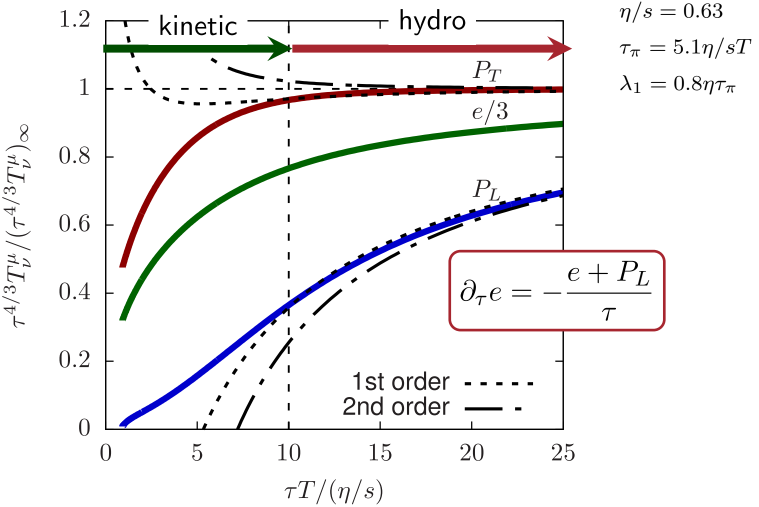
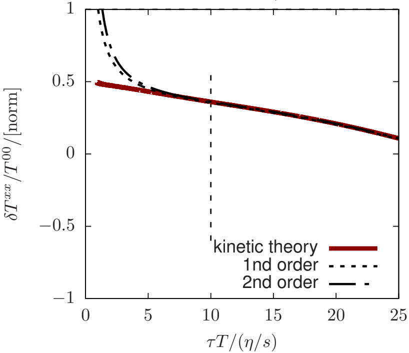


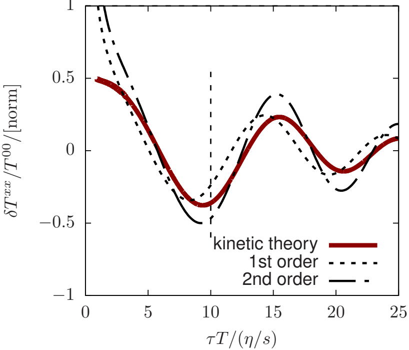
Equilibration of perturbations on expanding background

$$\delta T^{xx} = \underbrace{\frac{\delta e}{e}}_{\text{energy}} \left[\frac{1}{3}e + \frac{1}{3}\eta\tau_\pi k^2 + \frac{\eta}{2\tau} - \frac{2(\lambda_1 - \eta\tau_\pi)}{9\tau^2}\right] - \underbrace{\frac{ikg^x}{e}}_{\text{momentum}} \left[\eta - \frac{1}{\tau}\left(\frac{\eta^2}{2e} + \frac{\eta\tau_\pi}{2} - \frac{2}{3}\lambda_1\right)\right]$$

$k \sim 1/R_{\text{nucleus}}$



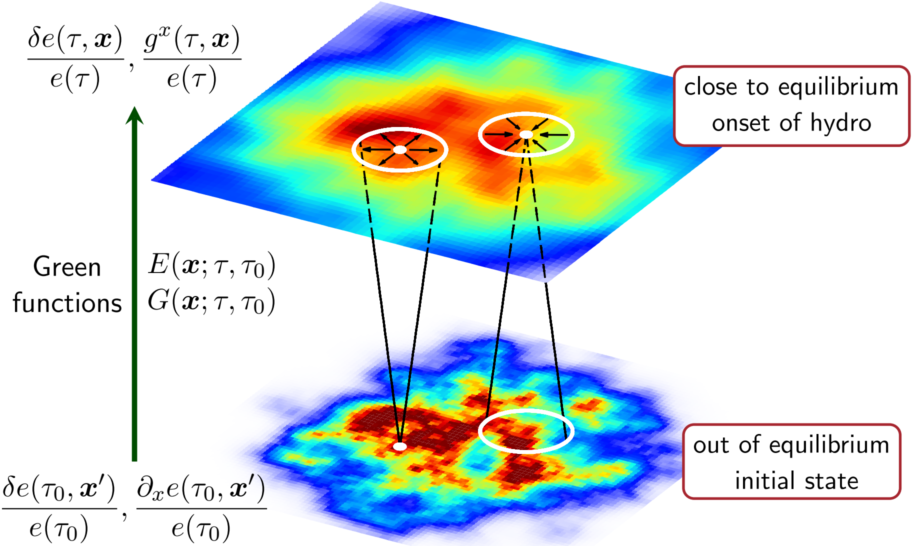
$k \sim 1/R_{\text{proton}}$



Green functions

Pre-equilibrium evolution from kinetic theory

Kinetic theory response functions to initial perturbations and gradients.



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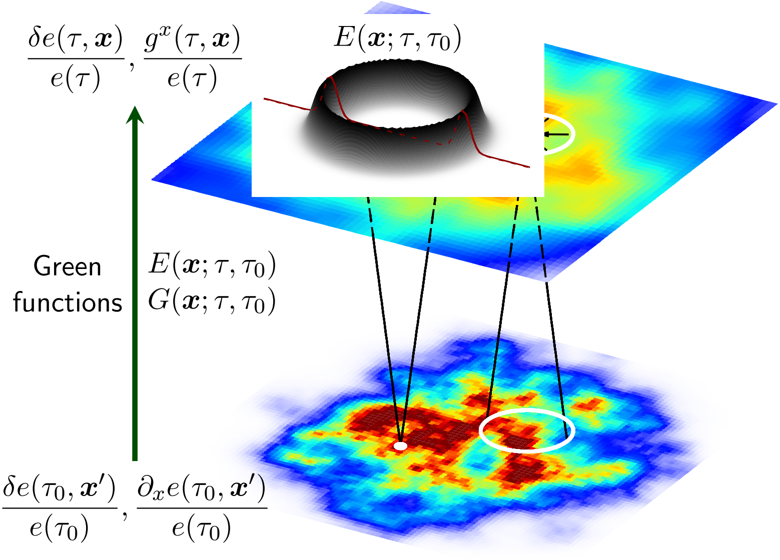
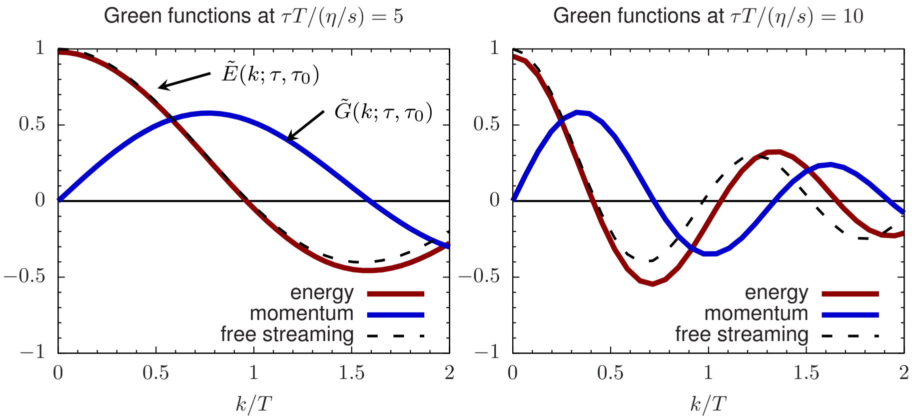


Green functions in $\boldsymbol{k}$ space

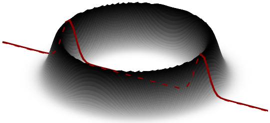
$$\frac{\delta e(\tau,k)}{e(\tau)} \equiv \underbrace{\tilde{E}(k;\tau,\tau_0)}_{\text{energy resp.}} \times \frac{\delta e(\tau_0,k)}{e(\tau_0)}, \qquad \frac{g^x(\tau,k)}{e(\tau)} \equiv -i \underbrace{\tilde{G}(k;\tau,\tau_0)}_{\text{momentum resp.}} \times \frac{\delta e(\tau_0,k)}{e(\tau_0)}.$$



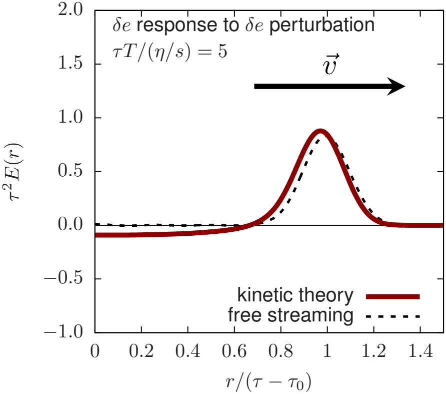
Now Fourier transform $\tilde{E}(k,\tau,\tau_0)$ to coordinate space.

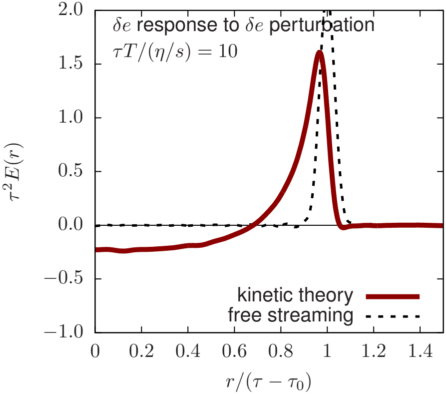
Energy Green function in coordinate space

Convolve energy perturbations with response kernel.



$$\frac{\delta e(\tau,\boldsymbol{x})}{e(\tau)} = \int d^2\boldsymbol{x}' E(|\boldsymbol{x}-\boldsymbol{x}'|;\tau,\tau_0) \times \frac{\delta e(\tau_0,\boldsymbol{x}')}{e(\tau_0)}$$





Energy Green function in coordinate space

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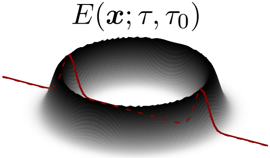
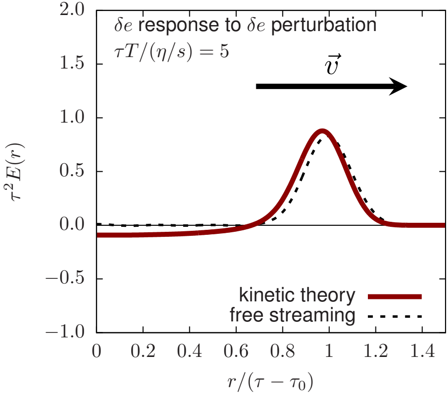
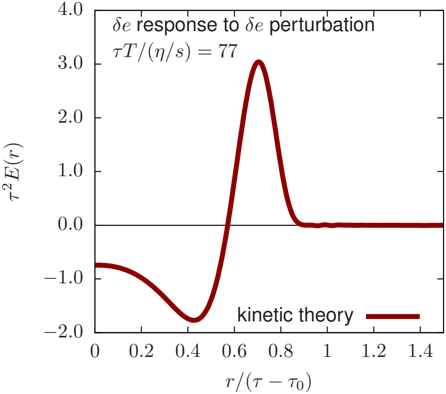
$$\frac{\delta e(\tau, \boldsymbol{x})}{e(\tau)} = \int d^2\boldsymbol{x}' E(|\boldsymbol{x} - \boldsymbol{x}'|; \tau, \tau_0) \times \frac{\delta e(\tau_0, \boldsymbol{x}')}{e(\tau_0)}$$





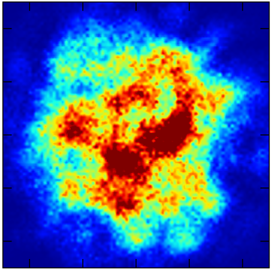


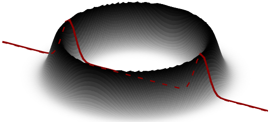
Conclusion

Effective kinetic theory:

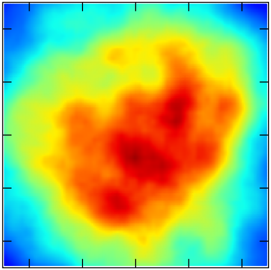
- Studied equilibration of transverse δe and g^x perturbations.
- Showed transition to hydro constituent equations $T^{ij}(e, \delta e, \vec{g})$.
- Constructed pre-equilibrium response functions.

$$\int d^2\boldsymbol{x}' \underbrace{\frac{\delta e(\tau_0,\boldsymbol{x}')}{e(\tau_0)}, \frac{\vec{\nabla} e(\tau_0,\boldsymbol{x}')}{e(\tau_0)}} \times \underbrace{E(|\boldsymbol{x}-\boldsymbol{x}'|;\tau,\tau_0)} = \underbrace{\frac{\delta e(\tau,\boldsymbol{x})}{e(\tau)}, \frac{\vec{g}(\tau,\boldsymbol{x})}{e(\tau)}}$$





Pre-equilibrium smearing and preflow generation



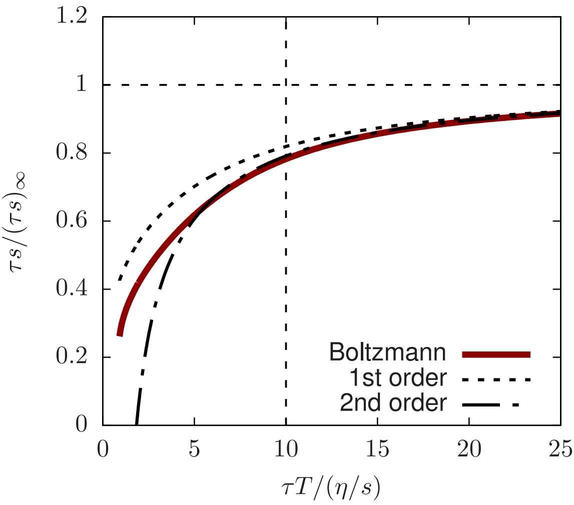
Outlook

- Implement kinetic pre-equilibrium in actual heavy ion simulations.
  - Isotropization of IPGlasma initial conditions.
  - Smooth transition to hydrodynamics.
  - Out of equilibrium preflow.
- Inclusion of quarks (chemical equilibration).
  - Out of equilibrium photon production.

Backup

Background entropy evolution

Boltzmann entropy $s = -\int \frac{d^3p}{(2\pi)^3}[f(\boldsymbol{p})\ln f(\boldsymbol{p}) - (1+f(\boldsymbol{p}))\ln(1+f(\boldsymbol{p}))]$.

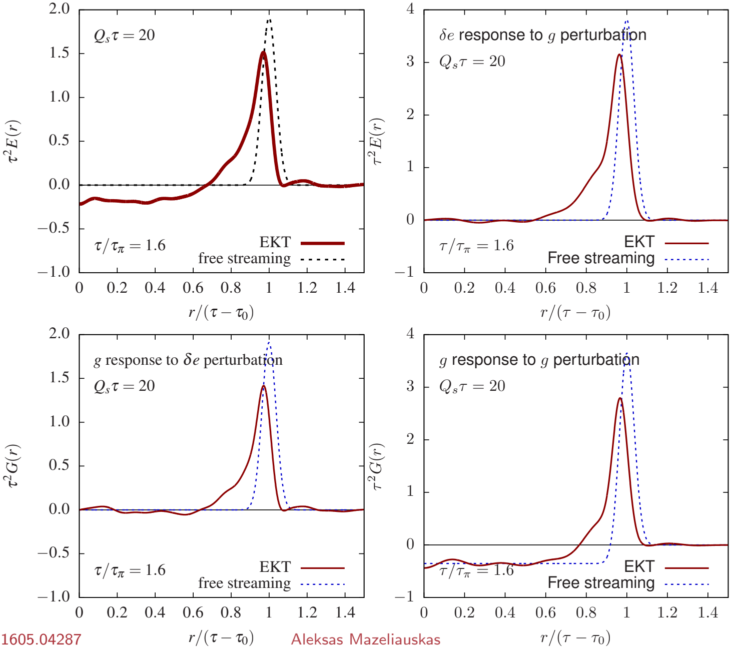


$\eta/s = 0.63$

$\tau_\pi = 5.1\eta/sT$

$\lambda_1 = 0.8\eta\tau_\pi$

Complete prethermal evolution map



Initial gluon distribution

Initial conditions motivated by classical simulations Lappi (2011)

$$f(p_z, p_\perp) = \frac{2}{\lambda} A f_0(p_z \xi / p_0, p_\perp / p_0),$$
$$f_0(\hat{p}_z, \hat{p}_\perp) = \frac{1}{\sqrt{\hat{p}_\perp^2 + \hat{p}_z^2}} e^{-2(\hat{p}_\perp^2 + \hat{p}_z^2)/3},$$

$p_0 = 1.8\, Q_s$, $\xi = 10$, and $\lambda = 10$.

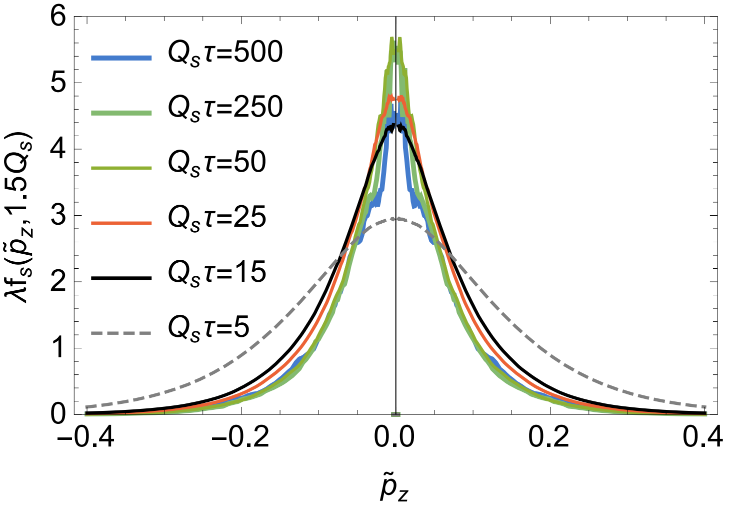
- $\sqrt{\langle p_T^2 \rangle} \approx 1.8\, Q_s$
- $\langle p_z^2 \rangle \ll \langle p_T^2 \rangle$
- A – energy per rapidity matches classical simultions

Energy perturbation $Q_s(\boldsymbol{x}_\perp) \sim Q_s + \delta Q_s e^{i \boldsymbol{k}_\perp \cdot \boldsymbol{x}_\perp}$.

$$\delta f^{(1)}_{\boldsymbol{k}_\perp, \boldsymbol{p}}(\tau_0) = -\frac{\delta Q_s}{Q_s} p\, \partial_p \bar{f}_{\boldsymbol{p}},$$

Scaling of distribution function

Scaling solution with $\tilde{p}_z = (Q_s\tau)^{1/3} p_z$ reached after $Q_s\tau = 15$
(c.f. Berges et al.) Kurkela and Zhu (2015)

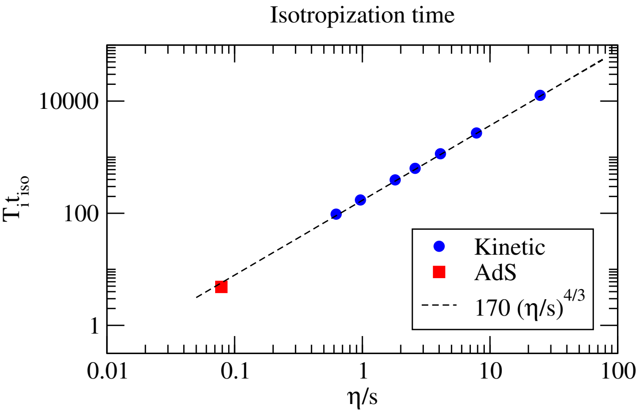


Scaling in weakly coupled theories

Isotropization time scales well with viscosity

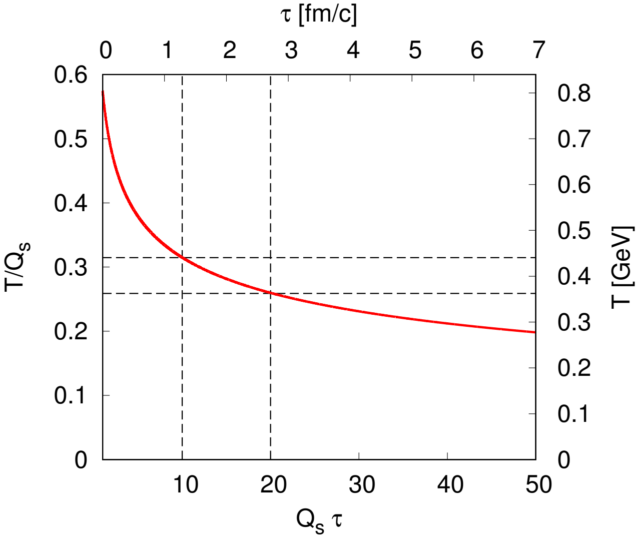
$$T_i t_{\mathsf{iso}} \approx 154 \ldots 183 (\eta/s)^{4/3} \tag{3}$$

L. Keegan, A. Kurkela, P. Romatschke, W. van der Schee and Y. Zhu (2015)



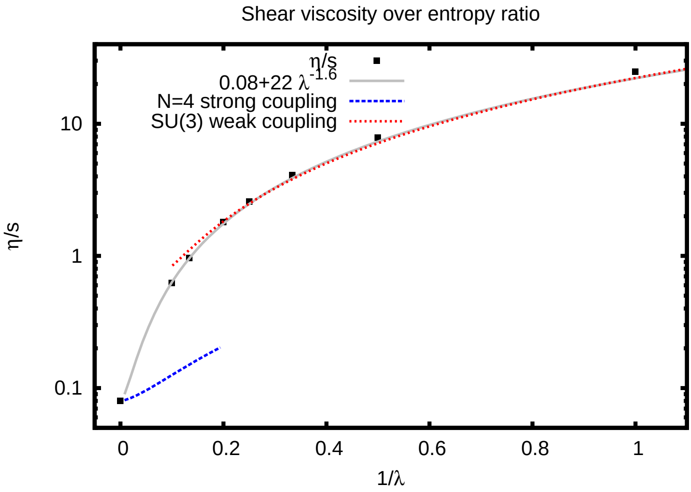
Temperature evolution and entropy matching

At switching time $T \sim 3T_c$



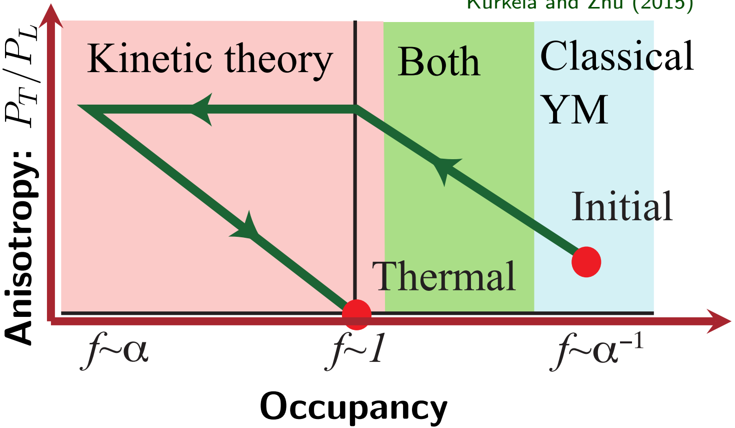
Equilibration in weak and strong coupling

L. Keegan, A. Kurkela, P. Romatschke, W. van der Schee and Y. Zhu (2015)

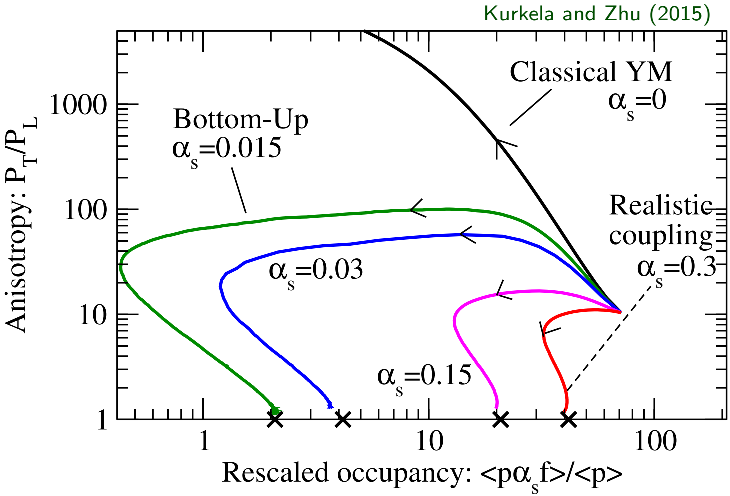


Isotropization of background gluonic fields

Kurkela and Zhu (2015)

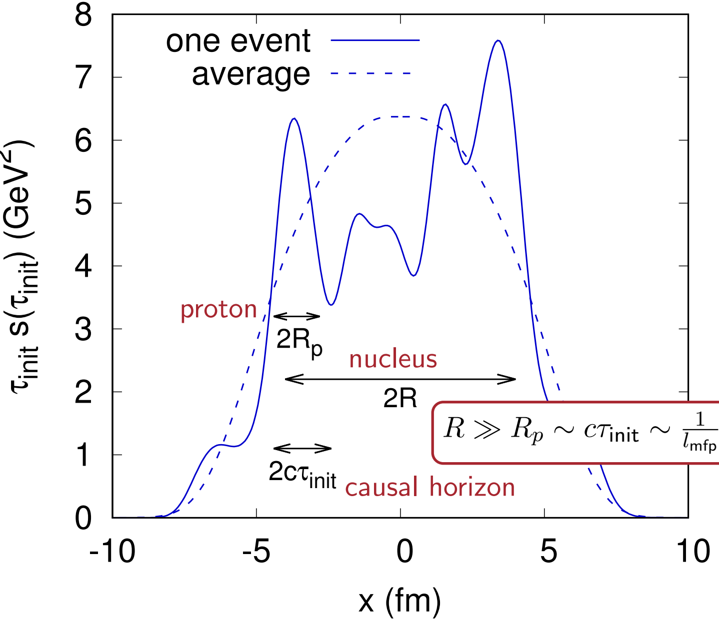


Isotropization of background gluonic fields



Perturbatively controlled approach to hydrodynamics!

Initial perturbations and preflow



$$R \gg R_p \sim c\tau_{\text{init}} \sim \frac{1}{l_{\text{mfp}}}$$

Far-from-equilibrium perturbations

Linearized kinetic theory

$$(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z})\bar{f}_{\boldsymbol{p}} = -\mathcal{C}[\bar{f}], \qquad \text{uniform background}$$

$$(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z} + \frac{i\boldsymbol{p}_\perp \cdot \boldsymbol{k}_\perp}{p})\delta f_{\boldsymbol{k}_\perp,\boldsymbol{p}} = -\delta\mathcal{C}[\bar{f},\delta f], \quad \text{transverse perturbation}$$

Out of equilibrium initial conditions at $\tau_0 = 1/Q_s$

- Background: anisotropic gluon distribution $\langle p_z^2 \rangle \ll \langle p_\perp^2 \rangle$

$$\bar{f}_{\boldsymbol{p}}(\tau_0) = f\left(\frac{p}{Q_s}\right), \quad Q_s \text{ – initial momentum scale}$$

- Perturbation: $Q_s(\boldsymbol{x}_\perp) = Q_s + \delta Q_s e^{i\boldsymbol{k}_\perp \cdot \boldsymbol{x}_\perp}$.

$$\delta f_{\boldsymbol{k}_\perp,\boldsymbol{p}}(\tau_0) = -\frac{\delta Q_s}{Q_s} p\,\partial_p \bar{f}_{\boldsymbol{p}}$$