Phenomenological Predictions of 3+1d Anisotropic Hydrodynamics

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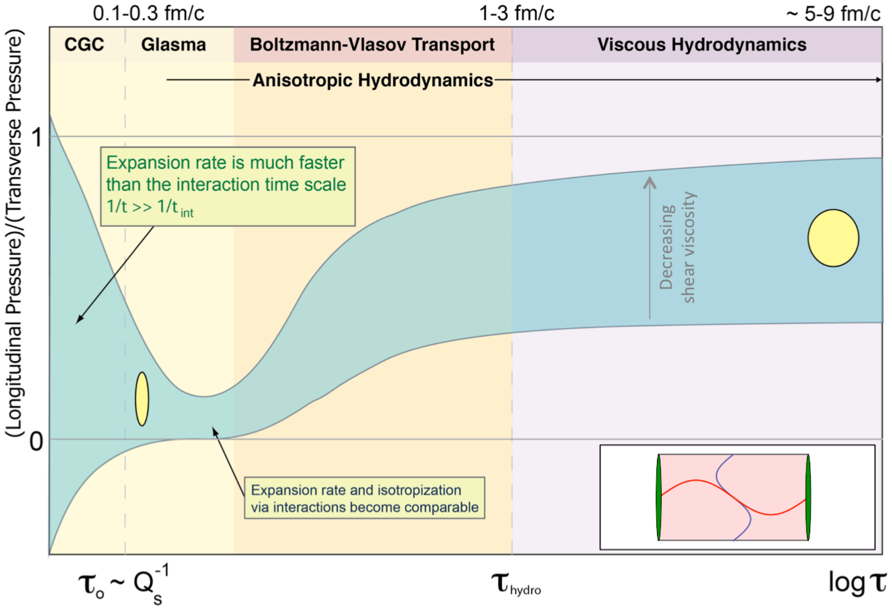
Motivation-1

- Large elliptic flow $\rightarrow$ strong hydrodynamics response of system to the geometry of initial state
- Relativistic Hydrodynamics is a remarkable tool to study QGP in heavy-ion collisions $\tau_0 < \tau < \tau_{\text{freezeout}}$
- Various Hydro approaches:
  - Ideal Hydro
  - vHydro: NS, IS, DNMR
- vHydro formalisms are based on linearization around isotropic equilibrium state
- QGP possesses strong momentum-space anisotropy in LRF (longitudinal expansion) and this casts down on the reliability the application of vHydro at early times, transverse edges, and so on

Motivation-2

- aHydro takes into account the momentum-space anisotropy in the local rest frame at leading order
- The goal of aHydro is to study
    - Early time dynamics
    - Small systems (p+A, p+p)
    - Dynamics near the transverse edges of the overlap region (dilute)
    - Dynamics at forward rapidity (dilute)
    - Temperature-dependent (and potentially large) η/s

QGP Momentum-Space Anisotropy



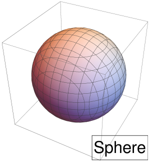
vHydro vs aHydro

$$\boxed{p^\mu \partial_\mu f = -\mathcal{C}[f]} \qquad \boxed{\mathcal{I}^{\mu_1\mu_2\ldots\mu_n} = \int_p p^{\mu_1} p^{\mu_2} .. p^{\mu_n} f(x,p)} \qquad \int_p \equiv \int \frac{d^3p}{(2\pi)^3 E}$$

- vHydro: Linearization around isotropic equilibrium state

$$f(x,p) = f_{\rm eq}\left(\frac{p\cdot u}{T}\right)(1 + \delta f(x,p))$$

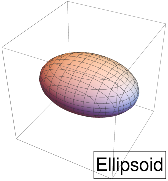
$$\to T^{\mu\nu} = \int_p p^\mu p^\nu f(x,p) = T^{\mu\nu}_{\rm eq} + \boxed{\Pi^{\mu\nu}} \Rightarrow \boxed{\pi^{\mu\nu} + \Delta^{\mu\nu}\Pi}$$



- aHydro: Linearization around anisotropic state

$$f(x,p) = f_{\rm aniso}\left(\frac{p\cdot u}{T}\right)\left[1 + \delta f(x,p)\right]$$

$$\to T^{\mu\nu} = \int_p p^\mu p^\nu f(x,p) = T^{\mu\nu}_{\rm aniso} + \tilde{\Pi}^{\mu\nu}$$



Anisotropic Hydrodynamics Setup

- aHydro distribution function is defined as [1]

$$\boxed{f(x,p) = f_{\rm eq}\left(\frac{1}{\lambda}\sqrt{p_\mu \Xi^{\mu\nu} p_\nu}\right)}$$

$$\boxed{\Xi^{\mu\nu} \equiv u^\mu u^\nu + \xi^{\mu\nu} - \Delta^{\mu\nu}\Phi}$$

$$\xi^{\mu\nu} \equiv \mathsf{diag}(0, \boldsymbol{\xi}),\ \xi^i_i = 0,\ u_\mu \xi^{\mu\nu} = 0.$$

- For simplicity, we take the collisional kernel in RTA

$$\boxed{\mathcal{C}[f] \equiv \frac{p\cdot u}{\tau_{\rm eq}}(f - f_{\rm eq})}$$

- Taking moments of BE obtains dynamical equations for $\boldsymbol{\xi}$, $\boldsymbol{u}$, λ, and T

$$\begin{aligned}
\partial_\mu J^\mu &= -\int dP\,\mathcal{C}[f] && \Rightarrow \text{one equation} \\
\partial_\mu T^{\mu\nu} &= -\int dP\,p^\nu \mathcal{C}[f] = 0 && \Rightarrow \text{four} + \text{one equations} \\
\partial_\mu \mathcal{I}^{\mu\nu\lambda} &= -\int dP\,p^\nu p^\lambda \mathcal{C}[f] && \Rightarrow \text{ten equations}
\end{aligned}$$

[1] chemical potential is assumed to be zero for now

Equation of State in Anisotropic Hydrodynamcis

$$\mathbf{vHydro} \rightarrow \quad T^{\mu\nu}_{\rm ideal} = \mathcal{E} u^\mu u^\nu \quad + \ \mathcal{P} X^\mu X^\nu + \mathcal{P} Y^\mu Y^\nu + \mathcal{P} Z^\mu Z^\nu$$

$$\mathbf{aHydro} \rightarrow \quad T^{\mu\nu}_{\rm aniso} = \mathcal{E}_{\rm aniso} u^\mu u^\nu + \mathcal{P}_x X^\mu X^\nu + \mathcal{P}_y Y^\mu Y^\nu + \mathcal{P}_z Z^\mu Z^\nu$$

- EoS is defined as the relation connecting **isotropic part of** energy density and pressure
- There are two typical approaches for imposing EoS in aHydro
  - Quasiparticle approach: Massive ahydro equations are suplemented by obtaining $m(T)$ from lattice data[2]
  - Standard approach: Massless ahydro equations are closed by obtaining $\mathcal{E}_{\rm iso}(\mathcal{P}_{\rm iso})$ from lattice data

$$\begin{aligned} \mathcal{E} &= \mathcal{E}_{\rm iso}(\lambda)\mathcal{H}(\boldsymbol{\xi}) \\ \mathcal{P}_i &= \mathcal{P}_{\rm iso}(\lambda)\mathcal{H}_i(\boldsymbol{\xi}) \end{aligned}$$

- Herein, we obtain EoS from Krakow parametrization of lattice QCD[3]

[2] See talk by Mubarak Alqahtani

[3] arXiv:0702030

"Cooper Frye" Freezeout in aHydro

$$\boxed{\left(p^0 \frac{dN}{dp^3}\right)_i = \frac{\mathcal{N}_i}{(2\pi)^3} \int f_i(x,p) p^\mu d\Sigma_\mu}$$

- $d\Sigma_\mu$ are constant-temperature 3d hypersurfaces $(T_{\rm FO} = 150 {\rm MeV})$
- The freezeout scheme is based on the ellipsoidal distribution function

$$\boxed{f(x,p) = f_{\rm eq}\left(\frac{1}{\lambda}\sqrt{p_\mu \Xi^{\mu\nu} p_\nu}\right)}$$

- The freezeout in $2^{\rm nd}$ order vHydro suffers from unphysical behaviors[4]

$$\boxed{f(x,p) = f_{\rm eq} + \delta f_\pi + \delta f_\Pi}$$

$$\delta f_\pi = f_{\rm eq}(1 - a f_{\rm eq}) \frac{p_\mu p_\nu \pi^{\mu\nu}}{2(\mathcal{E} + \mathcal{P})T^2}$$

$$\delta f_\Pi = -f_{\rm eq}(1 - a f_{\rm eq}) \left[\frac{m_i^2}{3E} - \left(\frac{1}{3} - c_s^2\right)\right] \frac{\Pi}{C_\Pi}$$

- Herein, we use the THERMINATOR-2 for freezeout by including 371 hadronic resonances $(M < 2.6\,{\rm GeV})$[5]

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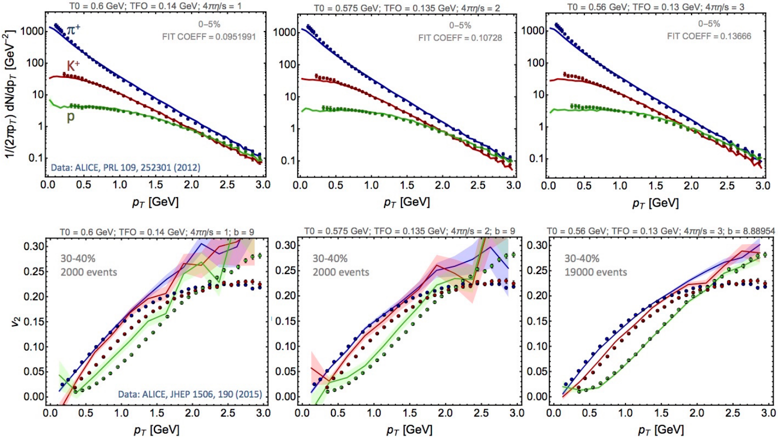
[4] arXiv:0301099 and arXiv:1408.0024

[5] arXiv:1102.0273

Preliminary results – Glauber ICs

Florkowski, Haque, Nopoush, Ryblewski, MS, forthcoming

LHC 2.76 TeV Pb+Pb collisions; top row shows spectra, bottom row shows differential v_2



Conclusions and Discussions

- We have built a 3+1d anisotropic hydrodynamics code for studying QGP with realistic EoS and consistent freezeout scheme
- We have some tuning parameters, i.e. T_0, T_{FO}, and η/s
- The fitting to the data for particle spectra and v_2 is quite good
- Our preliminary findings suggest that $\eta/s \simeq 0.23$ which is different from a vHydro study[6] with $\eta/s \simeq 0.095$
- Future plan: Including
  - off-diagonal components of anisotropy tensor
  - effect of finite chemical potential
  - NLO aHydro
  - fluctuating IC

[6] arXiv:1502.01675