Hot Quarks 2016

Non-equilibrium properties of the QGP in a magnetic field: a holographic approach

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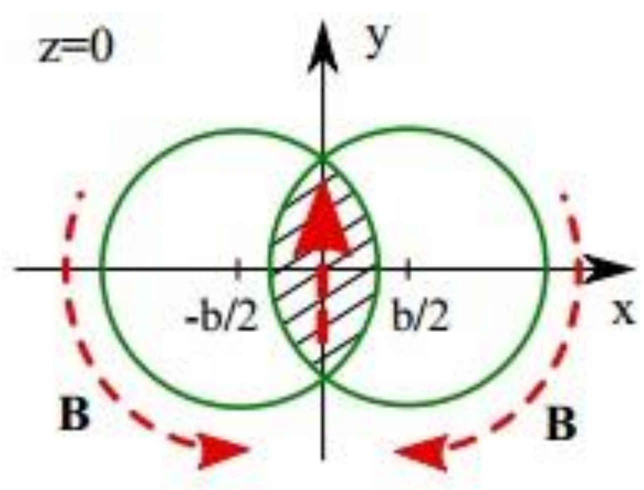
South Padre Island- TX, 12/09/2016.

Outline

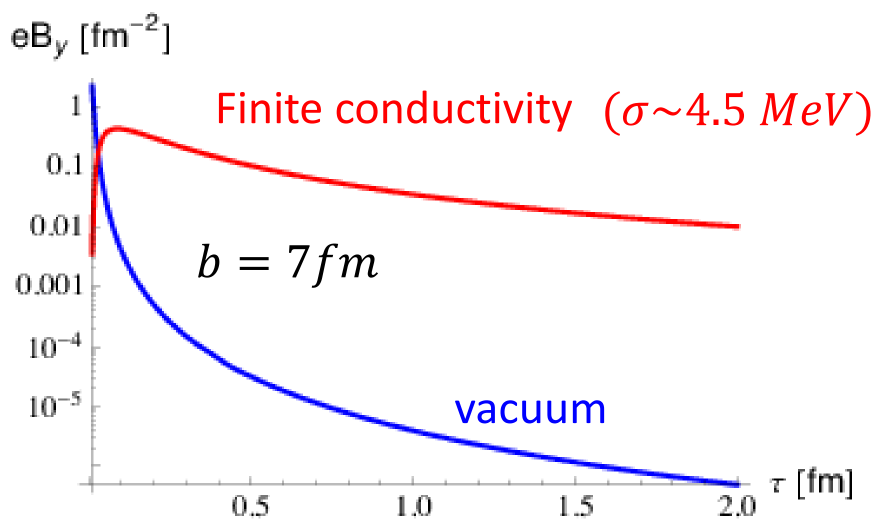
- **Motivation**
- **Holography and holographic constructions that incorporate the magnetic field**
- **Results for the thermodynamics**
- **Transport Coefficients**
- **Conclusions**

Magnetic fields in heavy ion collisions

Transverse plane of a (noncentral) heavy ion collision:



(arXiv:1103.4239)



arXiv:1401.3805

RHIC LHC

Generated magnetic fields: $\mathbf{eB \sim O(0.02\ GeV^2) - O(0.3\ GeV^2)}$

(In natural units)

The largest magnetic field found in nature!

- They probably decrease fast, though.

Holography

(As an effective theory for many-body phenomena)

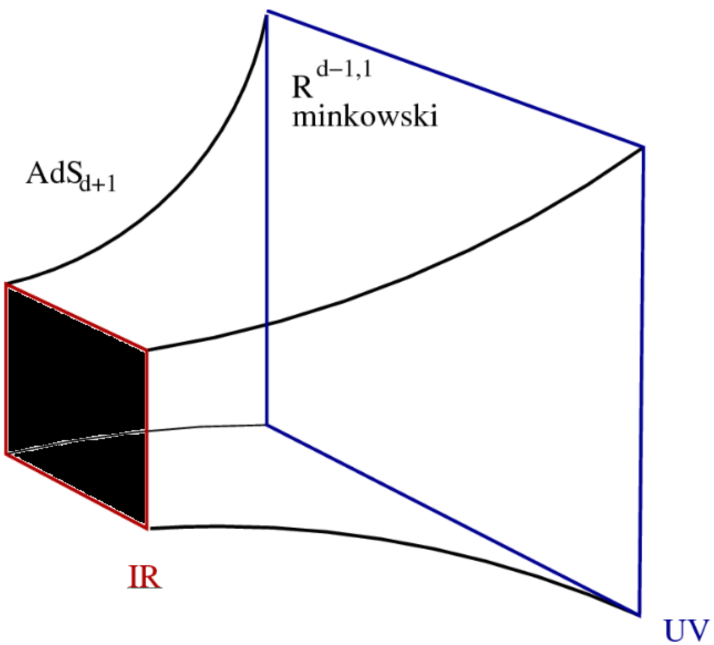
Holography in a nutshell

[Maldacena '97; Witten '98; Gubser, Polyakov, Klebanov '98]

- A (classical) gravitational theory in (d+1) dimension is dual to a quantum field theory in d dimensions.

In partiĉular, the **AdS/CFT (A**nti-de **S**itter/**C**onformal **T**heory**)** correspondence says:

$$AdS_5 \times S^5 \quad = \quad \mathcal{N} = 4 \text{ SU(N) Super Yang-Mills with } \lambda \to \infty.$$



"QCD" at strong coupling

↓

General Relativity

Selecting holographic constructions that incorporate the magnetic field:

1) The Magnetic Brane model (N=4 SYM plasma +magnetic field)

[D' Hoker, Kraus '09]

2) The phenomenological Einstein-Maxwell-Dilaton (EMD) model

[S. Gubser, A. Nellore '08] [RC et al '15-16]

The Magnetic Brane

[D' Hoker, Kraus '08]

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left[R - \frac{12}{L^2} - \boldsymbol{F^2}\right] + S_{boundary} + S_{topol}$$

- This action implements the effect of a **magnetic field** in our field theory.

The geometry of the magnetic brane:

$$ds^2 = -U(r)dt^2 + \frac{dr^2}{U(r)} + \boldsymbol{f(r)}(dx^2 + dy^2) + \boldsymbol{p(r)}dz^2$$

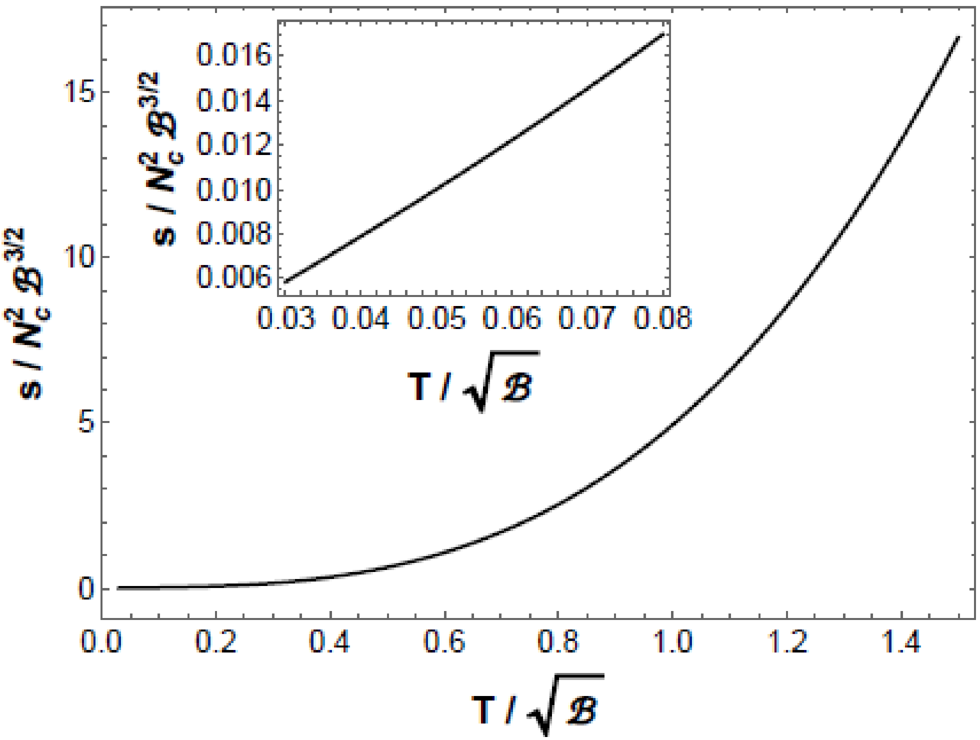
$$F = \boldsymbol{B}\, dx \wedge dy$$

SO(3) rotation symmetry broken down to SO(2)

Thermodynamics of the Magnetic brane

Entropy

[D' Hoker, Kraus '08]



- Really different than the QGP+B near the crossover region.

Need for a QCD-like model!

The EMD model

[Finazzo, RC, R. Rougemont, Noronha ' PRD 16]

Boundary terms

$$S = \frac{1}{16\pi G_5}\int_{\mathcal{M}_5} d^5x\sqrt{-g}\left[R - \frac{1}{2}(\partial_\mu\phi)^2 - V(\phi) - \frac{f(\phi)}{4}F_{\mu\nu}^2\right] + S_{\rm GHY} + S_{\rm CT}$$

Breaks conformal invariance! (like Λ_{QCD})

with

$$ds^2 = -U(r)dt^2 + \frac{dr^2}{U(r)} + \boldsymbol{f(r)}(dx^2 + dy^2) + \boldsymbol{p(r)}dz^2$$

$$F = \boldsymbol{B}\, dx \wedge dy$$

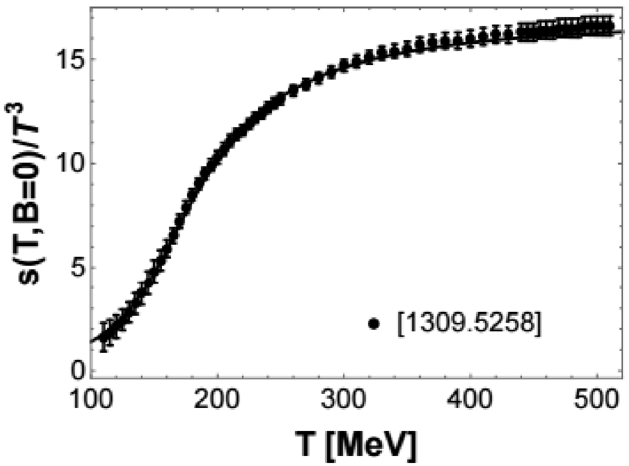
- Inputs to be fixed using lattice data:

$V(\phi), G_5$ → EoS (pressure, entropy, etc..)

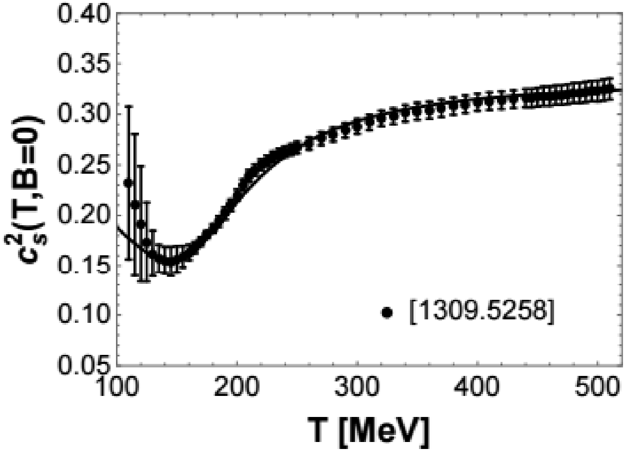
$f(\phi)$ → Magnetic susceptibility (at B=0!)

Thermodynamics at B=0

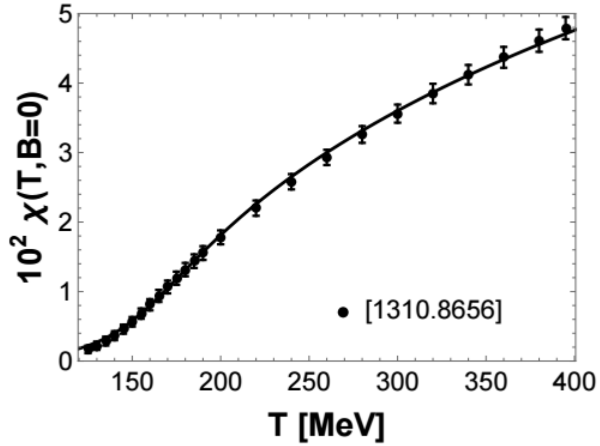
Entropy

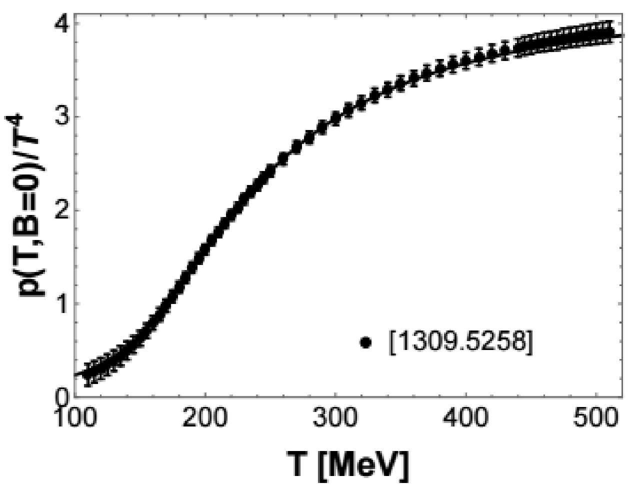


Speed of Sound

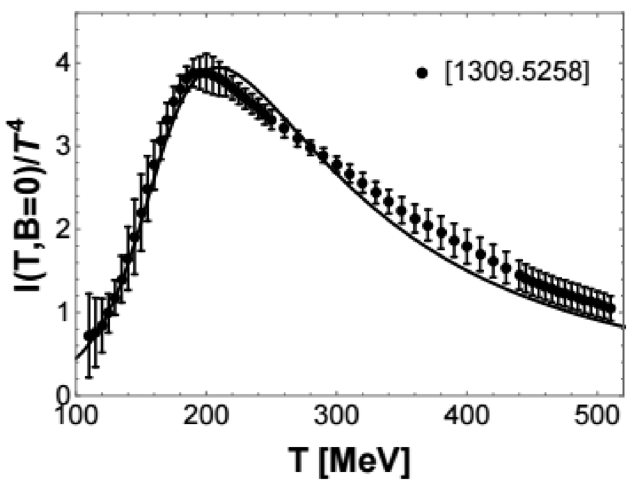


Magnetic susceptibility





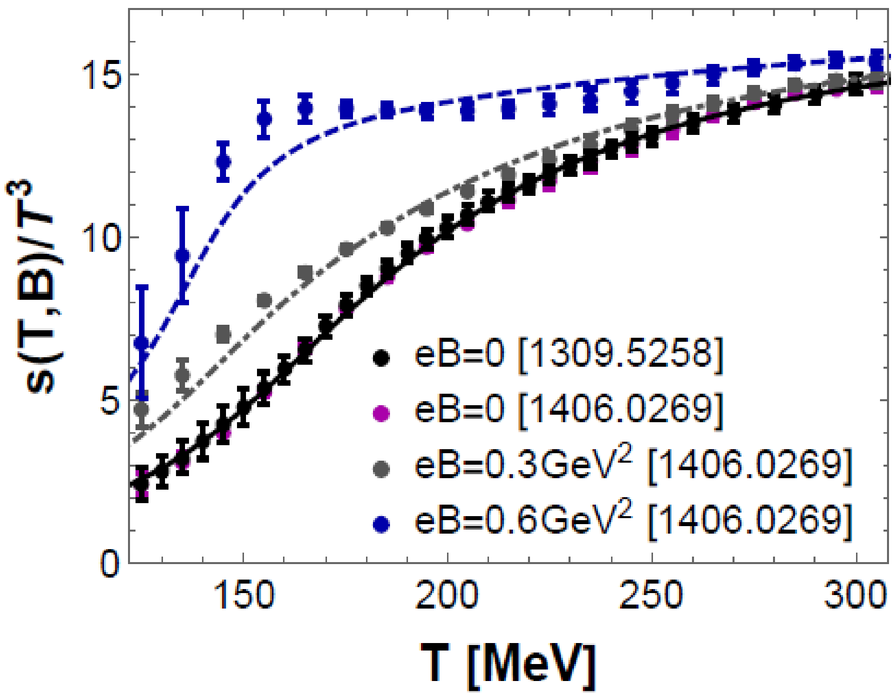
pressure



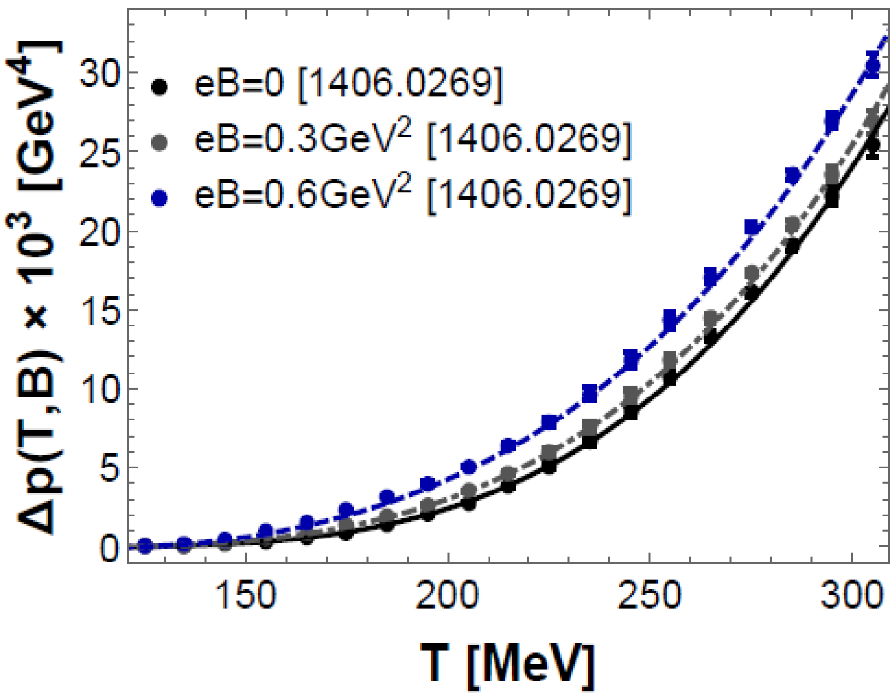
Trace anomaly

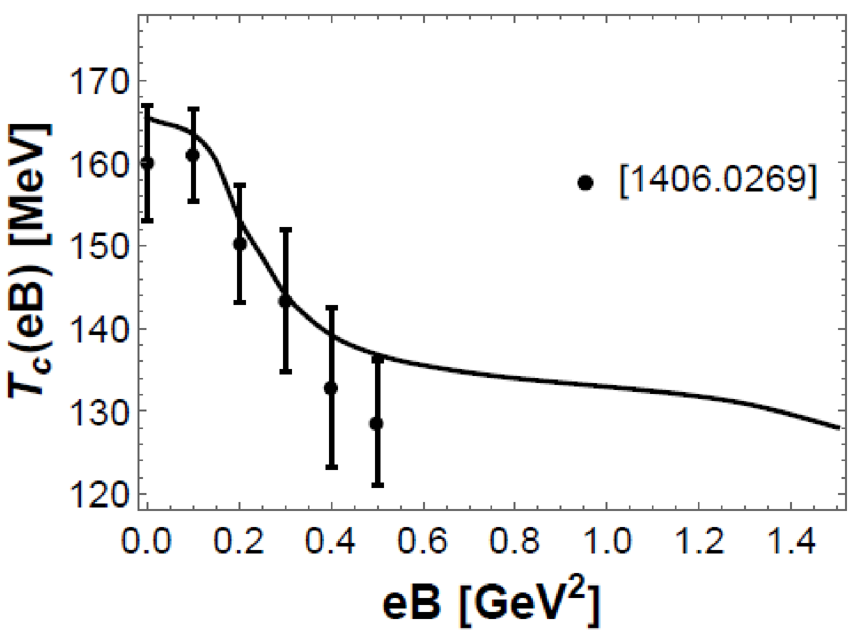
Predictions for thermodynamics at finite B:

Entropy



Pressure





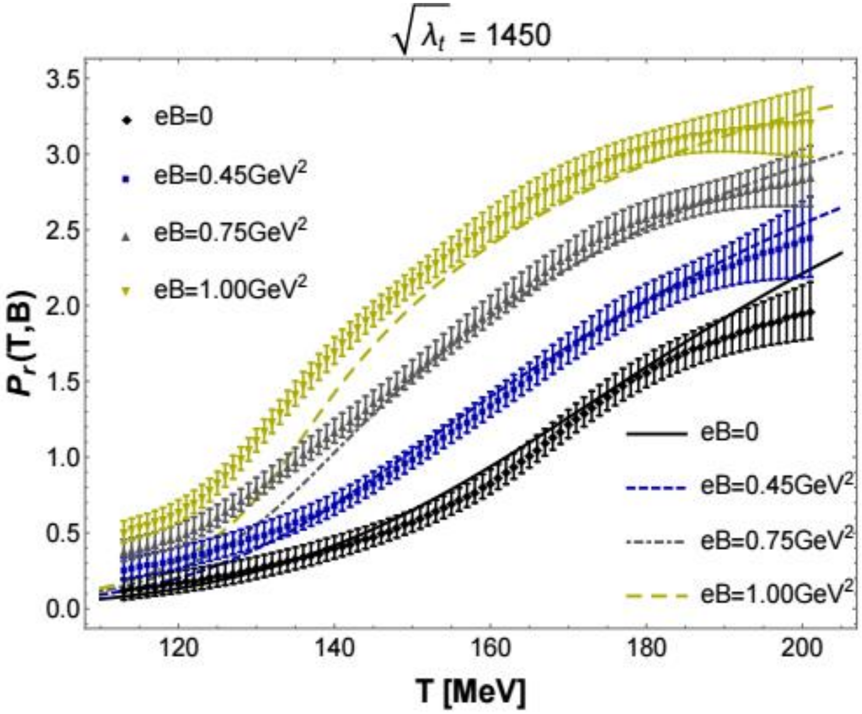
(Pseudo) Critical Temperature obtained from s/T^3

[Finazzo, RC, R. Rougemont, Noronha ' PRD 16]

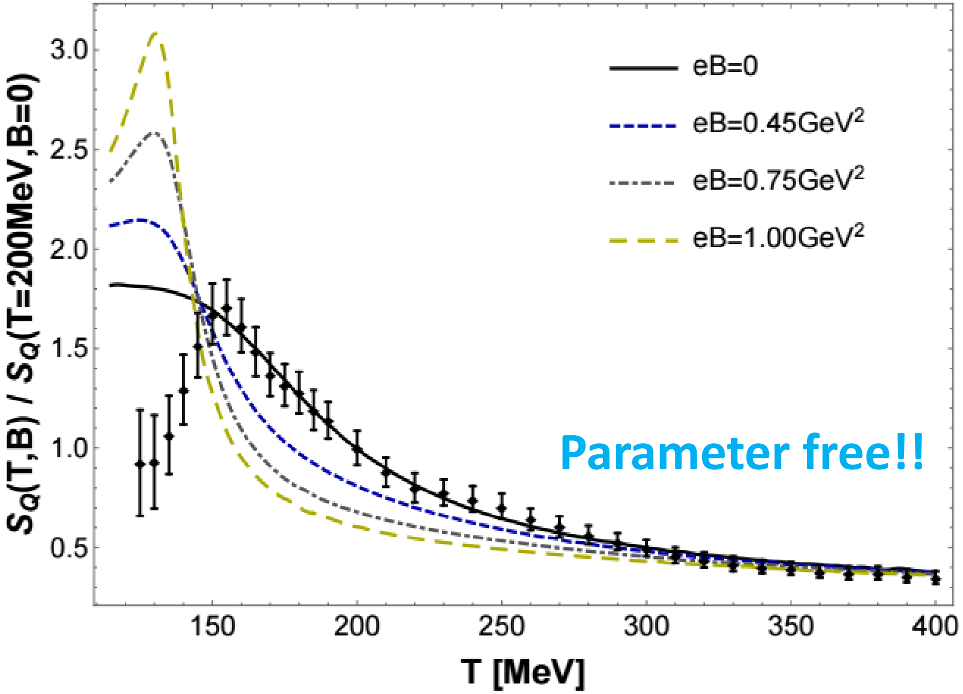
The Polyakov loop

[RC, R. Rougemont, S. Finazzo Noronha '16]

Polyakov loop



Heavy quark entropy



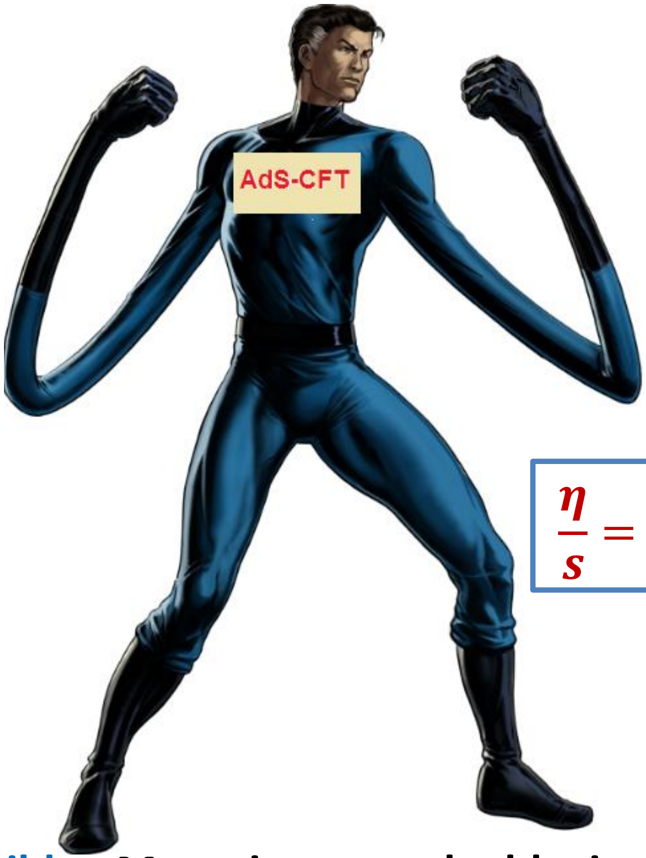
- Lattice data from [1303.3972], [1504.08280], [1603.06637])

To summarize...

LATTICE QCD:

Brute Force: **Evaluate numerically the QCD path integral.**

HOLOGRAPHY



$$\frac{\eta}{s} = \frac{1}{4\pi}$$

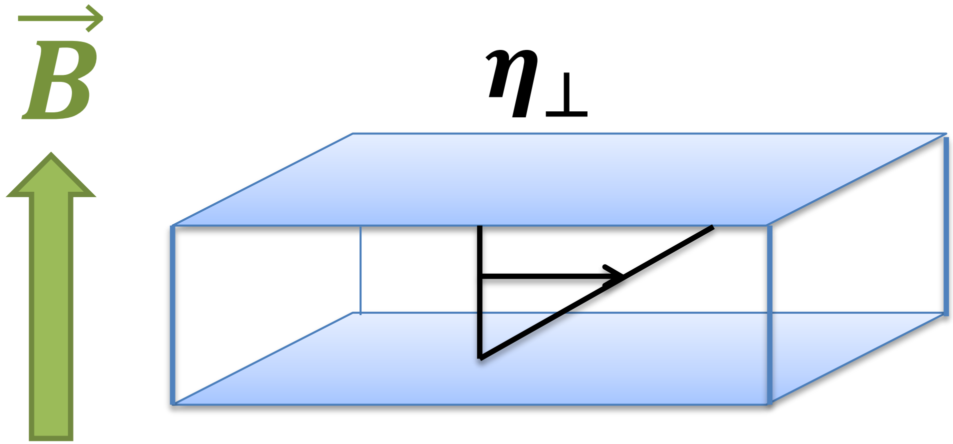
Flexible: **May give us valuable insights about equilibrium and non-equilibrium physics at strong coupling**

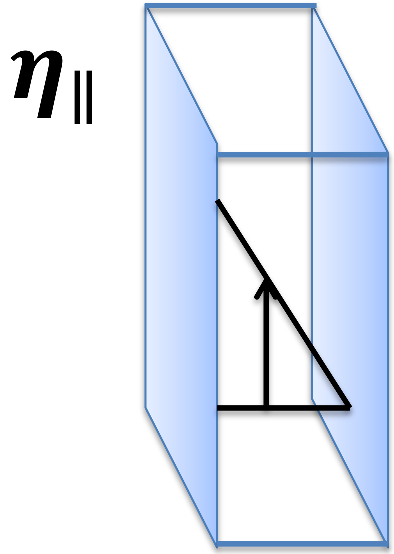
Transport coefficients:

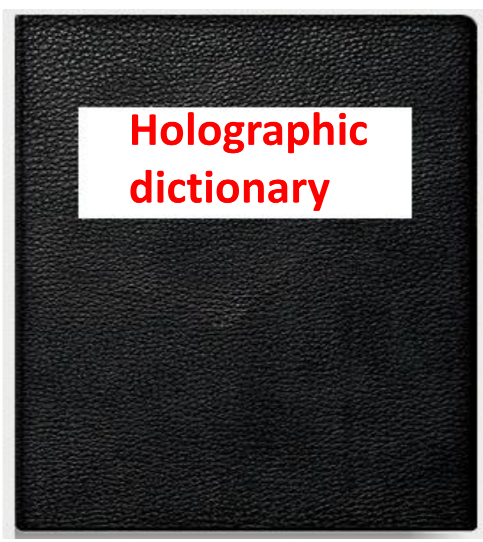
1.) How does **B** affect the shear viscosity?

Anisotropic shear viscosity

- The breaking of SO(3) rotational symmetry down to SO(2)!

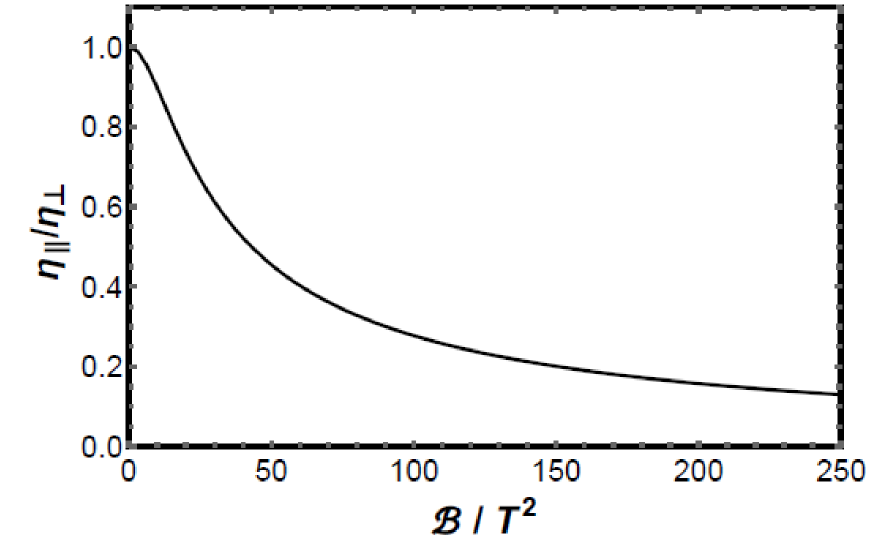






$$\frac{\eta_\perp}{s} = \frac{1}{4\pi},$$

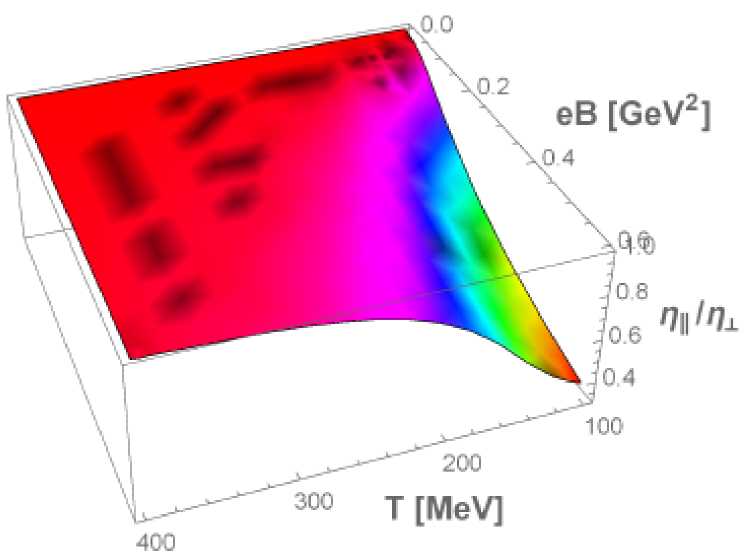
$$\frac{\eta_\parallel}{s} = \frac{1}{4\pi}\frac{g_{zz}(r_H)}{g_{xx}(r_H)}.$$

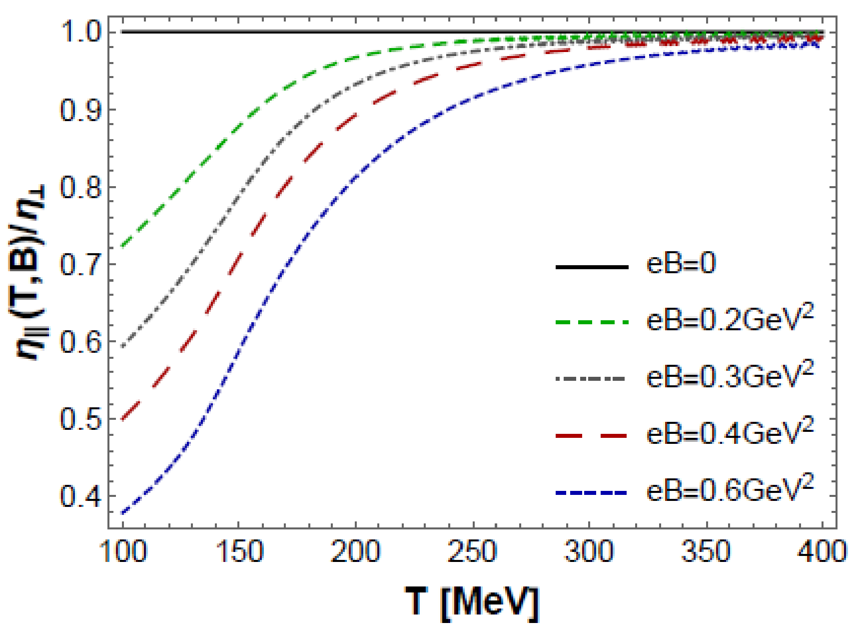


[RC, S. Finazzo, M. Zaniboni J. Noronha ' PRD 14]

$\eta_\parallel < \eta_\perp$ **for B>0**

Violates the "bound": $\frac{\eta}{s} = \frac{1}{4\pi}$





[S. Finazzo, RC, R. Rougemont, J. Noronha ' PRD 16]

2.)Heavy quark dynamics in the QGP from the holographic perspective

i.e. assuming strong coupling at all scales

Heavy quarks and the QGP

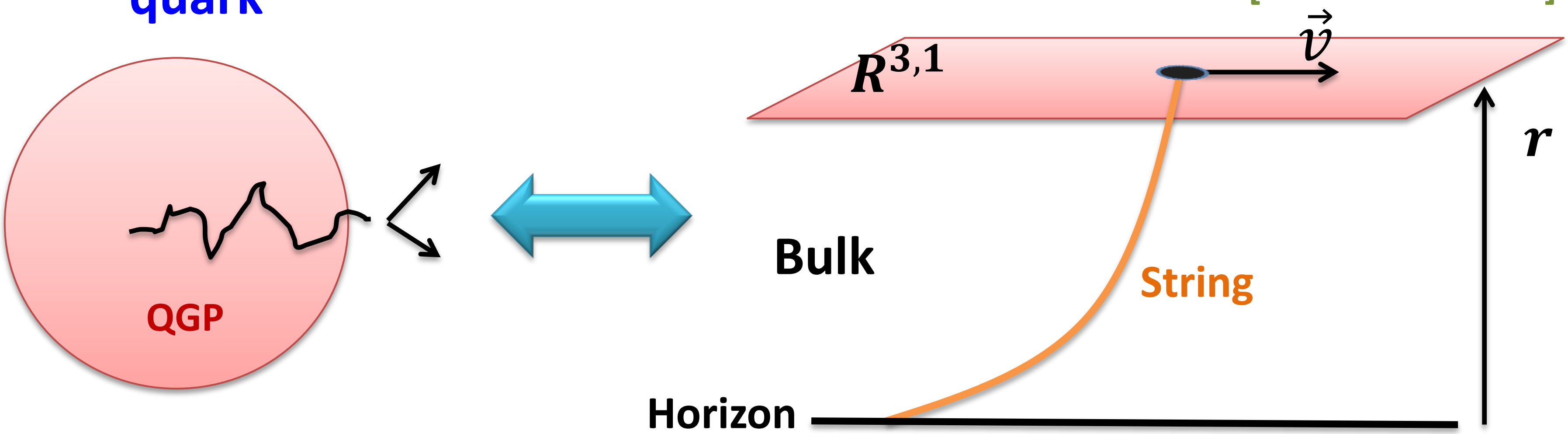
The heavy quark inside the QGP undergoes a Brownian motion. We have then:

- **energy loss;**
- **quenched momentum;**
- **momentum broadenning, etc..**

Motion of a heavy quark

AdS point of view

[Gubser et al ’06]

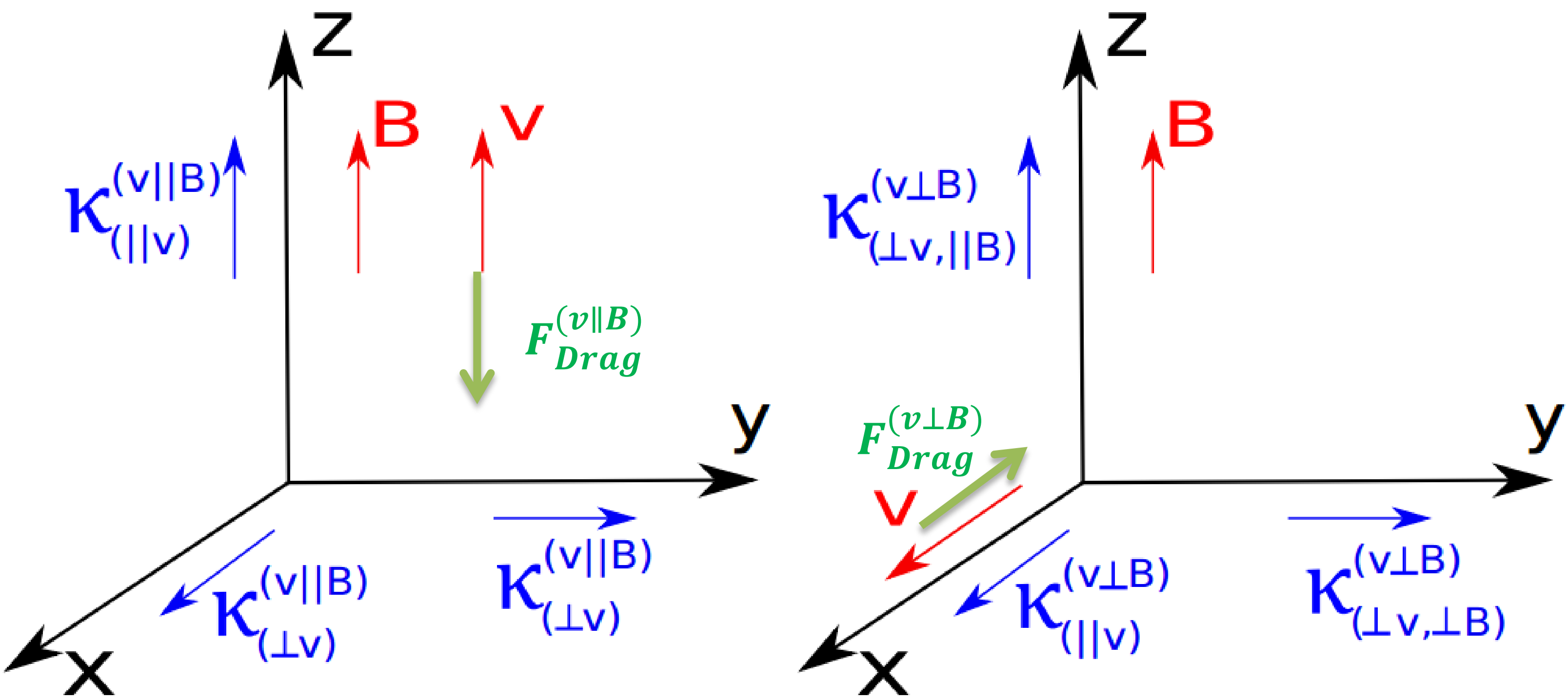


The Langevin equation

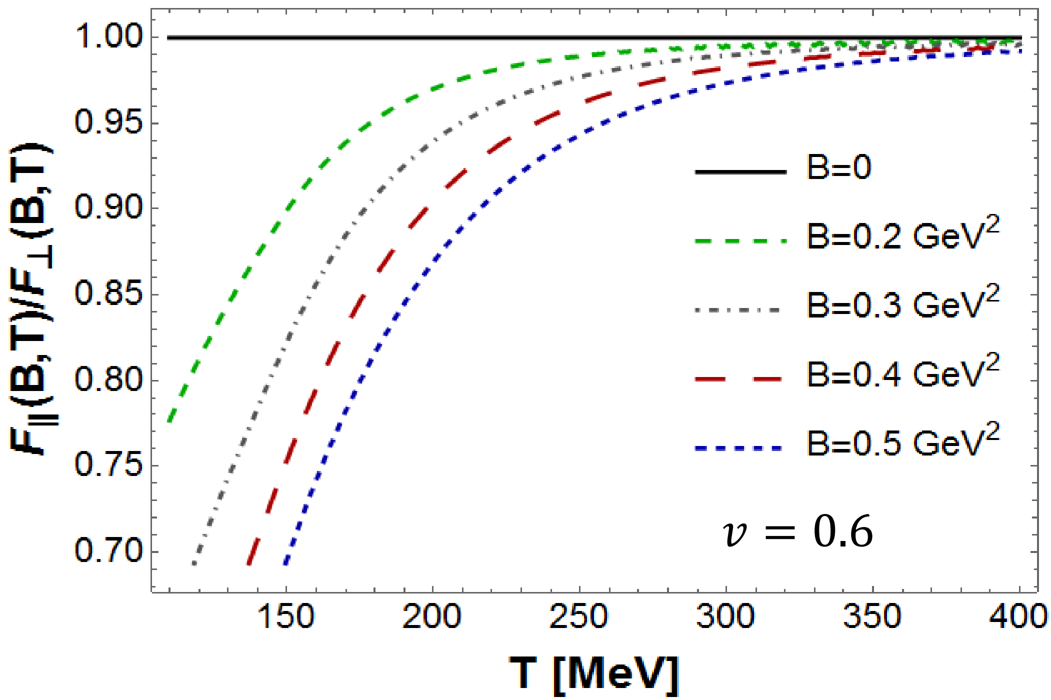
$$\frac{dp^i}{dt} = -F^{ij}_{Drag}p^j + \xi^i(t),\ \langle\xi^i(t)\rangle = 0,\ \langle\xi^i(t)\xi^j(t')\rangle = 2\kappa^{ij}\delta(t-t'),$$

Drag Force (Energy loss)

Diffusion coefficients



The Drag Force in the EMD model



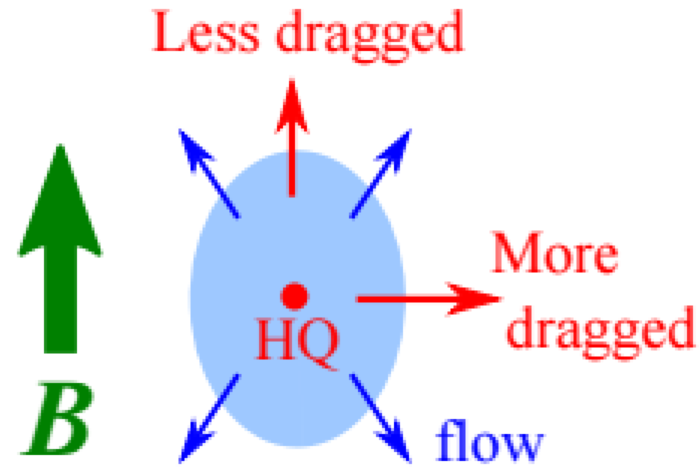


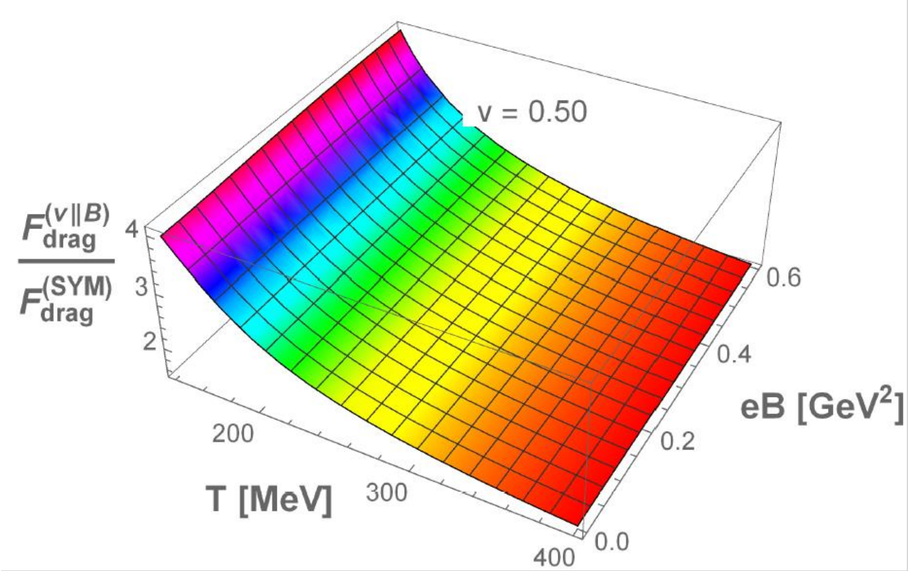
Figure taken from [arXiv:1512.03689]

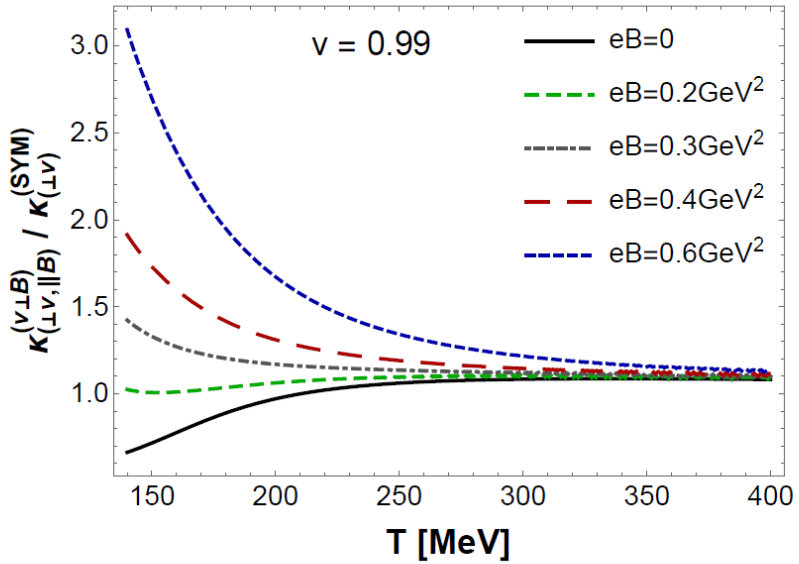
In general: $$F_{\rm drag}^{(v\perp B)} > F_{\rm drag}^{(v\parallel B)}$$

- Qualitative agreement with pQCD calculation [arXiv:1512.03689]

Many results

Example:





See https://arxiv.org/pdf/1605.06061v1.pdf to check all the results!

- The values of such coefficients found within the holographic context may be used as input in numerical codes

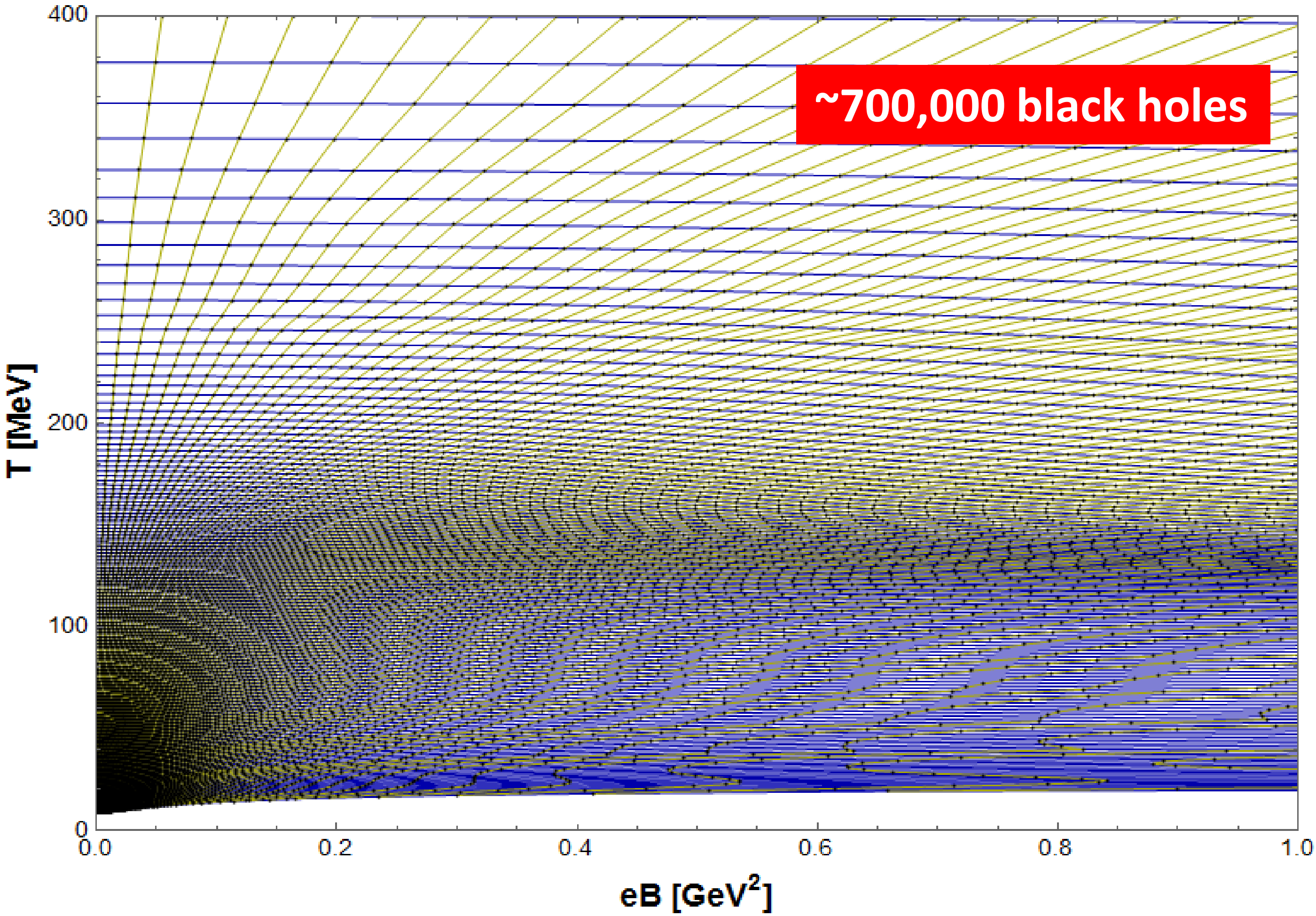
Conclusions and Outlook

- **Concerning holographic constructions with magnetic field:**
  - **The EMD+B model has a good quantitative agreement with lattice and gives us quantitative predictions.**
  - **The EMD holographic results may serve as input in numerical codes**

- **Some future steps**
  - **Calculate the two bulk viscosities of the EMD model.**
  - **Build a (causal and stable) relativistic dissipative viscous magnetohydrodynamics theory.**

Backup slides

Grid Structure of the EMD model



Each dot corresponds to a different geometry (black hole) related to some specific initial condition.

Brownian Motion

- We can describe the heavy quark motion via the Langevin equation

$$\frac{dp^i}{dt} = -\eta_D^{ij} p^j + \xi^i(t), \quad \langle \xi^i(t) \rangle = 0, \quad \langle \xi^i(t) \xi^j(t') \rangle = 2\kappa^{ij} \delta(t - t'),$$

where η_D^{ij} and $\xi^i(t)$ are the forces of the medium acting on the heavy quark.

- η_D^{ij} is a smooth dissipative force – a "drag force".
- $\xi^i(t)$ is a stochastic force around the average motion.
- κ^{ij} are the diffusion Langevin coefficients.

Final goal: $\eta_D^{ij}(B)$ and $\kappa^{ij}(B)$

Drag Force/Energy loss

- With the momentum Π_ξ in hand, we can compute now the **drag force**

$$F = \underbrace{\frac{dp_l}{dt} = \frac{dE}{dx^l}}_{\text{Energy loss}} = \Pi_\xi = \eta_D$$

(old notation)

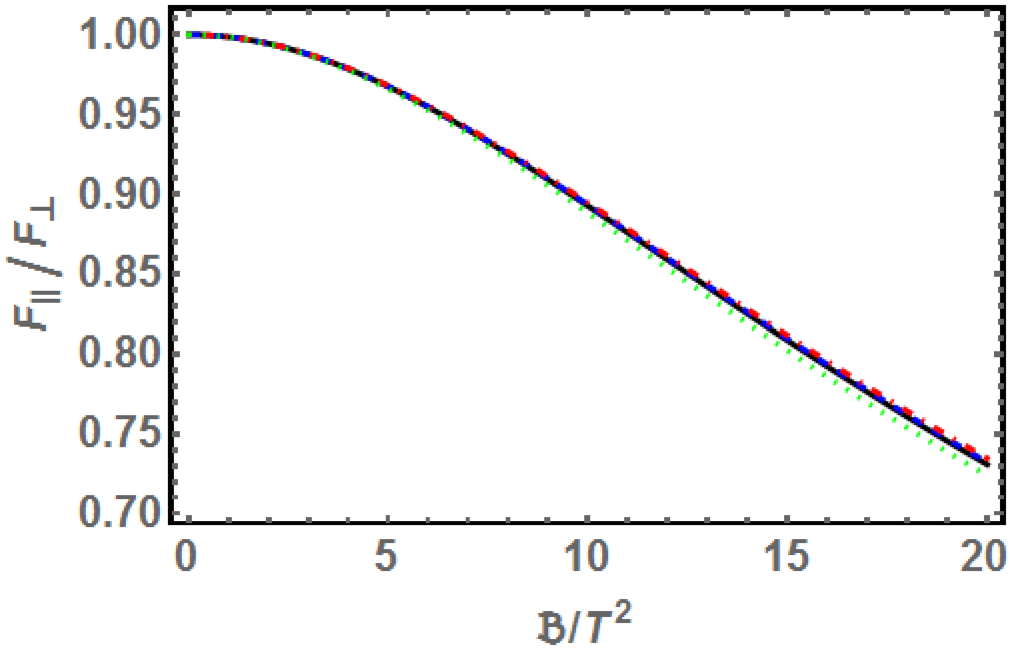
- When we include the magnetic field, the drag force will depend on the angle formed between the quark's velocity and the magnetic field.

So we define:

$F_\perp$ **= the drag force for** $\vec{v} \perp \vec{B}$

$F_\parallel$ **= the drag force for** $\vec{v} \parallel \vec{B}$

Comparing the parallel and perpendicular drag forces



For $\mathcal{B} \gg T^2$

$$\frac{F_{\parallel}}{F_{\perp}} \approx \frac{4\pi^2}{1-v^2}\frac{T^2}{\mathcal{B}}$$

The **weak coupling calculation** leads to [arXiv:1512.03689]:

$$\frac{F_{\parallel}}{F_{\perp}} \sim \frac{T^2}{eB}$$

→ Alleged impact on v_2 and R_AA!

The nearly "perfect fluid"

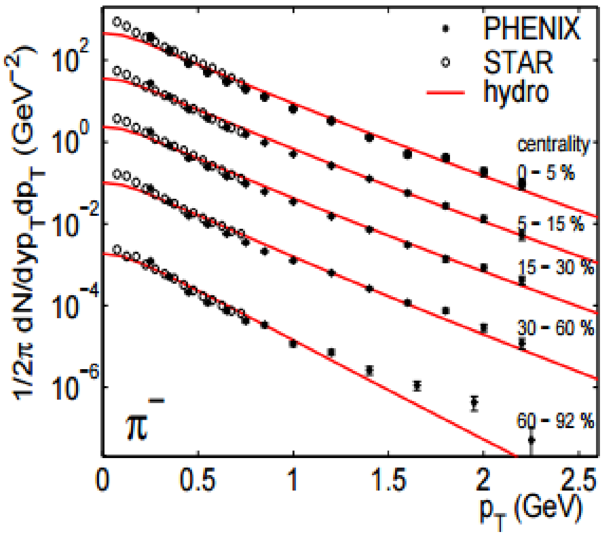
- The QGP behaves like a nearly "perfect fluid", in the sense that the shear viscosity to the entropy density is very close to the holographic result:

(in natural units)

Son, Kovtun, Starinets '05: $\frac{\eta}{s} = \frac{1}{4\pi}$ $\Rightarrow$ $\frac{\lambda_{micro}}{\lambda_{macro}} \ll 1$ $\Rightarrow$ Strongly Coupled (liquid regime)

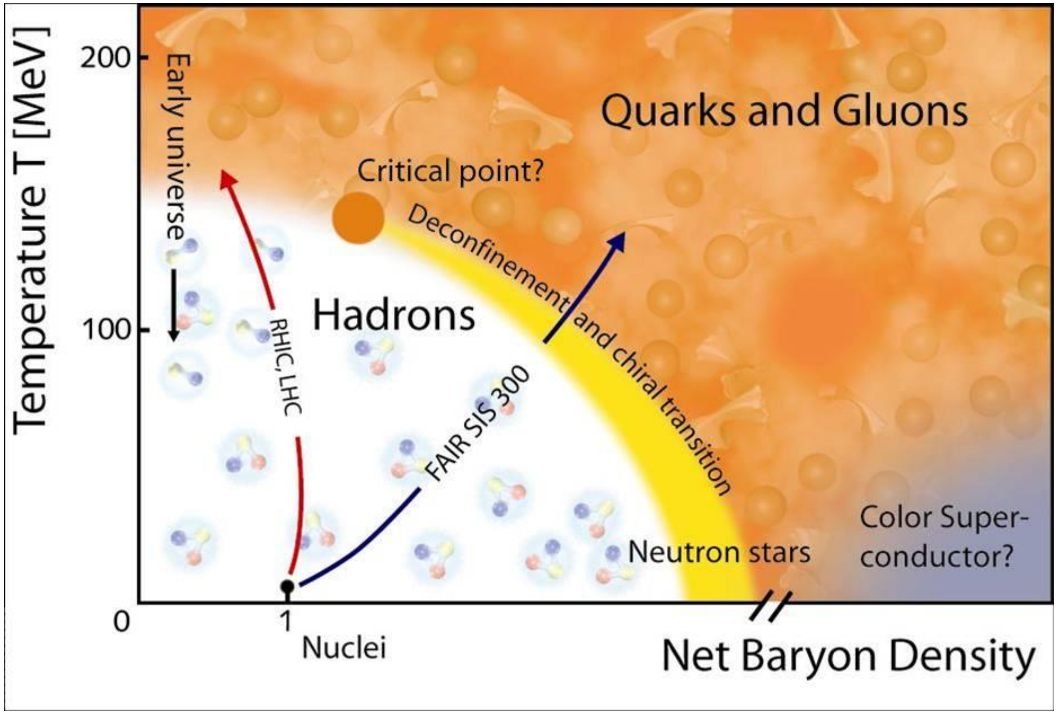
Therefore **we must go beyond pQCD**

- Remarkable success in treating the QGP as a nearly perfect fluid

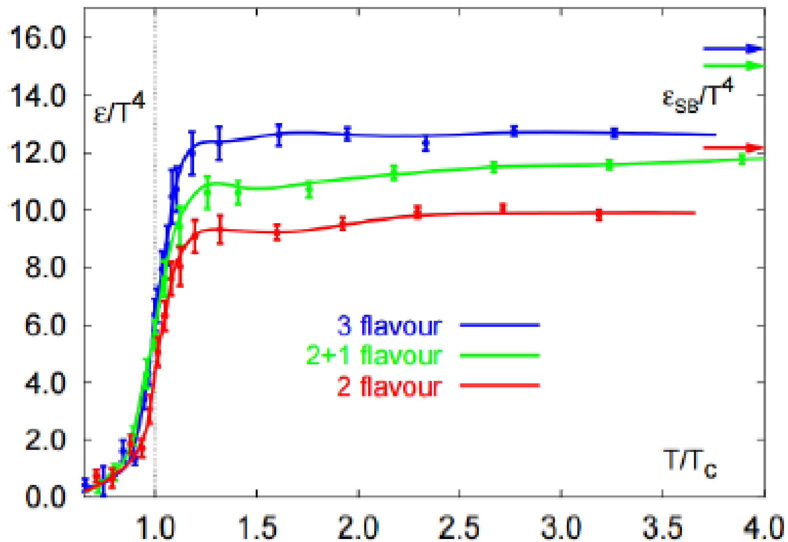


(arXiv:nucl-th/0305084)

The phase diagram for the hadronic matter



At low chemical potential, we have a crossover near $T_c \sim 150\ MeV$



arXiv:hep-lat/0106019

Deviations from the equilibrium

Gauge/Gravity usefulness increases

Equilibrium

- Temperature
- Pressure
- Entropy
- Etc..

Near-equilibrium

Transport Coefficients:

- Shear viscosity
- Conductivity
- Diffusion
- Etc..

Quasinormal modes (QNM)

Far-from-equilibrium

- QGP thermalization
- Shock waves
- Turbulence

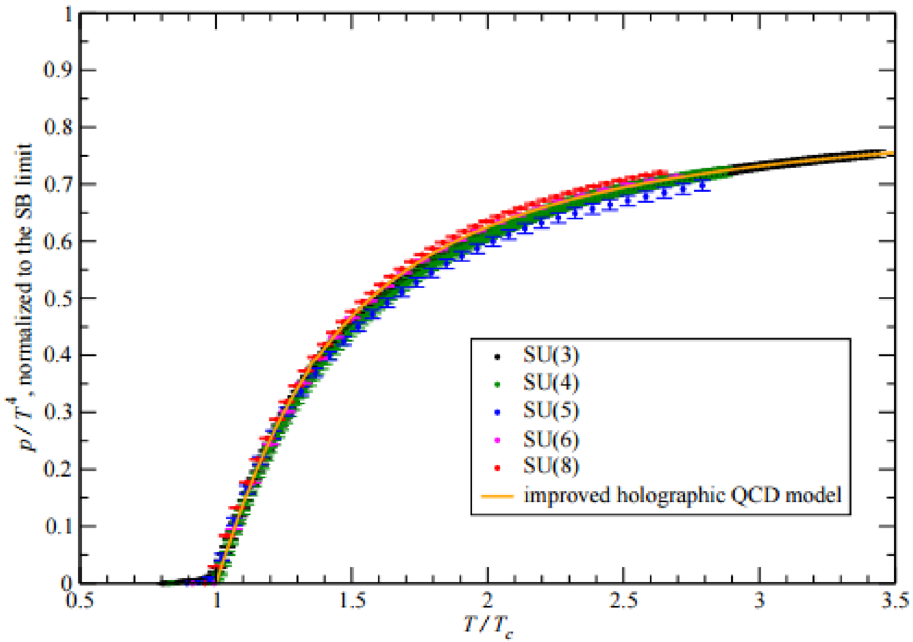
Uncharted territory

Difficulty Increases

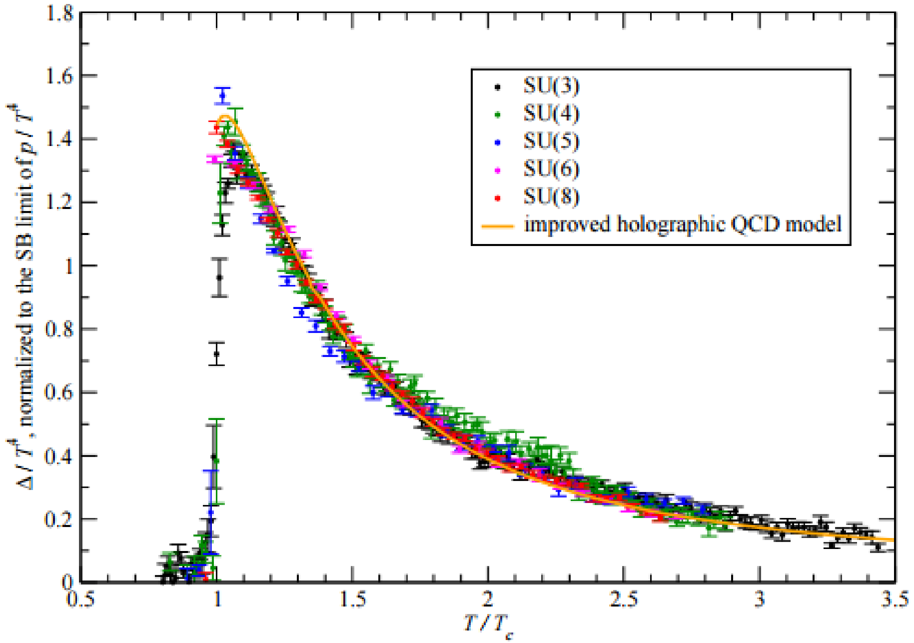
$N_c \to \infty$?

Pure glue YM EoS from the lattice [arXiv:0907.3719]

Pressure



Trace anomaly



- The "good'' observables to calculate in the context of the AdS/CFT duality are not suppresed by the rank of the gauge group.

- Examples of "bad'' quantities: Polyakov loop at finite μ_B, topological susceptibility, etc..

Holography in a nutshell

- A quantum field theory is dual to a gravitational theory in higher dimensions.

(In specific, the duality **A**nti-**d**e **S**itter = **C**onformal **F**ield **T**heory is a kind of holography)

$\mathcal{N} = 4$ SU(N) Super Yang-Mills = Type IIB Superstring in $AdS_5 \otimes S^5$

J.Maldacena, Adv.Theor.Math.Phys.2,231(1998)

- One hint for such correspondence comes from the global symmetries:

Symmetries of $\mathcal{N} = 4$ SYM SU(N) ⟷ Isometries of $AdS_5 \otimes S^5$

Conformal symmetry in 4D : SO(2,4)	AdS_5 isometry: SO(2,4)
R symmetry: SU(4) = SO(6)	S^5 isometry: SO(6)

- All the global symmetries are equal.

- The field theory has two parameters: N_c and g_{YM}. When N_c is large, it's the 't Hooft coupling $\lambda = g_{YM}^2 N_c$ that controls the perturbation theory.

- The parameters of the string theory are: string coupling g_s, string length l_s and the radius of the AdS space L.

The relations between them are:

$$g_{YM}^2 = 4\pi g_s$$
$$g_{YM}^2 N_c = \lambda = \frac{L^4}{l_s^4}$$

(**weakly**/**strongly** duality!)

This implies that

If $\lambda \to \infty$ (strong coupling), $\frac{L^4}{l_s^4} \to \infty$ (Supergravity regime)

$$\therefore S_{string} = \frac{1}{15\pi G_5} \int d^5x \sqrt{-g} \left(R + \frac{12}{L^2} \right)$$

(after some truncations/compactifications)

QCD *versus* N=4 SYM

	QCD	N=4 SYM
Confinement	✔	✘
SUSY	✘	✔
Quarks	✔	✘
Finite N_c (=3)	✔	✘
Discrete spectrum	✔	✘
Running coupling	✔	✘
Chiral condensate	✔	✘

However, if we include the temperature, we have (around Tc)

	QCD	N=4 SYM
Confinement	✗	✗
Susy	✗	✗
Strong Coupling	✓	✓
Debye Screening	✓	✓

Look for “universal” results when working with the N=4 SYM!

Effective theories for the QGP

The QGP created in a heavy ion collision is in a strongly coupled regime

We resort then to effective theories for low-energy QCD:

- Bag Model (Toy Model);
- Chiral Perturbation Theory;
- Nambu-Jona-Lasinio;
- **Gauge/Gravity duality;**
- Etc...

Integrated out gluons...

My choice

And what about the LATTICE QCD?!?!

No sign problem for the magnetic field in LQCD

$$\mathcal{Z} = \int \mathcal{D}A_\mu e^{-S_g} \det(\not{D} + m_f)$$

$$\det\left[XX^\dagger + m_f^2\right]$$

Manifestly positive! (Good for Monte-Carlo importance sampling)

where

$$S_g = \frac{1}{4}\int_0^\beta d\tau \int d^3x \, \mathrm{Tr}\left[G_{\mu\nu}G_{\mu\nu}\right]$$

$$S_g > 0$$

$$\not{D} = \begin{pmatrix} 0 & iX \\ iX^\dagger & 0 \end{pmatrix}, \quad iX = D_0 + i\boldsymbol{\sigma}\cdot\boldsymbol{D}$$

- If we include a finite chemical potential the fermionic determinant is not positive definite. This is the infamous sign problem.

- Ironically, it seems harder to access experimentally the magnetic effects compared with effects of finite baryonic chemical potential.

Fixing the parameters

OLD PARAMETRIZATION

$$f(\phi) = 1.12\,\mathrm{sech}(1.05\,\phi - 1.45)$$

$$V(\phi) = -12\cosh(0.606\,\phi) + 0.703\,\phi^2 - 0.1\,\phi^4 + 0.0034\,\phi^6, \quad \kappa^2 = 8\pi G_5 = 12.5$$

The old results were fixed using Lattice data from **[arXiv:1204.6710]** for $N_f = 2 + 1$.

NEW PARAMETRIZATION

$$V(\phi) = -12\cosh(0.63\phi) + 0.65\phi^2 - 0.05\phi^4 + 0.003\phi^6,$$

$$\kappa^2 = 8\pi G_5 = 8\pi(0.46),$$

$$f(\phi) = 0.95\,\mathrm{sech}(0.22\phi^2 - 0.15\phi - 0.32).$$

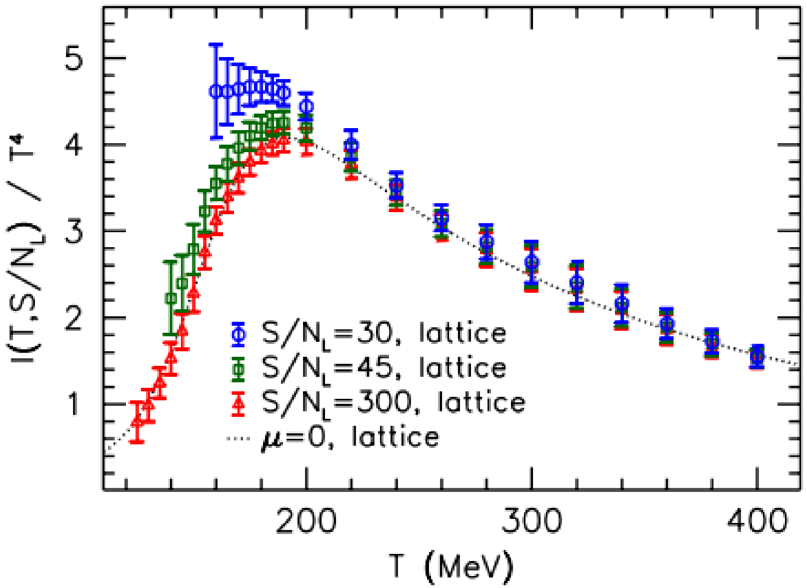
The new results were fixed using Lattice data from **[arXiv:1310.8656]**

The trace anomaly

- Whilst the QCD is not a conformal theory, the N=4 SYM is. Indeed, the trace anomaly for the QCD is given by

$$T_\mu^{QCD\,\mu} = \sum_q^{N_f} m_q \bar{q} q + \frac{\beta(g)}{2g^3} G^{a\mu\nu} G^a_{\mu\nu}$$

- Important in the crossover region!



Lattice result from [arXiv:1204.6710] for $N_f = 2 + 1$

- Although $\langle T_\mu^\mu \rangle \propto B^2$ for the magnetic brane, we want to do much better than this. We want a model which emulates the energy scale Λ_{QCD} of the QCD.

Matching the energy scale

- The recipe to get the correct energy scale Λ for the observables, is to compare the minimum of the speed of sound of the lattice with the black hole's calculation,

$$\lambda = \frac{T^{\text{lattice}}_{\text{min. } c_s^2}}{T^{\text{BH}}_{\text{min. } c_s^2}} \approx \frac{143.8\,\text{MeV}}{0.173} \approx 831\,\text{MeV}$$

- Therefore, if X is an observable such that $[X] = MeV^n$, then

$$X = \hat{X}\lambda^n$$

where $\hat{X}$ is the observable in black hole's units obtained from the EMD action.

Intermission:

How do we implement the anisotropy on the viscosities of a relativistic plasma?

The hydrodynamic model

Now, let us view somethings about the viscous tensor of rank 4, $\eta_{\alpha\beta\mu\nu}$.
The first question should be: What does this tensor mean?

Well, because of the "viscous" on its name, you may already have guessed that it's linked with dissipations, like heat, entropy, etc... To formulate this mathematicaly, we introduce a dissipation function R:

$$R = \frac{1}{2}\eta^{\alpha\beta\mu\nu} w_{\alpha\beta} w_{\mu\nu}$$

With

(we're in Minkowski here!)

$$w_{\alpha\beta} = \frac{1}{2}\left(\nabla_\alpha u_\beta + \nabla_\beta u_\alpha\right), \qquad \nabla_\alpha \equiv \Delta_{\alpha\beta}\partial^\beta, \qquad \Delta_{\alpha\beta} \equiv g_{\alpha\beta} - u_\alpha u_\beta$$

- If we take the derivative of R with respect of $w_{\mu\nu}$, we obtain the usual stress tensor $\tau^{\mu\nu}$:

$$\pi^{\mu\nu} = \eta^{\mu\nu\alpha\beta} w_{\alpha\beta}$$

Constructing $\eta_{\alpha\beta\mu\nu}$

- Assume isotropy first.
- For such task, we need to look the symmetries:

$$\eta_{\alpha\beta\mu\nu} = \eta_{\beta\alpha\mu\nu} = \eta_{\alpha\beta\nu\mu} = \eta_{\mu\nu\alpha\beta}$$

- Hence, we can construct $\eta_{\alpha\beta\mu\nu}$ with the objects:

(i) $\Delta^{\alpha\beta}\Delta^{\mu\nu}$ (ii) $\Delta^{\alpha\mu}\Delta^{\beta\nu} + \Delta^{\alpha\nu}\Delta^{\beta\mu}$

We have then two linearly independents coefficients, which are η and ζ;

$$\eta^{\alpha\beta\mu\nu} = \boldsymbol{\eta}\left(\Delta^{\alpha\mu}\Delta^{\beta\nu} + \Delta^{\alpha\nu}\Delta^{\beta\mu} - \frac{2}{3}\Delta^{\alpha\beta}\Delta^{\mu\nu}\right) + \boldsymbol{\zeta}\left(\Delta^{\alpha\beta}\Delta^{\mu\nu}\right)$$

Remember that the shear viscosity belongs to the traceless part of $\pi^{\mu\nu}$, while the bulk viscosity Is related to the non-zero part.

Constructing $\eta_{\alpha\beta\mu\nu}$

(X. Huang, M. Huang, D. Rischke, A. Sedrakian '10)

- Now, assume anisotropy due to a magnetic exernal field!

- This means that the magnetic field produces a privileged direction, thus, we shall have The following objects to make $\eta_{\alpha\beta\mu\nu}$:

(i) $\Delta^{\alpha\beta}\Delta^{\mu\nu}$

(ii) $\Delta^{\alpha\mu}\Delta^{\beta\nu} + \Delta^{\alpha\nu}\Delta^{\beta\mu}$

(iii) $\Delta^{\mu\nu}b^{\alpha}b^{\beta} + \Delta^{\alpha\beta}b^{\mu}b^{\nu}$

(iv) $b^{\mu}b^{\nu}b^{\alpha}b^{\beta}$

(v) $\Delta^{\mu\alpha}b^{\beta}b^{\nu} + \Delta^{\mu\beta}b^{\alpha}b^{\nu} + \Delta^{\nu\alpha}b^{\mu}b^{\beta} + \Delta^{\nu\beta}b^{\mu}b^{\alpha}$

(vi) $\Delta^{\mu\alpha}b^{\nu\beta} + \Delta^{\mu\beta}b^{\nu\alpha} + \Delta^{\nu\alpha}b^{\mu\beta} + \Delta^{\nu\beta}b^{\mu\alpha}$

(vii) $b^{\mu\alpha}b^{\beta}b^{\nu} + b^{\mu\beta}b^{\alpha}b^{\nu} + b^{\nu\alpha}b^{\mu}b^{\beta} + b^{\nu\beta}b^{\mu}b^{\alpha}$

(viii) $b^{\mu\alpha}b^{\nu\beta} + b^{\mu\beta}b^{\nu\alpha}$

7 parameters: 5 shear viscosities and 2 bulk viscosities!

- b^{μ} is the normal vector of the magnetic field

- $b^{\mu\nu} = \varepsilon^{\mu\nu\alpha\beta}b^{\alpha}u^{\beta}$

Constructing $\eta_{\alpha\beta\mu\nu}$

- Thus, to contruct $\eta_{\alpha\beta\mu\nu}$, we make linear combinations of those objects. In specific, we have:

$$\eta_{\alpha\beta\mu\nu} = \cdots + \eta_{\perp}(\mathrm{ii}) + \eta_{\parallel}(\mathrm{v}) + \cdots$$

where

$$\eta_{\perp} = \eta_{xyxy}, \quad \eta_{\parallel} = \eta_{yzyz} = \eta_{xzxz}$$

- Our objective is then to find $\eta_{\perp}$ and $\eta_{\parallel}$.

- Holographicaly, it means that we need to consider the modes h_{xy} and h_{yz}.

Bulk viscosity

-Associated with the **trace** of the dissipative part of the stress-energy tensor

Isotropic case:

$$\zeta = -\frac{4}{9}\lim_{\omega\to 0}\frac{1}{\omega}\,\mathrm{Im}\,G_R(\omega)$$

where

$$G_R(\omega) = -i\int dt\, d^3x\, e^{i\omega t}\theta(t)\langle[\tfrac{1}{2}T_i{}^i(t,\vec{x}), \tfrac{1}{2}T_k{}^k(0,0)]\rangle$$

→ **$\zeta = 0$ for conformal theories**

Anisotropic case:

$$\zeta_\perp = -\frac{2}{3}\lim_{\omega\to 0}\frac{1}{\omega}\left[\mathrm{Im}G^R_{P_\perp P_\perp}(\omega) + \mathrm{Im}G^R_{P_\parallel, P_\perp}(\omega)\right] \qquad P_\perp \equiv \frac{1}{2}T^a{}_a = \frac{1}{2}(T^x{}_x + T^y{}_y)$$

$$\zeta_\parallel = -\frac{4}{3}\lim_{\omega\to 0}\frac{1}{\omega}\left[\mathrm{Im}G^R_{P_\perp P_\parallel}(\omega) + \mathrm{Im}G^R_{P_\parallel, P_\parallel}(\omega)\right], \qquad P_\parallel \equiv \frac{1}{2}T^z{}_z$$

Anisotropic bulk viscosity from magnetic branes

- N=4 SYM is a conformal theory, so why did we bother to calculate the bulk viscosity? Because

$$\langle T^{\mu}_{\ \mu} \rangle = -\frac{\mathcal{B}^2}{24\pi G_5}$$

To compute the 2-point function we take the diagonal part of the fluctuated metric,

$$S = \frac{1}{16\pi G_5} \int_{\mathcal{M}_5} d^5x \mathcal{L}$$ where $$\mathcal{L} = \hat{\mathcal{L}} + \partial_t \hat{\mathcal{L}}^t + \partial_r \hat{\mathcal{L}}^r$$

$$\hat{\mathcal{L}} = \frac{1}{2}\partial_t \vec{H}^T \mathfrak{M}^{tt} \partial_t \vec{H} + \frac{1}{2}\partial_r \vec{H}^T \mathfrak{M}^{rr} \partial_r \vec{H} + \frac{1}{2}\vec{H}^T \mathfrak{M} \vec{H} + \partial_r \vec{H}^T \mathfrak{M}^r \vec{H}$$

$$\vec{H} = \begin{pmatrix} H_{tt} \\ H_{xx} \\ H_{zz} \end{pmatrix}$$

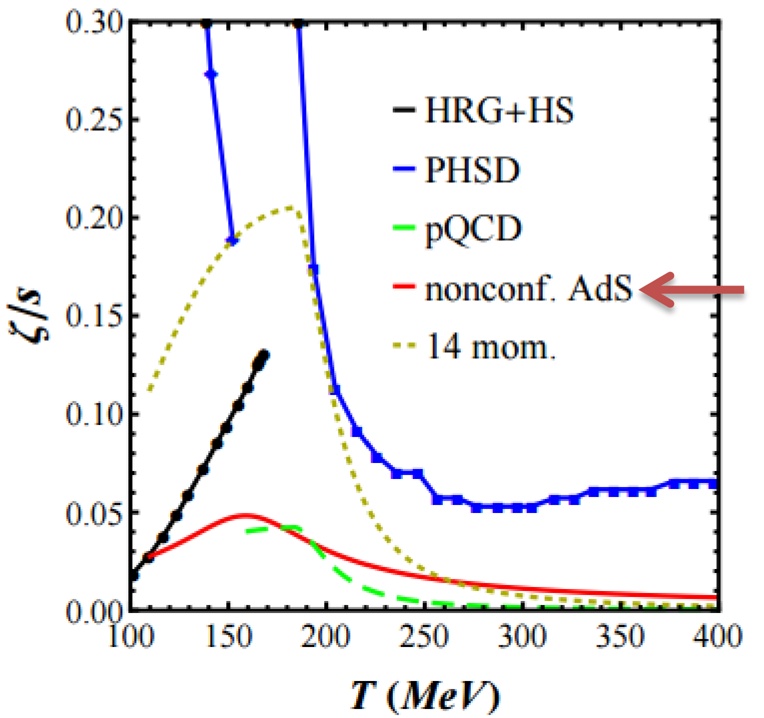
- After applying the rules to extract the retarded Green's function, we found (numerically)

$$\zeta_{\parallel} = \zeta_{\perp} = 0$$ → In the bag model, $T^a_a \neq 0$ and ζ=0 also (m=0).

Anisotropic bulk viscosity for the EMD model

- Not done yet!

- However, we have the result for the Einstein-Dilaton with the old parametrization.

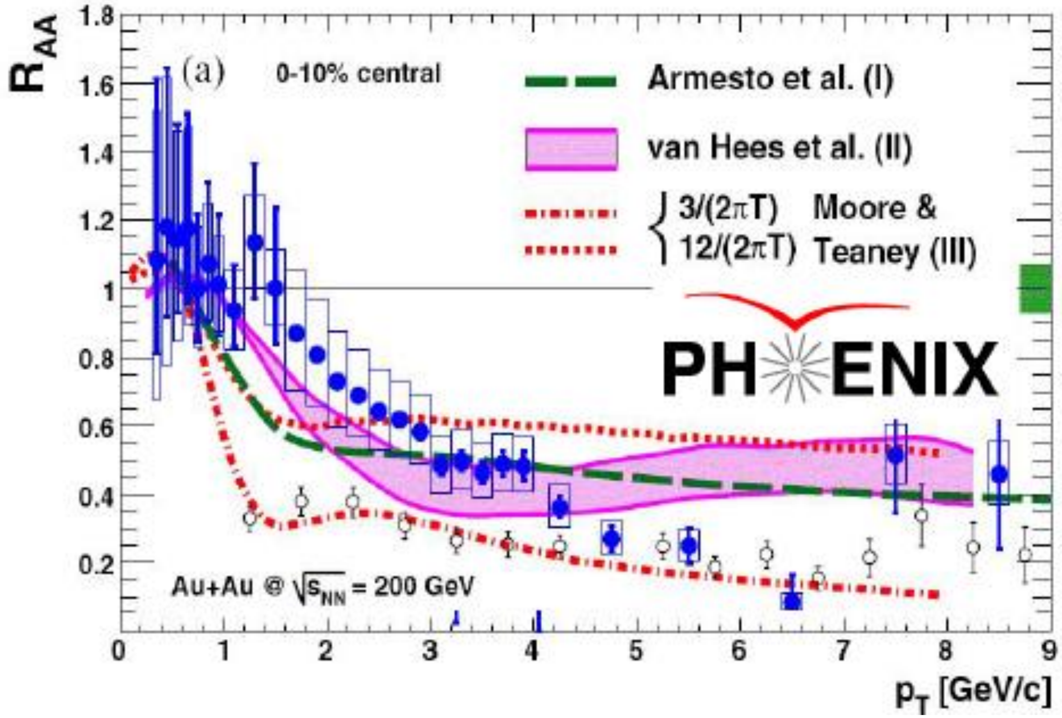


[Noronha-Hostler ’15]

The QGP is opaque for hard probes

Nuclear modification factor: $$R_{AA} = \frac{1}{N_{\rm coll}} \frac{dN_{\rm A+A}/dp_{\rm T}}{dN_{\rm p+p}/dp_{\rm T}}$$

Heavy flavor decays. E.g.: **$c(b) \to D(B) \to e^- + \nu_e + \pi$**



Solution of the Langevin Equation

Based on [arXiv:1006.3261]

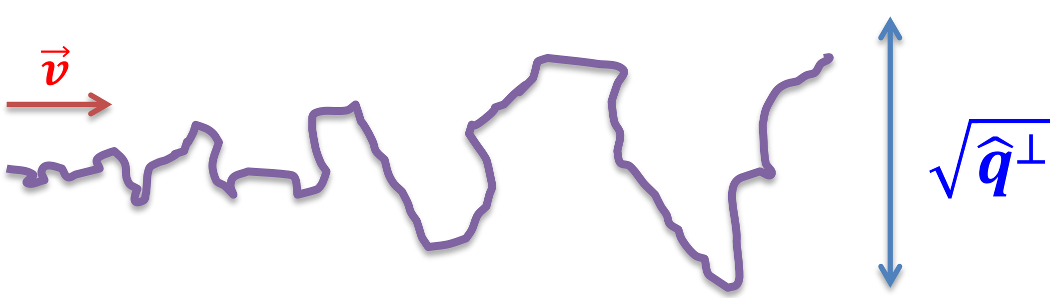
- Initial condition: $\vec{p}(0) = p_0 \frac{\vec{v}}{v}$

$$p^{\perp}(t) = \int_0^t dt' \, e^{\eta^{\perp}_{D,0}(t'-t)} \xi^{\perp}(t'),$$

$$p^{\parallel}(t) = p_0 e^{-\eta^{\parallel}_{D,0} t} + \int_0^t dt' \, e^{\tilde{\eta}^{\parallel}_{D,0}(t'-t)} \xi^{\parallel}(t'), \qquad \tilde{\eta}^{\parallel}_{D,0} \equiv \left[\eta^{\parallel}_D + p \left(\frac{\partial \eta^{\parallel}_D}{\partial p} \right) \right]_{p_0}$$

with

$$\langle (p^{\perp})^2 \rangle = 2\kappa^{\perp} t, \qquad \langle (\Delta p^{\parallel})^2 \rangle = \kappa^{\parallel} t$$

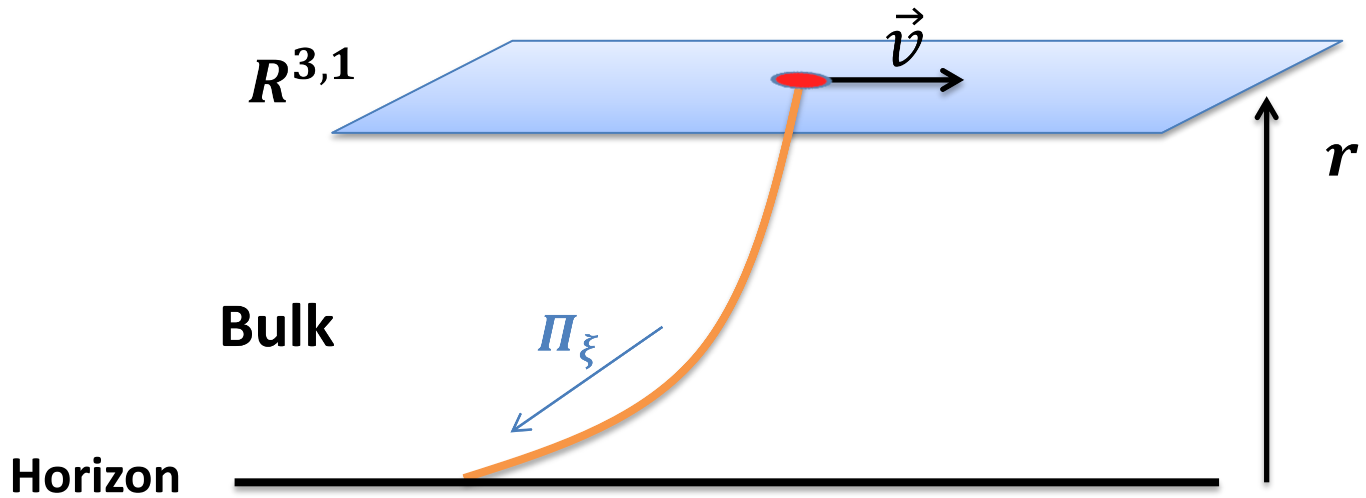


$$\hat{q}^{\perp} = \frac{\langle (p^{\perp})^2 \rangle}{vt} = 2\frac{\kappa^{\perp}}{v}$$

Modified jet-quenching parameter

The classical trailing string

- From the gauge/gravity point of view, we treat the heavy quark as a classical string with one endpoint attached on the boundary



- The string's dynamics is governed by the Nambu-Goto action

$$S_{\rm NG} = \frac{1}{2\pi\alpha'}\int d\tau d\sigma\, e^{\sqrt{\frac{2}{3}}\phi}\sqrt{-\det(\gamma_{ab})}, \quad \gamma_{ab} = g_{\mu\nu}\partial_a X^\mu \partial_b X^\nu, \;\; a,b \in \{\tau,\sigma\}$$

With the longitudinal component w.r.t the speed being

$$X^l = vt + \xi(r) \quad \Longrightarrow \quad \Pi_\xi = \frac{\delta S_{NG}}{\delta \xi'}$$