A Blast Wave Model With Viscous Corrections

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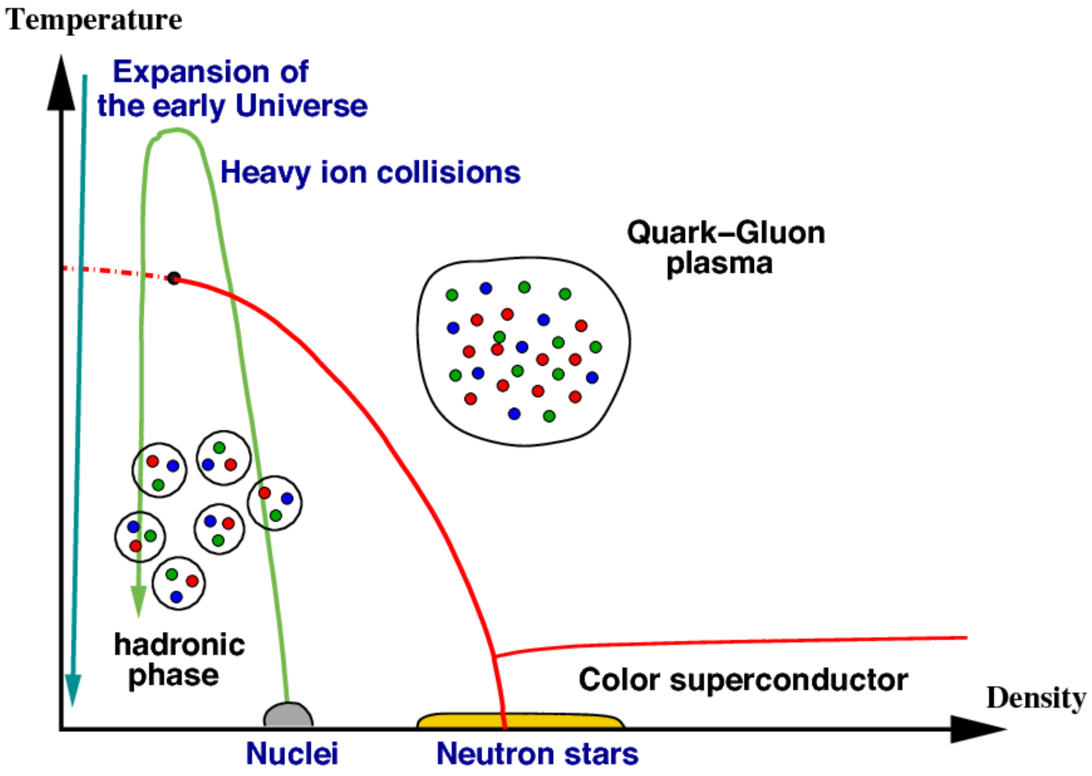
Outline

- Quark-gluon plasma(QGP)
- Hydrodynamics and blast wave
- Blast wave formalism with viscosity
- Results and discussion

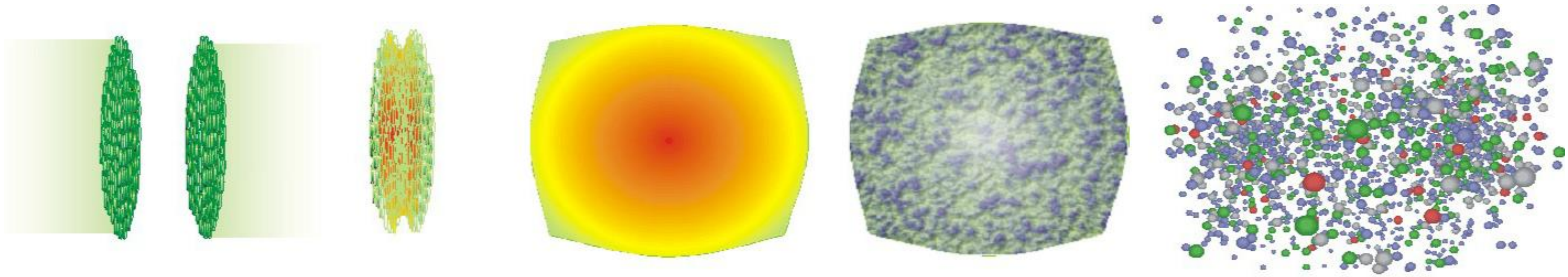
Quark-gluon Plasma(QGP)



- QCD suggests that ordinary nuclear matter has a phase transition at an extremely high temperature and density.
- Experiments: the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC)
- Preferred model: viscous hydrodynamics

Time Evolution

- Collisions generate a fireball (the hot center of the collision) with extremely high temperature and density
- The fireball cools rapidly and expands into the surrounding vacuum.
- Hadrons decouple at kinetic freeze-out and free-stream to the detectors.



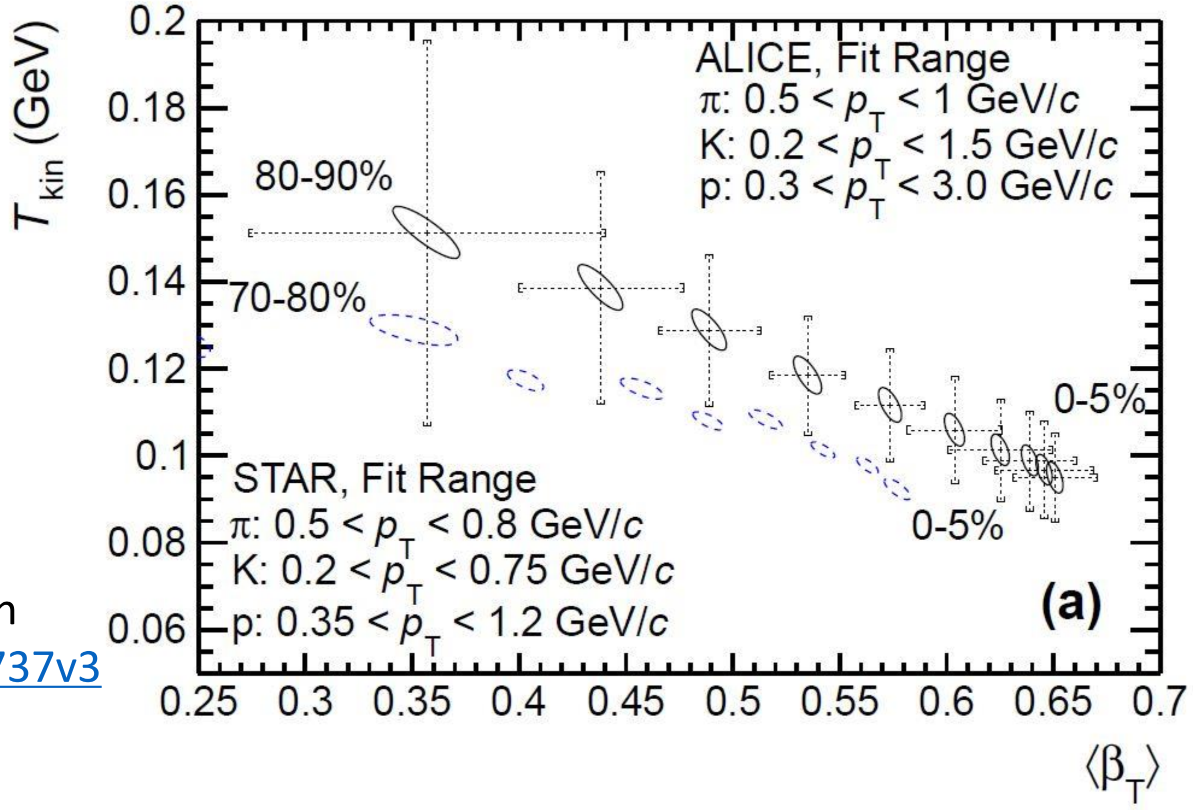
Fluid Dynamics and Blast Wave

Fluid dynamics

- Time evolution
- Determined by initial condition and equation of state
- Freeze-out hypersurface with flow field calculated consistently

Blast Wave

- Ansatz for freeze-out hypersurface and flow field, then fit final flow field

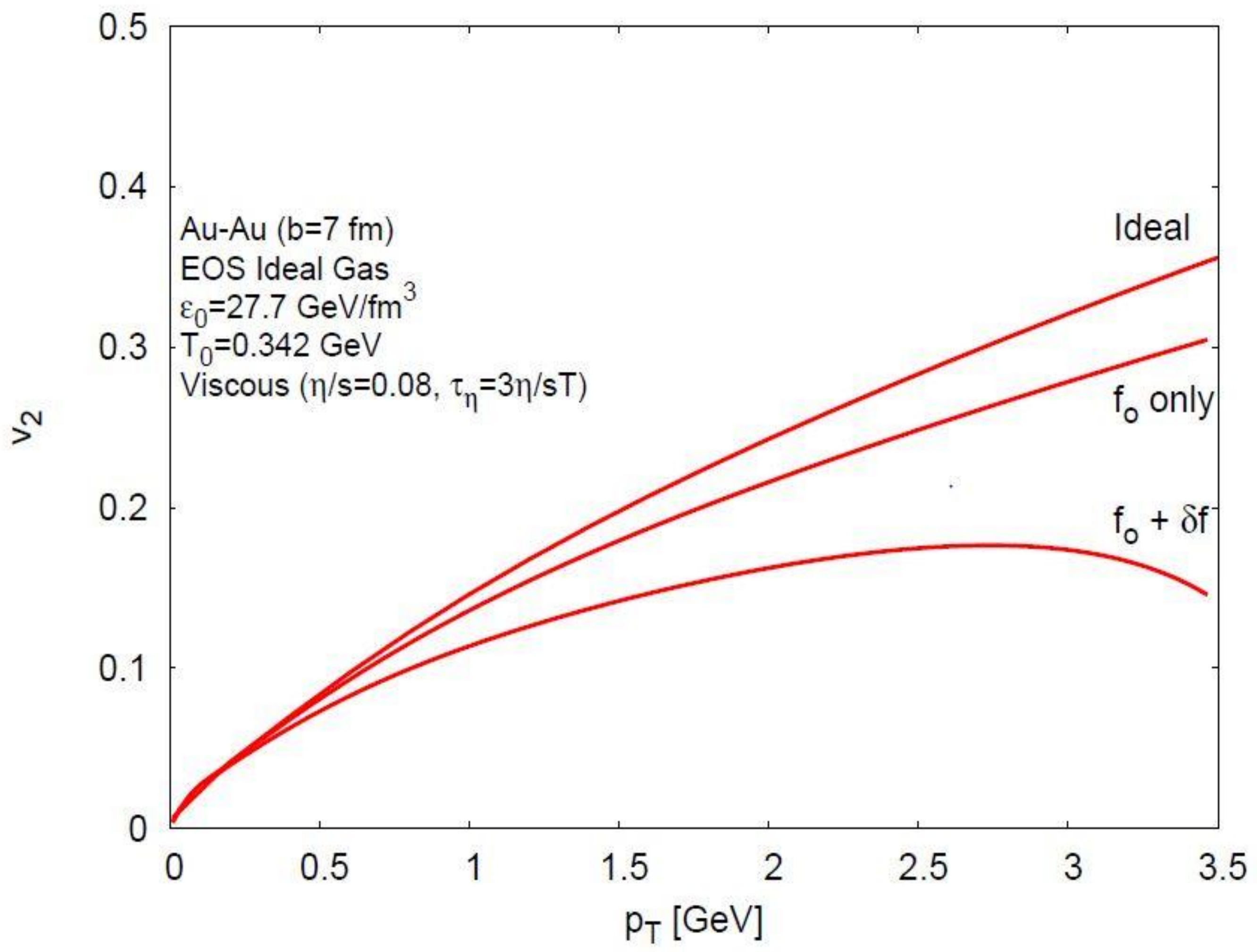


Picture by ALICE Collaboration
https://arxiv.org/abs/1303.0737v3

Add Viscosity

Picture by D. A. Teaney, http://arxiv.org/abs/0905.2433v1

- Limitation of ideal models: the predicted v2 is much above the measured result
- Add shear viscous corrections, $\frac{\eta}{s}$ enters as a new transport coefficient
- Extract parameters from experimental data



Blast Wave

- p_T spectra given by Cooper-Frye formula:

$$E\frac{d^3N}{dp^3} = \frac{g}{(2\pi)^3}\int_\sigma f(p,T)p^\mu\, d\sigma_\mu$$

where f is the distribution function, σ is the freeze-out hypersurface

- In equilibrium: $f_0 = \frac{1}{e^{p\cdot u/T} \mp 1}$
- With viscosity : $f = f_0 + \delta f$
- First-order(Navier-Stokes) approximation:

$$\delta f = \frac{1}{2}\frac{p_\mu p_\nu}{T^2}\frac{\pi^{\mu\nu}}{e+p} f_0$$

$$\pi^{\mu\nu} = 2\eta\langle\partial^\mu u^\nu\rangle \Rightarrow \delta f = \frac{1}{T^3}\frac{\eta}{s} f_0\, p_\mu p_\nu \langle\partial^\mu u^\nu\rangle$$

H. Song and U. Heinz, http://arxiv.org/abs/0712.3715v2

Blast Wave

- Hypersurface: τ=$\sqrt{t^2 - z^2}$ (constant τ)
  $$x^\mu = (t, x, y, z) = (\tau cosh\eta, x, y, \tau sinh\eta)$$
- Hadron momentum:
  $$p^\mu = (m_T coshY, p_T cos\theta, p_T sin\theta, m_T sinhY)$$
- Flow velocity:
  $$u^\mu = (cosh\eta_L cosh\eta_T, sinh\eta_T cos\phi_b, sinh\eta_T cos\phi_b, sinh\eta_L cosh\eta_T)$$
  where $v_L = tanh\eta_L, v_T = tanh\eta_T$
- Longitudinal flow: $v_L = \frac{z}{t} = tanh\eta$, η_L=η (boost invariance)

Blast Wave

F. Retiere and M. Annan Lisa, Phys.Rev.C 70,044907(2004)

- Radial flow:

$$\overrightarrow{v_T} = (v_T\cos\phi_b, v_T\sin\phi_b)$$

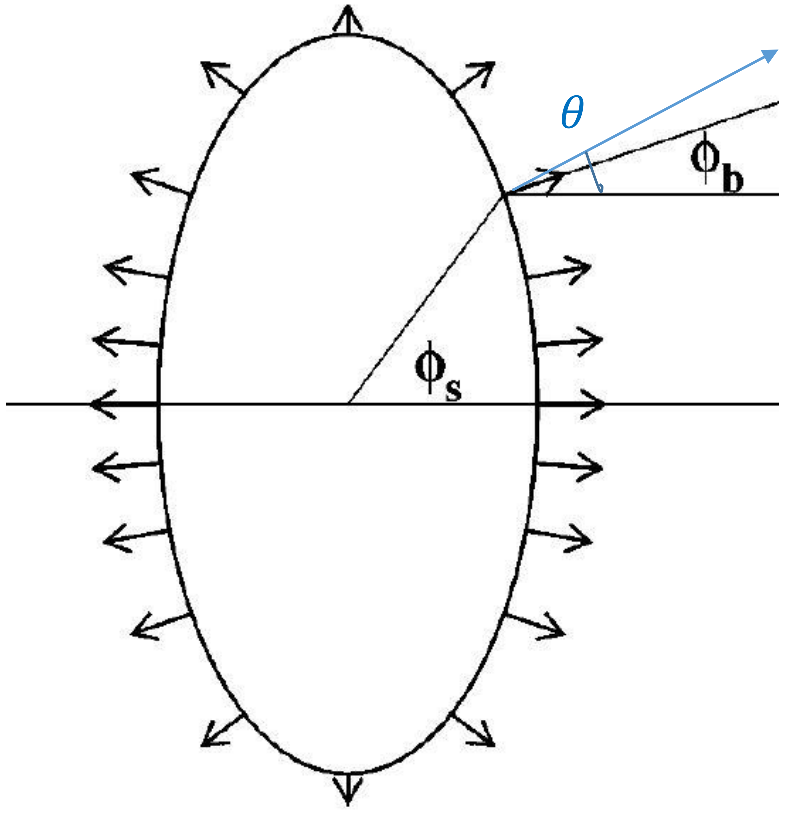
$$v_T = \alpha\rho^{n}$$

$$\alpha = \alpha_0 + \alpha_2\cos2\phi_b$$

$$\rho=\sqrt{\frac{x^2}{{R_x}^2}+\frac{y^2}{{R_y}^2}},\ x=\rho R_x\cos\phi,\ y=\rho R_y\sin\phi$$

- $tan\phi_b = \frac{R_x}{R_y} tan\phi$

- Parameters： $T, n, \alpha_0, \alpha_2, \frac{R_y}{R_x}, \frac{\eta}{s}$



$tan\phi_b = (\frac{R_x}{R_y})^2 tan\phi_s,\ tan\phi_s = \frac{y}{x}$

Shear Stress Tensor

- Definition: $\pi^{\mu\nu} = 2\eta \langle\partial^\mu u^\nu\rangle$

  $\langle\partial^\mu u^\nu\rangle \equiv [\frac{1}{2}(\Delta^\mu_\sigma \Delta^\nu_\tau + \Delta^\mu_\tau \Delta^\nu_\sigma) - \frac{1}{3}\Delta^{\mu\nu}\,\Delta_{\sigma\tau}]\partial^\sigma u^\tau$, $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$

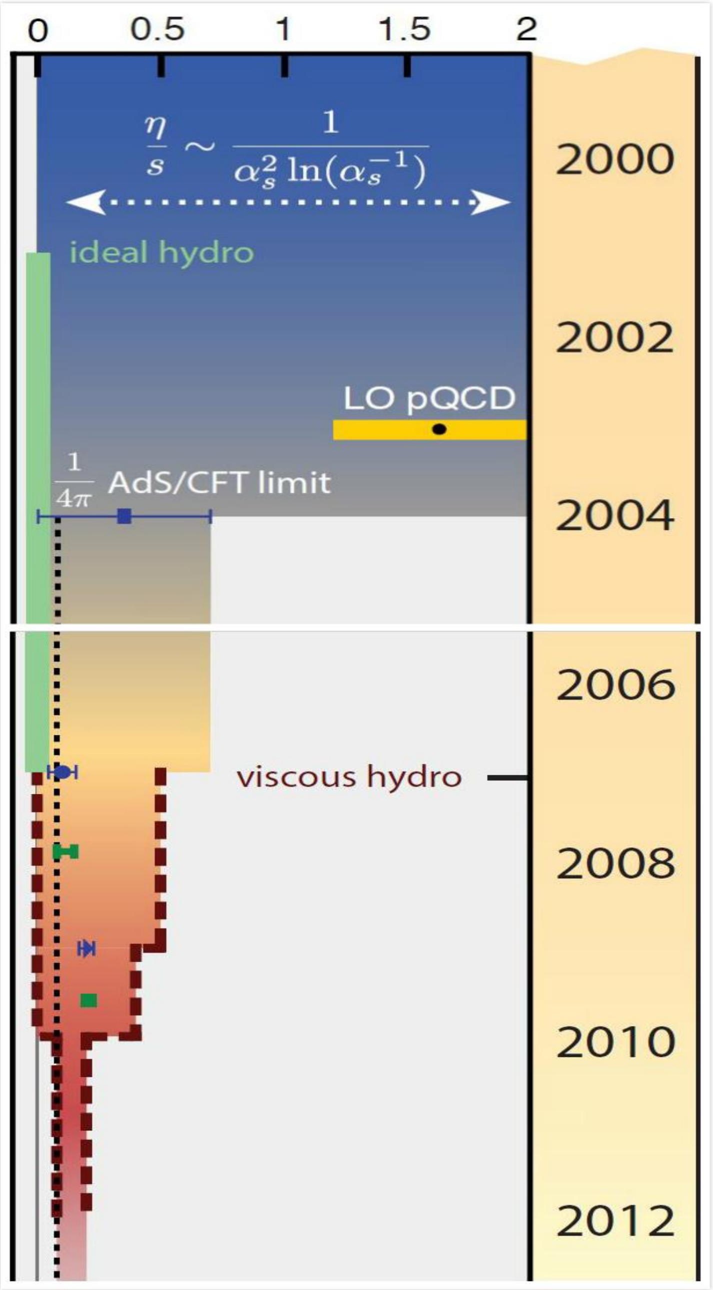
- One term for example:

$$\langle\partial^1 u^1\rangle = cosh^3(\alpha)cos(\phi_B)\Big(\frac{ntanh(\alpha)\cdot cos(\phi)}{r_x\rho} + \frac{4\alpha_i cos^2(\phi_B)sin^2(\phi_B)\rho^n}{r_x\rho cos\phi}\Big) - \frac{sinh(\alpha)sin^2(\phi_B)cos(\phi_B)}{r_x\rho cos(\phi)}$$

- Assumption: $\partial_\tau u^\mu = 0$

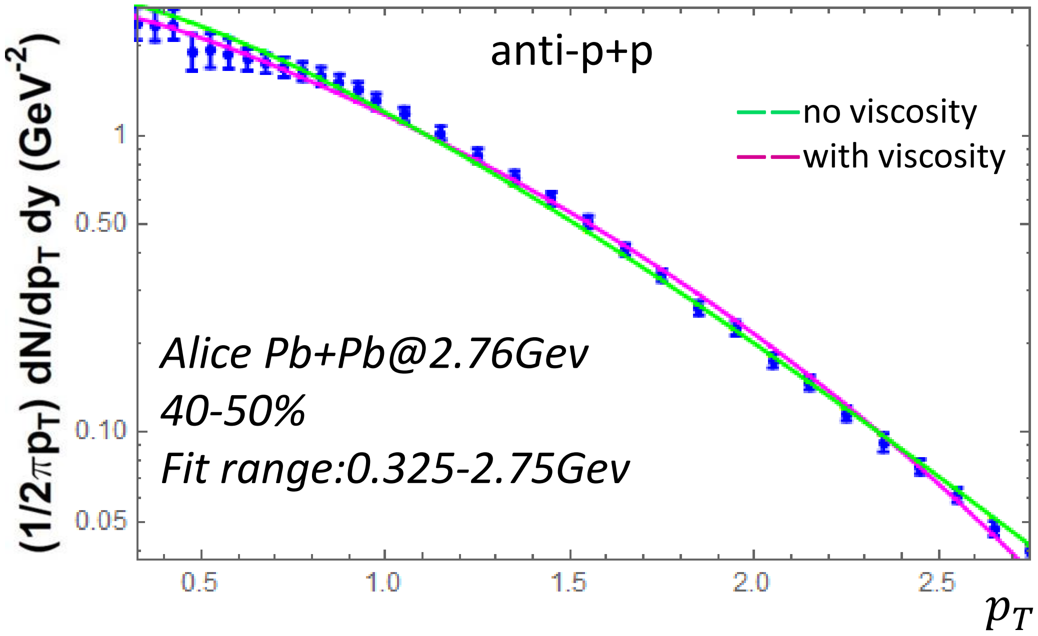
Application:
extract shear viscosity $\frac{\eta}{s}$ from data

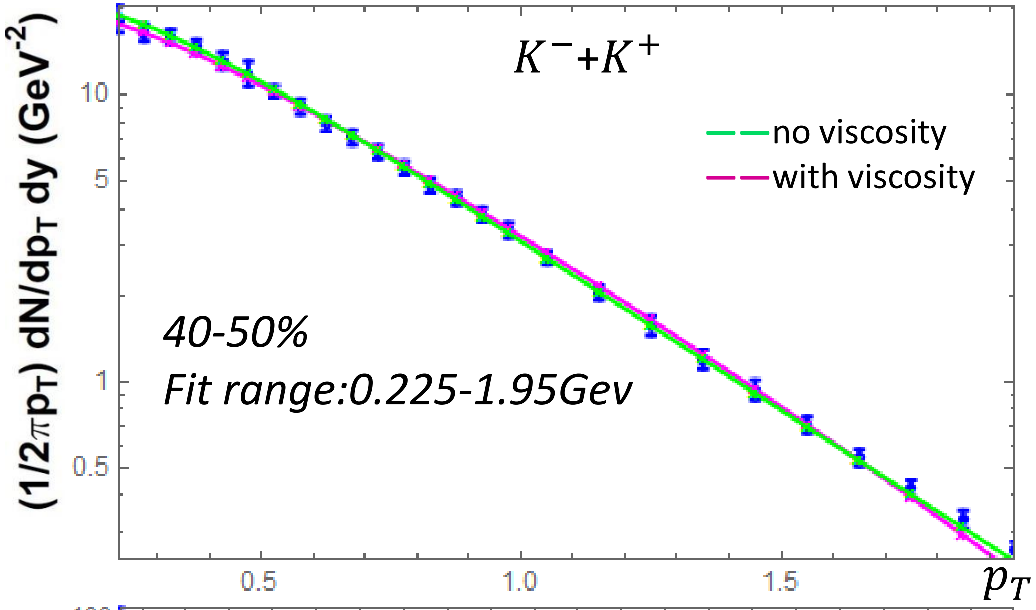
- Hydrodynamics
  - i). $\frac{\eta}{s}$ affects flow field
  - ii). $\frac{\eta}{s}$ affect freeze-out distribution function f_0
  - iii). Sensitive to temperature evolution
- Blast wave
  - i). Flow field from data
  - ii). $\frac{\eta}{s}$ Only affects freeze-out distribution function f_0
  - iii). Sensitive only to $\frac{\eta}{s}$ at freeze-out temperature

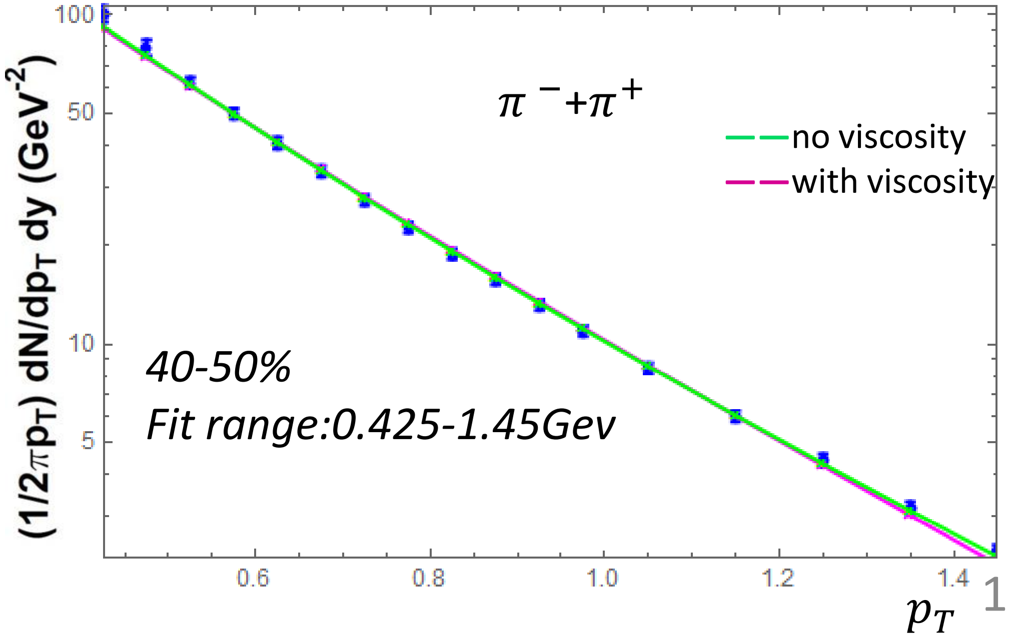


Picture by Charles Gale et al.
http://arxiv.org/pdf/1301.5893.pdf

χ^2fit to p_T spectra:





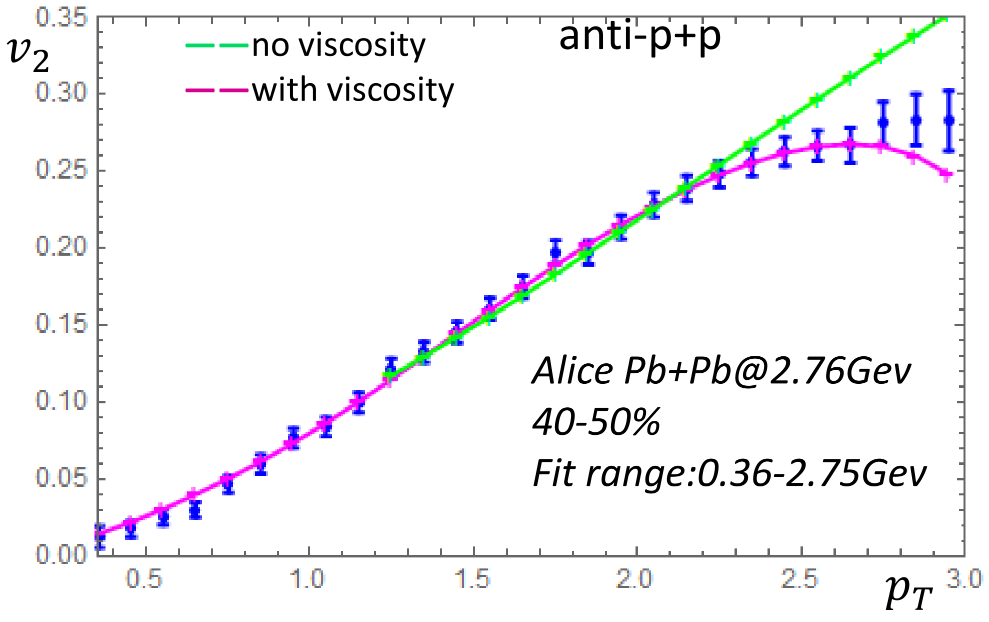


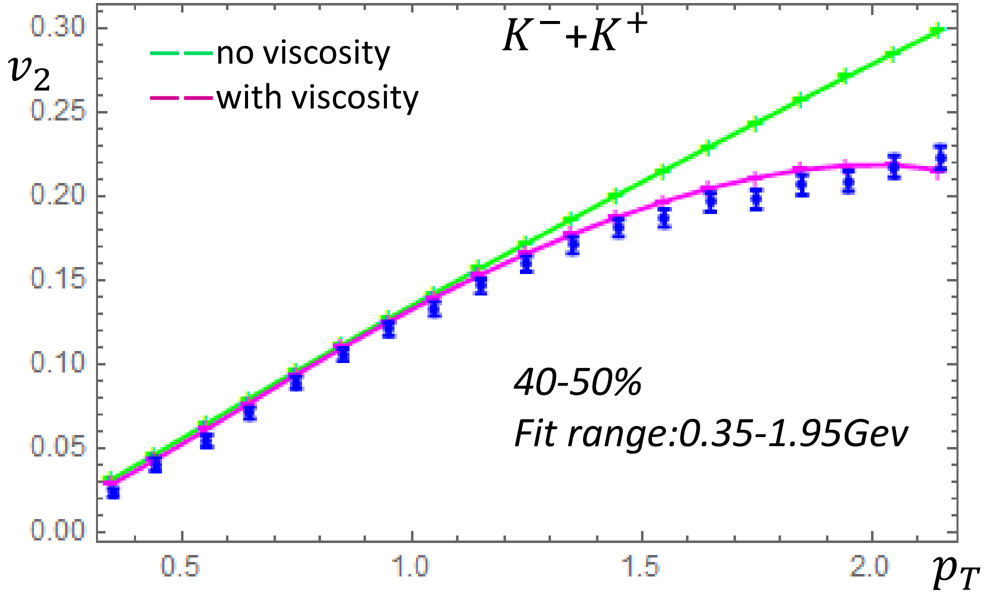
Set $\tau = 7fm, R_x R_y$=$12fm^2$

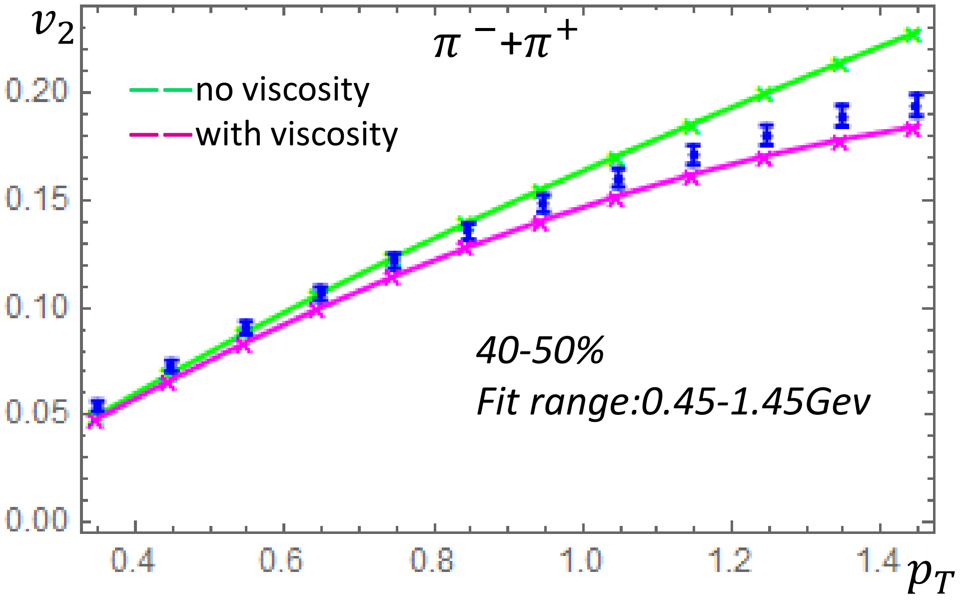
$$T = 114MeV, \alpha_0 = 0.846c, n = 1.08$$

$$\frac{R_y}{R_x} = 1.315, \alpha_2 = 0.0345c, \frac{\eta}{s} = 1.15 \cdot \frac{1}{4\pi}$$

χ^2fit to v_2:







χ^2=1.30 per degree of freedom(d.o.f)

Alternative fit only p and K v_2:

$v_2$of P,K,π	η/s=1.15/4π	χ^2=1.31 per d.o.f
v_2 of p and k, excludeπ	η/s =1.2/4π	χ^2=0.59 per d.o.f
v_2 of p,K,π,π with low temp	η/s =1.2/4π	χ^2=0.72 per d.o.f

Preliminary Result

Bayesian Package from The Models and Data Analysis Initiative (MADAI) Project:
http://stat.madai.us/

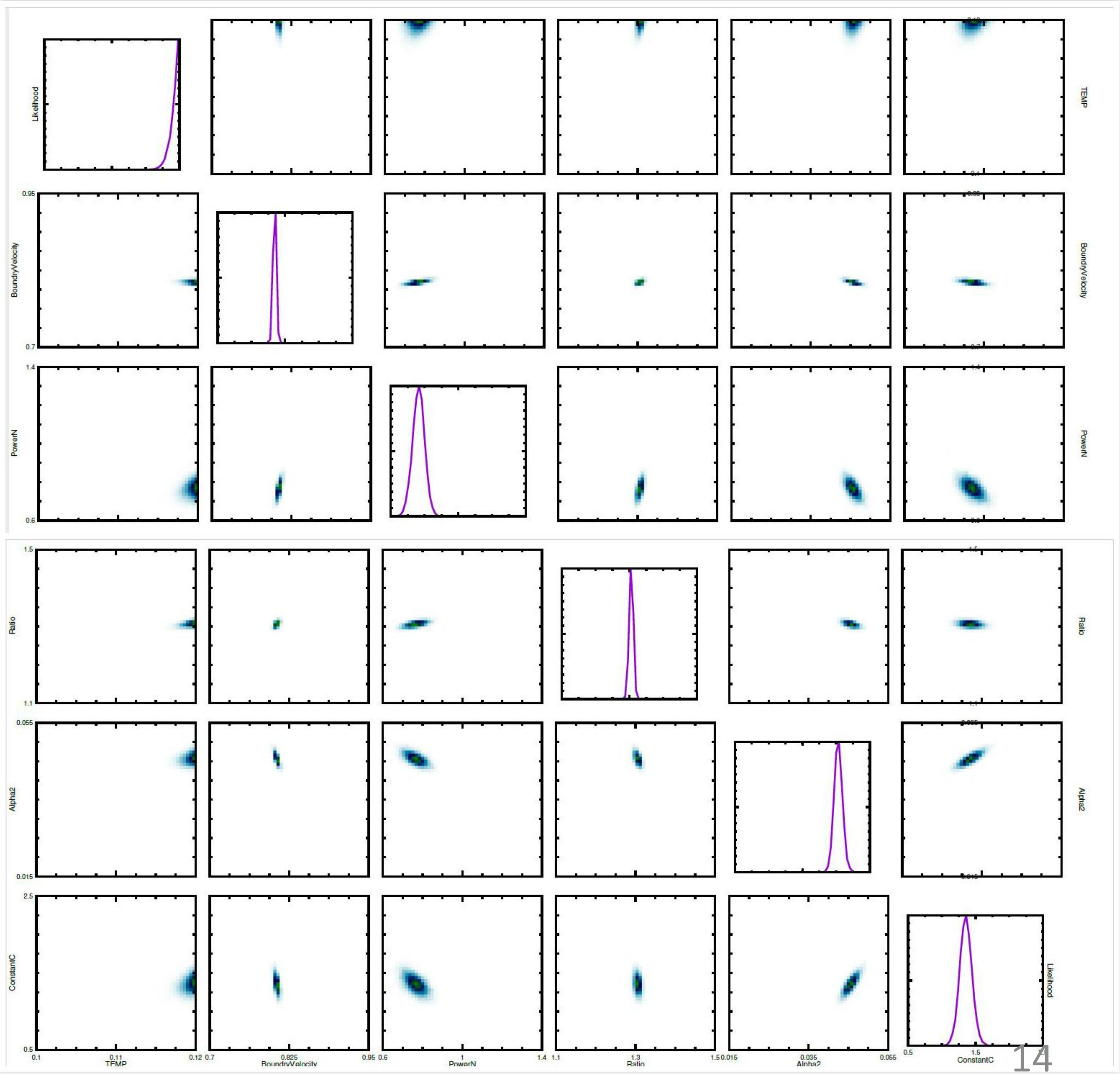
Work Process:
->generate sample points
->calculate values at sample points
->Gaussian Process Emulator
->likelihood estimate

$$T=120MeV,$$

$$n=0.762, \frac{R_y}{R_x}=1.307$$

$$\alpha_0=0.804c, \alpha_2=0.046c$$

$$\frac{\eta}{s}=1.39\cdot\frac{1}{4\pi}$$



Conclusion

- We develop a viscous blast wave and extract $\frac{\eta}{s}$ at kinetic freeze-out in a complimentary way
- Preliminary result shows $\frac{\eta}{s}$ is rather small
- Future work:

  i. Continue statistical analysis, more centrality bins

  ii. extract $\frac{\eta}{s}$ with uncertainty estimate

  iii. Use blast wave for other projects (quark recombination)