Spinodal Instability in Baryon-Rich Quark Matter

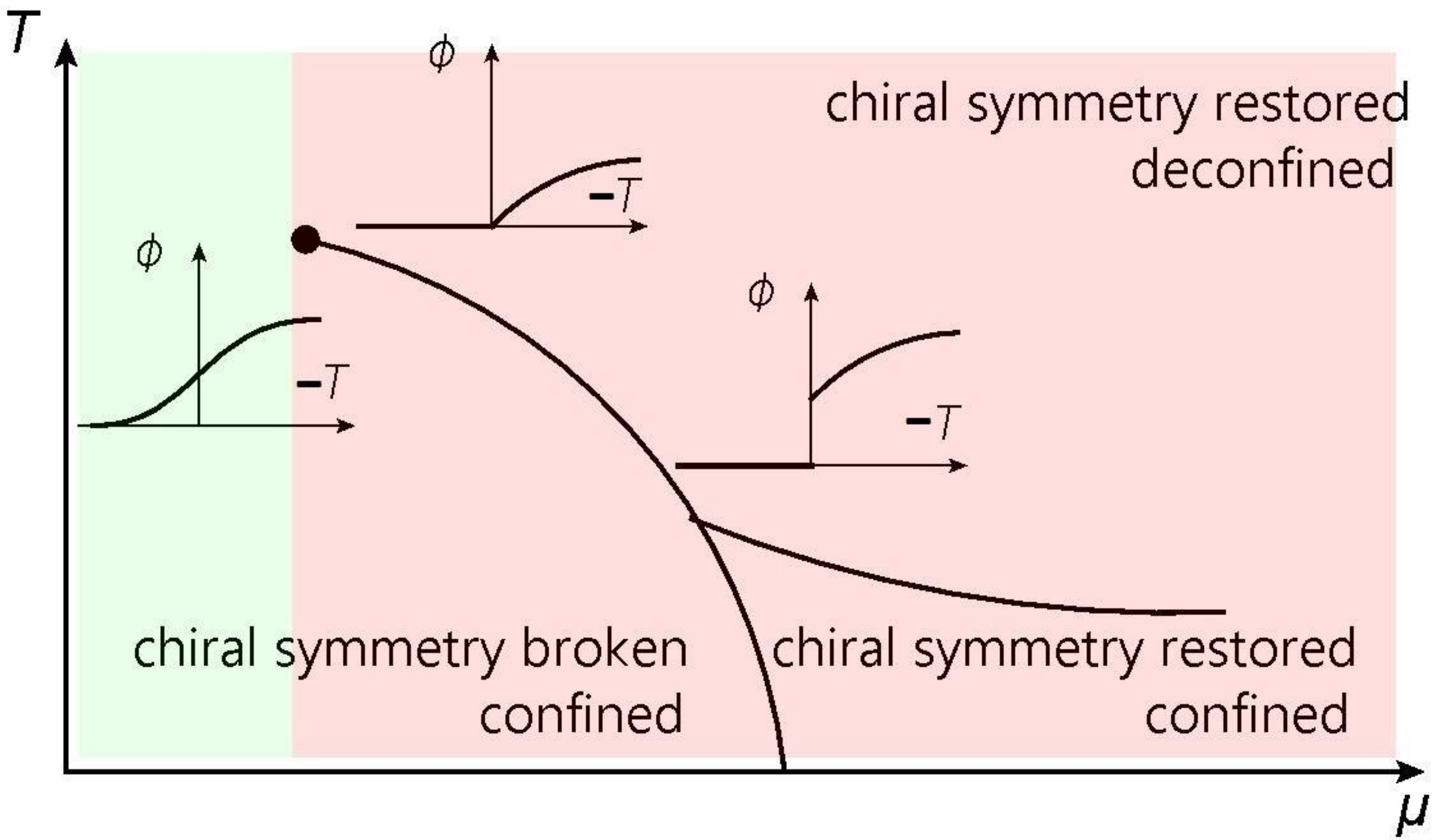
Feng Li and Che-Ming Ko

(arXiv: 1606.05012)

Outline

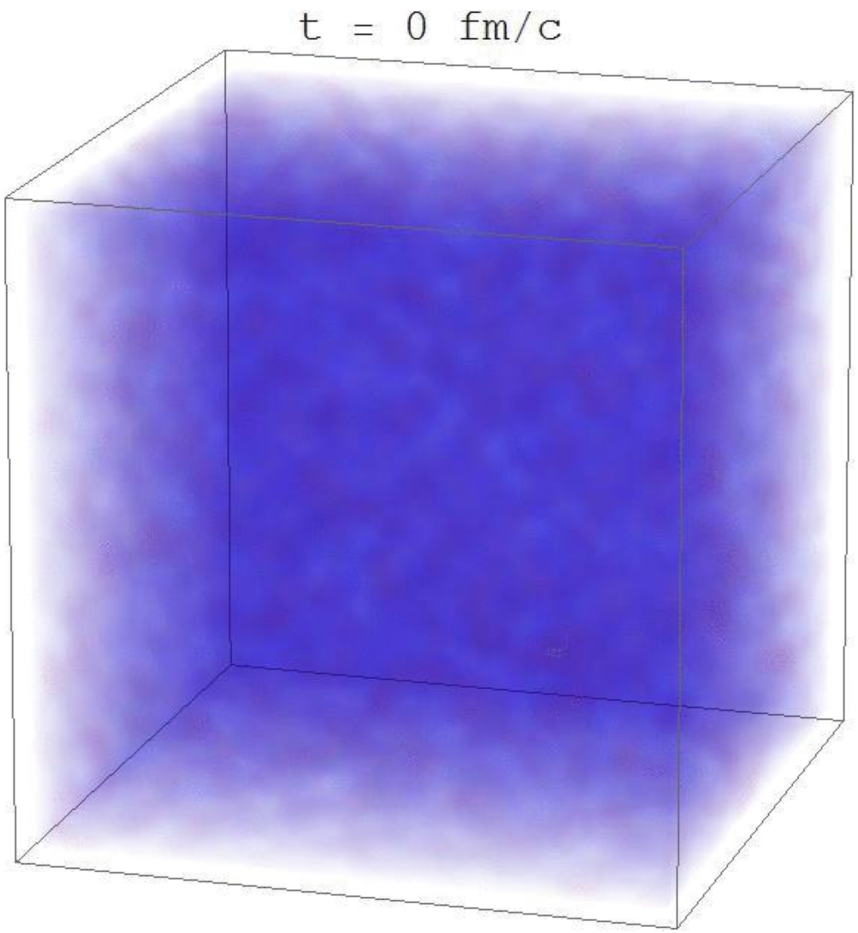
- Background (What is the spinodal instability? Why it matters?)
- Introduction to the Nambu-Jona-Lasino (NJL) model
- Quark matter in a box
- Quark matter in an expanding fireball
- Summary

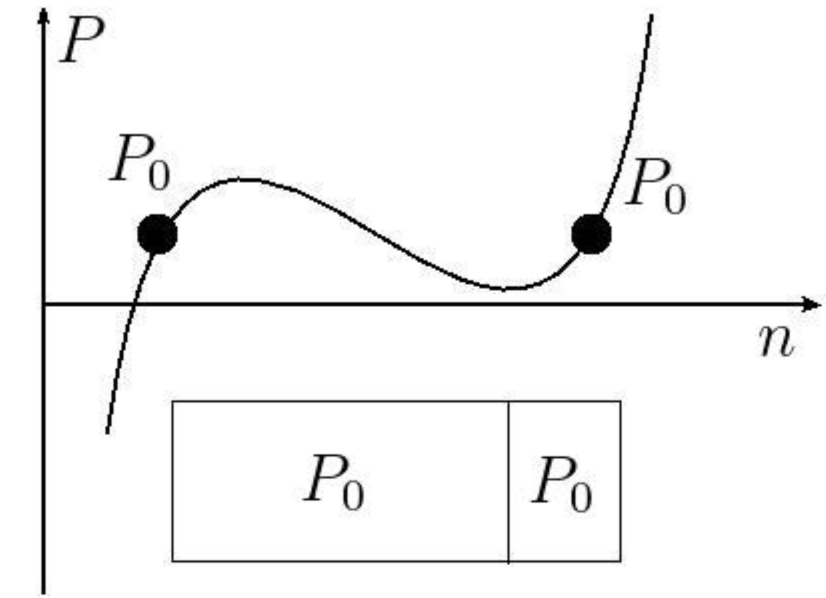
Order of Phase Transition



Y. Aoki, etc. Nature 443, 675 (2006)

Spinodal Instability





$$\left(\frac{\partial P}{\partial n}\right)_T < 0 \quad \text{or} \quad \left(\frac{\partial P}{\partial n}\right)_S < 0$$

The NJL Model (Action)

$$\mathcal{S}_{\mathrm{NJL}}[q,\bar{q}] = \int_0^\beta \mathrm{d}^4x \Big\{ \bar{q}(-\gamma^0\partial_\tau + i\boldsymbol{\gamma}\cdot\boldsymbol{\nabla} - m_0)q + \frac{G_S}{2}\sum_{a=0}^{8}\left[(\bar{q}\lambda^a q)^2 + (\bar{q}i\gamma_5\lambda^a q)^2\right] - G_V(\bar{q}\gamma_\mu q)^2 - K\left[\det{}_f\left(\bar{q}(1+\gamma_5)q\right) + \det{}_f\left(\bar{q}(1-\gamma_5)q\right)\right] \Big\}$$

- Quarks interact via contact interactions.
- The scalar interaction is attractive, while the vector one is repulsive.
- G_S and K are determined by fitting the meson masses and decaying constants. G_V is free to adjust.

T. Hatsuda and T. Kunihiro PLB 185, 304 (1987)

Spinodal Boundaries:

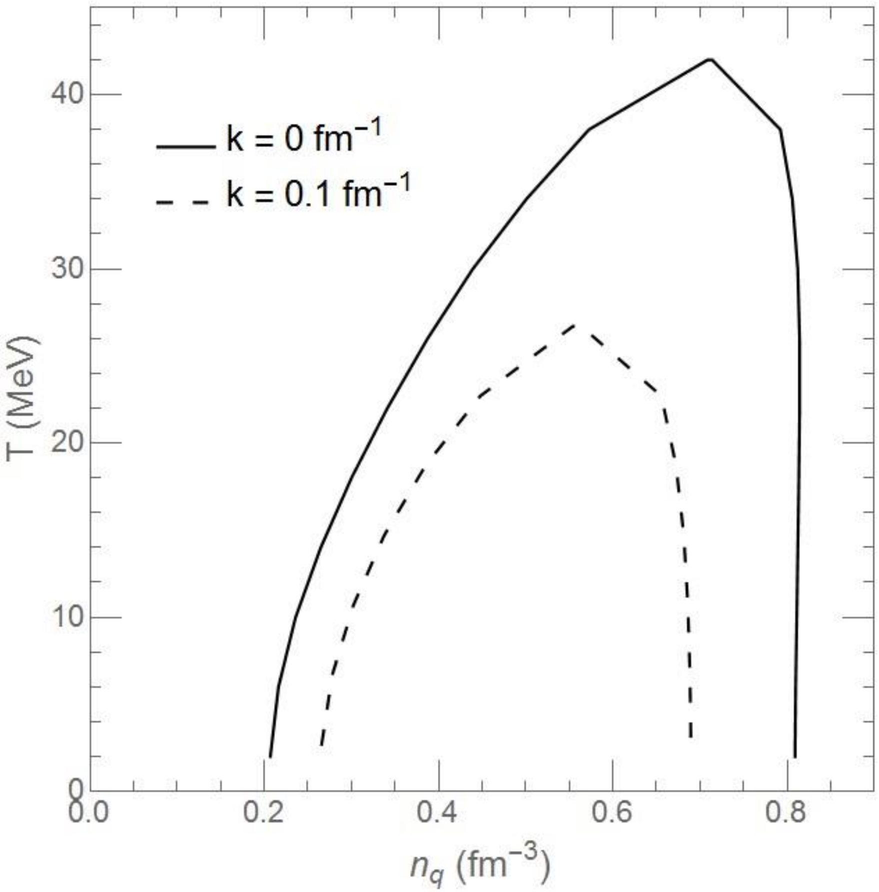
- Isothermal and isentropic spinodal boundaries in the case of Gv=0

  Tc is about 70 MeV

- Unstable modes of short-wavelength survive in the center of the spinodal instability region

- Isothermal spinodal boundaries in the case of Gv=0.2 Gs

  Supressed by the vector interaction



F. Li and C. M. Ko PRC 93, 035205 (2016)

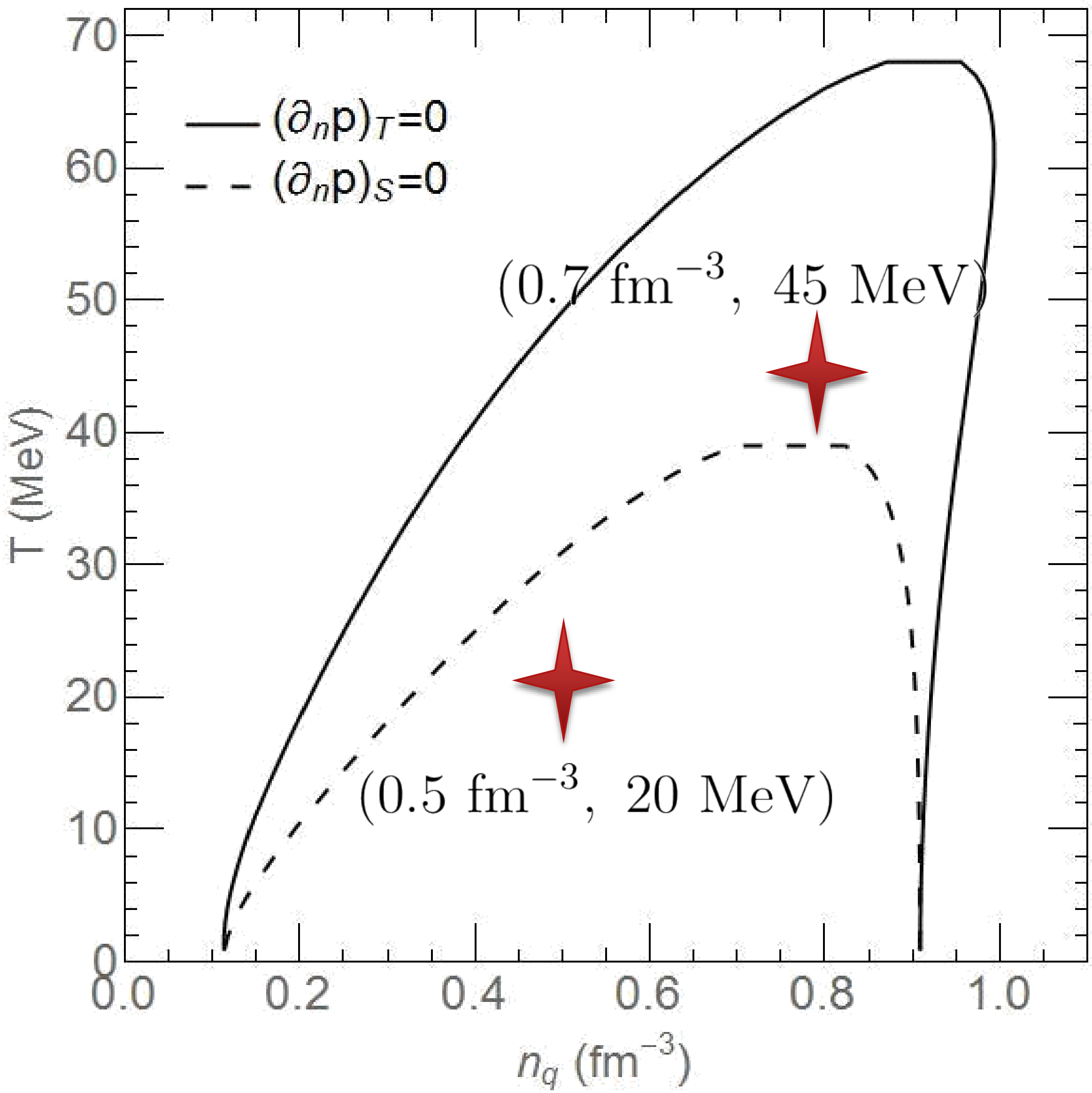
Solve the Boltzmann Equation

- Test Particle Method
- Equations of motion
- Collisions

C. Y. Wong, PRC 25, 1460 (1982)

- Cross sections are assumed constant (3mb)
- Treated geometrically
- Pauli blocking is taken into account

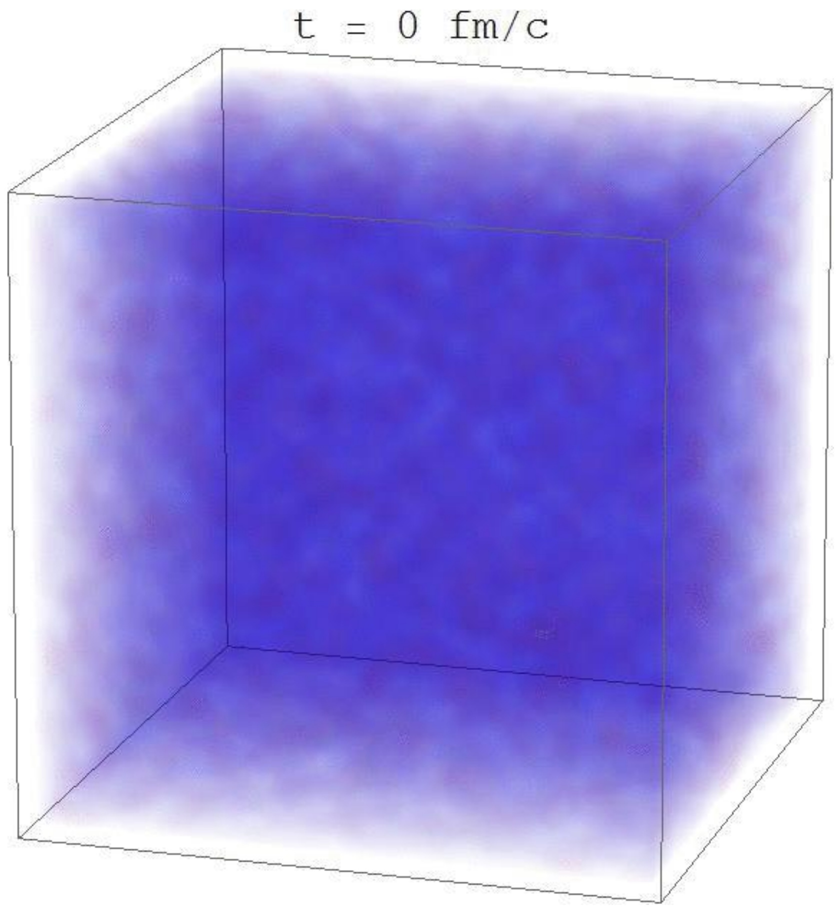
Quark Matter in a Box



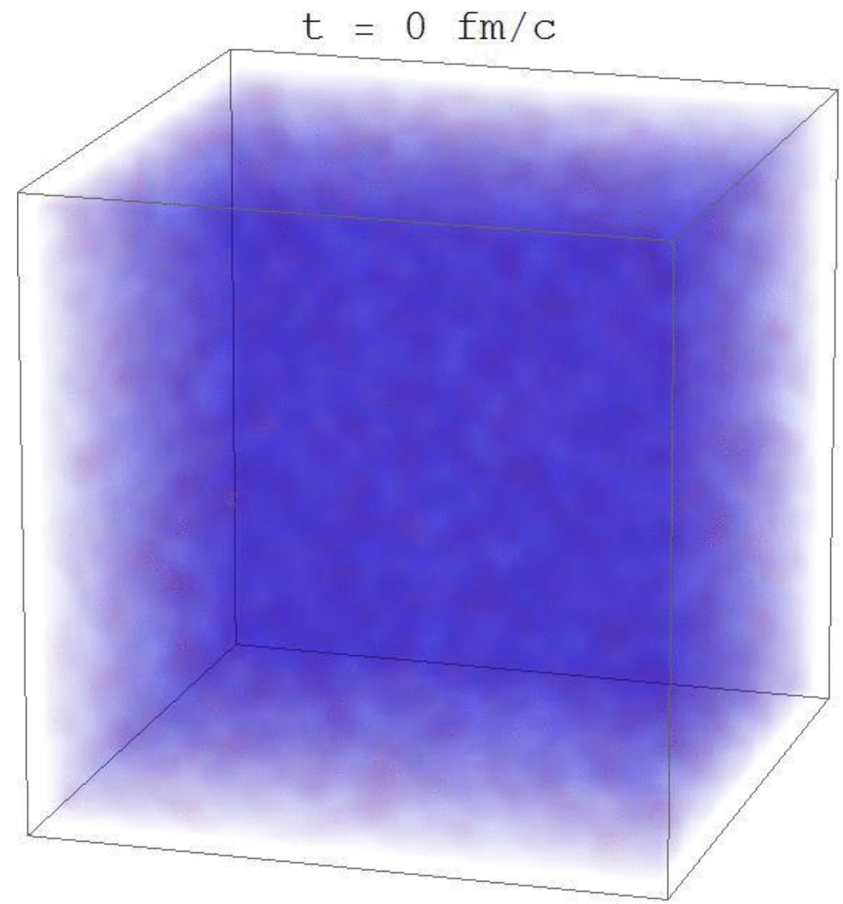
Almost uniform

$$G_V = 0$$

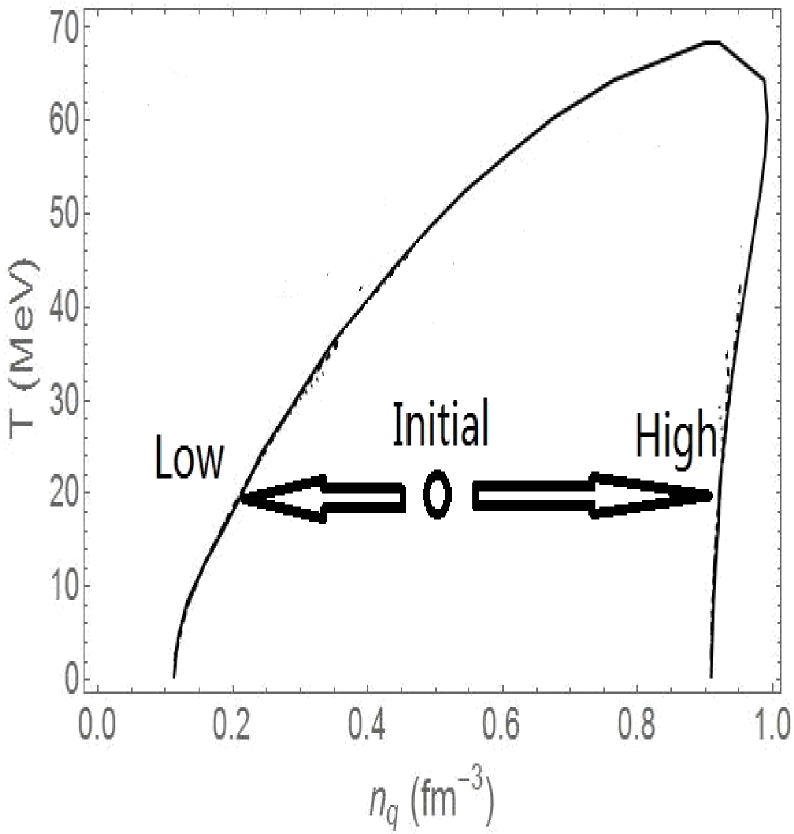
Quark Matter in a Box

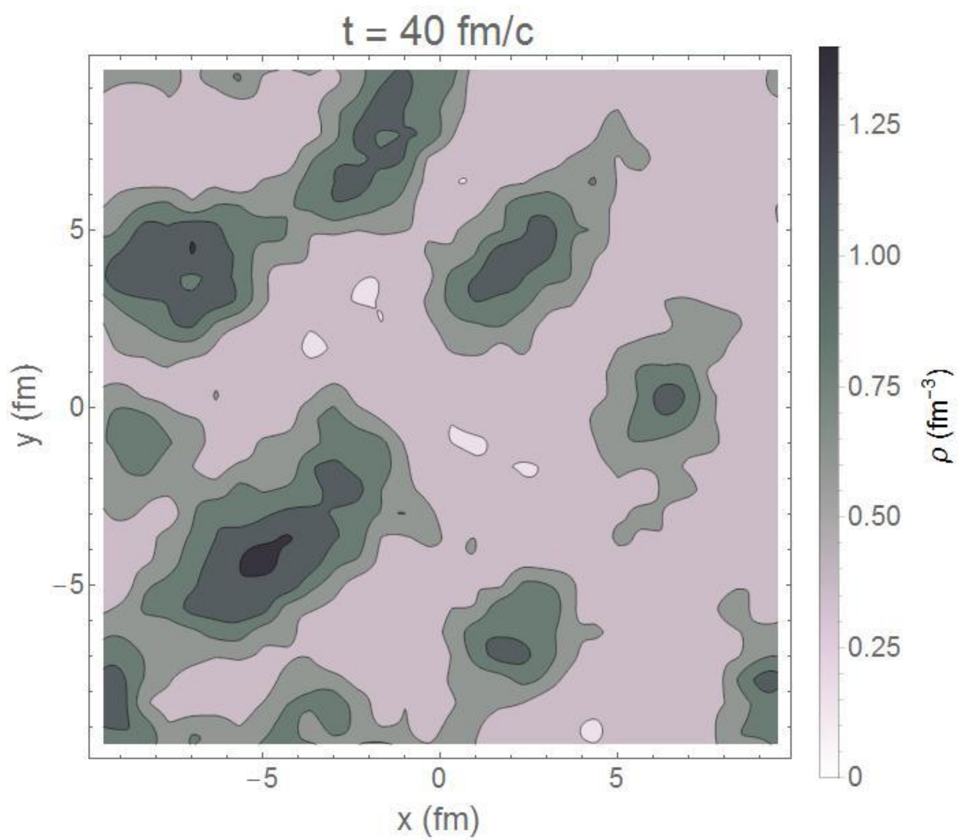


$T = 20$ MeV $n = 0.5$ fm^{-3}



$T = 45$ MeV $n = 0.7$ fm^{-3}


70
60
50
40
30
20
10
0
T (MeV)
0.0
0.2
0.4
0.6
0.8
1.0
n_q (fm^{-3})
Low
Initial
High


t = 40 fm/c
5
0
−5
y (fm)
−5
0
5
x (fm)
1.25
1.00
0.75
0.50
0.25
0
ρ (fm^{-3})

Density Moments

- Definition of Nth-order (scaled) density moments

  Equal to one for uniform density distribution

- Density distribution is not necessarily uniform if the scaled density moments equal to one

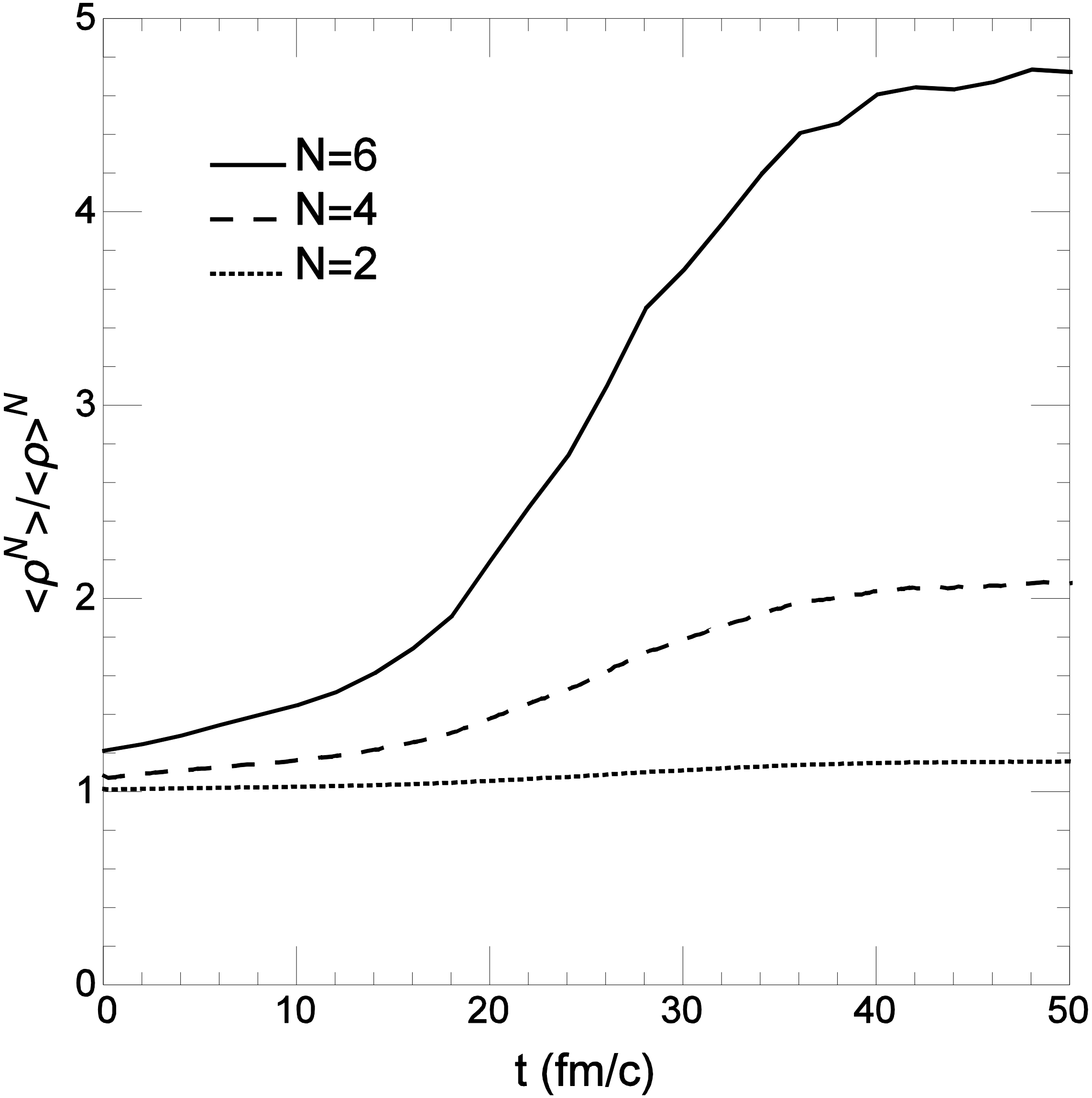
- Measure of the spectrum of density

- Estimate the scaled density moments in the mixed-phase region

$$\frac{\langle \rho^N \rangle}{\langle \rho \rangle^N} = \frac{[\rho_1^{N+1}(\rho_2 - \rho_0) + \rho_2^{N+1}(\rho_0 - \rho_1)][\rho_0(\rho_2 - \rho_1)]^{N-1}}{[\rho_1^2(\rho_2 - \rho_0) + \rho_2^2(\rho_0 - \rho_1)]^N}$$

$$\rho_0 = 0.5 \text{ fm}^{-3} \qquad \rho_1 \approx 0.25 \text{ fm}^{-3} \qquad \rho_2 \approx 1.0 \text{ fm}^{-3}$$

$$\langle \rho^2 \rangle / \langle \rho \rangle^2 \to 1.22 \quad \langle \rho^4 \rangle / \langle \rho \rangle^4 \to 2.11 \quad \langle \rho^6 \rangle / \langle \rho \rangle^6 \to 3.75$$


N=6
N=4
N=2
0
1
2
3
4
5
0
10
20
30
40
50
t (fm/c)
$<\rho^N>/<\rho>^N$

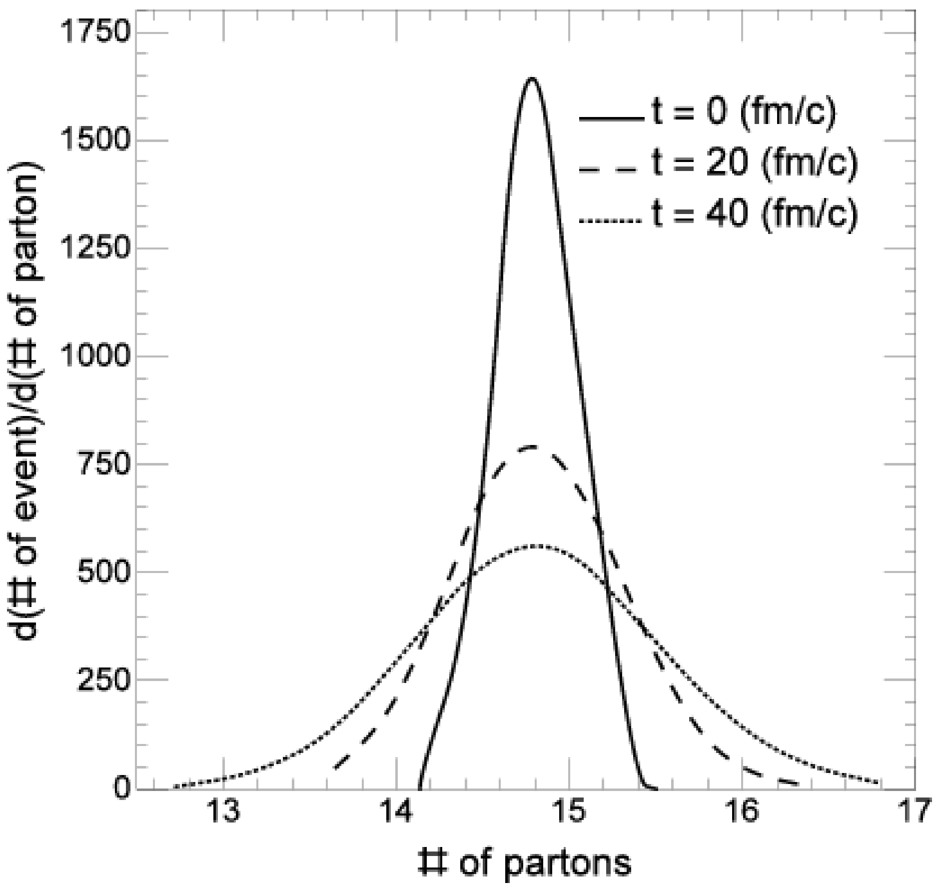
Number Fluctuation in a Sub-volume

- Size of the sub-volume = 0.8 fm^3, which is 1/10000 of the total volume. (Event number = 1000)

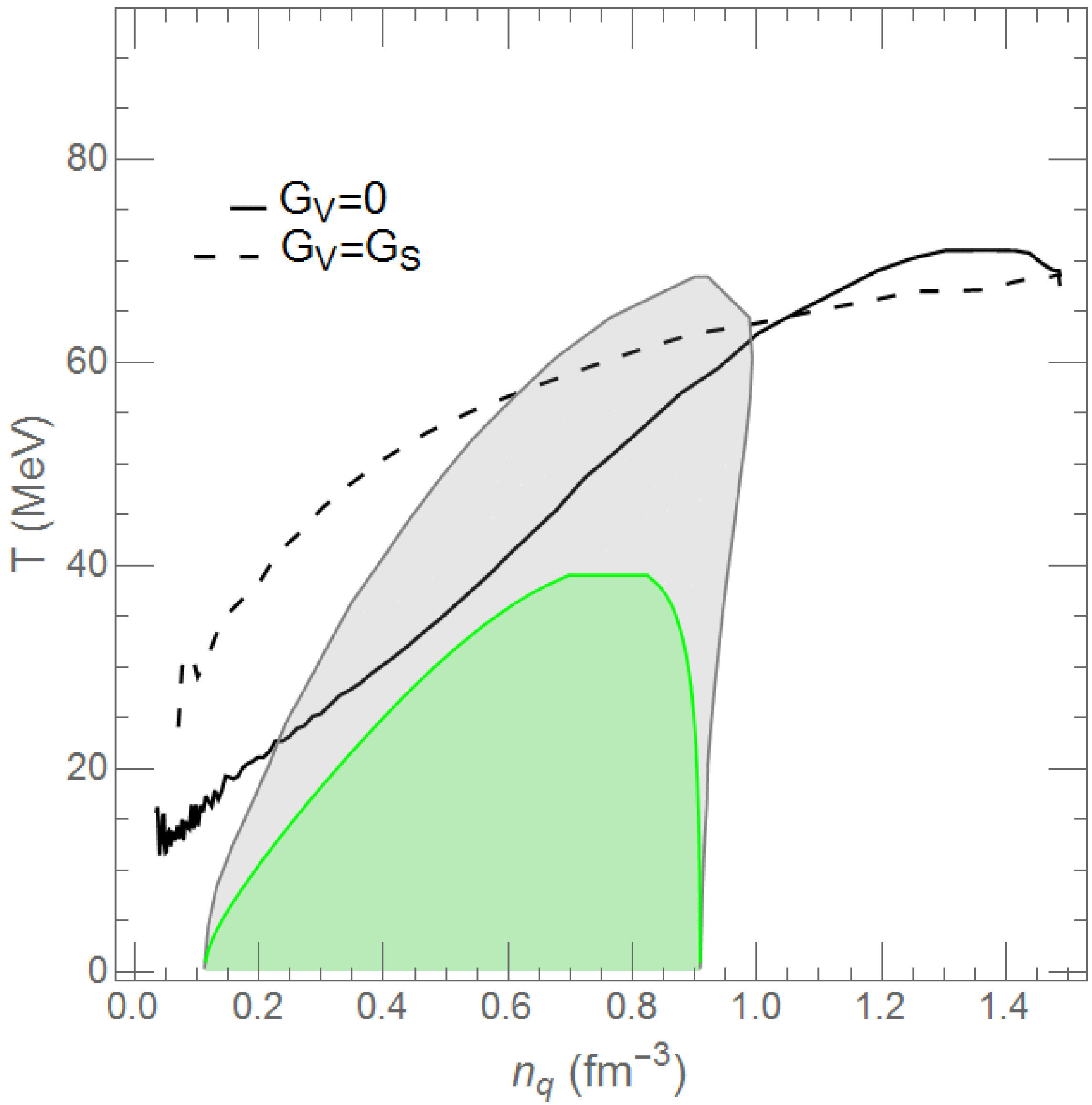
  Broaden and positively skewed with time

- Size of the sub-volume = 40 fm^3, which is 1/200 of the total volume

  Broaden but not skewed



Quark Matter in a Fireball

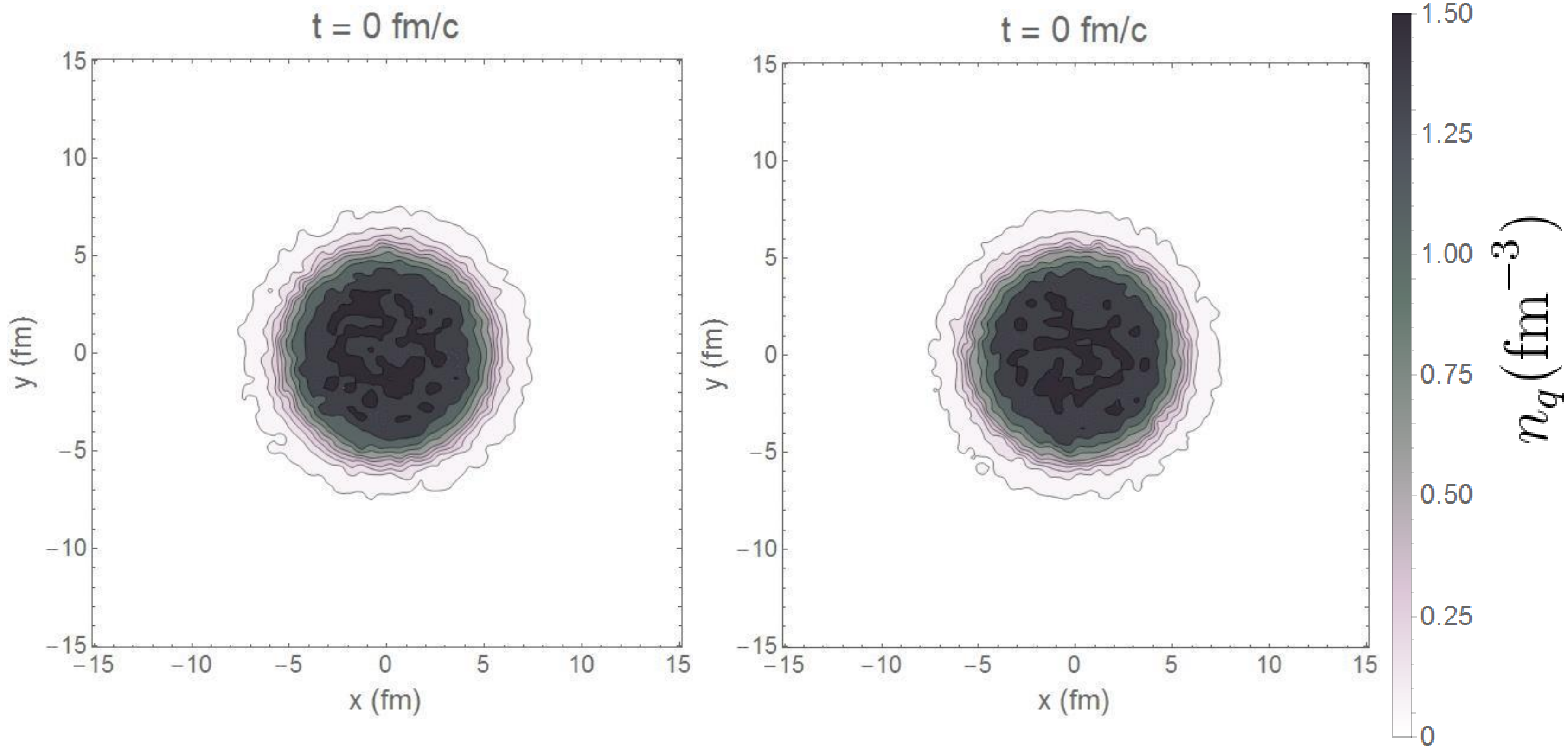


$$T_0 = 70 \text{ MeV}$$
$$n_0 = 1.5 \text{ fm}^{-3}$$

Phase trajectory of the central part of the fireball

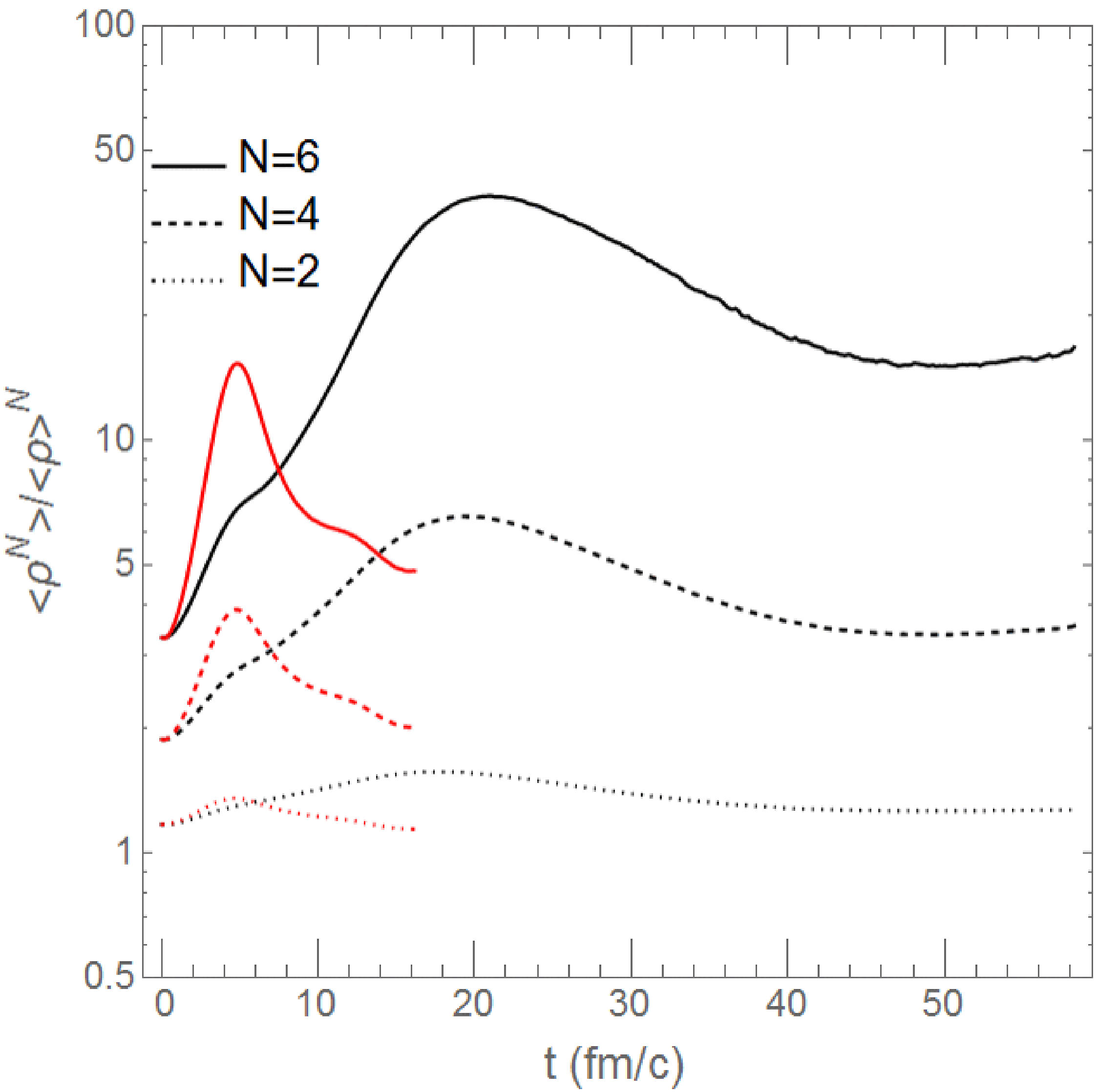
Quark Matter in a Fireball

$$T_0 = 70 \text{ MeV} \quad n_0 = 1.5 \text{ fm}^{-3}$$



$G_V = 0$ $\qquad$ $G_V = G_S$

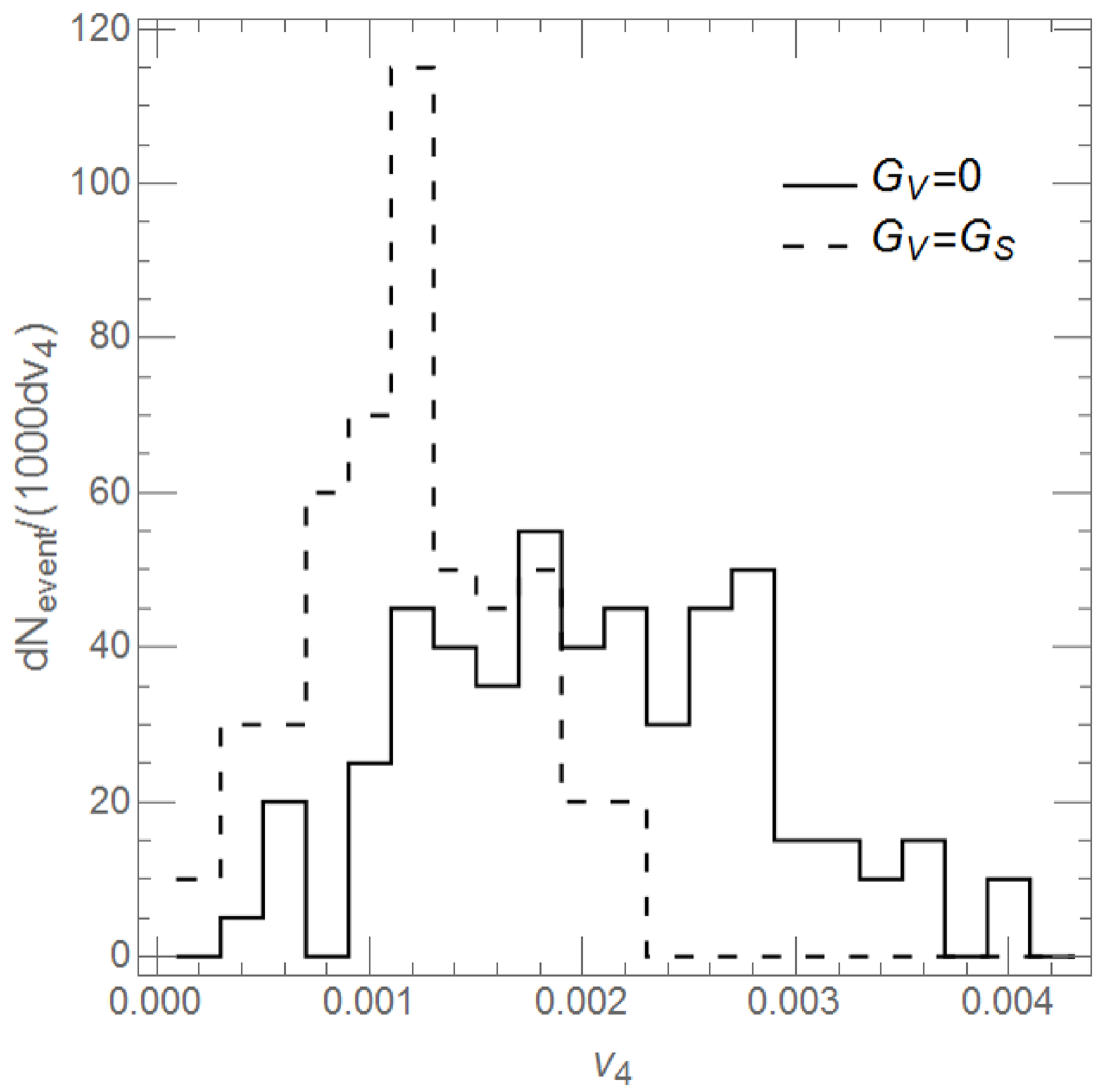
Density Moments



Black: $G_V=0$
Red: $G_V=G_S$

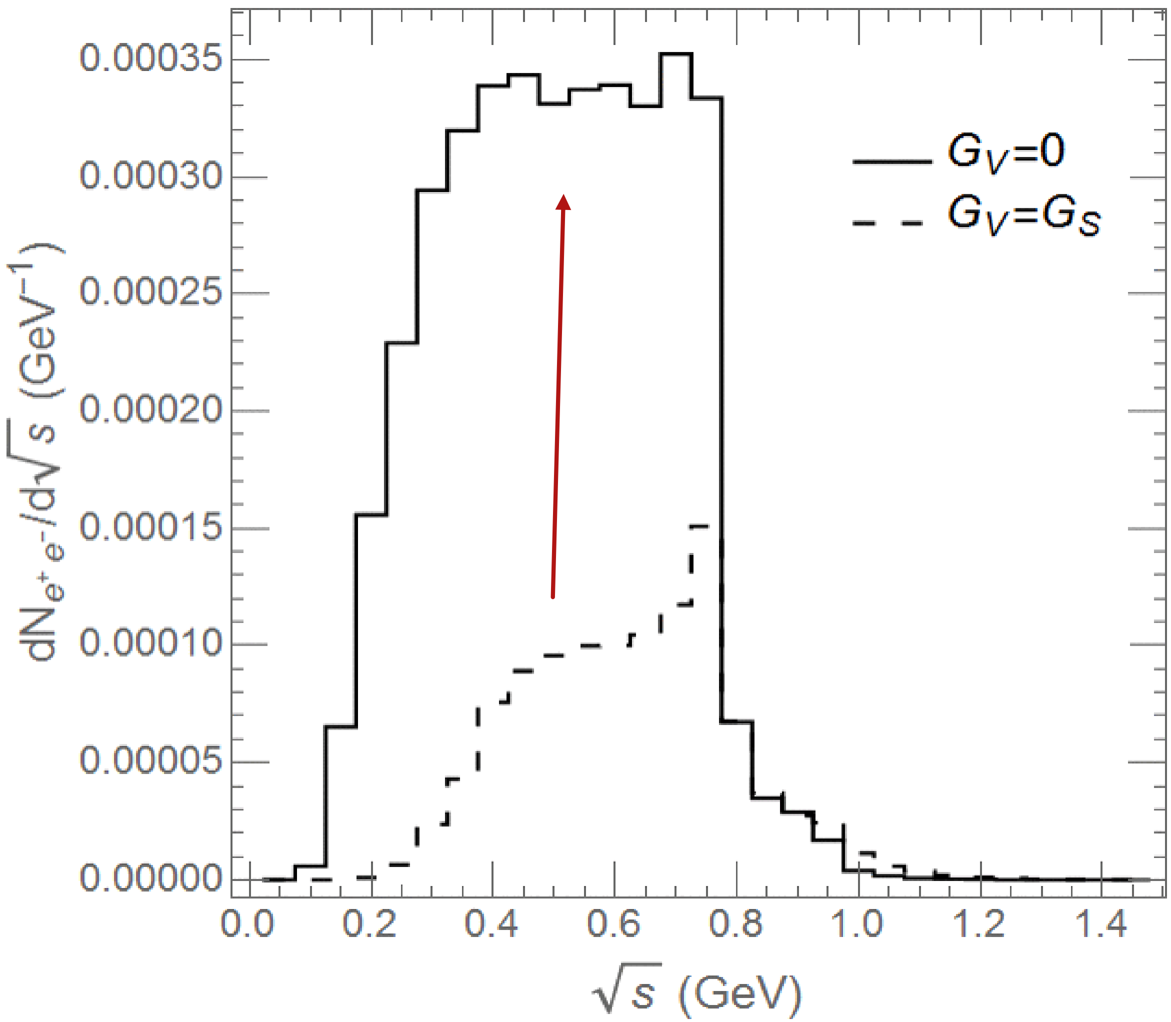
Anisotropic Flow

- Induced by the density fluctuations in the transverse plane
- Angular distribution in the transverse plane
- Two body correlation
- V2 event by event
- V4 event by event



Dilepton Yield

$q\bar{q} \rightarrow e^+e^-$ proportional to the square of the density



Summary

- The vector interaction suppress the spinodal instability.
- The instabilities of smaller wavelengths survive under lower temperature
- The high-order density moments increase and saturate at large values after phase separation.
- The event by event distribution of the particle number in the sub-volume is both broaden and positively skewed. Skewness disappears quickly as the volume size increases
- Expansions are slow in the presence of a first-order phase transition.
- Anisotropic flow and dilepton yield might be employed as the signals of the first order phase transition
- Calculations based on a more realistic model will be carried out.

Quark Matter in a Fireball (AMPT initial condition)

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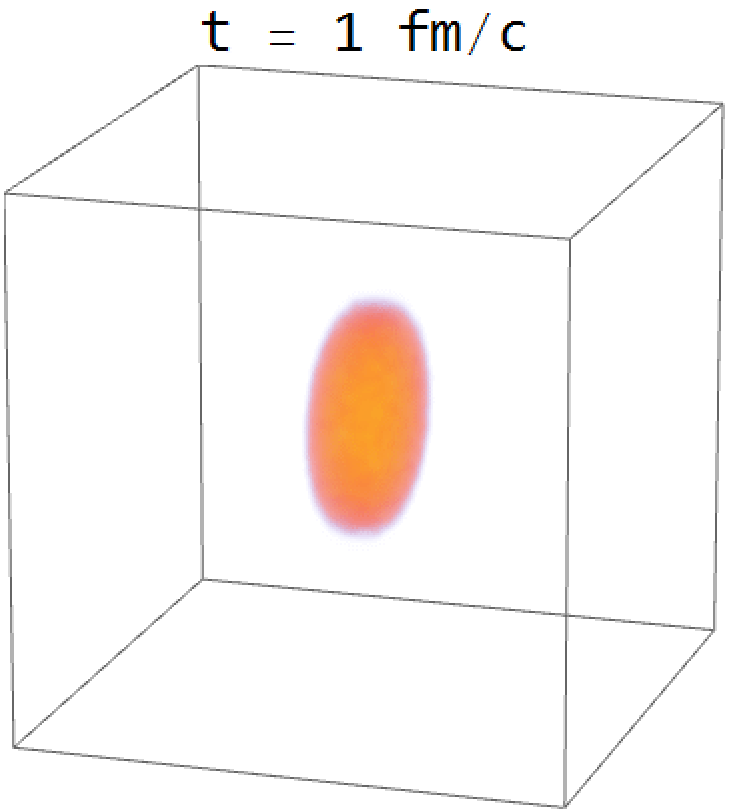

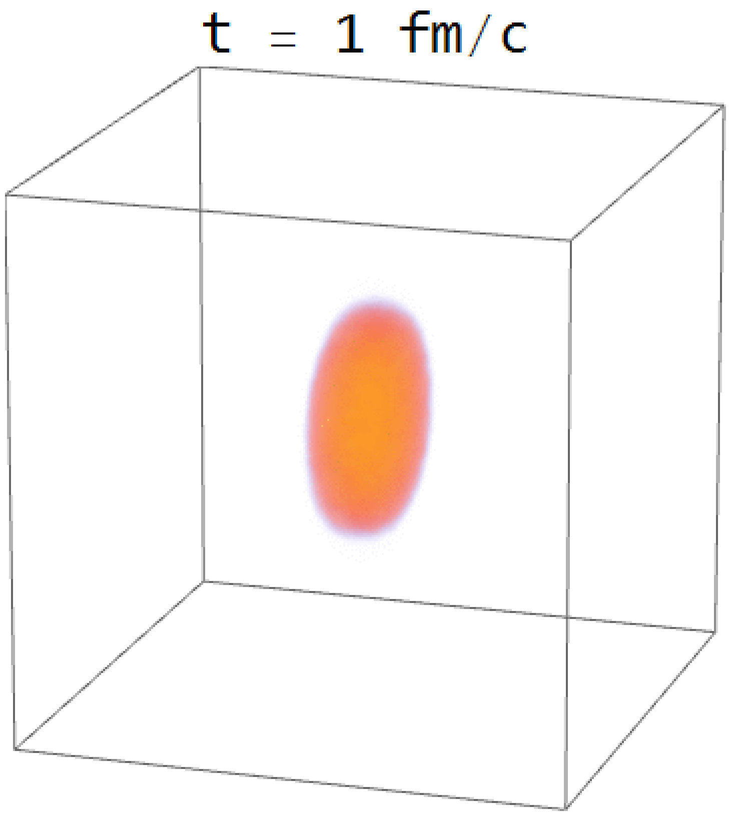

Quark Matter in a Fireball (AMPT initial condition)