Electric Conductivity of Hot Hadronic Matter

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Transport Coefficients

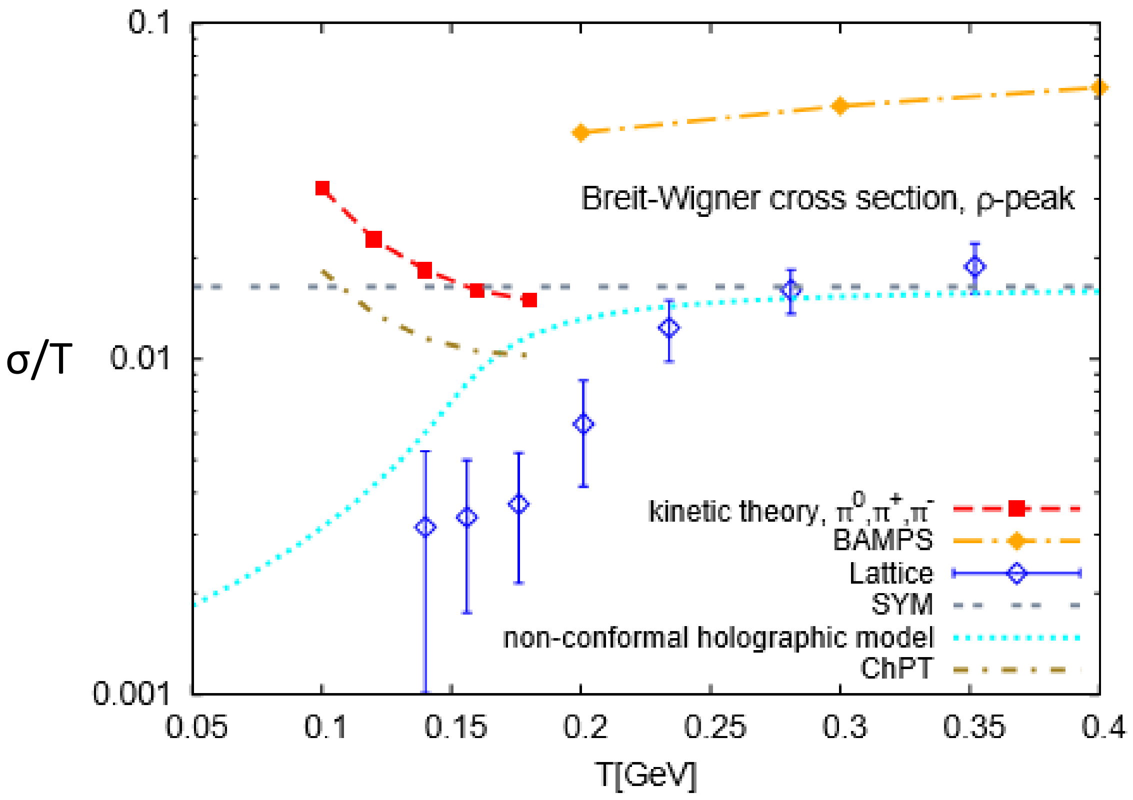
- One can characterize nuclear matter with transport coefficients

- Ex: η/s, σ/T, and $D_s(2\pi T)$

- These all characterize how quantities are transmitted through the medium, and probe the long wavelength limit of the medium.

Previous Calculations

- Previous calculations of the electric conductivity have widely varied.
- Many calculations disagree with the quantum lower bound.
- Here: perform a quantum many body calculation for a pion gas.
- Preserve unitarity and gauge invariance.



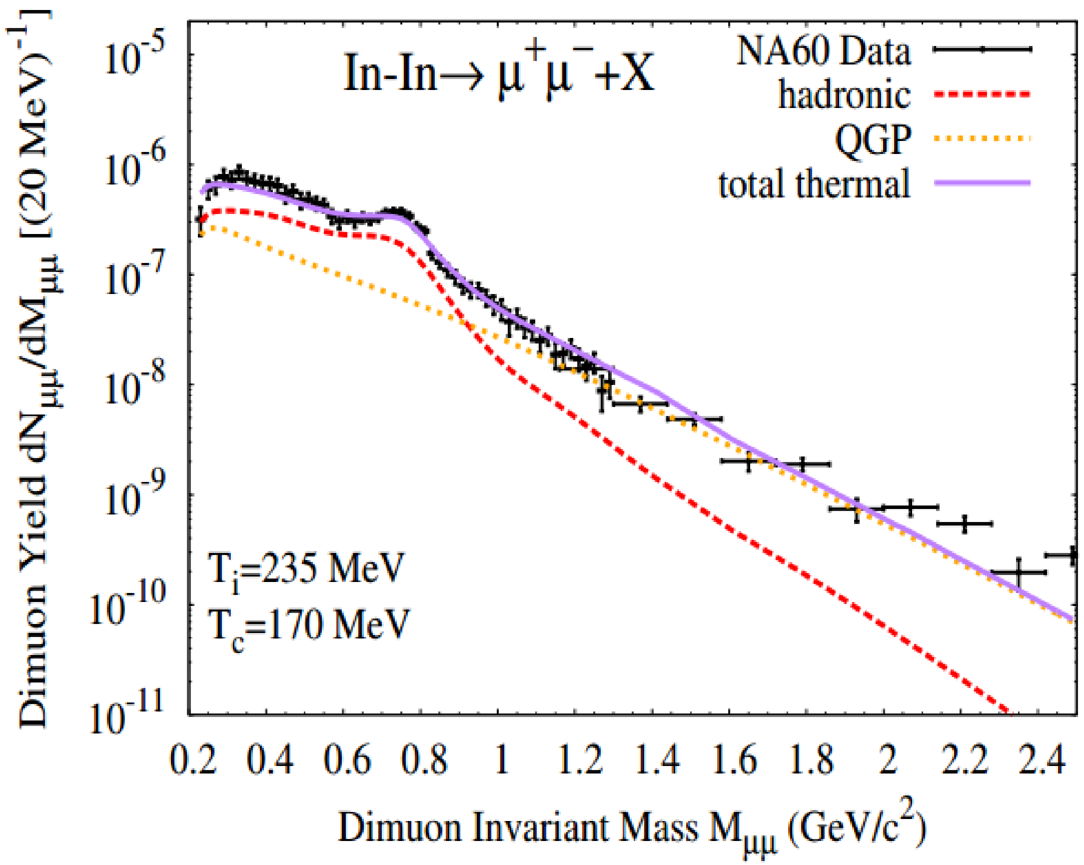
Greif et. al.

Dilepton Emission Spectra

- The calculation is rooted in dilepton production calculations, which have been successful.

- $\frac{d\epsilon_{l+l-}}{d^4q} = \frac{-\alpha_{EM}^2}{\pi^3 M^2} f_B(q_0,T) Im[\Pi_{em}(M,q,T,\mu_i)]$

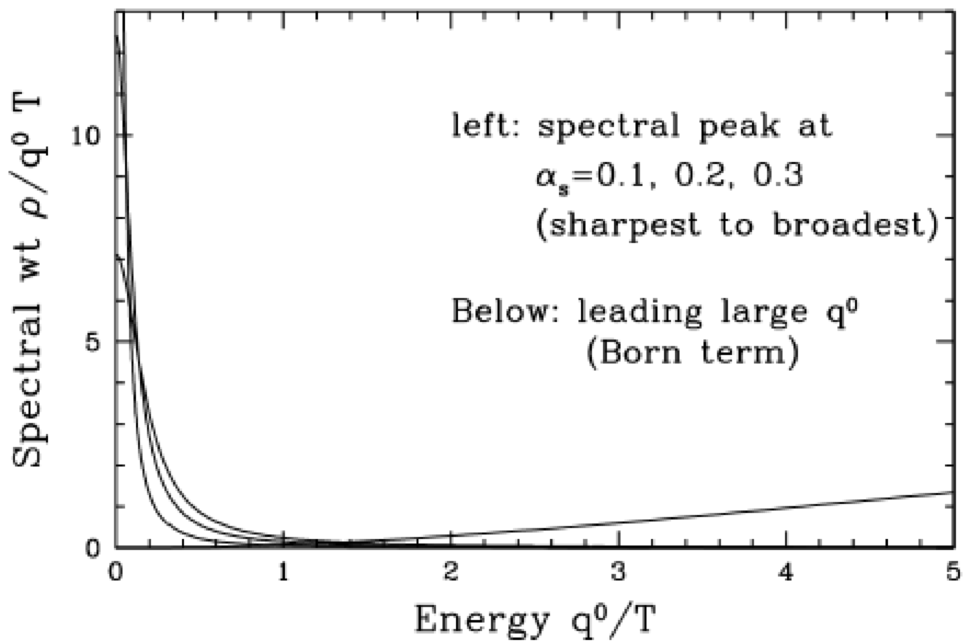
- σ(T)=$e^2 \lim_{q_0 \to 0} \frac{Im[\Pi_{em}(q_0,\vec{q}=0,T)]}{q_0}$



Rapp & van Hees et. al.

QCD Base Line

- Moore & Robert calculated the spectral weight for a 3-flavored plasma.
- σ(T)=$\frac{e^2}{6}\lim_{q_0\to 0}\frac{Im[\rho(q_0,\vec{q}=0,T)]}{q_0}$
- In the weakly coupled limit the conductivity approaches infinity, but spreads out into a Lorentzian like structure as α_s increases



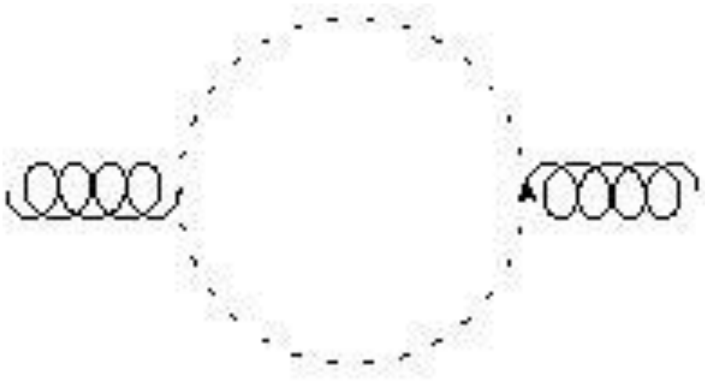
Moore et. al.

Vector Meson Dominance

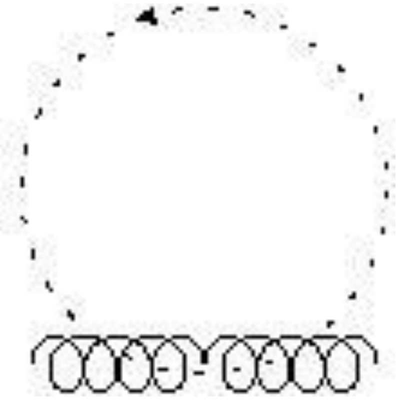
- Within the Vector Meson Dominance (VMD) model photon interactions are mediated by vector mesons (ρ,ω,ϕ).
- The primary contribution comes from the ρ meson, so one can relate Π_{em} to the ρ propagator
- $Im[\Pi_{em}(q_0,\vec{q})] = \frac{m_\rho^4}{g^2} Im[D_\rho(q_0,\vec{q})]$

Rho Self Energy

- I follow Urban's lead from *Nucl. Phys. A, 641* (1998), and Song's et. al. from Physical Review C, 54 (1996) , in calculating the ρ propagator.
- To calculate the ρ propagator one must first calculate the ρ meson self-energy.
- There are two self-energy diagrams necessary to maintain gauge invariance in vacuum.



Pi-Pi Loop



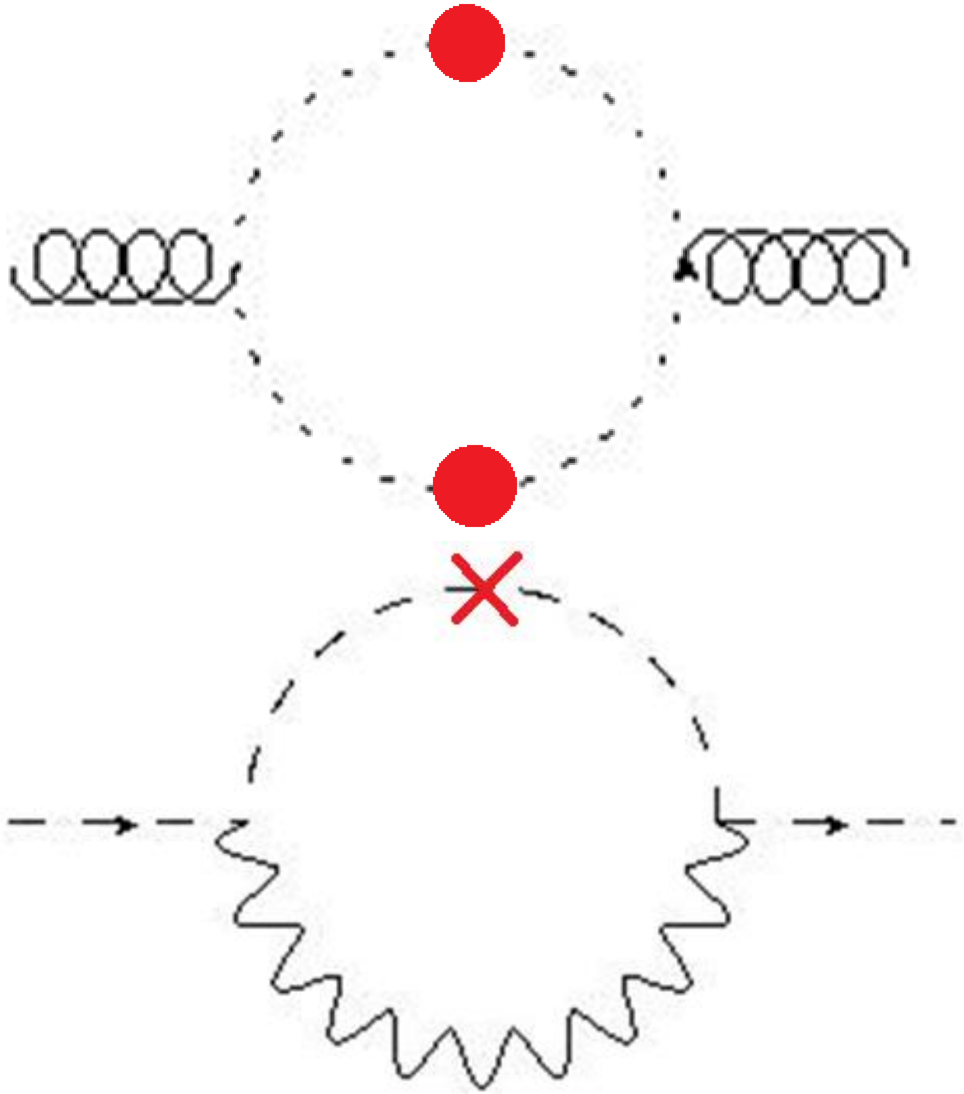
Tadpole Loop

Pion Self-Energy

- The pions in the ρ self-energy interact with the pion cloud.

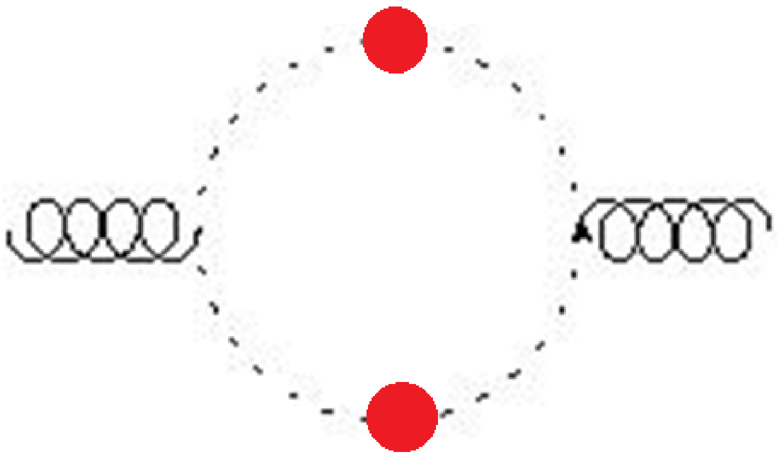
- These effects must be included to obtain a finite conductivity

- However, these effects violate gauge invariance, which must be compensated for by modifying the ρππ and the ρρππ vertex

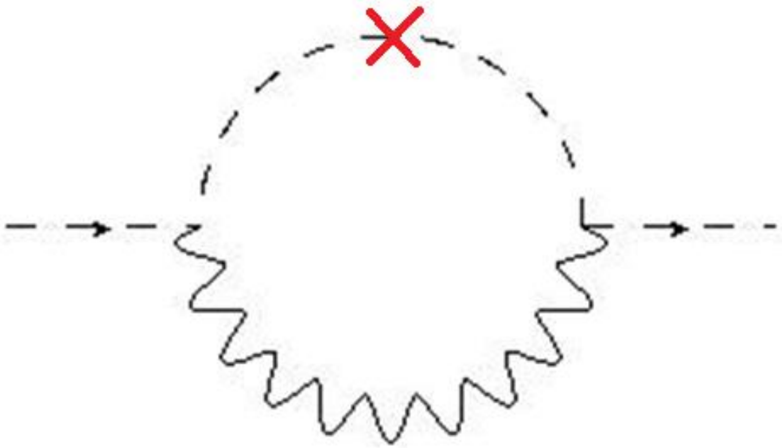


Resumed Pions

- I resum Σ_π resulting in a modification to the π propagator



$$ig^2 \int \frac{d^4k}{(2\pi)^4} \frac{\left((2k+q)_\mu (2k+q)_\nu\right)}{\left((k+q)^2 - m_\pi^2 - \Sigma_\pi(k+q)\right)\left(k^2 - m_\pi^2 - \Sigma_\pi(k)\right)}$$



$$\Sigma_\pi(k) = 2ig^2 \int \frac{d^4p}{(2\pi)^4} \frac{(2k+p)_\mu (2k+p)_\nu \left(-g^{\mu\nu} + \frac{p^\mu p^\nu}{m\rho^2}\right)}{\left((p+k)^2 - m_\pi^2\right)\left(p^2 - m_\rho^2\right)}$$

Vertex Corrections

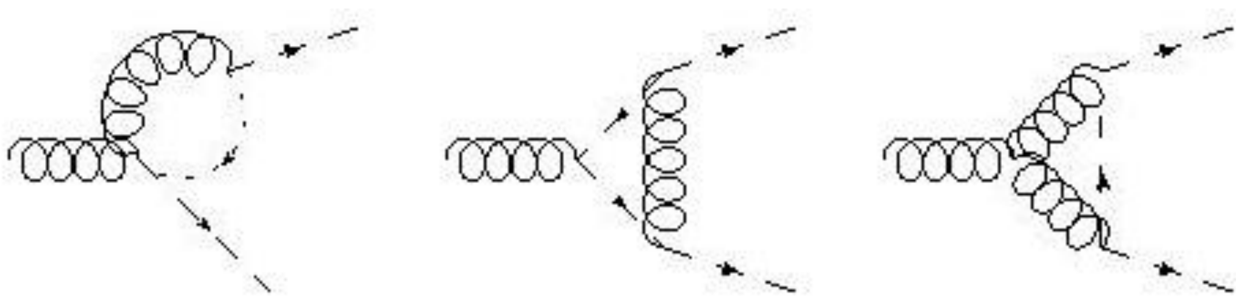
- The vertex corrections satisfy Ward-Takahashi identities:

$$q^{\mu}\Gamma^{(3)}_{\mu ab} = g\epsilon_{3ab}\left(-\Sigma_{\pi}(k+q) + \Sigma_{\pi}(k)\right)$$

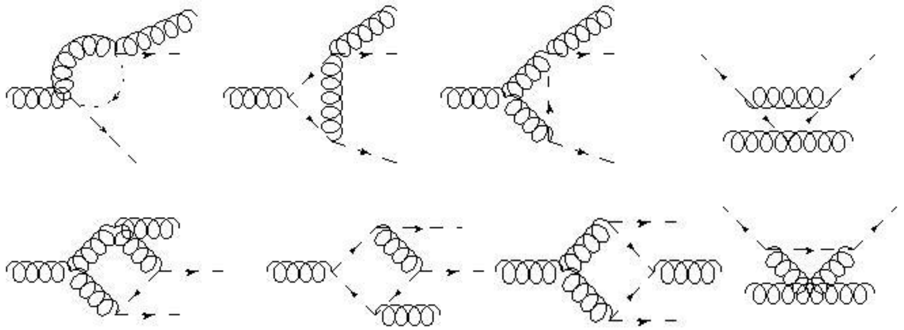
$$q^{\mu}\Gamma^{(4)}_{\mu\nu ab} = ig(\epsilon_{3ca}\Gamma^{(3)}_{\nu ab}(k,-q) - \epsilon_{3bc}\Gamma^{(3)}_{\nu ca}(k+q,-q))$$

- The corrections are given by:

ρππ

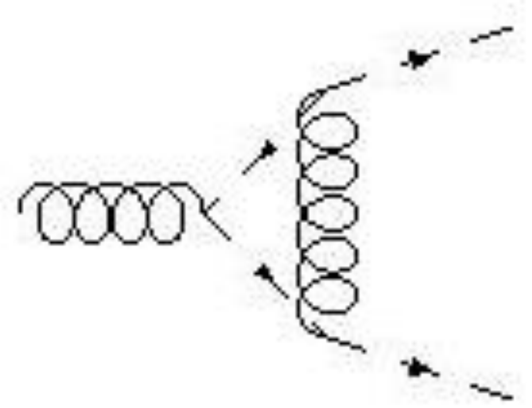


ρρππ

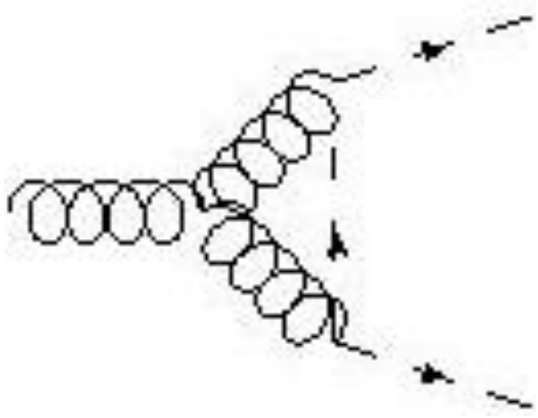


Dressing Vertex Correction Particles

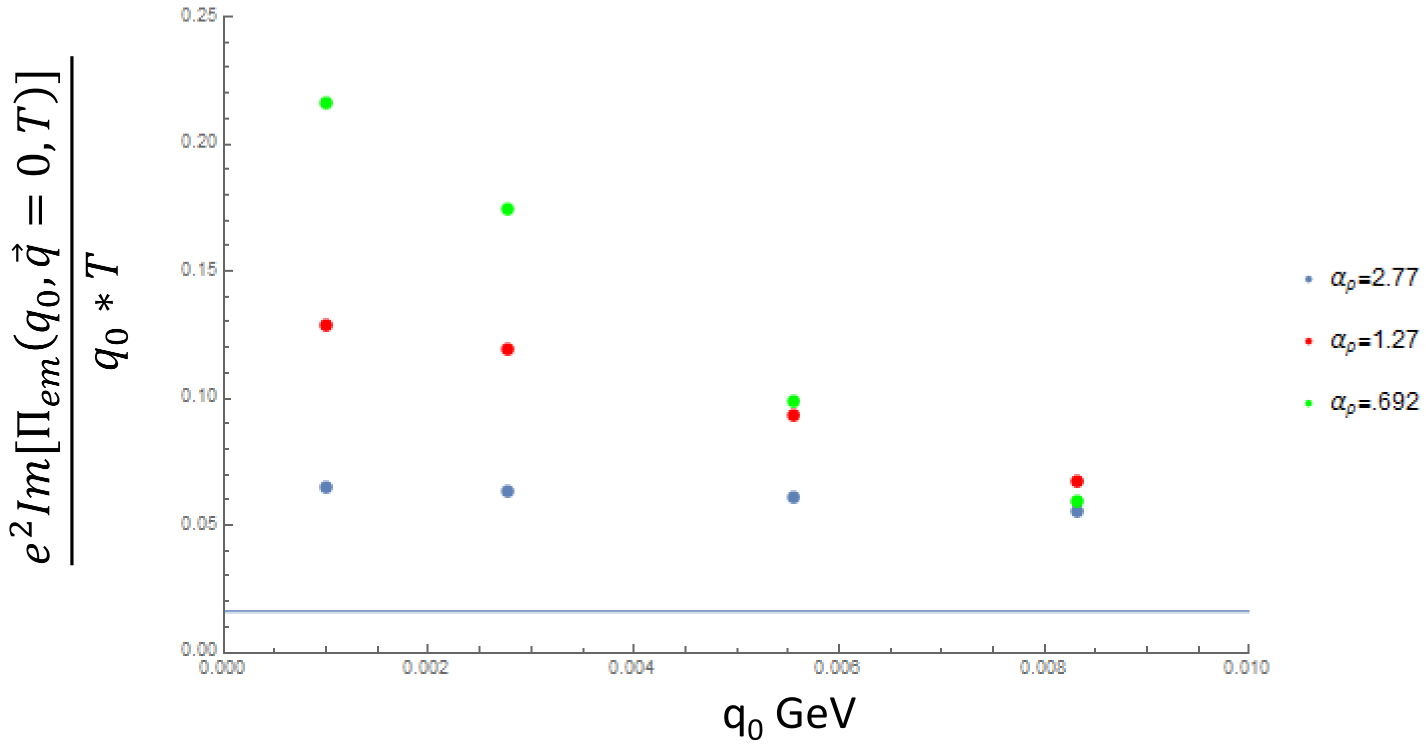
- Intermediate particle propagators in the vertex corrections must be dressed with a width to obtain a finite conductivity.



- It is still an open question of how to dress these propagators without violating gauge invariance.



Preliminary Results



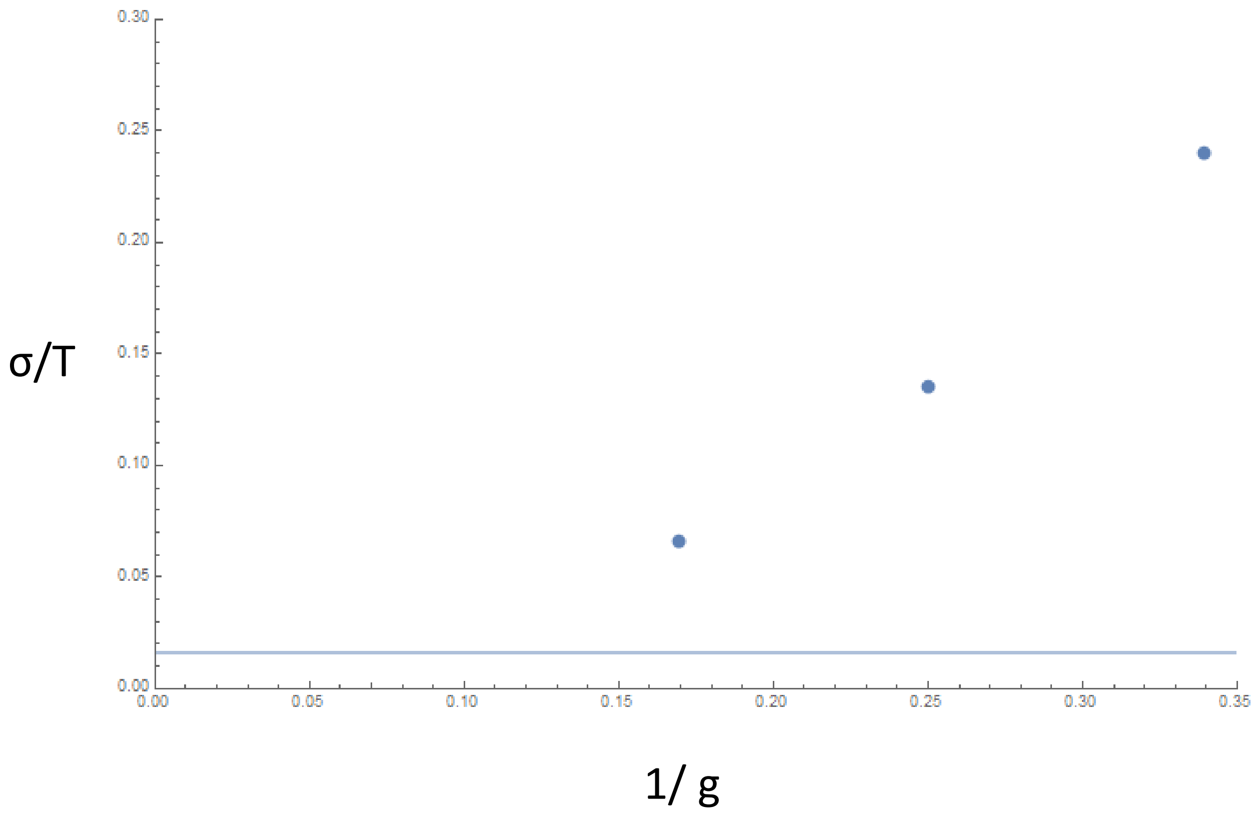
T = 150 MeV

- Suggests the pion gas is a strongly coupled medium.

Test Lower Bound

- By plotting the conductivity vs (1/ g) we attempt to extract the lower limit of the conductivity.

- More points are needed to reliably extract a value, but initial results are compatible with the quantum limit



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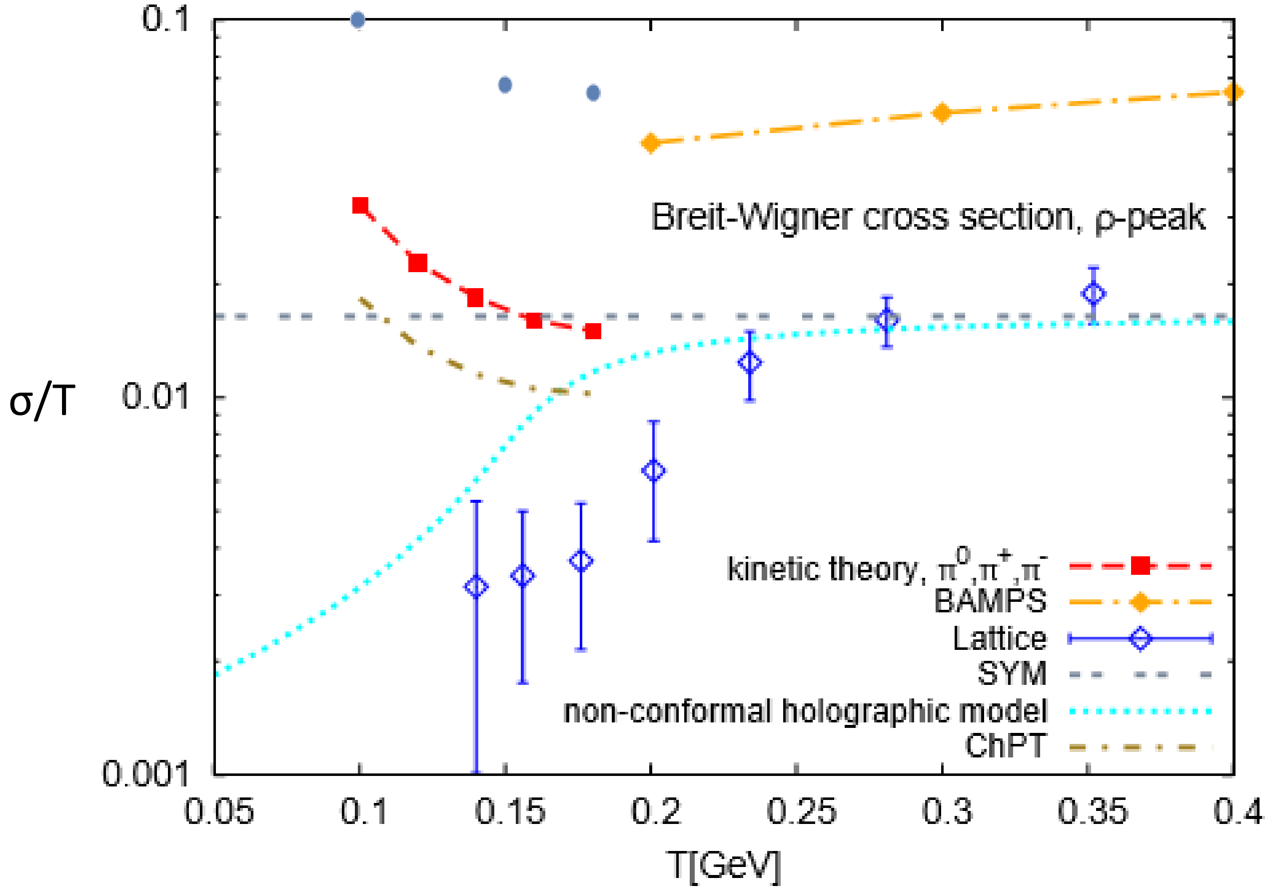

- Results are above the quantum lower bound.

Summary of Results

- We have calculated a conductivity, which is above the quantum limit using a unitary and approximately gauge invariant formalism

- The result suggests that the pion gas is a strongly coupled system.

- The strong coupling limit of our calculation approaches a value compatible with the quantum lower bound

- Future: - consolidate gauge invariance of results

  - Implement effects of baryons

References

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- [6] Urban, M., Buballa, M., & Wambach, J. (1998). *Nucl. Phys. A, 641*(4), 433-460.