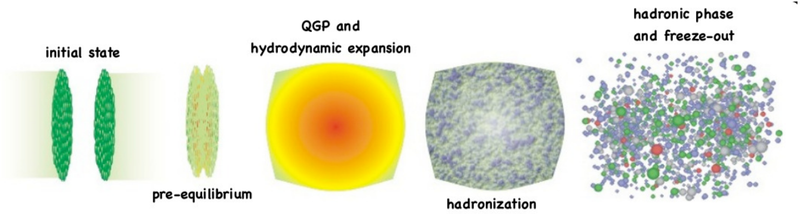
Hydrodynamics Overview

Jacquelyn Noronha-Hostler
University of Houston

Hot Quarks 2016- South Padre Island
Sept. 13^{th} 2016

"Standard Model" of Hydrodynamics in Heavy Ion Collisions



- Initial State
- Pre-equilibrium state (generate initial flow... full $T^{\mu\nu}$)
- Event-by-Event Relativistic Viscous Hydrodynamics (shear $\eta/s(T)$ and bulk $\zeta/s(T)$ viscosities)
- Hadronization via Cooper Frye Freeze-out
- Hadronic Afterburner (Decays, Transport)

Initial Conditions effects on Collective Flow

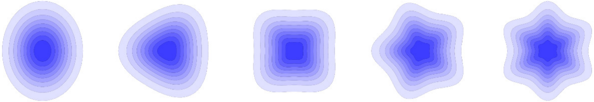
The distribution of particles can be written as a Fourier series

$$E\frac{d^3N}{d^3p} = \frac{1}{2\pi}\frac{d^2N}{p_T dp_T dy}\left[1 + \sum_n 2v_n \cos\left[n\left(\phi - \psi_n\right)\right]\right]$$

- Flow Harmonics at mid-rapidity

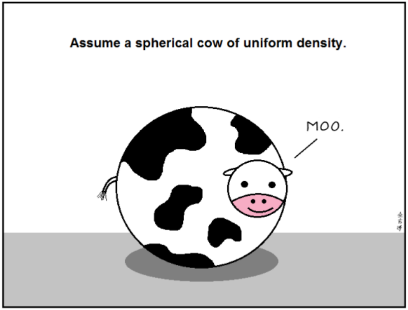
$$v_n(p_T) = \frac{\int_0^{2\pi} d\phi \frac{dN}{p_T dp_T d\phi} cos\left[n\left(\phi - \Psi_n\right)\right]}{\int_0^{2\pi} d\phi \frac{dN}{p_T dp_T d\phi}}$$

where $\Psi_n = \frac{1}{n}$ arctan $\frac{\langle sin[(n\phi)]\rangle}{\langle cos[(n\phi)]\rangle}$

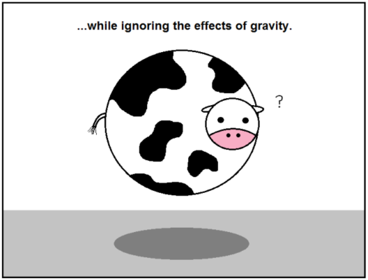


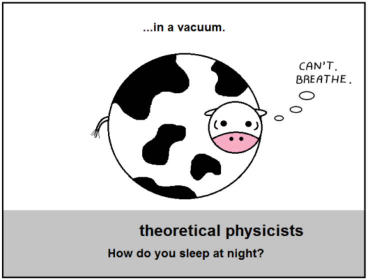
$n=2$ $n=3$ $n=4$ $n=5$ $n=6$

When you need to start somewhere...



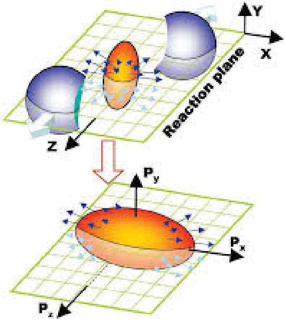
from Abstruse Goose



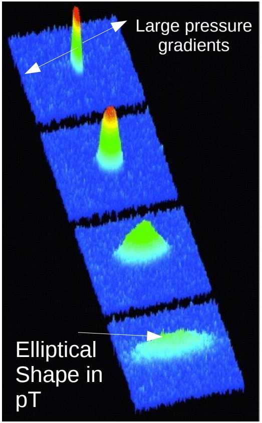


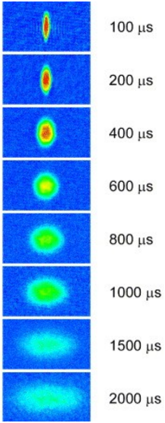
Hydrodynamics: eccentric initial state to elliptical flow

Assuming Gold ions are spheres...



Cold Atoms

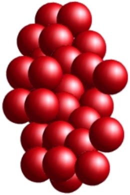




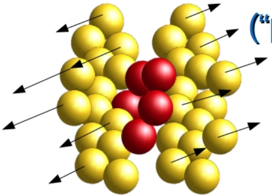
Centrality

Head on collisions ("central collisions")

J. Noronha-Hostler



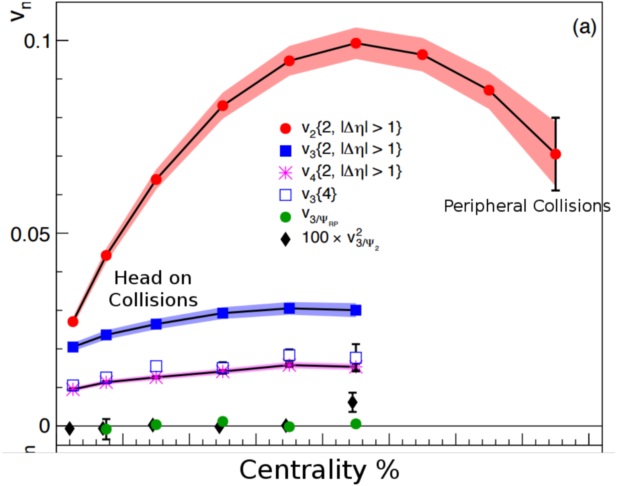
- More partons (protons and neutrons) = larger density
- More circular shape



Grazing collisions ("peripheral collisions")

- Fewer partons = small density
- Elliptical shape

Flow Harmonics across Centrality

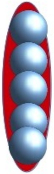


ALICE Phys.Rev.Lett. 107 (2011) 032301

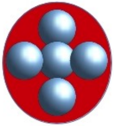
Same density (centrality), different shapes

Centrality bins by the density, but for the same density..

"**Event-by-Event**" Holding the number of partons (density) constant for the same types of collisions, different shapes can be formed.



For the same 5 participants



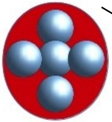
Ellipsoid= Large eccentricity (ε_2)

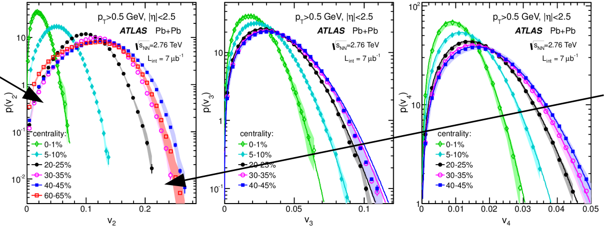
Circle= Small eccentricity (ε_2)

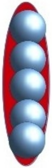
Triangles, squares etc can even appear...

Flow Harmonics distribution within a centrality

Large fluctuations in flow harmonics for the same denisty



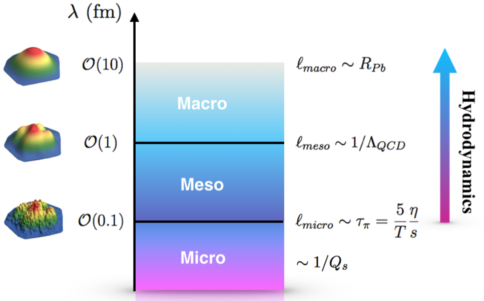




ATLAS JHEP 1311, 183 (2013)

Outline

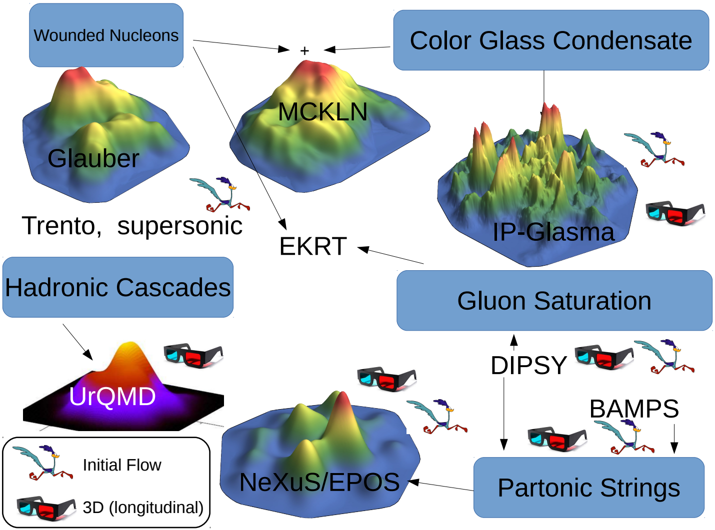
Energy Density Scales of the Initial Condition



JNH, Noronha, Gyulassy Phys.Rev. C93 (2016) no.2, 024909

- Sensitivity at most to sub-leading modes
  A. Mazeliauskas and D. Teaney, Phys. Rev. C 93, no. 2, 024913 (2016)
- Small scale fluctuations not relevant to final flow harmonics
  JNH, Noronha, Gyulassy Phys.Rev. C93 (2016) no.2, 024909
- Eccentricites best preditor of final flow harmonics
  Gardim et al,Phys.Rev. C85 (2012) 024908 ; Phys.Rev. C91 (2015) no.3, 034902, Niemi et al Phys.Rev. C87 (2013) no.5, 054901

Types of Initial Conditions



Eccentricities

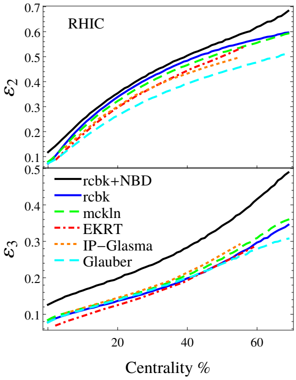
Eccentricities:

$$\varepsilon_{n,m} e^{i\Phi_{n,m}} = \frac{\int d^2\mathbf{r}\ r^m\ e^{in\phi}\ \epsilon(\tau_0, \mathbf{r})}{\int d^2\mathbf{r}\ r^m\ \epsilon(\tau_0, \mathbf{r})}$$

in the center of mass frame.
Also, define $\varepsilon_n = \varepsilon_{n,n}$

Variance in the Eccentricities

Leads to differences in the final flow harmonics. Different intial conditions leads to different parameters in the hydrodynamics itself.



Outline

Equations of Motion

Conservation of Energy and Momentum

$$\partial_\mu T^{\mu\nu} = 0$$

Ideal Energy momentum Tensor

$T^{\mu\nu} = \varepsilon u^\nu u^\nu - p\Delta^{\mu\nu}$

where $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$
Coordinate System: $x^\mu = (\tau, x, y, \eta)$ where $\tau = \sqrt{t^2 - z^2}$ and $\eta = 0.5 \ln\left(\frac{t+z}{t-z}\right)$

Equations of Motion

Conservation of Energy and Momentum

$$\partial_\mu T^{\mu\nu} = 0$$

Shear Energy momentum Tensor

$T^{\mu\nu} = \varepsilon u^\nu u^\nu - p\Delta^{\mu\nu} + \pi^{\mu\nu}$

with the shear stress tensor $\pi^{\mu\nu}$ where $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$
Coordinate System: $x^\mu = (\tau, x, y, \eta)$ where $\tau = \sqrt{t^2 - z^2}$ and $\eta = 0.5 \ln\left(\frac{t+z}{t-z}\right)$

Equations of Motion

Conservation of Energy and Momentum

$$\partial_\mu T^{\mu\nu} = 0$$

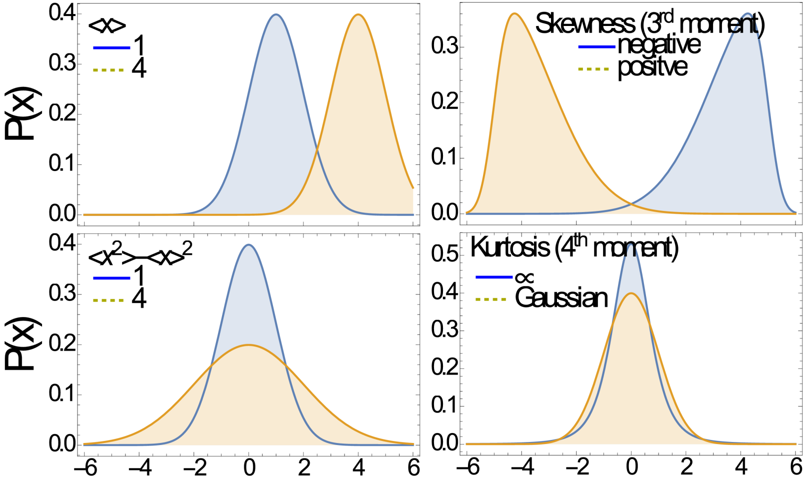
Shear+Bulk Energy momentum Tensor

$$T^{\mu\nu} = \varepsilon u^\nu u^\nu - (p + \Pi)\,\Delta^{\mu\nu} + \pi^{\mu\nu}$$

with the shear stress tensor $\pi^{\mu\nu}$ and bulk dissipative term Π
where $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$
Coordinate System: $x^\mu = (\tau, x, y, \eta)$ where $\tau = \sqrt{t^2 - z^2}$ and $\eta = 0.5 \ln\left(\frac{t+z}{t-z}\right)$

Outline

Dissecting the Flow Harmonic Distribution

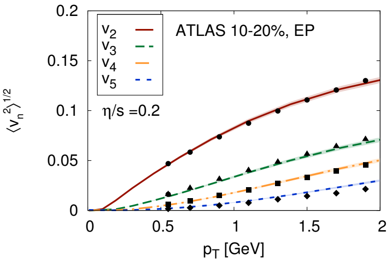


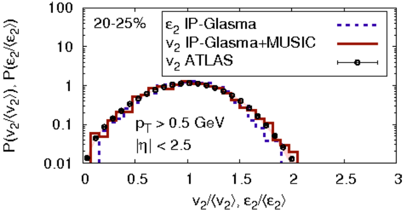
Reality is a bit more complicated...

- Experimentally, $v_2\{2\}$ is a two particle correlation where issues like non-flow effects (decays, jets etc) exist
- Rather, we use multiparticle cumulants $v_2\{n\}$ where $n = 2, 4, 6 \ldots$ indicate the number of correlated particles
- If non-flow can be eliminated (minimized via a rapidity gap) we expect $v_2\{4\} \sim v_2\{6\} \sim v_2\{8\} \ldots$
- Then, $v_2\{4\}/v_2\{2\}$ indicates magnitude of fluctuations

Good references: Ante Bilandzic Ph.D Thesis and Luzum and Petersen J.Phys. G41 (2014) 063102

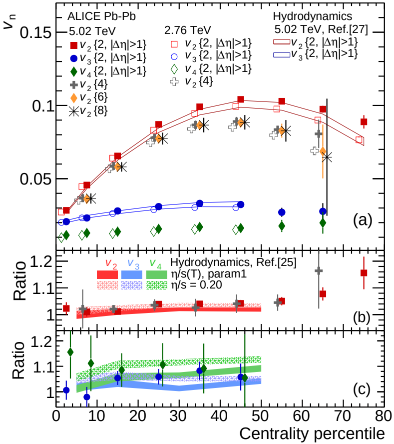
We fit all the things...





Gale et al, Phys.Rev.Lett. 110 (2013) no.1, 012302

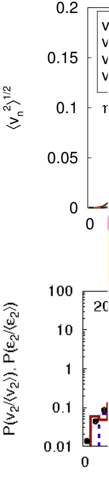
$\sim$ 5% changes across energies

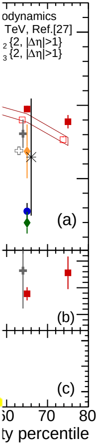


ALICE Phys.Rev.Lett. 116 (2016) no.13, 132302
Hydro: JNH, Luzum, Ollitrault Phys.Rev. C93 (2016) no.3, 034912 ; Niemi et al Phys.Rev. C93 (2016) no.1, 014912

We fit all the things...

$\sim$ 5% changes across energies





Gale et al, F

3, 132302

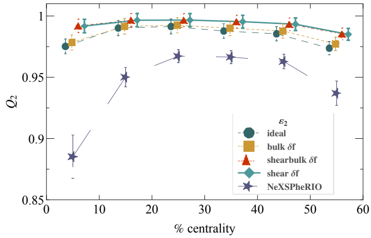
Hydro: JNH, Luzum, Ollitrault Phys.Rev. C93 (2016) no.3, 034912 ; Niemi et al Phys.Rev. C93 (2016) no.1, 014912

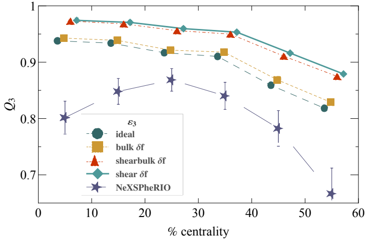
Taken from Hyperbole and a Half

But what can be learned from this?

Gardim,JNH,Luzum,Grassi PRC91(2015)3,034902

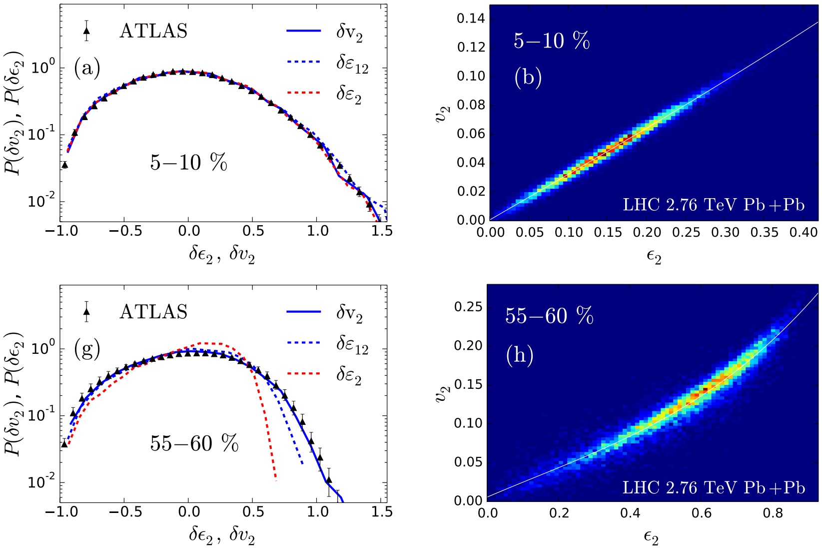
- ε_n and v_n are strongly correlated
  perfect estimation $Q_n \to 1$





Fitting the flow harmonics, gives us a good estimate of the initial state, however, viscosity also plays a role...

Complications with mapping between $\varepsilon_2 \rightarrow v_2$



Niemi,Eskola, Paatelainen,arXiv:1505.02677, also Schenke,Tribedy,Venugopalan, Nucl. Phys. A 926, 102 (2014)

Linear+Cubic Response

Linear response

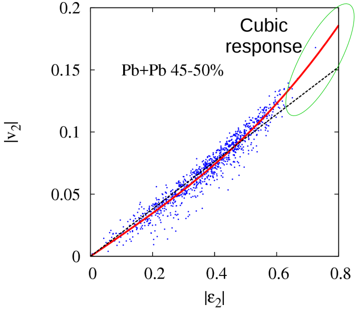
$$f(\varepsilon_n) = \kappa_n \varepsilon_n$$

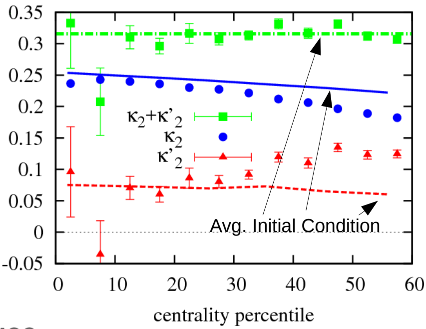
Teaney,Yan,PRC83(2011)064904;Gardim,et al,PRC85(2012)024908;PRC91(2015)3,034902

Linear+cubic response

$$f(\varepsilon_n) = \kappa_n \varepsilon_n + \kappa_n' |\varepsilon_n|^2 \varepsilon_n$$

JNH,Yan,Gardim,Ollitrault Phys.Rev. C93 (2016) no.1, 014909



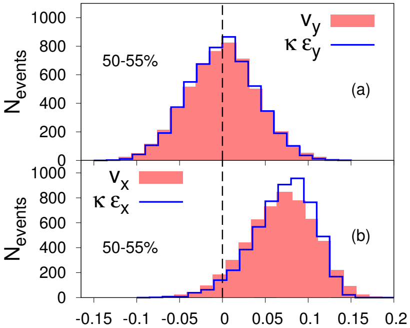


Skewness and higher order cumulants

Difference between cumulants due to skewness of v_x distribution

$$v_2\{4\} - v_2\{6\} = -\frac{s_1}{3\langle v_x\rangle^2}$$

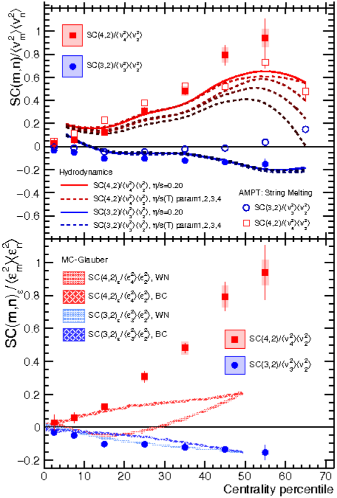
where $s_1 = \langle (v_x - \langle v_x\rangle)^3\rangle$



Giacolone et al arXiv:1608.01823

Event-by-event flow harmonics fluctuations

- $SC(n,m) = \langle v_n^2 v_m^2 \rangle - \langle v_n^2 \rangle \langle v_m^2 \rangle$
- Elliptical and triangular flow are anti-correlated
- $SC(3,2) \rightarrow$ initial state effect
- v_4 is correlated with v_2 and experiences non-linear effects
- $\eta/s(T)$ dependencies in SC(4,2) diminished by exp. effects Gardim et al arXiv:1608.02982
- Previous studies: ATLAS Phys.Rev. C92 (2015) no.3, 034903

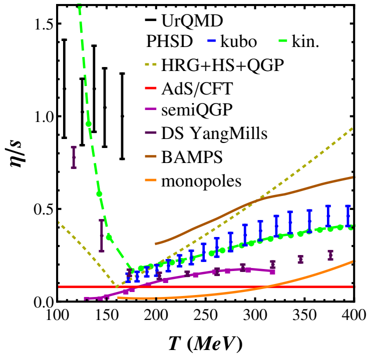


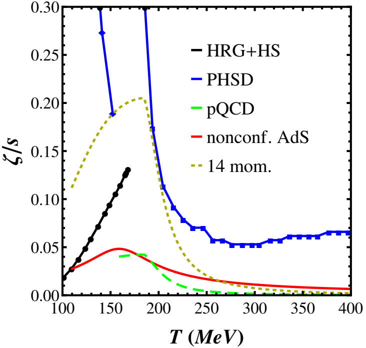
ALICE arXiv:1604.07663

Outline

Bulk Viscosity in Heavy-Ion Collisions

- Resistance against the deformation of a fluid $\Pi^{\mu\nu}_{Navier-Stokes} \sim \eta \partial^{\langle\mu} u^{\nu\rangle}$
- Resistance against the radial expansion of a fluid $\Pi_{Navier-Stokes} \sim -\zeta(\partial_\mu u^\mu)$

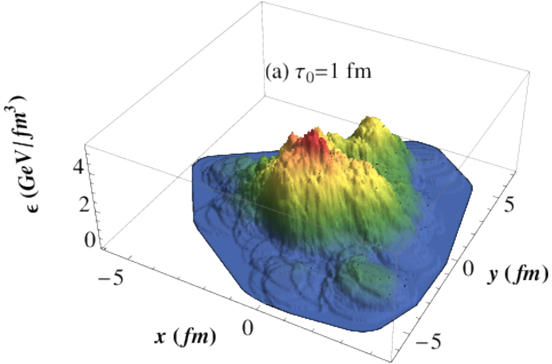




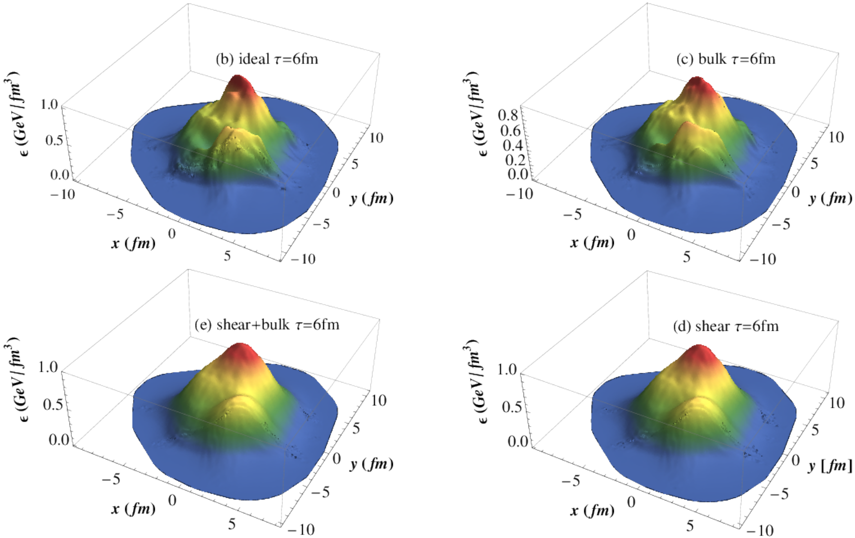
JNH arXiv:1512.06315 for references

Viscosity in Heavy-Ion Collisions

Given a lumpy initial condition $\tau = 1 fm$

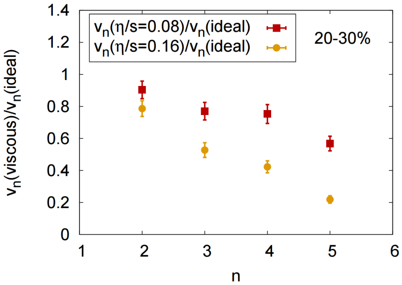


Viscosity in Heavy-Ion Collisions

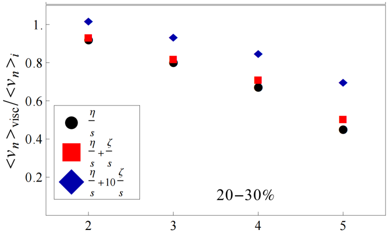


Viscosity kills off higher order flow harmonics

Higher order v_n's more strongly affected by η/s and ζ/s



Schenke, Jeon, Gale Phys.Rev. C85 (2012) 024901



JNH, Noronha, Grassi Phys.Rev. C90 (2014) no.3, 034907

Depends on correction terms, no correction also shows $\Uparrow$

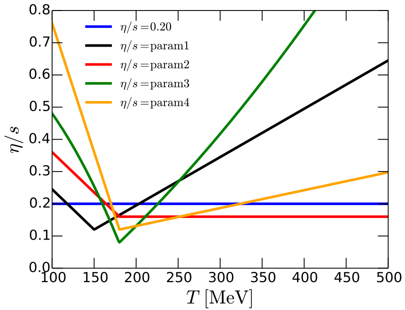
Bernhard et al Phys.Rev. C94 (2016) no.2, 024907

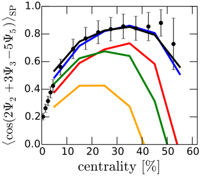
Another type shows $\Downarrow$

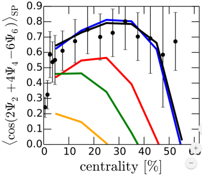
Ryu et al Phys.Rev.Lett. 115 (2015) no.13, 132301

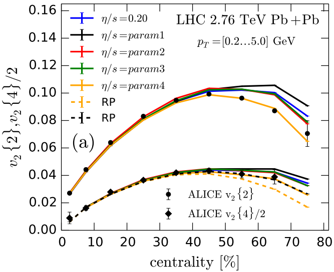
$\eta/s(T)$ sensitive observables

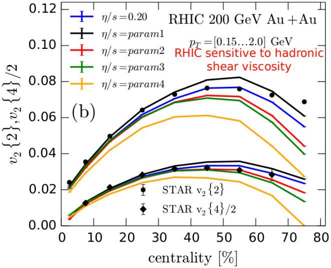
Varying η/s: Niemi et al. Phys.Rev. C93 (2016) no.2, 024907









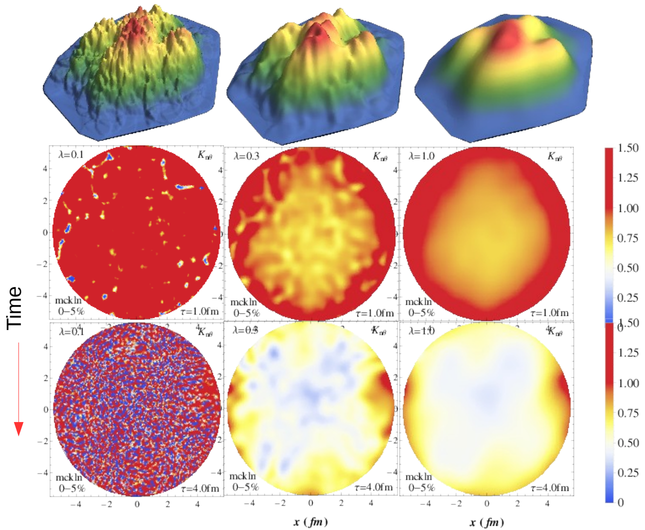


Event Plane Cor. sensitive to $\eta/s(T)$

Outline

Applicability of hydrodynamics in PbPb: Part I

Knudsen number in PbPb, JNH, Noronha, Gyulassy, Phys.Rev. C93 (2016) no.2, 024909

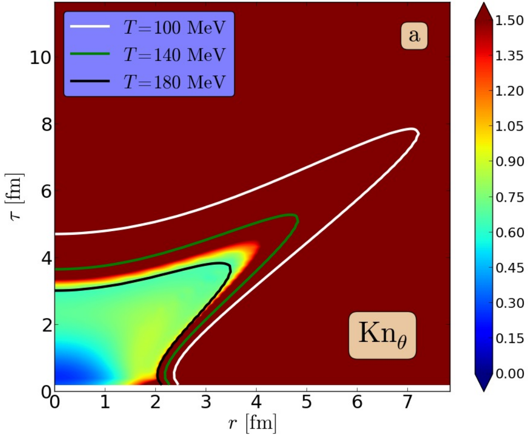


$K_{n\theta} = l_{micro}/L_{macro}$

$\rightarrow \tau_\pi\,\theta$

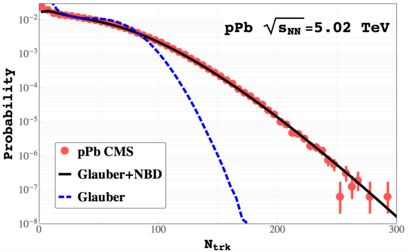
Applicability of hydrodynamics in pPb

Only very small region with a "good" Knudsen number

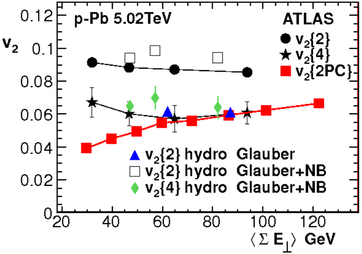


Niemi and Denicol arXiv:1404.7327

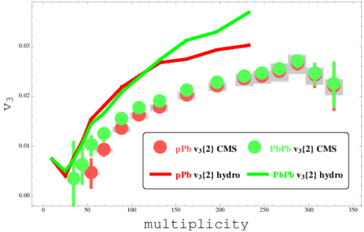
Yet, Glauber+Negative Binomial Distributions fit pPb



Kozlov, Luzum, Denicol, Jeon, Gale arXiv:1405.3976



Bozek and Broniowski Phys.Rev. C88 (2013) no.1, 014903

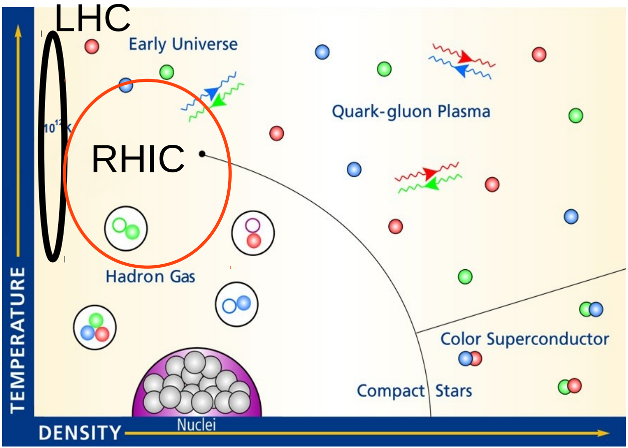


Kozlov, Luzum, Denicol, Jeon, Gale arXiv:1405.3976

Outline

Phase Diagram

What happens to hydrodynamics at finite μ_B?

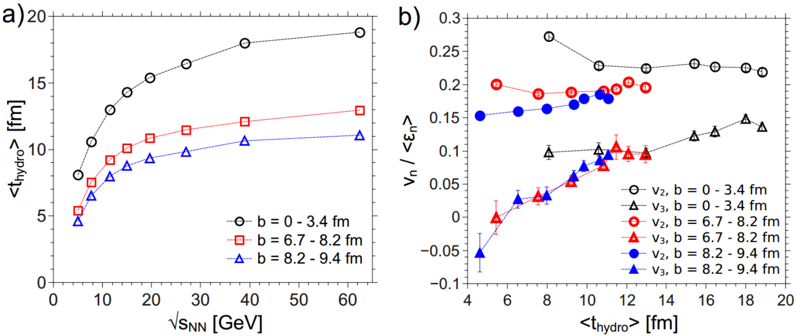


Hydrodynamics at Finite μ_B

- 2+1 Dimensions $\rightarrow$ 3+1 Dimensions
- Equation of State (Lattice QCD runs into the Fermi-Sign Problem)
  $\rightarrow$ Critical Point?
- Thermal fluctuations
- Baryon diffusion (and Strangeness diffusion and Charge diffusion)- Temperature dependence? Rougemont et al Phys.Rev.Lett. 115 (2015) no.20, 202301 , Lattice QCD: Aarts et al JHEP 1502 (2015) 186
- Transport Coefficients depend on T and μ_B,
  $\eta/s(T) \rightarrow \eta/s(T,\mu_B)$ and $\zeta/s(T) \rightarrow \zeta/s(T,\mu_B)$

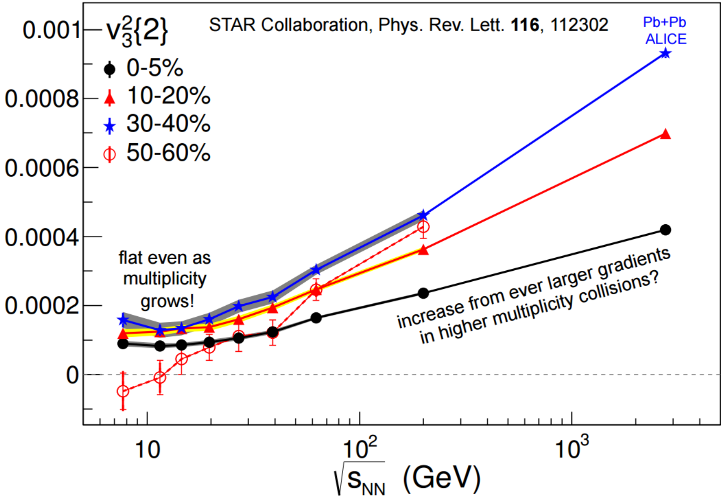
Triangular Flow at finite μ_B

v_3 disappears at low energies



Auvinen and Petersen Phys.Rev. C88 (2013) no.6, 064908

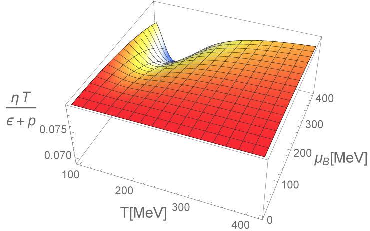
v_3 disappears at low energies

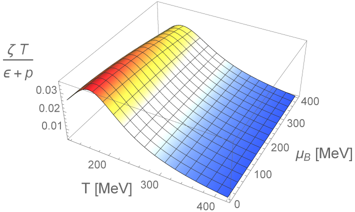


Do we still have perfect fluidity at finite μ_B?

Divergence of transport coef. depend on the universality class

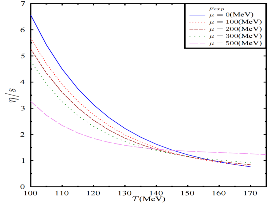
CP (Class B- no divergence)

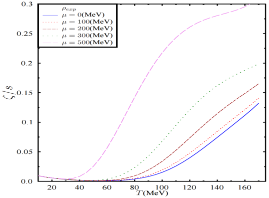




Rougemont, Noronha, JNH to appear shortly

No CP

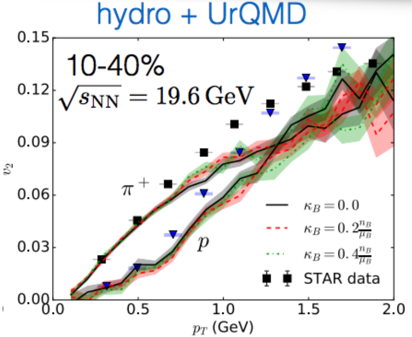




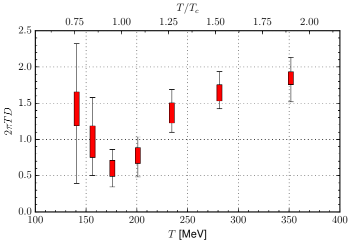
Kadam, Mishra Nucl.Phys. A934 (2014) 133-147
See also Denicol, Jeon, Gale, Noronha Phys.Rev. C88 (2013) no.6, 064901

Class H- diverges at CP

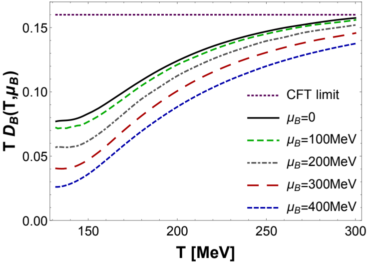
Baryon/Charge Diffusion



Talk from Chun Shen at the BEST collaboration



Aarts et al JHEP 1502 (2015) 186



Rougemont et al Phys.Rev.Lett. 115 (2015) no.20, 202301

- Still waiting to see how Diffusion effects v_2 vs. v_3
- Hadronic phase diminishes effects of Diffusion
- D(T) still needed

Conclusions

- Relativistic hydrodynamics has done an extraordinarily good job at describing flow harmonics, particle spectra, and event-by-event sensitive observables
- Many interesting open ended questions remain:
  - Can the initial state be determined separately from viscosity effects?
  - Are we seeing fluid-like behavior in small systems?
  - What are the implications of the vanishing v_3 at the Beam Energy Scan?
- Topics to be further explored at Hot Quarks: initial stages, transport coefficients, jets+hydrodynamics, anisotropic hydrodynamics, magnetohydrodynamics, and the chiral magnetic effect

Outline

Mapping of the Initial State onto the Flow Harmonics

Gardim et al, PRC85(2012)024908, Gardim,JNH,Luzum,Grassi PRC91(2015)3,034902

How do the eccentricities relate to the flow harmonics?

$$\varepsilon_{n,m} e^{i\Phi_{n,m}} = -\frac{\{r^m e^{in\phi}\}}{\{r^m\}}$$

Define Quality of Estimator:

$$Q_n = \frac{\mathrm{Re}\left\langle V_n V^*_{est,n} \right\rangle}{\sqrt{\langle |V_n|^2\rangle \langle |V_{est,n}|^2\rangle}}$$

Estimator as a perturbative series in powers of azimuthally asymmetric cumulants:

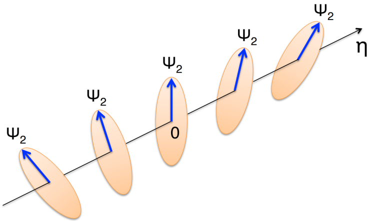
$$V_{est,n} = \sum_{m=n}^{m_{max}} k_{n,m} W_{n,m}$$

$$+ \sum_{l=1}^{m_{max}} \sum_{m=l}^{m_{max}} \sum_{m'=|n-l|}^{m_{max}} K_{l,m,m'} W_{l,m} W_{n-l,m'}$$

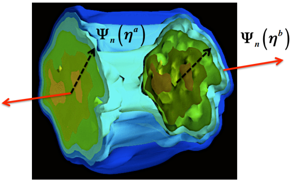
$$+ O(W^3)$$

As $Q_n \to 1$, the $\varepsilon_{n,m}$'s predict the corresponding v_n's

Longitudinal Twists



Taken from W. Li INT 2015 Talk



$$f(p_T,\phi,\eta) \sim 1 + 2\sum_{n=1}^{\infty} v_n(p_T,\eta)\cos\left[n\left(\phi - \Psi_n\left(p_T,\eta\right)\right)\right]$$

Equations of Motion

Projecting $T^{\mu\nu}$ in the parallel $u_\nu \partial_\mu T^{\mu\nu}$ and perpendicular $\Delta^\alpha_\nu \partial_\mu T^{\mu\nu}$ direction (for $\mu_B = 0$)

Israel-Stewart Equations

$$\begin{aligned} D\epsilon + (\epsilon + p)\partial_\mu u^\mu - \Pi^{\mu\nu}\nabla_{(\mu}u_{\nu)} &= 0\,, \\ (\epsilon + p)Du^\alpha - \nabla^\alpha p + \Delta^\alpha_\nu \partial_\mu \Pi^{\mu\nu} &= 0 \end{aligned} \tag{1}$$

where $\Pi^{\mu\nu}$ is the total viscous stress tensor (includes shear, bulk etc)

Second-order Transport Coefficients

Equations of Motion - 2nd order

Denicol et all, PRD85(2012)114047

$$\begin{aligned} \dot{\Pi} + \frac{\Pi}{\tau_\Pi} &= -\frac{\zeta/s}{\tau_\Pi}\theta + -\delta_{\Pi\Pi}\Pi\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu} \\ &+ \phi_1\Pi^2 + \phi_3\pi^{\mu\nu}\pi_{\mu\nu} \end{aligned} \quad (2)$$

$$\begin{aligned} \dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} &= \frac{2\eta/s}{\tau_\pi}\sigma^{\mu\nu} - \frac{4}{3}\pi^{\mu\nu}\theta \\ &+ 2\pi_\alpha^{\langle\mu}\omega^{\nu\rangle\alpha} + \phi_7\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} + \lambda_{\pi\Pi}\Pi\sigma_{\mu\nu} - \tau_{\pi\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} \\ &+ \phi_6\Pi\pi^{\mu\nu} \end{aligned} \quad (3)$$

Shear and Bulk only - in black

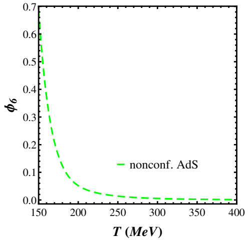
Currently used - red

Not yet tested - gray

Shear+Bulk Direct Coupling Terms

$\phi_6 \Pi \pi^{\mu\nu}$

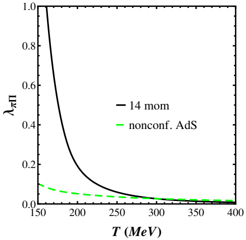
in $\pi^{\mu\nu}$ evolution



Finazzo et al, JHEP 1502 (2015) 051

$\lambda_{\pi\Pi} \Pi \sigma_{\mu\nu}$

in $\pi^{\mu\nu}$ evolution

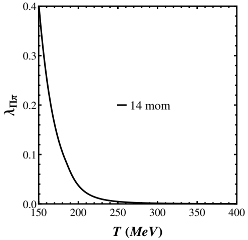


Denicol et al, PRC90(2014)024912

Molnar et al, PRD89(2014)074010

$\lambda_{\Pi\pi} \pi^{\mu\nu} \sigma_{\mu\nu}$

in Π evolution



Denicol et al, PRC90(2014)024912

Molnar et al, PRD89(2014)074010