Gravitational Waves from the Phase Transition of a Non-linearly Realised Electroweak Symmetry

Jason Yue

July 14, 2016

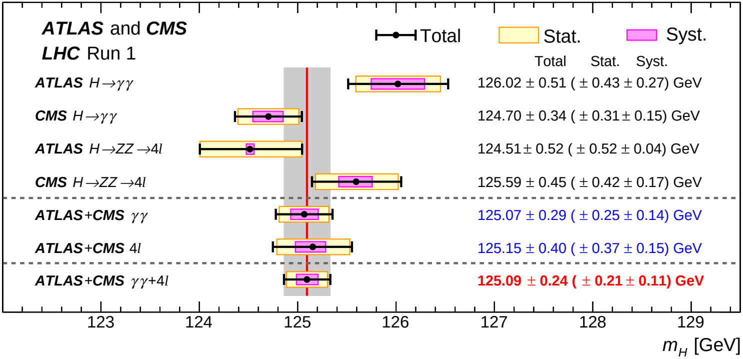
based on: work with A. Kobakhidze and A. Manning
[arXiv:1607.00883 [hep-ph]]

THE UNIVERSITY OF SYDNEY

CoEPP
ARC Centre of Excellence for Particle Physics at the Terascale

Outline

What to make of this?



[Phys. Rev. Lett. 114 (2015) 191803]

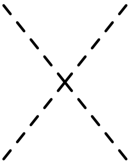
- Higgs boson discovered[a]!

$$m_h = 125.09 \pm 0.21(\text{stat.}) \pm 0.11(\text{syst.})\ \text{GeV}.$$

[a] ATLAS Collaboration, *Phys. Lett. B* **716** (2012) 1; CMS Collaboration, *Phys. Lett. B* **716** (2012) 30

Non-linear Realisation

Before the Higgs — Non-linear Realisation

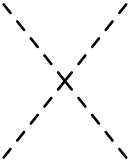


$$\mathcal{A}(\pi^+\pi^- \to \pi^+\pi^-) = \frac{1}{v^2}(s+t)$$

$$\mathcal{L} \supset \frac{v^2}{4}\mathrm{Tr}\left(D_\mu \Sigma D^\mu \Sigma^\dagger\right) - \frac{v}{\sqrt{2}} \sum_{i,j\in\{1,2,3\}} \begin{pmatrix} \overline{u}_L^i & \overline{d}_L^i \end{pmatrix} \Sigma \begin{pmatrix} \lambda^u_{ij} u^j_R \\ \lambda^d_{ij} d^j_R \end{pmatrix} + h.c.$$

$$\boxed{\Sigma := \exp\left[-i \underbrace{\left(\frac{\sigma^a}{2} - \frac{\delta^{a3}}{2}\mathbb{1}\right)}_{\text{broken } SU(2)_L \otimes U(1)_Y / U(1)_Q \text{ generators}} \pi^a\right]}$$

Before the Higgs — Non-linear Realisation



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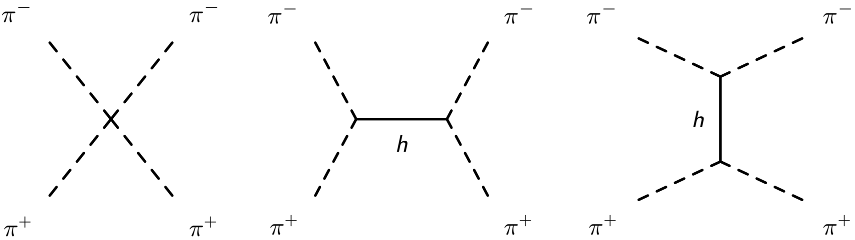
$$\mathcal{L} \supset \frac{v^2}{4}\mathrm{Tr}\left(D_\mu\Sigma D^\mu\Sigma^\dagger\right)\,(1 + \text{New Physics ??})$$
$$- \frac{v}{\sqrt{2}} \sum_{i,j\in\{1,2,3\}} \begin{pmatrix}\overline{u}_L^i & \overline{d}_L^i\end{pmatrix} \Sigma\,(1 + \text{New Physics ??}) \begin{pmatrix}\lambda_{ij}^u u_R^j \\ \lambda_{ij}^d d_R^j\end{pmatrix} + h.c.$$

New physics needed before

$E \lesssim 4\pi v \sim 3$ TeV

Specific UV completion — The SM Higgs



$$\boxed{\mathcal{A}(\pi^+\pi^- \to \pi^+\pi^-) = \frac{s+t}{v^2}(1-a^2) + \mathcal{O}\left(\frac{m_h^2}{E^2}\right)}$$

$$\mathcal{L} \supset \frac{v^2}{4}\mathrm{Tr}\left(D_\mu\Sigma^\dagger D^\mu\Sigma\right)\left(1 + 2a\frac{h}{v} + b\frac{h^2}{v^2} + b_3\frac{h^3}{v^3} + \dots\right)$$
$$-\frac{v}{\sqrt{2}}\sum_{i,j\in\{1,2,3\}}\left(\overline{u}_L^i\ \overline{d}_L^i\right)\Sigma\left(1 + c\frac{h}{v} + c_2\frac{h^2}{v^2} + \dots\right)\begin{pmatrix}\lambda_{ij}^u u_R^j \\ \lambda_{ij}^d d_R^j\end{pmatrix} + h.c.$$

Specific UV completion — The SM Higgs

$$H = \frac{\boxed{v + h(x)}}{\sqrt{2}} \Sigma \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$\boxed{\rho(x)}$

$a = b = c = 1,$

0 otherwise

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$$\mathcal{L} \supset D_\mu H^\dagger D^\mu H - \sum_{i,j \in \{1,2,3\}} \begin{pmatrix} \overline{u}^i_L & \overline{d}^i_L \end{pmatrix} \tilde{H} \lambda^u_{ij} u^j_R + h.c.$$

Realisation[a] of $SU(2)_L \otimes U(1)_Y$

- ***non-linearly realised***
  - ▹ physical Higgs not fixed in doublet with Goldstones embedded in:

$$\boxed{\Sigma \;:=\; \exp\left[-i\,\underbrace{\left(\frac{\sigma^a}{2} - \frac{\delta^{a3}}{2}\mathbb{1}\right)}_{\text{broken } SU(2)_L \otimes U(1)_Y / U(1)_Q \text{ generators}}\pi^a\right]}$$

  - ▹ operators constructed in terms of the singlet field $\rho(x) = v + h(x)$

- ***linearly realised*** (SM)
  - ▹ specific theory where operators have h fixed in doublet with Goldstones:

$$\boxed{H \;=\; \frac{\rho(x)}{\sqrt{2}}\Sigma\begin{pmatrix}0\\1\end{pmatrix}}$$

[a] see e.g. M. Gonzalez-Alonso, *et al.*, Eur. Phys. J. C **75** (2015) 3, 128

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non-linear realisation allows BSM interactions whilst keeping the SM particle content!

[a] see e.g. M. Gonzalez-Alonso, *et al.*, Eur. Phys. J. C **75** (2015) 3, 128

Minimal Setup — Gauge Sector

- Leave the gauge sector unmodified:

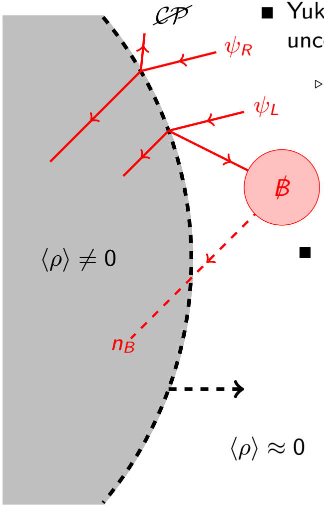
$$\mathcal{L} \supset \frac{\rho^2}{2}\mathrm{Tr}\left[(D_\mu\Sigma)^\dagger(D^\mu\Sigma)\right]$$

$$m_Z^2(\rho) = \frac{g_2^2+g_1^2}{4}\rho^2, \quad m_W^2(\rho) = \frac{g_2^2}{4}\rho^2$$

- Sign of vev?

$$m_W^2(v) = \frac{g_2^2}{4\sqrt{2}}G_F \quad \Longrightarrow \quad v \approx \pm 246\mathrm{GeV}$$

Minimal Setup — Yuakwa Sector and Higgs potential



- Yukawa sector and Higgs potential sector relatively unconstrained[a]
  - Successful electroweak baryogenesis[b]
    - (i) $\mathcal{CP}$-violating top-Higgs interaction
    - (ii) 1st order phase transition driven by anomalous cubic Higgs coupling

- For minimal considerations here, we only look at (ii):

$$V_{SM}(H^\dagger H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2$$

$$\downarrow$$

$$V_{\text{non-linear}}(\rho) = -\frac{\mu^2}{2}\rho^2 + \frac{\kappa}{3}\rho^3 + \frac{\lambda}{2}\rho^4$$

[a] see e.g. D. Binosi and A. Quadri, JHEP 1302 (2013) 020; A. Kobakhidze, arXiv:1208.5180 [hep-ph]; R. Contino *et al.*, JHEP 1005 (2010) 089

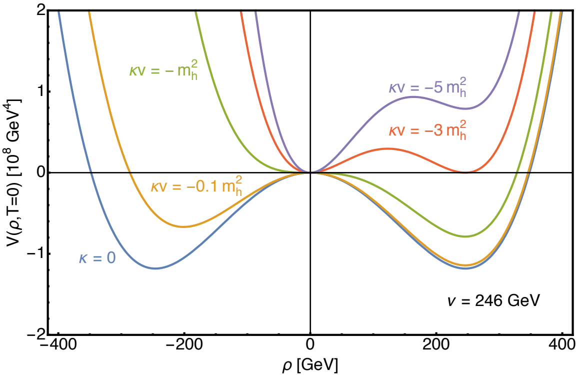
[b] A. Kobakhidze, L. Wu and J. Yue, JHEP 1604 (2016) 011

EWPT with Anomalous Higgs Cubic Interactions

Classical Potential Analysis

$$V'(v) = 0, \qquad V''(v) = m_h^2$$

- $\mu^2 > 0$ ($\rho = 0$ is a maximum)
  - $0 < |v\kappa| < m_h^2$
- $\mu^2 < 0$ ($\rho = 0$ is a minimum)
  - $m_h^2 < |v\kappa| < 3m_h^2$



Classical Potential Analysis

$$\mu^2 = \frac{1}{2}\left(m_h^2 + v\kappa\right), \quad \lambda = \frac{1}{2v^2}\left(m_h^2 - v\kappa\right) > 0$$

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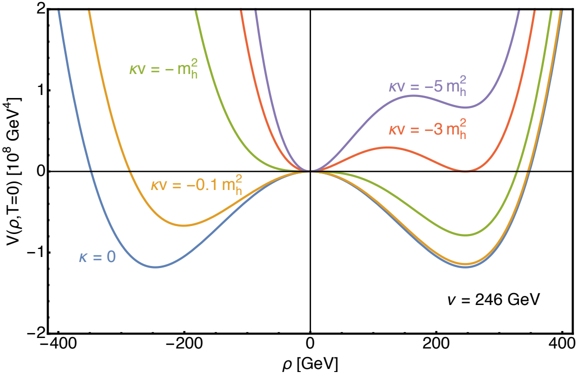
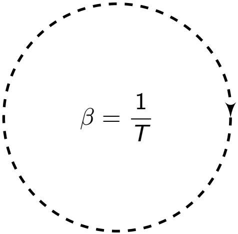


Finite temperature effects



$$p_E^0 = \omega_n = \begin{cases} 2n\pi T, & \text{bosons} \\ (2n+1)\pi T, & \text{fermions} \end{cases} \quad (n \in \mathbb{Z})$$

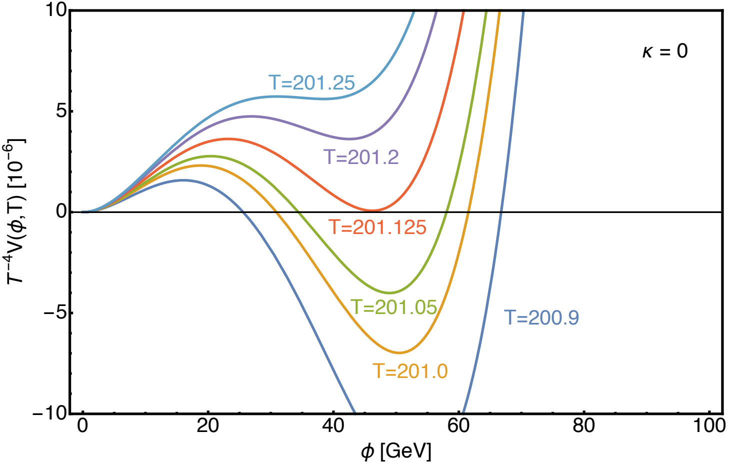
- Compactified time dimension

$$\int \frac{d^4 p}{(2\pi)^4} \to T \sum_{n=-\infty}^{\infty} \int \frac{d^3 p}{(2\pi)^3}$$

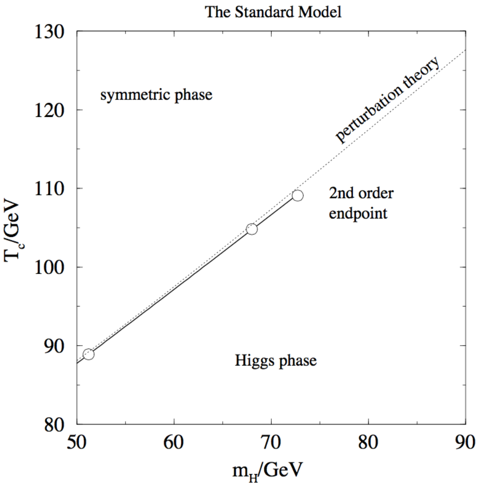
- Thermal function

$$\frac{T^4}{2\pi^2} J_i(\beta^2 m_i^2(\rho)) : \frac{T^4}{2\pi^2} \int_0^\infty dx \; x^2 \ln \left[1 - (-1)^{2s} e^{-\sqrt{x^2 + \beta^2 m_i^2(\rho)}} \right]$$

Thermal potential — SM

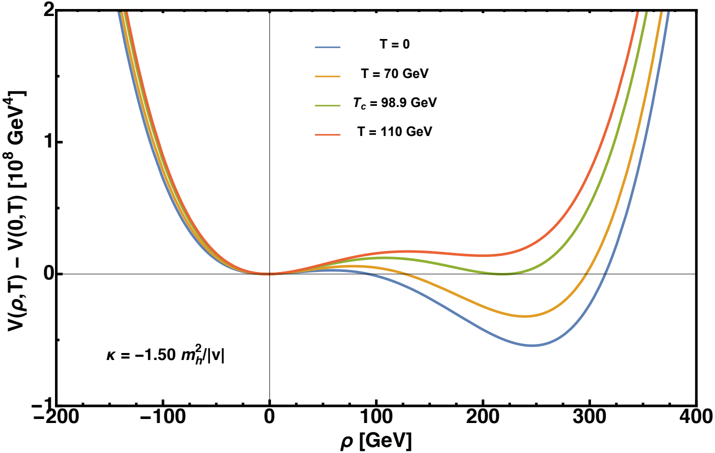


Thermal potential — SM



[M. Laine *et al.*, Nucl. Phys. Proc. Suppl. 73 (1999) 180]

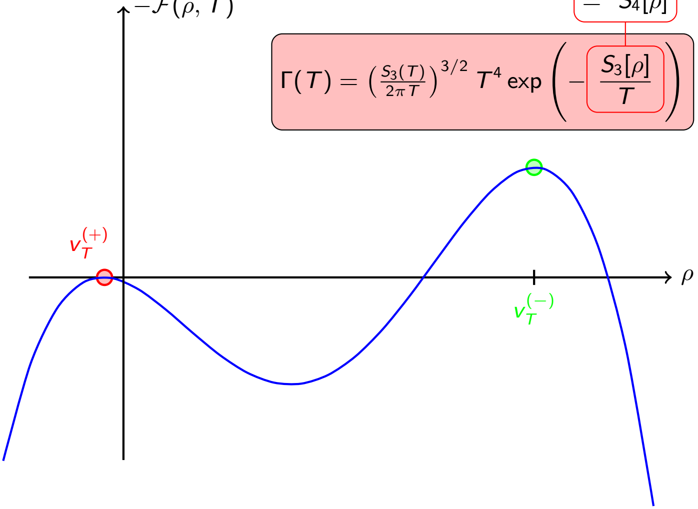
Finite Temperature Effective Potential



$$V_{\text{linear}}(\rho, T) \supset \kappa \frac{T^2 \rho}{12}$$

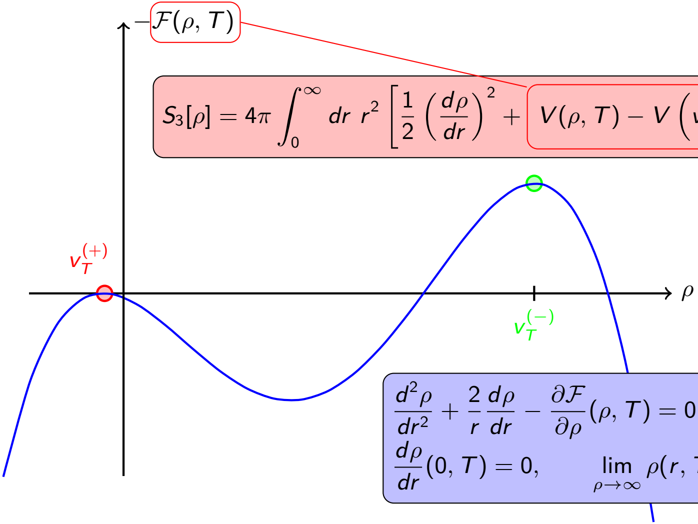
Vacuum transition

$$\Gamma(T) = \left(\frac{S_3(T)}{2\pi T}\right)^{3/2} T^4 \exp\left(-\overset{=^{a}\ S_4[\rho]}{\boxed{\frac{S_3[\rho]}{T}}}\right)$$



[a] A. D. Linde, Nucl. Phys. B216 (1983) 421.

Vacuum transition



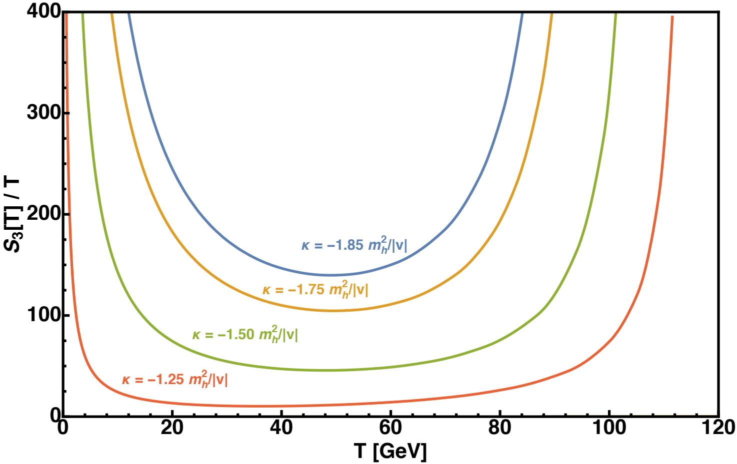
$$S_3[\rho] = 4\pi \int_0^\infty dr\ r^2 \left[\frac{1}{2} \left(\frac{d\rho}{dr} \right)^2 + V(\rho, T) - V\left(v_T^{(+)}, T\right) \right]$$

$$\frac{d^2\rho}{dr^2} + \frac{2}{r}\frac{d\rho}{dr} - \frac{\partial \mathcal{F}}{\partial \rho}(\rho, T) = 0$$

$$\frac{d\rho}{dr}(0, T) = 0, \qquad \lim_{\rho\to\infty} \rho(r, T) = v_T^{(+)},$$

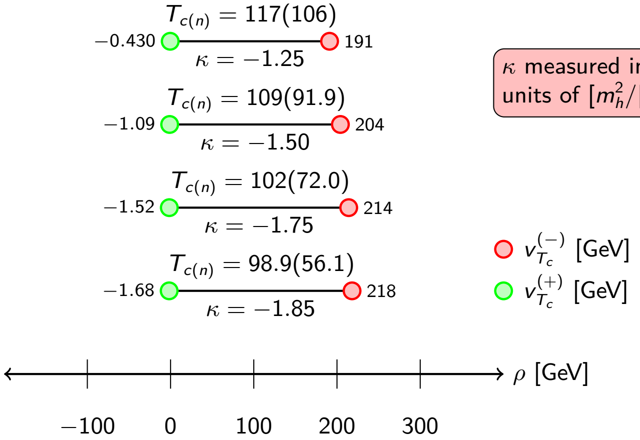
[a] A. D. Linde, Nucl. Phys. B216 (1983) 421.

Nucleation Temperature



$$\boxed{P \sim \int_{T_n}^{T_c} \left(\sqrt{\frac{45}{4\pi^3 g_*}} \frac{M_P}{T} \right)^4 e^{-S_3/T} \frac{dT}{T} \sim \mathcal{O}(1)}$$

First order phase transition



κ measured in units of $[m_h^2/|v|]$

Characterising Phase Transition via α and β parameters

- (normalised) latent heat released during PT:

$$\alpha := \Bigg(\underbrace{\frac{\pi^2}{30} g_* T_*^4}_{\rho_{rad}}\Bigg)^{-1} \left[-\Delta V + T_* \Delta\left(\tfrac{\partial V}{\partial T}\right)\right]$$

- (normalised) time scale of PT:

$$\Gamma(t) = \Gamma(t_*) e^{-\beta(t-t_*)+\ \dots} \quad \Longleftrightarrow \quad \frac{\beta}{H_*} = T_* \left.\frac{d}{dT}\right|_{T=T_*} \left(\frac{S_3(T)}{T}\right),$$

where $H_* = H(t_*)$ is the hubble rate measured at t_*, we will take $t_* = t_n$

Signatures from Gravitaitonal Waves and at the Colliders

Gravitational Waves

- Spectral density:

$$\propto \left\langle \dot{h}_{ij}^{TT}(\mathbf{k},\omega)\dot{h}_{ij}^{TT}(\mathbf{k},\omega)\right\rangle$$

$$h^2\Omega_{GW}(f) = \frac{h^2}{\rho_c}\frac{d\boxed{\rho_{GW}}}{d(\ln f)}$$

- Adiabatic expansion of universe:

$$s \sim a(t)^3 g(T)T^3$$

- Red-shift in energy $\sim a^4(t)$ and frequency $\sim a(t)$:

$$f_0 = f_* \left(\frac{a_0}{a_*}\right) = 1.65\times 10^{-7}\ \mathrm{Hz}\left(\frac{f_*}{H_*}\right)\left(\frac{T_*}{1\ \mathrm{GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6}$$

$$\Omega_{GW} = \Omega_{GW*}\left(\frac{a_0}{a_*}\right)^4\left(\frac{H_*}{H_0}\right)^2 = 1.67\times 10^{-5} h_0^{-2}\left(\frac{100}{g_*}\right)^{1/3}\Omega_{GW*}$$

Gravitational Waves from EWPT

- Energy budget[a]:

$$h^2\Omega_{\mathrm{GW}} \simeq h^2\Omega_{col} + h^2\Omega_{sw} + h^2\Omega_{\mathrm{MHD}}$$

- Accelerated wall criterion (satisfied by most κ):

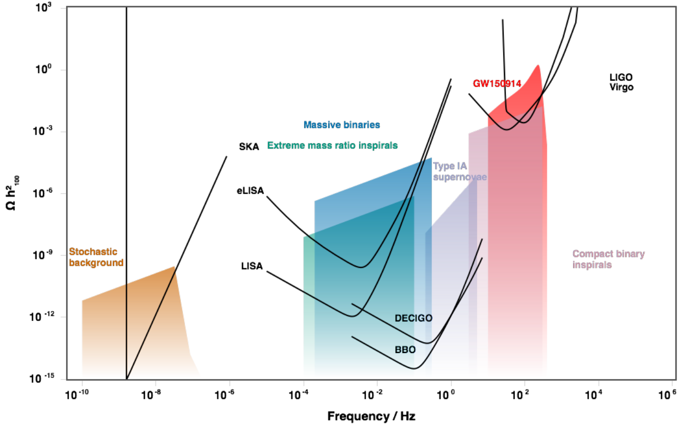
$$\alpha_* > \alpha_\infty :\approx 4.9 \times 10^{-3} \left(\frac{v_{T_*}^{(-)}}{T_*} \right)^2 \propto \Delta m_i^2$$

- Energy going into heating fluid + its motion, surplus into wall acceleration

$$\kappa_f = \frac{\alpha_\infty}{\alpha_*} \left(\frac{\alpha_\infty}{0.73 + 0.083\sqrt{\alpha_\infty} + \alpha_\infty} \right), \qquad \kappa_\rho = 1 - \frac{\alpha_\infty}{\alpha_*}.$$

[a] see e.g. J. Espinosa *et al.*, JCAP 1006 (2010) 028; C. Caprini *et al.*, JCAP 1604 (2016) 001

Gravitational Waves @ eLISA, DECIGO & BBO



[C. J. Moore, R. H. Cole and C. P. L. Berry, Class. Quant. Grav. **32** (2015) 015014]

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[a] (evolved) Laser Interferometer Space Antenna [(e)LISA] — P. Amaro-Seoane *et al.*, GW Notes **6** (2013) 4
Deci-Hertz Interferometer Gravitational-wave Observatory [DECIGO] — S. Kawamura *et al.*, Class. Quant. Grav. **28** (2011) 094011.
Big Bang observer [BBO] — E. S. Phinney *et al.*, Big Bang Observer Mission Concept Study (NASA) (2003)

[b] sensitivity curves from C. Caprini *et al.*, JCAP 1604 (2016) 001; K. Yagi and N. Seto, Phys. Rev. D **83** (2011) 044011

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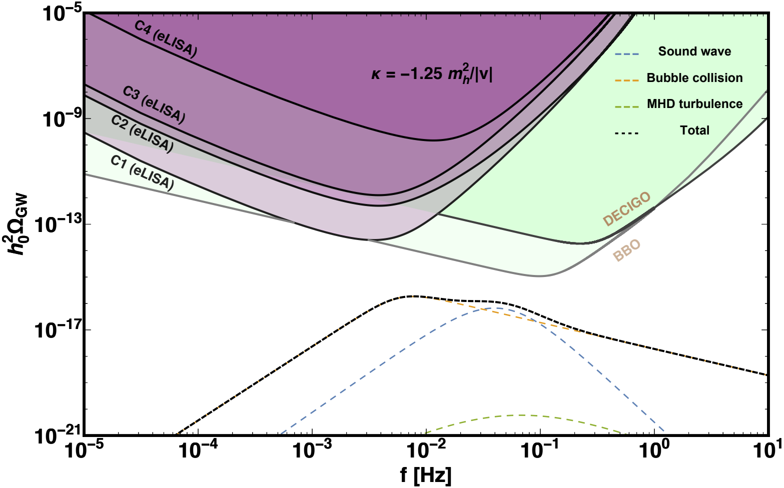

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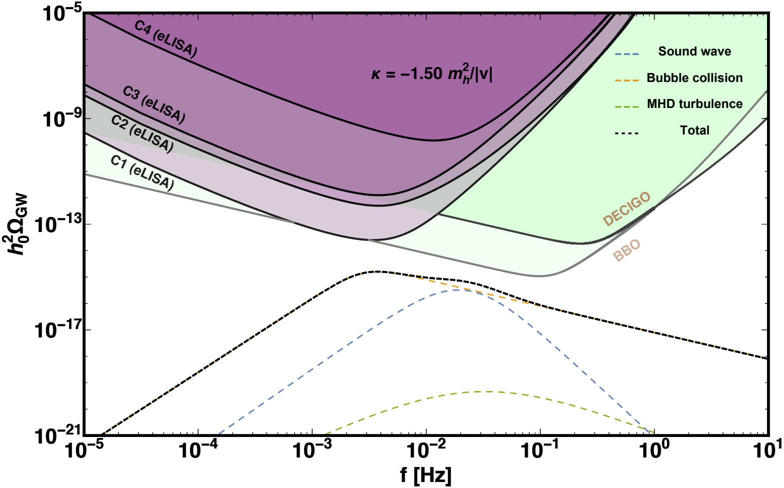

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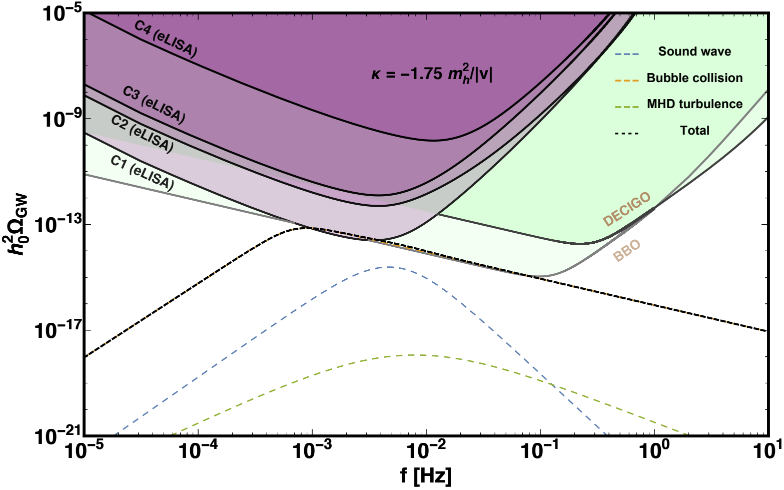

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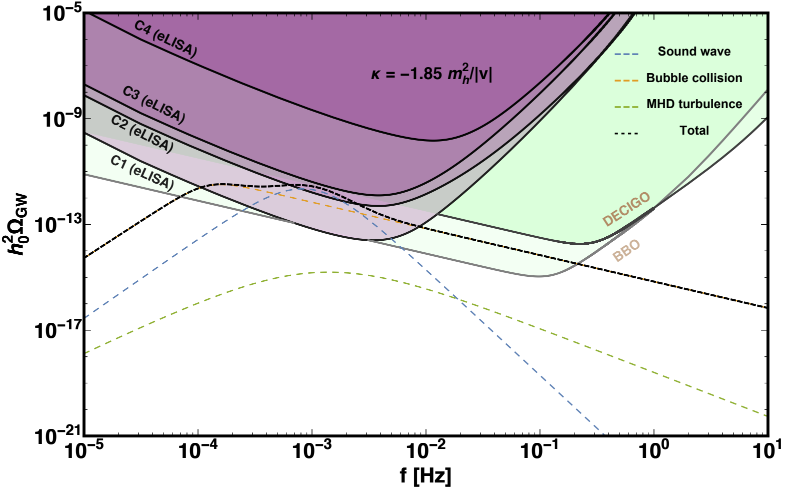

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Collider Signatures?

- Double Higgs production to probe the trilinear coupling[a]
- Sensitive to other anomalous couplings ($t\bar{t}h$, VVh, $VVhh$)

[a] see e.g. M. J. Dolan *et al.*, JHEP **1210** (2012) 112; J. Baglio *et al.*, JHEP **1304** (2013) 151; T. Liu and H. Zhang, arXiv:1410.1855 [hep-ph]; N. Liu *et al.*, JHEP **1501** (2015) 008; R. Contino *et al.*, JHEP **1208** (2012) 154

Collider Signatures?

- Double Higgs production to probe the trilinear coupling[a]

$$\lambda_3 := \left.\frac{d^3V}{d\rho^3}\right|_{\rho=v} = 6\lambda v + 2\kappa \qquad \Longrightarrow \qquad \lambda_3^{SM} = \frac{3m_h^2}{|v|}$$

- Sensitive to other anomalous couplings ($t\bar{t}h$, VVh, $VVhh$)

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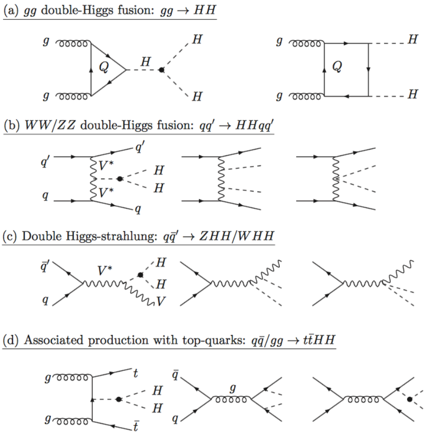

[JHEP 04 (2013) 151]

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- Double Higgs production to probe the trilinear coupling[a]
- Sensitive to other anomalous couplings ($t\bar{t}h$, VVh, $VVhh$)

Table 1-22. *Signal significance for $pp \to HH \to bb\gamma\gamma$ and percentage uncertainty on the Higgs self-coupling at future hadron colliders, from [102].*

	HL-LHC	HE-LHC	VLHC
$\sqrt{s}$ (TeV)	14	33	100
$\int \mathcal{L}dt$ (fb^{-1})	3000	3000	3000
$\sigma \cdot \mathrm{BR}(pp \to HH \to bb\gamma\gamma)$ (fb)	0.089	0.545	3.73
$S/\sqrt{B}$	2.3	6.2	15.0
λ (stat)	50%	20%	8%

[Snowmass Higgs Report 1310.8361]

[a] see e.g. M. J. Dolan *et al.*, JHEP **1210** (2012) 112; J. Baglio *et al.*, JHEP **1304** (2013) 151; T. Liu and H. Zhang, arXiv:1410.1855 [hep-ph]; N. Liu *et al.*, JHEP **1501** (2015) 008; R. Contino *et al.*, JHEP **1208** (2012) 154

Summary

- Non-linear realisation may lead to anomalous Higgs couplings that are not yet ruled out
- Gravitational waves may be used to infer information about the electroweak phase transition
  - Cubic Higgs self-coupling may contribute to first order phase transition
  - For, $\kappa \in [111, 118]$ GeV, $\sim \mu$Hz gravitational waves signatures may be detected at eLISA
  - Keep in mind that there is no fundamental reason to stick with our minimalistic scenario

Thank You!

Quantum corrections to the potential

- 1-loop correction

$$V_i^{(1)}(\phi) = \frac{(-1)^{2s_i}}{2} \int \frac{d^4 p}{(2\pi)^4} \ln \det K_i$$

- *Bosons*

$$K_i = (p^0)^2 + |\mathbf{p}|^2 + m_i^2(\rho)$$

- *Fermions*

$$K_i = \gamma_0 p_0 + \gamma_i p_i - (y_S(\rho) + m' + i\gamma^5 y_P(\rho)).$$

- Recover boson case with effective mass

$$m_t^2(\rho) = \left(m' + \frac{y_t}{\sqrt{2}}\rho\cos\xi\right)^2 + \left(\frac{y_t}{\sqrt{2}}\rho\sin\xi\right)^2$$

Quantum corrections to the potential

- $\overline{\text{DR}}$ scheme

$$V^{(1)}(\rho) = \sum_i \frac{n_i m_i^4(\rho)}{64\pi^2}\left(\ln\left(\frac{m_i^2(\rho)}{\mu_R^2}\right) - \frac{3}{2}\right)$$

- Thermal correction

$$V_T^{(1)}(\rho) = \frac{T^4}{2\pi^2}\sum_{i=W,Z,t,h} n_i J_i\left[\frac{m_i^2(\rho)}{T^2}\right]$$

- High temperature expansion $(m_i^2(\rho) \ll T^2)$

$$J_i(x) = \begin{cases} -\frac{\pi^4}{45} + \frac{\pi^2}{12}x - \frac{\pi}{6}x^{3/2} - \frac{x^2}{32}\ln\frac{x}{c_B} + \mathcal{O}\left(x^6\right), & \text{for bosons} \\ \frac{7\pi^4}{360} - \frac{\pi^2}{24}x - \frac{x^2}{32}\ln\frac{x}{c_F} + \mathcal{O}\left(x^6\right), & \text{for fermions} \end{cases}$$

$$c_B := 16\pi^2\exp(3/2 - 2\gamma_E) \quad \text{and} \quad c_F := \pi^2\exp(3/2 - 2\gamma_E)$$

Collision

$$h^2\Omega_{col}(f) = 1.67\times 10^{-5}\left(\frac{\beta}{H_n}\right)^{-2}\left(\frac{\kappa_v\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{1/3}\left(\frac{0.11v_w^3}{0.42+v_w^2}\right)S_\rho(f),$$

$$S_\rho(f) = \frac{3.8(f/f_\rho)^{2.8}}{1+2.8(f/f_\rho)^{3.8}},$$

$$f_\rho = 16.5\times 10^{-3}\left(\frac{0.62}{1.8-0.1v_w+v_w^2}\right)\left(\frac{\beta}{H_n}\right)\left(\frac{T_n}{100\ \mathrm{GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6}\ \mathrm{mHz}.$$

[a] see C. Caprini *et al.*, JCAP 1604 (2016) 001 and references therein

Sound wave

$$h^2\Omega_{sw}(f) = 2.65 \times 10^{-6} \left(\frac{\beta}{H_n}\right)^{-1} \left(\frac{\kappa_v \alpha}{1+\alpha}\right)^2 \left(\frac{100}{g_*}\right)^{1/3} v_w S_{sw}(f),$$

$$S_{sw}(f) = \left(\frac{f}{f_{sw}}\right)^3 \left(\frac{7}{4+3(f/f_{sw})^2}\right)^{7/2},$$

$$f_{sw} = 1.9 \times 10^{-2} v_w^{-1} \left(\frac{\beta}{H_n}\right) \left(\frac{T_n}{100 \text{ GeV}}\right) \left(\frac{g_*}{100}\right)^{1/6} \text{ mHz}.$$

[a] see C. Caprini *et al.*, JCAP 1604 (2016) 001 and references therein

MHD Turbulence

$$h^2\Omega_{\rm MHD}(f) = 3.35 \times 10^{-4} \left(\frac{\beta}{H_n}\right)^{-3/2} \left(\frac{\epsilon\kappa_v\alpha}{1+\alpha}\right)^2 \left(\frac{100}{g_*}\right)^{1/3} v_w S_{\rm MHD}(f),$$

$$S_{\rm MHD}(f) = \left(\frac{f}{f_{\rm MHD}}\right)^3 \left(\frac{1}{[1+(f/f_{\rm MHD})]^{11/3}(1+8\pi f/h_n)}\right),$$

$$f_{\rm MHD} = 2.7 \times 10^{-2} v_w^{-1} \left(\frac{\beta}{H_n}\right) \left(\frac{T_n}{100 \text{ GeV}}\right) \left(\frac{g_*}{100}\right)^{1/6} \text{ mHz}.$$

- Here $\epsilon \sim 5 - 10\%$ and we take the lower value for more conservative estimate.

[a] see C. Caprini *et al.*, JCAP 1604 (2016) 001 and references therein

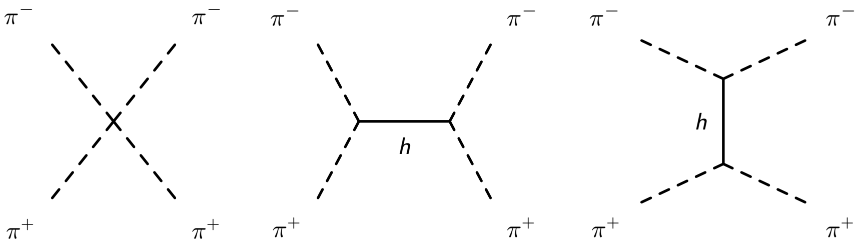
Characterising Phase Transition via α and β parameters

κ $[m_h^2/\lvert v\rvert]$	T_n GeV	α	β/H_n	$v_{T_n}^{(-)}/T_n$
-1.25	106.	0.037	1770	1.64
-1.50	91.9	0.057	989.	1.87
-1.75	72.0	0.11	308.	2.11
-1.85	56.1	0.24	69.5	4.33
-2.00	-	-	-	-

Specific UV completion — The SM Higgs

$$\mathcal{L} \supset \frac{v^2}{4}\mathrm{Tr}\left(D_\mu\Sigma^\dagger D^\mu\Sigma\right)\left(1 + 2a\frac{h}{v} + b\frac{h^2}{v^2} + b_3\frac{h^3}{v^3} + \dots\right)$$
$$- \frac{v}{\sqrt{2}} \sum_{i,j\in\{1,2,3\}} \left(\overline{u}_L^i \; \overline{d}_L^i\right)\tilde{\Sigma}\left(1 + c\frac{h}{v} + c_2\frac{h^2}{v^2} + \dots\right)\lambda_{ij}^u u_R^j + h.c.$$

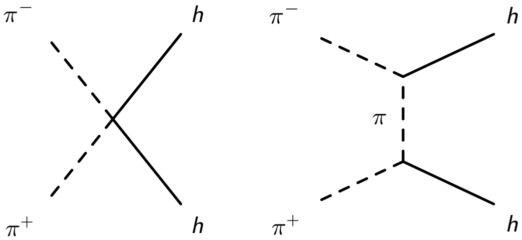
Specific UV completion — The SM Higgs



$$\boxed{\mathcal{A}(\pi^+\pi^- \to \pi^+\pi^-) = \frac{s+t}{v^2}(1-a^2) + \mathcal{O}\left(\frac{m_h^2}{E^2}\right)}$$

$$\mathcal{L} \supset \frac{v^2}{4}\mathrm{Tr}\left(D_\mu \Sigma^\dagger D^\mu \Sigma\right)\left(1 + 2a\frac{h}{v} + b\frac{h^2}{v^2} + b_3\frac{h^3}{v^3} + \dots\right)$$
$$- \frac{v}{\sqrt{2}} \sum_{i,j\in\{1,2,3\}} \left(\overline{u}_L^i \ \overline{d}_L^i\right) \tilde{\Sigma} \left(1 + c\frac{h}{v} + c_2\frac{h^2}{v^2} + \dots\right) \lambda_{ij}^u u_R^j + h.c.$$

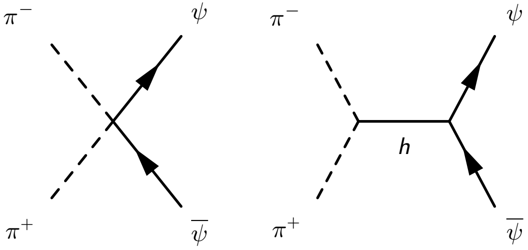
Specific UV completion — The SM Higgs



$$\boxed{\mathcal{A}(\pi^+\pi^- \to hh) = \frac{s}{v^2}(b - a^2) + \mathcal{O}\left(\frac{m_h^2}{E^2}\right)}$$

$$\mathcal{L} \supset \frac{v^2}{4}\mathrm{Tr}\left(D_\mu \Sigma^\dagger D^\mu \Sigma\right)\left(1 + 2a\frac{h}{v} + b\frac{h^2}{v^2} + b_3\frac{h^3}{v^3} + \dots\right)$$
$$- \frac{v}{\sqrt{2}} \sum_{i,j\in\{1,2,3\}} \left(\overline{u}_L^i \ \overline{d}_L^i\right) \tilde{\Sigma} \left(1 + c\frac{h}{v} + c_2\frac{h^2}{v^2} + \dots\right) \lambda_{ij}^u u_R^j + h.c.$$

Specific UV completion — The SM Higgs



$$\mathcal{A}(\pi^+\pi^- \to \overline{\psi}\psi) = \frac{m_\psi \sqrt{s}}{v^2}(1 - ac) + \mathcal{O}\left(\frac{m_h^2}{E^2}\right)$$

$$\mathcal{L} \supset \frac{v^2}{4}\mathrm{Tr}\left(D_\mu \Sigma^\dagger D^\mu \Sigma\right)\left(1 + 2a\frac{h}{v} + b\frac{h^2}{v^2} + b_3\frac{h^3}{v^3} + \dots\right)$$
$$- \frac{v}{\sqrt{2}} \sum_{i,j\in\{1,2,3\}} \left(\overline{u}_L^i \ \overline{d}_L^i\right) \tilde{\Sigma} \left(1 + c\frac{h}{v} + c_2\frac{h^2}{v^2} + \dots\right) \lambda_{ij}^u u_R^j + h.c.$$

Specific UV completion — The SM Higgs

$$\boxed{H = \frac{\boxed{v + h(x)}}{\sqrt{2}}\Sigma}$$

$$\rho(x)$$

$a = b = c = 1$,

0 otherwise

$$\mathcal{L} \supset \frac{v^2}{4}\mathrm{Tr}\left(D_\mu \Sigma^\dagger D^\mu \Sigma\right)\left(1 + 2\frac{h}{v} + \frac{h^2}{v^2}\right)$$

$$- \frac{v}{\sqrt{2}} \sum_{i,j\in\{1,2,3\}} \left(\overline{u}_L^i \; \overline{d}_L^i\right) \tilde{\Sigma} \left(1 + \frac{h}{v}\right) \lambda_{ij}^u u_R^j + h.c.$$

Specific UV completion — The SM Higgs

$$\rho(x)$$

$$\boxed{H \;=\; \frac{\boxed{v + h(x)}}{\sqrt{2}}\Sigma}$$

$$a = b = c = 1,$$

0 otherwise

$$\mathcal{L} \supset D_\mu H^\dagger D^\mu H - \sum_{i,j\in\{1,2,3\}} \left(\overline{u}_L^i \; \overline{d}_L^i\right) \tilde{H}\lambda_{ij}^u u_R^j + h.c.$$