Rapid thermal co-annihilation through bound states[1,2]

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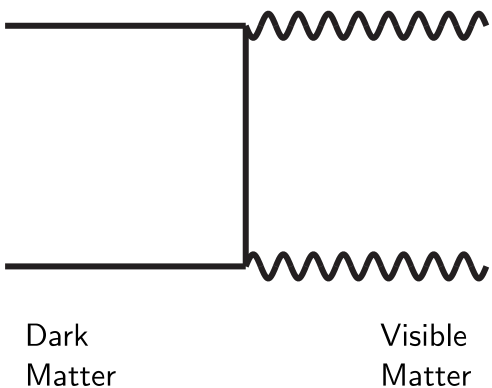
1 S. Kim and ML, 1602.08105.

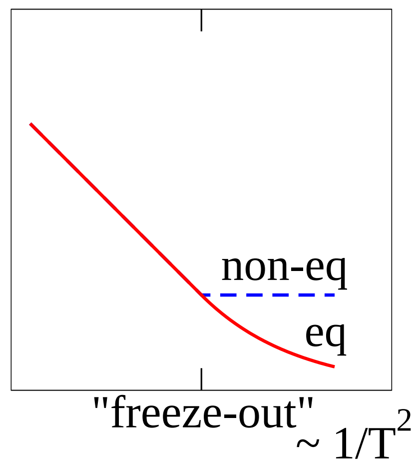
2 Supported by the SNF under grant 200020-155935.

Introduction

Could Weakly Interacting Massive Particles be dark matter?

An initially thermal system chemically decouples when pair annihilation is not fast enough to track the equilibrium distribution, which is $n_{\text{eq}} \sim (\frac{MT}{2\pi})^{3/2} e^{-M/T}$ at $T \ll M$.



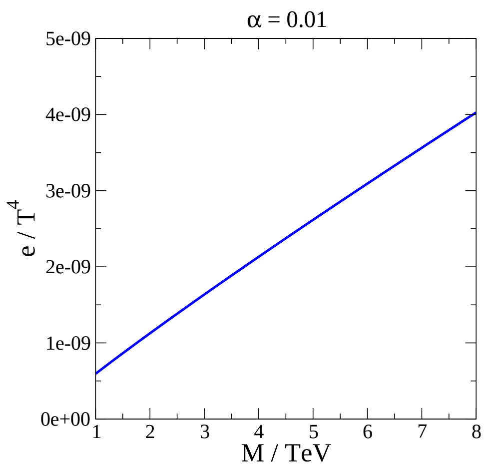


Back of the envelope estimate

Equate the Hubble rate with the co-annihilation rate:

$$H \sim n\langle\sigma v\rangle \;\Leftrightarrow\; \frac{T^2}{m_{\rm Pl}} \sim \left(\frac{MT}{2\pi}\right)^{3/2} e^{-M/T}\frac{\alpha^2}{M^2} \overset{\alpha\sim 0.01}{\Rightarrow} T \sim \frac{M}{25}\;.$$

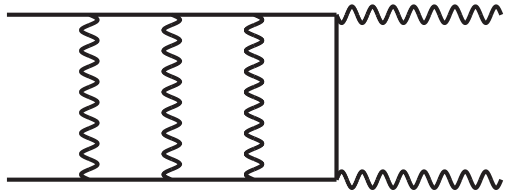
Compare $e \equiv Mn$ at the freeze-out with radiation $\sim T^4$:



LHC pushes up M, so there may be a danger "overclosure".

Could efficient decays help to avoid overclosure?

Indeed co-annihilating particles with $v \ll 1$ interact "strongly".



In particular the "Sommerfeld effect"[3] has been widely discussed.[4] It is an $\gtrsim O(1)$ correction for $T \lesssim \alpha^2 M$.

[3] L.D. Landau and E.M. Lifshitz, *Quantum Mechanics, Non-Relativistic Theory*, Third Edition, §136; V. Fadin, V. Khoze and T. Sjöstrand, *On the threshold behavior of heavy top production*, Z. Phys. C 48 (1990) 613.

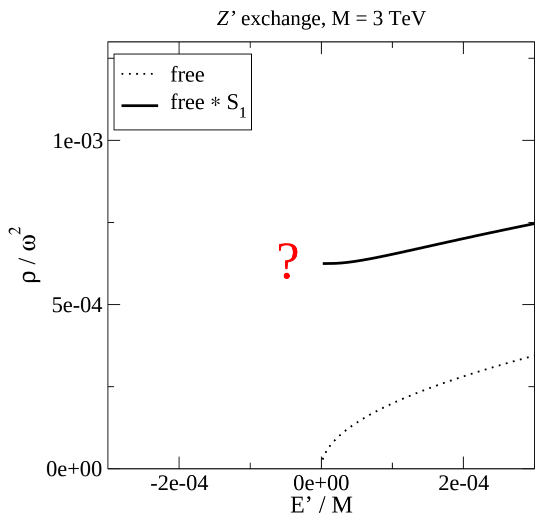
[4] e.g. J. Hisano, S. Matsumoto, M. Nagai, O. Saito and M. Senami, *Non-perturbative effect on thermal relic abundance of dark matter*, hep-ph/0610249; J.L. Feng, M. Kaplinghat and H.-B. Yu, *Sommerfeld Enhancements for Thermal Relic Dark Matter*, 1005.4678.

Rapid summary of the Sommerfeld effect

For attractive s-wave interaction:

$$S_1 = \frac{X_1}{1 - e^{-X_1}} \;, \quad X_1 = \frac{g^2 C_F}{4v} \;.$$

Corresponding spectral function ($E' \equiv \omega - 2M \equiv Mv^2$):



What happens below the threshold?

Perhaps there could be bound states?[5]

We are interested in **rare processes** where two dilute particles come together, i.e. $|\partial_t n| \sim e^{-2M/T}$. In a bound state they are "already" together, and with a less suppressed Boltzmann weight, because of a binding energy $\Delta E > 0$:

$$|\partial_t n_{\text{bound}}| \sim e^{-(2M-\Delta E)/T} \ .$$

If $T \lesssim \Delta E$, this contribution dominates the co-annihilation rate.

[5] e.g. B. von Harling and K. Petraki, *Bound-state formation for thermal relic dark matter and unitarity*, 1407.7874.

Formalism

Classic Boltzmann:

$$(\partial_t + 3H)\, n \approx -\langle \sigma v \rangle \, (n^2 - n^2_{\text{eq}}) \ .$$

Problem: by construction n contains only scattering states.

Boltzmann boosted by on-shell bound states.

Problem: How many? Thermal width? Melting?

General linearization:

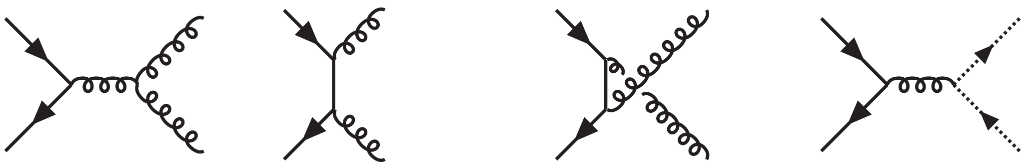
$$(\partial_t + 3H)\, n = -\Gamma_{\text{chem}}(n - n_{\text{eq}}) + \mathcal{O}(n - n_{\text{eq}})^2 \ .$$

$\Gamma_{\rm chem}$ is a "transport coefficient", and n is the total density ($n \equiv e/M$), including the contribution of bound states.[6]

[6] D. Bödeker and ML, *Heavy quark chemical equilibration rate as a transport coefficient,* 1205.4987; *Sommerfeld effect in heavy quark chemical equilibration,* 1210.6153.

The rate Γ_{chem} can be addressed within NRQCD

Let θ, η annihilate DM and DM'. Like in the optical theorem, decays are contained in an imaginary part of a 4-particle operator:[7]



$$\Rightarrow \quad \mathcal{O} = \frac{i c_1 \alpha^2 \, \theta^\dagger \eta^\dagger \, \eta\theta}{M^2} \, .$$

This yields $\Gamma_{\text{chem}} = \dfrac{8 c_1 \alpha^2}{M^2 n_{\text{eq}}} \dfrac{1}{\mathcal{Z}} \sum_{m,n} e^{-E_m/T} |\langle n | \eta\theta | m \rangle|^2$.

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7 G.T. Bodwin, E. Braaten and G.P. Lepage, *Rigorous QCD analysis of inclusive annihilation and production of heavy quarkonium*, hep-ph/9407339.

The thermal average can be resolved into a spectral fcn

$$\gamma \;\equiv\; \frac{1}{\mathcal{Z}} \sum_{m,n} e^{-E_m/T} \big|\langle n|\eta\theta|m\rangle\big|^2 \;,$$

$$\rho(\omega) \;\equiv\; \int_{-\infty}^{\infty} \mathrm{d}t\, e^{i\omega t} \left\langle \frac{1}{2} \big[(\eta\theta)(t),\, (\theta^\dagger \eta^\dagger)(0) \big] \right\rangle_T \;.$$

$\vdots$

$$\Rightarrow \quad \gamma = \int_{2M-\Lambda}^{\infty} \frac{\mathrm{d}\omega}{\pi}\, e^{-\omega/T} \rho(\omega) \;+\; \mathcal{O}\big(e^{-4M/T}\big) \;.$$

Spectral fcn is a cut of a Green's function

$$\begin{aligned}
\left[H - i\,\Gamma(r) - E'\right] G(E'; \mathbf{r}, \mathbf{r}') &= \delta^{(3)}(\mathbf{r} - \mathbf{r}') \ , \\
\lim_{\mathbf{r}, \mathbf{r}' \to \mathbf{0}} \mathrm{Im}\, G(E'; \mathbf{r}, \mathbf{r}') &= \rho(E') \ .
\end{aligned}$$

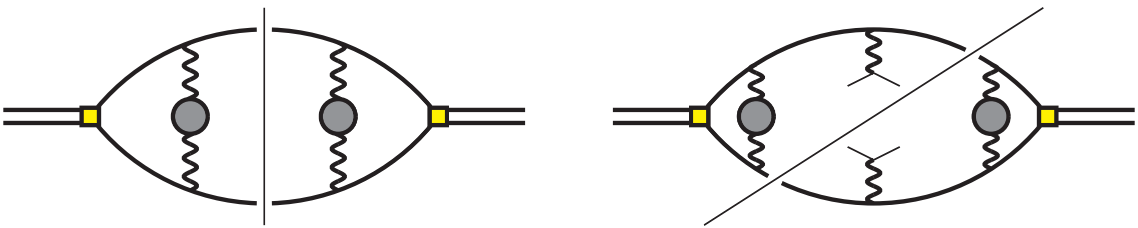
$$H = -\frac{\nabla_r^2}{M} + V(r) \ ,$$

$$E' \equiv \omega - 2M \ .$$

$V(r)$ and $\Gamma(r)$ emerge from gauge exchange

$$V(r) - i\,\Gamma(r) = g^2 \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \left(1 - e^{i\mathbf{k}\cdot\mathbf{r}}\right) i\Delta_{00\mathrm{T}}(0,k) \ .$$

The width represents real scatterings, present in a plasma:



$\sim V(r)$ $\qquad\qquad$ $\sim \Gamma(r)$

The total rate γ can also be defined non-perturbatively

$$\bar{S}_1 \equiv \frac{\gamma}{\gamma_{\text{free}}} \; .$$

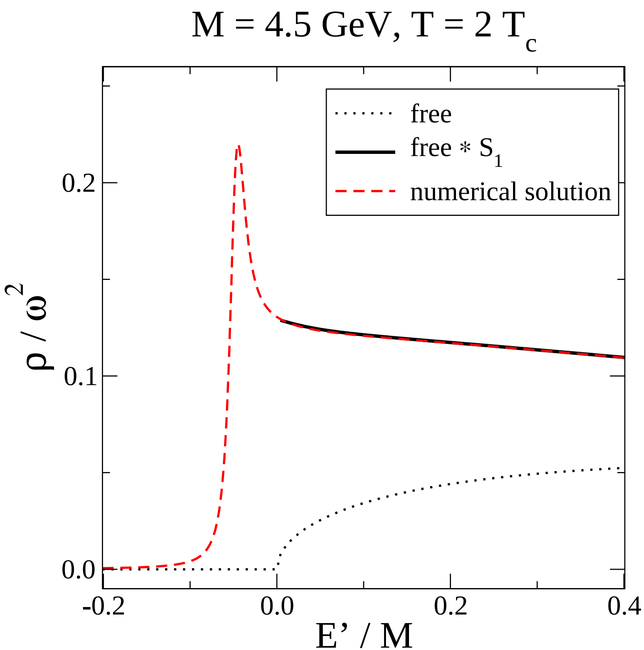
Let G^θ be the imaginary-time propagator of a heavy quark in NRQCD, and $\beta \equiv 1/T$. Then it can be shown that

$$\bar{S}_1 \; = \; \frac{\frac{1}{2N_c} \mathrm{Tr} \big\langle G^\theta(\beta;0) G^{\theta\dagger}(\beta;0) \big\rangle}{\big\{ \frac{1}{2N_c} \mathrm{Re}\, \mathrm{Tr} \big\langle G^\theta(\beta;0) \big\rangle \big\}^2} \; .$$

This is a gauge-invariant and UV-finite observable that can be directly measured within the lattice NRQCD framework.

Test results for QCD

On the perturbative side, use a real-time static potential [8]

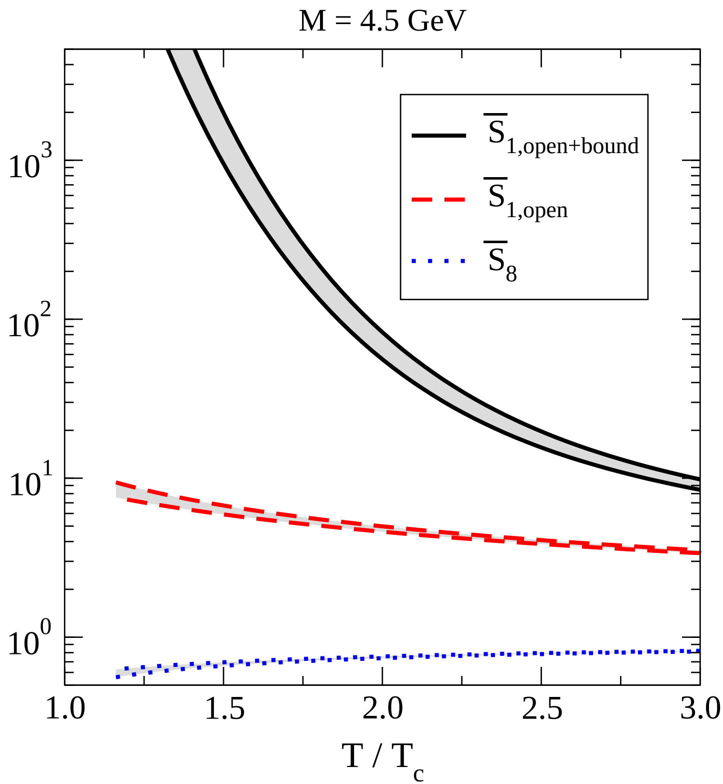


$$V(r) = -\alpha_s \left[m_D + \frac{\exp(-m_D r)}{r} \right] ,$$

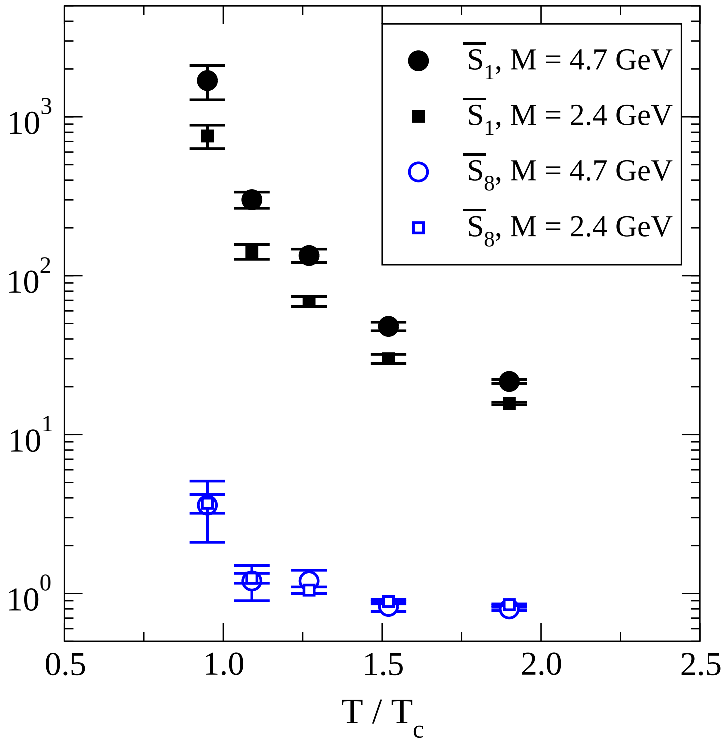
$$\Gamma(r) = 2\alpha_s T \int_0^\infty \frac{dx\, x}{(x^2+1)^2} \times \left[1 - \frac{\sin(x m_D r)}{x m_D r} \right] .$$

[8] ML, O. Philipsen, P. Romatschke and M. Tassler, *Real-time static potential in hot QCD*, hep-ph/0611300; A. Beraudo, J.-P. Blaizot and C. Ratti, *Real and imaginary-time $Q\overline{Q}$ correlators in a thermal medium*, 0712.4394; N. Brambilla, J. Ghiglieri, A. Vairo and P. Petreczky, *Static quark-antiquark pairs at finite temperature*, 0804.0993.

Thermal average ⇒ bound states dominate singlet decays

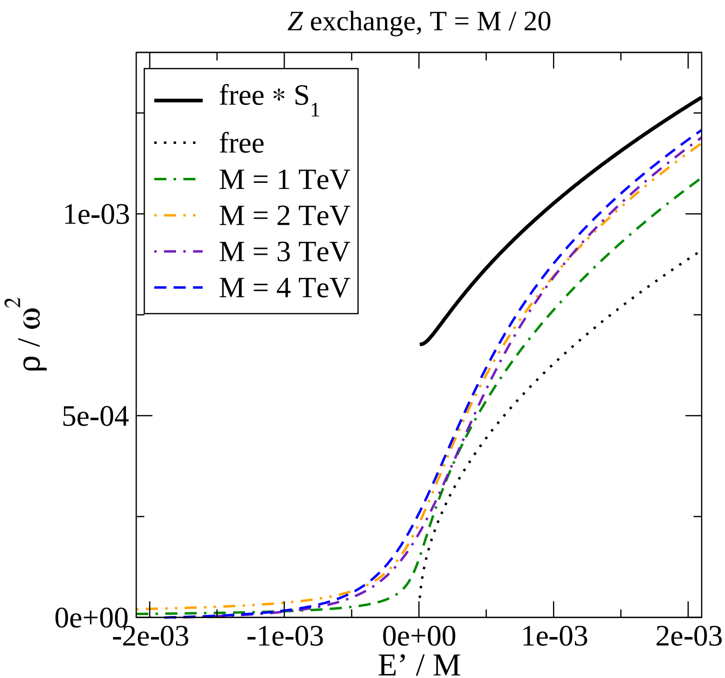


Lattice NRQCD confirms this on a qualitative level



First results for cosmology

Z exchange: no bound states are found



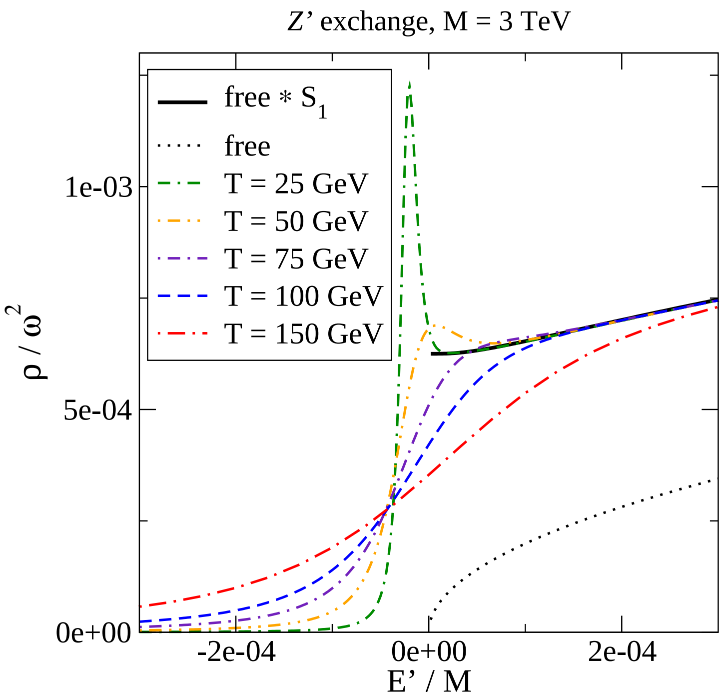
$$\alpha \equiv \frac{g_1^2 + g_2^2}{16\pi} \approx 0.01 \, ,$$

screening by m_Z

$$\simeq \text{91 GeV + Debye mass} \, .$$

Thermal average $\int \mathrm{d}E' \, e^{-E'/T} \ldots \Rightarrow$ Sommerfeld works well.

Z' exchange:[9] bound states melt below freeze-out



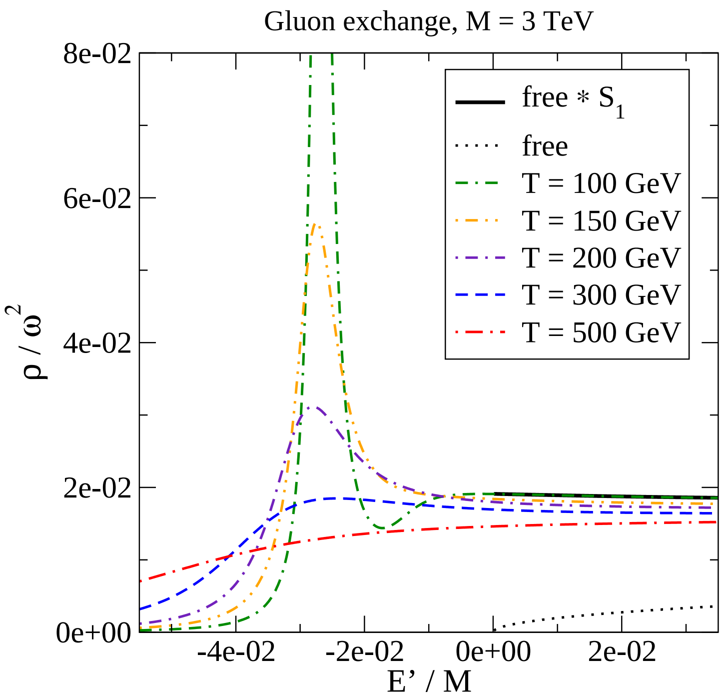
$$\alpha' \quad \equiv \quad (e')^2/(4\pi) \sim 0.01\ ,$$

screening by $m_{Z'}$

$$\approx \quad 1\ \text{GeV} + \text{Debye mass}\ .$$

[9] e.g. M. Pospelov, A. Ritz, M.B. Voloshin, *Secluded WIMP Dark Matter*, 0711.4866; W. Shepherd *et al*, *Bound states of weakly interacting dark matter*, 0901.2125.

Gluon exchange between gluinos:[10] like in QCD



$$m_{\rm D}^2 \equiv 2g_s^2T^2 \, ,$$

$$\alpha_3 \equiv \frac{3g_s^2}{4\pi} \, .$$

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[10] e.g. J. Ellis, F. Luo and K.A. Olive, *Gluino Coannihilation Revisited*, 1503.07142.

Summary

• With strong constraints from LHC and cosmology, there is a need to understand thermal corrections to dark matter freeze-out.

• Weak interactions: Sommerfeld effect seems to be sufficient.

• Strong interactions: the co-annihilation rate is much enhanced because of bound states. This may help to avoid overclosure.