Real-time dynamics of the Chiral Magnetic Effect

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based on work with

Soeren Schlichting, Mark Mace, Sayatan Sharma, Naoto Tanji, Jürgen Berges

Strong And Electroweak Matter 2016 – University of Stavanger, Norway

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Motivation

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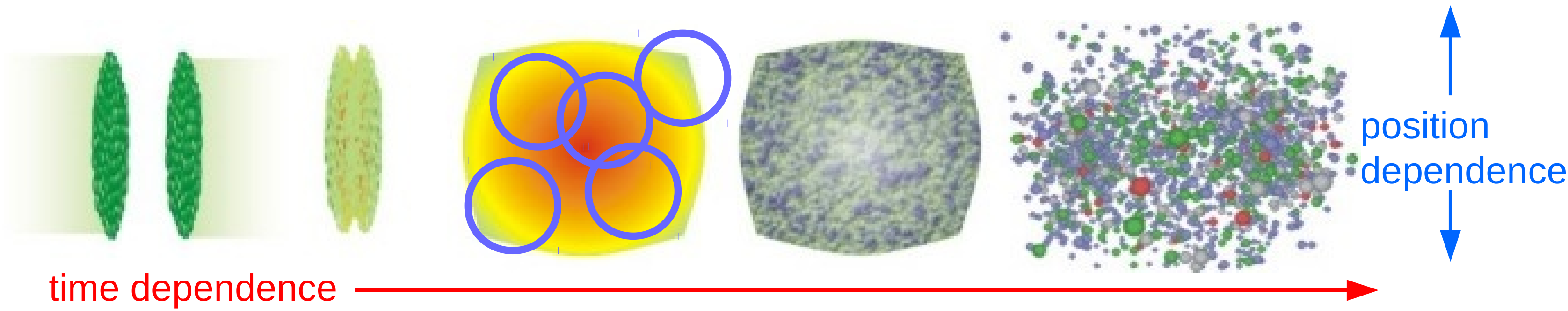
The hunt for CP- and P-odd phenomena in QCD

$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F^a_{\mu\nu}F^{a,\mu\nu} - \frac{\theta g^2}{32\pi^2}F^a_{\mu\nu}\tilde{F}^{a,\mu\nu}$$

So far the CP-odd θ term seems to be absent fundamentally
→ not the end of the game, QCD is a richer theory!
→ no explicit CP-odd effect, but **CP odd fluctuations !**

Heavy Ion Collisions great chance to test local CP-odd fluctuations of QCD, in the sense of finite time and volume (as opposed to 'S-matrix' experiments)



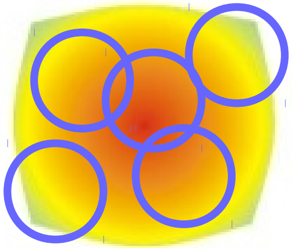
Interesting topological and anomalous phenomena emerge!

Motivation

A possible insight: Anomalous Transport Phenomena

(Kharzeev, McLerran, Fukushima, Warringa, Vilenkin ...)

Connecting local **CP-odd** fluctuations to observables!



Ingredients:

local CP-odd domains

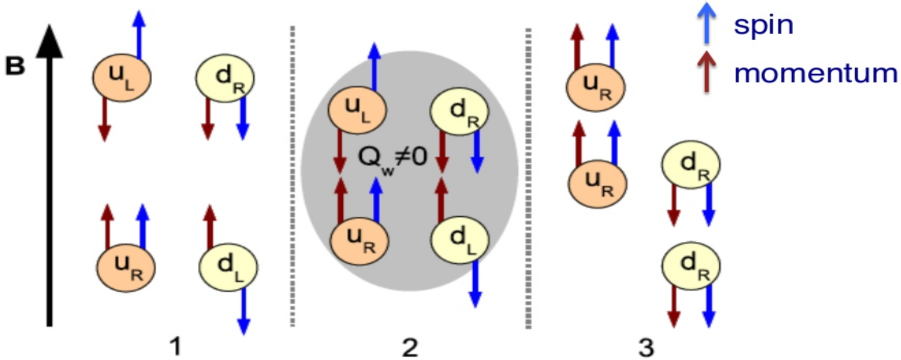
Axial Imbalance

$$\partial_\mu j_5^\mu = -\frac{g^2}{8\pi^2}\mathrm{Tr}F\tilde{F}$$

+ **Large Magnetic Fields** **=** **Electric currents (observable)**

Chiral Magnetic Effect: $\mathbf{J}_e \propto Q_5 \mathbf{B}$

Chiral Separation Effect: $\mathbf{J}_5 \propto Q_e \mathbf{B}$



Kharzeev, McLerran, Warringa 2007

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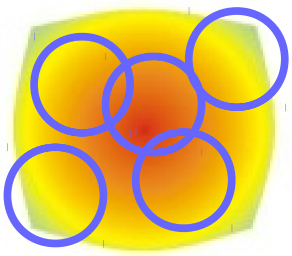

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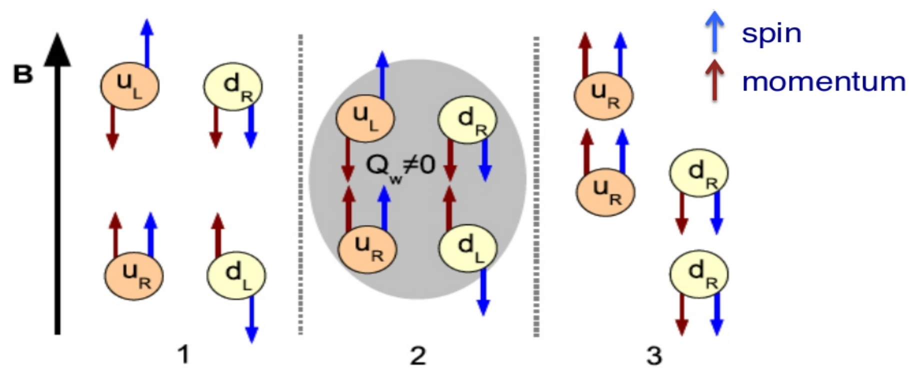

Kharzeev, McLerran, Warringa 2007

Real-time study in QCD+QED necessary!

Outline

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1. Anomalous Phenomena in a Heavy Ion Collision
2. The anomaly and chiral violation at very early times
3. Classical Statistical Simulations
4. Conclusions
5. Where to go from here?

1. Anomalous Phenomena during the stages of a Heavy Ion Collision

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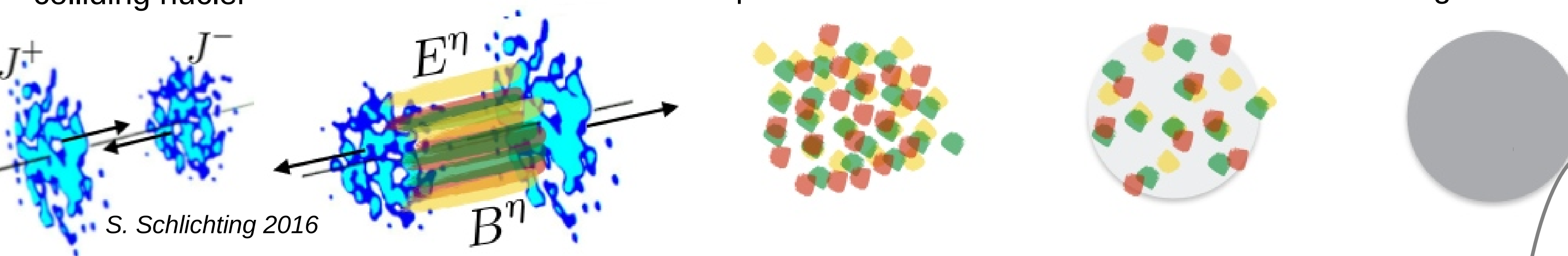
CGC colliding nuclei | flux tubes | over-occupied plasma | kinetic regime | hydrodynamic regime



non-equilibrium **anomalous fermion production** from coherent fields (Tanji et al. 2016) and sphaleron transitions (Mace et al. 2016)

large **magnetic fields** present
Anomalous Transport (CME, CSE and CMW)

?

Subsequent interactions in the fire ball, axial transport and relaxation

Thermalization / Freezout

classical statistical simulations + fermions

chiral kinetic theory

anomalous hydrodynamics

3. The anomaly and chiral violation at early times

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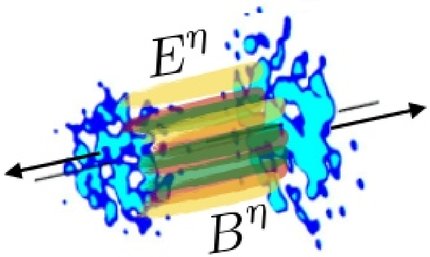
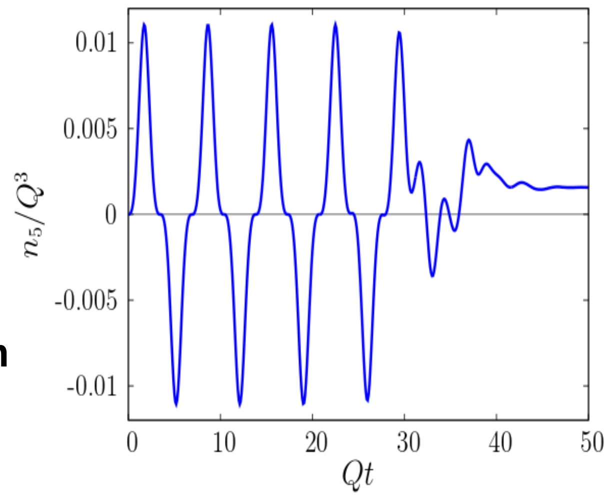
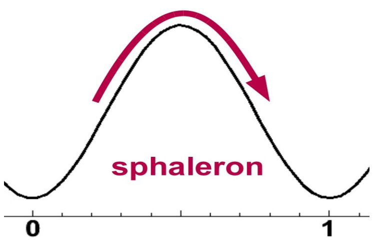
Very early times play a key role in the phenomenology of CP-odd phenomena in Heavy Ion Collisions



- **Color flux-tubes naturally provide large coherent** $\mathbf{E}^a \cdot \mathbf{B}^a$ (topological charge via field strength, *see e.g. N. Tanji, NM, J. Berges, Phys.Rev. D93 074507, arxiv:1603.03331*)





- **Sphaleron transition rates far from equilibrium can be huge!** *see M.Mace, S.Schlichting, R.Venugopalan, Phys.Rev. D93, 074036, arxiv: 1601.07342*

Persistence of background magnetic fields, important for anomalous transport, **at later times unclear.**

Will show you:
Can describe topological transitions, anomalous quark production and transport from first principles in the classical statistical regime

3. The anomaly and chiral violation at early times

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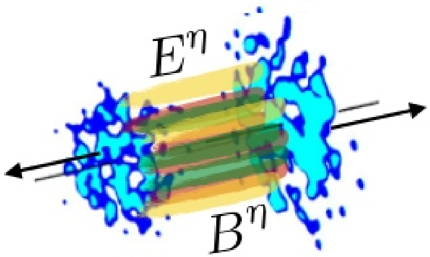

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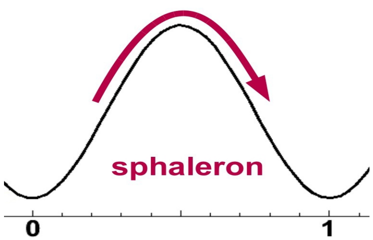

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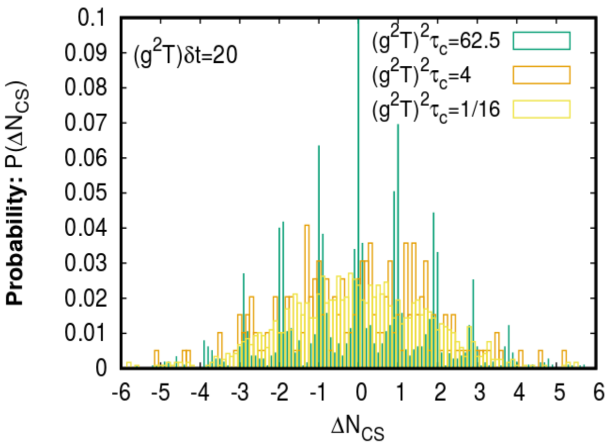

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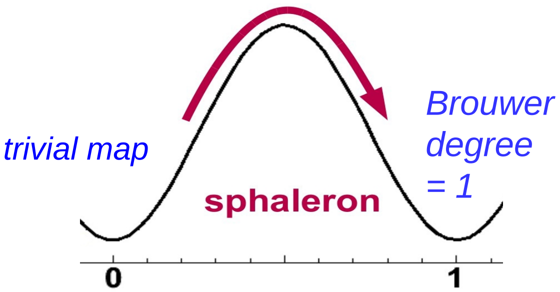
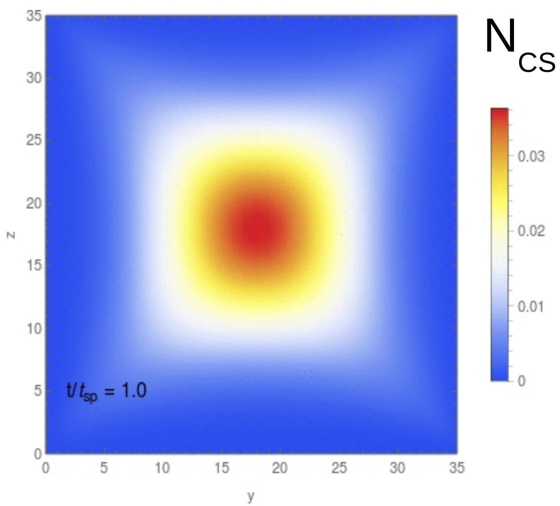
Understanding Fermion production during sphaleron transitions

NM, S.Schlichting, S.Sharma, *arXiv:1606.00342*



$$\boxed{\frac{dN_{CS}}{dt} = \frac{g^2}{8\pi^2}\int d^3x\ E_i^a(\mathbf{x})B_i^a(\mathbf{x})}$$



construct a SU(2) sphaleron transition explicitly

$$\mathbf{R^3} \cup \{\infty\} \rightarrow S^3_{SU(2)}$$

→ spatially distributed smooth and localized **'topological' lump**

→ similar study in the electroweak sector (Higgs winding) by Saffin & Tranberg, (JHEP 1202 (2012) 102)

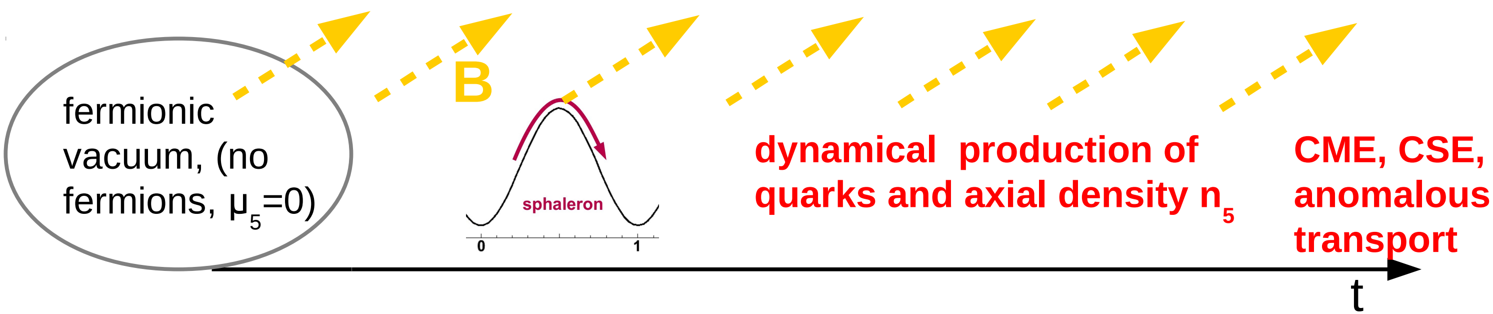
3. Classical Statistical Simulations

Simulating Fermions, topological transitions and magnetic fields in real-time

We study fermion production in SU(2) in the presence of background magnetic fields, employing **classical statistical lattice simulations.**

Will use Wilson fermions with very light and heavier masses, employing the **operator-improved modefunction method**

$$\boxed{i\gamma^0\partial_t\hat{\psi} = (-i\not{D}^s_W + m)\hat{\psi}}$$

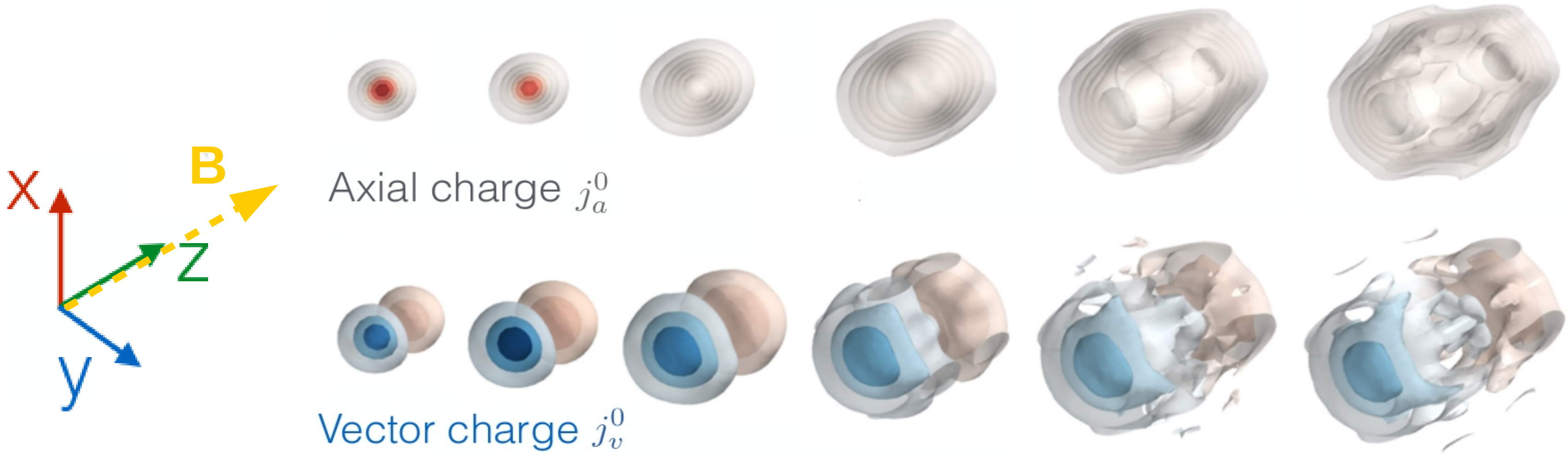


Very interesting also from a conceptual/technical point of view:

- Anomaly realization, fermion doubling and chiral symmetry on the lattice (Smit & Karsten 1983, also M.Mace, NM, S.Schlichting, S.Sharma: real-time study, in preparation)
- Abelian Magnetic Fields on the lattice (e.g. Bali et al., see talk by G. Endrodi)
- Algorithmic improvements (Fermions!)

Chiral Magnetic and Chiral Separation Effect

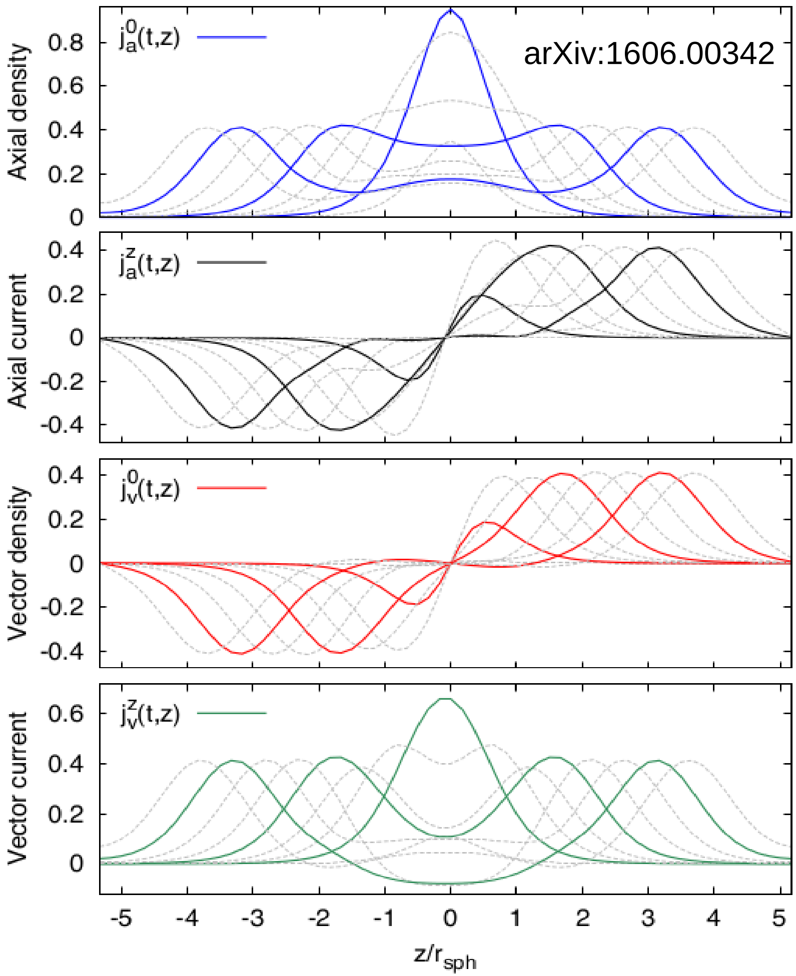
arXiv:1606.00342



Initially: Vacuum (no fermions, no axial charge)

Chiral Magnetic Effect: Electric current generated due to axial charge produced
Chiral Separation Effect: Axial current generated due to electric charge

Anomalous Transport



→ A gapless excitation, the **Chiral Magnetic Wave,** emerges dynamically

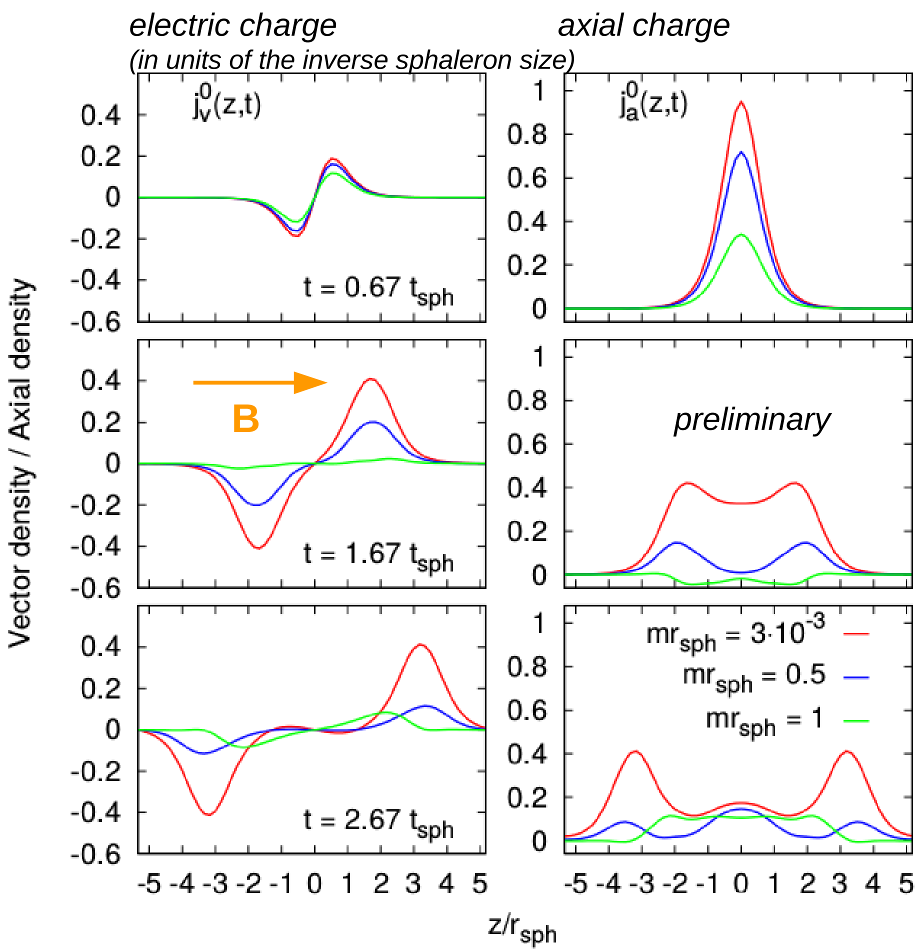
Can be well described in **hydrodynamic approximation** (see also talk by N. Yamamoto):

$$\boxed{j^0_{v,a}(t > t_{\rm sph}, z) = \frac{1}{2}\int_0^{t_{\rm sph}} dt' \left[S(t', z - c(t-t')) \mp S(t', z + c(t-t'))\right]}$$

$$S(z,t) = -\frac{g^2}{8\pi^2}\int d^2x_\perp \; F\cdot\tilde{F}$$

- can calculate transport coefficients!
  (see Mace, M, Schlichting, Sharma, in preparation)

Effects of finite quark mass



Fermion mass important for anomalous transport!

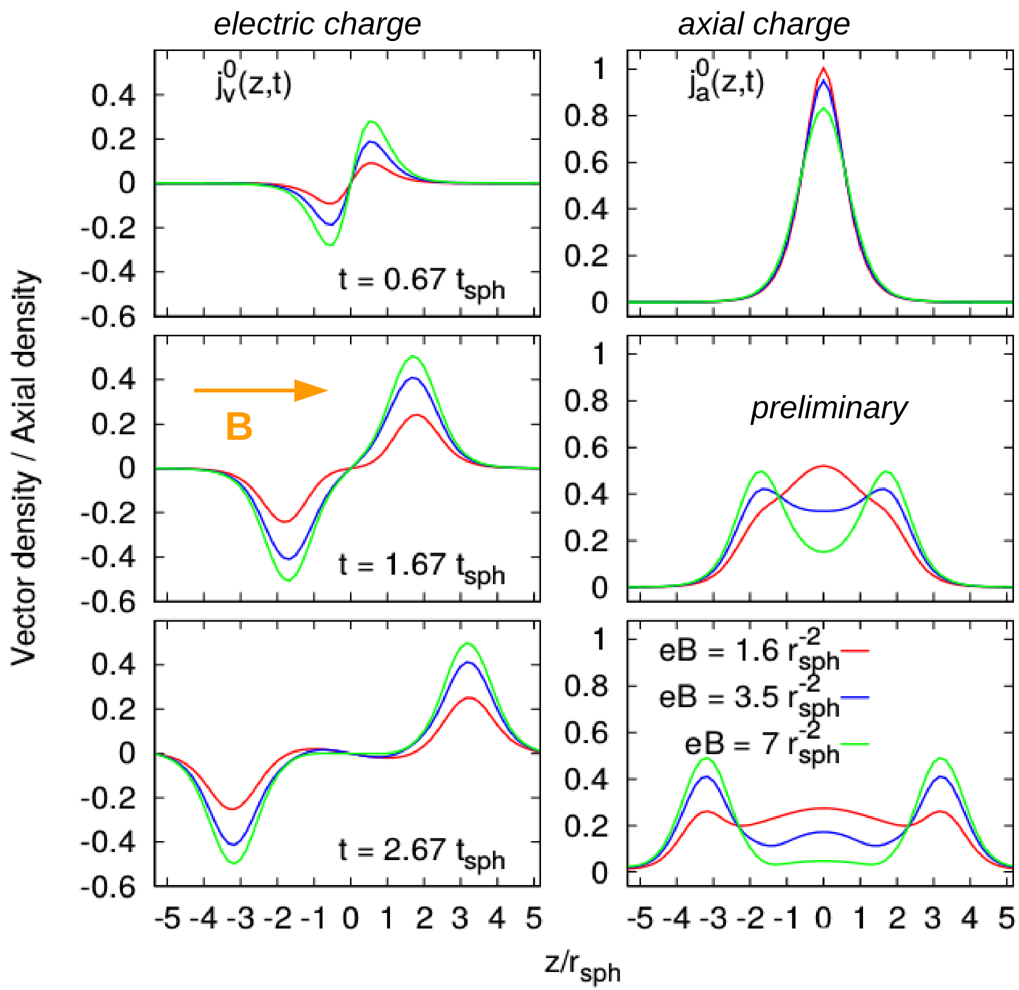
$$\partial_\mu j_a^\mu(x) = 2m\eta_a(x) - \frac{g^2}{8\pi^2}\mathrm{Tr}F_{\mu\nu}(x)\tilde{F}^{\mu\nu}(x)$$

- dissipation of axial charge

- counteracts anomalous transport of electric and axial charges

M. Mace, NM, S. Schlichting, S. Sharma, in preparation

Magnetic Field Dependence



Soon to come:
Transport coefficients, finite T

M. Mace, NM, S. Schlichting, S. Sharma, in preparation

4. Conclusions

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- I have shown you real-time classical statistical simulations of fermion production during sphaleron transitions in background magnetic fields

- Axial anomaly realized in lattice simulations using Wilson fermions

- Chiral Magnetic and Chiral Separation Effect emerge dynamically

- Observation of the Chiral Magnetic Wave

- Have investigated finite mass and magnetic field dependence. Finite quark mass plays an important role: dissipation of anomalous currents

Did not show you (yet):
Importance of topological transitions vs. coherent fields

5. Where to go from here?

Quantitative, first-principle predictions of fermion production and CP-odd phenomena and transport are possible!

→ can make phenomenology of anomalous transport precise

→ heavy ion collisions from the “local” perspective:
Phenomenology beyond anomalous transport possible (anomaly might have observable consequences even without background magnetic fields!)

→ local CP fluctuations might tell us about global CP-conservation and thus might relate to ones favorite explanation (axions etc.)

Real-time classical statistical simulations using chiral overlap-fermions:
M.Mace, NM, S. Schlichting, S.Sharma, in preparation

Decoherence color flux-tubes with non-zero EB in the early time regime:
N. Tanji, NM, J. Berges, Phys.Rev. D93 (2016) no.7, 074507

Anomalous effects in QED beyond the Schwinger limit:
NM, F. Hebestreit, J. Berges, arXiv:1605.01413, accepted for publication in PRL

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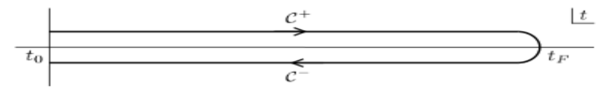
Thanks for your attention

Studienstiftung
des deutschen Volkes

Backup
A. Classical Statistical Simulations

(see for example Kasper et al. Phys.Rev. D90 (2014) 2, 025016)



typical situation: large **coherent or highly occupied** gauge fields
→ correspondence principle

$$A^+_{\mu,n} = \bar{A}_{\mu,n} + \frac{1}{2}\tilde{A}_{\mu,n}\,,$$
$$A^-_{\mu,n} = \bar{A}_{\mu,\bar{n}} - \frac{1}{2}\tilde{A}_{\mu,\bar{n}}\,,$$

'classical' 'quantum'

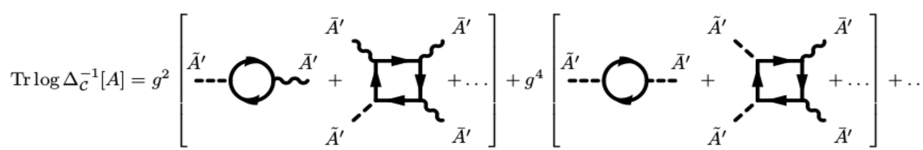
- initial stages of a heavy ion collision: A ~ 1/g
- colliding laser beams → large and coherent fields

The classical-statistical approximation is a systematic expansion of the 'quantum' fields around the 'classical' fields

Fermions are never “classical”

$$Z_{\mathcal{C}} = \int [dA]\, \rho_G(A)\, \exp\left(\mathrm{Tr}\log \Delta_{\mathcal{C}}[A]^{-1} + iS_G\right)$$

$$\boxed{i\gamma^0\partial_t\hat{\psi} = (-i\not{D}^s_W + m)\hat{\psi}}$$



Exact description via **modefunctions** up to **24x24x64** lattices

B. Backup
Algorithmic Improvements

Fermions: Exact description via **modefunctions** up to 24x24x64 lattices

Improved operators (NM, S.Schlichting, S.Sharma, arXiv:1606.00342)

We use a tree-level improved version of the lattice Hamiltonian, which takes the form

$$H = \sum_x \psi_x^\dagger m \gamma^0 \psi \quad + \quad \frac{1}{2} \sum_{n,x,i} C_n \psi_x^\dagger \gamma^0 \Big[\Big(-i\gamma^i - n r_w \Big) U_{x,+ni} \psi_{x+ni} + 2 n r_w \psi_x - \Big(-i\gamma^i + n r_w \Big) U_{x,-ni} \psi_{x-ni} \Big]$$

where r_w denotes the Wilson coefficient, the coefficients C_n are chosen to optimize the convergence, and we introduce the following short hand notation for the connecting gauge links

$$U_{x,+ni} = \prod_{k=0}^{n-1} U_{x+ki,i} \ , \qquad U_{x,-ni} = \prod_{k=1}^{n} U^\dagger_{x-ki,i} \tag{1}$$

Wilson-averaging (M.Mace, NM, S.Schlichting, S.Sharma, in preparation)

→ improvement of chiral properties
→ extremely important for larger fermion masses
→ average fermionic observables over Wilson parameters with opposite sign
→ leading order errors in the anomaly equation cancel

Backup
C. Magnetic Fields on the lattice

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Magnetic fields break translation invariance → magnetic translation group

- Magnetic fields on a torus very non-trivial
(see Al-Hashimi & Wiese "Accidental Symmetries", also Bali et al.)

$$U_{y,n} = \mathrm{e}^{\mathrm{i}a^2 \mathrm{q} \mathrm{B} \mathrm{n}_\mathrm{x}} \;;\; \mathrm{U}_{\mathrm{x},\mathrm{N}_\mathrm{x}-1,\mathrm{n}_\mathrm{y},\mathrm{n}_\mathrm{z}} = \mathrm{e}^{-\mathrm{i}a^2 \mathrm{q} \mathrm{B} \mathrm{N}_\mathrm{x} \mathrm{n}_\mathrm{y}}$$

$$U_{x,n} = \mathbf{1}, n_x \neq N_x - 1; U_{z,n} = \mathbf{1}$$

- Intriguing lattice artefacts!

→ spoil the low-cost method
--- while there probably are field configurations where low-cost works, this is certainly not the case in magnetic fields

Backup
D. Anomaly Realization on the Lattice

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Chiral Symmetry + Fermion doubling + Chiral Anomaly
= “one of the prettiest connections I have ever seen”

Lattice Fermions: Species Doubling, Chiral Invariance, and the Triangle Anomaly

Luuk H. Karsten (Stanford U., ITP) , Jan Smit (Amsterdam U.)

Sep 1980 - 38 pages

Nucl.Phys. B183 (1981) 103

In *Rebbi, C. (Ed.): Lattice Gauge Theories and Monte Carlo Simulations*, 495-532. (Nucl. Phys. B183 (1981) 103-140) and Stanford Univ. - ITP-677 (80,REC.NOV.) 71p (1981)

DOI: 10.1016/0550-3213(81)90549-6

Conference: C81-06-01, p.495-532

Proceedings

ITP-677-STANFORD

Contributions

- The axial anomaly and the fermion doubling problem are intimately related

- Lattice theory regularized on the basis of the action already

- Anomaly comes from the non-trivial continuum limit of any regulator you put in to remove doublers