Semi-holography for heavy ion collisions

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in collaboration with
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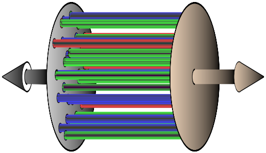
direct QCD approach to heavy ion collisions

glasma picture

$f \sim 1/\alpha_s$, $\alpha_s(Q_s) \ll 1$

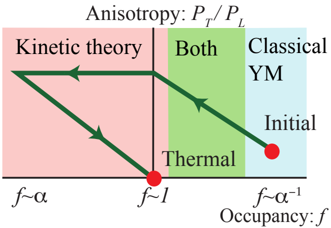
F. Gelis, E. Iancu, J. Jalilian-Marian, R. Venugopalan, *Ann.Rev.Nucl.Part.Sci.*60:463-489,2010



kinetic theory

takes over around $f \sim 1$

A. Kurkela, *QM 2015* arXiv:1601.03283

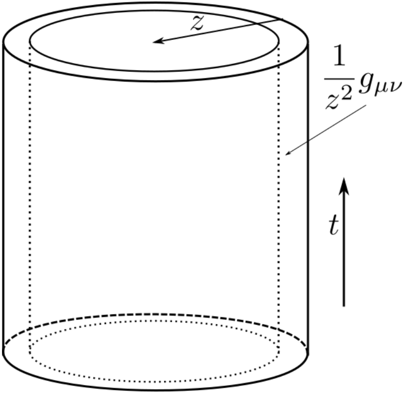


dual $\mathcal{N} = 4$ SYM approach to heavy ion collisions

shockwaves $\leftrightarrow$ nuclei, black hole formation $\leftrightarrow$ thermalization

$$ds^2 = \frac{1}{z^2}\left[dz^2 + \left(g^{(0)}_{\mu\nu} + z^2 g^{(2)}_{\mu\nu} + z^4 g^{(4)}_{\mu\nu} + z^4 \ln z \; h^{(4)}_{\mu\nu} + \mathcal{O}(z^6, z^6 \ln z)\right) dx^\mu dx^\nu\right]$$

$$g^{(0)}_{\mu\nu} = \eta_{\mu\nu}$$



Choose ds^2_{initial} such that
$t^{\mu\nu}(t_i) \propto g^{(4)}_{\mu\nu}(t_i)$ mimicks a CGC

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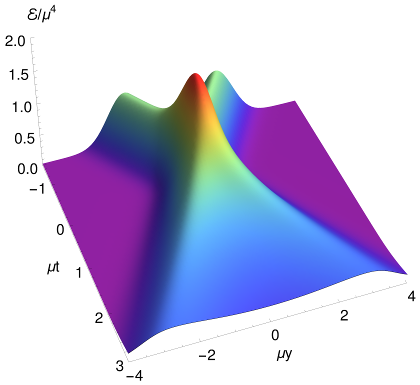

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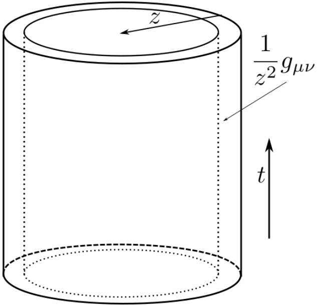
C. Ecker using a notebook by W.v.d. Schee (https://sites.google.com/site/wilkevanderschee/phd-thesis)

compare both approaches to heavy ion collisions

to get both on the same page one employs a geometric quench

$$ds^2 = \frac{1}{z^2}\left[dz^2 + \left(g^{(0)}_{\mu\nu} + z^2 g^{(2)}_{\mu\nu} + z^4 g^{(4)}_{\mu\nu} + z^4 \ln z\ h^{(4)}_{\mu\nu} + \mathcal{O}(z^6, z^6 \ln z)\right) dx^\mu dx^\nu\right]$$



$$g^{(0)}_{\mu\nu} dx^\mu dx^\nu = -dt^2 + dx^2 + dy^2 + dl^2 g(t)$$

$$g(-\infty) \to 1, \quad g(+\infty) \to t^2$$

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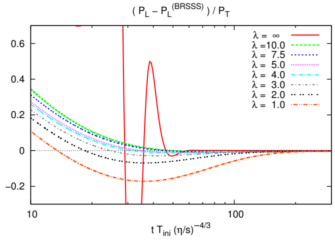

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L. Keegan, A. Kurkela, P. Romatschke, W.v.d. Schee, Y. Zhu, *JHEP* 1604 (2016) 031

But heavy ion collisions don't happen in curved space.

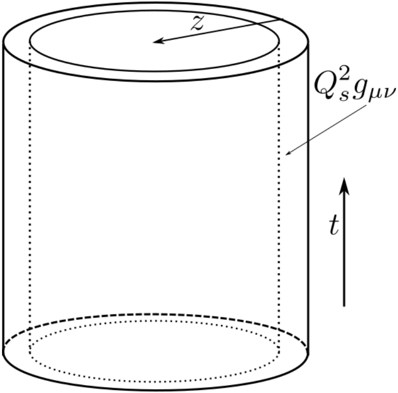
But heavy ion collisions don't happen in curved space.
Right?

semi-holographic approach to heavy ion collisions

the gravitational dual encodes the RG flow

N. Behr, A. Mukhopadhyay, *Phys. Rev. D 94*, 026002 (2016)

$$ds^2 = \frac{1}{z^2}\left[dz^2 + \left(g^{(0)}_{\mu\nu} + z^2 g^{(2)}_{\mu\nu} + z^4 g^{(4)}_{\mu\nu} + z^4 \ln z \; h^{(4)}_{\mu\nu} + \mathcal{O}(z^6, z^6 \ln z)\right) dx^\mu dx^\nu\right]$$



$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{2\pi^2}{N_c^2 Q_s^4} t^{\mathrm{UV}}_{\mu\nu} + \ldots$$

For the course grained $t_{\mu\nu}$ at the scale $z = Q_s^{-1}$

$$\nabla^\mu t^{Q_s}_{\mu\nu} = 0$$

semi-holographic proposal

E. Iancu, A. Mukhopadhyay, *JHEP* 1506 (2015) 003
A. Mukhopadhyay, FP, A. Rebhan, S.A. Stricker *JHEP* 1605 (2016) 141

solve gravity dual with boundary conditions (sources) at $z = 0$ deformed by gauge invariant operators (glasma)

$$\phi^{(b)} = \frac{\beta}{4N_c Q_s^4}\mathrm{Tr}\left(F_{\sigma\tau}F^{\sigma\tau}\right) \quad \chi^{(b)} = \frac{\alpha}{4N_c Q_s^4}\mathrm{Tr}\left(F_{\sigma\tau}\tilde{F}^{\sigma\tau}\right)$$

$$g_{\mu\nu}^{(b)} = \eta_{\mu\nu} + \frac{\gamma}{Q_s^4}\left[\frac{1}{N_c}\mathrm{Tr}\left(F_{\mu\alpha}F_{\nu}{}^{\alpha} - \frac{1}{4}\eta_{\mu\nu}F_{\sigma\tau}F^{\sigma\tau}\right)\right]$$

solve the UV theory deformed by marginal operators obtained from the gravity dual

$$S = -\frac{1}{4N_c}\int \mathrm{Tr}\left(F_{\sigma\tau}F^{\sigma\tau}\right) + W_{\mathrm{CFT}}\left[g_{\mu\nu}^{(b)}, \phi^{(b)}, \chi^{(b)}\right]$$

semi-holographic approach to HICs

equations of motion and conserved energy momentum tesor

$$D_\mu F^{\mu\nu} = \frac{\gamma}{Q_s^4} D_\mu \left(\hat{\mathcal{T}}^{\mu\alpha} F_\alpha{}^\nu - \hat{\mathcal{T}}^{\nu\alpha} F_\alpha{}^\mu - \frac{1}{2} \hat{\mathcal{T}}^{\alpha\beta} \eta_{\alpha\beta} F^{\mu\nu} \right) + \frac{\beta}{Q_s^4} D_\mu \left(\hat{\mathcal{H}} F^{\mu\nu} \right) + \frac{\alpha}{Q_s^4} \left(\partial_\mu \hat{\mathcal{A}} \right) \tilde{F}^{\mu\nu}$$

$$T^{\mu\nu} = t^{\mu\nu} + \hat{\mathcal{T}}^{\mu\nu} - \frac{\gamma}{Q_s^4 N_c} \hat{\mathcal{T}}^{\alpha\beta} \left[\mathrm{Tr}(F_\alpha{}^\mu F_\beta{}^\nu) - \frac{1}{2} \eta_{\alpha\beta} \mathrm{Tr}(F^{\mu\rho} F^\nu{}_\rho) + \frac{1}{4} \delta^\mu_{(\alpha} \delta^\nu_{\beta)} \mathrm{Tr}(F^2) \right] - \frac{\beta}{Q_s^4 N_c} \hat{\mathcal{H}}\, \mathrm{Tr}(F^{\mu\alpha} F^\nu{}_\alpha) - \frac{\alpha}{Q_s^4 N_c} \eta^{\mu\nu} \hat{\mathcal{A}}\, \mathrm{Tr}\left(F_{\sigma\tau} \tilde{F}^{\sigma\tau} \right).$$

$$\partial_\mu T^{\mu\nu} = 0$$

on shell and by the gravitational Ward identities.

Lets get familiar with the model

Simple toy example

Setup

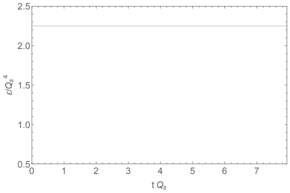
homogeneity, isotropy, $\alpha = \beta = 0$, $N_c = 2$, $A_0^a = 0$, $A_i^a = \delta_i^a f(t)$
$\Rightarrow$ $g_{\mu\nu}^{(b)} = \eta_{\mu\nu} + \frac{\gamma}{Q_s^4} t_{\mu\nu}$ is conformally flat, $p(t) = 1/3\varepsilon(t) = 1/2[f'(t)^2 + f(t)^4]$
$\Rightarrow$ the bulk metric is diffeomorphic to AdS-Schwarzschild with mass c

$$
\begin{aligned}
f''(t) \quad &+ \quad 2\frac{1 - \frac{1}{2}\frac{\gamma}{Q_s^4}(\hat{\mathcal{E}} + \hat{\mathcal{P}})}{1 + \frac{1}{2}\frac{\gamma}{Q_s^4}(\hat{\mathcal{E}} + \hat{\mathcal{P}})} f(t)^3 + \frac{1}{2}\frac{\gamma}{Q_s^4}\frac{(\hat{\mathcal{E}} + \hat{\mathcal{P}})'}{1 + \frac{1}{2}\frac{\gamma}{Q_s^4}(\hat{\mathcal{E}} + \hat{\mathcal{P}})} f'(t) = 0 \\
\hat{\mathcal{E}} + \hat{\mathcal{P}} \quad &= \quad \frac{N_c^2}{2\pi^2}\frac{c}{\sqrt{1 - 3\bar{\gamma}p(t)}[1 + \bar{\gamma}p(t)]^{3/2}} \\
&+ \quad \frac{N_c^2}{2\pi^2}\frac{\bar{\gamma}^3 p'(t)^2 \left(2[1 + \bar{\gamma}p(t)][3\bar{\gamma}p(t) - 1]p''(t) - \bar{\gamma}[1 + 6\bar{\gamma}p(t)]p'(t)^2\right)}{64[1 - 3\bar{\gamma}p(t)]^{5/2}[1 + \bar{\gamma}p(t)]^{7/2}}
\end{aligned}
$$

Simple toy example

Iterative algorithm

Initial values: $f(0) = (2p_0)^{1/4}$,
$f'(0) = 0$



$$
\begin{aligned}
f(t) &= (2p_0)^{1/4} cd(\omega| -1) \quad \omega = (2p_0)^{1/4} \\
&\Downarrow \\
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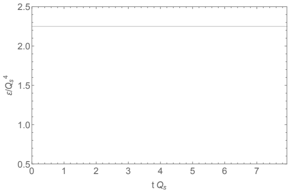

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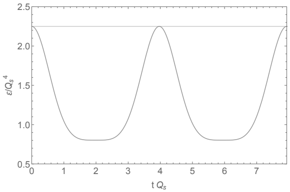

$$f(t) = (2p_0)^{1/4} cd(\omega| - 1) \quad \omega = (2p_0)^{1/4} \left(\frac{1 - \frac{1}{2}\frac{\gamma}{Q_s^4}(\hat{\mathcal{E}}_0 + \hat{\mathcal{P}}_0)}{1 + \frac{1}{2}\frac{\gamma}{Q_s^4}(\hat{\mathcal{E}} + \hat{\mathcal{P}})} \right)^{\frac{1}{2}}$$

$$\Uparrow$$

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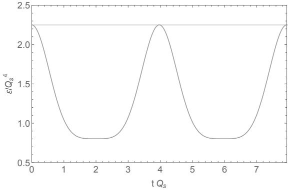

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\hat{\mathcal{E}} + \hat{\mathcal{P}} &= \text{lengthy}
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Simple toy example

Iterative algorithm

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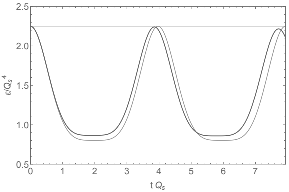

$$
\begin{aligned}
f(t) &= \text{numerical} \\
&\Uparrow \\
\hat{\mathcal{E}} + \hat{\mathcal{P}} &= \text{lengthy}
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$$

Simple toy example

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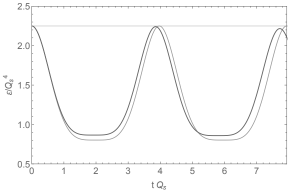

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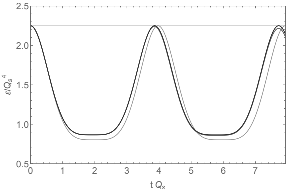

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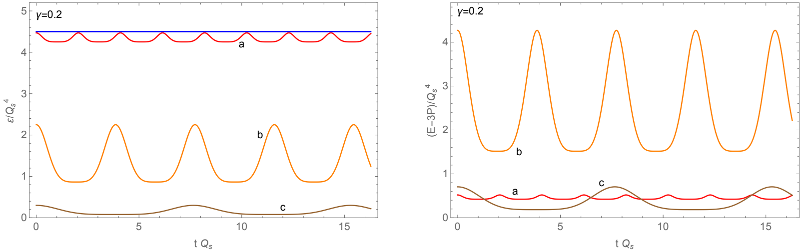
$$\Uparrow$$

$$\hat{\mathcal{E}} + \hat{\mathcal{P}} = \text{numerical}$$

Simple toy example

Results



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Simple toy example

Outlook

study isotropization/thermalization by including anisotropy or the dilaton

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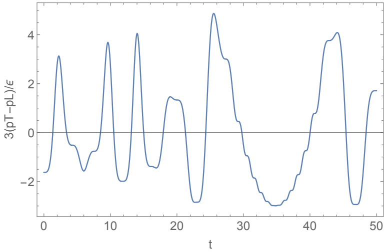

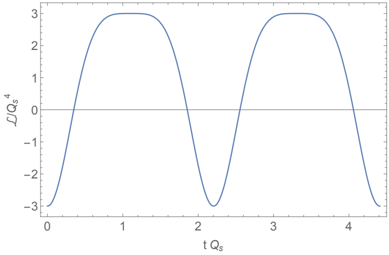

for late times replace glasma by kinetic theory

A. Mukhopadhyay, FP, A. Rebhan, A. Soloviev