

# Jet evolution in dense QCD matter

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THE OHIO STATE UNIVERSITY

Strong and Electroweak Matter 2016



Universitetet  
i Stavanger

# Outline

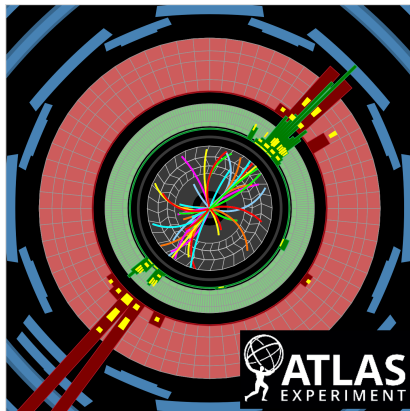
1. **Motivations**
2. **Radiative correction to jet quenching parameter**
3. **Multiple branching and thermalization**
4. **Summary**

Liou, Mueller and BW, Nucl. Phys. A 916 (2013) 102-125;

BW, JHEP **1110**, 029 (2011); JHEP **1412**, 081 (2014);

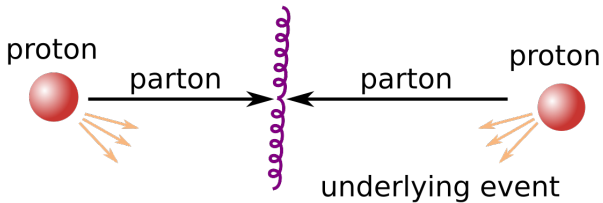
Iancu and BW, JHEP **1510**, 155 (2015).

# 1.1 Jets in proton-proton collisions



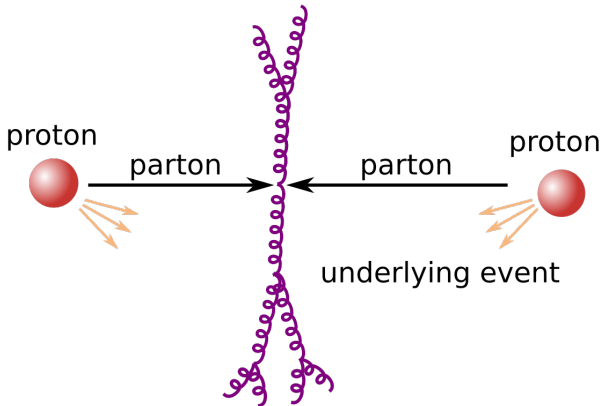
a dijet event recorded by ATLAS at the LHC

# 1.1 Jets in proton-proton collisions



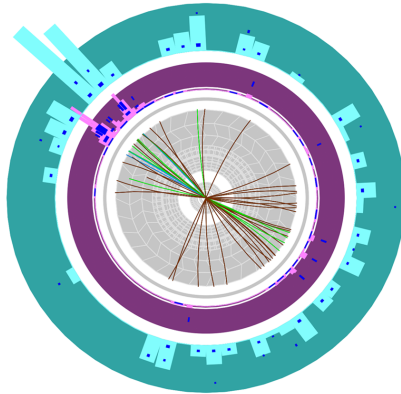
partons in a hard scattering process

# 1.1 Jets in proton-proton collisions



jet evolution in vacuum

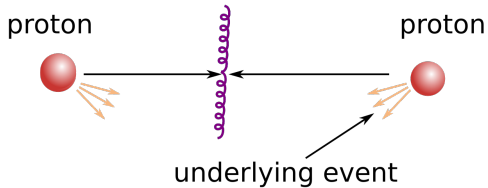
## 1.2 Jets in nucleus-nucleus collisions



an asymmetric dijet event in a PbPb collision

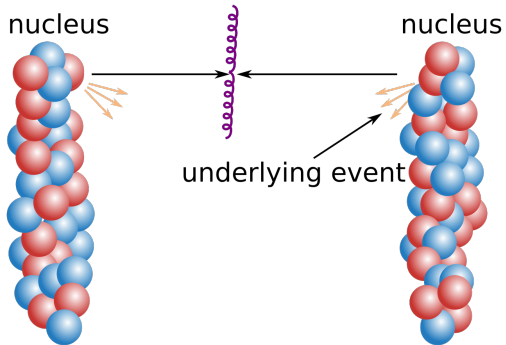
ATLAS, Phys. Rev. Lett. **105**, 252303 (2010).

## 1.2 Jets in nucleus-nucleus collisions



compare to a proton-proton collision

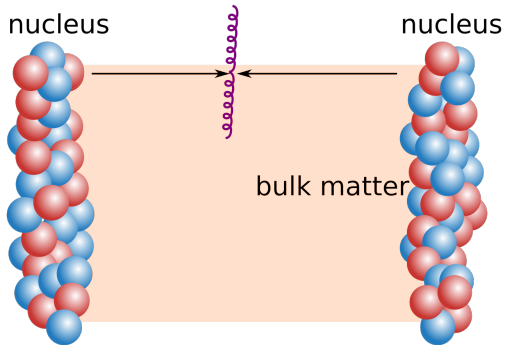
## 1.2 Jets in nucleus-nucleus collisions



partons in a hard scattering process

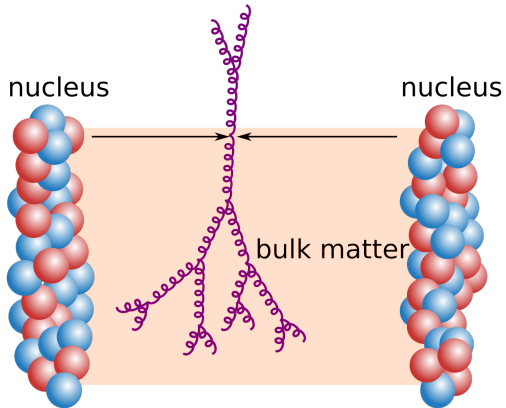


## 1.2 Jets in nucleus-nucleus collisions



underlying event  $\rightarrow$  bulk QCD matter

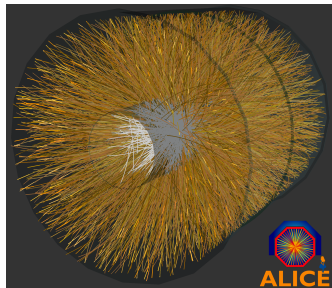
## 1.2 Jets in nucleus-nucleus collisions



jet evolution in bulk QCD matter

## 1.2 Jets in nucleus-nucleus collisions

### Bulk matter in central PbPb collisions



- **Particles produced**

$\approx 25,000$  at  $\sqrt{s_{NN}} = 2.76$  TeV

- **Thermalization**

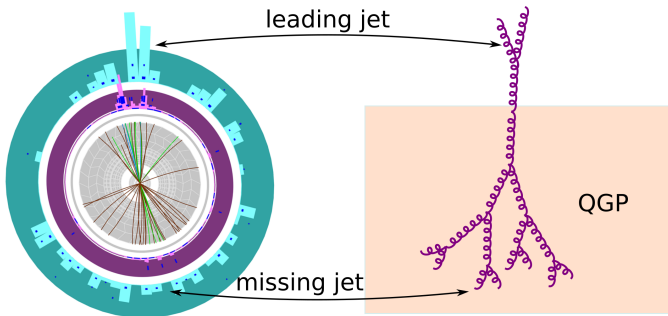
Talks by Venugopalan & Kurkela.

### Idealization

a quark-gluon plasma (QGP) with temperature  $T$

## 1.3 Jets in QCD matter

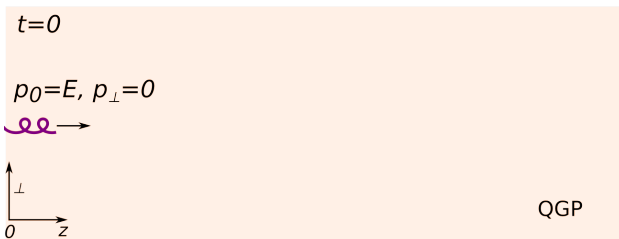
**Aim of this talk:** the entire evolution of a jet in a QGP



**What does this figure mean?**

# 1.3 Jets in QCD matter

What is the fate of a parton in a QGP?



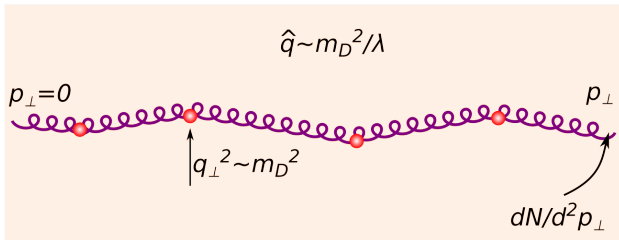
Properties of a hot QGP

$$m_D^2 \sim \alpha_s T^2, \quad \sigma \sim \frac{\alpha_s^2}{m_D^2}, \quad \lambda = \frac{1}{\rho\sigma} \sim \frac{1}{\alpha_s T}.$$

Arnold, Moore & Yaffe (2003).

## 1.3 Jets in QCD matter

### 1. Multiple scattering:

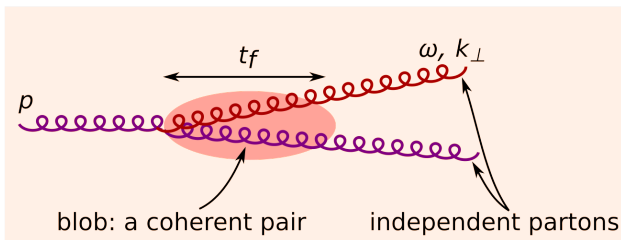


$$\frac{dN}{d^2 p_{\perp}} = \frac{1}{\pi \hat{q} t} e^{-\frac{p_{\perp}^2}{\hat{q} t}} \Rightarrow \langle p_{\perp}^2 \rangle = \hat{q} t$$

Here,  $\hat{q}$  is called **jet quenching parameter**.

# 1.3 Jets in QCD matter

## 2. Gluon radiation



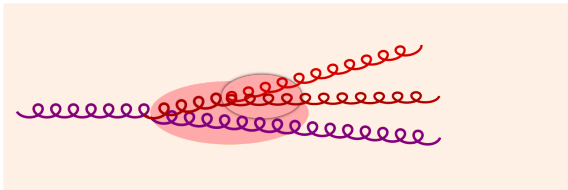
radiation occurs within the formation time

$$t_f = \frac{2\omega}{k_{\perp}^2}$$

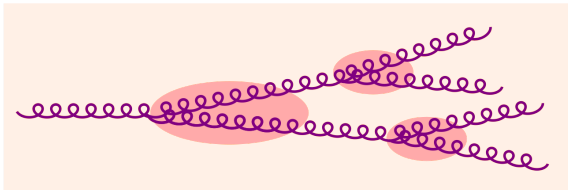
# 1.3 Jets in QCD matter

## Topics of this talk

1. **Fully-overlapping emission:** radiative correction to  $\hat{q}$



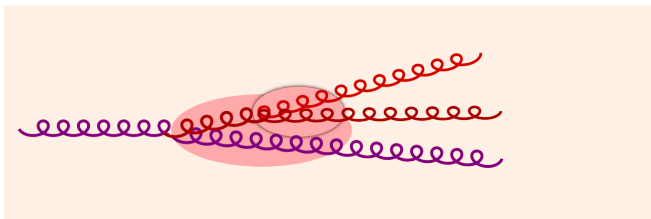
2. **Independent emission:** multiple branching & thermalization





## 2 Radiative correction to $\hat{q}$

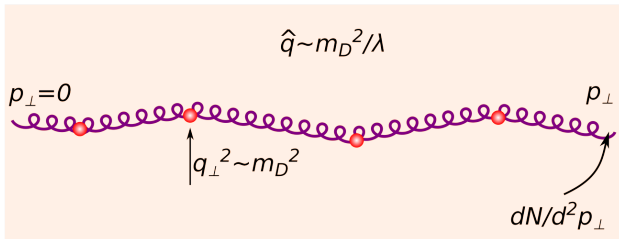
$p_{\perp}$ -broadening: diffusion + the recoil of gluon radiation



Liou, Mueller and BW (2013); BW (2011, 2014).

## 2 Radiative correction to $\hat{q}$

**Setup the problem:** calculate typical  $\langle p_{\perp}^2 \rangle$

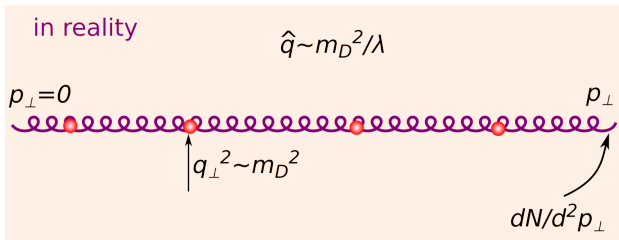


$$\frac{dN}{d^2p_{\perp}} = \frac{1}{\pi \langle p_{\perp}^2 \rangle} e^{-\frac{p_{\perp}^2}{\langle p_{\perp}^2 \rangle}} \quad \text{with} \quad \langle p_{\perp}^2 \rangle = \hat{q}t$$

**Multiple soft scatterings ( $m_D \ll E$ )**

## 2 Radiative correction to $\hat{q}$

**Setup the problem:** calculate typical  $\langle p_{\perp}^2 \rangle$

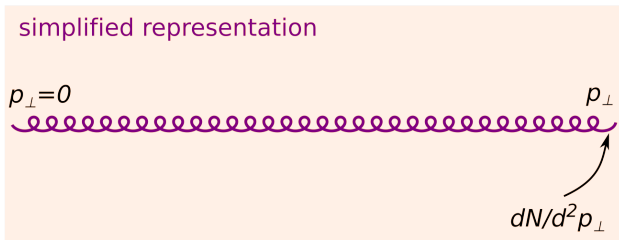


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**Small angle scatterings**

## 2 Radiative correction to $\hat{q}$

**Setup the problem:** calculate typical  $\langle p_{\perp}^2 \rangle$

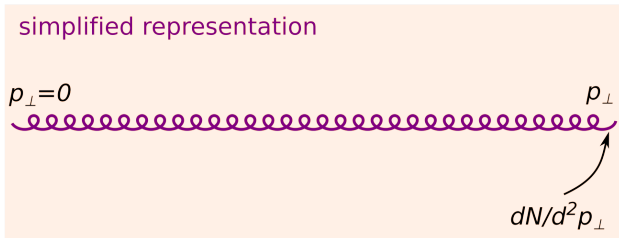


$$\frac{dN}{d^2 p_{\perp}} = \frac{1}{\pi \langle p_{\perp}^2 \rangle} e^{-\frac{p_{\perp}^2}{\langle p_{\perp}^2 \rangle}} \quad \text{with } \langle p_{\perp}^2 \rangle = \hat{q}t$$

**Multiple soft scatterings**

## 2 Radiative correction to $\hat{q}$

**Setup the problem:** calculate typical  $\langle p_{\perp}^2 \rangle$

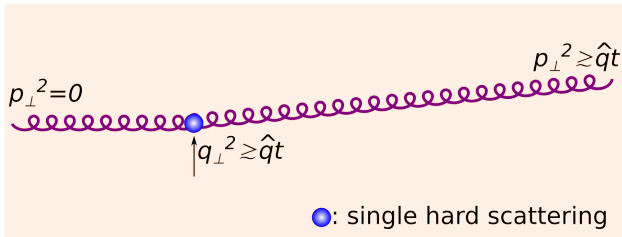


$$\frac{dN}{d^2 p_{\perp}} = \frac{1}{\pi \langle p_{\perp}^2 \rangle} e^{-\frac{p_{\perp}^2}{\langle p_{\perp}^2 \rangle}} \quad \text{with } \langle p_{\perp}^2 \rangle = \hat{q}t$$

**What is the contribution from recoil of gluon radiation?**

## 2 Radiative correction to $\hat{q}$

Missing effect #1: (rare) single hard scattering

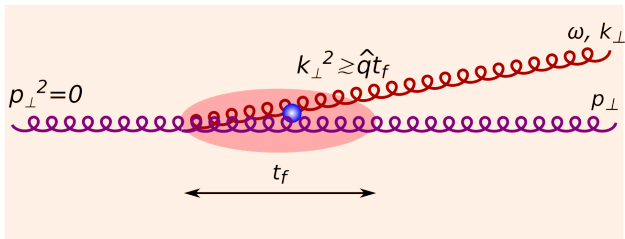


$$\frac{dN}{d^2p_{\perp}} \approx \frac{t}{\lambda} \frac{1}{\sigma} \frac{d\sigma}{d^2p_{\perp}} \propto p_{\perp}^{-4} \quad \text{for } p_{\perp}^2 \gtrsim \hat{q}t$$

Does single scattering play any important role?

## 2 Radiative correction to $\hat{q}$

Single scattering of the coherent pair within  $t_f$

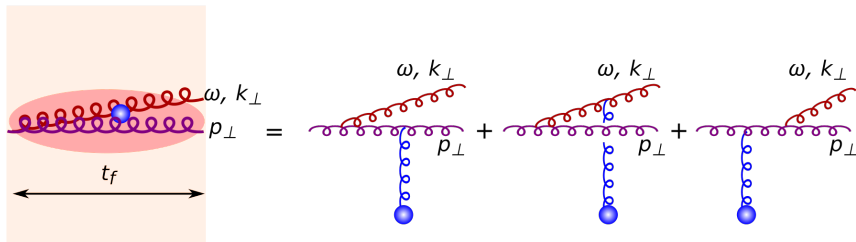


- kinematic region ( $t_f = \frac{2\omega}{k_{\perp}^2}$ )

$$k_{\perp}^2 \gtrsim \hat{q} t_f$$

## 2 Radiative correction to $\hat{q}$

Single scattering of the coherent pair within  $t_f$



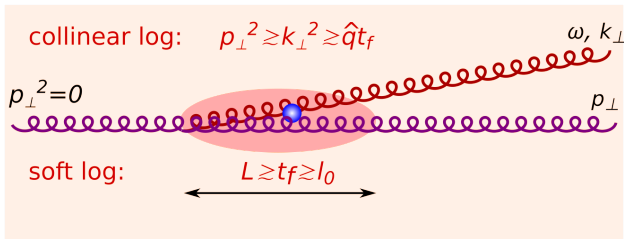
Soft & collinear divergences

$$\langle p_{\perp}^2 \rangle = \alpha \hat{q} L \underbrace{\int \frac{d\omega}{\omega}}_{\text{soft}} \underbrace{\int \frac{dk_{\perp}^2}{k_{\perp}^2}}_{\text{collinear}} \quad \text{with } \alpha \equiv \frac{\alpha_s N_c}{\pi}.$$



## 2 Radiative correction to $\hat{q}$

Single scattering of the coherent pair within  $t_f$

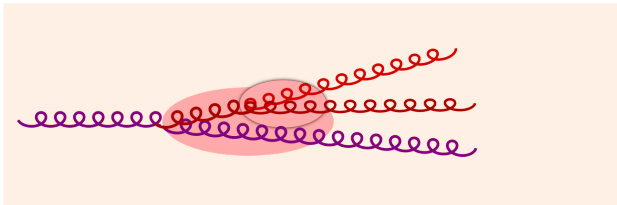


Double logarithmic enhanced contribution

$$\langle p_{\perp}^2 \rangle_{rad} = \alpha \hat{q} L \int_{l_0}^L \frac{dt_f}{t_f} \int_{\hat{q} z}^{\hat{q} L} \frac{dk_{\perp}^2}{k_{\perp}^2} = \frac{\alpha}{2} \hat{q} L \ln^2 \frac{L}{l_0}$$

## 2 Radiative correction to $\hat{q}$

**Leading log resummation:** arbitrary  $n$ -gluon emission



$$\langle p_{\perp}^2 \rangle_{tot} \approx \hat{q}L \sum_{n=0}^{\infty} \frac{(\alpha \ln^2 \frac{L}{l_0})^n}{(n+1)!n!} = \hat{q}L \frac{I_1(2\sqrt{\alpha} \ln \frac{L}{l_0})}{(\sqrt{\alpha} \ln \frac{L}{l_0})}$$

**The lead log result of average energy loss**

$$\Delta E_{tot} \approx \frac{\alpha_s N_c}{12} \langle p_{\perp}^2 \rangle_{tot} L$$

## 2 Radiative correction to $\hat{q}$

### In summary

$$\frac{dN}{d^2p_{\perp}} = \frac{1}{\pi \langle p_{\perp}^2 \rangle_{tot}} e^{-\frac{p_{\perp}^2}{\langle p_{\perp}^2 \rangle_{tot}}} \text{ with } \langle p_{\perp}^2 \rangle_{tot} \approx \hat{q}L \left[ 1 + \frac{\alpha_s N_c}{2\pi} \ln^2 \frac{L}{l_0} \right]$$

**Physical interpretation:** renormalized  $\hat{q}$

$$\hat{q} \rightarrow \hat{q}_{tot} = \frac{\langle p_{\perp}^2 \rangle_{tot}}{L}$$

**One gluon spectrum shall be modified accordingly.**

Blaizot & Mehtar-Tani (2014); Iancu (2014).

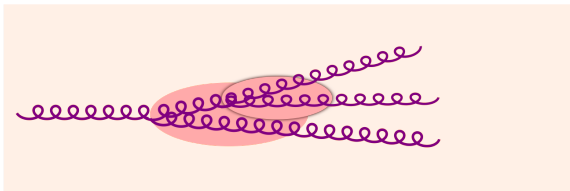
## 2 Radiative correction to $\hat{q}$

### Further discussions

- **Running coupling effects:** see also [Iancu & Triantafyllopoulos \(2014\)](#)

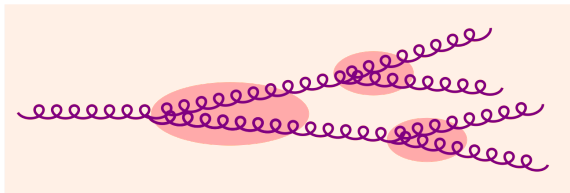
$$\hat{q}_{tot} \approx \hat{q} \left[ 1 + \frac{\alpha_s(Q_s^2) N_c}{\pi} \ln^2 \frac{L}{l_0} \right] \quad \text{with } Q_s^2 \equiv \hat{q}L.$$

- **Interplay with vacuum double log:** [Mueller, BW, Xiao & Yuan \(2016\)](#)
- **Partially overlapping emission:** [Arnold, Chang & Iqbal \(2015-2016\)](#)



### 3 Multiple branching and thermalization

**Jet evolution:** a game of diffusion, drag and branching

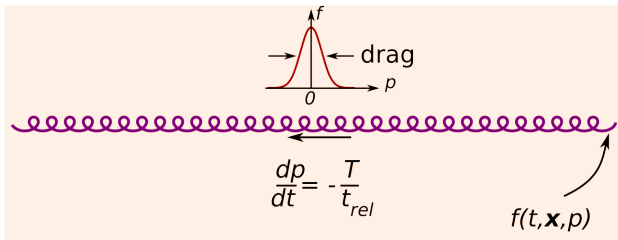


**Focus on distribution in the longitudinal phase space!**

Iancu and BW (2015).

## 3.1 Diffusion and drag

### Missing effect # 2: drag

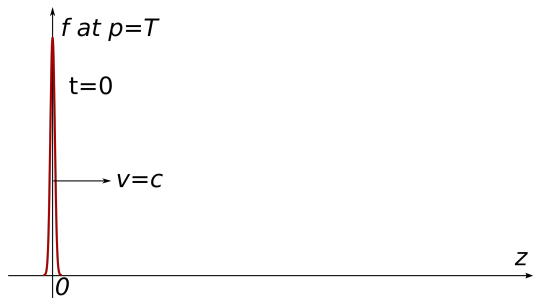


- **The "drag" time:** time for a gluon with momentum  $p$  to stop

$$t_{\text{drag}}(p) \equiv \frac{p}{T} t_{\text{rel}} \quad \text{with the relaxation time } t_{\text{rel}} = \frac{4T^2}{\hat{q}}$$

## 3.1 Diffusion and drag

What happens to a gluon initially with  $p = T$ ?



- at  $t = 0$

$$f = \delta(z)\delta(p - T)$$

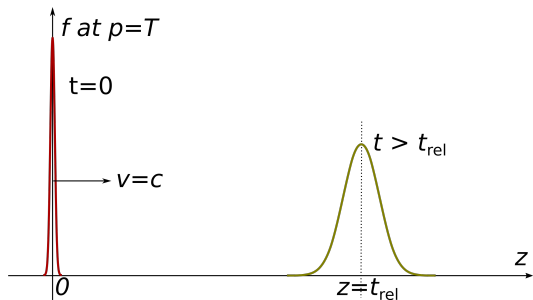
- at  $t > t_{\text{rel}} = t_{\text{drag}}$

$$f \simeq \underbrace{\frac{e^{-\frac{|p|}{T}}}{2}}_{\text{thermal}} \underbrace{\frac{e^{-\frac{(z-t_{\text{rel}})^2}{4t}}}{2\sqrt{\pi t/t_{\text{rel}}}}}_{\text{diffusion in space}}$$

**It relaxes into local thermal equilibrium within  $\sim t_{\text{rel}}$ .**

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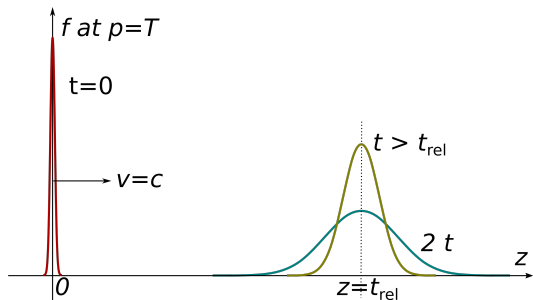
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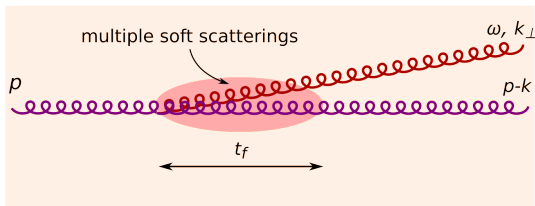
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## 3.2 Branching

### The Landau-Pomeranchuk-Migdal (LPM) effect



Formation time:

$$t_f(\omega) = \frac{2\omega}{k_{\perp}^2} \simeq \frac{\omega}{\hat{q}t_f}$$
$$\Leftrightarrow$$
$$t_f \simeq \sqrt{\frac{\omega}{\hat{q}}}$$

The branching time:  $t_{\text{br}}(\omega) \equiv \frac{t_f(\omega)}{\alpha} \simeq \frac{1}{\alpha} \sqrt{\frac{\omega}{\hat{q}}}$

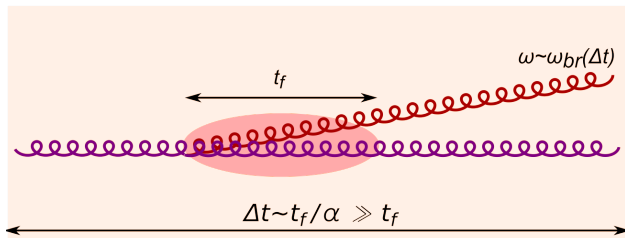
Probability for emitting a gluon with energy  $\omega$  within  $t$

$$P(\omega, t) \sim \frac{t}{t_{\text{br}}(\omega)} \simeq \alpha \sqrt{\frac{\omega t^2}{\hat{q}}}$$

Baier, Dokshitzer, Mueller, Peigne & Schiff & Zakharov (1996-1998).

## 3.2 Branching

Most probable radiation pattern within  $\Delta t$



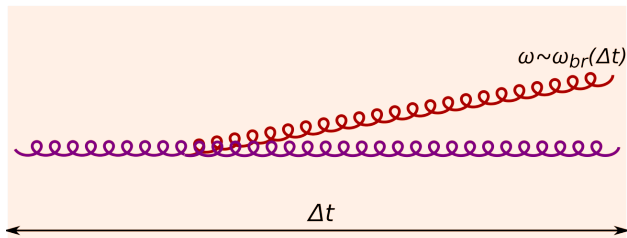
Radiated gluon with  $\sim \omega_{br}(\Delta t) \equiv \alpha^2 \hat{q} \Delta t^2$

probability  $\sim \Delta t / t_{br}(\omega_{br}) \sim 1$

Baier, Dokshitzer, Mueller & Schiff (2001).

## 3.2 Branching

Most probable radiation pattern within  $\Delta t$



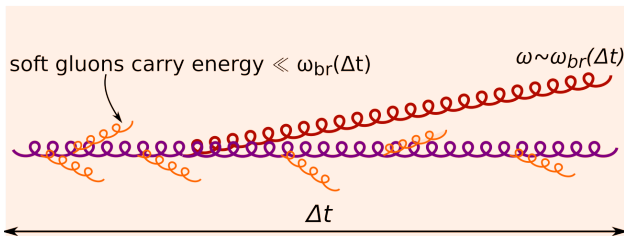
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Most probable radiation pattern within  $\Delta t$

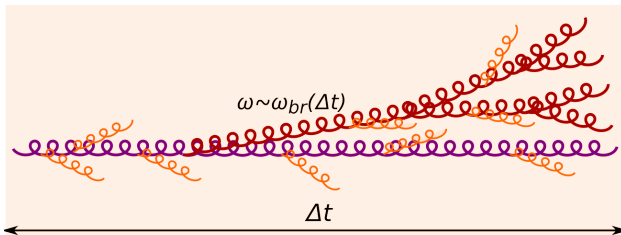


$\Rightarrow$  **Typical energy loss**  $\sim \omega_{\text{br}}(\Delta t) \equiv \alpha^2 \hat{q} \Delta t^2$

Baier, Dokshitzer, Mueller & Schiff (2001).

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Most probable radiation pattern within  $\Delta t$

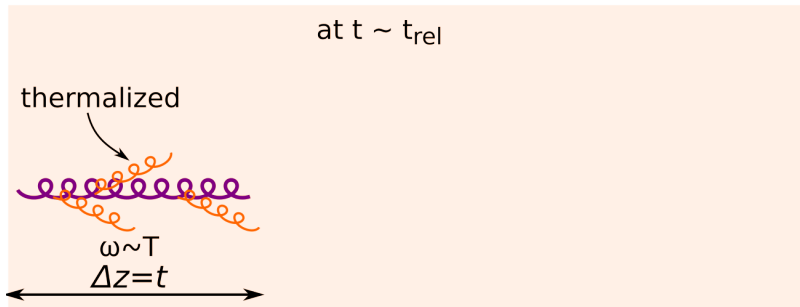


$\Rightarrow$  **Typical energy loss**  $\sim \omega_{br}(\Delta t) \equiv \alpha^2 \hat{q} \Delta t^2$

Blaizot, Iancu & Mehtar-Tani (2013); Fister & Iancu (2015).

### 3.4 At $t < t_{\text{br}}(E)$

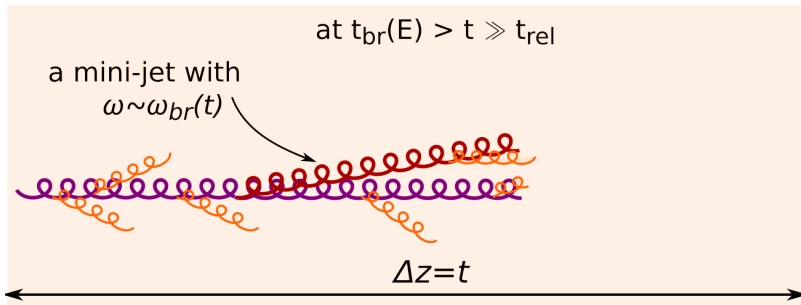
**Thermalization of mini-jets with  $\omega \lesssim \omega_{\text{br}}(t) < E$**



Gluons with  $\omega \sim T$  are radiated and thermalized within  $\sim t_{\text{rel}}$ .

### 3.4 At $t < t_{\text{br}}(E)$

**Thermalization of mini-jets with  $\omega \lesssim \omega_{\text{br}}(t) < E$**

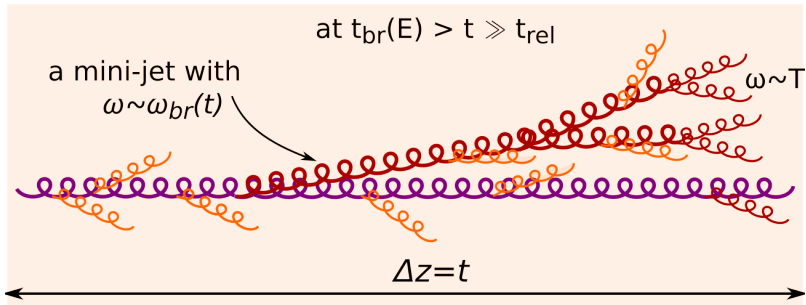


Within  $t$ , a mini-jet (a gluon with  $\omega \sim \omega_{\text{br}}(t)$ ) is emitted.



### 3.4 At $t < t_{br}(E)$

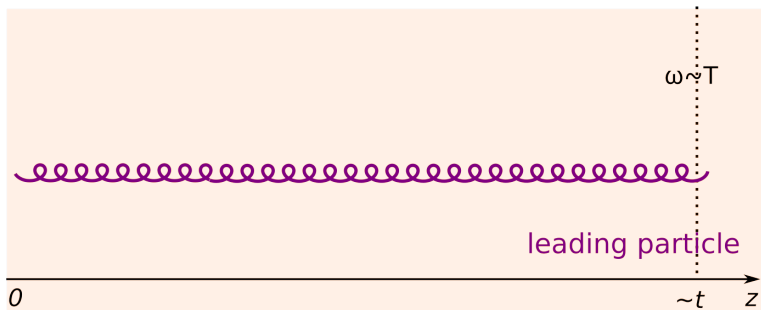
**Thermalization of mini-jets with  $\omega \lesssim \omega_{br}(t) < E$**



The mini-jet branches into soft gluons with  $\omega \sim T$  within  $\sim t$ .

### 3.4 At $t < t_{\text{br}}(E)$

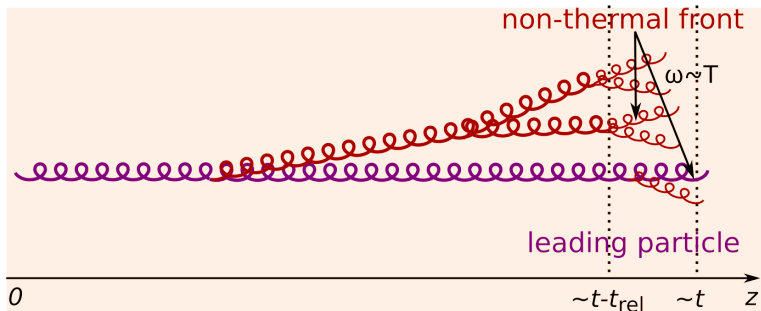
#### Categorization of gluons:



non-thermal front with **leading particle** and thermal tail

### 3.4 At $t < t_{\text{br}}(E)$

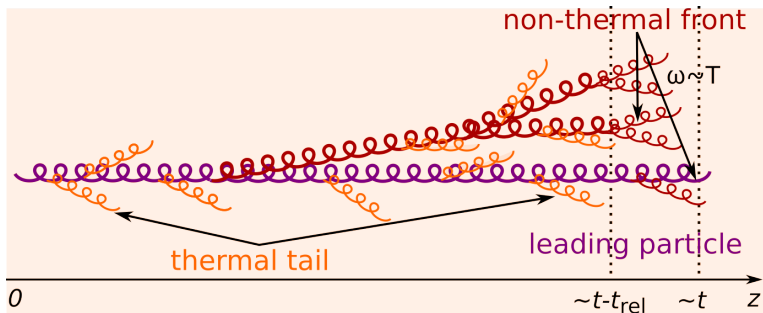
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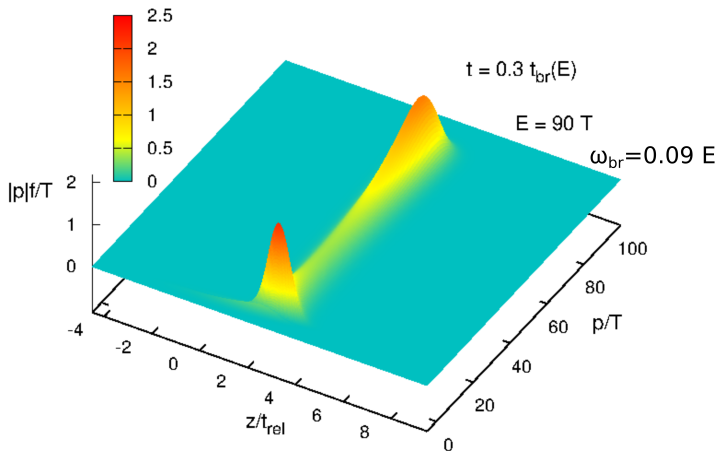
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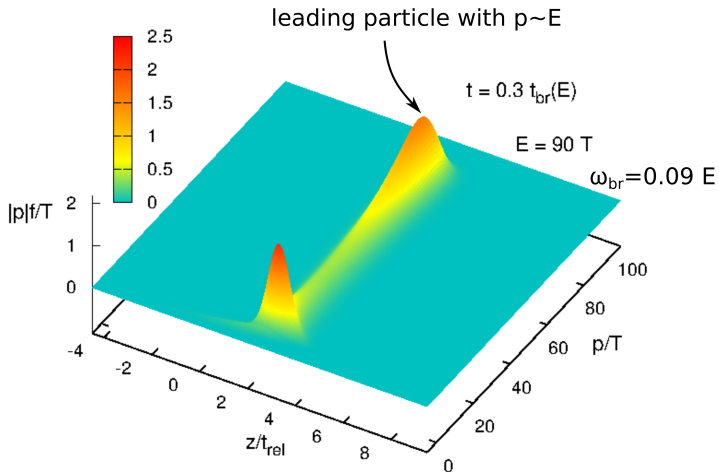
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The distribution  $f$  at  $t = 0.3t_{\text{br}}(E)$  with  $E = 90 T$



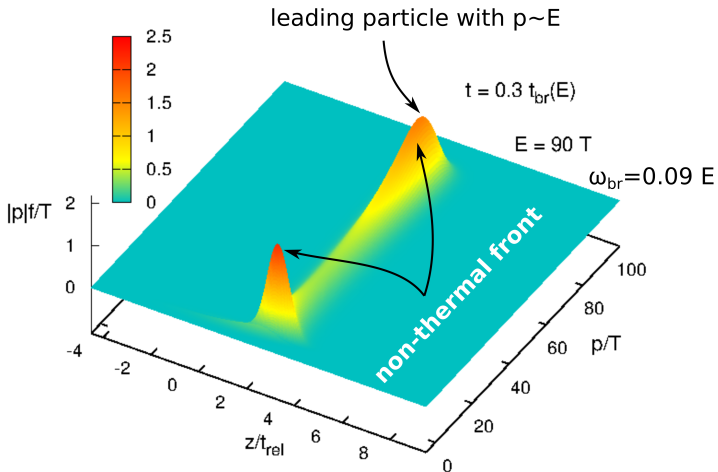
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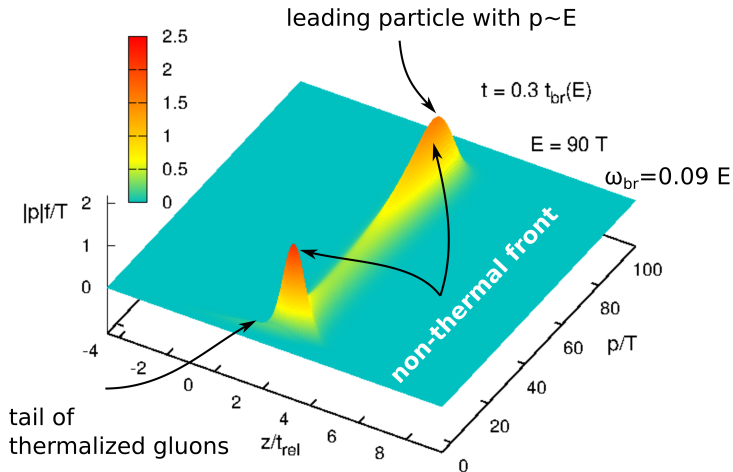
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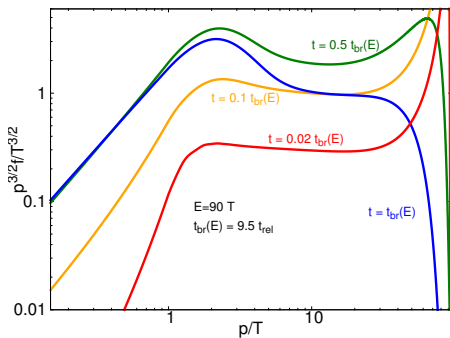
The distribution  $f$  at  $t = 0.3 t_{\text{br}}(E)$  with  $E = 90 T$





### 3.4 At $t < t_{\text{br}}(E)$

Leading particle peak at  $z = t$

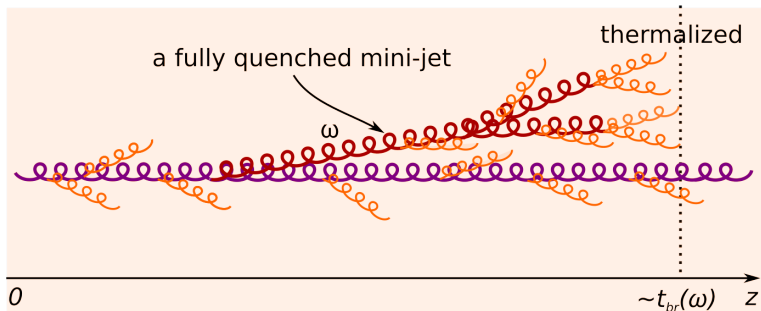


**Scaling behavior  $f \propto p^{-\frac{3}{2}}$  exists at  $z \simeq t \lesssim 0.5 t_{\text{br}}(E)$**

See also: Mueller, Schiff & Son (2001); Blaizot, Iancu & Mehtar-Tani (2013); Kurkela & Lu (2014).

### 3.4 At $t < t_{\text{br}}(E)$

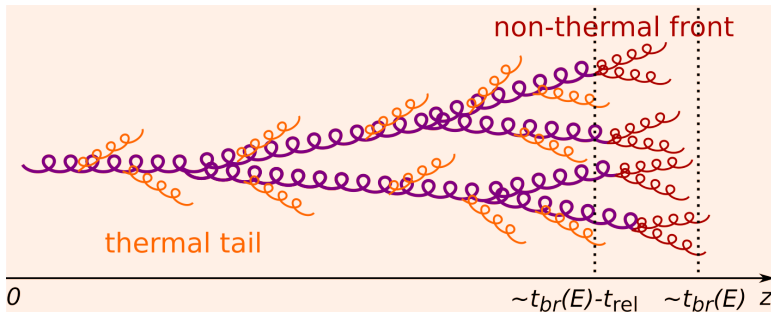
What happens to the mini-jet afterwards?



All its branching products thermalize at  $z \sim t_{\text{br}}(\omega)$ .

### 3.5 At $t \simeq t_{\text{br}}(E)$

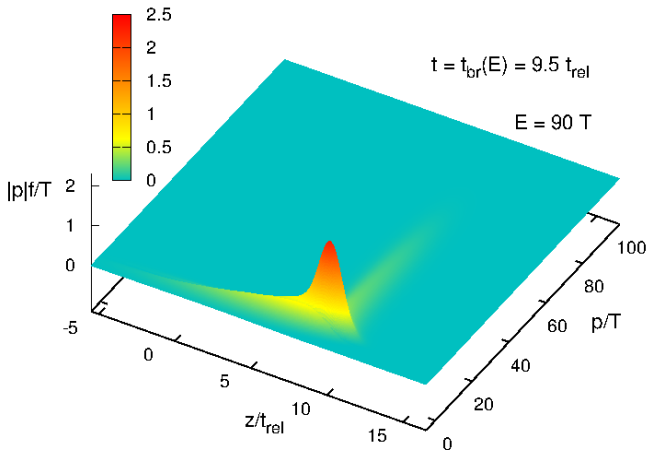
The gluon itself is a mini-jet:



**non-thermal front and thermal tail.**

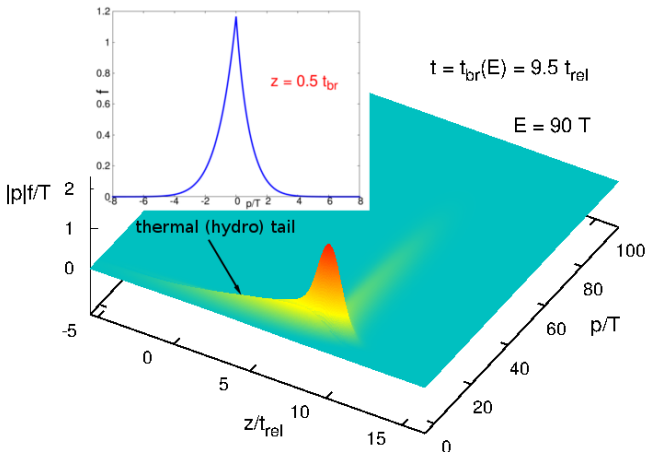
### 3.5 At $t \simeq t_{\text{br}}(E)$

Most gluons with  $p \sim T$ : non-thermal front + thermal tail



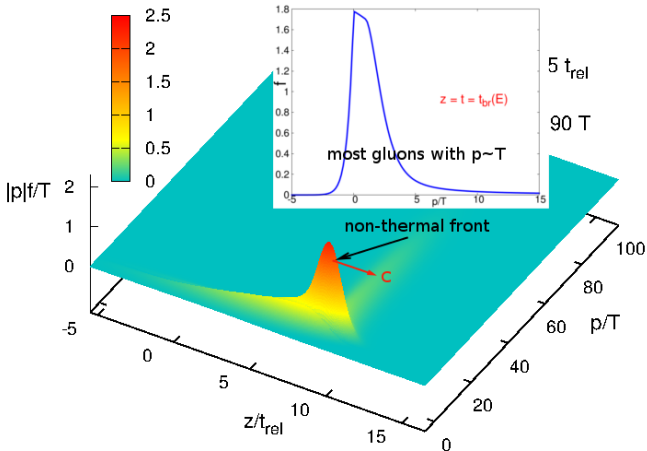
### 3.5 At $t \simeq t_{\text{br}}(E)$

Most gluons with  $p \sim T$ : non-thermal front + thermal tail



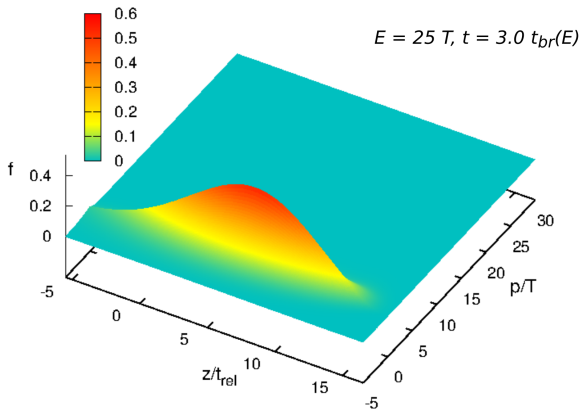
### 3.6 At $t \simeq t_{\text{br}}(E)$

Most gluons with  $p \sim T$ : non-thermal front + thermal tail



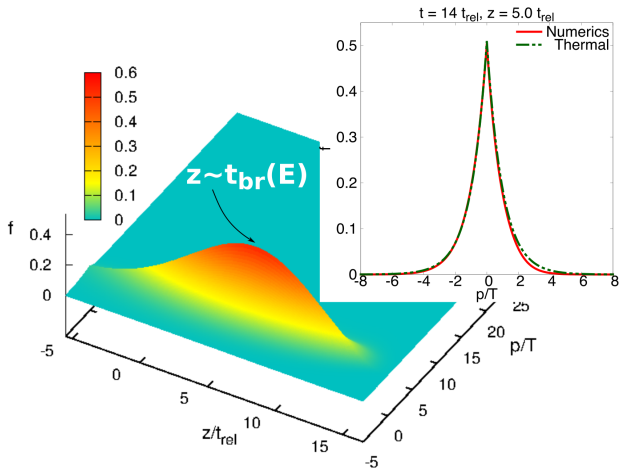
### 3.6 At $t > t_{br}(E)$

The jet itself is a fully quenched mini-jet.



### 3.6 At $t > t_{br}(E)$

The jet itself is a fully quenched mini-jet.

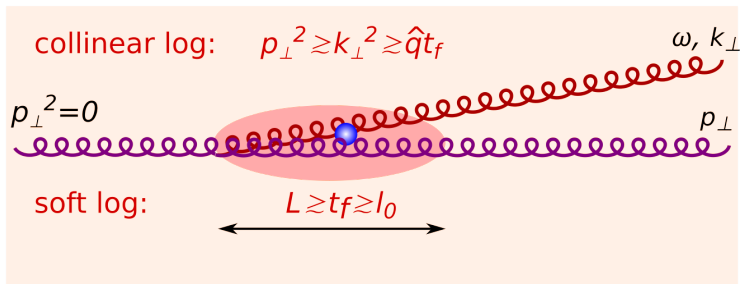




# Summary

## Radiative correction to $\hat{q}$

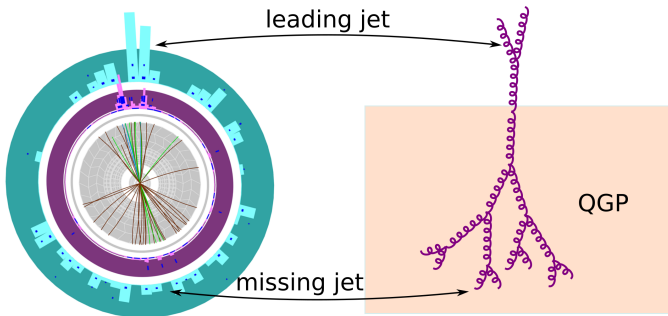
$$\hat{q}_{tot} \approx \hat{q} \sum_{n=0}^{\infty} \frac{(\alpha \ln^2 \frac{L}{l_0})^n}{(n+1)!n!}$$



$$\hat{q}_{tot} \approx 1.8\hat{q} \text{ for } \alpha_s = 1/3, L = 5 \text{ fm} \ \& \ 1/l_0 = 0.3 \text{ GeV.}$$

## 1.3 Jets in QCD matter

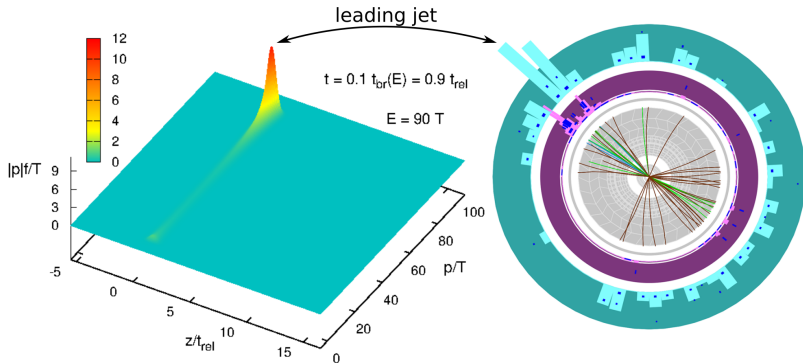
**Aim of this talk:** the entire evolution of a jet in a QGP



**What does this figure mean?**

# Summary

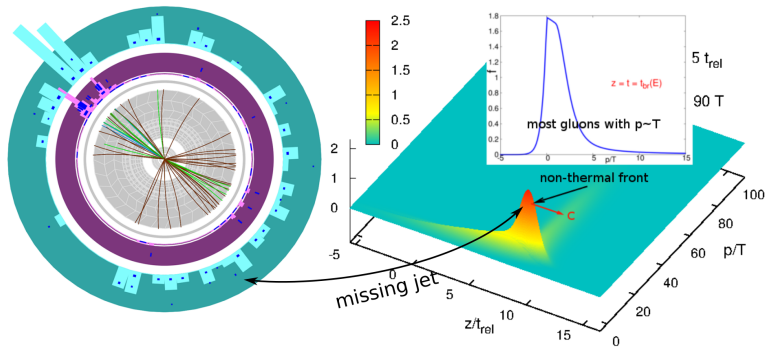
## The leading jet



$$\text{path length } L \ll t_{\text{br}}(E) \equiv \frac{1}{\alpha} \sqrt{E/\hat{q}}$$

# Summary

## The missing recoiling jet



$$\text{path length } L \simeq t_{br}(E) \equiv \frac{1}{\alpha} \sqrt{E/\hat{q}}$$