

Electroweak physics in hadronic collisions

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Some useful references

- C. Roda's Hasco 2015 lectures: [Day 1](#) and [Day 2](#)
- [P. Layfer's lecture](#) with $X = 1, \dots, 18$
- J. Kopp's [Quantum Field Theory Lecture notes](#)
- [ATLAS](#) and [CMS](#) Standard Model Physics results.
- Particle Data Group publication and [web](#)

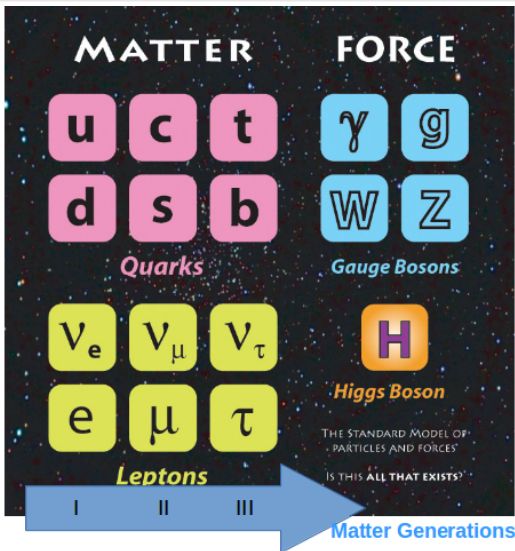
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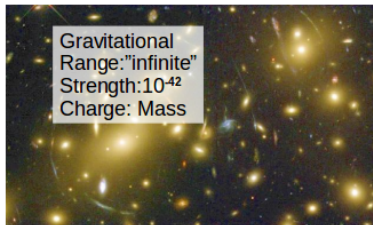
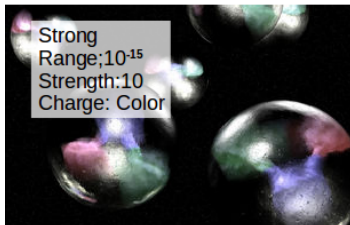
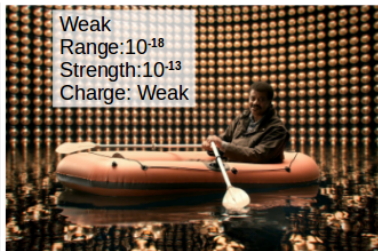
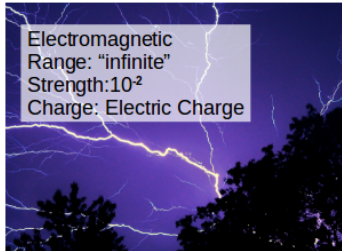
The Standard Model

- **The Standard Model** → **The Standard Model of Particle Physics**
- Many lectures in this school are about **Standard Model**: QCD from C. Doglioni, Top from E. Yazgan and Higgs from A. Knue
- Focus on **Electroweak Physics**
- **SM** is a very well assessed theory and, so far, very much in agreement with experimental measurements. So why bother with that?
 - “Laboratory” where to watch Quantum Field Theory in action!
 - All SM processes are the background for Beyond Standard Model searches ⇒ better know your enemy!
 - Perform precision measurements to find discrepancies with precise theory predictions ⇒ Indirect hints of Beyond Standard Model Physics.

The Standard Model: Quarks, Leptons and Interactions



Four Interactions



ElectroWeak Theory

Electro

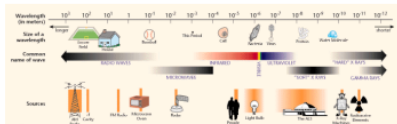
Weak



$$\begin{aligned}\nabla \cdot \mathbf{E} &= \frac{\rho}{\epsilon_0} \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} &= \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}\end{aligned}$$



Solution of Maxwell equations in vacuum is an oscillating field.
Electromagnetic radiation
LIGHT



Unification of electricity and magnetism

- Electric and magnetic forces are caused by the same **fields**
- Electromagnetism

by Maxwell mid-19th century

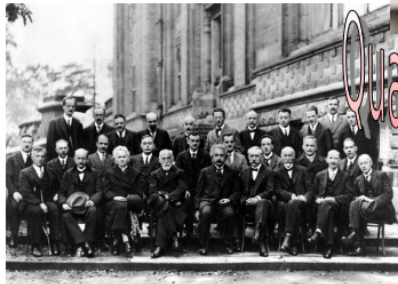
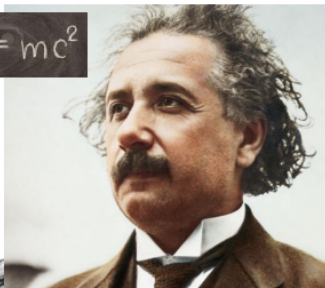
Introduction of **fields** (scalar, pseudoscalars, vectorial, axial)

- Allows separation of the object that produced the force, with the object that feels it

XXth Century Revolutions

Relativity (Special)

$$E=mc^2$$



Quantum Mechanics

$$i\hbar \frac{\partial}{\partial t} \Psi = H\Psi$$

Special Relativity

- Postulates: Speed of light in vacuum is a **universal** constant **AND** all inertial reference frame systems are equivalent.
 - Space and time are “categories” that mix together.
 - Four-dimensional space-time: $(\vec{x}, t) \rightarrow (x_0, x_1, x_2, x_3) \rightarrow x$
 - Changing reference frame: contraction of length, expansion of time intervals
 - Mass and energy transform one in the other
 - speed of light in vacuum is a universal constant
- ⇒ Classical physics has to be modified when objects travel close to speed light.

$$E = mc^2$$

Electromagnetism

Maxwell equations **do NOT seem to be relativistic invariant** (Lorentz invariant).

Use potentials $A(\vec{x}, t)$, $A(x)$ instead of Electrical $\vec{E}(\vec{x}, t)$ and Magnetic $\vec{B}(\vec{x}, t)$ field.

$$\vec{E}(\vec{x}, t) = -\vec{\nabla}V(\vec{x}, t) - \frac{\partial \vec{A}(\vec{x}, t)}{\partial t},$$

$$\vec{B}(\vec{x}, t) = \vec{\nabla} \times \vec{A}(\vec{x}, t)$$

Covariant indices: $\mu = 0, 1, 2, 3$

Relativistic potential $A^\mu = (V, \vec{A})$

Relativistic current $J^\mu = (\rho, \vec{J})$

Electromagnetic strength field tensor

F

$$F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}, \tilde{F}^{\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

Covariant form of Maxwell equations:

$$\partial_\mu \tilde{F}^{\mu\nu} = 0 \quad \partial_\mu F^{\mu\nu} = J^\nu.$$

Gauge Transformations

Conservation of electromagnetic current:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot J = 0 \Rightarrow \partial_\nu J^\nu = 0;$$

$$\partial_\nu \partial_\mu F^{\mu\nu} = 0$$

In terms of potential A , $\partial_\mu F^{\mu\nu} = J^\nu$ becomes:

$$\square A^\nu - \partial^\nu (\partial_\mu A^\mu) = J^\nu$$

Note that the same dynamic can be described by the different potentials. Same field strength tensor $F^{\mu\nu}$ but different potentials A provided that:

$$A^\mu \rightarrow A'^\mu = A^\mu + \partial_\mu \Lambda$$

This is called gauge invariance.

We can choose the Lorentz gauge $\partial_\mu A^\mu = 0$ such that:

$$\square A^\nu - \partial^\nu (\partial_\mu A^\mu) = J^\nu \rightarrow \square A^\nu = J^\nu$$

Quantum Mechanics

Quantum Mechanics Göttingen is one of the birthplaces of QM!

- Microscopic world requires a different kind of model to describe reality.
- In particular wave \leftrightarrow particle dualism:
 - Light is made both of wave (interference, diffraction) and particle (photoelectric effect. Photon quanta of energy $E = h\nu$)
 - Matter is also wave with wave length $\lambda = \hbar/p \Rightarrow$ matter also shows typical quantum mechanical behavior. (e.g. interference and diffraction of electrons).

How to describe both particle and wave behavior of and electron?

Schrödinger Equation

De Broglie proposed to describe electrons with momentum p with a wave with wavelength (De Broglie wavelength 1923):

$$\lambda = \frac{\hbar}{p}$$

In 1926 Schrödinger proposed a mathematical approach to describe a particle with momentum p and mass m evolving in a potential $V(\vec{r}, t)$.

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi(\vec{r}, t) + V(\vec{r}, t) \psi(\vec{r}, t)$$

Schrödinger Equation

What is $\psi(\vec{r}, t)$?

Wave Function

Max Born (Göttingen again!) proposed that the square of $\psi(\vec{r}, t)$ represents the probability to find a particle in a definite state:

$$\int_r^{r+dr} |\psi(\vec{r}, t)|^2 = \text{Prob.}(\text{find a particle} \in [r, r + dr], t)$$

Deterministic (Classic) \Rightarrow **Probabilistic (quantum)**

However note that Schrödinger equation is NOT relativistic invariant

How to deal with fast and microscopic particles?

Klein-Gordon Equation

Reminder: Relativistic motion for particle of mass m and momentum \vec{p}
(using natural units $\hbar = c = 1$)

$$E^2 - |\vec{p}|^2 = m^2 = p_\mu p^\mu$$

Quantum Mechanics substitution:

$$E \rightarrow i\hbar \frac{\partial}{\partial t} \quad \text{and} \quad \vec{p} \rightarrow i\hbar \vec{\nabla} \quad \Rightarrow \quad p_\mu \rightarrow i\hbar \partial_\mu$$

gives the Klein-Gordon equation:

$$\left(-\frac{\partial^2}{\partial t^2} + \nabla^2 \right) \psi = m^2 \psi \quad (\square^2 - m^2)\psi = 0$$

Solutions:

$$\psi(x_\mu) \propto e^{-ip_\mu x^\mu} = e^{-i(Et - \vec{p} \cdot \vec{x})}$$

Positive and **Negative** energy solution. **What are “negative” energy solutions?**

$$E = \pm \sqrt{p^2 + m^2}$$

Dirac equation: Consequences

In 1928 **P.A.M. Dirac** first successful attempt to put together special relativity and quantum mechanics. Trying to get “the square root” of Klein-Gordon Equation:

$$(\square^2 - m^2)\psi = 0 \rightarrow \left(i\gamma^0 \frac{\partial}{\partial t} + i\vec{\gamma}\vec{\nabla} - m \right) \psi = 0$$

$$\Rightarrow ((\square^2 - m^2)\psi = (i\gamma^\mu \partial_\mu - m)\psi = 0$$

γ^μ cannot be *simple* numbers (e.g. scalars). They have to satisfy:

$$\left(-i\gamma^0 \frac{\partial}{\partial t} + i\vec{\gamma}\vec{\nabla} - m \right) \left(i\gamma^0 \frac{\partial}{\partial t} - i\vec{\gamma}\vec{\nabla} - m \right) = 0$$

Therefore, they have to satisfy:

$$(\gamma^0)^2 = 1 \quad (\gamma^1)^2 = (\gamma^2)^2 = (\gamma^3)^2 = -1 \quad \text{Unitarity}$$

$$\gamma^i \gamma^j + \gamma^j \gamma^i = 0 \quad i \neq j \quad \text{anticommutation}$$

$$\Rightarrow \gamma^i \gamma^j = g^{ij}$$

$$[\gamma^i, \gamma^j] = 2\sigma^{ij}$$

Dirac equation: Solutions

The simplest solution is with $D = 4$ (i.e. $\psi = (\psi_0, \psi_1, \psi_2, \psi_3)$). (D is **NOT** the space-time dimension) \Rightarrow *spinor* has 4 components. And:

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{I} \\ \mathbb{I} & 0 \end{pmatrix} \quad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \quad i = 1, 2, 3$$

where \mathbb{I} and \mathbb{O} are 2×2 matrices

$$\mathbb{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \mathbb{O} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

where the σ_i are the 2×2 Pauli spin matrices

Dirac Spinors

The wavefunctions can be written as:

$$\psi \propto u(p)e^{-ip_\mu \cdot x^\mu}$$

This is a plane wave multiplied by a **four component spinor** $u(p)$

Note that the spinor depends on four momentum p^μ

For a particle at rest $\vec{p} = 0$ the Dirac equation becomes:

$$\left(i\gamma^0 \frac{\delta}{\delta t} - m \right) \psi = (i\gamma^0(-iE) - m) \psi = 0$$

$$Eu = \begin{pmatrix} m\mathbf{I} & 0 \\ 0 & -m\mathbf{I} \end{pmatrix} u$$

There are **four** eigenstates, two with $E = m$ and two with $E = -m$.

Solutions of Dirac Equation

The spinors associated with the four eigenstates are:

$$u^1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad u^2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad u^3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad u^4 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

and the wavefunctions are:

$$\psi^1 = e^{-imt} u^1 \quad \psi^2 = e^{-imt} u^2 \quad \psi^3 = e^{+imt} u^3 \quad \psi^4 = e^{+imt} u^4$$

Solutions of Dirac Equation: Rest

ψ^1 describes an $S=1/2$ fermion of mass m with spin \uparrow

ψ^2 describes an $S=1/2$ fermion of mass m with spin \downarrow

ψ^3 describes an $S=1/2$ antifermion of mass m with spin \uparrow

ψ^4 describes an $S=1/2$ antifermion of mass m with spin \downarrow

Fermions have exponents $-imt$, antifermions have $+imt$

Negative energy solutions $E = -m$ are either:

Fermions travelling backwards in time

Antifermions travelling forwards in time

Solutions of Dirac Equation: Motion

Fermions:

$$u^1 = \begin{pmatrix} 1 \\ 0 \\ p_z/(E+m) \\ (p_x + ip_y)/(E+m) \end{pmatrix} \quad u^2 = \begin{pmatrix} 0 \\ 1 \\ (p_x - ip_y)/(E+m) \\ -p_z/(E+m) \end{pmatrix}$$

Antifermions:

$$v^2 = \begin{pmatrix} p_z/(E+m) \\ (p_x + ip_y)/(E+m) \\ 1 \\ 0 \end{pmatrix} \quad v^1 = \begin{pmatrix} (p_x - ip_y)/(E+m) \\ -p_z/(E+m) \\ 0 \\ 1 \end{pmatrix}$$

Wavefunctions: Electron and Positron

Electron with energy E and momentum \vec{p}

$$\psi = u^1(p)e^{-ip \cdot x} \quad \uparrow$$

$$\psi = u^2(p)e^{-ip \cdot x} \quad \downarrow$$

Positron with energy E and momentum \vec{p}

$$\psi = v^1(p)e^{ip \cdot x} = u^4(-p)e^{-i(-p) \cdot x} \quad \uparrow$$

$$\psi = v^2(p)e^{ip \cdot x} = u^3(-p)e^{-i(-p) \cdot x} \quad \downarrow$$

Note the reversal of the sign of p in both parts of the antifermion wavefunction and the change from u to v spinors

Helicity

- Useful to introduce *chirality* operators:

$$\gamma_5 \equiv i\gamma_0\gamma_1\gamma_2\gamma_3 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \quad (\gamma_5)^2 = 1$$

$$P_L \equiv \frac{1 - \gamma_5}{2} \quad P_R \equiv \frac{1 + \gamma_5}{2} \quad \Rightarrow \quad P_{(L,R)}^2 = P_{(L,R)} \quad P_L P_R = P_R P_L = 0$$

$$\psi(x) = [P_L + P_R]\psi(x) \equiv \psi_L(x) + \psi_R(x)$$

$$\mathcal{H} = \frac{\vec{\sigma} \cdot \vec{p}}{|\vec{\sigma}||\vec{p}|} \quad \text{Helicity}$$

These are also called *fermion helicities*.

- Straightforward to identify two components of spin \Rightarrow quantum mechanical and relativistic description of an electron (with correct spin assignment) $S = \frac{1}{2}$.

Dirac equation: Consequences

The Dirac equation is able to give the correct description of an electron moving at high energy (including the magnetic properties e.g. spin) however ... this equation allowed a second solution for a particle with mass equal to the electron mass but with opposite charge.

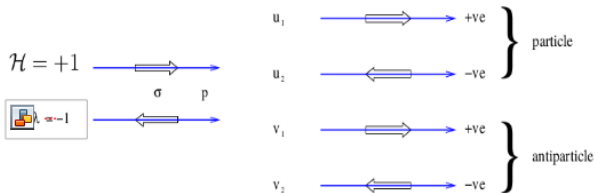
This second solution produced three years of confusion...

In 1931 Dirac gives the key input for the interpretation of this second solution: *“if this second particle existed it would be a particle of a new type, unknown to the experimental physics, having the same mass of the electron and opposite charge”*

Helicity

Operator P_L projects out Left Handed (LH) Helicity $\mathcal{H} = -1$

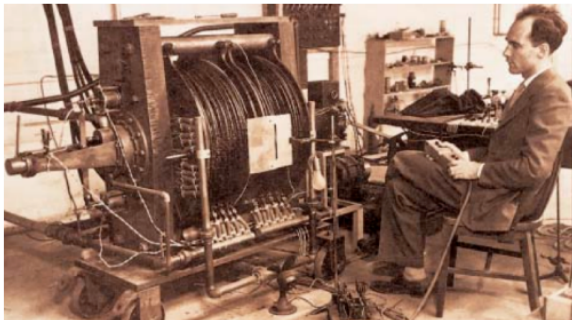
Operator P_R projects out Right Handed (RH) Helicity $\mathcal{H} = +1$



Massless fermions with $p=E$ are purely Left Handed
 Massless antifermion with $p=E$ are purely Right Handed

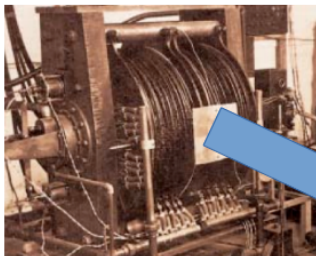
Anti-Electron Discovery

One year after Dirac's positron hypothesis, Anderson, a student working for his PhD with Millikan discovered a particle with same mass but opposite charge than the electron



Look at the control room!

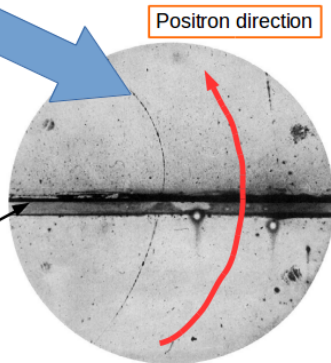
Cloud Chamber



Inside the electromagnet

6 mm Lead

Discovery of the Dirac positron



Lagrangian

As in analytical mechanics, Dirac and Klein-Gordon equations are equation of motions

⇒ they are the **Euler-Lagrange** of “some” Lagrangian $\mathcal{L}(\phi, \partial_\mu\phi)$
Euler-Lagrange equations:

$$\partial_\mu \frac{\delta \mathcal{L}}{\delta(\partial_\mu \phi)} = \frac{\delta \mathcal{L}}{\delta \phi}$$

The \mathcal{L} that gives the Dirac equation is:

$$\mathcal{L} = \bar{\psi}(i\partial - m)\psi$$

Symmetries and Conservation Laws



***Richard Feynman quotes
Prof. Hermann Weyl:***

**“a thing is symmetrical if one
can subject it to a certain
operation,
and it appears exactly the
same after the operation.”**

Symmetries are one of the most important driving ideas in physics.
Two categories of symmetries:

- **Continuous** symmetries
- **Discrete** symmetries

Continuous Symmetries



Göttingen again!

Imagine a \mathcal{L} that is invariant under continuous transformations of the fields (parameter α):

$$\phi(x) \rightarrow \phi'(x) = T(\alpha; \phi(x))$$

$$\phi(x) \rightarrow \phi'(x) = \phi(x) + \alpha \Delta(\phi(x)) \text{ For small } \alpha$$

If Lagrangian is conserved (actually it is $\int \mathcal{L} d^4x$ that has to be conserved):

$$\mathcal{L} \rightarrow \mathcal{L}' = \mathcal{L} + \alpha \Delta \mathcal{L} = \mathcal{L}$$

$$\partial_\mu j^\mu = 0$$

$$j_\mu = \frac{\delta \mathcal{L}}{\delta(\partial_\mu \phi)} \Delta \phi$$

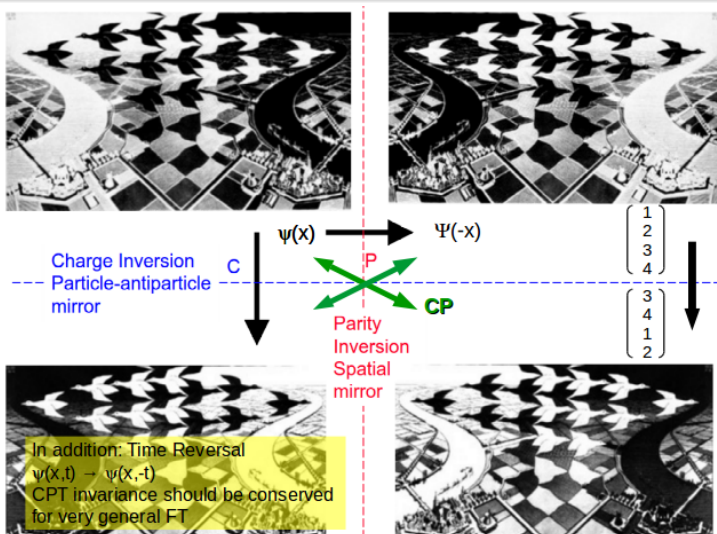
Noether's theorem. j_μ is also called conserved current. For every continuous symmetry in nature, there is a corresponding conservation law (and viceversa!) ☰

Examples

What is the conservation law:

- associated to the **gauge symmetry** of the electromagnetic field?
- associated to the **translation invariance**?
- associated with **rotational invariance**?

Discrete Symmetries: Parity, Charge Conjugation and Time Reversal



Dirac Fields and Transformations

How do the different Dirac field bilinears transform under parity?

	$\bar{\psi}\psi$	$i\bar{\psi}\gamma^5\psi$	$\bar{\psi}\gamma^\mu\psi$	$\bar{\psi}\gamma^\mu\gamma^5\psi$	$\bar{\psi}\sigma^{\mu\nu}\psi$
P	+1	-1	$(-1)^\mu$	$-(-1)^\mu$	$(-1)^\mu(-1)^\nu$
T	+1	-1	$(-1)^\mu$	$(-1)^\mu$	$-(-1)^\mu(-1)^\nu$
C	+1	+1	-1	+1	-1

Scalar Vector Tensor
 Pseudo-Scalar Axial Vector

How do the following quantities transform under discrete symmetries?

Temperature
Helicity
Momentum
Spin

Any idea of an example of a tensor field?

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Towards a Quantum Field Theory model of Electromagnetism

- The successful use of quantum mechanics and relativity started with the Dirac equation found his completion in the **quantum field theory** (QFT) describing particle interactions.
- The key ingredient in this theory is the concept of **field**, introduced by Maxwell, and modified to respect the new concepts introduced by quantum mechanics and relativity.
- After quantization, the fields are not anymore continuous but they are decompsed in quantum of energy that are what we indicated with “particles” and that are indeed the manifestation of the quantistic fields.

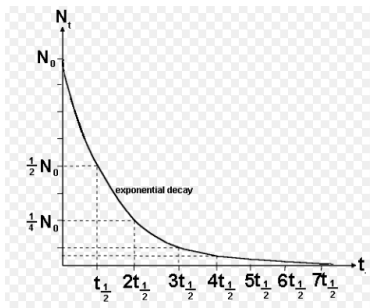
Cross section, decay rates,...

Decay rate Γ is the probability per unit time that a given type of particle will disintegrate.

$$N(t) - N(t + \Delta t) = -N\Gamma\Delta t$$

$$dN = \Gamma N dt$$

$$N(t) = N(0)e^{-\Gamma t}$$



If more than one decay mode the total decay rate is given by the sum of all possible decay rates.

$$\Gamma_{tot} = \sum_{i=1}^n \Gamma_i$$

Branching ratios are obtained from: $BR_i = \Gamma_i / \Gamma_{tot}$. The mean lifetime is:

$$\tau = \frac{1}{\Gamma_{tot}}$$

When extremely short life time (e.g. cannot directly measure the decay time) Γ is called **Decay Width**. Γ_i are **Partial Widths**

Cross sections

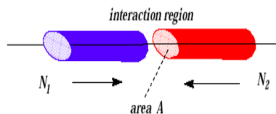
Cross sections are connected to the probability that a certain process happens.

Example: we have two beams with opposite directions of electrons and positrons we want to know how many $\mu^+\mu^-$ events we will measure. This will depend both on the $e^+e^- \rightarrow \mu^+\mu^-$ dynamics and on the number of collisions we produce.

$$\begin{aligned}\frac{dN}{dt} &= \sigma(e^+e^- \rightarrow \mu^+\mu^-) \times \mathcal{L} \\ N &= \sigma(e^+e^- \rightarrow \mu^+\mu^-) \times \int \mathcal{L} dt\end{aligned}$$

- Inclusive cross section e.g. $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$
- Differential cross section $\frac{d\sigma}{d\Omega}$ (e.g. $\frac{d\sigma}{dp}$ and $\frac{d\sigma}{d \cos \theta}$ and $\frac{d\sigma}{dN_{jet}}$)

Luminosity



- $N_{1,2}$ particles in bunch 1, 2
- f_{rev} revolution frequency
- A transverse dimension of beam (equivalent $4\pi\sigma_x\sigma_y$)
- n_b number of colliding bunches

$$\mathcal{L} = \frac{n_b N_1 N_2}{A T_{rev}} = \frac{n_b N_1 N_2}{A} f_{rev}$$

$$\mathcal{L} = [L]^{-2} [T]^{-1}$$

- m^2 is by far a too large unit.
- Units for cross sections in particle physics are barns:

$$1\text{barn} = 10^{-28}\text{m}^2 = 10^{-24}\text{cm}^2$$

Theory vs Experiment

In order to calculate decay rates and the cross-sections we need two ingredients:

- **Matrix element** that contains the dynamic of the interaction \Rightarrow Feynman diagrams
- **Phase space**: contains masses, momenta, energy and it reflects the possible kinematic allowed space for the interaction. For example if the process is not allowed because the energy of the final state would be higher than the energy of the initial state it is this part of the calculation that is 0.

Cross sections and decay widths with Golden Rule

Decay Rate

Suppose the particle 1 is at rest and decays in n particles the decay rate Γ is:

$$1 \rightarrow 2 + 3 + 4 + \dots + n$$

The decay rate is given by:

$$\Gamma = \frac{S}{2m_1} \int |\mathcal{M}|^2 (2\pi)^4 \delta^4(p_1 - p_2 - \dots - p_n) \times \prod_{j=2}^n 2\pi \delta(p_j^2 - m_j^2) \theta(p_j^0) \frac{d^4 p_j}{(2\pi)^4}$$

Simplest case $1 \rightarrow 2 \ 3$:

$$\Gamma = \frac{S|\vec{p}|}{8\pi m_1^2} |\mathcal{M}|^2 \quad S \text{ is a factor for identical particles}$$

$|\vec{p}|$ is the particle of the outgoing of the momenta. In particle 1 rest frame $\vec{p}_2 = -\vec{p}_3$. Remember that in natural units:

$$\hbar = c = 1$$

$$[L] = [T] = [E]^{-1} = [M]^{-1}$$

$$\Rightarrow [\mathcal{M}] = [E]$$

Scattering

Suppose that two particles colliding 1 and 2 produce particles 3 + 4 + ... + n. Cross section is given by:

$$\sigma = \frac{S}{4\sqrt{(p_1 p_2)^2 - (m_1 m_2)^2}} \int |\mathcal{M}|^2 (2\pi)^4 \delta^4(p_1 + p_2 - p_3 \dots - p_n) \times \prod_{j=3}^n 2\pi \delta(p_j^2 - m_j^2) \theta(p_j^0) \frac{d^4 p_j}{(2\pi)^4}$$

Simplest case 1 + 2 → 3 + 4:

$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{8\pi}\right)^2 \frac{S |\mathcal{M}|^2}{(E_1 + E_2)^2} \frac{|\vec{p}_f|}{|\vec{p}_i|}$$

$|\vec{p}_i|$ in the rest frame of (1, 2) $p_{i(f)}$ is the incoming (outgoing) particle momentum.

In this case \mathcal{M} is dimensionless.

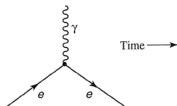
How to calculate \mathcal{M} ?

- \mathcal{M} represents the probability amplitude between an initial state and a final state.
- \Rightarrow contains the interaction.
- it is then integrated out and summed over all the polarizations (unless you are able to produce polarized beams or you can measure the polarization of decay products)

Each diagram represents a function of the kinematic variables of the initial and final state particles that is used to calculate the probability with which a certain process occurs

Simplest interaction is electron with an electromagnetic field

In terms of particles:



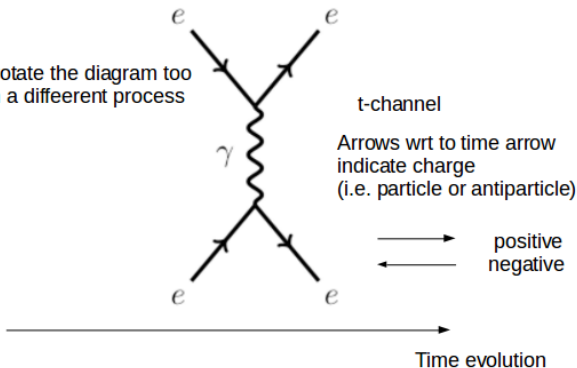
\Rightarrow these are Feynman Diagrams
Feynman diagrams are NOT just drawings!

They are symbolic calculations

Feynman Diagrams

Coulomb scattering (classical) $e^+e^+ \rightarrow e^+e^+$
Møller scattering

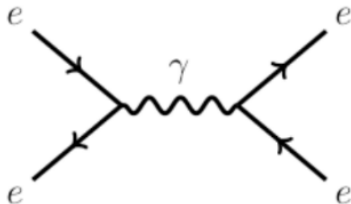
We can rotate the diagram too
To obtain a different process



Electron-Positron

Description of electron-positron scattering

$e^+e^- \rightarrow$
 e^+e^-

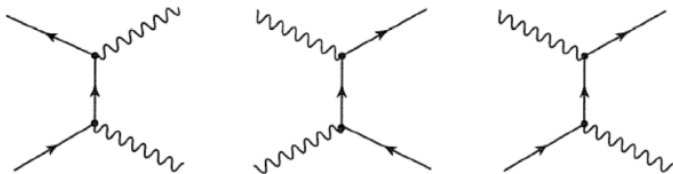


s-channel

Electron-positron
colliders
Some examples of
electron positron
colliders in the past?
Present? Future?

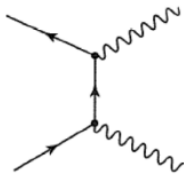
Time evolution

Exercise



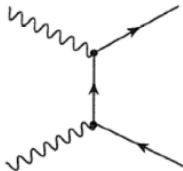
Do you recognize these processes?
(only electrons and photons involved here)

Exercise



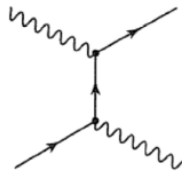
Electron-positron annihilation

$$e^+e^- \rightarrow \gamma\gamma$$



Pair production

$$\gamma\gamma \rightarrow e^+e^-$$

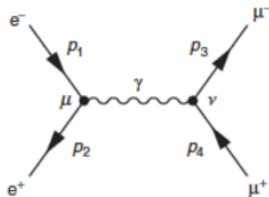


Coulomb scattering

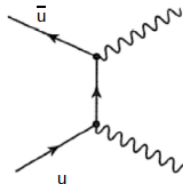
$$e^+\gamma \rightarrow e^+\gamma$$

Adding muons, quarks,....

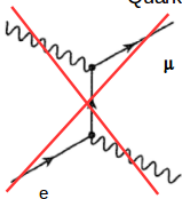
By substituting electrons with muons we can describe different processes



Muon-Antimuon
production

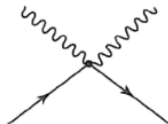
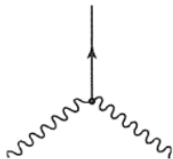


Quark-Antiquark annihilation



Is not allowed!

A Funny Game?



Are not allowed

Why?

Calculating \mathcal{M}

Let us start with the vertex:

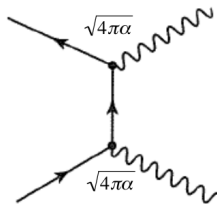
The vertex $e - \gamma - e$ is related to the strength of the interaction and

on the electric charge. $-q\sqrt{\frac{4\pi}{\hbar c}}$

For electrons: $\rightarrow \sqrt{4\pi\alpha} = g_e$

For quark u-type $\frac{2}{3}\sqrt{4\pi\alpha}$

Reminder: $\alpha_{em} = 1/137$




For electrons:

$$|\mathcal{M}|^2 \propto 16\pi^2\alpha^2$$


Each additional vertex “adds” up $q\sqrt{\alpha}$ to \mathcal{M} (and thus $q^2\alpha$ to any observable)

Feynman Rules for QED

Incoming fermion  = $u^s(p)$


Incoming antifermion  = $\bar{v}^s(p)$

Outgoing fermion  = $\bar{u}^s(p)$


Outgoing antifermion  = $v^s(p)$

Incoming photon  = ϵ^μ

Outgoing photon  = $\epsilon^{\mu*}$

Photon propagator  = $\frac{-ig^{\mu\nu}}{p^2 + i\epsilon}$ (5.2)

Fermion propagator  = $\frac{i(\not{p} + m)}{p^2 - m^2 + i\epsilon}$ (5.2)

Vertex  = $-ie\gamma^\mu$ (5.2)

- Impose 4-momentum conservation at each vertex.
- Integrate over momenta not determined by 10.:
- Figure out the overall sign of the diagram.

$$\int \frac{d^4 p}{(2\pi)^4}$$

External lines represent "real" particles: $E^2 = p^2 + m^2$

Internal lines are called "propagators" and mediate electromagnetic interaction.

There are "virtual" particles: $E^2 - p^2 = q^2$ different from m^2 !

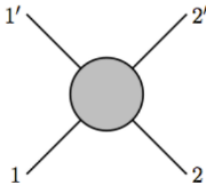
In particular for photon propagators $\rightarrow 1/q^2$

Matrix element amplitude is inversely proportional to the "momentum/energy" transferred by the propagator

Mandelstam Variables

$$1+2 \rightarrow 1'+2'$$

Incoming particles
have momenta p_1, p_2
Outgoing particles
have momenta p_1', p_2'



Time
evolution

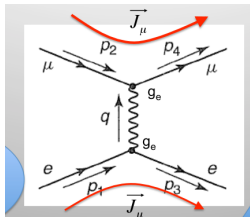
$$s = (p_1 + p_2)^2 = (p_1' + p_2')^2,$$

$$t = (p_1 - p_1')^2 = (p_2' - p_2)^2,$$

$$u = (p_1 - p_2')^2 = (p_1' - p_2)^2.$$

Mandelstam
variables

Workout: Electron-Muon Scattering



$$\begin{aligned}
 \mathcal{M} &= [\bar{u}_e(p_3)\gamma^\nu \frac{ie\gamma^\nu}{q} u_e(p_1)][\bar{u}_\mu(p_4)\gamma^\mu \frac{ie\gamma^\mu}{q} u_\mu(p_2)] \\
 &= -\frac{e^2}{q^2} [\bar{u}_e(p_3)\gamma^\mu u_e(p_1)][\bar{u}_\mu(p_4)\gamma^\mu u_\mu(p_2)] \\
 |\bar{\mathcal{M}}|^2 &= \frac{1}{(2s_1+1)(2s_2+1)} \sum_{\text{spins}} |\bar{\mathcal{M}}|^2 = 2e^4 \frac{s^2+u^2}{t^2}
 \end{aligned}$$

$$q^2 = (p_1 - p_3)^2 = (p_2 - p_4)^2 = t$$

- $u(e/\mu)$ destroys an electron/muon or creates a positron/ μ^+ .
- $\bar{u}(e, \mu)$ creates an electron/muon or destroys a positron/ μ^+
- Factor e^2 from the two vertices
- $\frac{1}{q^2}$ from fermionic propagator
- $J^\mu = \bar{u}_i\gamma^\mu u_i$ is the electromagnetic current

Leading Order QED

TABLE 6.1
 Leading Order Contributions to Representative QED Processes

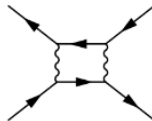
	Feynman Diagrams		$ \mathcal{M} ^2/2e^4$		
	Forward peak	Backward peak	Forward	Interference	Backward
Moller scattering $e^- e^- \rightarrow e^- e^-$			$\frac{s^2 + u^2}{t^2}$	$\frac{2s^2}{tu}$	$\frac{s^2 + t^2}{u^2}$
(Crossing $s \leftrightarrow u$)			(u ↔ t symmetric)		
Bhabha scattering $e^- e^+ \rightarrow e^- e^+$	Forward	"Time-like"	Forward	Interference	Time-like
			$\frac{s^2 + u^2}{t^2}$	$\frac{2u^2}{ts}$	$\frac{u^2 + t^2}{s^2}$
$e^- \mu^- \rightarrow e^- \mu^-$			$\frac{s^2 + u^2}{t^2}$		
(Crossing $s \leftrightarrow t$)					$\frac{u^2 + t^2}{s^2}$

Halzen & Martin P.129

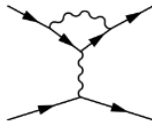
Loops and Higher Order



Higher Order corrections to $e^+e^- \rightarrow e^+e^-$



Box diagrams



Penguin diagrams

If we increase the number of possible vertices (higher order) the numbers of diagrams between initial and final state increases.

Higher Orders

“Dressed” fermions



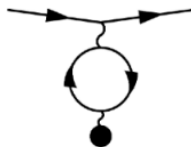
A real (or virtual) fermion emits and reabsorbs a virtual photon.

Vertex corrections



A virtual photon connects fermions across a previous vertex.

“Bubble” propagators



A real (or virtual) photon creates fermion/antifermion pairs.

Each pair of vertices + virtual particle adds a factor $\alpha = 1/137$

Sum of higher order QED corrections converges!

Renormalization

UV Divergence

Higher order diagrams with large virtual 4-momentum $k \rightarrow \infty$ transfer give divergent integrals

This is a problem with Feynman diagrams calculation

$$\mathcal{M} \propto \int d^4k \frac{\not{k} + m}{k^2 - m^2} \frac{(\not{q} - \not{k}) - m^2}{(q - k)^2 - m^2} \propto \int k^3 dk \frac{k^2}{k^4} \approx \int k dk$$

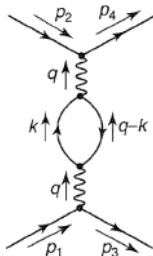
The solution is the technique called renormalization that redefines coupling, masses using a cut-off (M) ([Mass regularization](#))

Renormalization: redefinition of masses, charges, spinors,....:

$$e \rightarrow e_R = \left(1 - \frac{\alpha}{3\pi} \ln \left(\frac{\Lambda^2}{m^2} \right) + \mathcal{O}(\alpha^2) \right)^2$$

Generally in QED it is safe to ignore terms $\mathcal{O}(\alpha^2)$,

Renormalized current: $J^\mu \rightarrow J_r^\mu = e_r (\bar{u} \gamma^\mu u)$



α running

A consequence of renormalisation is that the value of the coupling constant α_{em} becomes a function of q^2 (the scale of energy of the interaction):

$$\alpha(q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi} \ln\left(\frac{q^2}{\mu^2}\right)}$$

where μ is a reference 4-momentum transfer which is used to remove the dependence on the cutoff parameter Λ .

At low energies $\alpha = 1/137$,

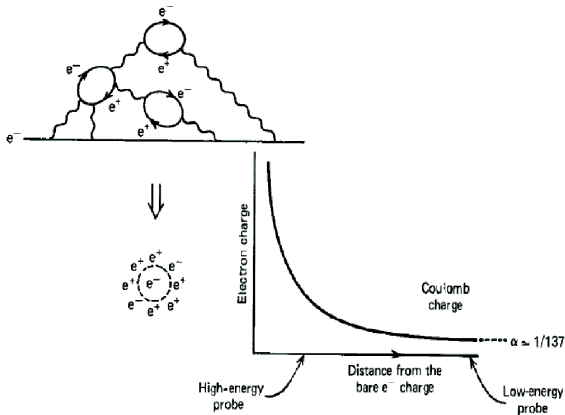
At $q^2 \sim M_Z = 90$ GeV $\alpha = 1/128$,

Can be thought of as a correction to the “bare” electric charge to account for “screening” by higher order diagrams with virtual photons and fermion/antifermion pairs.

Vacuum polarization

In QED electron and positron *virtual* clouds effectively screen the electric charge:

- Probe close \Rightarrow Large effective charge
- Probe far \Rightarrow small effective charge



Anomalous Magnetic Moment

The first application of renormalization in QED was the anomalous coupling of the electron

Magnetic moment can be broken in:

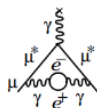
$$\mu = (1 + a) \frac{q\hbar}{2m} \quad \text{where} \quad a = \frac{g - 2}{2}.$$



Dirac



Schwinger



$$a_e = \frac{\alpha}{2\pi} + C_2 \left(\frac{\alpha}{\pi}\right)^2 + C_3 \left(\frac{\alpha}{\pi}\right)^3 + C_4 \left(\frac{\alpha}{\pi}\right)^4 + \dots$$

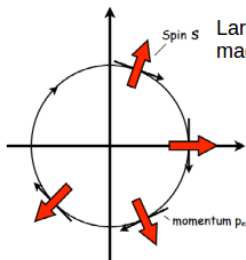
1st order

2nd order

3rd order

4th order

Strategy for measuring g

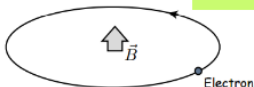


Larmor frequency of a magnetic moment in a magnetic field

$$\omega = g_e \frac{e}{2m} B$$

Cyclotron frequency:

$$\omega_c = \frac{e}{m} B$$

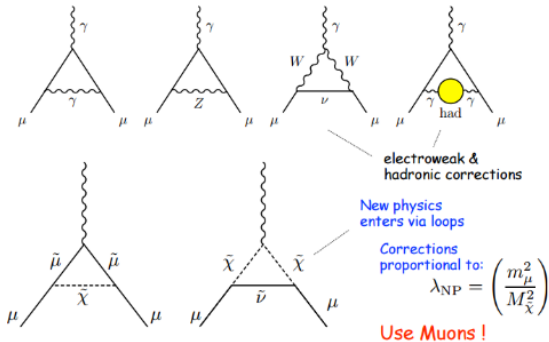


For $g=2$ Larmor frequency equals Cyclotron frequency

Theoretical Predictions

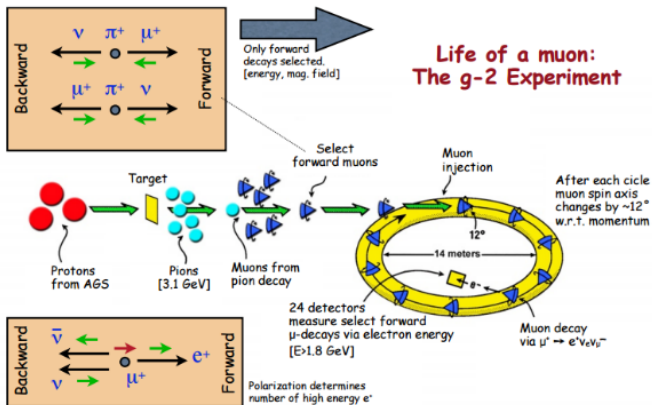
Instead of measuring the anomalous magnetic moment for the electron, concentrate on the measurement of anomalous magnetic moment for the muon

Why it is interesting?

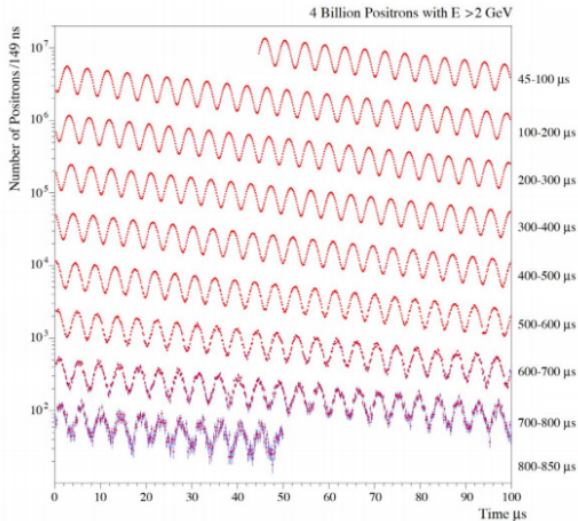


Measuring Muon $g - 2$

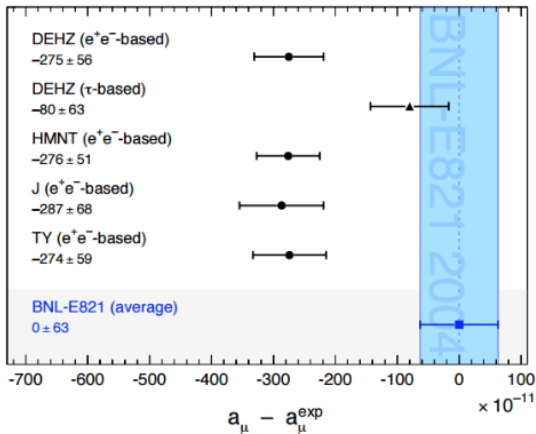
From a beam of pion it is possible to create a 95% polarized muon beam



Measuring Muon $g - 2$



Results Muon $g - 2$



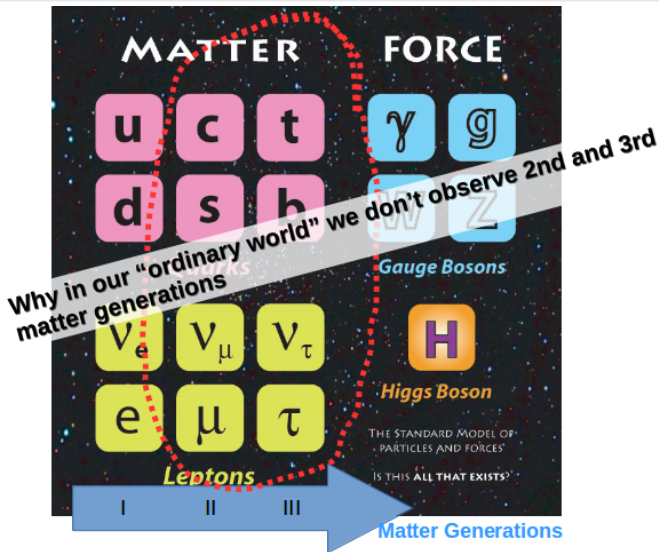
Moving to FNAL: Summer 2013



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- 2 Quantum Electrodynamics
 - Experimental observables
 - Amplitude Matrix Calculation
 - Renormalization
- 3 Weak Interactions
- 4 ElectroWeak Unification
 - Experimental Checks
- 5 Multiboson Production

Matter Generations



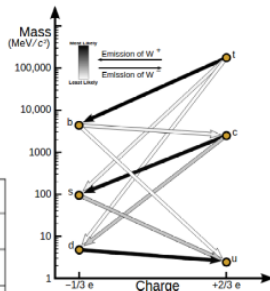
Decays and Conservation Laws

- The photon is **stable**: there is nothing lighter to decay...
- The electron is **stable**, is the lighter charged particle
- The proton is **stable**, it is the lightest baryon and baryon number is conserved (more on this later)
- The positron and antiproton are also **stable** for the same reasons as above (unless they come in contact)
- Also the neutron in the “protected” environment of the nucleus can become **stable**
- Our world (the matter) is populated by electrons, protons, neutrons and neutrinos (lepton number)
- More exotic particles can be created but they decay transforming to more stable particles

Interactions and conservation laws

Quantity	Strong	Electro-magnetic	Weak
Electric Charge	Y	Y	Y
Color	Y	Y	Y
Baryon Number	Y	Y	Y
Lepton Number	Y	Y	Y*
Flavour	Y	Y	N

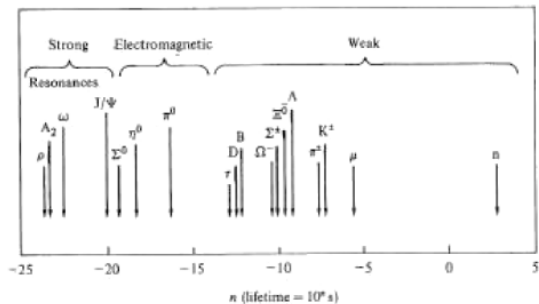
	Weak	EM	Strong
Quarks	+	+	+
Charged leptons	+	+	-
Neutral leptons	+	-	-



Weak force is effective only when the other interactions cannot occur for conservation laws

Lifetimes and Interactions

Lifetime is one of the main characteristics of a particle.
 We would like to understand the interaction involved in the particle decay
 from its lifetime

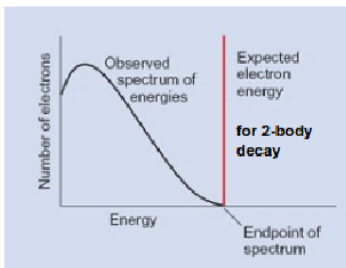


Neutron decay

In the 30' neutron decay was causing serious problems.

If $n \rightarrow e^- + p$ than it would have been a 2-body decay with E_e peaked at one value

At the contrary a continously decaying distribution was observed

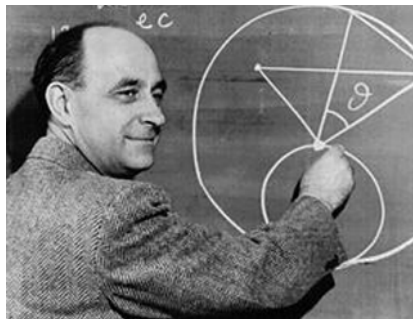
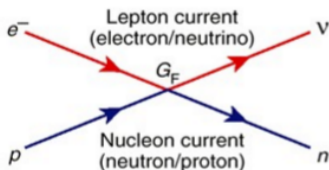


W. Pauli (Goettingen again!)
proposed that a 3rd particle went
undetected: the neutrino
 $n \rightarrow p + e^- + \bar{\nu}$

Pauli's hypothesis: **undetected neutrino** has a very small mass, no electric charge (which is not enough: the photon can be perfectly identified!)

Weak Interaction: First Attempt

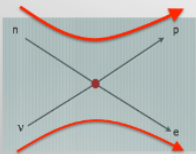
E. Fermi made a successful theory of “weak interactions”



What is the interaction term of such interaction?
Today we call that an Effective Field Theory

Fermi Theory: First Attempt

First attempt to describe weak interaction as a Field Theory



Matrix element form for em scattering

$$M_{em} = -\frac{g_e^2}{q^2} [\bar{u}(p_3)\gamma^\mu u(p_1)][\bar{u}(p_4)\gamma_\mu u(p_2)]$$

Fermi's contact interaction for the weak interaction

$$M_{weak} = \frac{G_F}{\sqrt{2}} [\bar{u}(p)\gamma^\mu u(n)][\bar{u}(e)\gamma_\mu u(v)]$$

Notes:

Electromagnetic current \rightarrow **Weak** current (still a vector)

Fermi constant G_F characterize the strength of the interaction

$\rightarrow G_F = 1.17 \cdot 10^{-5} \text{ GeV}^{-2}$

The weak current changes the electric charge $\Delta Q = \pm 1$

\rightarrow **Weak Charged Current**

Weak current involves composite particle (but Fermi didn't know that!)

Matrix element for $\bar{\nu}n \rightarrow pe$ and for neutron decay $n \rightarrow p + e + \bar{\nu}$ are equal

The cross section of the process diverges as E_ν^2 (Effective Field Theory)

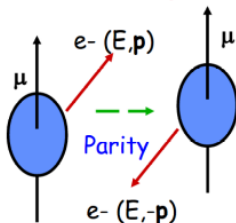
Need to incorporate **cut off scale**

Parity Violation

In 1956 Lee-Yang proposed to perform some experiment to test Parity conservation of weak interaction.

In 1957 Madame Wu (Beta decay of Co^{60}) experiment and Garwin Lederman Weinrich (pion decay) shown that parity is **maximally violated** by weak interactions

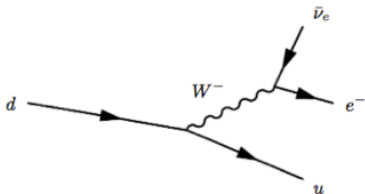
↑
 B field
 |
 Co^{60} Nuclei
 spin aligned
 Beta decay to
 Ni^{*60}



Madame Wu's experiment

Need to incorporate in Fermi's theory Parity violation

Second Attempt



$$\mathcal{M} = \left(\frac{g}{\sqrt{2}} \bar{u}_d \gamma^\mu \frac{1}{2} (1 - \gamma^5) u_u \right) \frac{1}{M_W^2 - q^2} \left(\frac{g}{\sqrt{2}} \bar{u}_{\nu_e} \gamma^\mu \frac{1}{2} (1 - \gamma^5) u_e \right)$$

Vectorial
Axial
Cut-off scale

V-A theory of weak interactions

What is M_W ? It is the mass of the **boson** that carries the weak interaction

Strength of Weak Force

Finding the relation between g and G_F :

$$\mathcal{M} = \left(\frac{g}{\sqrt{2}} \bar{u}_d \gamma^\mu \frac{1}{2} (1 - \gamma^5) u_u \right) \frac{1}{M_W^2 - q^2} \left(\frac{g}{\sqrt{2}} \bar{u}_{\nu_e} \gamma^\mu \frac{1}{2} (1 - \gamma^5) u_e \right)$$

$$M_{\text{weak}} = \frac{G_F}{\sqrt{2}} [\bar{u}(p) \gamma^\mu u(n)] [\bar{u}(e) \gamma_\mu u(\nu)]$$

$$\frac{G_F}{\sqrt{2}} = \frac{g}{\sqrt{2}} \times \frac{g}{\sqrt{2}} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{M_W^2 - q^2} \rightarrow \lim_{q^2 \ll M_W^2} \frac{g^2}{8M_W^2}$$

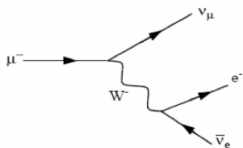
with $M_W \sim 80 \text{ GeV}$, $G_F = 1.12 \cdot 10^{-5} \text{ GeV}^{-2} \Rightarrow g = 0.65$

$$\alpha_{\text{Weak}} = \frac{g^2}{4\pi} = \frac{1}{29.5}$$

Not order of magnitudes from $\alpha_{EM} = \frac{1}{137}$!

Weak currents are weak because of the mass of the propagator, **NOT BECAUSE** of the small coupling!

Muon Decay



Reminder:

For a decay $\mu \rightarrow e + \bar{\nu}_e + \nu_\mu$:

$$d\Gamma = \frac{1}{2(2\pi^5)} \frac{|\bar{\mathcal{M}}|^2}{2|m_\mu|} \delta^{(4)}(q_e + k'_{\bar{\nu}_e} + k_{\nu_\mu} + p_\mu)$$

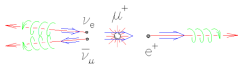
$$\frac{d^3k}{2E_k} \frac{d^3q}{2E_q} \frac{d^3k'}{2E_{k'}}$$

$$\mathcal{M} = \left(\frac{G_F}{\sqrt{2}} \bar{u}_d \gamma^\mu \frac{1}{2} (1 - \gamma^5) u_u \right)$$

$$\mathcal{M} \propto G_F^2 m_\mu^2$$

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} \propto G_F^2 m_\mu^2$$

The most probable configuration is when the neutrinos recoil against the electron taking $p_e^* = m_\mu/2$



With dimensional arguments:

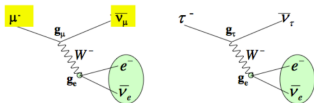
$$\Gamma_\mu = G_F^2 m_\mu^n$$

$$[E] = ([E]^{-2})^2 [E]^n$$

$$n = 5$$

Universality

Universality means that for all matter generations (leptons and quarks) the weak couplings are the same.
 First check with lepton τ decay.



Measuring the two lifetimes and the branching ratio (and taking into account small phase space difference $\frac{\rho_\tau}{\rho_\mu}$) we get:

$$\frac{g_\mu}{g_\tau} = 1.001 \pm 0.003$$

$$\Gamma(\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu) = \frac{1}{\tau_\mu}$$

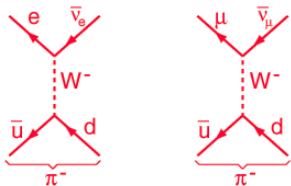
$$\Gamma(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau) = \frac{BR(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)}{\tau_\tau}$$

$$\frac{\Gamma(\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu)}{\Gamma(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)} = \frac{1}{\tau_\mu} \frac{\tau_\tau}{BR(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)} \exp$$

$$\frac{\Gamma(\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu)}{\Gamma(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)} = \frac{g_e^2 g_\mu^2 m_\mu^5 \rho_\mu}{g_e^2 g_\tau^2 m_\tau^5 \rho_\tau} = \frac{g_\mu^2 m_\mu^5 \rho_\mu}{g_\tau^2 m_\tau^5 \rho_\tau} \quad V - A$$

Pion Decay

Understanding the interplay of Parity violation and momentum conservation. Consider charged pion decay:



For the moment we don't know how W boson couples with π meson. Describe it with a Form Factor $F^\mu = f_\pi p^\mu$

$$\Gamma = \frac{|p_\nu|}{8\pi m_\pi^2} (|\mathcal{M}|^2)$$

$$\langle |\mathcal{M}|^2 \rangle = \left(\frac{g_W}{2M_W} \right)^4 f_\pi^2 m_\ell^2 (m_\pi^2 - m_\ell^2)$$

$$\Gamma = \frac{f_\pi^2}{\pi m_\pi^3} \left(\frac{g_W}{4M_W} \right)^4 m_\ell^2 (m_\pi^2 - m_\ell^2)$$

We don't know f_π^2 but we can calculate the ratio of BR:

$$\frac{\Gamma(\pi \rightarrow e\nu_e)}{\Gamma(\pi \rightarrow \mu\nu_\mu)} = \frac{m_e^2(m_\pi^2 - m_e^2)^2}{m_\mu^2(m_\pi^2 - m_\mu^2)^2}$$

Pion Decay

Two competing factors:

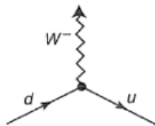
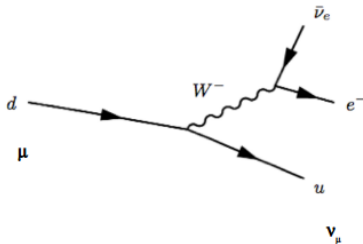
- larger phase space for electron decay
- Conservation of angular momentum $\text{Spin}(\pi^+)$ says that ℓ and $\bar{\nu}_\ell$ have opposite spin directions. $\bar{\nu}$ is always RH (positive helicity) thus also ℓ should have positive helicity which is possible only for very limited phase space of the electron (larger for muon)

Universality of Weak interaction: Hadrons

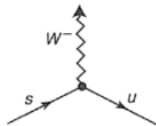
It is very appealing to have universality of weak interactions for quark sector too.

It works almost fine to replace $\mu \rightarrow d$ quark and $\nu_\mu \rightarrow u$ with u

But naive approach doesn't work (i.e. decay rates are not the ones predicted) for s quarks



$$\frac{-ig_w}{2\sqrt{2}} \gamma^\mu (1 - \gamma^5) \cos \theta_C$$



$$\frac{-ig_w}{2\sqrt{2}} \gamma^\mu (1 - \gamma^5) \sin \theta_C$$

Coupling modification proposed by Cabibbo

Comparing with experimental results

(e.g. $\Lambda \rightarrow p e \bar{\nu}_e$)

$\theta_C = 12.7$ deg

$\sin(\theta_C) = 0.220$ (Cabibbo suppressed)

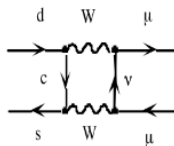
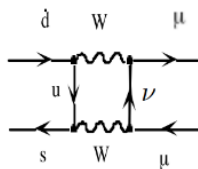
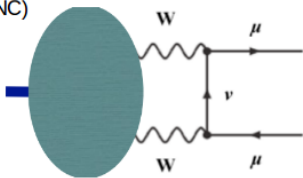
$\cos(\theta_C) = 0.976$ (Cabibbo allowed)

GIM Mechanism

Again issues in the strange sector.

Neutral K (meson composed by s and \bar{d}) do not decay in $\mu^+\mu^-$ pairs.

$DS=1$ but involving neutral particles \rightarrow Flavour Changing Neutral Currents (FCNC)



To reconcile Cabibbo angles and absence of FCNC

Glashow, Iliopoulos and Maiani postulated the existence of a 4th quark: charm that couples to W boson in a Cabibbo favoured way

Contribution of diagram without charm

$$u\bar{u} + d\bar{d} \cos^2 \theta + s\bar{s} \sin^2 \theta + (s\bar{d} + \bar{s}d) \sin \theta \cos \theta - (\bar{s}d + s\bar{d}) \sin \theta \cos \theta$$

FCNC Today

Experimental searches for FCNC are still a very important tool to have hints of Physics Beyond Standard Model processes

- *Standard model prediction*

$$(B_s \rightarrow \mu^+ \mu^-) = (3.2 \pm 0.2) \times 10^{-9}$$

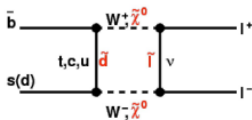
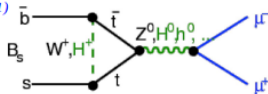
- *New Physics models*

- *Virtual SM particles in loops could be replaced by heavy NP particles and thus significantly enhance the branching ratio*

- *Search for New Physics*

- *Due to its small and precisely calculated branching ratio $B_s \rightarrow \mu^+ \mu^-$ is a very sensitive mode for NP at very high masses*
- *Search is complementary to direct searches at the energy frontier*

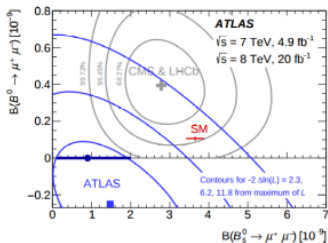
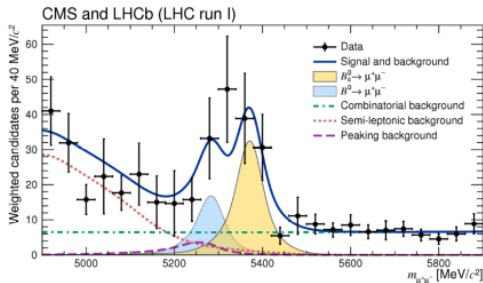
Buras et al., PLB 694, 402 (2011)



In general, heavy particles
 → large contribution

$B_S \rightarrow \mu\mu$ and $B_d \rightarrow \mu\mu$

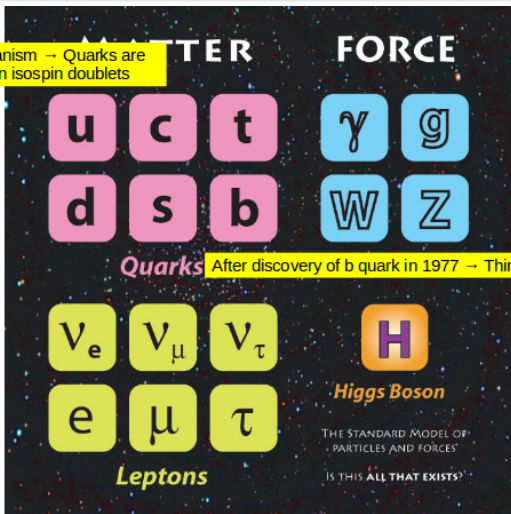
At LHC:
 $B_s \rightarrow \mu\mu$ and $B_d \rightarrow \mu\mu$
 $\mu\mu$
 Branching ratios
 have been
 measured (wrt B_d
 $\rightarrow J/\psi$ Ks decays)



CMS and LHCb: Nature 522 (2015) 68
 and arXiv:1411.4413
 ATLAS: arXiv:1604.04263

Third Generation

GIM Mechanism → Quarks are organized in isospin doublets



CKM Matrix

If three generations

Cabibbo angle (2x2 Matrix) is replaced by the Cabibbo-Kobayashi-Maskawa (3x3 Matrix)

$$J_{\mu}^+ \propto (\bar{u}, \bar{c}, \bar{t})(1 - \gamma_5) \gamma_{\mu} V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Two generations:
1 real parameter

Three generations:
3 real parameter
1 phase

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Unique way to have CP Violation in SM

Cabibbo angle
with 2 generations

$$V_{CKM} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

Unitarity triangle

6 Unitarity constrains \rightarrow 6 triangles

$$V_{CKM} V_{CKM}^+ = V_{CKM}^+ V_{CKM} = 1$$

The most famous
 unitarity triangle

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

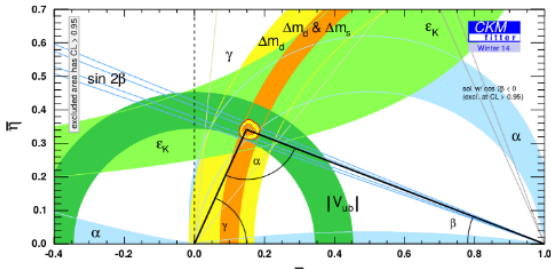
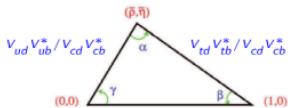
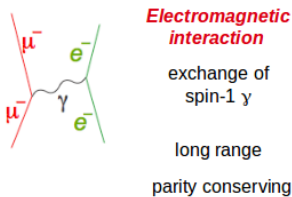


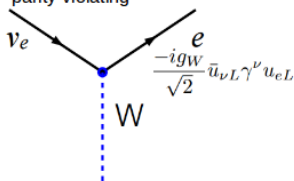
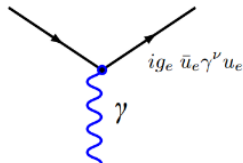
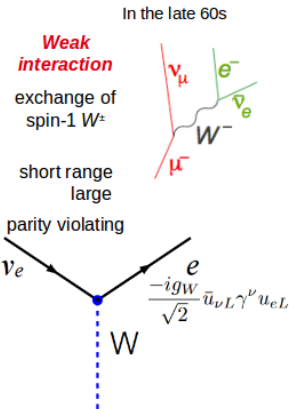
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Electromagnetic and Weak Interactions



But

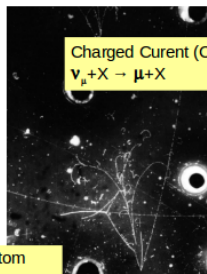
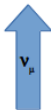
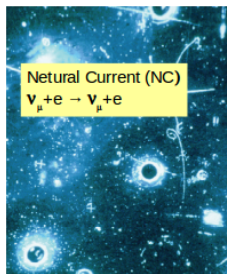
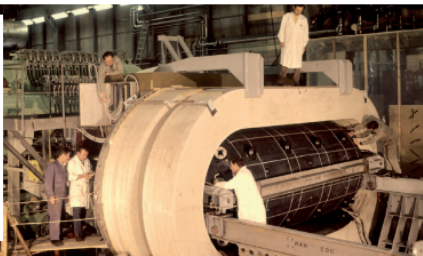


So: Can there be a **symmetry** relating γ and W^\pm ?

If so it must be **broken**

Discovery of Weak Neural Currents

and their secondaries. The projected bubble chamber was called Gargamelle, from the name of the giant mother of Gargantua in novels by François Rabelais (16th century). After an agreement in 1965 between the CEA-Saclay for building the chamber, and CERN to operate it in a neutrino beam, the Gargamelle collaboration was formed in 1967 by seven laboratories: Aachen, Brussels, CERN, Paris, Milano, Orsay and London. Gargamelle was designed and built under the leadership of André Lagarrigue, and assembled and operated at CERN (fig. 1) by a team including Paul Musset and André Rousset. But in the White report written by the collaboration to establish a shopping list of reactions to study, the neutral currents had only the 10th priority!



Neutrino flux from bottom

Adding NC to Weak Interaction

Let us define a **doublet** structure that contains the fermions

$$\chi_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$$

Define two operators

$$\tau_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \tau_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

Weak Current

$$j_\mu^- = \bar{\nu}_L \gamma_\mu e_L = \bar{\chi}_L \gamma_\mu \tau^+ \chi_L$$

$$j_\mu^+ = \bar{e}_L \gamma_\mu \nu_L = \bar{\chi}_L \gamma_\mu \tau^- \chi_L$$

τ matrices can be expressed as Pauli matrices

$$\tau^\pm = \tau^1 \pm i\tau^2$$

Can we do something with τ_3 ?

$$\tau^3 = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\bar{\chi}_L \gamma_\mu \tau^3 \chi_L = (\bar{\nu}_L, \bar{e}_L) \gamma_\mu \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

YES!
A Third current!

$$\frac{1}{2} \bar{\nu}_L \gamma_\mu \nu_L - \frac{1}{2} \bar{e}_L \gamma_\mu e_L$$

Adding NC to Weak Interaction

Note that electromagnetic current cannot be J^μ_3

$$j_\mu^{em} = -\bar{e}_L \gamma_\mu e_L - \bar{e}_R \gamma_\mu e_R$$

And we have also the "orthogonal" current to J^μ_3

$$-(\bar{\nu}_L \gamma_\mu \nu_L + \bar{e}_L \gamma_\mu e_L)$$

We have:

A triplet (Spin 1)

$$j_\mu^+ = \bar{e}_L \gamma_\mu \nu_L = \bar{\chi}_L \gamma_\mu \tau^+ \chi_L$$

$$j_\mu^- = \bar{e}_L \gamma_\mu \nu_L = \bar{\chi}_L \gamma_\mu \tau^- \chi_L$$

$$j_\mu^3 = \bar{\chi}_L \gamma_\mu \tau^3 \chi_L$$

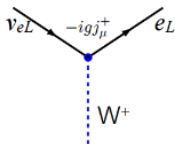
A singlet (Spin 0)

$$-(\bar{\nu}_L \gamma_\mu \nu_L + \bar{e}_L \gamma_\mu e_L)$$

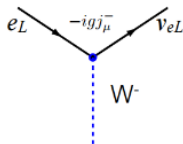
And another singlet (Spin 0) (for j e.m)

$$-(\bar{e}_R \gamma_\mu e_R)$$

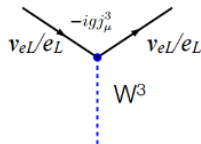
Diagrams



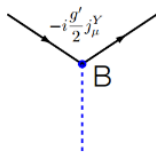
$$j_\mu^+ = \bar{\chi}_L \gamma_\mu \tau^+ \chi_L$$



$$j_\mu^- = \bar{\chi}_L \gamma_\mu \tau^- \chi_L$$



$$j_\mu^3 = \frac{1}{2} \bar{\nu}_L \gamma_\mu \nu_L - \frac{1}{2} \bar{e}_L \gamma_\mu e_L$$



$$j_\mu^Y = -2\bar{e}_R \gamma_\mu e_R - \bar{e}_L \gamma_\mu e_L - \bar{\nu}_L \gamma_\mu \nu_L$$

W_3 and B cannot be identified as photon and Z boson.
 Need some additional work.....

Weinberg Angle

Let us define A and Z as linear combination of W_3 and B

$$\begin{aligned} A_\mu &= B_\mu \cos \theta_W + W_\mu^3 \sin \theta_W \\ Z_\mu &= -B_\mu \sin \theta_W + W_\mu^3 \cos \theta_W \end{aligned}$$

$\sin \theta_W$ is called Weinberg angle

A vertex is:

$$\begin{aligned} & -i \frac{g'}{2} \cos \theta_W (-2\bar{e}_R \gamma_\mu e_R - \bar{e}_L \gamma_\mu e_L - \bar{\nu}_L \gamma_\mu \nu_L) \\ & - ig \sin \theta_W \left(\frac{1}{2} \bar{\nu}_L \gamma_\mu \nu_L - \frac{1}{2} \bar{e}_L \gamma_\mu e_L \right) \end{aligned}$$



$$ig_e (\bar{e} \gamma_\mu e)$$

To make it a "photon" we need:
 No coupling to photon
 Equal coupling between L and R
 $g \sin \theta_W = g' \cos \theta_W = g_e$

Z Boson

Z vertex

$$i\frac{g'}{2} \sin \theta_W (-2\bar{e}_R \gamma_\mu e_R - \bar{e}_L \gamma_\mu e_L - \bar{\nu}_L \gamma_\mu \nu_L) \\ - ig \cos \theta_W \left(\frac{1}{2} \bar{\nu}_L \gamma_\mu \nu_L - \frac{1}{2} \bar{e}_L \gamma_\mu e_L \right)$$

- if we define

$$g_Z = \frac{g_e}{\sin \theta_W \cos \theta_W} = \frac{g}{\cos \theta_W} = \frac{g'}{\sin \theta_W}$$

Z vertex

$$ig_Z \sin^2 \theta_W (-\bar{e}_R \gamma_\mu e_R - \frac{1}{2} \bar{e}_L \gamma_\mu e_L - \frac{1}{2} \bar{\nu}_L \gamma_\mu \nu_L) \\ - ig_Z \cos^2 \theta_W \left(\frac{1}{2} \bar{\nu}_L \gamma_\mu \nu_L - \frac{1}{2} \bar{e}_L \gamma_\mu e_L \right) \\ - ig_Z \left(\frac{1}{2} \bar{\nu}_L \gamma_\mu \nu_L - \frac{1}{2} \bar{e}_L \gamma_\mu e_L + \sin^2 \theta_W (\bar{e}_R \gamma_\mu e_R + \bar{e}_L \gamma_\mu e_L) \right)$$

Electroweak Unification

We have "unified" Electromagnetic and Weak interaction.

It isn't just a "recast" of fields → Electromagnetic and Weak interactions are related by a "single" θ_W parameter $\sin^2 \theta_W = 0.22$

Left doublets interact via weak CC

Right component are singlets and DO NOT interact via weak CC

Pauli matrices are the generators of SU(2) symmetry

Electromagnetism is a gauge theory (symmetry group is U(1))

Electroweak theory is based on gauge theory SU(2) x U(1)

And the currents are just the Noether conserved current from the invariance under local gauge transformations

Quarks can be added

Remember the CKM Matrix $\begin{pmatrix} u_L \\ d'_L \end{pmatrix}, \begin{pmatrix} c_L \\ s'_L \end{pmatrix}, \begin{pmatrix} t_L \\ b'_L \end{pmatrix}$
 And color factors for quarks

Couplings $\frac{g_Z}{2} \bar{u}(f) \gamma^\mu (c_V^f - c_A^f \gamma^5) u(f)$

Lepton	$2c_V$	$2c_A$	Quark	$2c_V$	$2c_A$
ν_e, ν_μ, ν_τ	1	1	u, c, t	$1 - \frac{8}{3} \sin^2 \theta_W$	1
e, μ, τ	$-1 + 4 \sin^2 \theta_W$	-1	d, s, b	$-1 + \frac{4}{3} \sin^2 \theta_W$	-1

Summary

Charged Current (CC):

$$J_{CC,\ell}^{\mu} = \frac{g}{s\sqrt{2}} \bar{\ell} \gamma^{\mu} (1 - \gamma^5) \nu_{\ell}$$

$$J_{CC,\text{quarks}}^{\mu} = \frac{g}{s\sqrt{2}} \bar{u}_i \gamma^{\mu} (1 - \gamma^5) V_{CKM,i,j} d_j$$

Neutral Current (NC):

$$J_{NC}^{\mu} = \frac{g}{2 \cos \theta_W} \bar{f} \gamma^{\mu} (C_V^f - C_A^f \gamma^5) f$$

$$e = g \sin \theta_W$$

$$M_Z = \frac{M_W}{\cos \theta_W}$$

$$M_W^2 \sin^2 \theta_W = \frac{\pi \alpha}{\sqrt{2} G_F}$$

fermion	C_V		C_A	
ν_{ℓ}	$\frac{1}{2}$	0.5	$\frac{1}{2}$	0.5
ℓ	$-\frac{1}{2} + 2 \sin^2 \theta_W$	-0.04	$-\frac{1}{2}$	-0.5
u,c,t quarks	$\frac{1}{2} - \frac{4}{3} \sin^2 \theta_W$	0.19	$\frac{1}{2}$	0.5
d,s,b quarks	$-\frac{1}{2} + \frac{2}{3} \sin^2 \theta_W$	-0.35	$-\frac{1}{2}$	-0.5

Discovery of W and Z Bosons

Just a beautiful theory?

Discovery of W and Z bosons

Afer the construction of the SppS at CERN it was possible to collide protons with antiprotons at $\sqrt{s}=630$ GeV

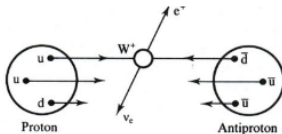
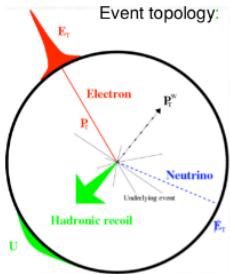
Previous generation of colliding hadrons was ISR (pp at $\sqrt{s}=62$ GeV)

W boson production mechanism at hadron colliders: Drell-Yan processes

$$u + \bar{d} \rightarrow W^+ \rightarrow e^+ + \nu_e, \mu^+ + \nu_\mu$$

$$\bar{u} + d \rightarrow W^- \rightarrow e^- + \bar{\nu}_e, \mu^- + \bar{\nu}_\mu$$

Event topology:



$\sigma(pp(\sqrt{s}=630 \text{ GeV}) \rightarrow \text{jet-jet}) \sim 100 \text{ nb}$

$\sigma(pp(\sqrt{s}=630 \text{ GeV}) \rightarrow W) \sim 6 \text{ nb}$

Hopeless to identify hadronic decays of W

→ Look for W boson production in lepton decays (e, μ) lepton τ are more difficult

Characteristics of W bosons decays

Taking into account color factors and possible decay modes of W boson we have:

$$\Gamma(W \rightarrow e\bar{\nu}_e) = \Gamma(W \rightarrow \mu\bar{\nu}_\mu) = \Gamma(W \rightarrow \tau\bar{\nu}_\tau) = \Gamma(W \rightarrow \ell\nu_\ell)$$

$$\Gamma(W \rightarrow u\bar{d}) = \Gamma(W \rightarrow c\bar{s}) = 3\Gamma(W \rightarrow \ell\nu_\ell)$$

$$\Gamma(W \rightarrow \ell\nu_\ell) = \left(\frac{g}{\sqrt{2}}\right)^2 \frac{M_W}{24\pi} = \frac{1}{2} \frac{G_F M_W^3}{3\pi\sqrt{2}} \approx 225 \text{ MeV}$$

- **One large momentum lepton** (High- P_T) (electron or muon)
- **One neutrino \rightarrow undetected** \rightarrow unbalance of momentum in transverse plane
 \Rightarrow Missing transverse energy (or momentum) \cancel{E}_T

How much is "High- P_T "?. In W rest frame (θ is the polar angle wrt to the beam)

$$p_{T\ell}^2 = \frac{\hat{s}}{4} \sin^2 \theta \quad \cos \theta = \sqrt{1 - \frac{4p_{T\ell}}{\hat{s}}} \quad \frac{d \cos \theta}{dp_{T\ell}^2} = \frac{2}{\hat{s}} \frac{1}{\cos \theta}$$

$$\frac{d\sigma}{dp_{T\ell}^2} = \frac{d\sigma}{d \cos \theta} \times \frac{d \cos \theta}{dp_{T\ell}^2} \sim \frac{d\sigma}{d \cos \theta} \times \frac{1}{\cos \theta}$$

W boson kinematics

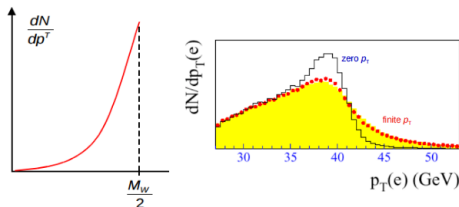
Singularity for $\theta = \pi/2 \rightarrow$ jacobian peak. Jacobian peak is spoiled by W boson transverse boost, measurement

Invariant mass of W boson cannot be reconstructed (neutrino in final state).

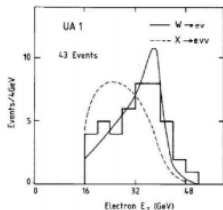
A related quantity is transverse mass ("Invariant mass in the transverse plane"):

$$M_T^{\ell\nu} = \sqrt{2p_T^\ell p_\ell^\nu (1 - \cos \phi)}$$

Less sensitive to momentum of W boson

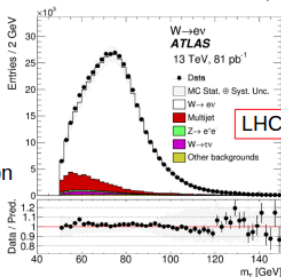
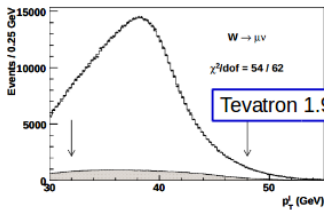


SppS, Tevatron, LHC



Electron momentum

SppS (530 and 630 GeV)



LHC 13 TeV

Main background “QCD Multijet”
 Dijet events with one jet faking a lepton

Standard Model

At tree level W boson mass, top quark mass and Higgs boson mass are free parameters of the SM theory

With 1-loop dependencies show up

$$m_W^2 = \frac{\frac{\pi\alpha}{\sqrt{2}G_F}}{\sin^2\theta_W(1-\Delta r)}$$

$$(\Delta r)_{\text{top}} \simeq -\frac{3G_F}{8\sqrt{2}\pi^2 \tan^2\theta_W} m_t^2$$

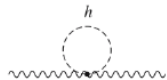
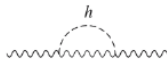
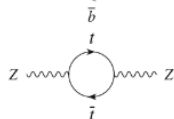
Quadratic dependence on M_{top}

$$(\Delta r)_{\text{Higgs}} \simeq \frac{3G_F m_W^2}{8\sqrt{2}\pi^2} \left(\ln \frac{m_H^2}{m_Z^2} - \frac{5}{6} \right)$$

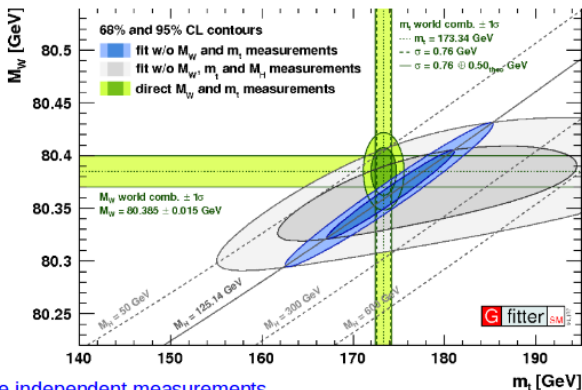
Logarithmic dependence on M_{Higgs}

Measuring M_W , M_{top} and M_{Higgs} one can test the consistency of SM

- Inconsistencies would be an indirect probe of New Physics (BSM particles in loops)
- Reach is much larger than direct searches (for example as resonance produced $pp \rightarrow X \rightarrow \gamma\gamma$)



SM Electroweak Fits



Three independent measurements

To give useful limit W boson mass should be measured much more precisely than M_{top}

M_W should be known ~ 6 MeV (!) to match current uncertainties on M_H and

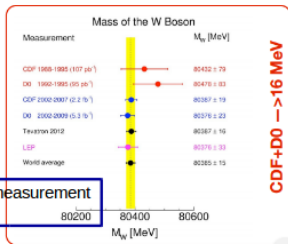
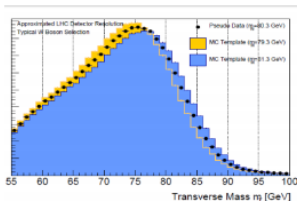
M_{top}

M_W is the leading uncertainty

Measuring W Boson Mass

Method for measuring W boson mass is to fit all three kinematical distributions:

$P_{T,l'}$, $P_{T,\nu}$ and M_{TW}



No LHC measurement so far

CDF+D0 → > 16 MeV

From CDF Run2 Paper

Source	Uncertainty (MeV)
Lepton energy scale and resolution	7 <i>exp</i>
Recoil energy scale and resolution	6 <i>exp</i>
Lepton removal	2 <i>exp</i>
Backgrounds	3 <i>exp</i>
$p_T(W)$ model	5 <i>th</i>
Parton distributions	10 <i>th</i>
QED radiation	4 <i>th</i>
W-boson statistics	12 <i>stat</i>
Total	19

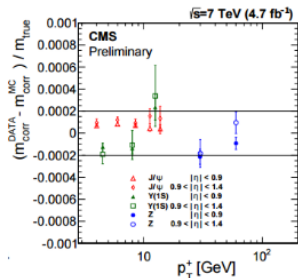
Key is to reduce the uncertainties.
 Challenges from theory:

- PDF
- QCD Initial State Radiation/boson P_T
- QED Final State Radiation affects lepton P_T
- Experimentally “in-situ” calibration for lepton momentum and recoil resolution

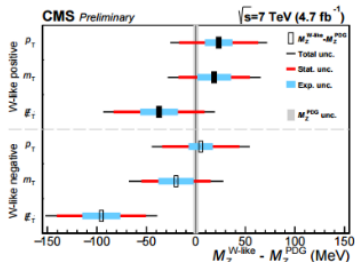
Possible to constrain theory by simultaneous measurements for different \sqrt{s} and for different rapidity (LHCb)

W Boson Mass: Experimental Challenges

Difficulties for precision measurement of W boson mass:
 Experimental uncertainties → lepton momentum scale, modeling of recoil
 Theoretical uncertainties → PDF functions, P_{TW} boson modeling



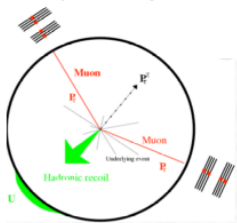
Z boson represents the primary calibration reference ($Z \rightarrow ee, Z \rightarrow \mu\mu$). Use also J/ψ and Y for lower momentum



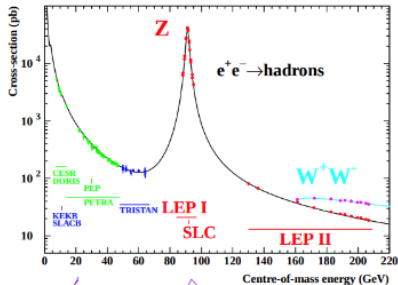
Z boson can also be used to create a W-like by removing the energy deposits of one of the leptons (creating thus a fake Missing Transverse Energy)

Z boson

As for W boson, Z boson can be identified at hadron colliders only in dilepton decays



Measurements at hadron colliders cannot compete with LEP measurements (with some exceptions)



LEP was able to collide electrons with positrons at various centre of mass energies:
 $\sqrt{s} = M_Z$ up to ~ 205 GeV

Z Boson width

Z boson can decay in all known fermions except top quarks
Partial widths $\Gamma(Z \rightarrow f\bar{f})$ can be obtained by taking:

$$\Gamma(W \rightarrow \ell\nu_\ell) = \left(\frac{g}{\sqrt{2}}\right)^2 \frac{M_W}{24\pi} g \rightarrow \frac{g}{\cos\theta_W} \text{ and } M_W \rightarrow M_Z$$

and multiplying $[C_V^{f^2} + C_A^{f^2}]$

$$\Gamma(Z \rightarrow \nu\bar{\nu}) = \frac{g^2}{\cos^2\theta_W} \frac{M_Z}{48\pi} [C_V^{f^2} + C_A^{f^2}]$$

$$\Gamma(Z \rightarrow e^+e^-) = \Gamma(Z \rightarrow \mu^+\mu^-) = \Gamma(Z \rightarrow \tau^+\tau^-) = 84\text{MeV}$$

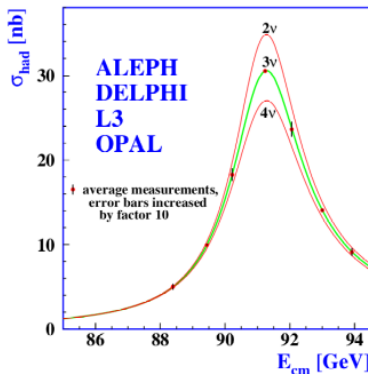
$$\Gamma(Z \rightarrow \nu_e\bar{\nu}_e) = \Gamma(Z \rightarrow \nu_\mu\bar{\nu}_\mu) = \Gamma(Z \rightarrow \nu_\tau\bar{\nu}_\tau)$$

$$\Gamma(Z \rightarrow d\bar{d}) = \Gamma(Z \rightarrow s\bar{s}) = \Gamma(Z \rightarrow b\bar{b}) = 118\text{MeV}$$

$$\Gamma(Z \rightarrow u\bar{u}) = \Gamma(Z \rightarrow c\bar{c}) = 92\text{MeV}$$

Number of neutrinos

$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{\text{had}} + \Gamma_{\text{inv}}$$



$$\sigma_{\text{had}}^0 = \frac{12\pi\Gamma_e\Gamma_{\text{had}}}{m_Z^2\Gamma_Z^2}$$

Measuring Γ_{inv} gives
 the number of
 neutrinos or any
 additional "invisible"
 particle coupled with Z
 $\Gamma_{\text{inv}} = N_\nu \times \Gamma(Z \rightarrow \nu\nu)$

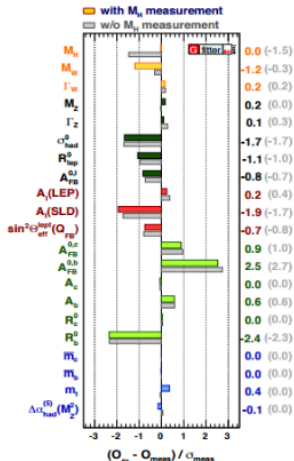
$$N_\nu = 2.9840 \pm 0.0082$$

Does this close any
 possibility to for additional
 neutrino generations?

Z Lineshape at LEP

Measured electroweak observables and fits

M_H [GeV] ^(o)	125.7 ± 0.4	LHC
M_W [GeV]	80.385 ± 0.015	Tevatron
Γ_W [GeV]	2.085 ± 0.042	Tevatron
M_Z [GeV]	91.1875 ± 0.0021	LEP
Γ_Z [GeV]	2.4952 ± 0.0023	LEP
σ_{had}^0 [nb]	41.540 ± 0.037	LEP
R_ℓ^0	20.767 ± 0.025	LEP
$A_{\text{FB}}^{0,\ell}$	0.0171 ± 0.0010	LEP
$A_\ell^{(*)}$	0.1499 ± 0.0018	SLC
$\sin^2\theta_{\text{eff}}^\ell(Q_{\text{FB}})$	0.2324 ± 0.0012	SLC
A_c	0.670 ± 0.027	SLC
A_b	0.923 ± 0.020	SLC
$A_{\text{FB}}^{0,c}$	0.0707 ± 0.0035	LEP
$A_{\text{FB}}^{0,b}$	0.0992 ± 0.0016	LEP
R_c^0	0.1721 ± 0.0030	LEP
R_b^0	0.21629 ± 0.00066	LEP
\bar{m}_c [GeV]	$1.27^{+0.07}_{-0.11}$	Tevatron
\bar{m}_b [GeV]	$4.20^{+0.17}_{-0.07}$	Tevatron
m_t [GeV]	173.18 ± 0.94	Tevatron
$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) (\Delta\nabla)$	2757 ± 10	Tevatron



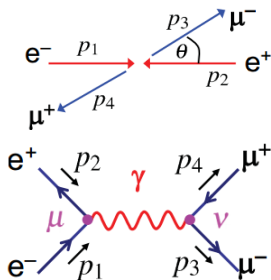
Forward-Backward Asymmetries

Let us revisit process like $e^+e^- \rightarrow \mu^+\mu^-$.
 For QED (exchange of γ boson):

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}(1 + \cos^2 \theta)$$

\Rightarrow symmetrical in θ .

In addition to γ there is the exchange of a
 Z boson (both vector and axial couplings).



Forward-Backward Lepton Asymmetries

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha}{s^2} \left[(1 + \cos^2\theta) + F_{\gamma Z}(\cos\theta) \frac{s(s - M_Z^2)}{(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2} + F_Z(\cos\theta) \frac{s^2}{(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2} \right]$$

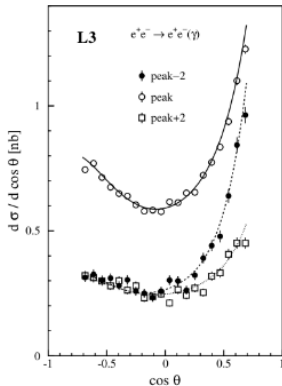
$$F_{\gamma Z}(\cos\theta) = \frac{Q_e Q_\mu}{4 \sin^2\theta_W \cos^2\theta_W} [2g_V^e g_V^\mu (1 + \cos^2\theta) + 4g_Z^e g_A^\mu \cos\theta]$$

$$F_Z(\cos\theta) = \frac{1}{16 \sin^4\theta_W \cos^4\theta_W} [(g_V^{e2} + g_A^{e2})(g_V^{\mu2} + g_A^{\mu2})(1 + \cos^2\theta) + 8g_V^e g_A^e g_V^\mu g_A^\mu \cos\theta]$$

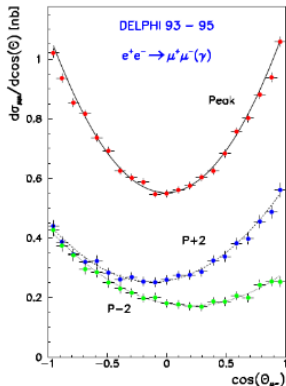
Asymmetrical term $\propto \cos\theta$ appears.

- On resonance $\sqrt{s} = M_Z$:
 - $\gamma^* Z$ interference term vanishes
 - γ term contributes $\sim 1\%$
 - Z contribution dominates
- Off resonance: $s = (M_Z - 3\text{GeV})^2$ $\gamma^* Z$ counts 0.2%

Forward Backward asymmetry at LEP



$d\sigma(ee \rightarrow ee)/d\cos\theta$
 measured at LEP at different \sqrt{s}



$d\sigma(ee \rightarrow \mu\mu)/d\cos\theta$ measured
 at LEP at different \sqrt{s}

Why the two cross sections are different?

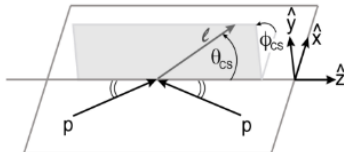
Forward Backward asymmetry at LHC

Similar treatment for $pp \rightarrow \mu\mu$

Additional complications:

Initial states are quarks with pdf

In pp collisions cannot identify polar angle \rightarrow use Collins Soper reference frame



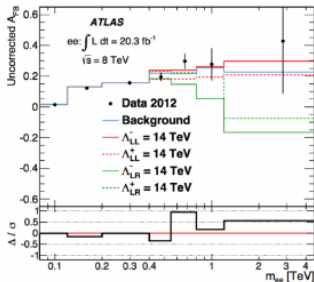
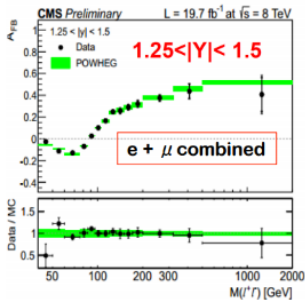
Measure Forward Backward asymmetries

$$A_{FB} = \frac{N_{\cos \theta_{CS}^e \geq 0} - N_{\cos \theta_{CS}^e < 0}}{N_{\cos \theta_{CS}^e \geq 0} + N_{\cos \theta_{CS}^e < 0}}$$

Function of $\sin^2 \theta_W$

$$A_{FB} = \frac{16}{3} \cdot \frac{(1 - 4|Q_f| \sin^2 \theta_W)}{1 + (1 - 4|Q_f| \sin^2 \theta_W)^2} \cdot \frac{(1 - 4|Q_f'| \sin^2 \theta_W)}{1 + (1 - 4|Q_f'| \sin^2 \theta_W)^2}$$

$$\sin^2 \theta_{W,eff}$$



At low masses \rightarrow Consistency of the Standard Model

Parity Violating properties

Test of Vector and Axial couplings of EW

At $M_Z \rightarrow$ Measure $\sin^2 \theta_{W,eff}$ \rightarrow Consistency of SM

At Large invariant masses \rightarrow Sensitivity to physics

BSM and additional gauge bosons

Measurements of $\sin^2 \theta_{W,eff}$

		$\sin^2 \theta_{eff}^{lept}$	Δ/σ (wrt LEP+SLC)
pp	ATLAS	0.2308 ± 0.0012	-0.6
	CMS [6]	0.2287 ± 0.0032	-0.9
ppbar	D0 [5]	0.23146 ± 0.00047	-0.1
	CDF [4]	0.2315 ± 0.0010	-0.03
e ⁺ e ⁻	LEP, $A_{FB}^{0,b}$ [3]	0.23221 ± 0.00029	-
	LEP, $A_{FB}^{b,l}$ [3]	0.23099 ± 0.00053	-
	SLC, A_{LR} [3]	0.23098 ± 0.00026	-
	LEP+SLC [3]	0.23153 ± 0.00016	-
	PDG global fit [46]	0.23146 ± 0.00012	-0.4

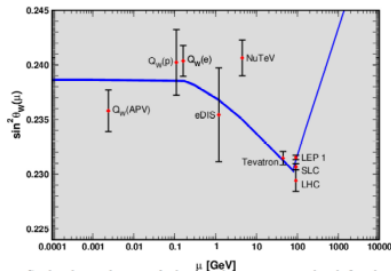
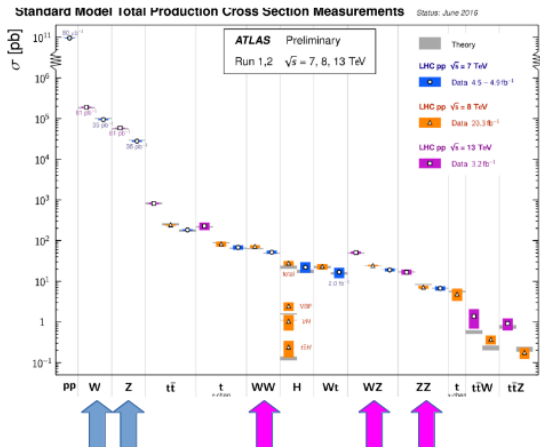


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- 1 Introduction
- 2 Quantum Electrodynamics
 - Experimental observables
 - Amplitude Matrix Calculation
 - Renormalization
- 3 Weak Interactions
- 4 ElectroWeak Unification
 - Experimental Checks
- 5 Multiboson Production

SM Measurements at LHC

Exploit the huge cross section at LHC and statistics to test all the corners of Standard Model

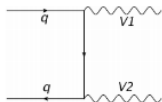


Diboson Measurements

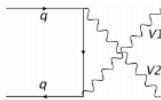
Multiboson production are processes where the peculiar feature of the $SU(2) \times U(1)$ non abelian symmetry can be tested

→ Triple and Quartic vertices not allowed in QED for photon

Inclusive diboson production

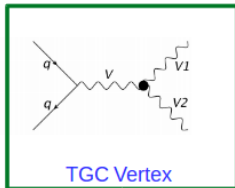


t-channel



u-channel

Production dominated by qq annihilation



- Diboson measurements are an important test of the **Standard Model** and perturbative QCD at TeV scale
- Confirm irreducible background for Higgs analysis (WW, ZZ, Z γ)
- Diboson processes are the backgrounds for New Physics
- Measurement of **anomalous** triple and quartic gauge boson couplings (aTGC and aQGC) is an indirect search for New Physics

WW Measurements

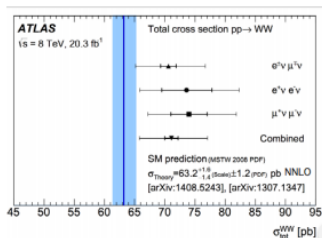
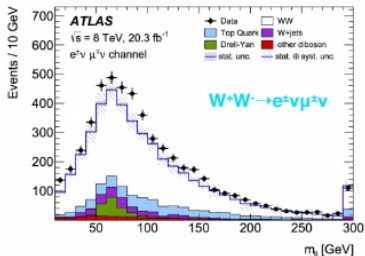
$W+W \rightarrow l\nu l\nu$ dilepton channel

Final state (2 high P_T leptons + $\cancel{E_T}$) is very similar to $t\bar{t}$ decays in dileptonic channel \rightarrow veto on additional jet activity

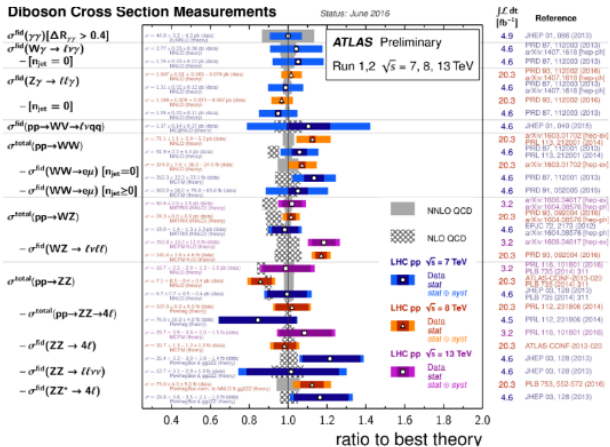
Measurements on "fiducial" cross section, full phase space cross section and differential cross section \rightarrow set anomalous Triple Gauge Couplings (aTGC)

Background dominated by $t\bar{t}$ events ($\sim 20\%$)

Prediction undershoots data in ATLAS \rightarrow Shape is OK but overall normalization is off by 2σ \rightarrow Systematic shift on jet veto efficiency (from NLO to NNLO)



Diboson Measurements



Effective Field Theory Framework

Effective Lagrangians can be used to probe for new physics at energy scale Λ in a model independent way.

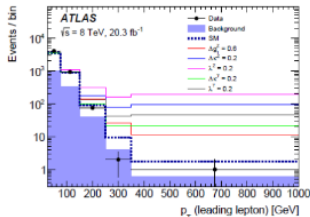
Assume Λ lies above energy range of experiments

$$\mathcal{L}_{eff} = \sum_n \frac{1}{\Lambda^n} \sum_i \alpha_i^{(n)} \mathcal{O}_i^{(n)}$$

$\alpha_i^{(n)}$ - coupling coefficients

$\mathcal{O}_i^{(n)}$ - operators of dimension mass^{4+n}

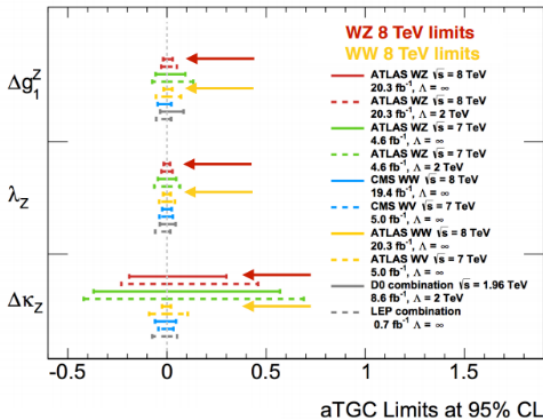
Neutral aTGCs (ZZV) forbidden in SM
 Additional couplings for QGC's and
 neutral couplings from dim.8 operators.



$$\mathcal{L} = -ig_{WWV} [g_1^V (W_{\mu\nu}^\dagger W^{\mu\nu} - W^{\dagger\mu} W_{\mu\nu}) V^\nu + \kappa^V W_\mu^\dagger W_\nu V^{\mu\nu} + \frac{\lambda^V}{m_W^2} W_{\rho\mu}^\dagger W_\nu^\mu V^{\nu\rho}]$$

$$\text{SM} : g_1^V = \kappa_V = 1; \lambda_V = 0$$

anomalous Triple Gauge Couplings

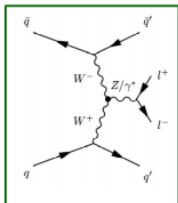


Vector Boson Scattering Processes

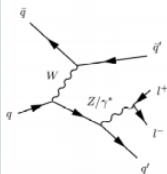
Identify more **exclusive** processes involving **Triple Boson** vertices

Vector Boson Fusion (VBF) production of Z boson

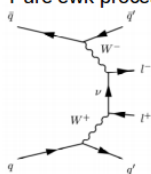
Pure ewk processes diagrams



Signal



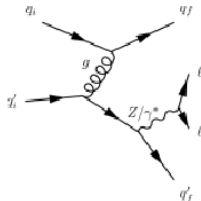
Z-Brehmstrahlung-like



Mutiperipheral

VBF diagram cannot be calculated independently from Z-brehm or mutiperipheral without breaking gauge invariance

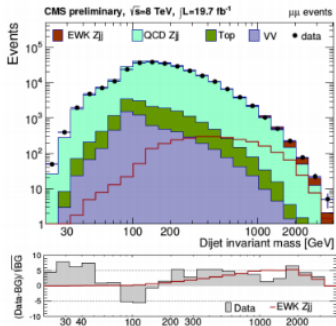
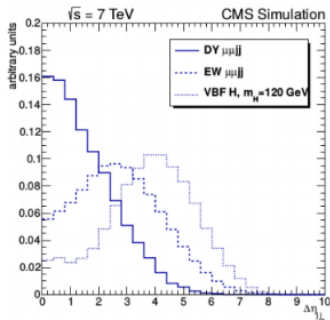
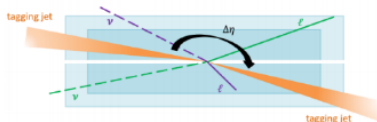
Main background Z+2 jet is reduced by topological selections



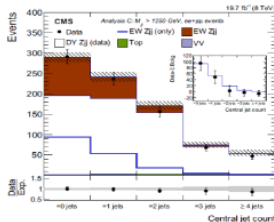
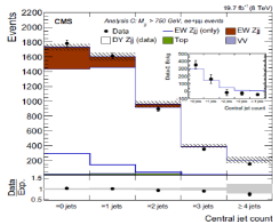
Vector Boson Fusion Processes

Two VBF jets are different wrt jets from Z+jet:

- Large M_{jj} and $|\Delta\eta_{jj}|$
- Low central hadronic activity



Vector Boson Fusion Processes

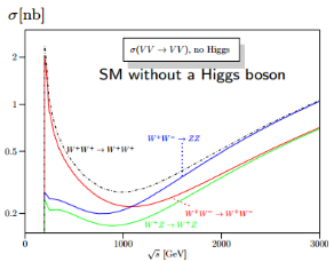


	Fiducial dijet cuts	EWK cross section [fb]	
		Data	Theory
ATLAS Zjj	$p_T > 55, 45 \text{ GeV}, y < 4.4;$ $m_{jj} > 250 \text{ GeV};$ jet veto; $p_{T, \text{balance}} < 0.15$	$54.7 \pm 4.6 \text{ (stat)}^{+9.8}_{-10.4} \text{ (syst)} \pm 1.5 \text{ (lumi)}$	46.1 ± 1.0 [Powheg+Py]
CMS Zjj	$p_T > 25 \text{ GeV}, \eta < 5;$ $m_{jj} > 120 \text{ GeV};$	$174 \pm 15 \text{ (stat)} \pm 40 \text{ (syst)}$	208 ± 18 [LO MG+Py]
CMS Wjj	$p_T > 60, 50 \text{ GeV}, \eta < 4.7;$ $m_{jj} > 1 \text{ TeV}$	$420 \pm 40 \text{ (stat)} \pm 90 \text{ (syst)}$	500 ± 30 [LO MG+Py]

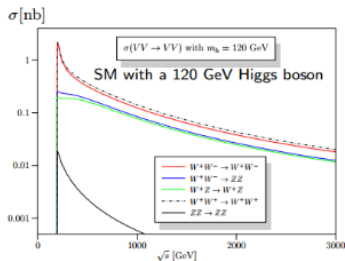
Vector Boson Fusion Processes

Vector Boson Scattering is a key process to experimentally probe the EWSB

total cross sections for $VV \rightarrow VV$ as a function of M_{VV}



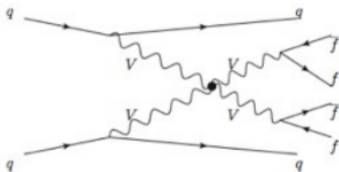
Unitarity violated for $\sqrt{s} \rightarrow \sim 1$ TeV



Restored with Higgs boson

Vector Boson Fusion Processes

In **Vector Boson Scattering (VBS)** processes LHC proton beam serves as source of V bosons



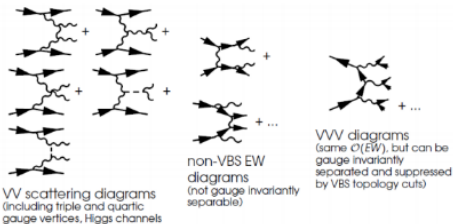
Final state	Process	VVjj-EW	VVjj-QCD
$\ell^\pm \nu \ell'^\pm \nu' jj$ (same sign, arbitrary flavor)	$W^\pm W^\pm$	19.5 fb	18.8 fb
$\ell^\pm \nu \ell'^\mp \nu' jj$ (opposite sign)	$W^\pm W^\mp$	91.3 fb	3030 fb
$\ell^+ \ell^- \nu' \nu' jj$	ZZ	2.4 fb	162 fb
$\ell^\pm \ell^\mp \ell'^\pm \nu' jj$	$W^\pm Z$	30.2 fb	687 fb
$\ell^\pm \ell^\mp \ell'^\pm \ell'^\mp jj$	ZZ	1.5 fb	106 fb



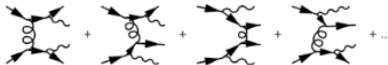
Extremely small cross section. Even smaller than ttH production cross section

Vector Boson Fusion Processes

Signal Processes (EW)



Background Processes (QCD)



Vector Boson Fusion Processes

Selection:

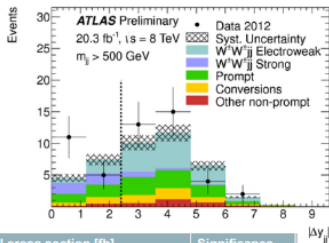
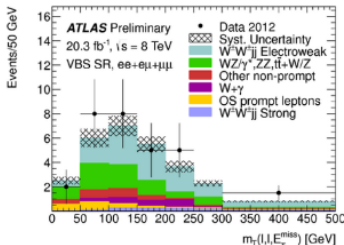
2 Same Sign leptons with
 $P_T > 25$ GeV
 B-jet veto $m_{jj} > 20$ GeV

3rd lepton veto $P_T > 7$ GeV

$|M_{j1} - M_{j2}| > 10$ GeV, $E_{T\text{Miss}} > 40$ GeV

$M_{jj} > 500$ GeV (inclusive)

$M_{jj} > 500$ GeV + $|\Delta\eta_{jj}| > 2.4$ (VBS enhanced)



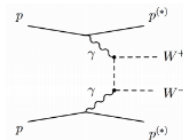
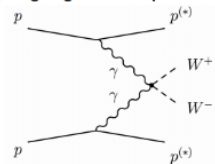
	Fiducial dijet cuts	Fiducial cross section [fb]	Significance		
		Data	Theory	Obs.	Exp'd
CMS	incl. W^*W^{*ij} $p_T > 20$ GeV, $ \eta < 5$, $m_{jj} > 300$ GeV, $ \Delta\eta_{jj} > 2.5$	$4.0^{+2.4}_{-2.0}$ (stat) $^{1.1}_{-1.0}$ (syst)	5.8 ± 1.2 [VBFNLO]	2.0σ	3.1σ
ATLAS	incl. W^*W^{*ij} $p_T > 30$ GeV, $ \eta < 4.5$, $m_{jj} > 500$ GeV	2.1 ± 0.5 (stat) ± 0.3 (syst)	1.52 ± 0.11 [Powheg+Py8]	4.5σ	3.4σ
ATLAS	EWK W^*W^{*ij} as above + $ \Delta\eta_{jj} > 2.4$	1.3 ± 0.4 (stat) ± 0.2 (syst)	0.95 ± 0.06 [Powheg+Py8]	3.6σ	2.8σ

Vector Boson Fusion Processes

Vector Boson Scattering is one of the processes that can prove aQGC.

The others are:

- Exclusive $\gamma\gamma \rightarrow WW$ production
- Triple gauge boson production

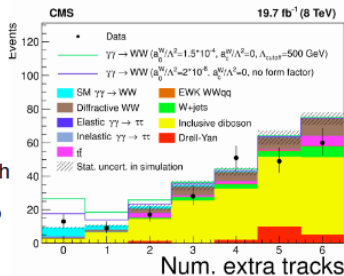


Protons in the final state do not dissociate

Very clean final state \rightarrow

No additional tracks (but remember pileup)

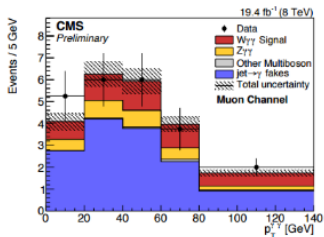
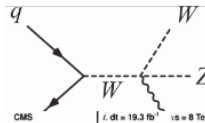
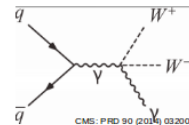
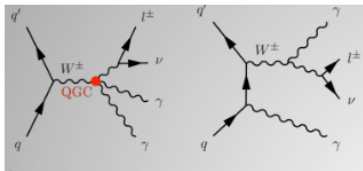
\rightarrow Possible to tag forward proton with small t using dedicated forward detectors (TOTEM for CMS and AFP for ATLAS)



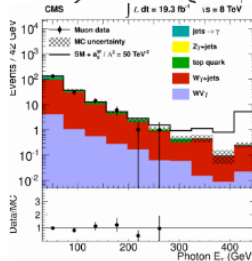
Vector Boson Fusion Processes

Triple Boson production

Processes measured so far: $W\gamma\gamma$, $Z\gamma\gamma$



CMS-SMP-15-008 Feb. 2016



Vector Boson Fusion Processes

N.B.: For full fiducial definition, see papers

Observation of $Z\gamma\gamma$ production!

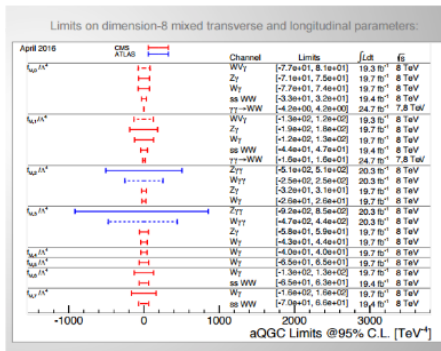
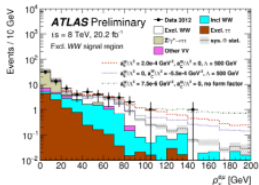
	Fiducial cuts	Fiducial cross section [fb]		Sig.
		Data	Theory	
ATLAS $Z\gamma\gamma$ ($Z \rightarrow l^+l^-$)	Z selection ($p_T^l > 25$ GeV) + 2γ : $p_T > 15$ GeV, $ \eta < 2.37$, isolated; $ \Delta R_{\gamma,l} > 0.4$; $ \Delta R_{\gamma,\gamma} > 0.4$	$5.07^{+0.73}_{-0.68}$ (stat) $^{+0.41}_{-0.38}$ (syst) ± 0.1 (lumi)	$3.70^{+0.21}_{-0.11}$ [NLO MCFM]	6.3 σ
CMS $Z\gamma\gamma$ ($Z \rightarrow l^+l^-$)	Z selection ($p_T^l > 10, 15$ GeV) + 2γ : $p_T > 15, 20$ GeV, $ \eta < 2.5$; $ \Delta R_{\gamma,l} > 0.4$; $ \Delta R_{\gamma,\gamma} > 0.4$	12.7 ± 1.4 (stat) ± 1.8 (syst) ± 0.3 (lumi)	12.95 ± 1.47 [aMC@NLO+Py8]	5.9 σ
ATLAS $Z\gamma\gamma$ ($Z \rightarrow \nu\nu$)	$E_{\text{miss}} > 110$ GeV + 2γ : $p_T > 22$ GeV, $ \eta < 2.37$, isolated; $ \Delta R_{\gamma,\gamma} > 0.4$	$2.5^{+1.0}_{-0.9}$ (stat) ± 1.1 (syst) ± 0.1 (lumi)	$0.737^{+0.006}_{-0.032}$ [NLO MCFM]	
ATLAS $W\gamma\gamma$ ($W \rightarrow l\nu$)	W selection ($p_T^l > 20$ GeV) + 2γ : $p_T > 20$ GeV, $ \eta < 2.37$, isolated; $ \Delta R_{\gamma,l} > 0.7$; $ \Delta R_{\gamma,\gamma} > 0.4$	$6.1^{+1.1}_{-1.0}$ (stat) ± 1.2 (syst) ± 0.2 (lumi)	2.90 ± 0.16 [NLO MCFM]	3 σ
CMS $W\gamma\gamma$ ($W \rightarrow l\nu$)	W selection ($p_T^l > 25$ GeV) + 2γ : $p_T > 25$ GeV, $ \eta < 2.5$; $ \Delta R_{\gamma,l} > 0.4$; $ \Delta R_{\gamma,\gamma} > 0.4$	6.0 ± 1.8 (stat) ± 2.3 (syst) ± 0.2 (lumi)	4.76 ± 0.53 [aMC@NLO+Py8]	2.4 σ

Evidence for $W\gamma\gamma$ production!

Vector Boson Fusion Processes

Possible BSM physics can be expressed by higher-dimensional effective operators supplementing the SM Lagrangian

- Dimension-8 theories are the lowest order leading to aQGCs
- The theories lead to operators contributing to the different couplings, with coefficients that are constrained with this data



What you should know

- Revised the basic of the EWK theory – basic principle and how it was built
- Revised few fundamental measurements
- Understand what are the EWK measurements relevant for LHC

Thank you for your attention !