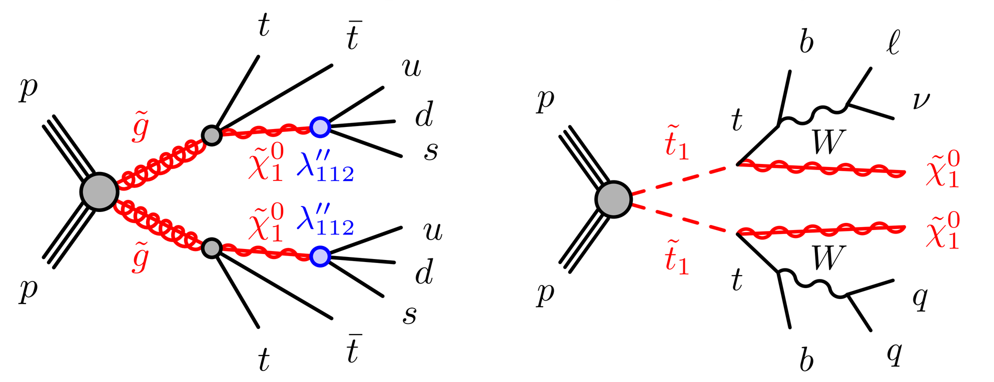
Towards a unitarity based computation of NLO corrections in QCD with heavy quarks.

Vasily Sotnikov, University of Freiburg

Motivation

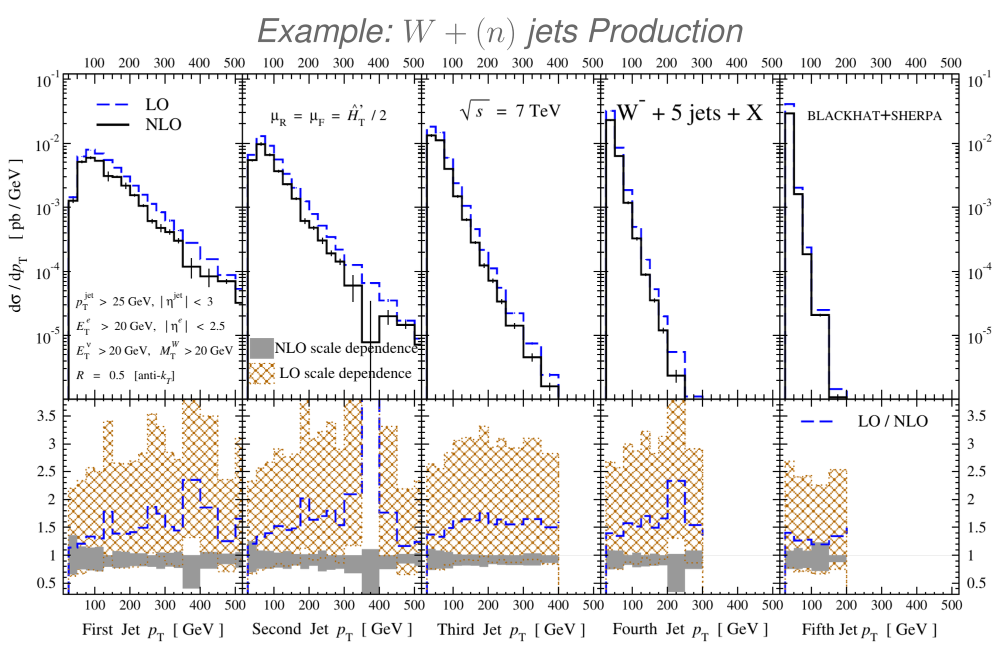
Associated production of heavy b, t quarks with vector bosons (W,Z) and jets constitutes main irreducible background to a number of relevant processes at the LHC. For example WH and ZH production, single top production, and new physics searches, where heavy new particles decay into a spray of light ones, e.g. *SUSY* searches for the 3rd generation squark pair production.



Our aim is to provide QCD NLO radiative corrections to this process class, extending existing predictions for $pp \to W(l\nu)b\bar{b}$, $pp \to W(l\nu)b\bar{b}j$ [1] , and top pair production $t\bar{t} + (\leq 3)j$ [2, 3]. In order to provide these predictions, virtual corrections to the processes have to be calculated. In our approach, we employ generalized D-dimensional unitarity, where loop amplitudes are constructed out of on-shell tree amplitudes. Unitarity methods have been successfully applied to the production of W/Z with up to five light jets [4]. In this work, the objective is to extend the framework to processes with massive final states in order to obtain NLO predictions including bottom and top quark mass effects.

Impact of Quantum Corrections

Whereas the leading perturbative order in the coupling constant gives a good qualitative prediction, a quantitatively reliable prediction requires the inclusion of at least next-to-leading order quantum corrections in the strong coupling constant. Thereby increasing the precision of the computations and, in addition, reducing the dependence on unphysical factorization and renormalization scales. Displayed are predictions [4] for a W-boson produced in association with up to 5 jets. The transverse momentum distribution of the softest jet is given. The scale bands in the lower panels shrink significantly when quantum corrections are included.



Generalized Unitarity Framework

The traditional approach to evaluation of 1-loop amplitudes relies on direct computation of all Feynman diagrams for a given process. Each diagram is a loop integral of the form

$$I_N = \int [dl] \frac{\mathcal{N}(p_1, p_2, \ldots, p_N; l)}{d_1 d_2 \ldots d_N}, \qquad (*)$$

with N being the total number of legs, and is reduced using *Passarino-Veltman decomposition* to a set of master scalar integrals. While being perfectly adequate for dealing with relatively low multiplicities, this approach is challenging when dealing with large multiplicities because of the growth of complexity of each diagram, factorial growth of the total number of diagrams, spurious singularities and gauge cancellations.

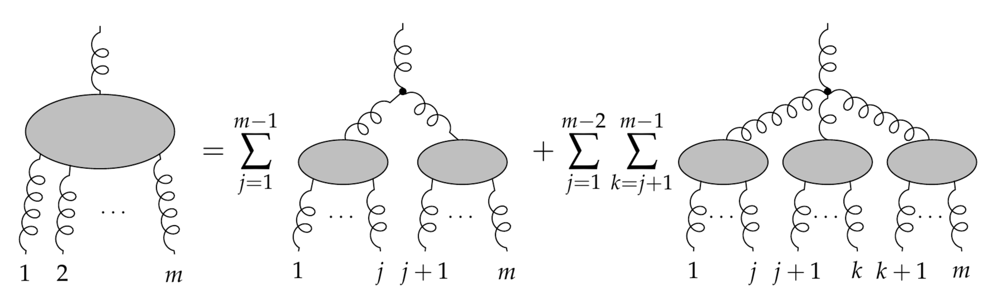
An alternative framework for computing 1-loop amplitudes is *Generalized Unitarity*, which heavily exploits properties of amplitudes as an analytic function of particles' momenta such as 1-particle ***poles*** and ***branch cuts***. The discontinuities across this branch cuts are given in terms of scattering amplitudes of lower order in the perturbation expansion. This information can be used to reconstruct the full one-loop amplitude from tree amplitudes. The Generalized Unitarity framework consists of several independently useful ingredients:

- colour decomposition [5];
- efficient numerical tree amplitude generation;
- basis of 1-loop master integrals[6];
- reduction of the 1-loop amplitude at the ***integrand*** level [7–9].

Colour decomposition [5] disentangles colour and kinematics and produces a minimal set of independent gauge-invariant contributions to the full 1-loop amplitude, known as ***colour ordered primitive*** amplitudes. These then can be independently evaluated using tree level *colour ordered* amplitudes, thus ensuring individual gauge independence.

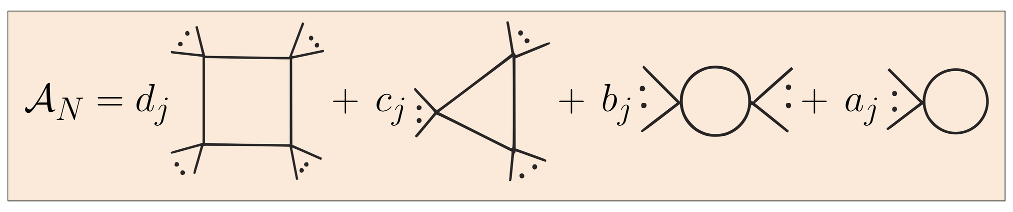
Off-shell Recursion Relations

To calculate tree amplitudes in a flexible and a robust way (requiring good numerically efficiency, D-dimensional evaluation, massive particles, complex kinematics), we make use of ***off-shell recursion*** relations, introduced by Berends and Giele [10]. The building blocks are currents (amplitudes with one off-shell leg) and the interactions(Feynman rules) provided by the theory.



Integrand Level Reduction

Instead of computing each Feynman diagram $(*)$ we view the whole (colour ordered) amplitude as a sum of all possible box, triangle, bubble and tadpole master integrals with some coefficients, proportional to the products of tree amplitudes.



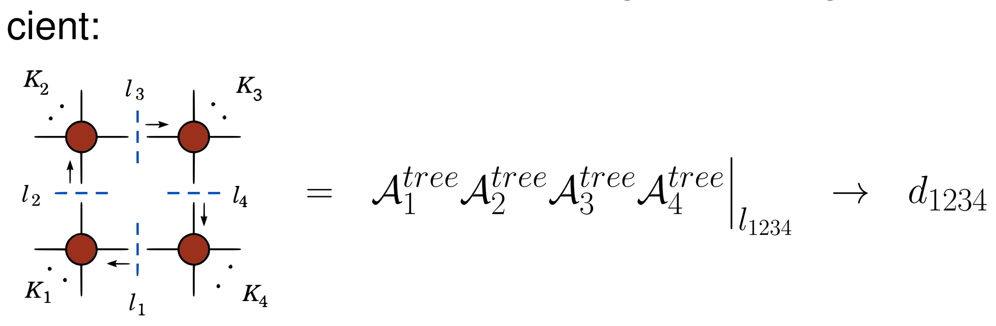
We can extract this coefficients numerically one by one by considering the decomposition of the ***integrand*** of the amplitude above [8]:

$$A_N(l) = \sum_{i_1 i_2 i_3 i_4} \frac{\bar{d}_{i_1 i_2 i_3 i_4}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} + \sum_{i_1 i_2 i_3} \frac{\bar{c}_{i_1 i_2 i_3}(l)}{d_{i_1} d_{i_2} d_{i_3}} + \sum_{i_1 i_2} \frac{\bar{b}_{i_1 i_2}(l)}{d_{i_1} d_{i_2}} + \sum_{i_1} \frac{\bar{a}_{i_1}(l)}{d_{i_1}},$$

where each term corresponds to one of the basis integrals (d_i are the inverse scalar propagators). Scalar integral coefficients are now contained in the numerator functions, which are polynomials in loop momenta and can be reconstructed by performing ***unitarity cuts***: we choose loop momenta $l_{ij\ldots k}$ such that some of the particles in the loop go on shell,

$$d_i(l_{ij\ldots k}) = d_j(l_{ij\ldots k}) = \cdots = d_k(l_{ij\ldots k}) = 0,$$

the integrand of the loop amplitude then factorizes to the product of tree amplitudes (summed over internal polarization states). For example, performing a quadruple cut in four dimensions will isolate a single box integral coefficient:



Cutting fewer propagators isolates a coefficient of the lower point integral, the contributions from higher point integrals however have to be identified and subtracted during evaluation to avoid double counting.

Generally all possible cuts have to be considered, but in the case, when there are no massive particles inside loops, tadpole and 1-leg bubble integrals vanish, which presents an opportunity for simplifications.

Rational Part

Using only cuts in four dimensions we are able only to compute the cut-constructable part of the amplitude, since we drop possible dependence on the dimensionality of the integral coefficients. There exist many different methods for extracting this missing piece of the amplitude. The most straightforward approach is the generalization of the 4-dimensional cuts to D dimensions in dimensional regularization [11] and applying the same reduction procedure twice in two different ***integer*** dimensions. The rational part is then reconstructed by coefficients of higher dimensional basis integrals. Going to higher dimensions however introduces performance penalties because of larger polarization sums.

Alternatively one can make use of a special set of Feynman rules [12], *SUSY*-inspired decompositions [13], on-shell recursion [14], etc. Most of these methods however either do not fit naturally in the numerical unitarity framework or are not set up to deal with massive particles inside loops.

Conclusions and Outlook

Generalized Unitarity provides a complete general framework for numerical evaluation of loop amplitudes. The current version of the BLACKHAT library [7] implements these ideas in the case of massless particles inside loops and is able to compute high multiplicity NLO amplitudes.

We are working on the extension of the library to include the effects of heavy quarks, which requires the following ingredients to be implemented:

- the loop momentum parametrizations required to put internal particles on their mass shell;
- the extraction of the tadpole and 1-leg bubble coefficients;
- some additional special cases for the triple and quadruple cuts;
- the rational part computation;
- resolve the conflict between double cuts and renormalization induced by self-energy insertions on the external legs;
- import missing master integrals.

We then can use the library first to verify existing results and finally to produce new phenomenology for processes with heavy jets and high multiplicity.

References

1. Luisoni, G., Oleari, C. & Tramontano, F. $Wb\bar{b}j$ production at NLO with POWHEG+MiNLO. ***JHEP*** **04,** 161 (2015).
2. Höche, S., Maierhoefer, P., Moretti, N., Pozzorini, S. & Siegert, F. Next-to-leading order QCD predictions for top-quark pair production with up to three jets. arXiv: `1607.06934 [hep-ph]` (2016).
3. Ellis, R. K., Giele, W. T., Kunszt, Z. & Melnikov, K. Masses, fermions and generalized D-dimensional unitarity. ***Nucl. Phys.*** **B822,** 270–282 (2009).
4. Bern, Z. ***et al.*** Next-to-leading order $W + 5$-jet production at the LHC. ***Phys. Rev. D*** **88,** 014025 (1 2013).
5. Ita, H. & Ozeren, K. Colour Decompositions of Multi-quark One-loop QCD Amplitudes. ***JHEP*** **02,** 118 (2012).
6. Ellis, R. K. & Zanderighi, G. Scalar one-loop integrals for QCD. ***JHEP*** **02,** 002 (2008).
7. Berger, C. F. ***et al.*** Automated implementation of on-shell methods for one-loop amplitudes. ***Phys. Rev. D*** **78,** 036003 (3 2008).
8. Ossola, G., Papadopoulos, C. G. & Pittau, R. Reducing full one-loop amplitudes to scalar integrals at the integrand level. ***Nucl. Phys.*** **B763,** 147–169 (2007).
9. Ellis, R. K., Giele, W. T. & Kunszt, Z. A Numerical Unitarity Formalism for Evaluating One-Loop Amplitudes. ***JHEP*** **03,** 003 (2008).
10. Berends, F. A. & Giele, W. T. Recursive Calculations for Processes with n Gluons. ***Nucl. Phys.*** **B306,** 759–808 (1988).
11. Giele, W. T., Kunszt, Z. & Melnikov, K. Full one-loop amplitudes from tree amplitudes. ***JHEP*** **04,** 049 (2008).
12. Garzelli, M. V., Malamos, I. & Pittau, R. Feynman rules for the rational part of the Electroweak 1-loop amplitudes. ***JHEP*** **01.** [Erratum: JHEP10,097(2010)], 040 (2010).
13. Badger, S. D. Direct Extraction Of One Loop Rational Terms. ***JHEP*** **01,** 049 (2009).
14. Bern, Z., Dixon, L. J. & Kosower, D. A. Bootstrapping multi-parton loop amplitudes in QCD. ***Phys. Rev.*** **D73,** 065013 (2006).