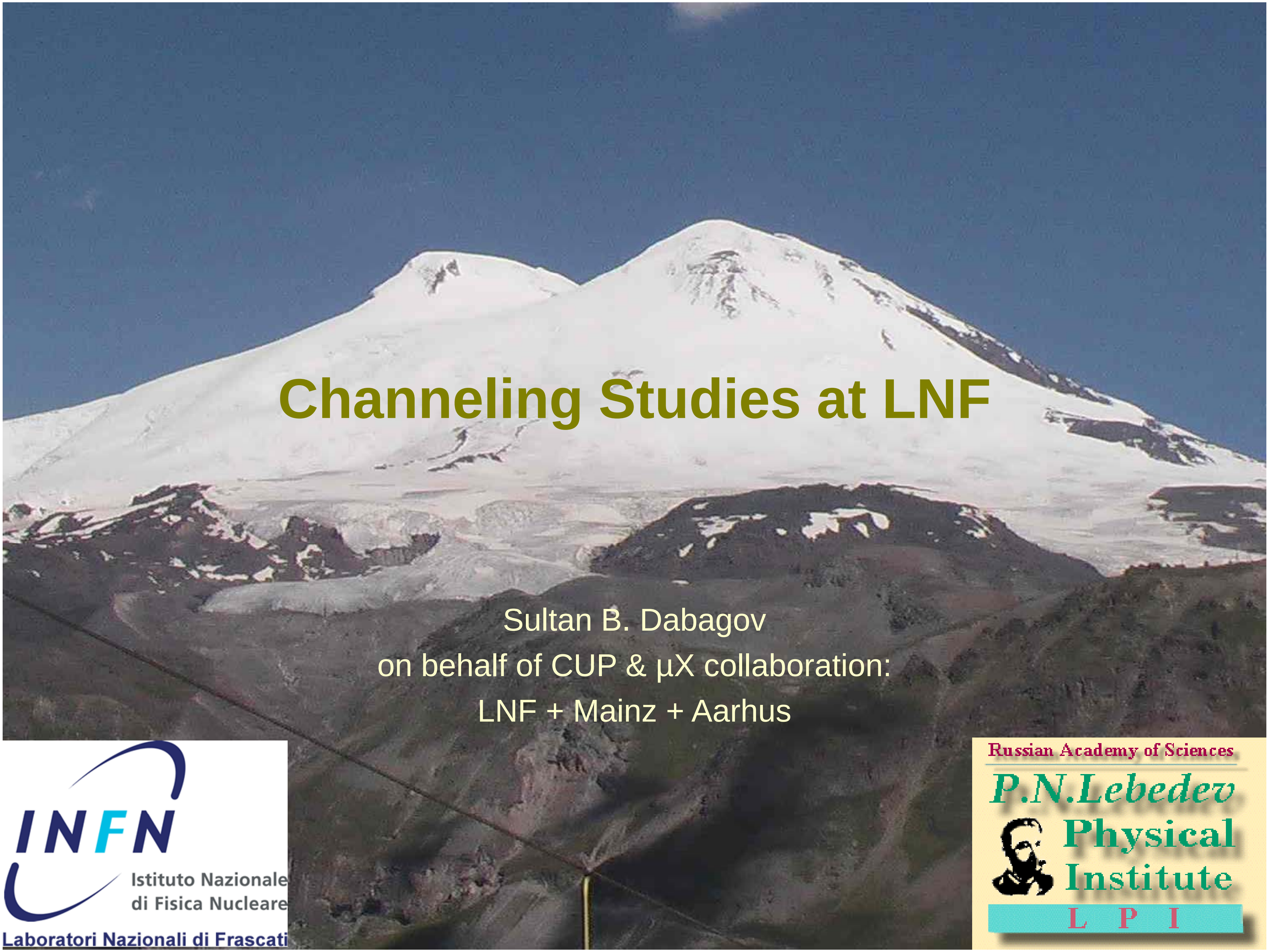


Channeling Studies at LNF

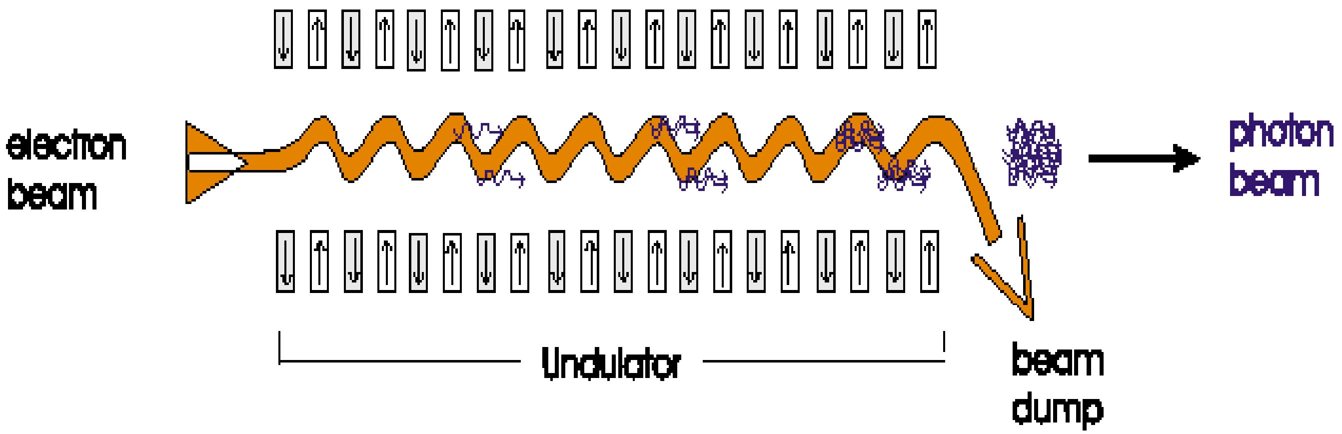
Sultan B. Dabagov
on behalf of CUP & µX collaboration:
LNF + Mainz + Aarhus

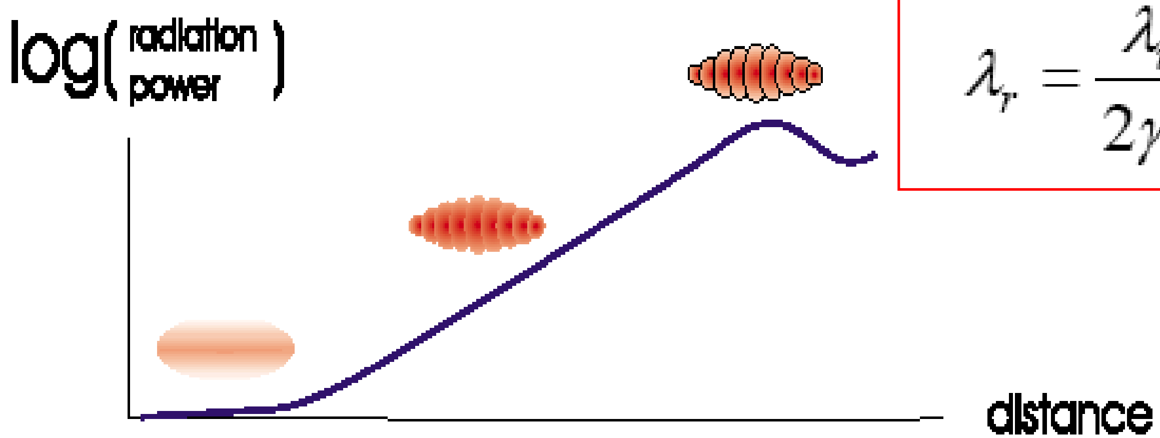
Istituto Nazionale di Fisica Nucleare
Laboratori Nazionali di Frascati

Russian Academy of Sciences
P.N.Lebedev Physical Institute
LPI

@ Free Electron Laser

Self-Amplified-Spontaneous-Emission
(No Mirrors - Tunability – Harmonics)





$$\lambda_r = \frac{\lambda_u}{2\gamma^2}(1 + a_u{}^2 + \gamma^2\theta^2)$$

@ FEL activity at LNF

SPARC

Thomson → MAMBO …

SPARC → Channeling

SPARC → Plasmon

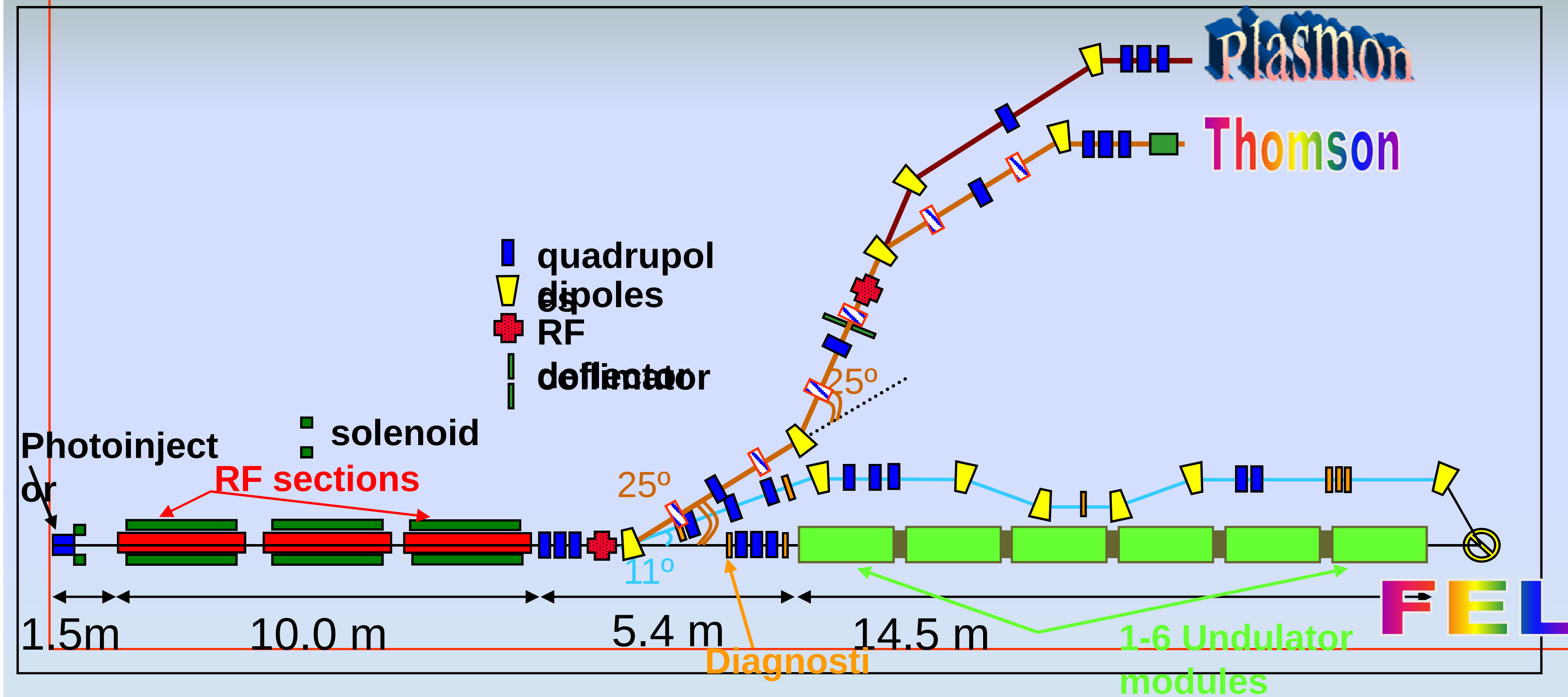
SPARC → SEEDING

FEL project at Frascati

Source
Pulsed
Self Amplified
Radiation
Coherent

sparXfel

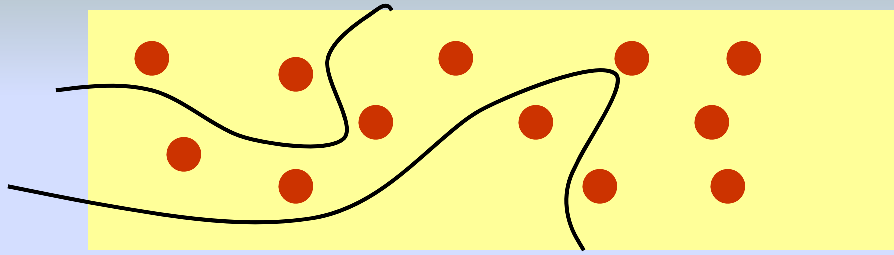
@ SPARC & Applications



Application	Bunch charge (nC)	Energy (MeV)	Bunch length rms (ps)	Norm. rms emittance (μm)	Energy Spread (%)
PlasmonX	0.025	100-200	0.025	0.1	0.2
Thomson	1-3	28-200	3	2-5	0.2-0.1

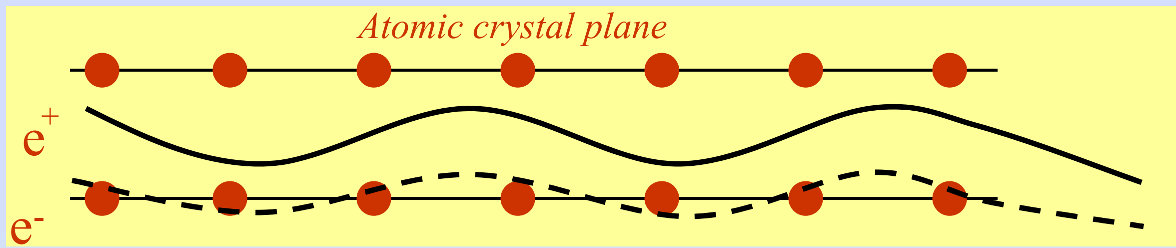
@ Channeling of Charged Particles

@ Amorphous:

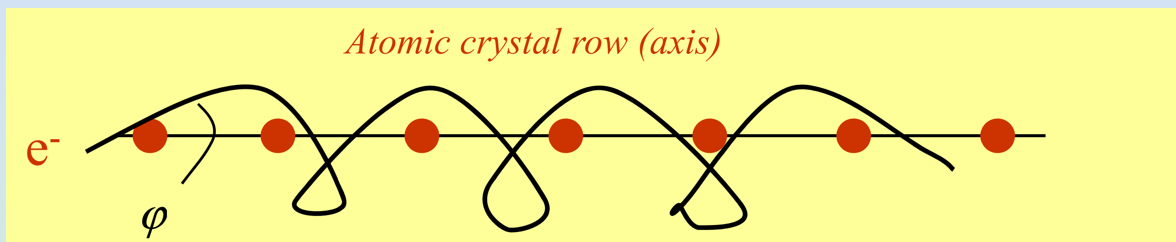


@ Channeling:

planar channeling



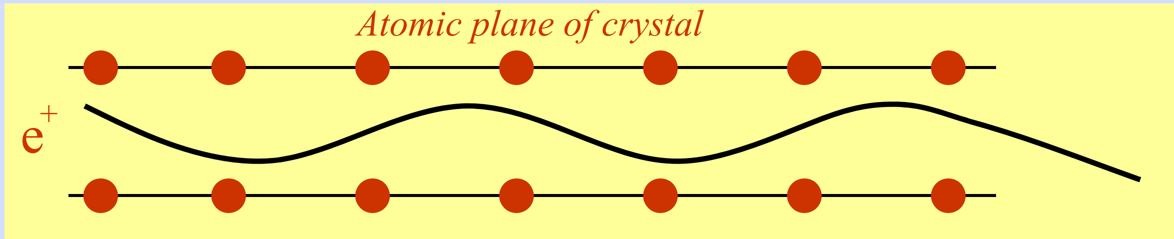
axial channeling



$\varphi << 1$ $\quad (\varphi < \varphi_L \sim \sqrt{U/E})$ *- the Lindhard angle is the critical angle for the channeling*

@ Channeling of Charged Particles & Channeling Radiation

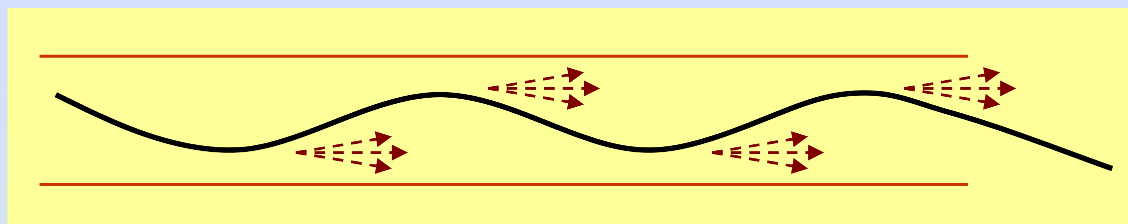
@ Channeling:



$\varphi << 1 \qquad (\varphi < \varphi_L \sim \sqrt{U/E})$ - *the Lindhard angle is the critical angle for the channeling*

@ Channeling Radiation:

$$\omega = \omega(\theta) = \frac{\omega_{fi}}{1 - \beta_{\parallel} \cos\theta}$$



ω_{fi} - *optical frequency* $\longrightarrow$ *Doppler effect* $\longrightarrow$ $\omega_0 \gamma^{3/2}$ - $\omega_0 \gamma^2$

$(\gamma >> 1)$

Powerful radiation source of X-rays and γ-rays:

- polarized
- Tunable (keV - MeV)
- narrow forwarded

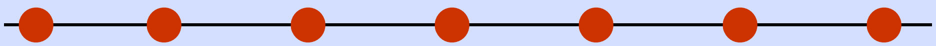
@ Channeling: Continuum model

$$V(r) = \frac{Z_1 Z_2 e^2}{r} \varphi(r/a)$$

screening function of Thomas-Fermi type

$$a = .8853 a_0 \left(Z_1^{1/2} + Z_2^{1/2}\right)^{-2/3}$$

screening length



$$\varphi(r/a): \quad \sum_{i=1}^{3} \alpha_i \exp(-\beta_i r/a)$$ Molier's potential

$$1 - \left[1 + \frac{Ca}{r^2}\right]^{-1/2} \qquad C^2 \approx 3$$ Lindhard potential

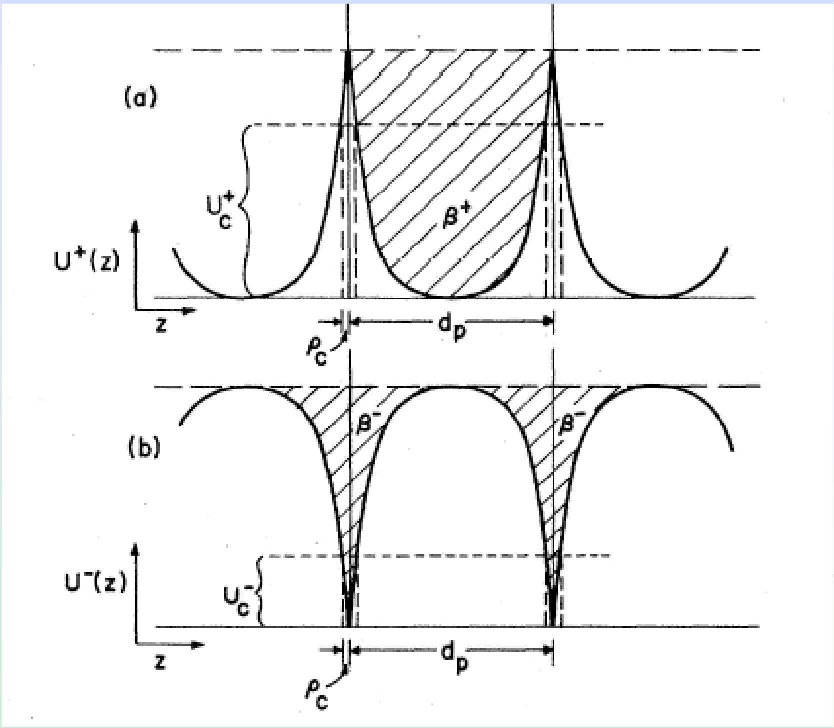
…… Firsov, Doyle-Turner, etc.

Lindhard:

Continuum model –

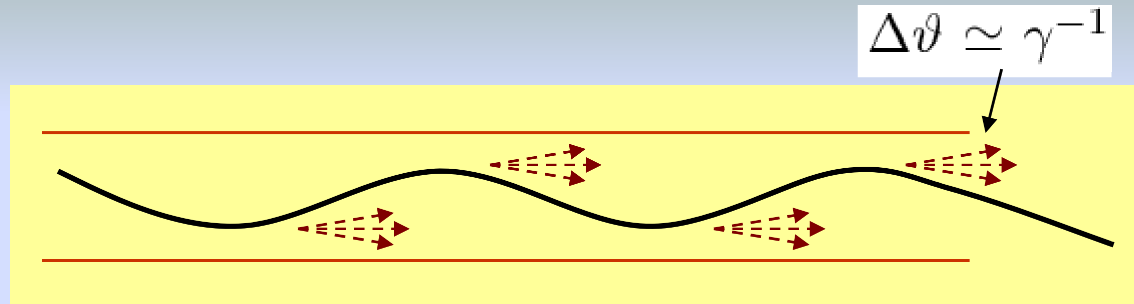
continuum atomic plane/axis potential

$$V_{RS}(\rho) = \frac{1}{d} \int_{-\infty}^{+\infty} V\left(\sqrt{\rho^2 + x^2}\right) dx$$



@ Channeling Radiation

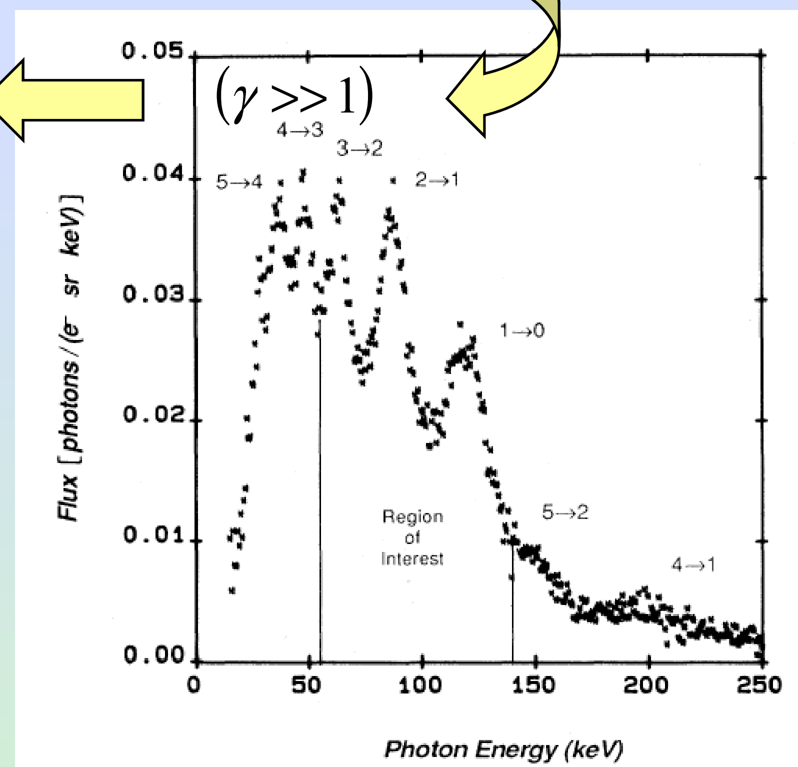
@ Channeling Radiation:

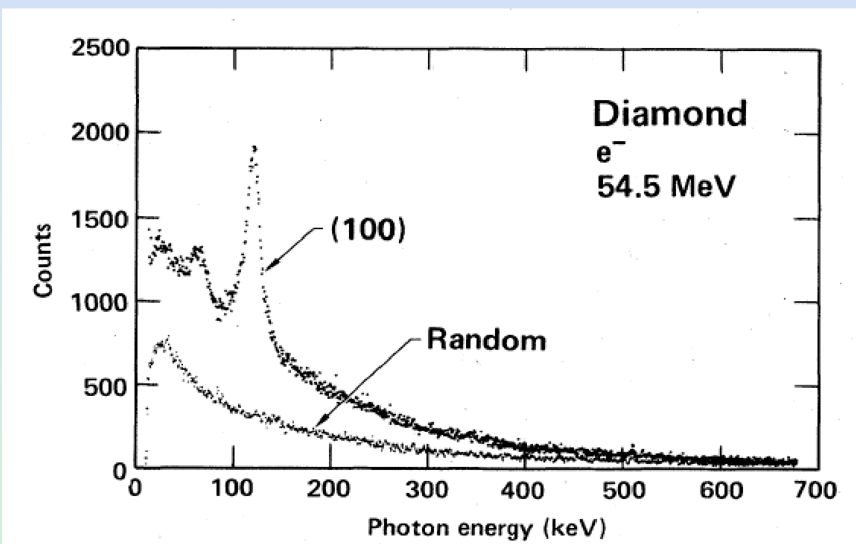


$$\omega = \omega(\theta) = \frac{\omega_{fi}}{1 - \beta_{\parallel} \cos\theta}$$

ω_{fi} - *optical frequency* $\longrightarrow$ *Doppler effect* $\longrightarrow$ $\omega_0 \gamma^{3/2} \div \omega_0 \gamma^2$

Powerful radiation source of X-rays and γ-rays:
- •polarized
  - •tunable
    - •narrow forwarded





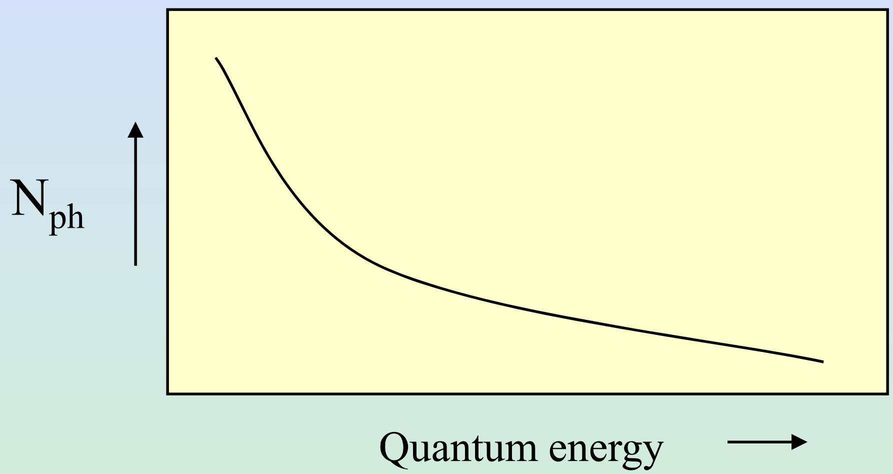
@ Bremsstrahlung & Coherent Bremsstrahlung vs Channeling Radiation

@ amorphous - electron:

- Radiation as sum of independent impacts with atoms
- Effective radius of interaction – a_{TF}
- Coherent radiation length $l_{coh} >> a_{TF}$
- Deviations in trajectory less than effective radiation angles:

$$\Delta\theta \propto a_{TF} / p \qquad \Delta\vartheta \simeq \gamma^{-1}$$

$$\left(\frac{d^2 I}{d\omega\Omega}\right)_{BR} \simeq (\pi L_R)^{-1}\gamma^2 \frac{1+\gamma^4\theta^4}{(1+\gamma^2\theta^2)^4} \longrightarrow \left(\frac{dI}{d\omega}\right)_{BR} \simeq \frac{4}{3} L_R^{-1}$$



@ Bremsstrahlung & Coherent Bremsstrahlung vs Channeling Radiation

@ interference of consequent radiation events:

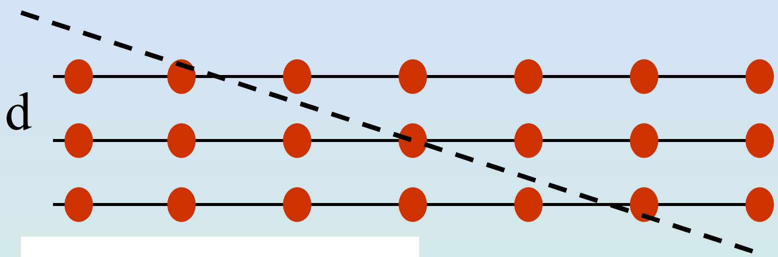
phase of radiation wave $\longrightarrow$ $(\omega t - \mathbf{kr}(t))$

Radiation field as interference of radiated waves:

$$l_{coh} \approx \frac{v}{\omega - \mathbf{k}\dot{\mathbf{r}}} = \frac{\lambda\beta}{1 - \beta\cos\theta} \longrightarrow l_{coh} \propto \gamma^2\lambda$$

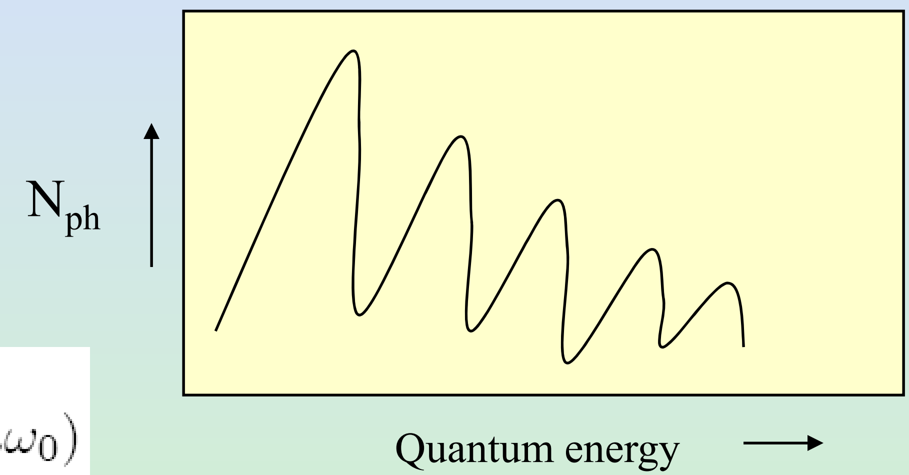
Coherent radiation length can be rather large even for short wavelength

@ crystal: $l_1 = n l_{coh}$ $\quad l = d/\sin\alpha$ $\quad l_1 = \dfrac{n\lambda\beta}{1 - \beta\cos\theta}$ $\quad \omega_0 \equiv \beta/l_1$



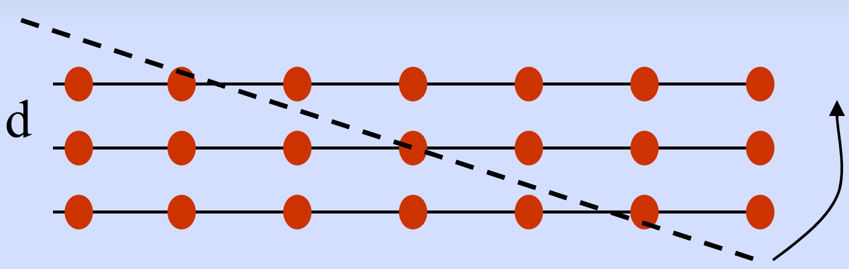
$$\omega = \frac{n\omega_0}{1 - \beta\cos\theta}$$

$$\left(\frac{d^2 I}{d\omega\Omega}\right)_{CBR} \propto \delta\left(\omega(1 - \beta\cos\theta) - n\omega_0\right)$$



@ Bremsstrahlung & Coherent Bremsstrahlung vs Channeling Radiation

@ crystal: $\alpha \to 0$



$\longrightarrow$ channeling

$$\omega = \frac{\omega_{fi}}{1 - \beta_{||} \cos\theta}$$

$$\left(\frac{dI}{d\omega}\right)_{CR} \propto \omega \left[1 - 2\left(\frac{\omega}{\omega_m}\right) + 2\left(\frac{\omega}{\omega_m}\right)^2\right], \quad \omega \le \omega_m \simeq 2\gamma^2\omega_{fi}$$

$\dfrac{ChR}{B} \propto \gamma^{1/2} Z^{-2/3}$ at definite conditions channeling radiation can be significantly powerful than bremsstrahlung

B:	CB:	ChR:
	NZe	$N \leftrightarrow \quad l_{coh} \propto \gamma^2/\omega \quad N_{eff}$
$\propto NZ^2$	$\propto (NZ)^2$	$\propto (N_{eff} Z)^2$

@ Channeling Radiation & Thomson Scattering

$$\omega_{lab}^{ChR} \approx \frac{2\gamma^2}{1+\theta^2\gamma^2}\omega_0^{ChR}$$ - radiation frequency - $$\omega_{lab}^{TS}\left\{\begin{array}{c}\vartheta = 0\\ \vartheta = \pi/2\\ \vartheta = \pi\end{array}\right\} \simeq \left\{\begin{array}{c}1\\2\\4\end{array}\right\}\frac{\gamma^2}{1+\vartheta^2\gamma^2}\omega_0^{TS}$$

$\propto \gamma^{3/2}$ $\qquad$ $\propto \gamma^2$

$$\left(\frac{dN_{ph}}{dt}\right)_{ChR} \propto \gamma^{1/2}$$ - number of photons per unit of time - $$\left(\frac{dN_{ph}}{dt}\right)_{TS} \propto Const$$

$P \propto \gamma^2$ - radiation power - $P \propto \gamma^2$

@ comparison factor: $$f \simeq \frac{\mathbf{A}_{Ch}^2}{\mathbf{A}_{TS}^2}\frac{L_{Ch}}{L_{TS}}$$ $$L_{Ch}(z) \simeq \int_0^z N_{ch}(z)\,dz$$

Laser beam size & mutual orientation

@ strength parameters – crystal & field:

	Si $\langle 110\rangle$	C $\langle 100\rangle$	W $\langle 111\rangle$
$\mathbf{A}_{Ch}^2$, eV/Å^3	~ 520	~ 580	~ 10000

$\mathbf{A}_{TS}^2$ ~ 700 eV/Å^3 for the 10 TW laser with a beam diameter of 0.1 mm

@ Channeling Radiation & Thomson Scattering

For X-ray frequencies: **100 MeV** electrons **channeled** in 105 μm Si (110) emit ~ 10^{-3} ph/e^-

corresponding to a Photon Flux ~ 10^8 ph/sec

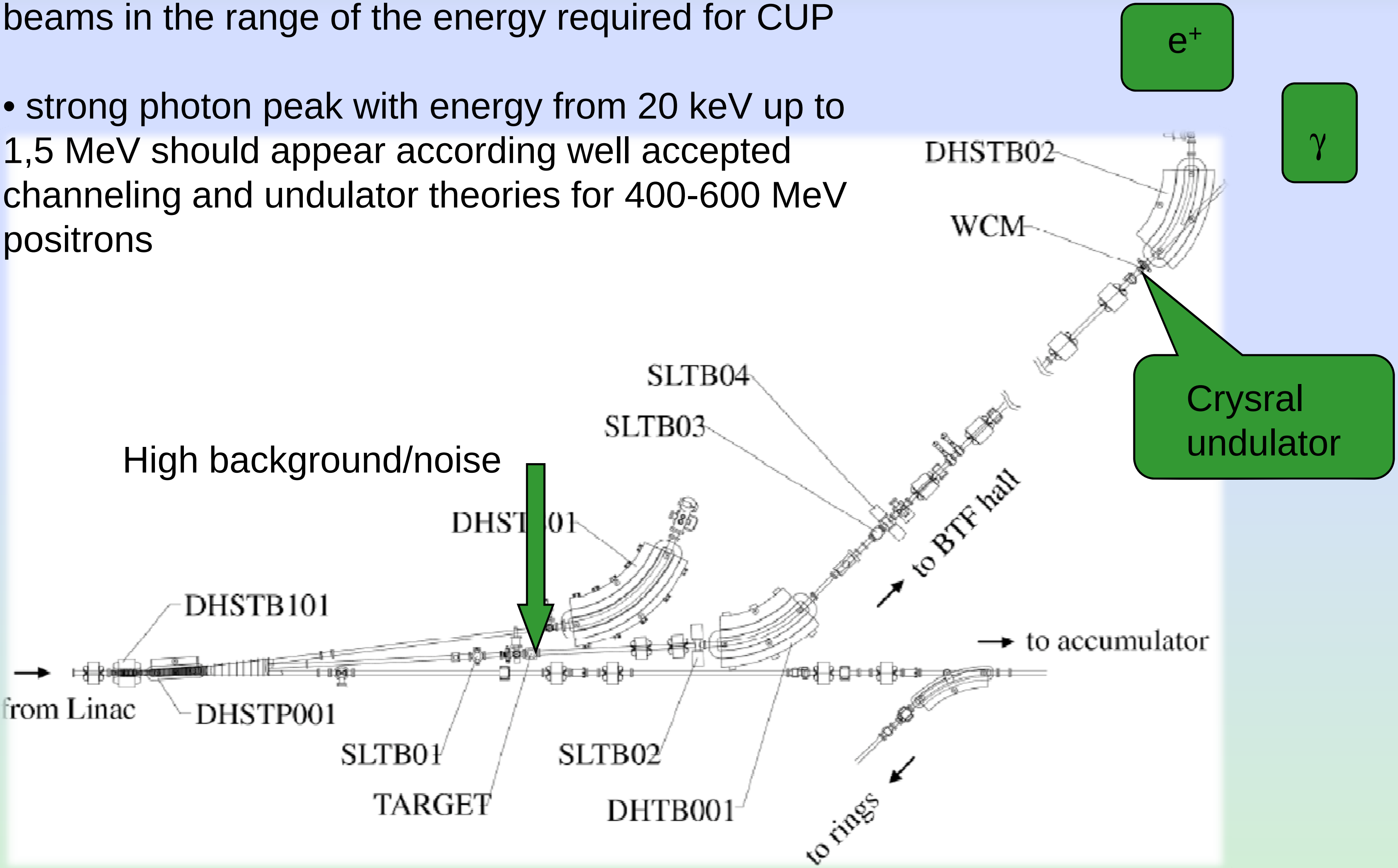
ChR – effective source of photons in very wide frequency range:

- in x-ray range – higher than B, CB, and TS
- however, TS provides a higher degree of monochromatization and TS is not undergone incoherent background, which always takes place at ChR

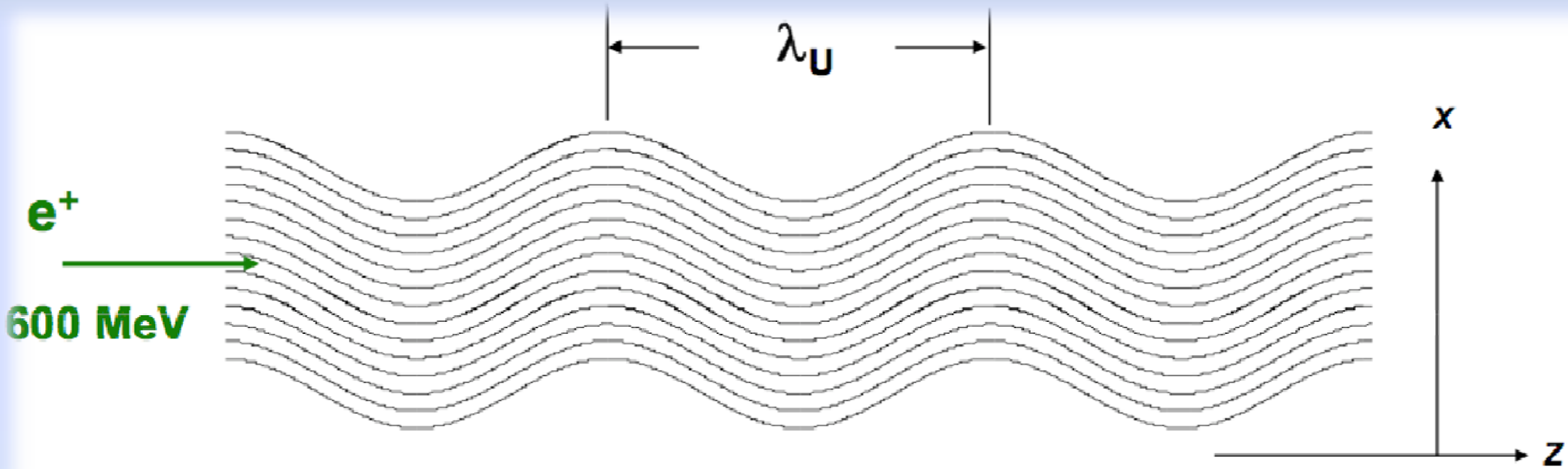
@ BTF Layout

• BTF as unique European facility to deliver positron beams in the range of the energy required for CUP

• strong photon peak with energy from 20 keV up to 1,5 MeV should appear according well accepted channeling and undulator theories for 400-600 MeV positrons



@ Positron Channeling in Si-Ge Undulator



$$x = A \cdot \cos\left(\frac{2\pi}{\lambda_U} z\right) \quad A = 9\ \text{Å}, \quad \lambda_U = 50\ \mu\text{m}, \quad N_U = 4$$

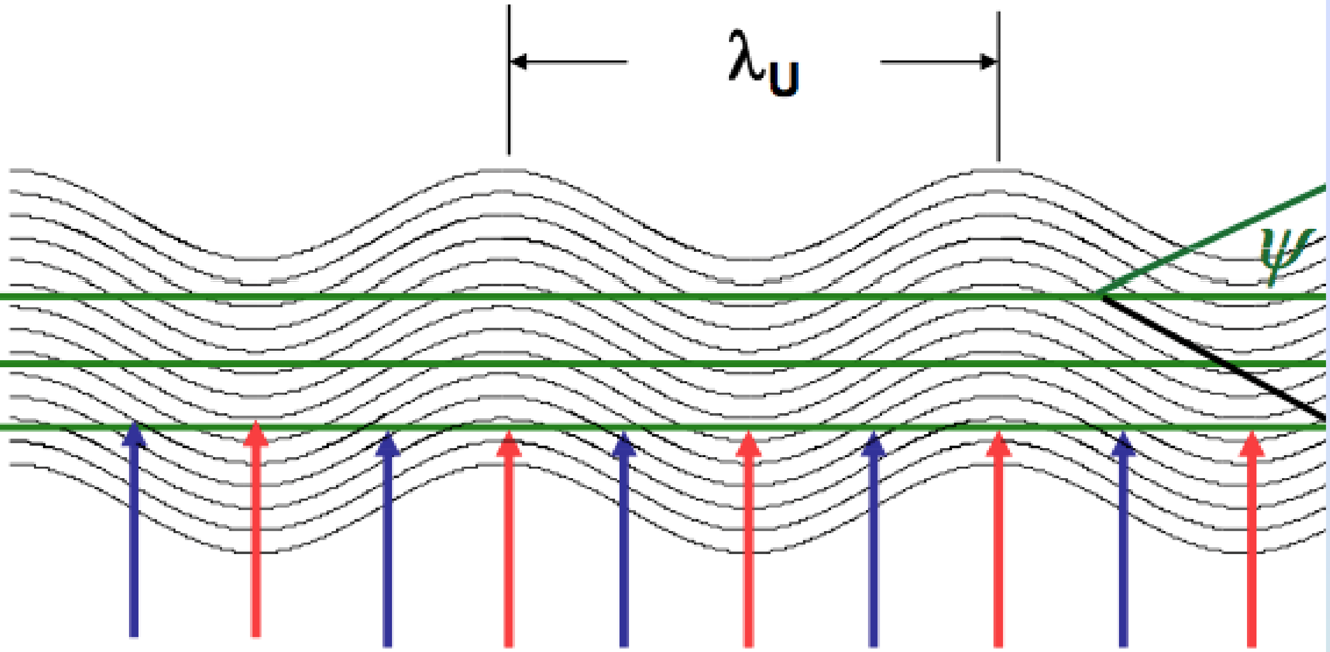
Beam Energy E = 600 MeV, γ = 1175.2 $\quad K = \gamma \cdot A \cdot \frac{2\pi}{\lambda_U} = 0.133$

Photon energy $\quad \hbar\omega = k \frac{4\pi \cdot \gamma^2 \hbar c}{\lambda_U (1 + K^2/2 + \gamma^2(\theta_x^2 + \theta_y^2))} = 67.9\ \text{keV}$

at $\theta_x = \theta_y = 0$, and first order $k = 1$

@ Positron Channeling in Si-Ge Undulator

N.F. Shul`ga, V. Boiko, JETP Letters 84 (2006) 305



Planar channeling regions

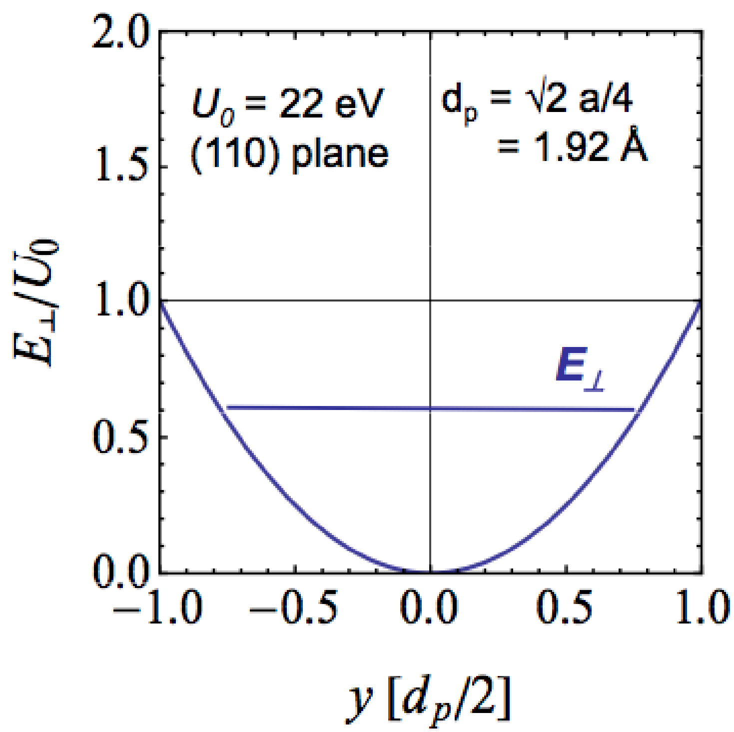
Coherent bremsstrahlung regions

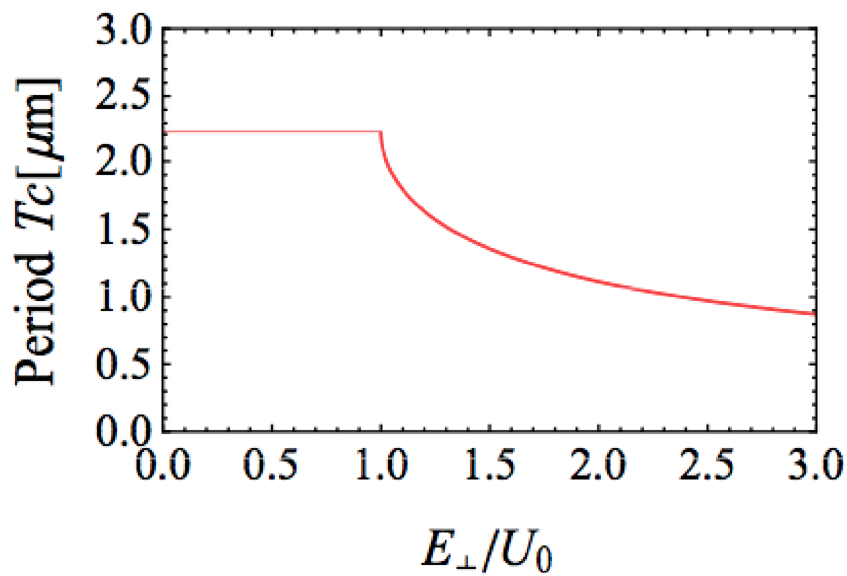
@ Dechanneling of positrons

$$\frac{\partial F(x,E_\perp)}{\partial x} = \frac{\partial^2}{\partial E_\perp^{\ 2}} \left\langle 2\overline{\frac{dE_\perp}{dx}}(E_\perp - U(y)) \right\rangle_T F(x,E_\perp) - \frac{\partial}{\partial E_\perp} \left\langle \overline{\frac{\Delta E_\perp}{\Delta x}} \right\rangle_T F(x,E_\perp)$$

with $D_e^{(1)}(E_\perp) = \left\langle \overline{\frac{dE_\perp}{dx}} \right\rangle_T$ the mean transverse energy increase

$D_e^{(2)}(E_\perp) = \left\langle 2\overline{\frac{dE_\perp}{dx}}(E_\perp - U(y)) \right\rangle_T$ the diffusion coefficient $T = T(E_\perp)$ the oscillation period





@ Theoretical estimations: need for small divergence

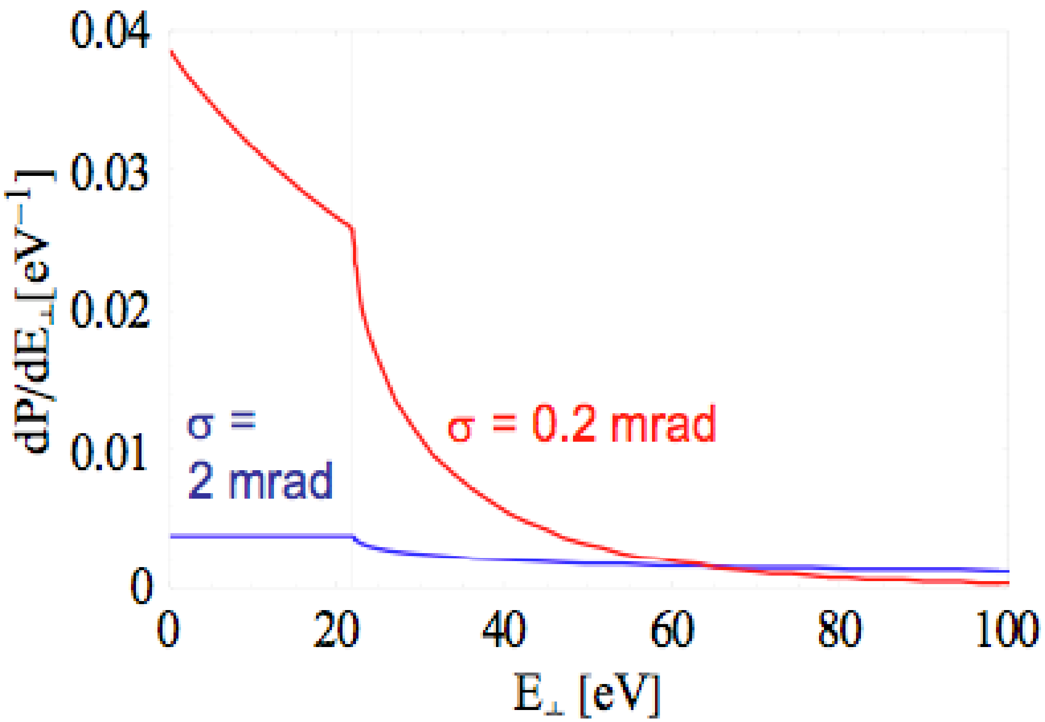


Fig. 1: Initial occupation probabilities as function of the transverse energy. Bound states in the potential well are located below $U_0 = 22$ eV. Curves are shown for two angular divergences σ of the positron beam as indicated. Integrals between $0 \leq E_\perp \leq 0.5\,U_0$ are 0.043 and 0.38 for $\sigma = 2$ mrad and $\sigma = 0.2$ mrad, respectively

@ Theoretical estimations: feasibility of observation

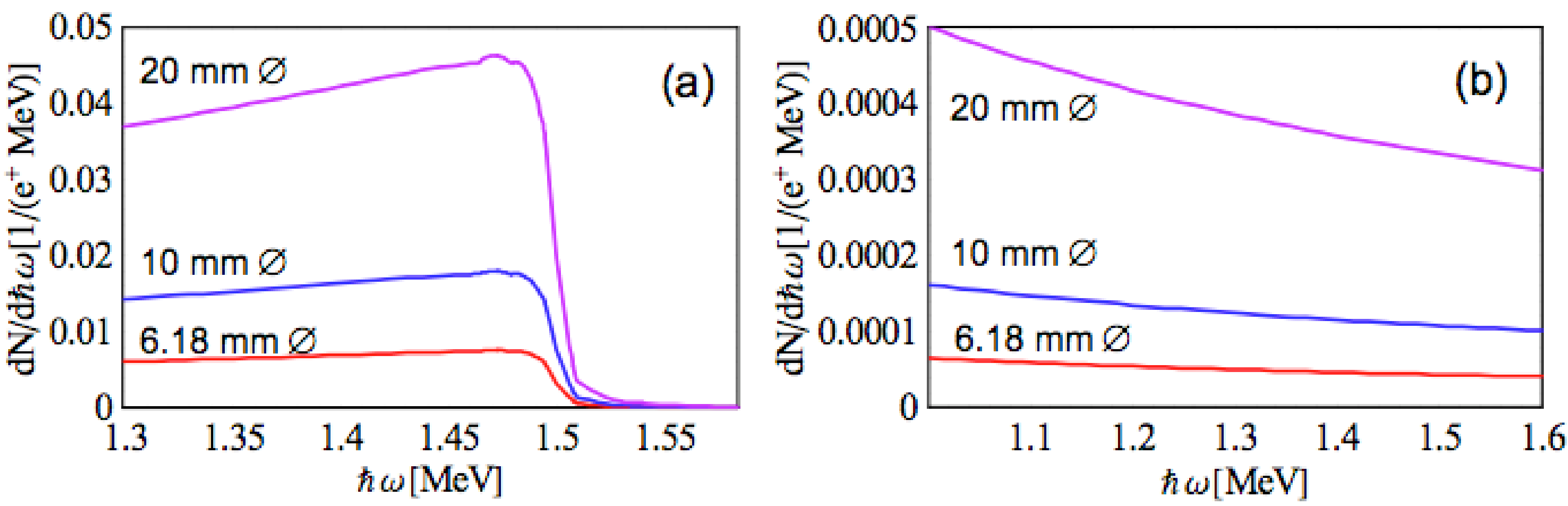
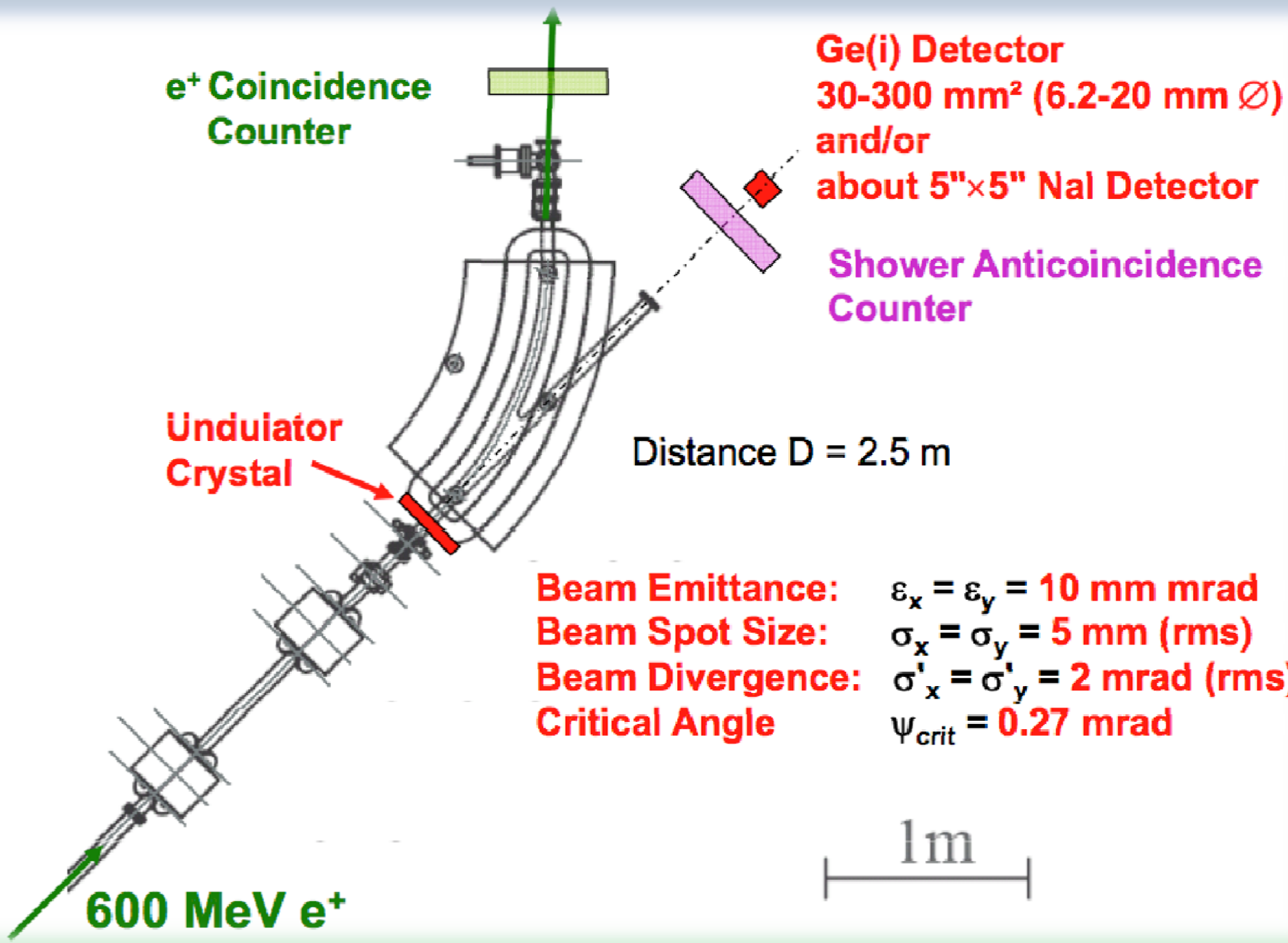


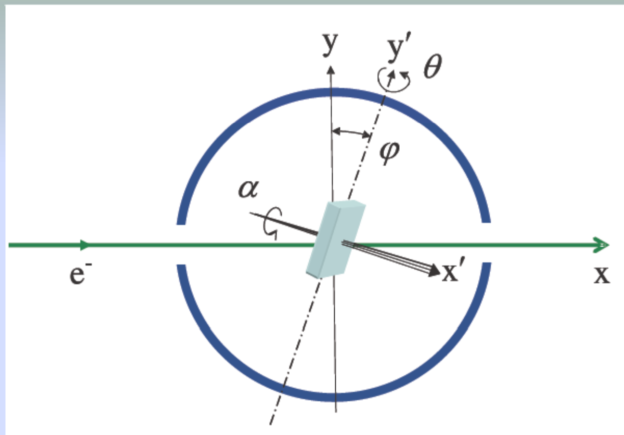
Fig. 2: (a) Calculated positron spectra for various detector apertures as indicated, (b) corresponding bremsstrahlung spectrum.

@ Experimental Setup at BTF



Beam Emittance:	$\varepsilon_x = \varepsilon_y = 10$ mm mrad
Beam Spot Size:	$\sigma_x = \sigma_y = 5$ mm (rms)
Beam Divergence:	$\sigma'_x = \sigma'_y = 2$ mrad (rms)
Critical Angle	$\psi_{crit} = 0.27$ mrad

@ Crystal characterization: MAMI 855 MeV e⁻



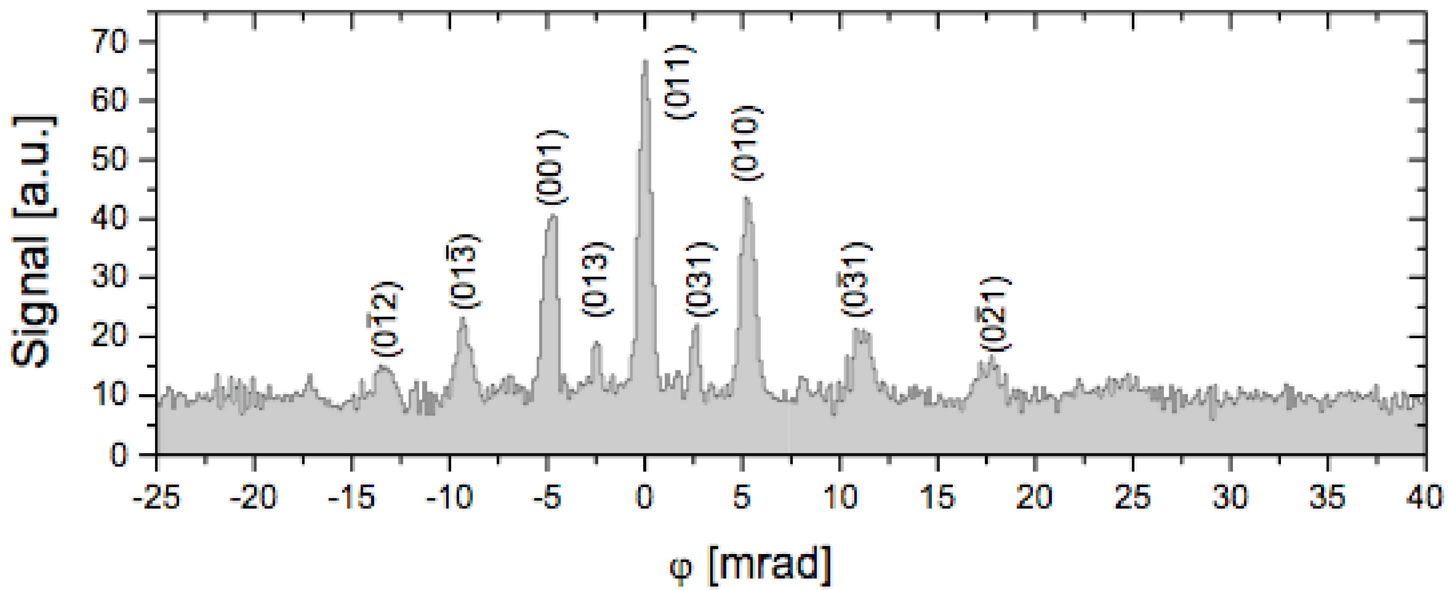
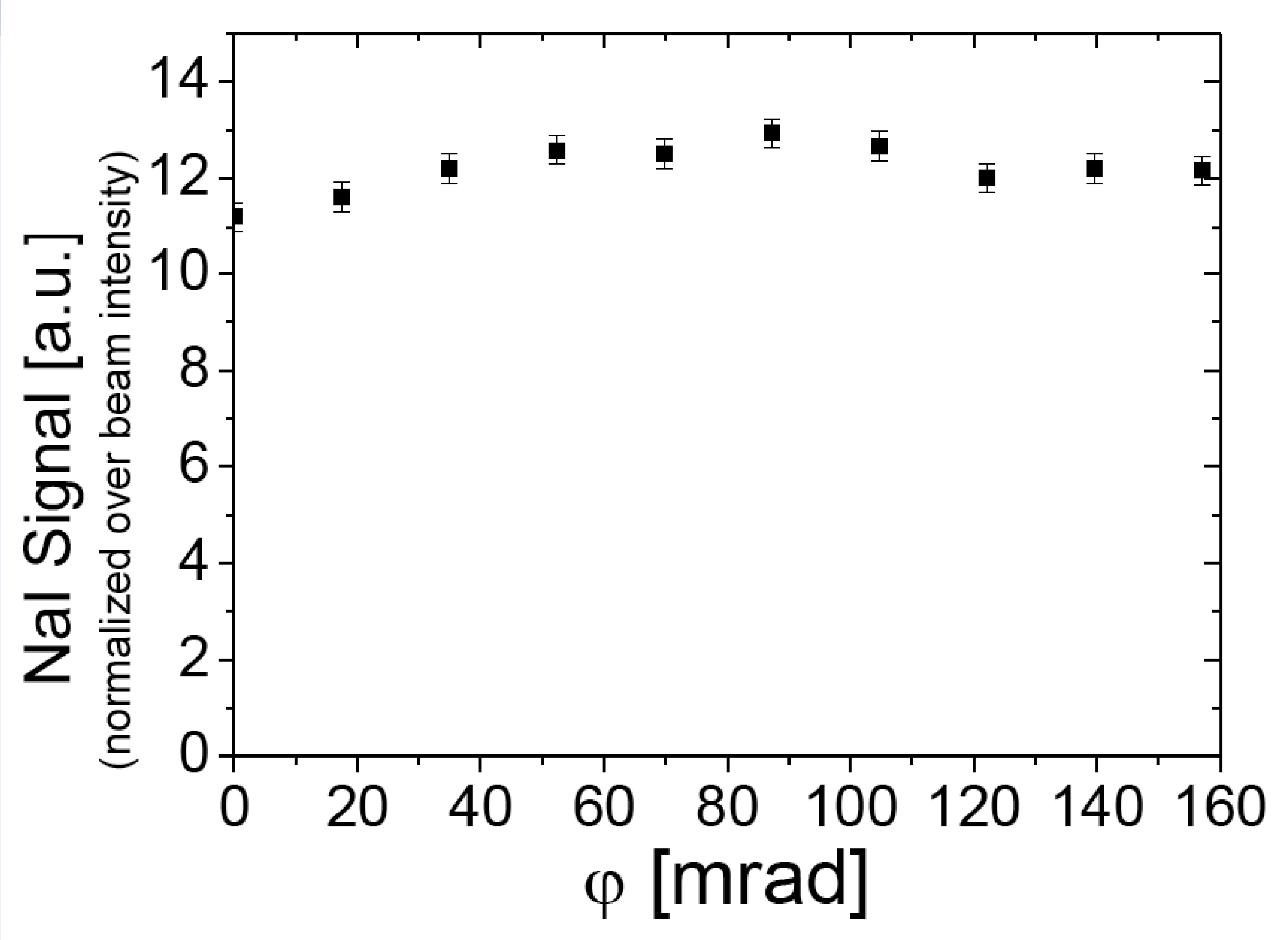


Fig. 4: Electron channeling measurement at MAMI by a scan over the azimuthal angle φ of a Si single crystal ($d = 14.7\ \mu$m) at a beam energy of 855 MeV. The crystal was tilted by the polar angle $\theta = 0.255°$. Accumulated events in the energy range 4.1 – 10.2 MeV.

@ Radiation record at BTF 600 MeV e^+



No evidence for channeling / channeling radiation

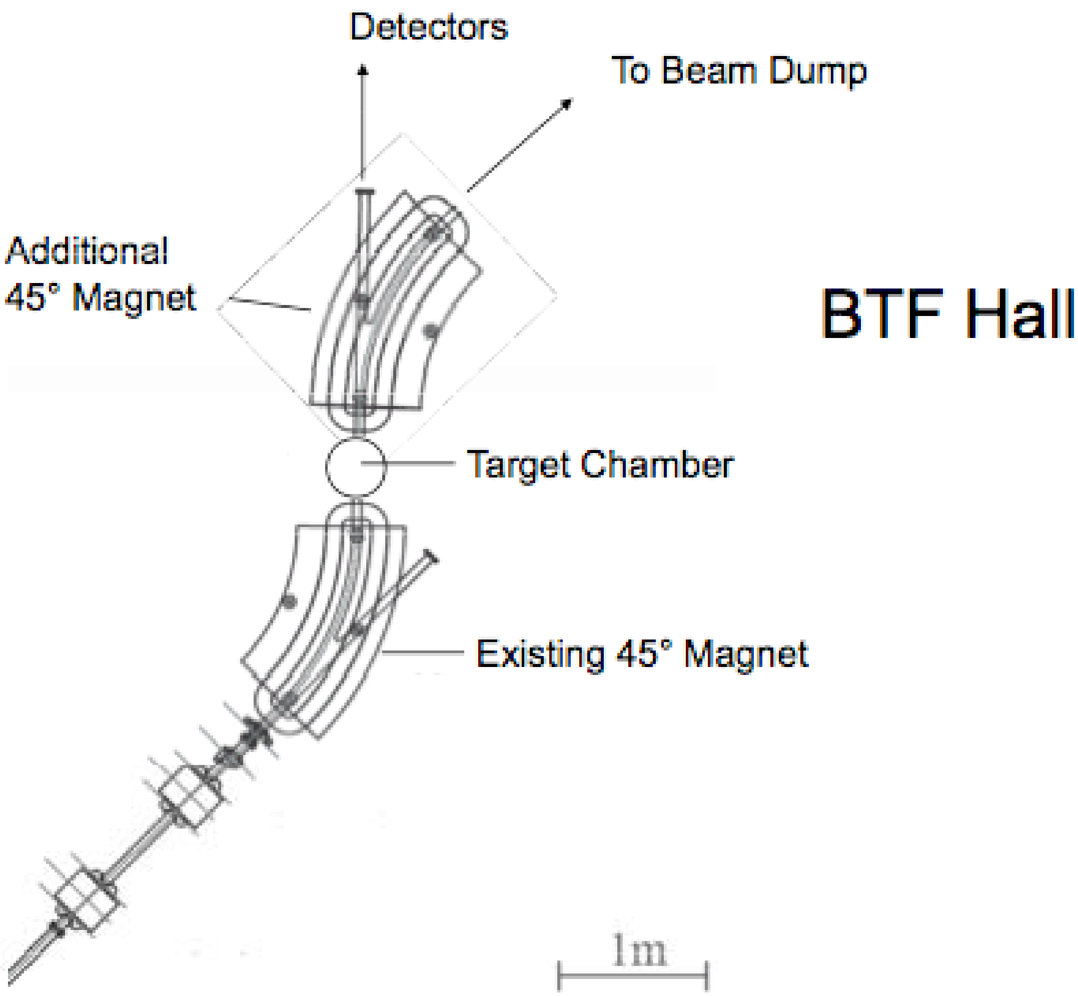
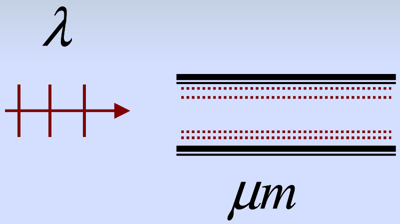


Fig. 9: Proposed modifications to the beamline in the BTF hall.

X-ray channeling:

flux peaking

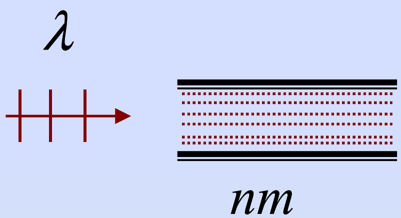
@ down to bulk x-ray channeling



$\theta << 1 \quad (\theta_c \sim 10^{-3})$ *: grazing incidence optics*

$\lambda \quad \to \quad \lambda_\perp >> \lambda$ *: from nm to μm*

$d_0 \sim 1\mu m \div 10\mu m: \quad \lambda_\perp << d_0$ *: surface channeling*



$\theta_d = \lambda / d_0 \sim \theta_c$ *: diffraction angle approaches Fresnel angle*

$\lambda_\perp / d_0 \sim 1$ *: bulk channeling*

@ Simulations for x channeling (straight & bent)

Angular distributions

Spatial

Coherent scattering:
$0\text{-}L_0$

Multiple reflections:
$L_0\text{-}20L_0\text{-}10^3L_0$

Angle of incidence – 0.5 critical angle of channeling

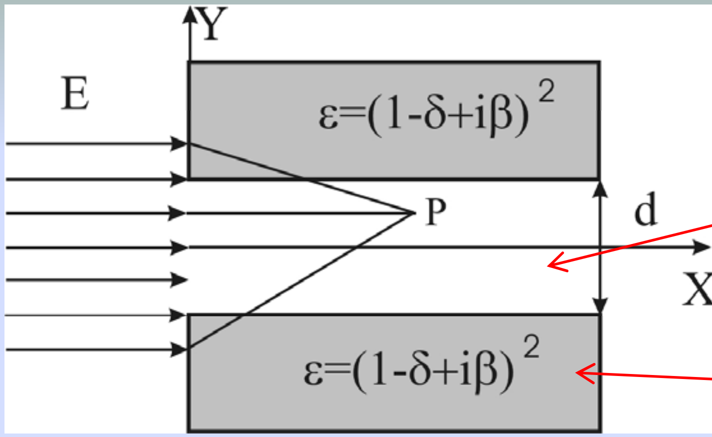
@ Simulations for x channeling: bending of radiation

Evolution of angular distribution

$r_{curv} \sim 2\ m$:

Strong bending effect

@ wave field formation in a planar waveguide

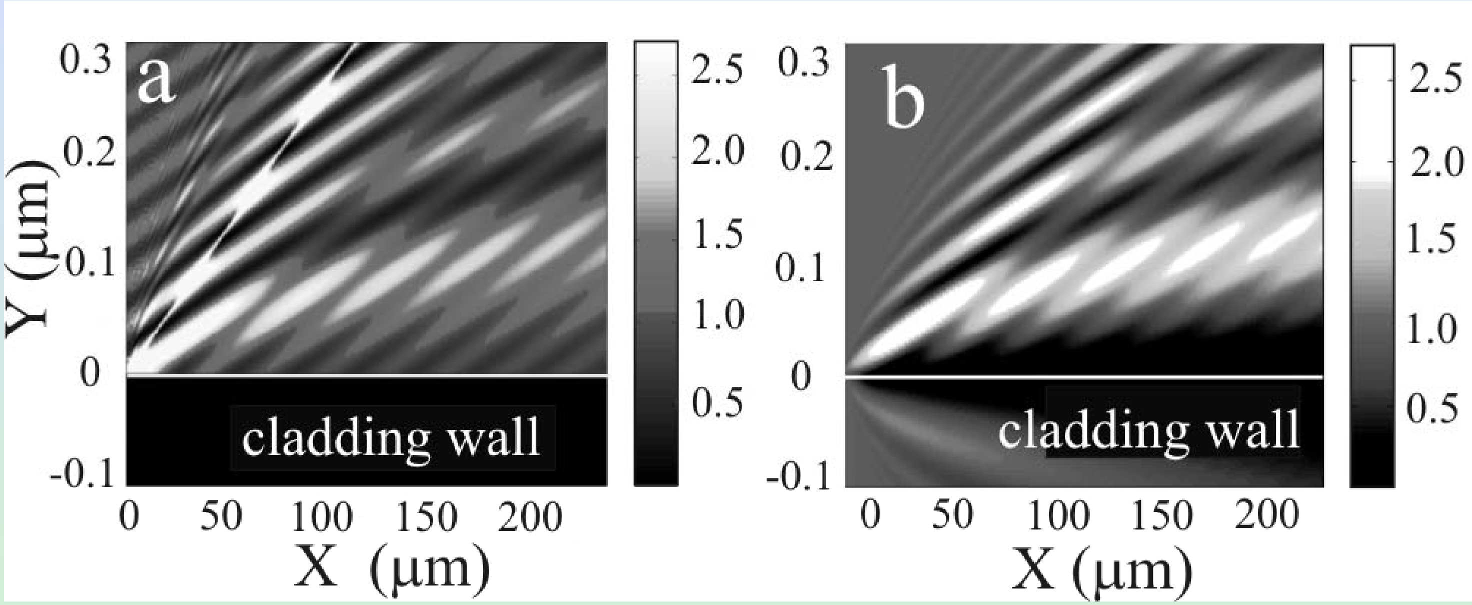


$$E_m(x) \propto \begin{cases} \cos(k\vartheta_m x) & , \text{ even mode} \\ \sin(k\vartheta_m x) & , \text{ odd mode} \end{cases}$$

$$|E_m(x)| \propto e^{-k\sqrt{\vartheta_c^2-\vartheta_m^2}|x|}$$

Diffraction from Si corner (gap 30 nm; λ=0.1 nm):

(a) analytical solution; *(b) computer simulation*



@ planar n-guide

$$\tan(k\vartheta_m d) = \begin{cases} \left(\frac{\vartheta_c^2}{\vartheta_m^2} - 1\right)^{1/2}, & \text{even mode} \\ -\left(\frac{\vartheta_c^2}{\vartheta_m^2} - 1\right)^{-1/2}, & \text{odd mode} \end{cases}$$ *:: quantum states of channeling*

$\Delta\varphi \equiv k\vartheta_c d$ *:: character of radiation transmission ::*

$d/\lambda_\perp \gg 1 \longrightarrow \Delta\varphi \gg 2\pi$ *:: the ray optics approach for describing radiation propagation*

- $\zeta = 2\pi\theta_c a/\lambda = 2\pi a/\lambda_{\perp c}$ $[\lambda_{\perp c} = \lambda/\theta_c]$ *:: the number of bound modes N.*
- $\zeta \gg 2\pi$ $N \gg 1$ *:: the geometric optics approximation*

  $\zeta \gg 2\pi$ $a \gg \lambda_{\perp c}$ [*glass* $\lambda_{\perp c}$ *= 40 nm*]
- *Thus for a wide waveguide a >> 40 nm there are* many bound modes *and multiple reflection of rays can be used.*
- *In the other limit* $\zeta \ll 1$ *or a << 7nm* one even bound mode

@ planar n-guide

Expansion in a set of guiding modes :: $\Psi_{in}(x) = \sum_m Q_m \Psi_m(x),$

$Q_m = \int_{-d}^{d} e^{ik_{\perp in}x} \Psi_m(x)dx$ *:: population of m-th mode*

- *For an ultra narrow waveguide with aperture ::*

$2a = 5.2$ nm ($\zeta = 1/\mathrm{sqrt}(6)$)

the integral intensity of the bound mode ::

$P = Q_1^2 = 25a \quad k_{\perp in}=0$

Normal incidence of the incoming beam ::

Effective aperture:: about one order larger than the geometrical one

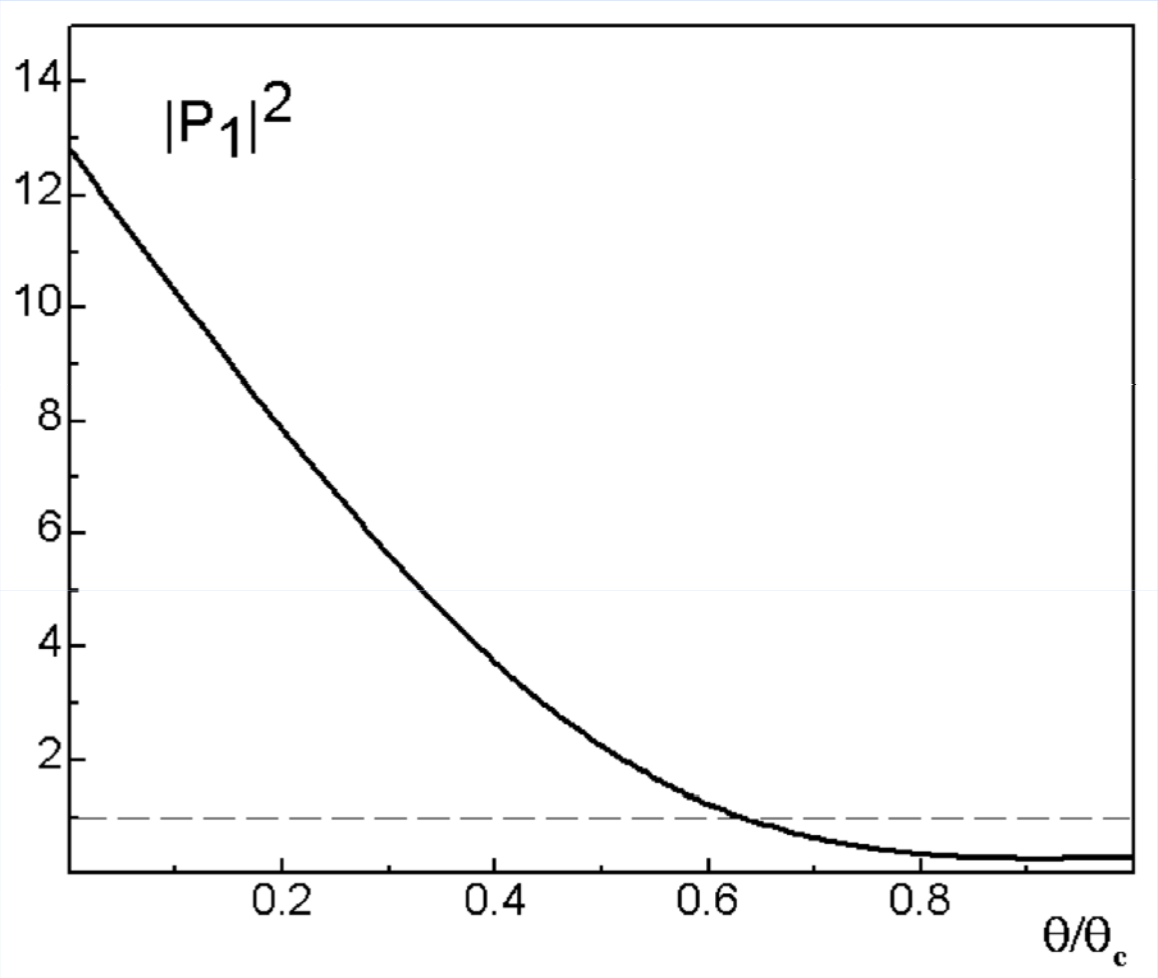
- Strong dependence on the incidence angle:

$Q_1^2 = 4.5a$ at $\theta_{in} = k_{\perp in}/k = 0.5 \bullet \theta_c$
$Q_1^2 = 0.6a$ at $\theta_{in} = \theta_c$.

@ planar n-guide

Angular dependence for the population of a single mode propagation in ultra narrow planar waveguide.

The dependence is normalized to geometrical acceptance of the waveguide.



@ flux peaking effect

- *Integral within* $-\theta_m \leq \theta_{in} \leq \theta_m$ $\longrightarrow$ $$P = P(0)\left[\frac{1}{2} + \frac{\sin(2\varphi_m)}{4\varphi_m}\right]$$

$$\varphi_m = \arctan\frac{\theta_m\sqrt{6}}{\theta_c}$$

- *For the considered ultra narrow waveguide the intensity at the gap center*

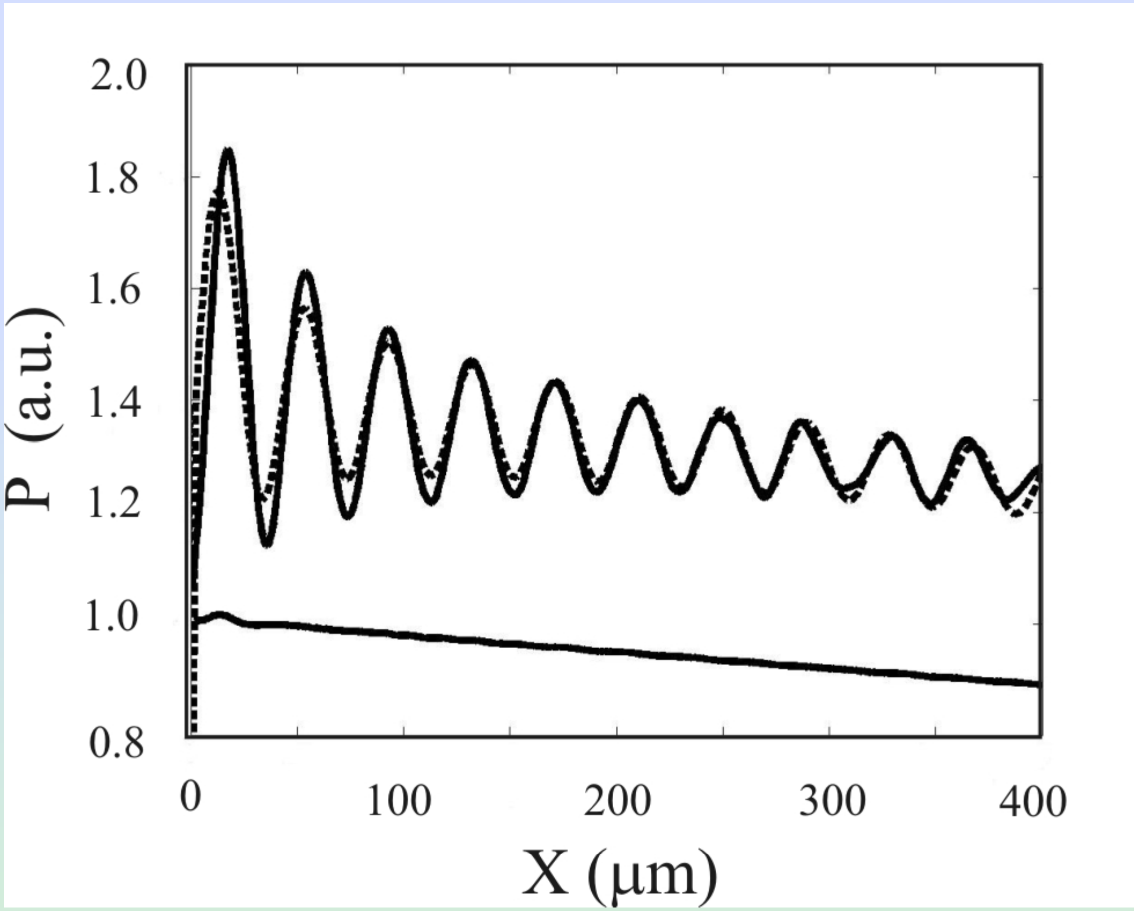
$I = 3.5\ I_0$ at $\theta_{in} = 0$ - **flux peaking effect**

- *Attenuation with penetration due to absorption! In the glass cladding 73 % of the wave intensity (tunneling length ~3a ≈ 8 nm)*

… the strong tunneling effect…

@ total transmitted power for a planar n-guide

*Normalized value of radiation power integrated within vacuum gap vs. coordinate x calculated for step-like entrance function (***bottom*** *lines) and for total calculated field (***top*** *lines). Solid lines are the result of computer simulation and* ***dashed*** *lines are the result of asymptotic solution*



@ circular n-guide

$$\rho^2 \frac{d^2 u}{d\rho^2} + \rho \frac{du}{d\rho} + \left(k^2(\vartheta^2 - \vartheta_c^2)\rho^2 - m^2\right) u = 0$$

wave equation in circular guide

$$u(\rho) \propto \begin{cases} J_m(k\vartheta_m \rho) \ , \ \rho \le d \\ K_m\left(k\rho\sqrt{\vartheta_c^2 - \vartheta_m^2}\right) \ , \ \rho > d \end{cases}$$

J_{m}(y) and K_{m}(y) –
the 1st kind and the modified 2nd kind Bessel functions

Two limits:

inside *the core* ⟶ $J_m(\rho) \to \rho^{-1/2} \cos \alpha\rho$

the cladding ⟶ $\rho > d$ as $K_m(\rho) \to \rho^{-1/2} e^{-\rho}$

$$\frac{(\partial_\rho J_m / J_m)|_{\rho=d}}{(\partial_\rho K_m / K_m)|_{\rho=d}} = \left(\frac{\vartheta_c^2}{\vartheta_m^2} - 1\right)^{1/2}$$

dispersion equation

@ circular n-guide

@ effective aperture is larger than the geometric one in **~10^{10}** *times;*

due to very large tunneling length, which is equal to $a\exp(11)\sqrt{6} = 0.39mm$

no absorption*!*

@ the average effective aperture at $\theta_m=\theta_c$ only **24 times** *is larger than the geometric one.*

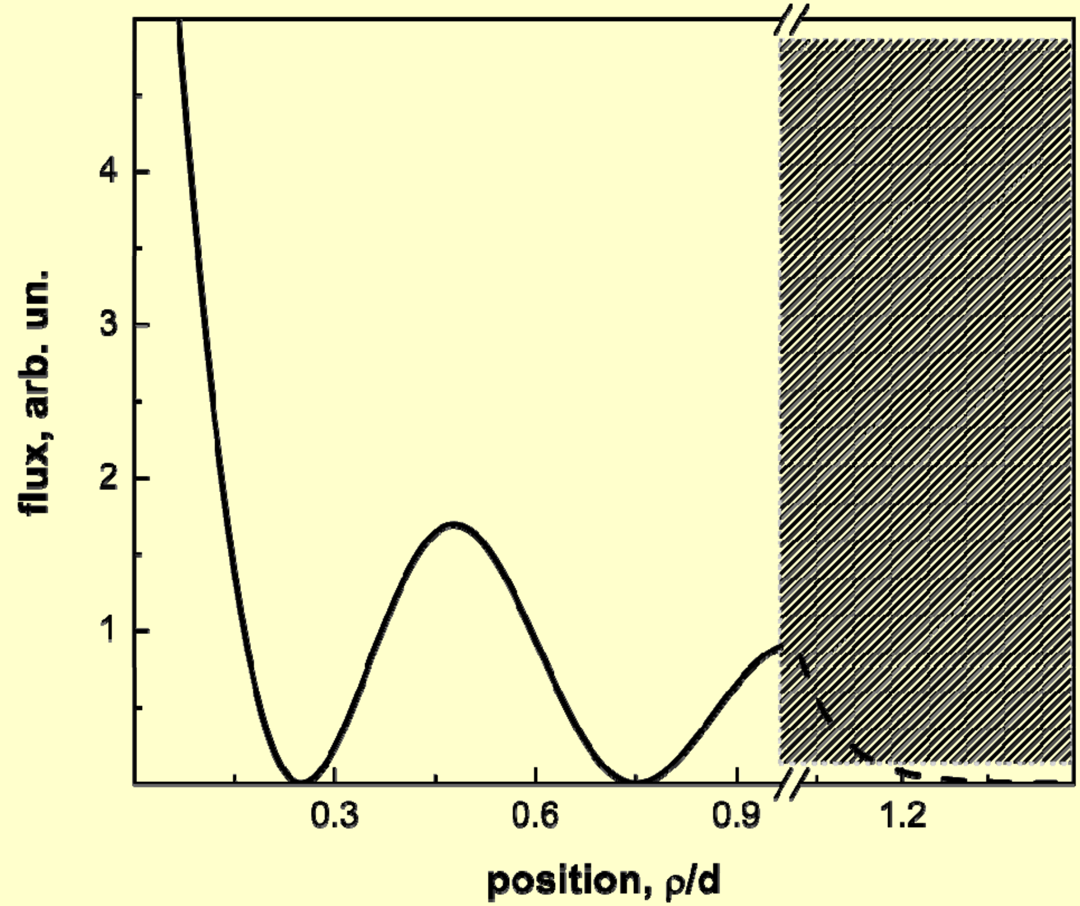
@ the **flux peaking** *is also strong – the intensity of the bound mode at the center of wave guide at $\theta_{in} = 0$ is equal $I(0) = Q_{01}^2 \bullet A^2 = 576$.*

Intensity increase is **576** *times due to the logarithmic singularity of the function $K_0(x)|_{x \to 0}$*

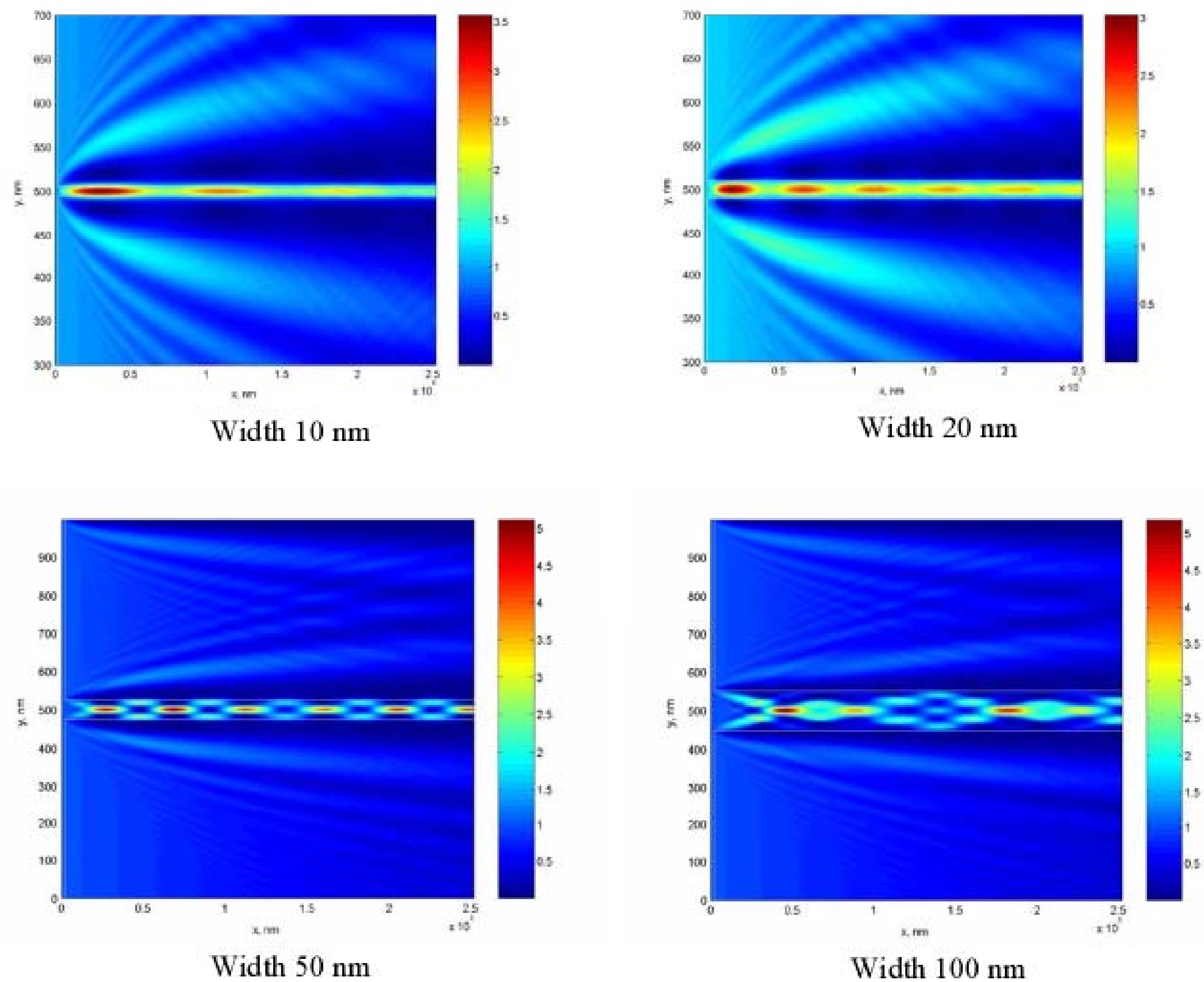
@ flux peaking effect

Flux peaking effect for a circular guide.

Due to the flux peaking effect, at the guide center, the radiation intensity may overcome in 2 orders the intensity of the incoming beam



@ nanocapillaries



Wavelength 0.1nm, material Si, length 2*Labsorb=2.5e5 nm

$\zeta = 2\pi\theta_c a/\lambda = 2\pi a/\lambda_{\perp c}$

- number of modes

$\lambda_{\perp c} = \lambda/\theta_c$

~ 40 nm for glass

Tunneling length ~ 8 nm

@ Resume for x channeling

Analysis of radiation propagation through the guides of various shapes, above presented, has shown that all the observed features can be described within unified theory of X-ray channeling:

- **surface channeling in μ-size guides**
- **bulk channeling in n-size guides**.

The main criterion defining character of radiation propagation is the ratio between the transverse wavelength of radiation and the effective size of a guide, i.e.

$\lambda_{\perp}/d \equiv \vartheta_d/\vartheta_c$ - **the ratio between the diffraction and Fresnel angles**.

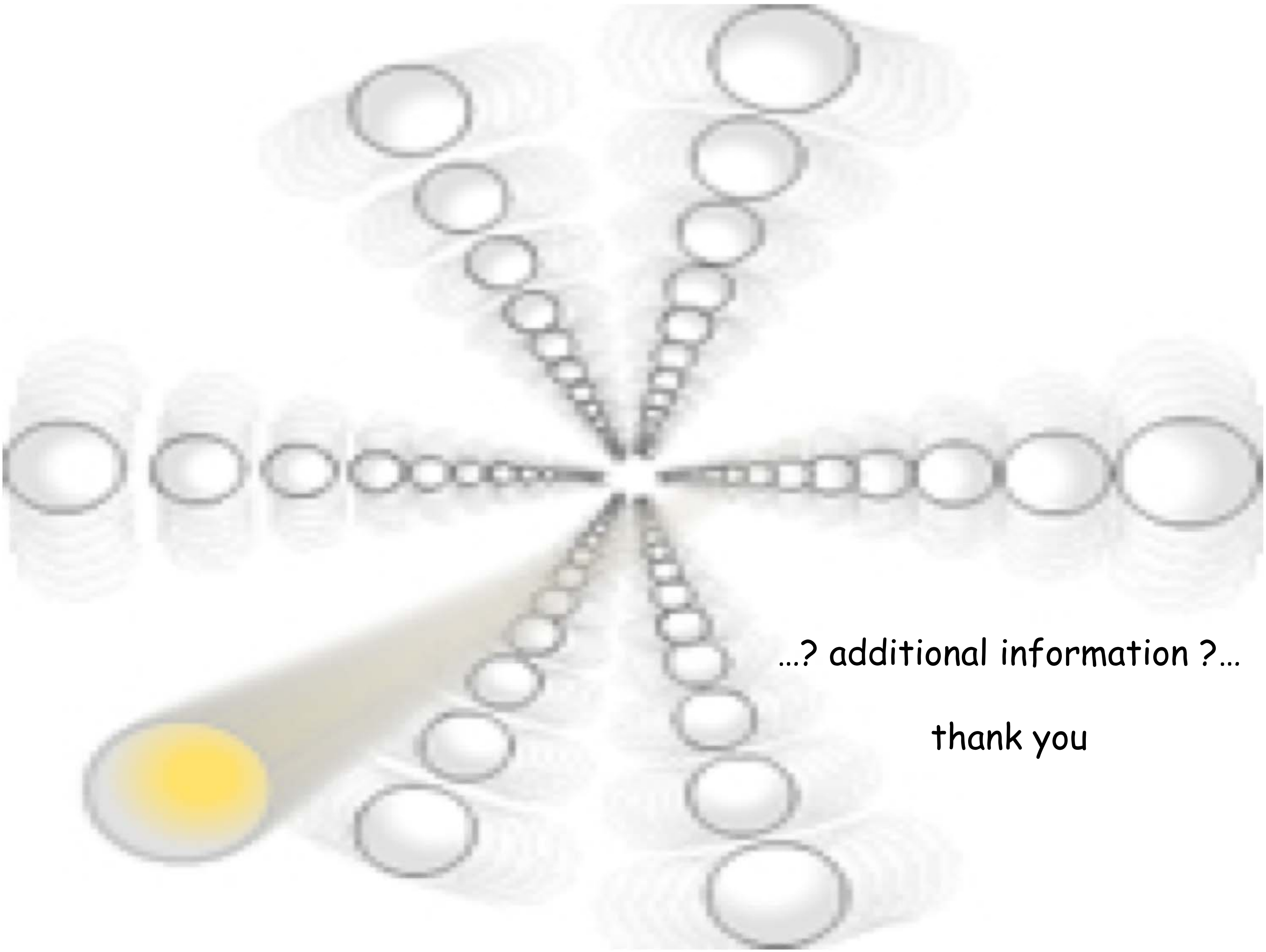
@ *this ratio is rather* **small**, *i.e. when the number of bound states is large, the* **ray optics** *approximation*

- **$\lambda_{\perp} \simeq$ d**, a **few modes** *will be formed in a quantum well;*
- **$\lambda_{\perp} \gg$ d** - *just a* **single mode** .

@ **flux peaking of X radiation**, *i.e. the increase of the channeling state intensity at the center of a guide*

- *a* ***proper channeling effect*** *that can be explained only by the modal regime of radiation propagation, and may find an interesting application for the purposes of extreme focusing.*

@ *all the considerations taken for X-rays should be valid for* **thermal neutrons**.



…? additional information ?…

thank you