# Why Accelerators?
# Introduction to Accelerator Physics

by

Verena Kain CERN BE-OP

# Why accelerators?

Accelerators are instruments that increase the energy of particles to study smaller and smaller structures and create heavy short-lived objects in collision with

$$E = m \cdot c^2$$

Wavelength of probe radiation needs to be smaller than object to be resolved

$$l = \frac{h}{p} = \frac{h \times c}{E}$$

| Object | size | Radiation energy |
|---|---|---|
| Atom | $10^{-10}$ m | 0.00001 GeV |
| Nucleus | $10^{-14}$ m | 0.01 GeV |
| Nucleon | $10^{-15}$ m | 0.1 GeV |
| Quarks | - | > 1 GeV |

# The units we are using...

Energy: in units of eV:

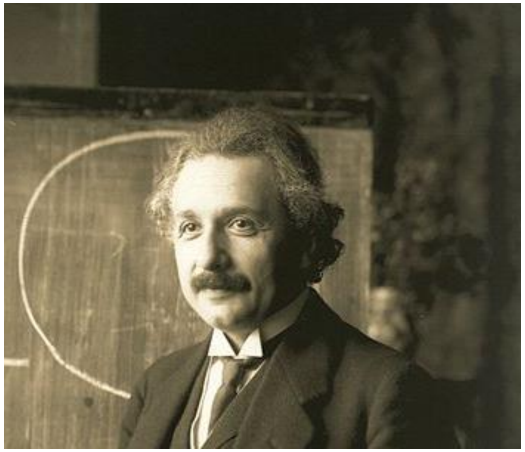
corresponds to the energy gained by charge of a single electron moved across a potential difference of one volt.

1 eV = 1.602176565(35) × $10^{-19}$ × 1 J

This comes from electrostatic particle accelerators.

Unit of mass m: we use $E = mc^2$

➔ Unit of mass is eV/$c^2$



Unit of momentum p:

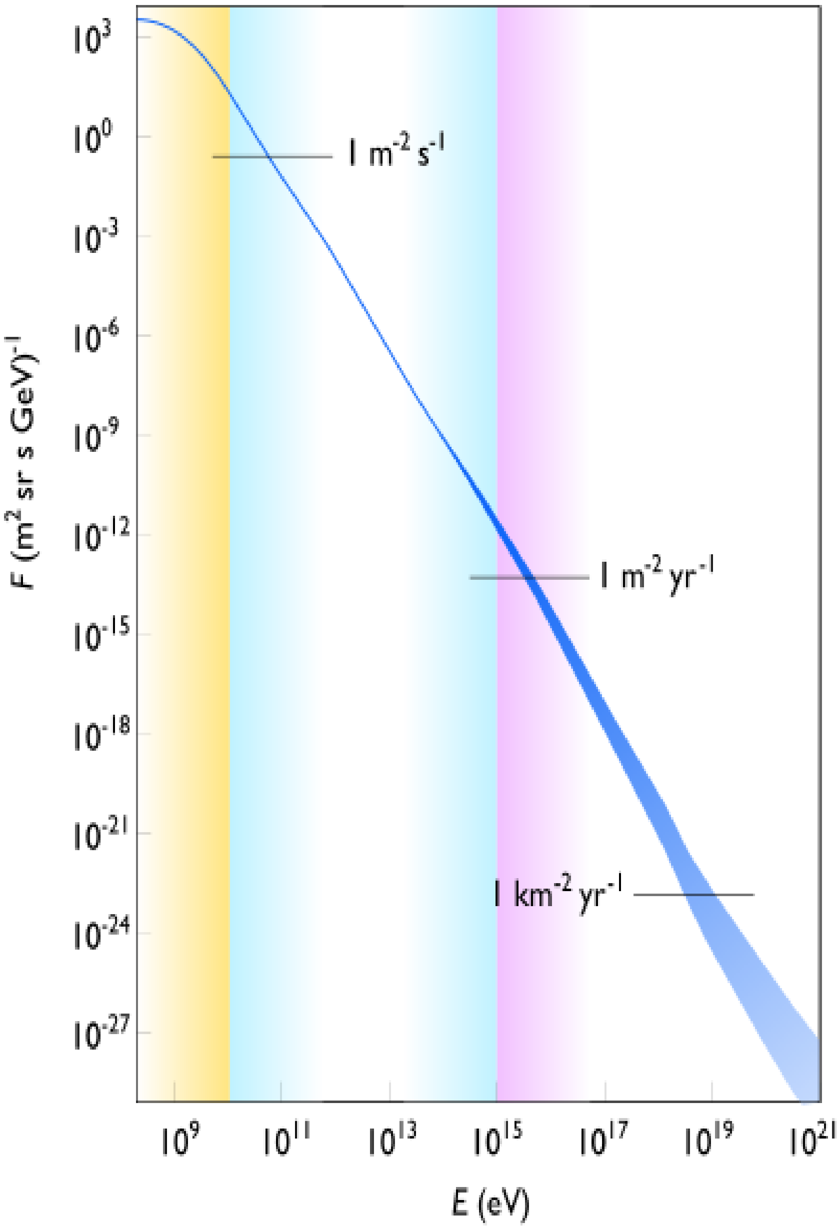
with $E^2 = (mc^2)^2 + p^2c^2$

➔ Unit of momentum is eV/c

# Natural Accelerators

- Radioactive Accelerators
  - Rutherford experiment 1911
  - Used $\alpha$ particles tunneling through the Coulomb barrier of Ra and Th to investigate the inner structure of atoms
  - Existence of positively charged nucleus, d ~ $10^{-13}$ m
  - $\alpha$ particle kinetic energy ~ 6 MeV

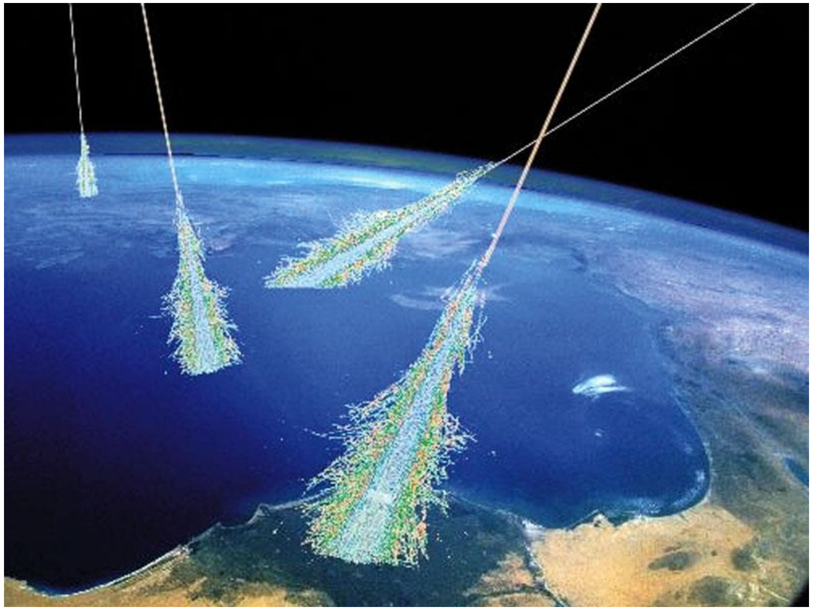
- Cosmic rays
  - Energies up to 3 x $10^{20}$ eV for heavy elements have been measured. ~ 40 x $10^{6}$ times what the LHC can do.
  - “Ultra high energy” cosmic rays are rare…



# Why accelerators then….?

“Our” accelerators have the advantage:

High energies, high fluxes of a given particle species, controlled energies at a specific location where a detector can be installed.



# How can we increase the energy of a particle?

Use electro-magnetic fields. Can increase the energy of CHARGED particles

- Increase energy

$$\mathrm{D}E = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\, d\vec{r} = q \int_{\vec{r}_1}^{\vec{r}_2} (\vec{E} + \vec{v} \times \vec{B})\, d\vec{r}$$

- The particle trajectory direction d**r** parallel to **v**

$$\mathrm{D}E = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\, d\vec{r} = q \int_{\vec{r}_1}^{\vec{r}_2} \vec{E}\, d\vec{r} = qU$$

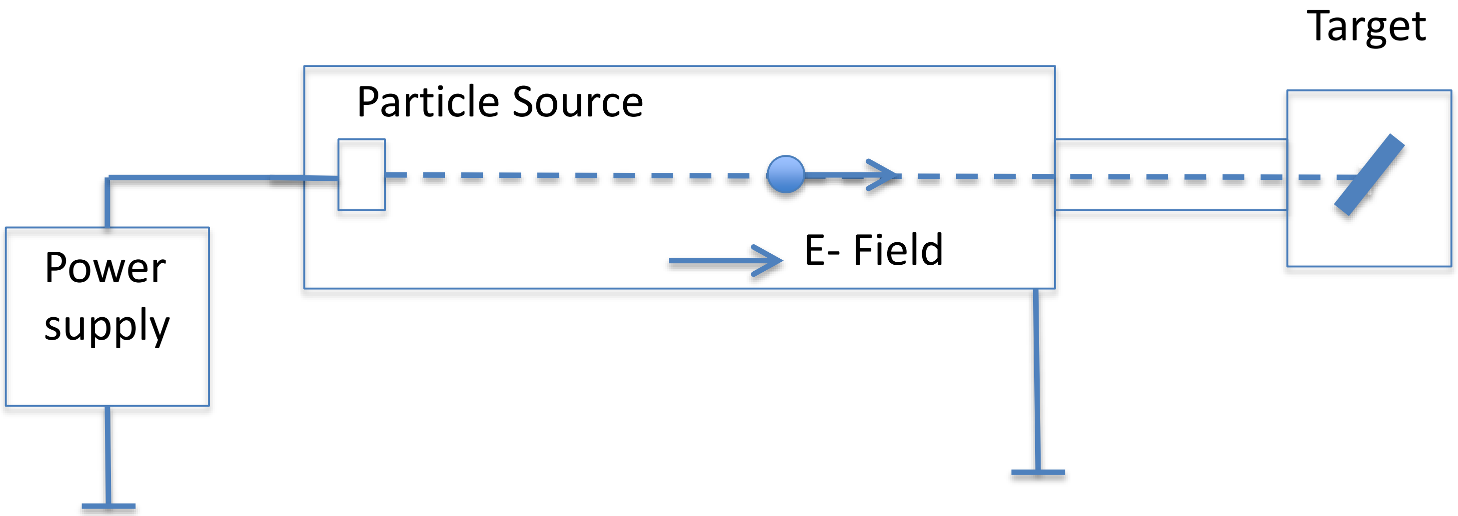
- **…**increase of energy with electric fields

- (Magnetic fields are needed for control of trajectories.)

# The basic accelerator

Electrostatic accelerator:

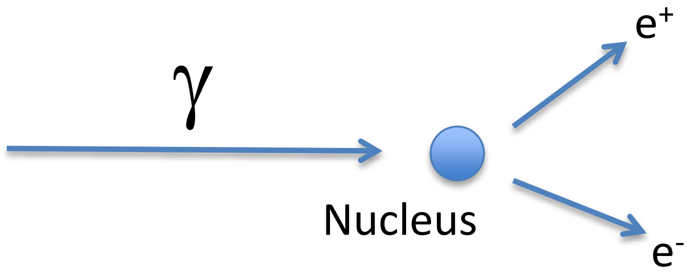
Charged particles go through the accelerating voltage gap **once** and then hit the target.



Limited by the maximum reachable voltage: ~ 10 MV

# Why collisions?

- Conservation laws: e.g. momentum and energy conservation



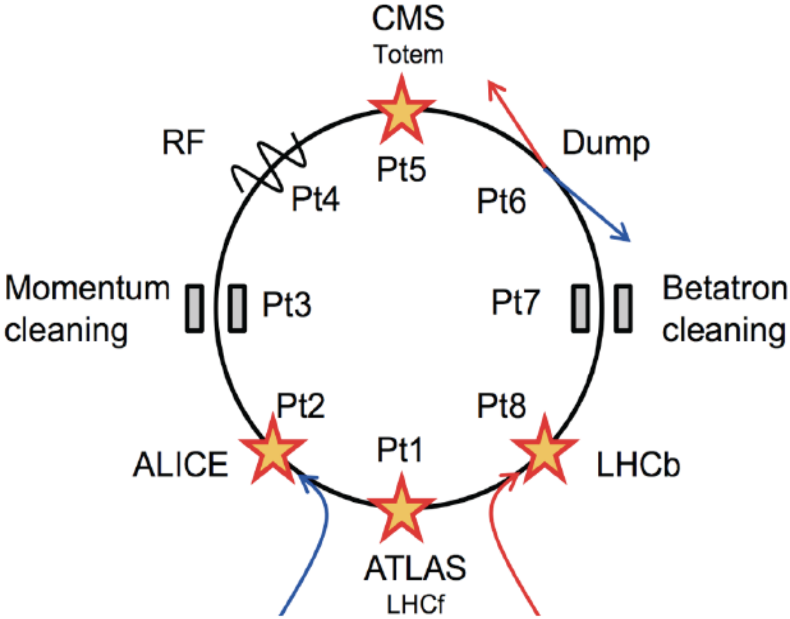
Photon into $e^+$,$e^-$ only in proximity of nucleus. Nucleus takes part of momentum (and part of available energy…)

- Center-of-mass Frame and Center-of-mass Energy ($E_{CM}$)
  - Center-of-mass frame defined where: $\sum \vec{p}_i = \vec{0}$
  - The energy available for creation of particles corresponds to $E_{CM}$

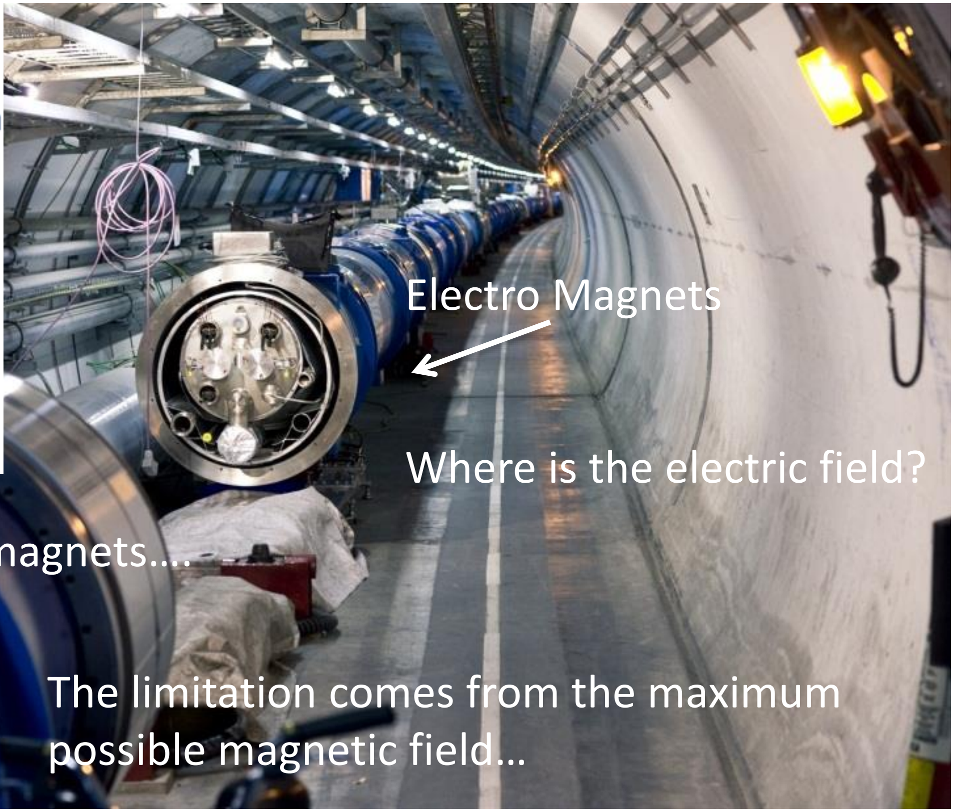
# The Large Hadron Collider

It is a circular 2-beam accelerator. No targets. Collider.

Passing through accelerating gap of same voltage over and over again.



LHC protons are accelerated to 7000000 MeV



Where is the electric field?

27 km of magnets….

The limitation comes from the maximum possible magnetic field…

# The Key Concepts for High Energy Accelerators

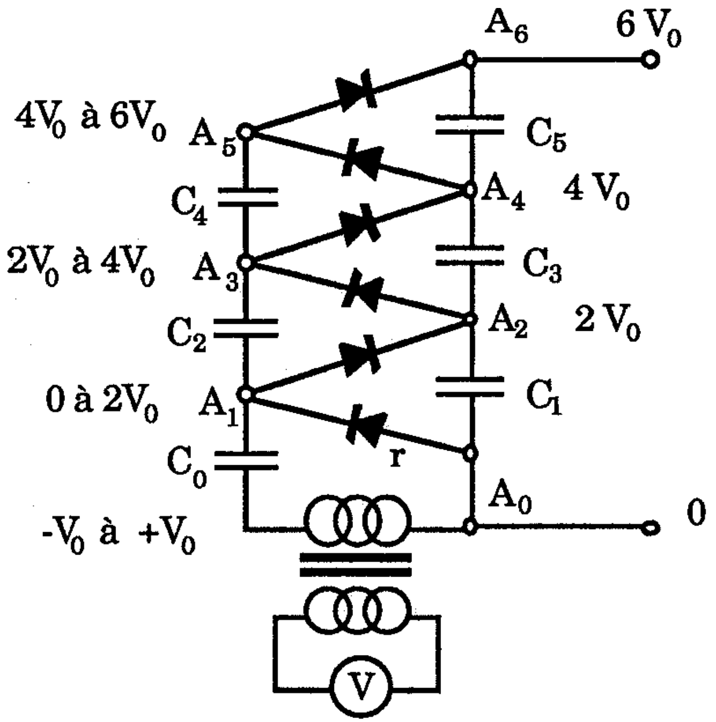
- RF Resonant Acceleration

- Strong Focusing – alternating gradient focusing
  - Keep the beam size under control

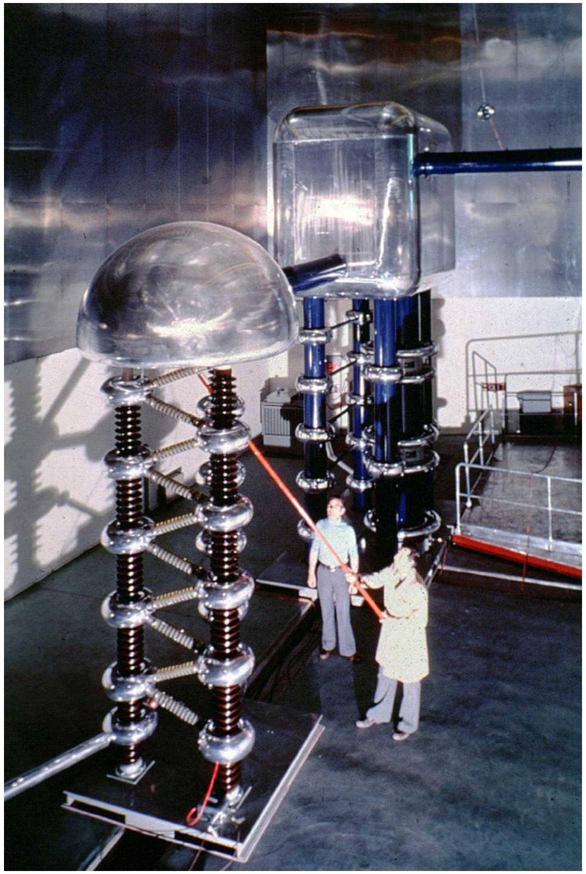
# HISTORY OF ACCELERATORS

# Electrostatic Accelerators – 1930s

- Cockcroft-Walton electrostatic accelerator
  - High voltage source by using high voltage rectifier units
  - High voltage limited due to sparking in air. Limit ~ 1 MV

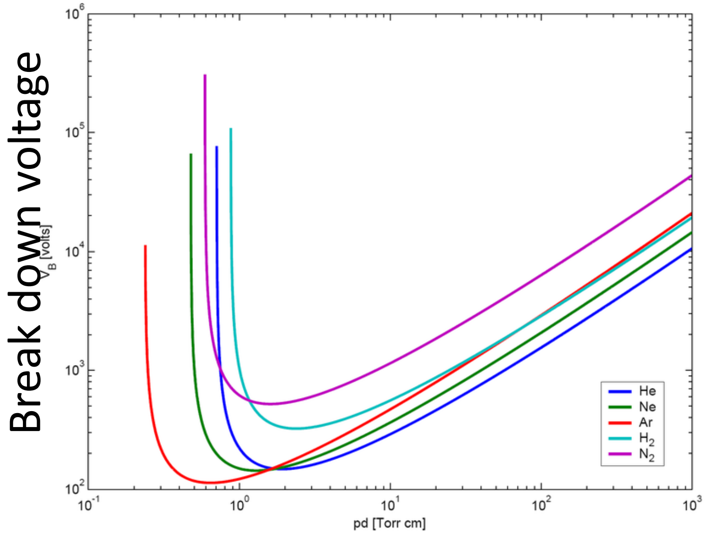


CERN used until 1993 as ion-source: 750 kV



# Electrostatic Accelerators

- Limit of 1 MV overcome: placing the electrodes under high pressure gas. Paschen's law

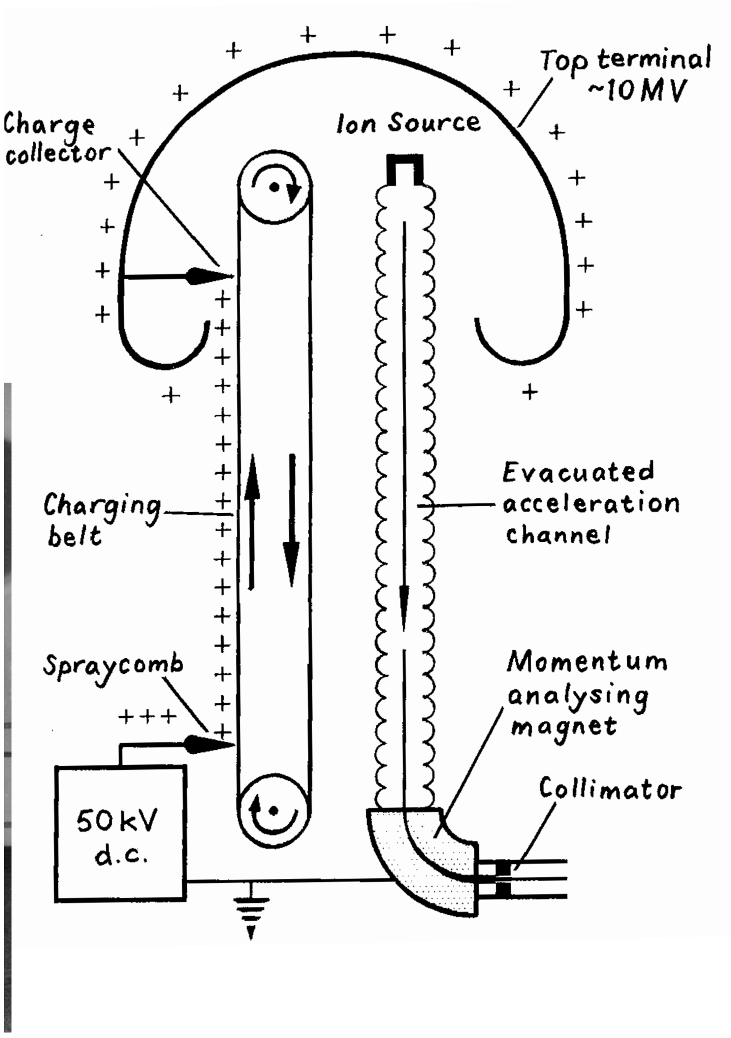


Break down voltage

Product of pressure x gap

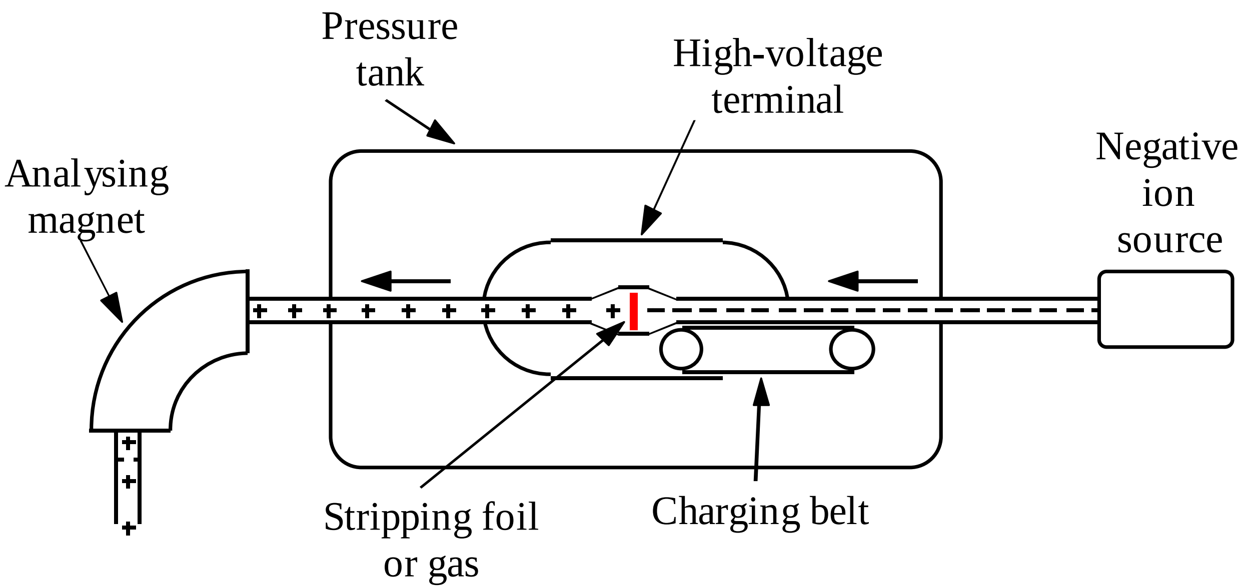
Break down voltage depends on gas pressure and gap between electrodes.

- ➔ Van De Graaf generator
  - 1 – 10 MV



# Tandem Van de Graaf Generator

- …use the accelerating voltage twice

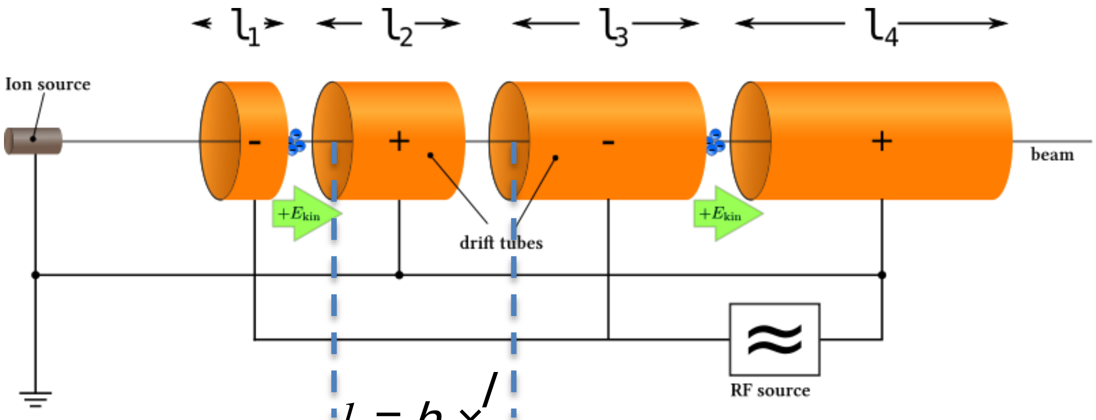


- Up to 25 MV
- Advantages of Van de Graaf:
  - Great variety of ion beams
  - Very good energy precision, small energy spread
- Applications in nuclear physics, accelerator mass spectroscopy,…

# RF Acceleration – the Revolution

Electrostatic accelerator limitation: maximum voltage before sparking for acceleration over single gap

- ➔ pass through acceleration gap of same voltage many times (Ising)
- 1928 Wideroe: first working RF accelerator



$$l_i = b_i \times \frac{l}{2}$$

Energy gain per gap:

$$E = q\,V_{RF}\sin(\phi_s)$$

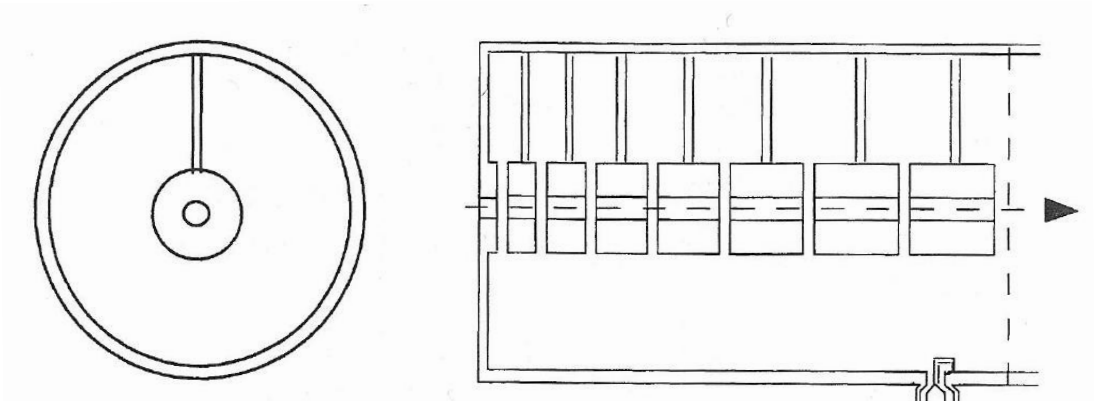
$\Phi$s...phase wrt to RF field

- Particle synchronous with field. In shielding tube when field has opposite sign. Voltage across each cell the same.

- Remark: tubes have to become longer and longer, as particles become faster and faster
- or higher frequency $\lambda = c/f_{RF}$
- But radiation power loss: $P = \omega_{RF} C V_{RF}^2$, C gap capacitance

# Alvarez Linac or Drift Tube Linac

- Eliminate power loss: drift tube placed in cavity
  - Electromagnetic field oscillating in cavity. Standing wave, TM mode ( longitudinal E-Field, transverse B-Field)
  - Resonant frequency of cavity = accelerating field frequency
  - Reduce power loss
  - Exploit Farraday’s law: $\nabla \times \vec{E} = -\frac{\partial}{\partial t}\vec{B}$



# Circular Accelerators

- Linear accelerators can in principle accelerate to arbitrarily high energies.
- ….but become longer and longer

- ➔ Particles on circular paths to pass accelerating gap over and over again

- ➔ Cyclotron proposed by E.O. Lawrence in 1929 and built by Livingston in 1931.

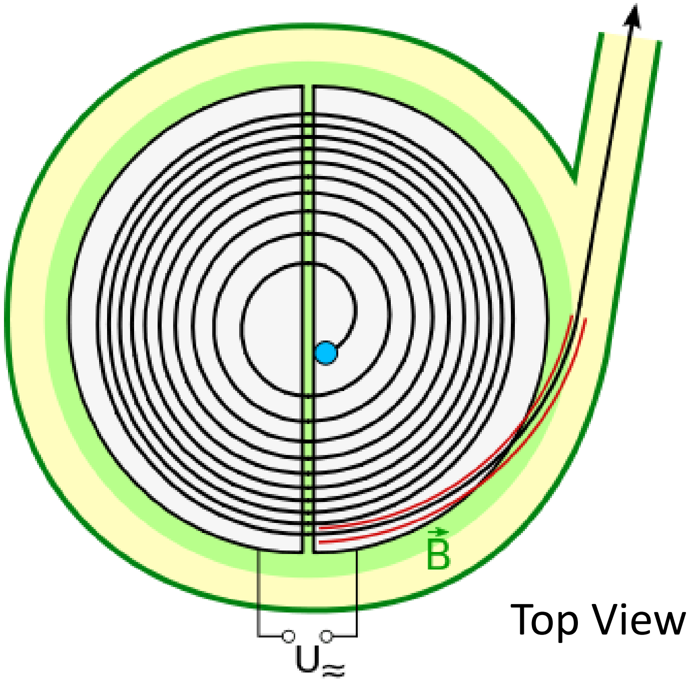
# Cyclotron

Particle Source in the middle

Between the two “Dees” acceleration gap connected to RF source. $\omega_{RF} = \omega_{cyclotron}$

Vertical magnetic field to guide the particles in the horizontal plane. The radius of particle trajectory becomes larger and larger with larger energy

Particles extracted with a deflector magnet or an electrode.



Top View

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \longrightarrow F_L = q\,v\,B \longrightarrow$$ Vertical B
No E

$$F_c = m\frac{v^2}{r} \longrightarrow$$ centrifugal force

$$F_L = F_c \longrightarrow \omega = \frac{v}{r} = \frac{qB}{m} \longrightarrow$$ revolution period

# Cyclotron Limitation

- Cyclotron frequency is constant for constant mass
- For relativistic particles mass is not constant

$$w = \frac{v}{r} = \frac{Bq}{m} = \frac{Bq}{m(E)}$$

- The classical cyclotron only valid for particles up to few % of speed of light
  - Not useful for electrons…already relativistic at 500 keV

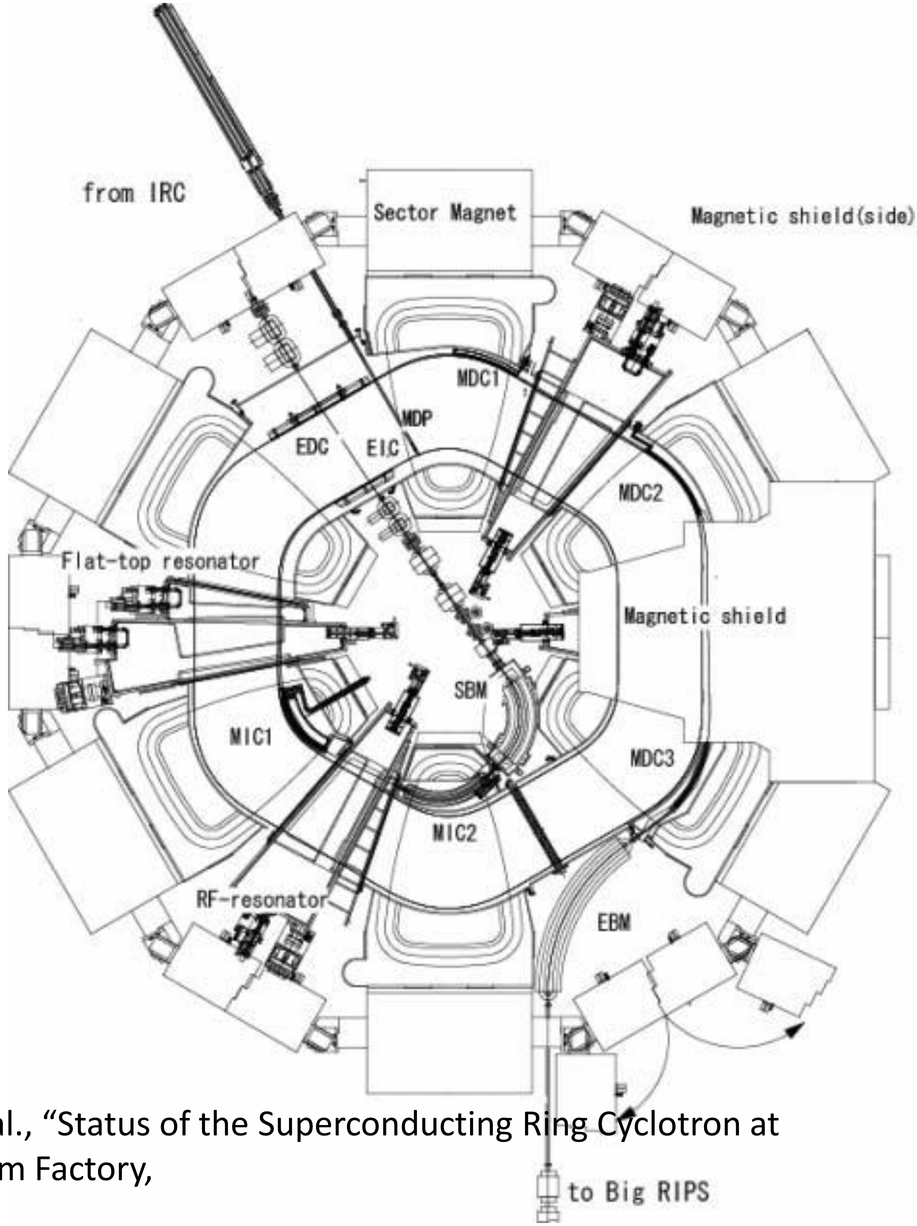
- Possibilities: synchrocyclotrons (change frequency (and magnetic field) with energy) or isochronous cyclotrons (increase magnetic field with r, frequency constant)

- Modern cyclotrons can reach > 500 MeV (PSI, TRIUMF, RIKEN)

# Biggest Cyclotron in the world

- RIKEN, Japan
- 19 m diameter, 8 m high
- 6 superconducting sector magnets, 3.8 T
- Heavy ion acceleration

- Uranium ions accelerated up to 345 MeV/u



K. Yamda et al., “Status of the Superconducting Ring Cyclotron at RIKEN RI Beam Factory,
EPAC 2008

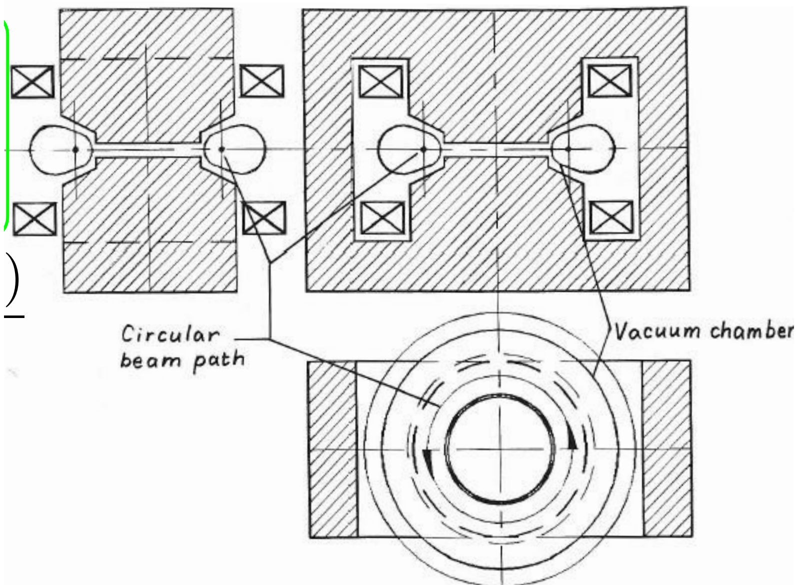
# PSI cyclotron

High intensity $P_{max}$ = 1300 kW

# Betatron - 1940

1942 – 6 MeV

- Another early circular accelerator
- Idea by Wideroe in 1923. Kerst builds the first working betatron in 1940.
- Difference: constant radius and magnetic field changing with time to keep radius constant.
- Accelerating E-field generated through induction, no external E-field
- $e^-$ accelerated to 2.3 MeV
- "Betatron" for beta rays ($e^-$)



# Synchrotron

- Higher and higher energies – larger and larger radii, limited B fields – cannot stay compact

- Fix trajectory ➔ R = constant; R can be large
- Dipole magnets with field only where the beam is
  - “small” magnets

- R= constant ➔B field increases **synchronously** with beam energy

- Synchrotron - all big modern machines are synchrotrons

# Cosmotron – BNL – 1952 – 3 GeV

3 GeV $p^+$ synchrotron

Particle rigidity: $B\rho = \frac{p}{e}$

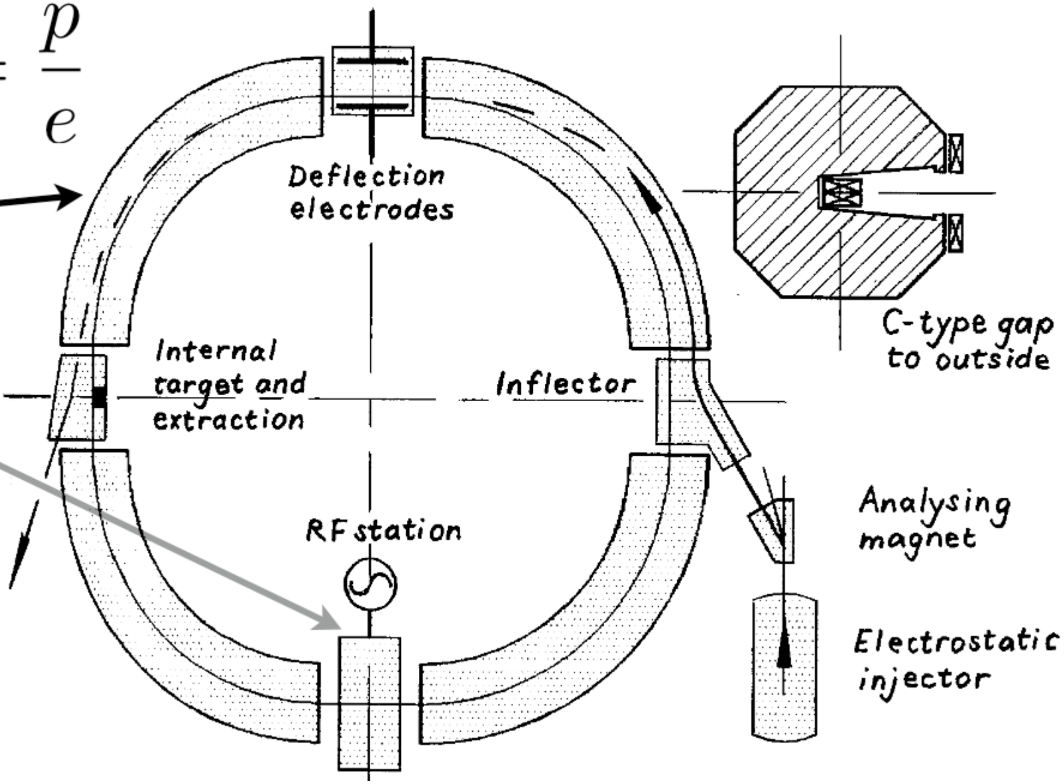
B = B(t) magnetic field from the bending magnets

p = p(t) particle momentum varies by the RF cavity

e electric charge

ρ constant radius of curvature

**New magnetic elements for injection and extraction.**



Particles do many 1000 turns – trajectories have a angular spread ➔ divergence

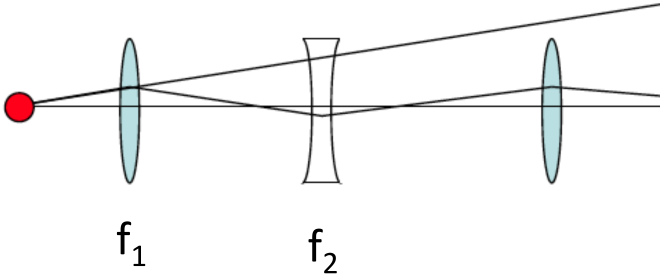
Need focusing elements. Cosmotron **weak focusing** machine

# Strong Focusing

Idea by E. Courant, M. Livingston, H. Snyder in 1952 and earlier by Christofilos

## Alternating gradient focusing

- Analogous to geometrical optics: a series of alternating focusing and defocusing lenses will focus.



$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

Consider $f_1=f$, $f_2 = -f$ ➔ $\boxed{F = f^2/d > 0}$

In our case the lenses will be magnets with alternating gradients QUADRUPOLES

# The first alternating gradient synchrotrons

Alternating gradient focusing was quickly adopted by synchrotrons and transfer lines.

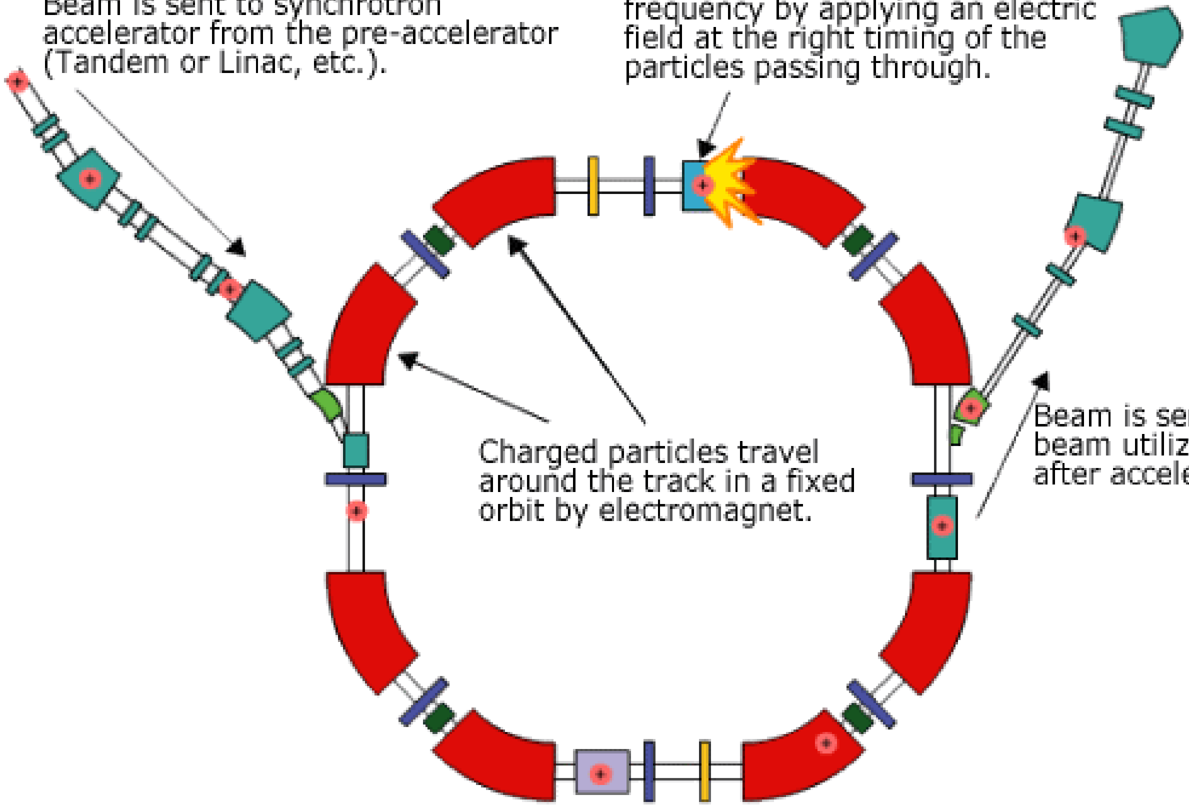
- 1954: Cornell University, $e^-$ accelerated to 1.5 GeV (Wilson et al.)

The following two machines are still in operation.

They use combined function magnets.

- 1959: CERN Proton Synchrotron (PS) accelerated protons to 28 GeV
- 1960: Brookhaven Alternating Gradient Synchrotron (AGS) accelerated protons to 33 GeV

# The basic Layout of a Alternating Gradient Synchrotron



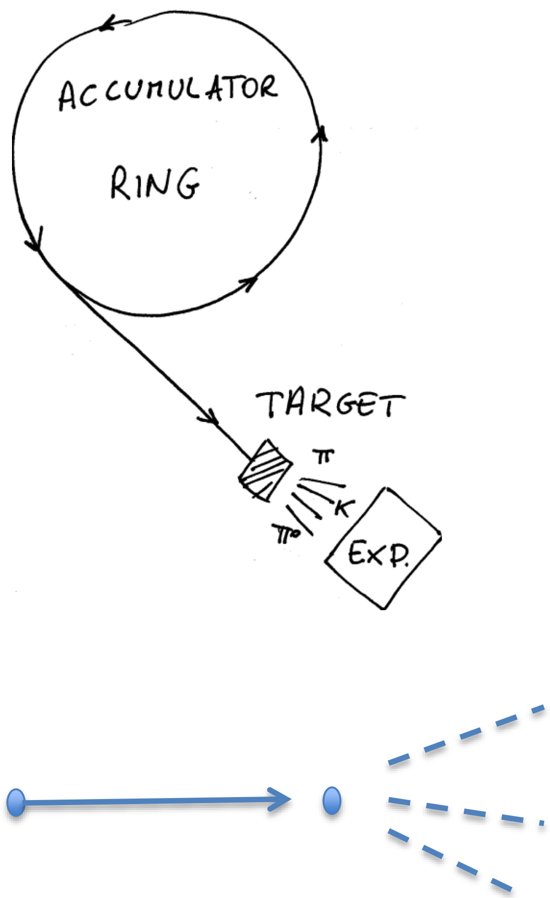
Accelerating cavity
It accelerates particles with high frequency by applying an electric field at the right timing of the particles passing through.

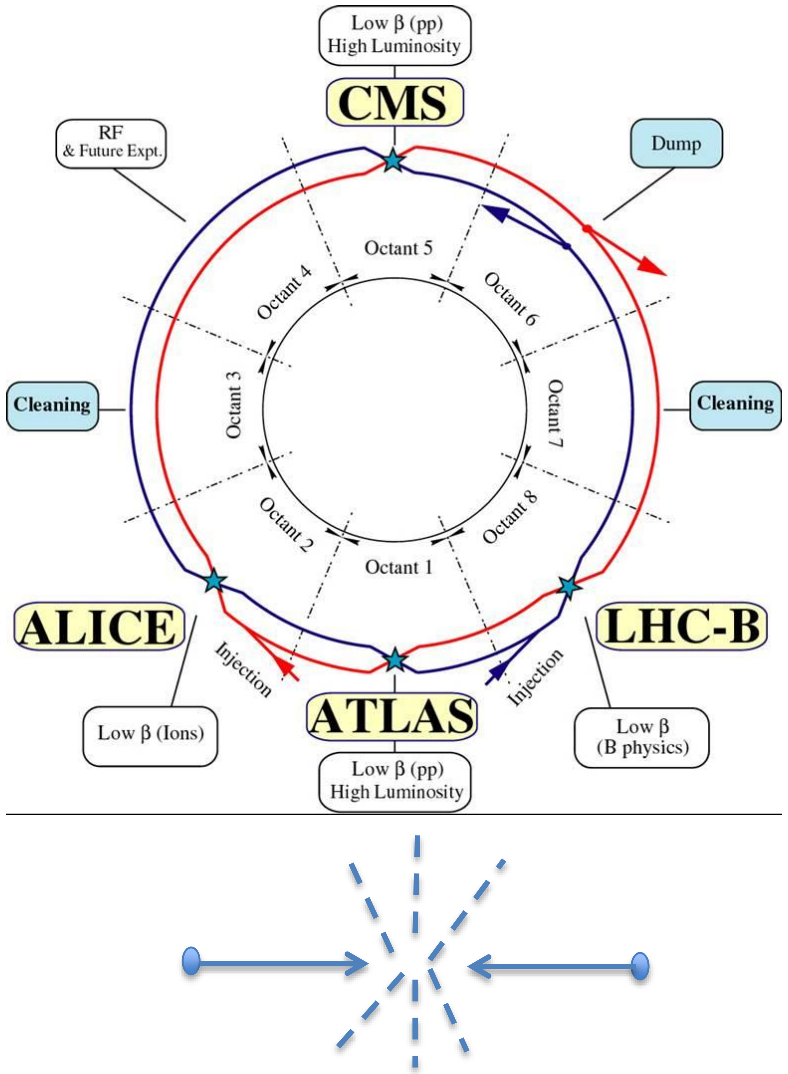
Beam is sent to synchrotron accelerator from the pre-accelerator (Tandem or Linac, etc.).

Charged particles travel around the track in a fixed orbit by electromagnet.

Beam is sent to the beam utilizing course after acceleration.

# Fixed target vs. Colliders





# Fixed target vs. Colliders – Center-of-mass Energy

- Center-of-mass Frame and Center-of-mass Energy ($E_{CM}$)
  - Center-of-mass frame defined where: $\sum \vec{p}_i = \vec{0}$
  - The energy available for creation of particles corresponds to $E_{CM}$

# Center-of-Mass Energy

Transformation to center-of-mass frame: Lorentz transformation

$$p^\mu = (E/c, \vec{p})$$

4-momentum

$$p^\mu p_\mu = \frac{E^2}{c^2} - \vec{p}^2$$

$$p'^\mu = L^\nu_\mu p^\nu$$

Lorentz Transformation

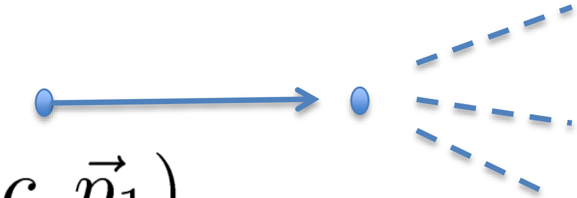
$$p^\mu p_\mu = p'^\mu p'_\mu$$

The norm: is Lorentz invariant

$$\frac{E^2_{CM}}{c^2} - \vec{0}^2 = \frac{E^2_{tot}}{c^2} - \vec{p}^2_{tot}$$

$$\frac{E^2_{CM}}{c^2} = \frac{E^2_{tot}}{c^2} - \vec{p}^2_{tot}$$

# $E_{CM}$ in Fixed Target Experiment



$$p_2 = (m_2 c, \vec{0})$$

$$p_1 = (E_1/c, \vec{p_1})$$

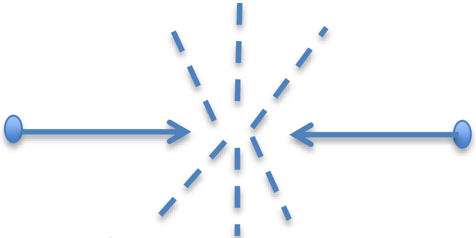
$$p_{tot} = (E_1/c + m_2 c, \vec{p_1})$$

$$E_{CM}^2 = (m_1^2 + m_2^2)c^4 + 2E_1 m_2 c^2$$

$$E_{CM} \propto \sqrt{E_1}$$

# $E_{CM}$ in Collider Experiment

Laboratory Frame = CM Frame



$$p_1 = (E_1/c, \vec{p}_1) \qquad p_2 = (E_2/c, -\vec{p}_1)$$

$$E_{CM} = E_1 + E_2$$

➔ Collider more energy efficient;
But also more complex: two beams to be accelerated and to be brought into collision

# The next step: storage ring colliders

Make use of all the particles’ energy. 2-beam synchrotrons.

The first one: Ada (Frascatti), 1961-64, e+,e-, 250 MeV, 3m circumference

Many examples to come at DESY, SLAC, KEK, Fermilab with the Tevatron (980 GeV), BNL with RHIC

1971-1984: ISR (CERN), p+,p+, 31.5 GeV, 948 m circumference

1981-1991: SPS running as SppS, p+, p-, 270 – 315 GeV, 6.9 km circumference; discovery of W and Z Bosons

1989-2000: LEP highest energy electron synchrotron, e+,e-, 104 GeV, 27 km circumference; three generations of quarks, gluons and leptons

2008 - : LHC highest energy proton synchrotron, p+,p+, heavy ions, 6.5 TeV (2.76 TeV per nucleon for $^{208}Pb^{82+}$); Discovery of Higgs

# THE MAIN MAGNETS

# How can we keep the particles on a circular trajectory?

- Usually use only magnetic fields for transverse control

$$\boxed{\vec{F} = q \cdot (\vec{E} + \vec{v} \times \vec{B})}$$

Lorentz Force

- What is the equivalent E field of B = 1 T?
  - Ultra-relativistic: $|\vec{v}| \approx c \approx 3 \times 10^8 m/s$

$$\begin{aligned} F &= q \cdot 3 \cdot 10^8 \frac{m}{s} \cdot 1T \\ &= q \cdot 3 \cdot 10^8 \frac{m}{s} \cdot \frac{Vs}{m^2} \\ &= q \cdot 300 \frac{MV}{m} \end{aligned}$$

Equivalent electric field!!:

$$|\vec{E}| = 300 \frac{MV}{m}$$

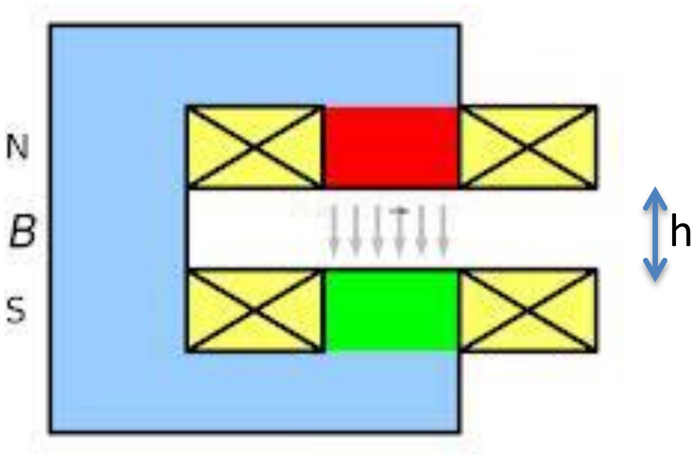
- ➔ To guide the particles we use magnetic fields from electro-magnets.

# Dipole magnets: guiding magnets

- Vertical magnetic field to bend in the horizontal plane

- Dipole electro-magnets:

$$\boxed{\vec{F} = q \cdot \vec{v} \times \vec{B}}$$

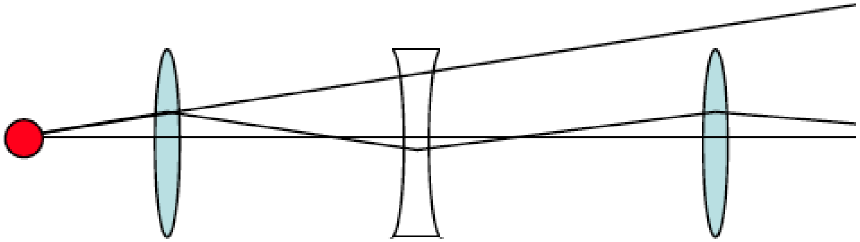


$$B = \frac{\mu_0 n I}{h}$$

# Focusing is mandatory for stability

Define design trajectory with dipole magnets

Trajectories of particles in beam will deviate from design trajectory



- ➔ Focusing
  - Particles should feel restoring force when deviating from design trajectory horizontally or vertically

# Focusing with Quadrupole Magnets

- Requirement: Lorentz force increases as a function of distance from design trajectory

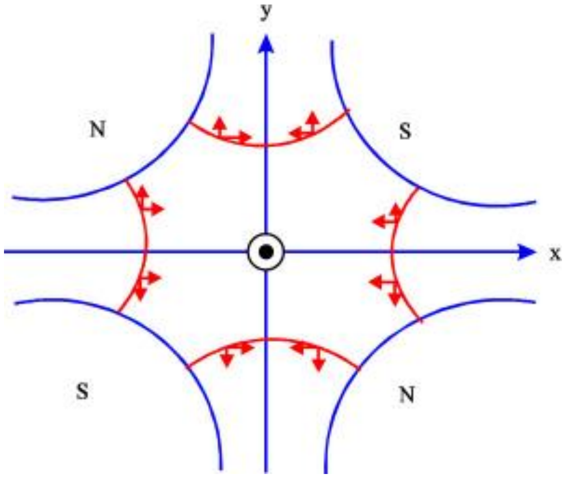
- E.g. in the horizontal plane

$$F(x) = q \cdot v \cdot B(x)$$

- We want a magnetic field that

$$B_y = g \cdot x \qquad B_x = g \cdot y$$

➔ Quadrupole magnet



The red arrows show the direction of the force on the particle

- **Gradient** of quadrupole

$$g = \frac{2\mu_0 n I}{r^2} \left[\frac{T}{m}\right]$$

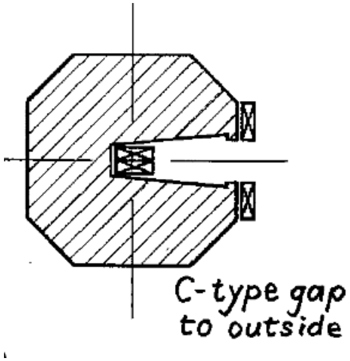
Normalized gradient, focusing strength

$$k = \frac{g}{p/e} [m^{-2}]$$

# Strong focusing

- Initially weak focusing or constant-gradient focusing.

- In early cyclotrons needed to introduce magnet shims to distort the guide field such that it was decreasing with R. A gradient.



Example: bending magnets in the Cosmotron, the first synchrotron.

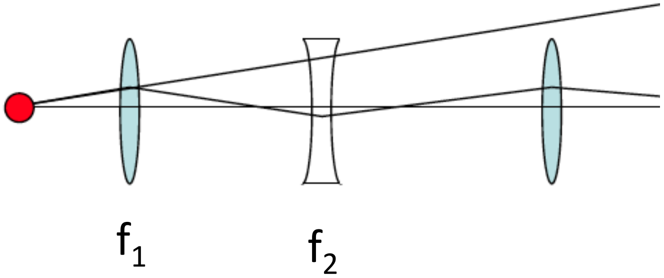
Apertures very large.
Energy limit was believed to be 10 GeV.

- The new idea: instead of constant-gradient: alternating gradient
- allowed for smaller apertures and much, much higher energies.

- Analogous to geometrical optics with light lenses.

# Strong focusing

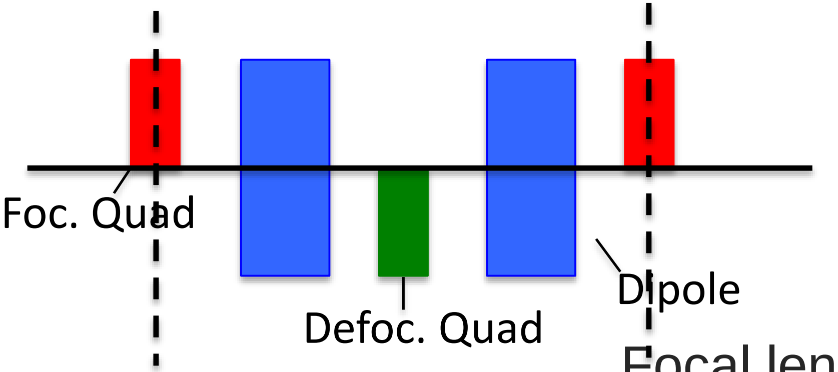
- Light lenses:



$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

Consider $f_1=f$, $f_2 = - f$ ➔ $\boxed{F = f^2/d > 0}$

- In a synchrotron: the lenses are the quadrupoles

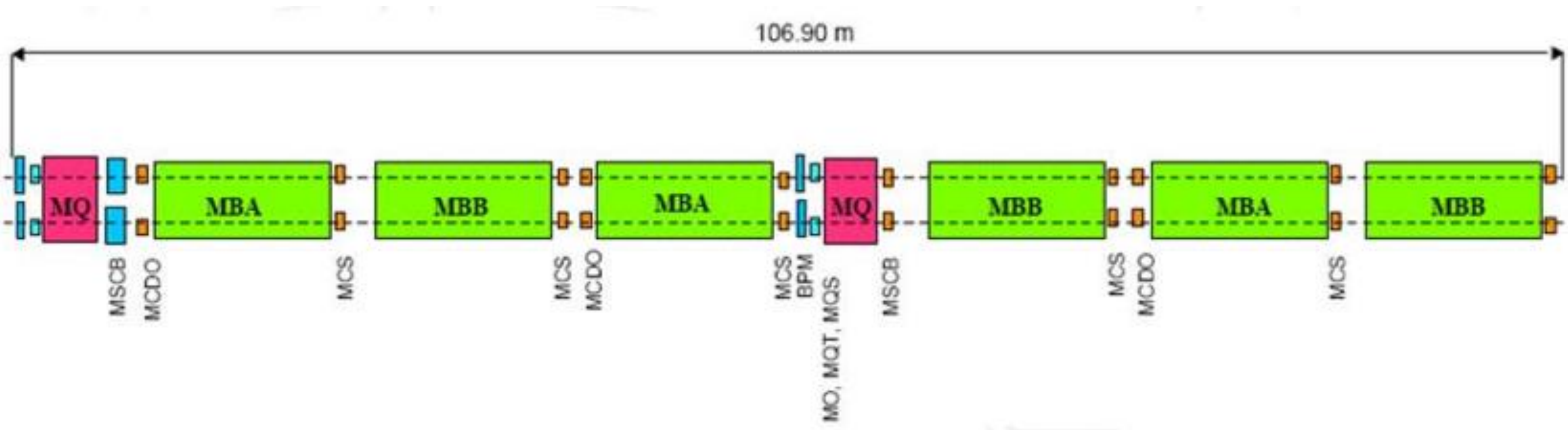


FODO Cell: F = Focusing, 0=nothing (bend, RF,..),
D = Defocusing

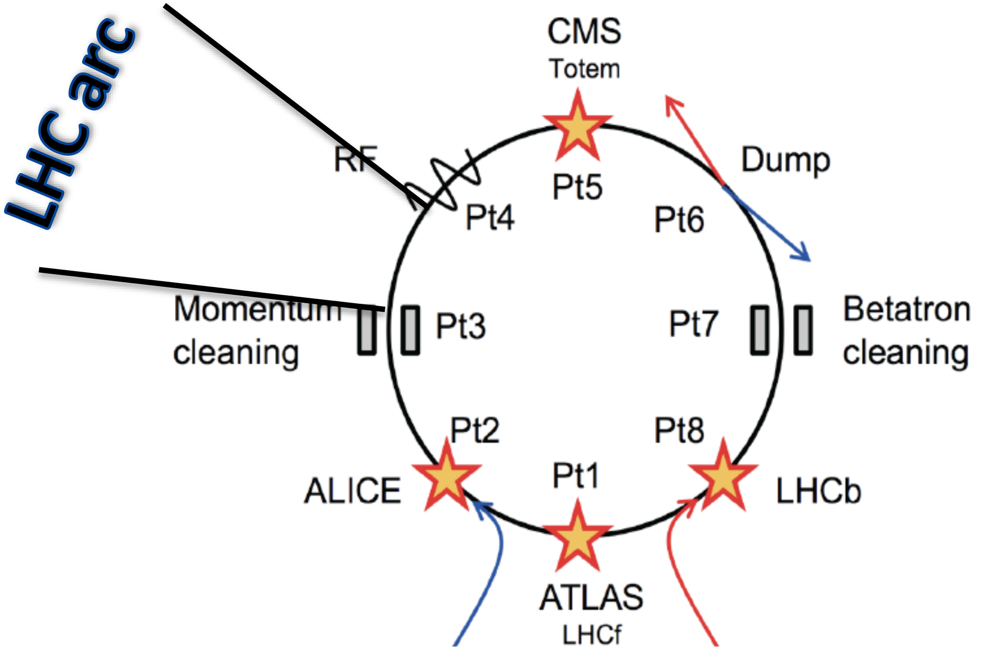
- Focal length of quadrupole

$$f = \frac{1}{k \cdot l_Q}$$

# The LHC FODO cell

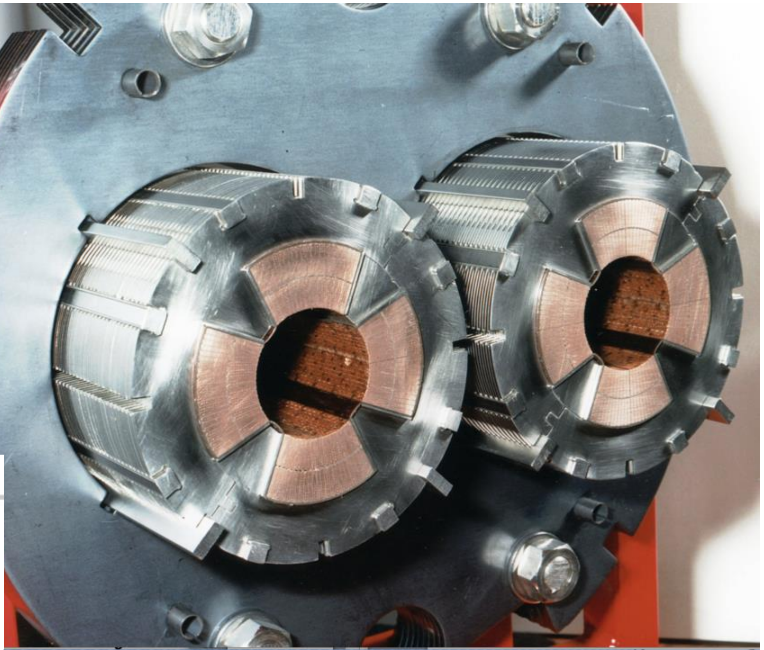


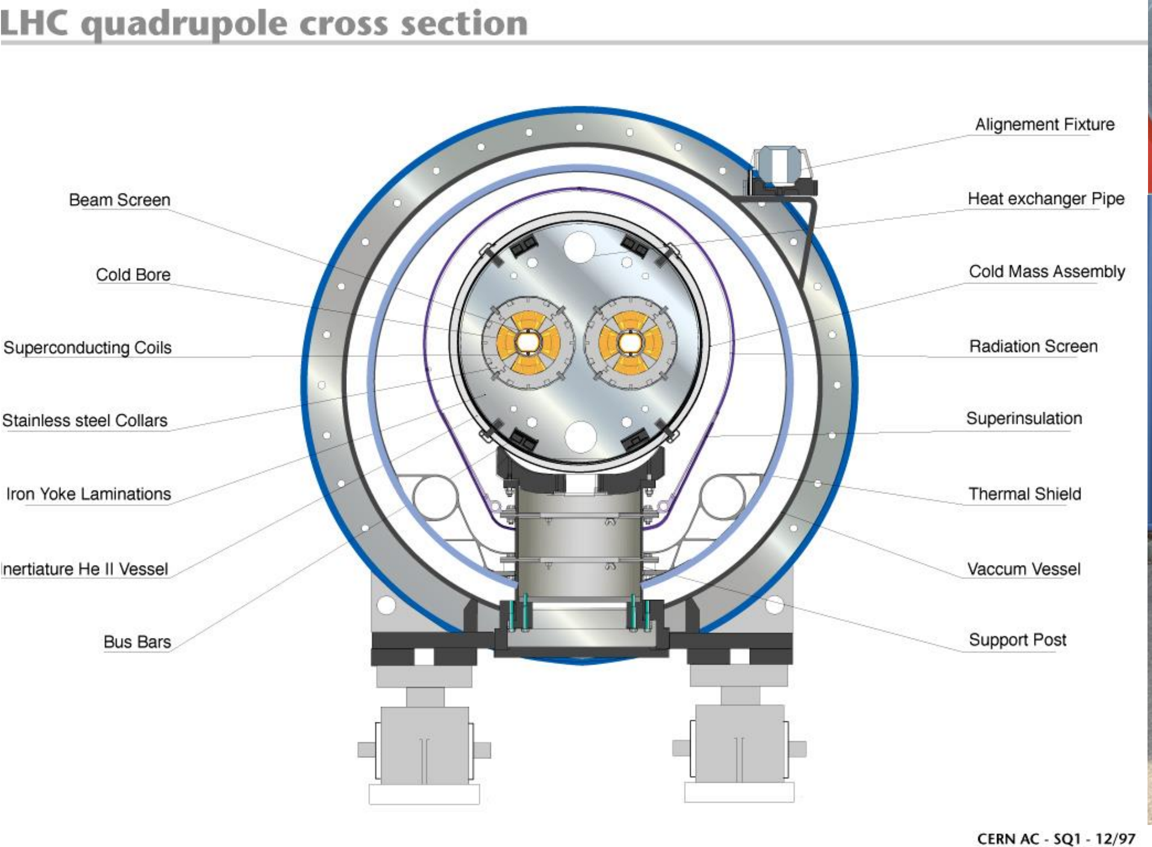
- Each LHC arc consists of
- 23 FODO cells



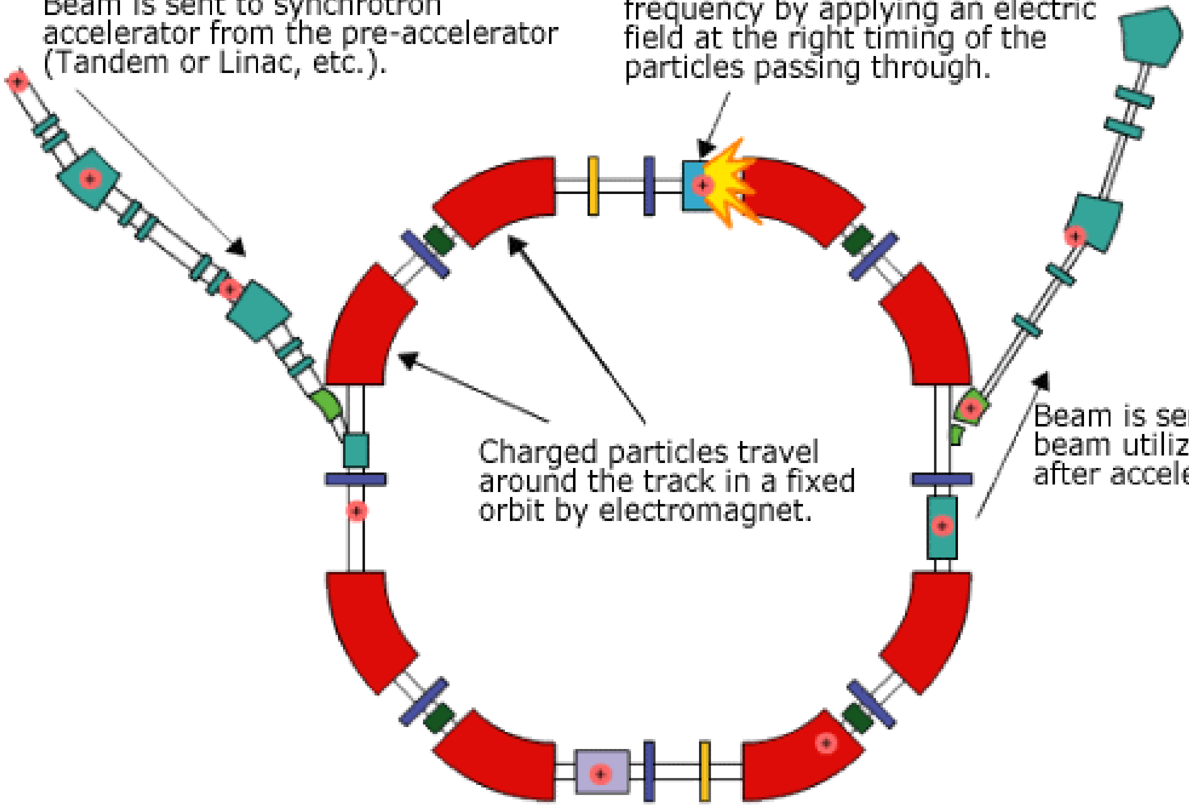
# The LHC main quadrupole magnet

- Length = 3.2 m
- Gradient = 223 T/m
- Peak field 6.83 T
- Total number in LHC: 392





# Synchrotron



Beam is sent to synchrotron accelerator from the pre-accelerator (Tandem or Linac, etc.).

Accelerating cavity

It accelerates particles with high frequency by applying an electric field at the right timing of the particles passing through.

Charged particles travel around the track in a fixed orbit by electromagnet.

Beam is sent to the beam utilizing course after acceleration.

# ACCELERATION

# LHC superconducting cavities

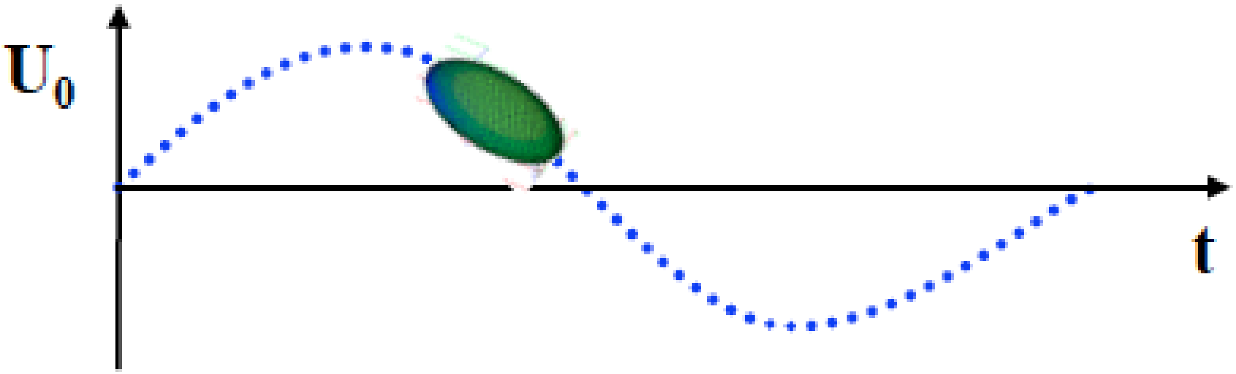
- 8 cavities per beam in 2 cryo-modules per beam.
- Can deliver 2 MV per cavity. Accelerating field 5 MV/m
- RF frequency: 400 MHz.

# Acceleration

- Using RF acceleration: multiple application of the same accelerating voltage.
- Brilliant idea to gain higher energies

- …but accelerating voltage is changing with time while particles are going through the RF system.

- ➔ Longitudinal dynamics



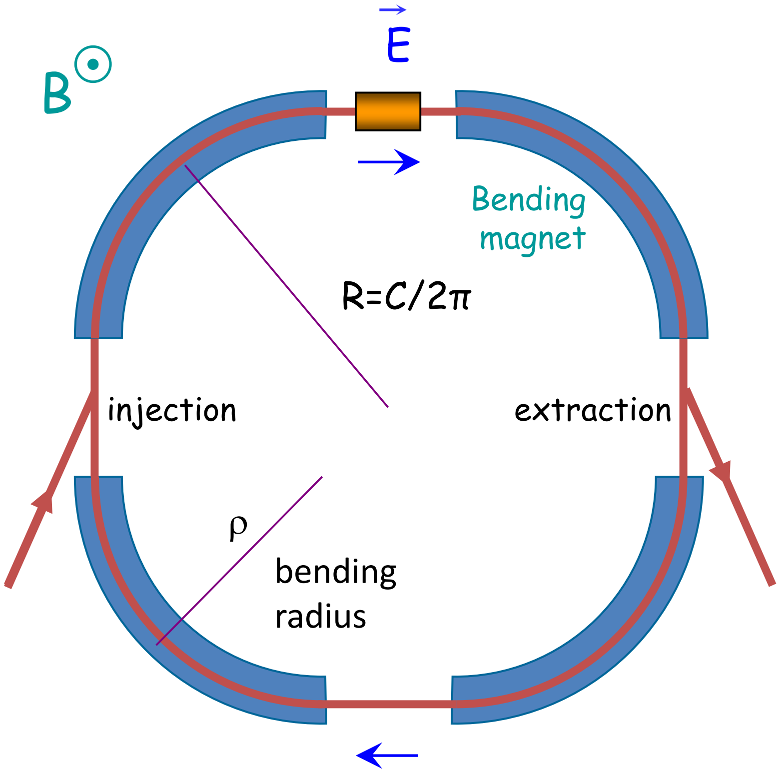
Not all particles arrive at the same time.

Not all particles will receive the same energy gain.

Not all particles will have the same energy.

# Acceleration in a Synchrotron

- Synchrotron: there is a synchronous RF phase of the RF field for which the energy gain fits the increase of the magnetic field



Energy gain per turn $eV\sin\phi = eV\sin\omega_{RF}t$

Reference particle,
synchronous particle $\phi = \phi_s = const$

RF synchronism: the RF frequency must be locked to the revolution frequency $\omega_{RF} = h\omega_{rev}$

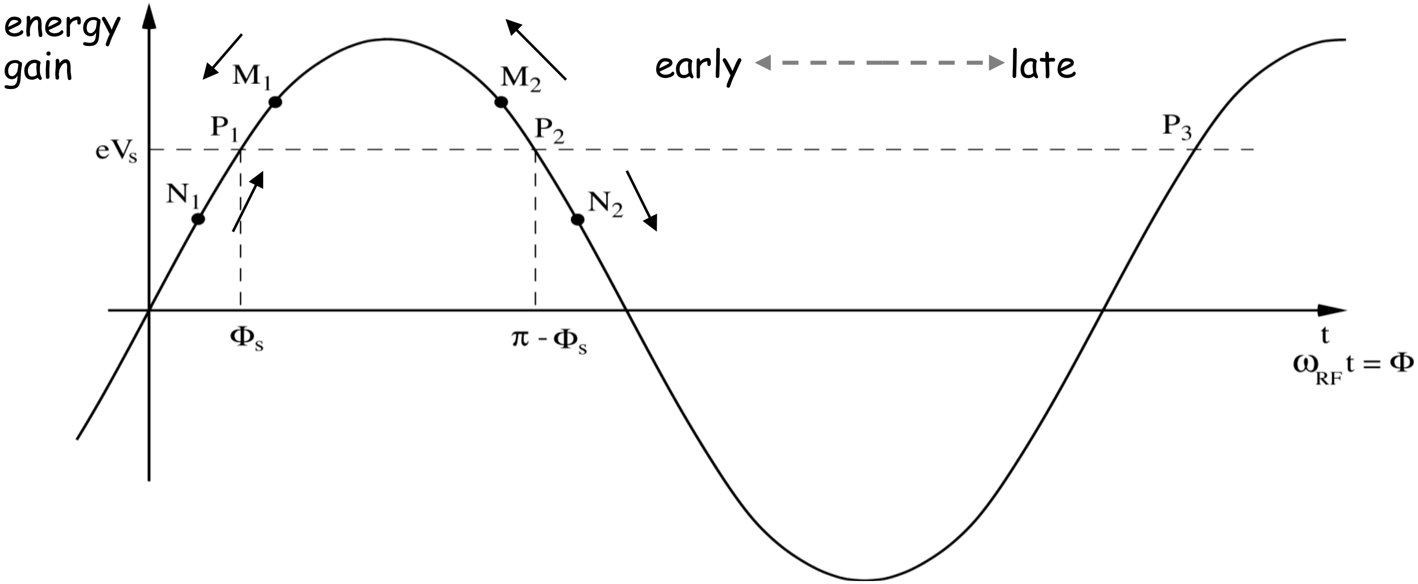
h…harmonic number

constant orbit, bending radius

variable magnetic field

# Principle of Phase Stability

- Assume the situation where energy increase is transferred into a velocity increase
- Particles $P_1$, $P_2$ have the synchronous phase.



$M_1$ & $N_1$ will move towards $P_1$ => stable

$M_2$ & $N_2$ will go away from $P_2$ => unstable (and finally be lost)