

# Introduction to Transverse Beam Dynamics

Bernhard Holzer, CERN

## 1.) the basic ideas

„ ... in the end and after all it should be a kind of circular machine“  
 → need transverse deflecting force

Lorentz force       $\vec{F} = q * (\cancel{\vec{v}} + \vec{v} \times \vec{B})$

typical velocity in high energy machines:       $v \approx c \approx 3 * 10^8 \text{ m/s}$

Example:

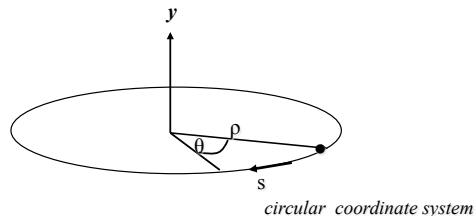
$B = 1 \text{ T}$	$\rightarrow F = q * 3 * 10^8 \frac{m}{s} * 1 \frac{V_s}{m^2}$	technical limit for el. field
	$F = q * 300 \frac{MV}{m}$	
equivalent el. field ...	$E$	$E \leq 1 \frac{MV}{m}$

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old greek dictum of wisdom:

if you are clever, you use magnetic fields in an accelerator wherever it is possible.

The ideal circular orbit



condition for circular orbit:

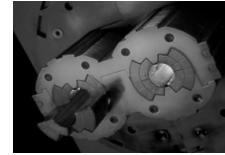
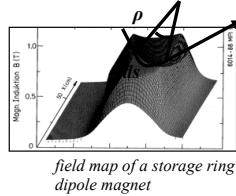
Lorentz force	$F_L = e v B$	}	$\frac{p}{e} = B \rho$
centrifugal force	$F_{centr} = \frac{\gamma m_0 v^2}{\rho}$		
$\frac{\gamma m_0 v}{\rho} = e v B$			

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## 2.) The Magnetic Guide Field

**Dipole Magnets:**

define the ideal orbit  
homogeneous field created  
by two flat pole shoes



s.c. LHC dipole

Normalise magnetic field to momentum:

$$\frac{p}{e} = B \rho \quad \longrightarrow \quad \frac{1}{\rho} = \frac{e B}{p}$$

convenient units:

$$B = [T] = \left[ \frac{Vs}{m^2} \right] \quad p = \left[ \frac{GeV}{c} \right]$$

$$B \approx 1 \dots 8 \text{ T}$$

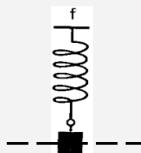
Example LHC:

$$\left. \begin{array}{l} B = 8.3 \text{ T} \\ p = 7000 \frac{\text{GeV}}{c} \end{array} \right\} \quad \begin{aligned} \frac{1}{\rho} &= e \frac{8.3 \frac{Vs}{m^2}}{7000 * 10^9 \frac{eV}{c}} = \frac{8.3 * 3 * 10^8 \frac{m}{s}}{7000 * 10^9 \frac{m^2}{s}} \\ &\rho = 2.81 \text{ km} \end{aligned}$$

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## Focusing Properties and Quadrupole Magnets

classical mechanics:  
pendulum



there is a restoring force, proportional  
to the elongation x:

$$m * \frac{d^2x}{dt^2} = -c * x$$

general solution: free harmonic oscillation

$$x(t) = A * \cos(\omega t + \varphi)$$

this is how grandma's Kuckuck's clock is working!!!

Storage Rings: linear increasing Lorentz force to keep trajectories in vicinity of  
the ideal orbit  
linear increasing magnetic field  $B_y = g x \quad B_x = g y$

$$F(x) = q * v * B(x)$$

as in the dipole case we normalise to the beam rigidity

$$k = \frac{g}{B\rho} = \frac{g}{p/q}$$

LHC main quadrupole magnet  $g \approx 25 \dots 220 \text{ T/m}$



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#### 4.) The equation of motion:

**Linear approximation:**

- \* ideal particle  $\rightarrow$  design orbit
- \* any other particle  $\rightarrow$  coordinates  $x, y$  small quantities  
 $x, y \ll \rho$
- $\rightarrow$  magnetic guide field: only linear terms in  $x$  &  $y$  of  $B$   
have to be taken into account

**Taylor Expansion of the  $B$  field ...**      normalised to momentum  $p/e = B\rho$   
and only terms linear in  $x, y$  taken into account  
dipole fields / quadrupole fields

$$\frac{B(x)}{p/e} = \frac{B_0}{B_0\rho} + \frac{g^*x}{p/e} + \cancel{\frac{1}{2}\frac{ex}{p/e}} + \cancel{\frac{1}{3!}\frac{ex^2}{p/e}} + \dots$$

$$= \frac{1}{\rho} + k^*x$$

**Separate Function Machines:**

Split the magnets and optimise  
them according to their job:  
bending, focusing etc

Example:  
heavy ion storage ring TSR



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#### Equation of Motion:

**Remember:**

**Hamiltonian for ideal particle,  $\delta = 0$**

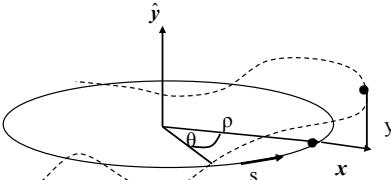
$$H = \frac{p_x^2 + p_y^2}{2} - \frac{x^2}{2\rho(s)^2} + \frac{k_1(s)}{2}(x^2 - y^2)$$

with  $k$  and  $\rho$  representing the normalised quadrupole and dipole fields

**putting into Hamiltonian equations**

$$\frac{\partial H}{\partial x} = \frac{-dp_x}{ds}, \quad \frac{\partial H}{\partial p_x} = \frac{dx}{ds}$$

... see e.g. Goldstein p 241



**we get the equation of motion**

$$\frac{d^2x}{ds^2} + \left\{ \frac{1}{\rho(s)^2} - k_1(s) \right\}^* x = 0, \quad \frac{d^2y}{ds^2} + k_1(s)^* y = 0$$

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### Equation of Motion:

In linear approximation ( $x, y \ll \rho$  and only dipole & quadrupole fields)  
we can derive a differential equation for the transverse motion of the particles

\* Equation for the horizontal motion:

$$x'' + x \left( \frac{1}{\rho^2} - k \right) = 0$$

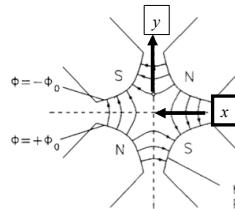
Under the influence of the focusing fields from the quadrupoles „k“ and dipoles  $1/\rho^2$  the transverse movement of the particles inside looks like a harmonic oscillation

\* Equation for the vertical motion:

$$\frac{1}{\rho^2} = 0 \quad \text{no dipoles ... in general ...}$$

$k \leftrightarrow -k$  quadrupole field changes sign

$$y'' + k y = 0$$



... mmmpff ... just another differential equation .... but it does not look sooo comfortable.

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### 5.) Solution of Trajectory Equations

$$\left. \begin{array}{l} \text{Define ... hor. plane: } K = 1/\rho^2 - k \\ \text{... vert. Plane: } K = k \end{array} \right\} \quad x'' + K x = 0$$

Differential Equation of harmonic oscillator ... with spring constant  $K > 0 \rightarrow$  focusing case

$$\text{Ansatz: } x(s) = a_1 \cdot \cos(\omega s) + a_2 \cdot \sin(\omega s)$$

general solution: linear combination of two independent solutions

$$x'(s) = -a_1 \omega \sin(\omega s) + a_2 \omega \cos(\omega s)$$

$$x''(s) = -a_1 \omega^2 \cos(\omega s) - a_2 \omega^2 \sin(\omega s) = -\omega^2 x(s) \quad \longrightarrow \quad \omega = \sqrt{K}$$

general solution:

$$x(s) = a_1 \cos(\sqrt{K}s) + a_2 \sin(\sqrt{K}s)$$

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determine  $a_1, a_2$  by boundary conditions:

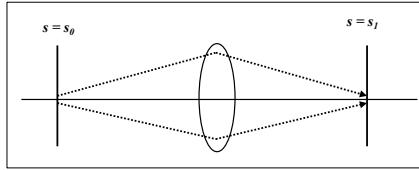
$$s = 0 \quad \longrightarrow \quad \begin{cases} x(0) = x_0 & , \quad a_1 = x_0 \\ x'(0) = x'_0 & , \quad a_2 = \frac{x'_0}{\sqrt{|K|}} \end{cases}$$

*Hor. Focusing Quadrupole  $K > 0$ :*

$$\begin{aligned} x(s) &= x_0 \cdot \cos(\sqrt{|K|}s) + x'_0 \cdot \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}s) \\ x'(s) &= -x_0 \cdot \sqrt{|K|} \cdot \sin(\sqrt{|K|}s) + x'_0 \cdot \cos(\sqrt{|K|}s) \end{aligned}$$

For convenience expressed in matrix formalism:

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{foc} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$

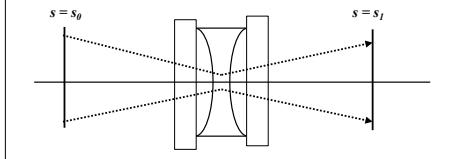


$$M_{foc} = \begin{pmatrix} \cos(\sqrt{|K|}s) & \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}s) \\ -\sqrt{|K|} \sin(\sqrt{|K|}s) & \cos(\sqrt{|K|}s) \end{pmatrix}_0$$

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*hor. defocusing quadrupole:*

$$x'' - K x = 0$$



Remember from school:

$$f(s) = \cosh(s) \quad , \quad f'(s) = \sinh(s)$$

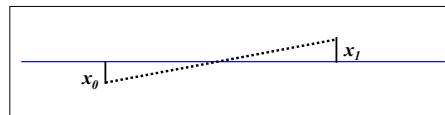
Ansatz:  $x(s) = a_1 \cdot \cosh(\omega s) + a_2 \cdot \sinh(\omega s)$

$$M_{defoc} = \begin{pmatrix} \cosh \sqrt{|K|}l & \frac{1}{\sqrt{|K|}} \sinh \sqrt{|K|}l \\ \sqrt{|K|} \sinh \sqrt{|K|}l & \cosh \sqrt{|K|}l \end{pmatrix}$$

*drift space:*

$$K = 0$$

$$x_1 = x_0 + x'_0 * l$$



$$M_{drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

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### Combining the two planes:

Clear enough (hopefully ... ?): a quadrupole magnet that is focussing in one plane acts as defocusing lens in the other plane ... et vice versa.

hor foc. quadrupole lens

$$M_{foc} = \begin{pmatrix} \cos(\sqrt{|K|}l) & \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}l) \\ -\sqrt{|K|} \sin(\sqrt{|K|}l) & \cos(\sqrt{|K|}l) \end{pmatrix}$$

matrix of the same magnet in the vert. plane:

$$M_{defoc} = \begin{pmatrix} \cosh \sqrt{|K|}l & \frac{1}{\sqrt{|K|}} \sinh \sqrt{|K|}l \\ \sqrt{|K|} \sinh \sqrt{|K|}l & \cosh \sqrt{|K|}l \end{pmatrix}$$

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_f = \begin{pmatrix} \cos(\sqrt{|k|}l) & \frac{1}{\sqrt{|k|}} \sin(\sqrt{|k|}l) \\ -\sqrt{|k|} \sin(\sqrt{|k|}l) & \cos(\sqrt{|k|}l) \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \cosh(\sqrt{|k|}l) & \frac{1}{\sqrt{|k|}} \sinh(\sqrt{|k|}l) \\ \sqrt{|k|} \sinh(\sqrt{|k|}l) & \cosh(\sqrt{|k|}l) \end{pmatrix} \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_i$$

! with the assumptions made, the motion in the horizontal and vertical planes are independent „... the particle motion in x & y is uncoupled“ !

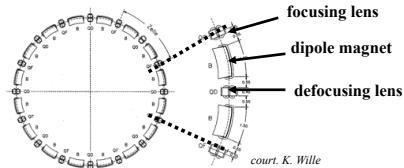
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### Transformation through a system of lattice elements

combine the single element solutions by multiplication of the matrices

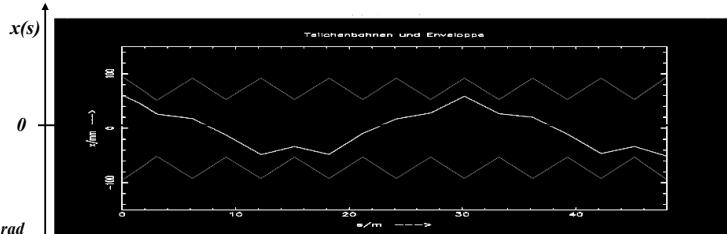
$$M_{total} = M_{QF} * M_D * M_{QD} * M_{Bend} * M_{D*} \dots$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s2} = M(S_2, S_1) * \begin{pmatrix} x \\ x' \end{pmatrix}_{sl}$$



in each accelerator element the particle trajectory corresponds to the movement of a harmonic oscillator „

typical values  
in a strong  
foc. machine:  
 $x \approx \text{mm}$ ,  $x' \leq \text{mrad}$



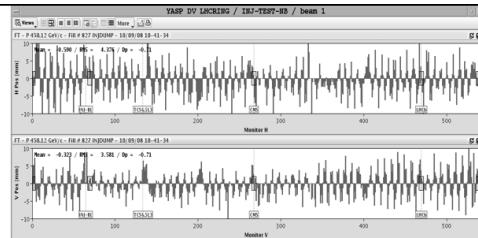
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## 6.) Orbit & Tune:

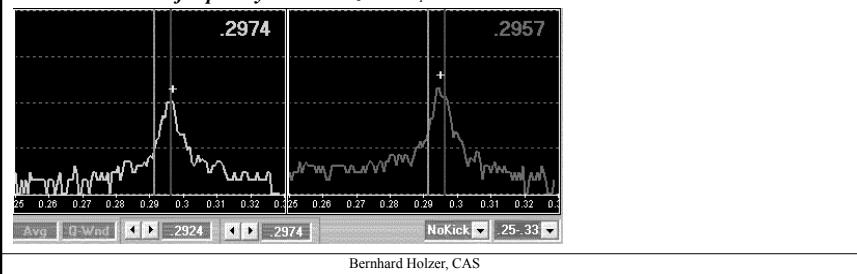
**Tune: number of oscillations per turn**

64.31  
59.32

**Relevant for beam stability:**  
*non integer part*



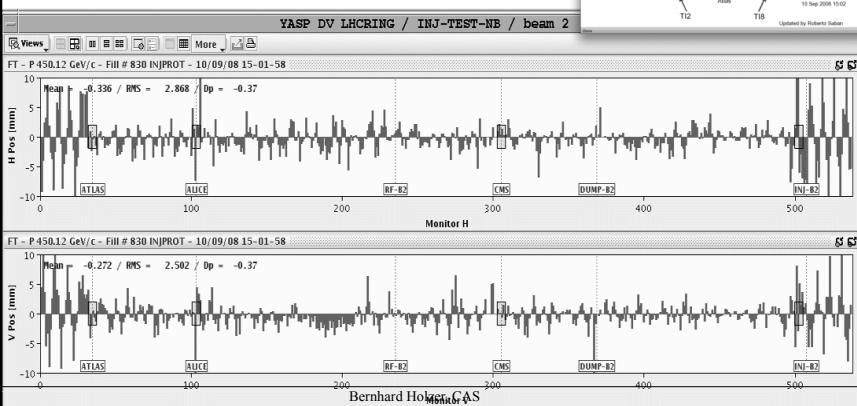
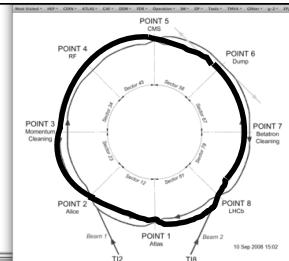
**LHC revolution frequency: 11.3 kHz**       $f_q = 0.31 * 11.3 = 3.5 \text{ kHz}$



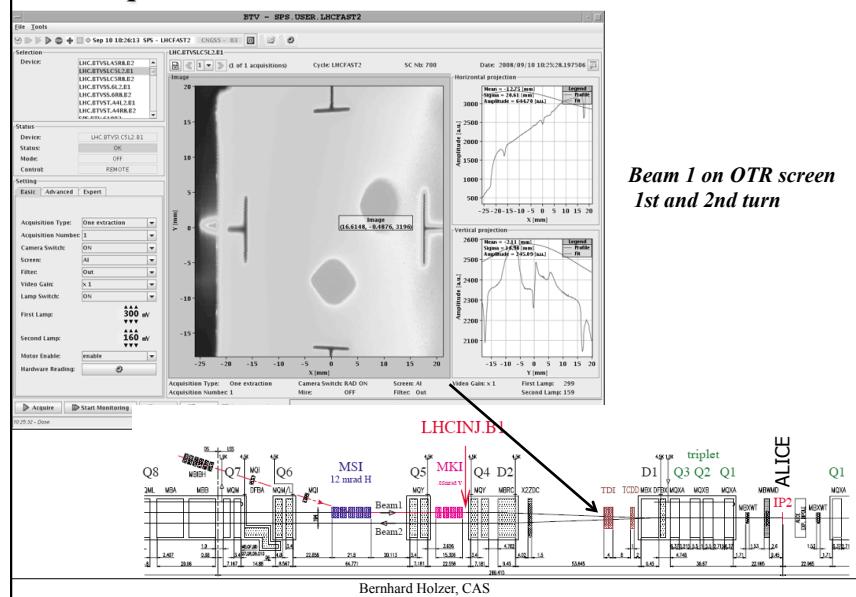
## LHC Operation: Beam Commissioning

**First turn steering "by sector:"**

- ❑ One beam at the time
- ❑ Beam through 1 sector (1/8 ring),  
correct trajectory, open collimator and move on.

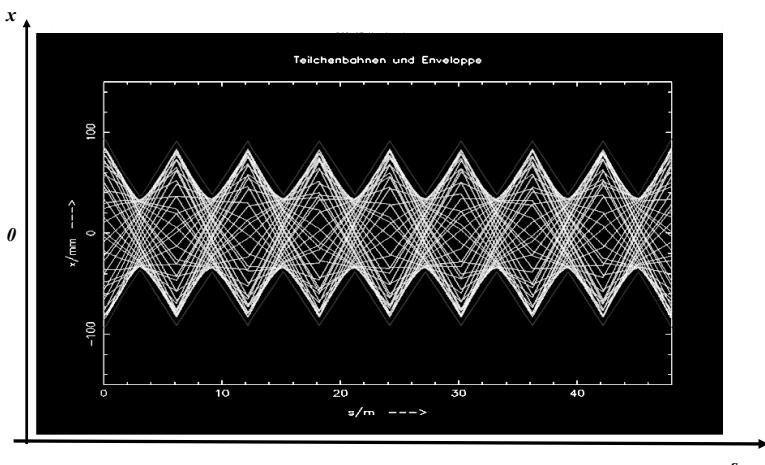


## LHC Operation: the First Beam



Question: what will happen, if the particle performs a second turn ?

... or a third one or ...  $10^{10}$  turns



## 7.) The Beta Function

*General solution of Hill's equation:*

$$(i) \quad x(s) = \sqrt{\varepsilon} \sqrt{\beta(s)} \cdot \cos(\psi(s) + \phi)$$

$\varepsilon, \phi$  = integration constants determined by initial conditions

$\beta(s)$  periodic function given by focusing properties of the lattice  $\leftrightarrow$  quadrupoles

$$\beta(s+L) = \beta(s)$$

Inserting (i) into the equation of motion ...

$$\psi(s) = \int_0^s \frac{ds}{\beta(s)}$$

$\Psi(s)$  = „phase advance“ of the oscillation between point „0“ and „s“ in the lattice.  
For one complete revolution: number of oscillations per turn „Tune“

$$Q_y = \frac{1}{2\pi} \oint \frac{ds}{\beta(s)}$$

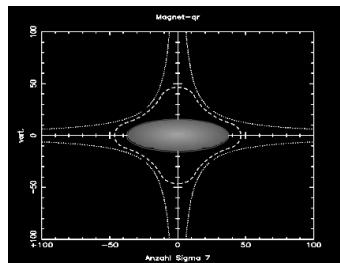
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## The Beta Function

*Amplitude of a particle trajectory:*

$$x(s) = \sqrt{\varepsilon} * \sqrt{\beta(s)} * \cos(\psi(s) + \varphi)$$

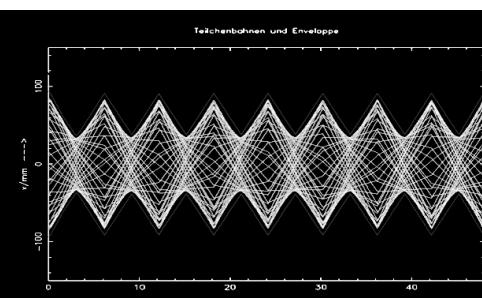
*Maximum size of a particle amplitude*



$\hat{x}(s) = \sqrt{\varepsilon} \sqrt{\beta(s)}$

$\beta$  determines the beam size  
(... the envelope of all particle trajectories at a given position "s" in the storage ring.

It reflects the periodicity of the magnet structure.



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## 8.) Beam Emittance and Phase Space Ellipse

general solution of Hill equation

$$\left\{ \begin{array}{ll} (1) & x(s) = \sqrt{\epsilon} \sqrt{\beta(s)} \cos(\psi(s) + \phi) \\ (2) & x'(s) = -\frac{\sqrt{\epsilon}}{\sqrt{\beta(s)}} \{ \alpha(s) \cos(\psi(s) + \phi) + \sin(\psi(s) + \phi) \} \end{array} \right.$$

from (1) we get

$$\cos(\psi(s) + \phi) = \frac{x(s)}{\sqrt{\epsilon} \sqrt{\beta(s)}}$$

Insert into (2) and solve for  $\epsilon$

using the parameter definitions

$$\alpha(s) = \frac{-1}{2} \beta'(s)$$

$$\gamma(s) = \frac{1 + \alpha(s)^2}{\beta(s)}$$

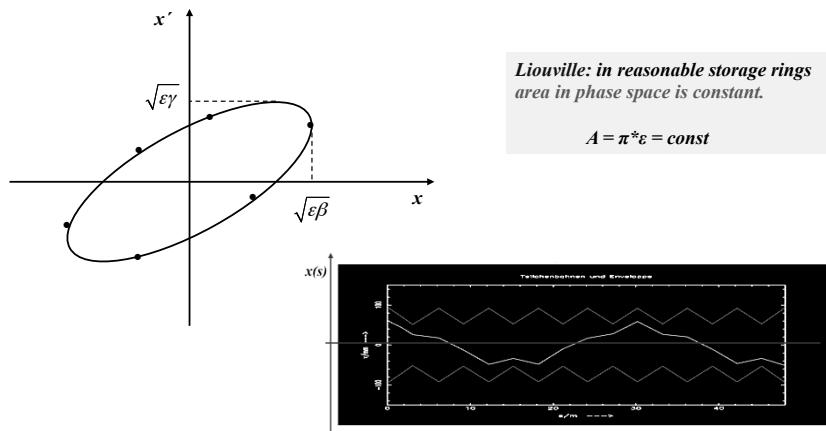
$$\epsilon = \gamma(s) x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s) x'^2(s)$$

- \*  $\epsilon$  is a constant of the motion ... it is independent of „s“
- \* parametric representation of an ellipse in the  $x$ - $x'$  space
- \* shape and orientation of ellipse are given by  $\alpha$ ,  $\beta$ ,  $\gamma$

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## Beam Emittance and Phase Space Ellipse

$$\epsilon = \gamma(s) x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s) x'^2(s)$$



$\epsilon$  beam emittance = wozilicity of the particle ensemble, intrinsic beam parameter,  
cannot be changed by the foc. properties.

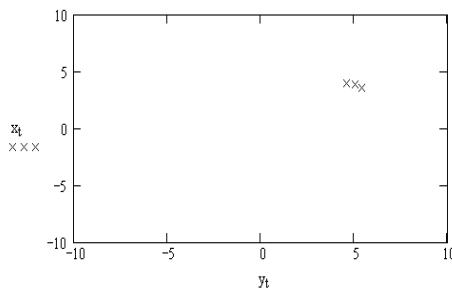
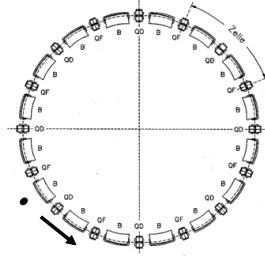
Scientifiquely speaking: it is the area covered in transverse  $x$ ,  $x'$  phase space ... and it is constant !!!

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### Particle Tracking in a Storage Ring

Calculate  $x, x'$  for each linear accelerator element according to matrix formalism

plot  $x, x'$  as a function of „ $s$ “

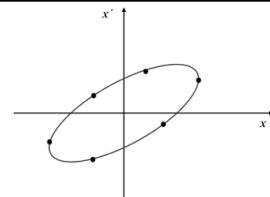


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### Phase Space Ellipse

particle trajectory:  $x(s) = \sqrt{\varepsilon} \sqrt{\beta(s)} \cos\{\psi(s) + \phi\}$

max. Amplitude:  $\hat{x}(s) = \sqrt{\varepsilon\beta}$   $\longrightarrow$  determine  $x'$  at that position ...



... put  $\hat{x}(s)$  into  $\varepsilon = \gamma(s) x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s) x'^2(s)$  and solve for  $x'$

$$\varepsilon = \gamma \cdot \varepsilon \beta + 2\alpha \sqrt{\varepsilon \beta} \cdot x' + \beta x'^2$$

$$\longrightarrow x' = -\alpha \cdot \sqrt{\varepsilon / \beta}$$

\* A high  $\beta$ -function means a large beam size and a small beam divergence. !  
... et vice versa !!!

\* In the middle of a quadrupole  $\beta = \text{maximum}$ ,  $\alpha = \text{zero}$  }  $x' = 0$   
... and the ellipse is flat

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**Phase Space Ellipse**

$$\varepsilon = \gamma(s) x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)x'^2(s)$$

$$\alpha(s) = \frac{-1}{2}\beta'(s)$$

$$\gamma(s) = \frac{1+\alpha(s)^2}{\beta(s)}$$

$$\longrightarrow \varepsilon = \frac{x^2}{\beta} + \frac{\alpha^2 x^2}{\beta} + 2\alpha \cdot x x' + \beta \cdot x'^2$$

... solve for  $x'$   $x'_{1,2} = \frac{-\alpha \cdot x \pm \sqrt{\varepsilon\beta - x^2}}{\beta}$

... and determine  $\hat{x}'$  via:  $\frac{dx'}{dx} = 0$

$$\longrightarrow \hat{x}' = \sqrt{\varepsilon\gamma}$$

$$\longrightarrow \hat{x} = \pm\alpha\sqrt{\frac{\varepsilon}{\gamma}}$$

shape and orientation of the phase space ellipse depend on the Twiss parameters  $\beta \alpha \gamma$

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**Emissivity of the Particle Ensemble:**

$$x(s) = \sqrt{\varepsilon} \sqrt{\beta(s)} \cdot \cos(\Psi(s) + \phi)$$

$$\hat{x}(s) = \sqrt{\varepsilon} \sqrt{\beta(s)}$$

Gauß Particle Distribution:  $\rho(x) = \frac{N \cdot e}{\sqrt{2\pi}\sigma_x} \cdot e^{-\frac{1}{2}\frac{x^2}{\sigma_x^2}}$

particle at distance  $1\sigma$  from centre  
↔ 68.3 % of all beam particles

single particle trajectories,  $N \approx 10^{11}$  per bunch

LHC:  $\beta = 180\text{ m}$   
 $\varepsilon = 5 * 10^{-10}\text{ m rad}$

$$\sigma = \sqrt{\varepsilon * \beta} = \sqrt{5 * 10^{-10}\text{ m} * 180\text{ m}} = 0.3\text{ mm}$$

aperture requirements:  $r_\theta = 12 * \sigma$

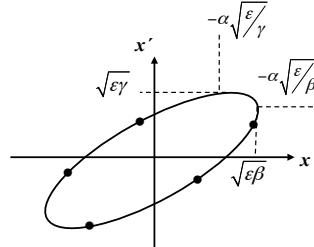
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### 13.) Liouville during Acceleration

$$\varepsilon = \gamma(s) x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)x'^2(s)$$

*Beam Emittance corresponds to the area covered in the x, x' Phase Space Ellipse*

*Liouville: Area in phase space is constant.*



**But so sorry ...  $\varepsilon \neq \text{const} !$**

*Classical Mechanics:*

*phase space = diagram of the two canonical variables*

*position & momentum*

$x$                    $p_x$

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*According to Hamiltonian mechanics:  
phase space diagram relates the variables x and  $p_x$*

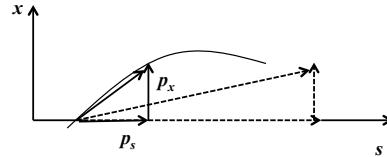
*Liouville's Theorem:*  $\int p_x dx = \text{const}$

*for convenience (i.e. because we are lazy bones) we use in accelerator theory  
x' instead of  $p_x$*

$$x' = \frac{dx}{ds} = \frac{dx}{dt} \frac{dt}{ds} = \frac{\beta_x}{\beta} = \frac{p_x}{p} \quad \text{where } p \sim p_s$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} ; \quad \beta_s = \frac{\dot{x}}{c}$$

$$\underbrace{\int x' dx}_{\varepsilon} = \frac{\int p_x dx}{p} \propto \frac{1}{m_0 c \cdot \gamma \beta}$$



$$\Rightarrow \varepsilon = \int x' dx \propto \frac{1}{\beta \gamma}$$

*the beam emittance shrinks during acceleration  
 $\varepsilon \sim 1/\gamma$*

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**Nota bene:**

1.) A proton machine ... or an electron linac ... needs the highest aperture at injection energy !!!  
as soon as we start to accelerate the beam size shrinks as  $\gamma^{-1/2}$  in both planes.

$$\sigma = \sqrt{\varepsilon \beta}$$

2.) To confuse the students we introduce often a "normalized" emittance  $\varepsilon_n$   
... which is energy independent

$$\varepsilon_n = \varepsilon_0 * \beta \gamma$$

Example: HERA proton ring

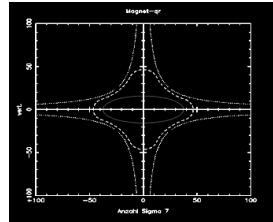
$$\varepsilon_n = 5.0 * 10^{-6} \text{ mrad}$$

$$\text{injection energy: } 40 \text{ GeV} \quad \gamma = 43$$

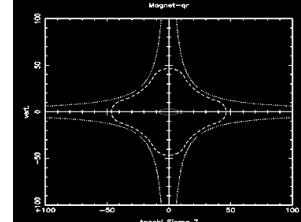
$$\varepsilon_0 (40 \text{ GeV}) = 1.2 * 10^{-7} \text{ mrad}$$

$$\text{flat top energy: } 920 \text{ GeV} \quad \gamma = 980$$

$$\varepsilon_0 (920 \text{ GeV}) = 5.1 * 10^{-9} \text{ mrad}$$



7  $\sigma$  beam envelope at  $E = 40 \text{ GeV}$



... and at  $E = 920 \text{ GeV}$

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## 11.) Résumé

1.) Beam rigidity

$$\frac{p}{e} = B \rho$$

2.) Equation of motion

$$x'' + x \left( \frac{1}{\rho^2} - k \right) = 0 \quad y'' + k y = 0$$

3.) Transfer matrix foc. quadrupole

$$M_{foc} = \begin{pmatrix} \cos(\sqrt{|K|}s) & \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}s) \\ -\sqrt{|K|} \sin(\sqrt{|K|}s) & \cos(\sqrt{|K|}s) \end{pmatrix}_0$$

defoc. quadrupole

$$M_{defoc} = \begin{pmatrix} \cosh \sqrt{|K|}l & \frac{1}{\sqrt{|K|}} \sinh \sqrt{|K|}l \\ \sqrt{|K|} \sinh \sqrt{|K|}l & \cosh \sqrt{|K|}l \end{pmatrix}$$

drift

$$M_{drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

4.) general solution of Hill's equation

$$x(s) = \sqrt{\varepsilon} \sqrt{\beta(s)} \cdot \cos(\psi(s) + \phi)$$

5.) Tune

$$Q_s = \frac{1}{2\pi} \oint \frac{ds}{\beta(s)}$$

6.) Emittance as phase space ellipse

$$\varepsilon = \gamma(s) x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s) x'^2(s)$$

Bernhard Holzer, CAS

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Bernhard Holzer, CAS