

# Instabilities Part I: Introduction – multiparticle systems, macroparticle models and wake functions

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#### Outline



We will look conceptually into **collective effects** and their **impact on beams**. We will first introduce **multiparticle systems** and investigate **multiparticle effects**. This will be important to effectively describe collective effects. We will then introduce the concept of **wake fields** as one very important collective effect.

- Part 1: Introduction multiparticle systems, macroparticle models and wake functions
  - Introduction to beam instabilities
  - · Basic concepts
    - Particles and macroparticles macroparticle distributions
    - · Beam matching
    - Multiparticle effects filamentation and decoherence



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#### What are collective effects?



 We will study the dynamics of charged particle beams in a particle accelerator environment, taking into account the beam self-induced electromagnetic fields, i.e. not only the impact of the machine onto the beam but also the impact of the beam onto the machine.



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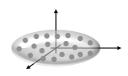
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## What are collective effects?



- A charged particle beam is generally described as a multiparticle system via the generalized coordinates and canonically conjugate momenta of all of its particles

   this makes up a distribution in the 6-dimensional beam phase space which can be described by a particle distribution function.
- Hence, we will study the **evolution of the beam phase space** (or particle distribution function):



 $\frac{\partial}{\partial s} \psi (x, x', y, y', z, \delta, s)$ 



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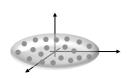
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#### What are collective effects?

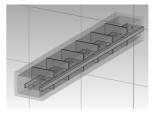


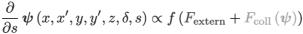
- A charged particle beam is generally described as a multiparticle system via the generalized coordinates and canonically conjugate momenta of all of its particles

   this makes up a distribution in the 6-dimensional beam phase space which can be described by a particle distribution function.
- Hence, we will study the **evolution of the beam phase space** (or particle distribution function):
  - o Optics defined by the machine lattice provides the **external force fields** (magnets, electrostatic fields, RF fields), e.g. for guidance and focusing
  - Collective effects add to this distribution dependent force fields (space charge, wake fields)







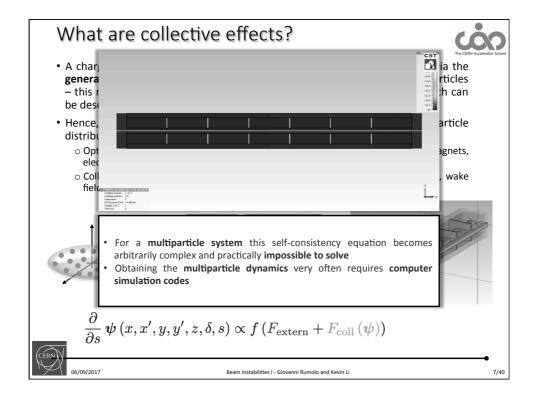




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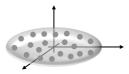
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## What is a beam instability?



• A beam becomes unstable when a **moment of its distribution** exhibits an **exponential growth** (e.g. mean positions, standard deviations, etc.), resulting into beam loss or emittance growth!

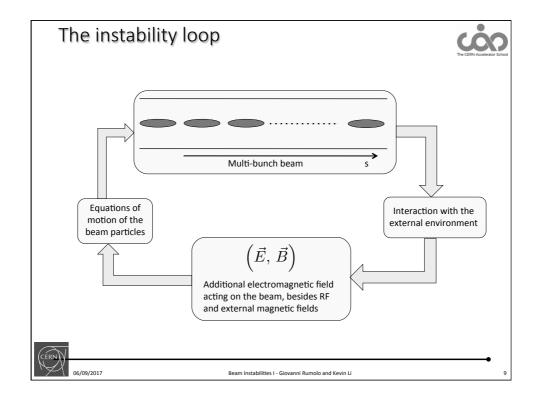


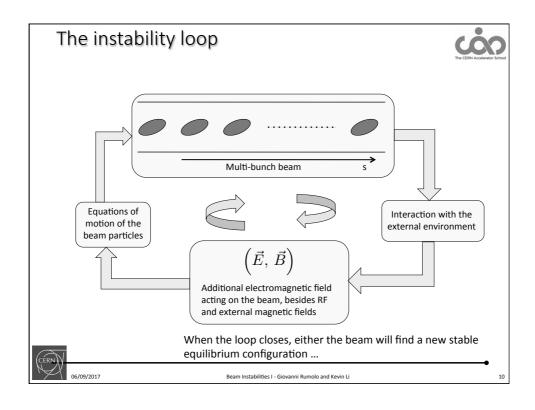
$$\begin{split} N &= \int \psi\left(x, x', y, y', z, \delta\right) \, dx dx' dy dy' dz d\delta \\ \langle x \rangle &= \frac{1}{N} \int x \cdot \psi\left(x, x', y, y', z, \delta\right) \, dx dx' dy dy' dz d\delta \\ \sigma_x^2 &= \frac{1}{N} \int \left(x - \langle x \rangle\right)^2 \cdot \psi\left(x, x', y, y', z, \delta\right) \, dx dx' dy dy' dz d\delta \end{split}$$

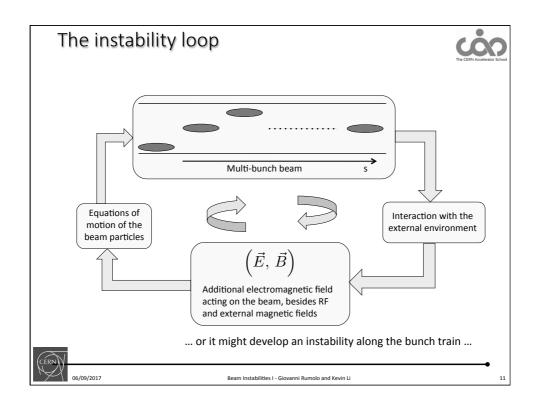
and similar definitions for  $\langle y \rangle, \sigma_y, \langle z \rangle, \sigma_z$ 

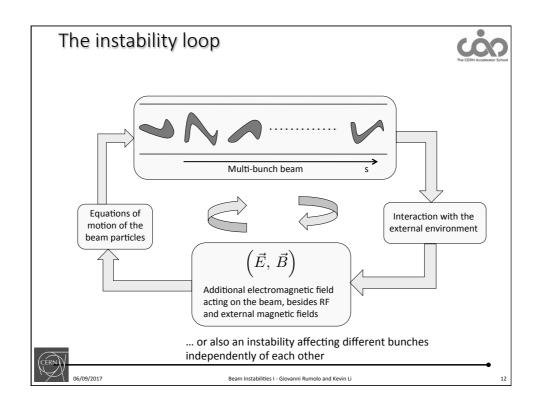


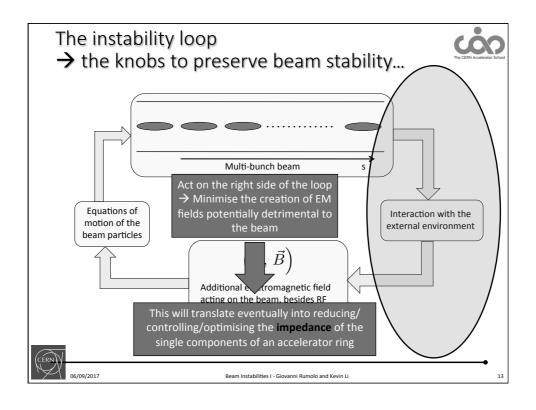
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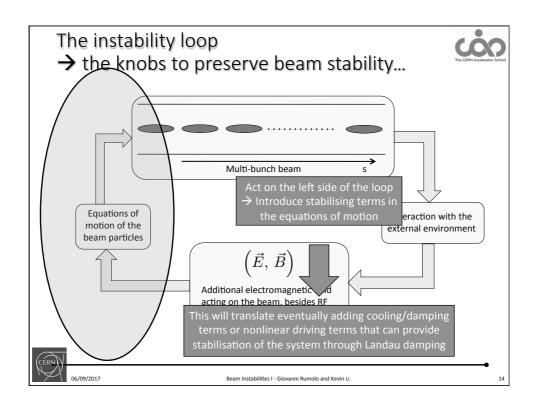


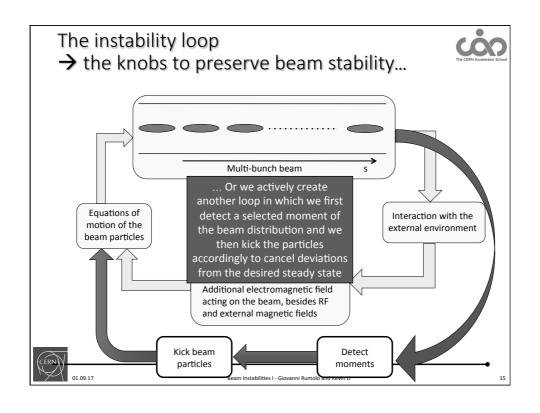












#### Formal description of a beam instability



• Formally, instead of investigating the full system of equations for a multiparticle system, we typically instead describe the latter by a single particle distribution function:

$$\psi = \psi(x, x', y, y', z, \delta, s)$$

where

$$dN(s) = \psi(x, x', y, y', z, \delta, s) dxdx'dydy'dzd\delta$$

• The accelerator environment constitutes a Hamiltonian system for which:

$$\frac{\partial x}{\partial s} = \frac{\partial H}{\partial x'}, \quad \frac{\partial x'}{\partial s} = -\frac{\partial H}{\partial x}, \quad \frac{d}{ds}\psi = 0$$
 Vlasov equation

• It follows for the evolution of this particle distribution function:

$$\frac{d}{ds}\psi = \frac{\partial\psi}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial\psi}{\partial x}\frac{\partial x'}{\partial s} + \frac{\partial}{\partial s}\psi$$
$$= \underbrace{\frac{\partial\psi}{\partial x}\frac{\partial H}{\partial x'} - \frac{\partial\psi}{\partial x}\frac{\partial H}{\partial x}}_{} + \underbrace{\frac{\partial}{\partial s}\psi}_{} = 0$$





## Formal description of a beam instability

• The evolution of a multiparticle system is given by the evolution of its particle distribution function

$$\frac{\partial}{\partial s} \boldsymbol{\psi} = [\boldsymbol{H}, \boldsymbol{\psi}]$$

• With the Hamiltonian composed of an external and a collective part, and the particle distribution function decomposed into an unperturbed part and a small perturbation one can write

$$\frac{\partial}{\partial s}\psi = [\boldsymbol{H_0} + \boldsymbol{H_1}, \psi_0 + \psi_1]$$

• This becomes to first order

$$\frac{\partial}{\partial s} \psi_1 = \underbrace{[\boldsymbol{H_0}, \psi_1] + [\boldsymbol{H_1}(\psi_0 + \psi_1), \psi_0]}_{\text{Linearization in } \boldsymbol{\psi_1}: \quad \dots \propto \boxed{\boldsymbol{\hat{\Lambda}\psi_1} = -i\frac{\Omega}{\beta c}\psi_1}$$

$$\implies \psi_1(s) = \exp\left(-i\frac{\Omega}{\beta c}s\right)\psi_1(0) \qquad \text{We are looking for the EV of the evolution } \rightarrow \text{becomes an EV problem!}$$



#### Formal description of a beam instability



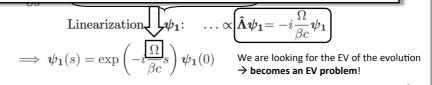
• The evolution of a **multiparticle system** is given by the evolution of its **particle distribution function** 

$$\frac{\partial}{\partial s} \psi = [\boldsymbol{H}, \psi]$$

We call these distinct eigenvalues  $\psi_1$  a bunch or a beam mode.

The mode and thus for example also an instability is fully characterized by a single number:

the complex tune shift  $\Omega$ 





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## Formal description of a beam instability



• The evolution of a **multiparticle system** is given by the evolution of its **particle distribution function** 

$$\frac{\partial}{\partial s} \boldsymbol{\psi} = [\boldsymbol{H}, \boldsymbol{\psi}]$$

Remark:

ullet The stationary distribution  $\psi_0$  is the distribution where

$$\frac{\partial}{\partial s}\psi_{\mathbf{0}} = [\boldsymbol{H}_{\mathbf{0}}, \psi_{\mathbf{0}}] = 0$$

• In particular, a distribution is always stationary if

$$\psi_0 = \psi_0(H_0), \text{ as } [H_0, \psi_0(H_0)] = 0$$

Solving for or finding the stationary solution for a given H<sub>0</sub> (which in fact represents the machine ,potential') will be later referred to as **matching**.

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## Why worry about beam instabilities?

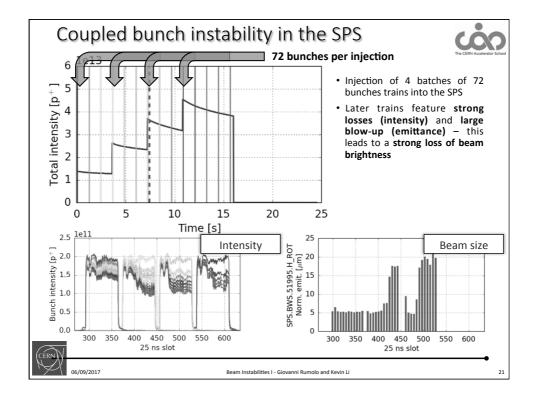


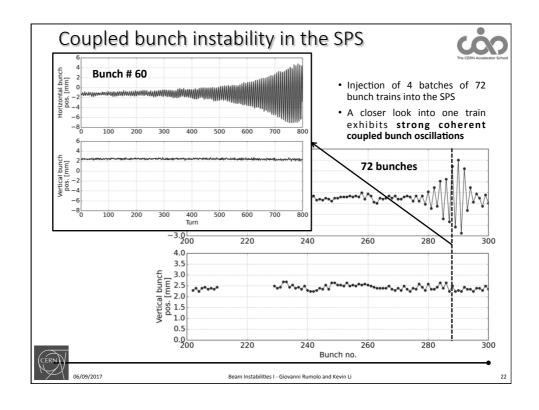
- Why study beam instabilities?
  - The onset of a beam instability usually determines the maximum beam intensity that a machine can store/accelerate (performance limitation)
  - Understanding the type of instability limiting the performance, and its underlying mechanism, is essential because it:
    - Allows identifying the source and possible measures to mitigate/suppress the effect
    - Allows dimensioning an active feedback system to prevent the instability

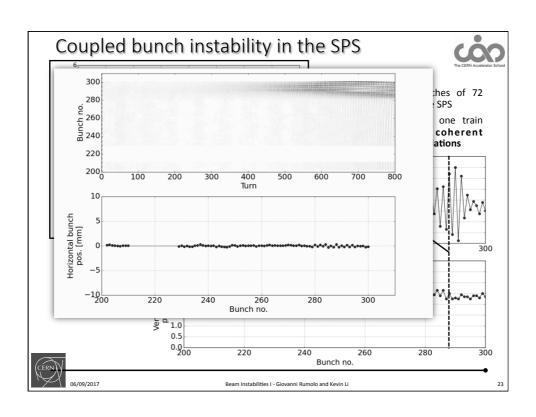


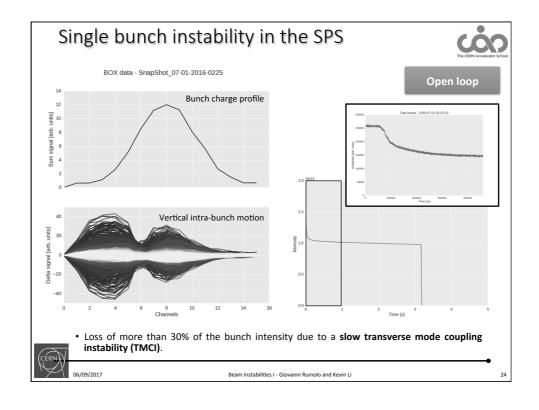
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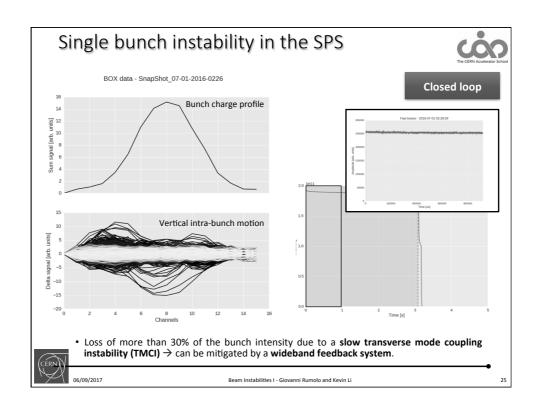
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- We have seen the difference between external forces and self-induced forces which lead to collective effects.
- We have seen schematically how these collective effects can induce coherent beam instabilities and some knobs to avoid them.
- We have seen **examples of beam instabilities** and have understood how they can lead to serious **performance limitations**.
- Part 1: Introduction multiparticle systems, macroparticle models and wake functions
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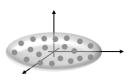
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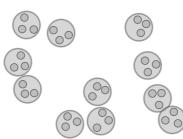
## The particle description



- As seen earlier, and especially for the analytical treatment, we can represent a charged particle beam via a particle distribution function.
- In computer simulations, a charged particle beam is still represented as a multiparticle system. However, to be **compatible with computational resources**, we need to rely on **macroparticle models**.
- A macroparticle is a numerical representation of a cluster of neighbouring physical particles.
- Thus, instead of solving the system for the N (~10<sup>11</sup>) physical particles one can significantly reduce the number of degrees of freedom to N<sub>MP</sub> (~10<sup>6</sup>). At the same time one must be aware that this increases of the granularity of the system which gives rise to numerical noise.



 $\Psi\left(x,x^{\prime},y,y^{\prime},z,\delta
ight)$ 







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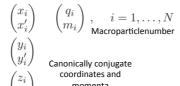
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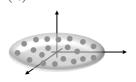
# Macroparticle representation of the beam



 Macroparticle models permit a seamless mapping of realistic systems into a computational environment – they are fairly easy to implement

Beam:





$$\Psi\left(x,x',y,y',z,\delta\right)$$

	df							
Out[6]:		dp	x	хр	у	ур		
	0	0.001590	0.000566	-2.285393e-05	-0.001980	4.283152e-0		

	-P	l		,	7.	-
0	0.001590	0.000566	-2.285393e-05	-0.001980	4.283152e-06	0.353427
1	0.001978	0.000370	1.954404e-05	-0.000359	5.543904e-05	0.159670
2	0.003492	-0.000829	-2.773707e-05	0.000291	6.627340e-05	-0.251489
3	0.002195	-0.001668	-2.317633e-05	0.001878	-1.870926e-05	-0.038597
4	0.000572	0.000990	5.493907e-05	0.000152	-1.951051e-05	0.492968
5	-0.000418	0.001088	4.778027e-05	0.003320	-7.716856e-06	0.415582
6	-0.000114	-0.000194	1.065400e-05	0.001798	-4.984276e-07	-0.349064
7	0.001100	-0.001257	-6.873217e-05	-0.002374	5.657645e-06	-0.023157
8	0.002706	0.005351	-1.867898e-07	-0.000765	3.012523e-05	-0.291095
9	0.003508	0.000499	1.865768e-05	-0.001032	-5.363820e-05	0.211726
10	-0.001711	-0.003168	4.372560e-05	-0.001933	-2.151020e-05	-0.145358
11	-0.002150	-0.000565	-1.853825e-05	-0.003895	-6.192450e-06	0.072499
12	0.002059	0.003453	-3.808703e-05	0.000118	3.179588e-05	-0.001816
13	0.002709	0.000241	-3.457535e-05	0.000474	5.057865e-05	-0.005464
14	-0.001593	0.000711	-1.667091e-05	-0.002523	-3.804168e-05	
15	-0.000830	-0.000393	-7.473946e-05	-0.003633		

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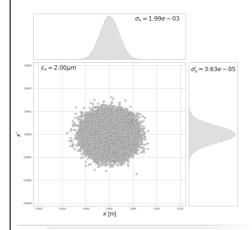
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## Macroparticle representation of the beam





In [6]:	<pre>df = pd.DataFrame(bunch.get_coords_n_momenta_dict()) df</pre>							
Out[6]:		dp	x	хр	у	ур	z	
	0	0.001590	0.000566	-2.285393e-05	-0.001980	4.283152e-06		
	1	0.001978	0.000370	1.954404e-05	-0.000359	5,543904		
	2	0.003492	-0.000829	-2.773707e-05	0.000291			
	3	0.002105	0.001669	2 2176220 DE	0.001070			

• Initial conditions of the beam/particles

Profile	Size	Matching
Gaussian	Emittance	Optics
Parabolic		
Flat		

- We use random number generators to obtain random distributions of coordinates and momenta
- Example transverse Gaussian beam in the SPS with normalized emittance of 2 um (0.35 eVs longitudinal)

$$\varepsilon_{\perp} = \beta \gamma \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}$$
$$= \beta \gamma \sigma_x \sigma_{x'}$$

 $\varepsilon_{\parallel} = 4\pi\sigma_z\sigma_\delta \frac{p_0}{e}$ 



- We have learned about the particle description of a beam.
- We have seen macroparticles and macroparticle models.
- We have seen how macroparticle models are mapped and represented in a computational environment.
- Part 1: Introduction multiparticle systems, macroparticle models and wake functions
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#### Beam matching

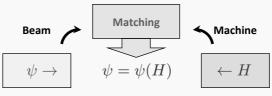


• As seen earlier, given a particle distribution function and a machine (described by a Hamiltonian H) the stationary solution is given by:

$$\frac{\partial}{\partial s} \boldsymbol{\psi} = [\boldsymbol{H}, \boldsymbol{\psi}] = 0$$

and can be constructed via matching:

- In real life, an injected beam ought to be **matched to the machine** for best performance.
- Given a particle distribution function and a machine optics locally described by a Hamiltonian we ensure matching by targeting for:



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### Matching examples



We take the example of Gaussian distribution functions

$$\psi(H) = \exp\left(\frac{H}{H_0}\right)$$

• Betatron motion

$$H = \frac{1}{2} x'^2 + \left(\frac{Q_x}{R}\right)^2 x^2$$

$$H_0 = \sigma_{x'}^2 = \left(\frac{Q_x}{R}\right)^2 \sigma_x^2 \Longrightarrow \boxed{\frac{\sigma_x}{\sigma_{x'}} = \frac{R}{Q_x} = \beta_x}$$

• Synchrotron motion - linear

$$H(z,\delta) = -\frac{1}{2}\eta\beta c\,\delta^2 + \frac{eVh}{4\pi R^2 p_0}\,z^2$$
 
$$H_0 = \eta\beta c\,\sigma_\delta^2 = \frac{eVh}{2\pi R^2 p_0}\,\sigma_z^2 \implies \boxed{\frac{\sigma_z}{\sigma_\delta} = R\eta\,\sqrt{\frac{2\pi\beta^2 E_0}{eV\eta h}} = \frac{R\eta}{Q_s}\,\sigma_\delta = \beta_z}$$



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## Matching examples



We take the example of Gaussian distribution functions

$$\psi(H) = \exp\left(\frac{H}{H_0}\right)$$

In reality the synchrotron motion is described by the Hamiltonian:

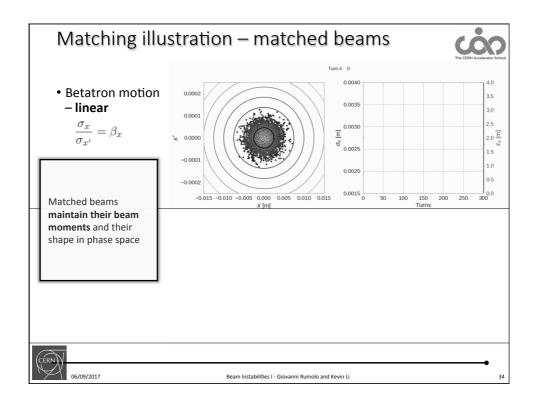
$$H(z,\delta) = -\frac{1}{2}\eta\beta c\,\delta^2 + \frac{eV}{2\pi h p_0} \left(\cos\left(\frac{hz}{R}\right) - \cos\left(\frac{hz_c}{R}\right) + \frac{\Delta E}{eV} \left(\frac{hz}{R} - \frac{hz_c}{R}\right)\right)$$

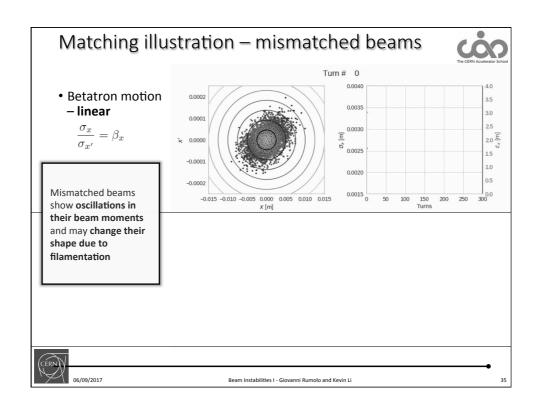
This leads to **nonlinear equations** and the matching procedure becomes more involved.

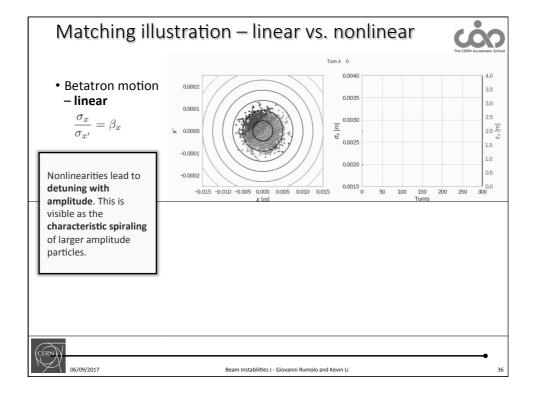


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# Signpost 📑



- We have learned about the **meaning of matching** a beam to the machine optics.
- We have seen how to formally match a beam to a given description of a machine.
- We have seen **examples of matched and mismatched beams** and have seen the difference between **linear and non-linear motion**.
- Part 1: Introduction multiparticle systems, macroparticle models and wake functions
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## Sources and impact of transverse nonlinearities

- We have learned or we may know from operational experience that there are a set of crucial machine parameters to influence beam stability – among them chromaticity and amplitude detuning
- Chromaticity
  - Controlled with sextupoles provides chromatic shift of bunch spectrum wrt. impedance
  - o Changes interaction of beam with impedance
  - o Damping or excitation of headtail modes
- Amplitude detuning
  - o Controlled with octupoles provides (incoherent) tune spread
  - $\circ$  Leads to absorption of coherent power into the incoherent spectrum ightarrow Landau damping
- model fortunately, this is pretty simple!

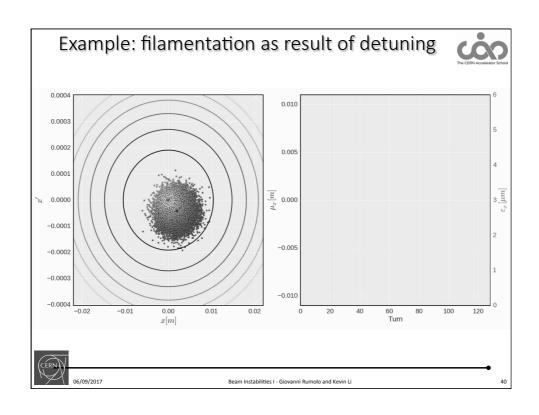


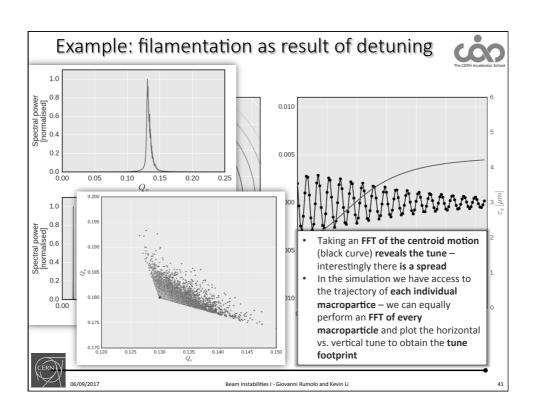
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Filamentation and decoherence

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- Source for transverse nonlinearities are **chromaticity** and **detuning with amplitude** from octupoles, for example.
- Transverse nonlinearities can lead to **decoherence** and **emittance blow-up**.
- The effects seen so far are chacteristics for multiparticle systems but are not collective effects.
- Part 1: Introduction multiparticle systems, macroparticle models and wake functions
  - Introduction to beam instabilities
  - · Basic concepts
    - Particles and macroparticles macroparticle distributions
    - · Beam matching
    - Multiparticle effects filamentation and decoherence
    - · Wakefields as sources of collective effects



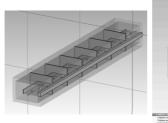
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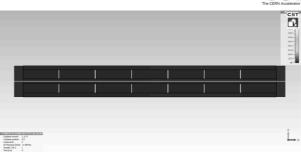
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## Wakefields as sources of collective effects





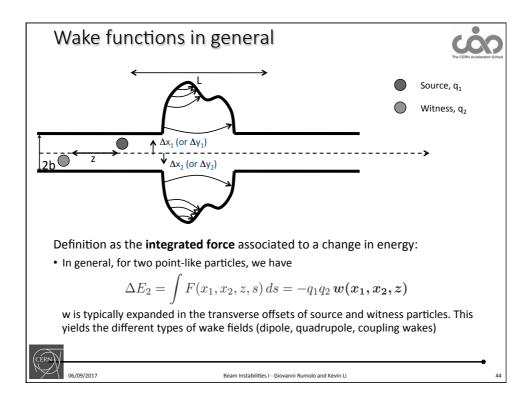


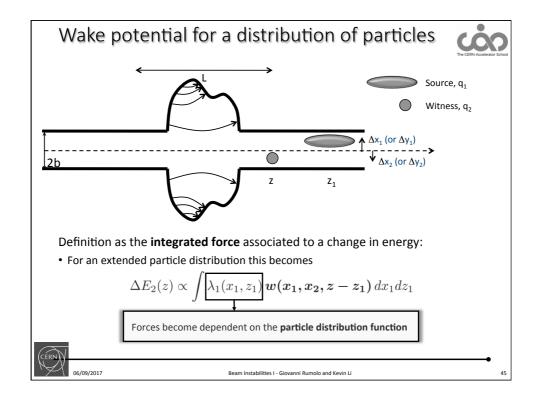
- The wake function is the electromagnetic response of an object to a charge pulse. It is an intrinsic property of any such object.
- The wake function **couples two charge distributions** as a function of the distance between them.
- The response depends on the boundary conditions and can occur e.g. due to finite conductivity (resistive wall) or more or less sudden changes in the geometry (e.g. resonator) of a structure.



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# Wake fields – impact on the equations of motion

$$\Delta E_2(z) \propto \int \lambda_1(x_1, z_1) \boldsymbol{w(x_1, x_2, z - z_1)} dx_1 dz_1$$

• We include the impact of wake field into the standard Hamiltonian for linear betatron (or synchrotron motion):

$$H = \frac{1}{2}x' + \frac{1}{2}\left(\frac{Q_x}{R}\right)^2 x^2 + \frac{e^2}{\beta^2 EC} \int \lambda_1(x_1, z_1) w(x_1, x, z - z_1) dx_1 dz_1 dx$$

• The equations of motion become:

$$x'' + \left(\frac{Q_x}{R}\right)^2 x + \frac{e^2}{\beta^2 EC} \int \lambda_1(x_1, z_1) w(x_1, x, z - z_1) dx_1 dz_1 = 0$$

The presence of wake fields adds an additional excitation which depends on

- 1. The moments of the beam distribution
- 2. The shape and the order of the wake function



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• Analytical or semi-analytical approach, when geometry is simple (or simplified)

How are wakes and impedances computed?

- Solve Maxwell's equations with the correct source terms, geometries and boundary conditions up to an advanced stage (e.g. resistive wall for axisymmetric chambers)
- Find closed expressions or execute the last steps numerically to derive wakes and impedances
- Numerical approach
  - Different codes have been developed over the years to solve numerically Maxwell's equations in arbitrarily complicated structures
  - Examples are CST Studio Suite (Particle Studio, Microwave Studio), ABCI, GdFidL, HFSS, ECHO2(3)D. Exhaustive list can be found from the program of the <a href="ICFA">ICFA</a> mini-Workshop on "Electromagnetic wake fields and impedances in particle accelerators", Erice, Sicily, 23-28 April, 2014
- Bench measurements based on transmission/reflection measurements with stretched wires
  - Seldom used independently to assess impedances, usefulness mainly lies in that they can be used for validating 3D EM models for simulations



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- We have learned about the concept of particles, distributions and macroparticles as well as some peculiarities of multiparticle dynamics in accelerators, decoherence, filamentation.
- We have learned about the basic **concept of wake fields** and how these can be characterized as a **collective effect** in that they depend on the particle distribution.
- We now have a basic understanding of multiparticle systems and wakefields and are now ready to look at the **impact of these** in the longitudinal and transverse planes.
- Part 1: Introduction multiparticle systems, macroparticle models and wake functions
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## End part 1





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