Instabilities Part II: Longitudinal wake fields – impact on machine elements and beam dynamics

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Outline

We will close in into the description and the impact of **longitudinal wake fields**. We will discuss the **energy balance** and then show some examples of phenomena linked to **longitudinal wake fields** such as beam induced heating, potential well distortion, microwave and Robinson instabilities.

Part 2: Longitudinal wakefields –
impact on machine elements and beam dynamics

- Longitudinal wake function and impedance
- Energy loss – beam induced heating and stable phase shift
- Potential well distortion, bunch lengthening and microwave instability
- Robinson instability

Signpost

- We have learned about the concept of **particles**, **distributions** and **macroparticles** as well as some **peculiarities of multiparticle dynamics** in accelerators, decoherence, filamentation.
- We have learned about the basic **concept of wake fields** and how these can be characterized as a **collective effect** in that they depend on the particle distribution.
- We now have a basic understanding of multiparticle systems and wakefields and are now ready to look at the **impact of these** in the longitudinal and transverse planes.

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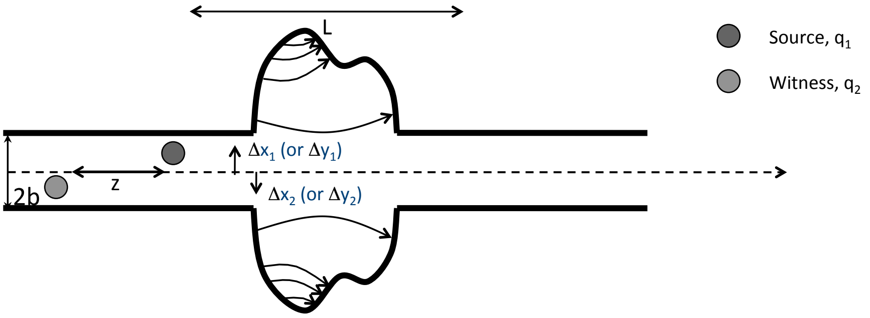
Recap: wake functions in general



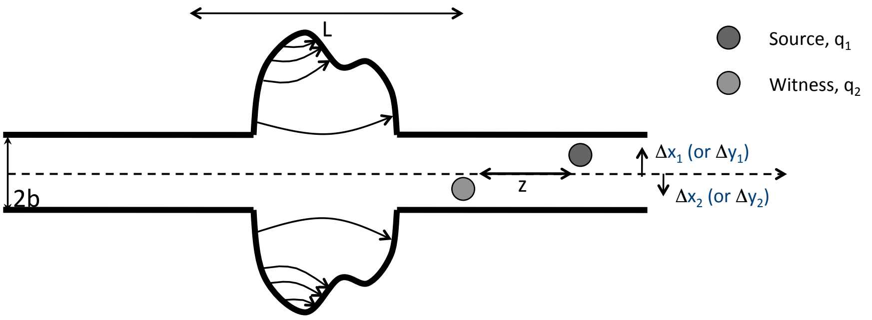
Definition as the **integrated force** associated to a change in energy:

- In general, for two point-like particles, we have

$$\Delta E_2 = \int F(x_1, x_2, z, s)\, ds = -q_1 q_2\, \boldsymbol{w(x_1, x_2, z)}$$

w is typically expanded in the transverse offsets of source and witness particles. This yields the different types of wake fields (dipole, quadrupole, coupling wakes)

Longitudinal wake function



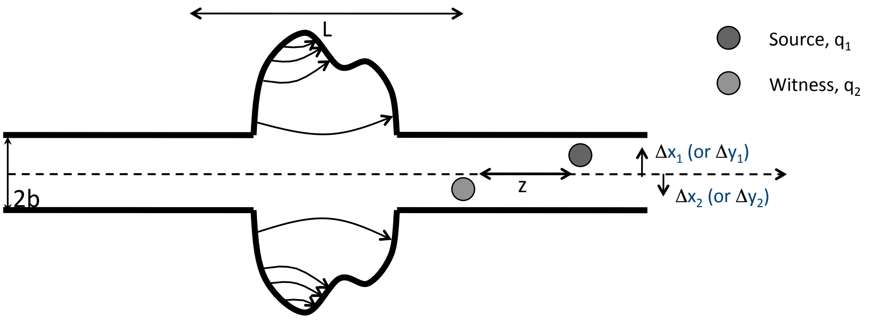
- Longitudinal wake function

$$\Delta E_2 = \int F_z(x_1, x_2, z, s)\, ds = -q_1 q_2 \left(\boxed{W_{\|}(z)} + \boxed{O(\Delta x_1) + O(\Delta x_2)}\right)$$

Zeroth order with source and test centred
usually dominant

Higher order terms
Usually negligible for small offsets

Longitudinal wake function



- Longitudinal wake function

$$\Delta E_2 = \int F_z(z, s)\, ds = -q_1 q_2\, W_{\|}(z)$$

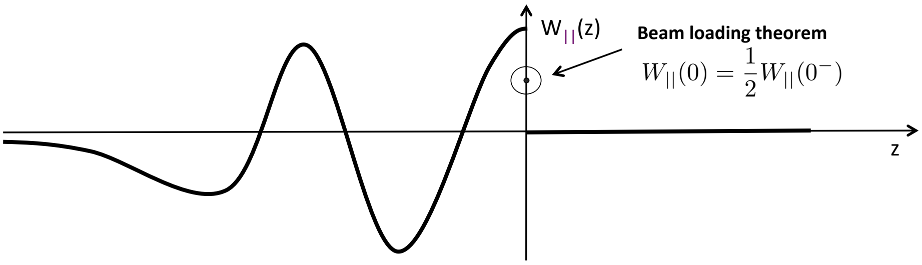
$$\longrightarrow \frac{\Delta E_2}{E_0} = \left(\frac{\gamma^2 - 1}{\gamma}\right) \frac{\Delta p_2}{p_0}$$

Energy kick of the witness particle from longitudinal wakes

Longitudinal wake function

$$W_{\parallel}(z) = -\frac{\Delta E_2}{q_1 q_2} \xrightarrow[q_2 \to q_1]{z \to 0} W_{\parallel}(0) = -\frac{\Delta E_1}{q_1^2}$$

- The value of the wake function in z=0 is related to the **energy lost by the source particle** in the creation of the wake
- ***$W_{||}(0)>0$*** since ***$\Delta E_1<0$***
- ***$W_{||}(z)$*** is discontinuous in z=0 and it vanishes for all z>0 because of the ultra-relativistic approximation



Beam loading theorem

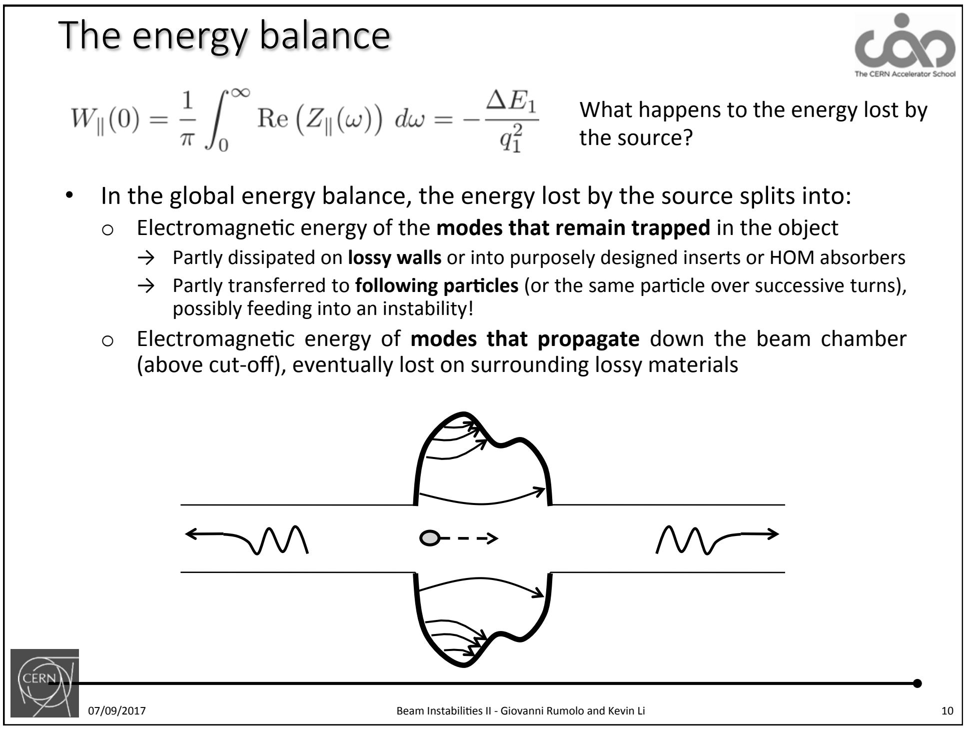
$$W_{||}(0) = \frac{1}{2} W_{||}(0^-)$$

Longitudinal impedance

$$W_{\parallel}(z) = -\frac{\Delta E_2}{q_1 q_2} \xrightarrow[q_2 \to q_1]{z \to 0} W_{\parallel}(0) = -\frac{\Delta E_1}{q_1^2}$$

- The **wake function** of an accelerator component is basically its **Green function in time domain** (i.e., its response to a pulse excitation)
  - → Very useful for macroparticle models and simulations, because it can be used to describe the driving terms in the single particle equations of motion!
- We can also describe it as a **transfer function in frequency domain**
  - → This is the definition of **longitudinal beam coupling impedance** of the element under study

$$\underset{[\Omega]}{\boxed{Z_{\parallel}(\omega)}} = \int_{-\infty}^{\infty} \underset{[\Omega/s]}{\boxed{W_{\parallel}(z)}} \exp\left(-\frac{i\omega z}{c}\right) \frac{dz}{c}$$

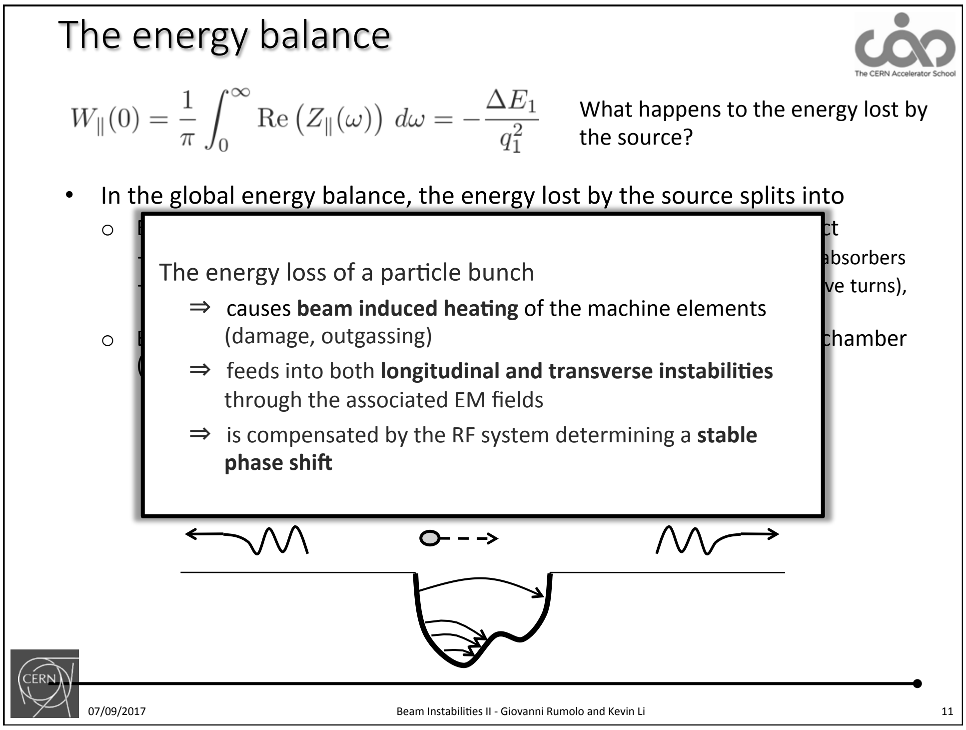


The energy balance

$$W_\parallel(0) = \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left(Z_\parallel(\omega)\right)\,d\omega = -\frac{\Delta E_1}{q_1^2}$$

What happens to the energy lost by the source?

- In the global energy balance, the energy lost by the source splits into:
  - Electromagnetic energy of the **modes that remain trapped** in the object
    - → Partly dissipated on **lossy walls** or into purposely designed inserts or HOM absorbers
    - → Partly transferred to **following particles** (or the same particle over successive turns), possibly feeding into an instability!
  - Electromagnetic energy of **modes that propagate** down the beam chamber (above cut-off), eventually lost on surrounding lossy materials



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What happens to the energy lost by the source?

- In the global energy balance, the energy lost by the source splits into

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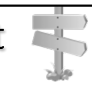
The energy loss of a particle bunch

⇒ causes **beam induced heating** of the machine elements (damage, outgassing)

⇒ feeds into both **longitudinal and transverse instabilities** through the associated EM fields

⇒ is compensated by the RF system determining a **stable phase shift**

Signpost



- We have specialized the general definition of the wake function to the specific case of the **purely longitudinal wake function**.
- We have seen how longitudinal wake functions are related to the **energy loss** of the source particles.
- We have discussed the **energy balance** which containes all the **fundamental underlying mechanisms** for collective effects related to wake fields and impedances.

Part 2: Longitudinal wakefields –
impact on machine elements and beam dynamics

- Longitudinal wake function and impedance
- Energy loss – beam induced heating and stable phase shift
- Potential well distortion, bunch lengthening and microwave instability
- Robinson instability

Bunch energy loss per turn

- We remember the energy loss for two particles due to a longitudinal wake field:

$$\Delta E_2 = -\, q_1 q_2 \, W_{\|}(z)$$

- This can be generalized to an energy loss for a multi particle distribution for a single passage:

$$\Delta E_{\mathrm{total}} = -\frac{e^2}{2\pi} \int \lambda(z) \underbrace{\int \lambda(z')\, W_{\|}(z-z')\, dz'}_{\propto \Delta E(z)} \, dz$$

- which in frequency domain becomes

$$\Delta E = -\frac{e^2}{2\pi} \int \left|\hat{\lambda}(\omega)\right|^2 \mathrm{Re}\left[Z_{\|}(\omega)\right] \, d\omega$$

- If instead, we consider a multi particle distribution over multiple passages spaced by $2\pi/\omega_0$, we arrive at

$$\Delta E = -\frac{e^2 \omega_0}{2\pi} \sum_{p=-\infty}^{\infty} \left|\hat{\lambda}(p\omega_0)\right|^2 \mathrm{Re}\left[Z_{\|}(p\omega_0)\right]$$

Beam energy loss per turn

The bunch energy loss is given by the **bunch/beam spectrum** and the real part of the machine **longitudinal impedance**

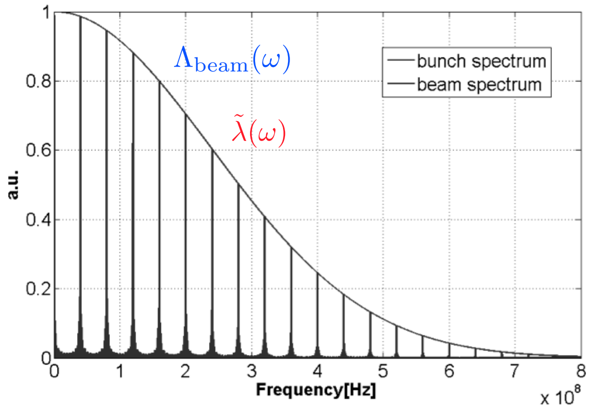
$$\Delta E = -\frac{e^2\omega_0}{2\pi}\sum_{p=-\infty}^{\infty}\left|\hat{\lambda}(p\omega_0)\right|^2 \mathrm{Re}\left[Z_{\parallel}(p\omega_0)\right]$$

Bunch spectrum

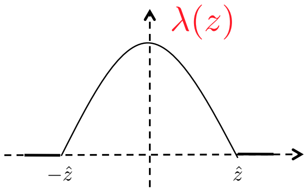
$\tilde{\lambda}(\omega)$

Beam spectrum

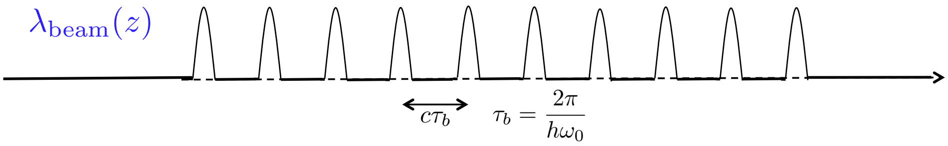
$\Lambda_{\text{beam}}(\omega)$



Energy loss of a train of bunches

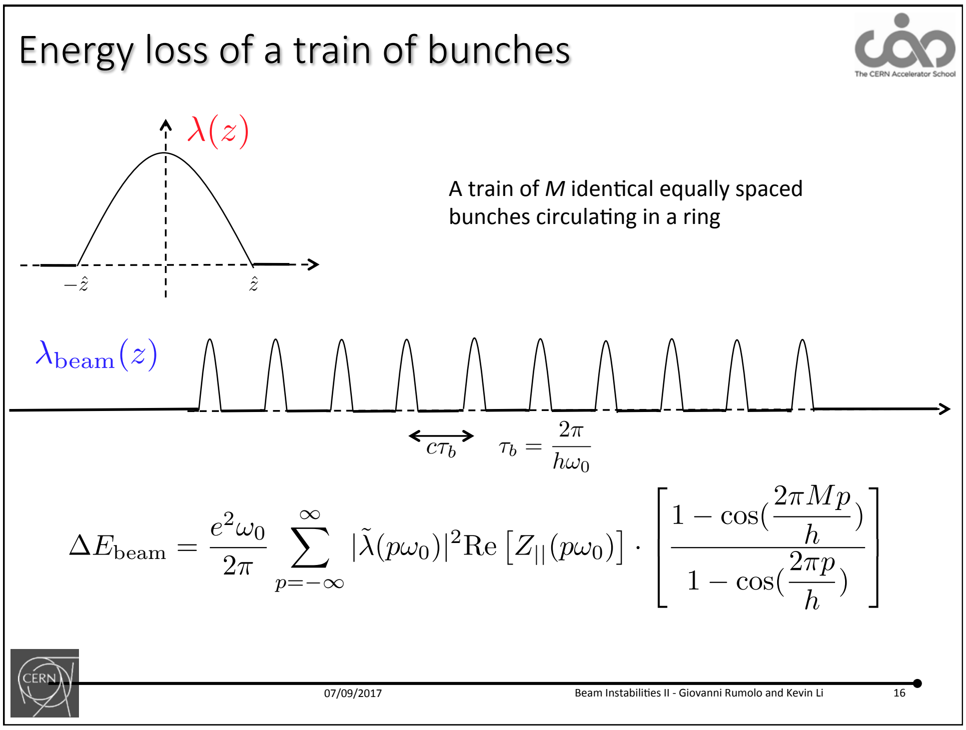


A train of *M* identical equally spaced bunches circulating in a ring



$$\tau_b = \frac{2\pi}{h\omega_0}$$

$$\lambda_{\text{beam}}(z) = \sum_{n=0}^{M-1}\lambda(z - nc\tau_b) \quad \overset{\mathcal{F}}{\Longleftrightarrow} \quad \boxed{\Lambda_{\text{beam}}(\omega) = \tilde{\lambda}(\omega)\sum_{n=0}^{M-1}\exp(-in\omega\tau_b)}$$



Energy loss of a train of bunches

A train of *M* identical equally spaced bunches circulating in a ring

$$\Delta E_{\rm beam} = \frac{e^2\omega_0}{2\pi}\sum_{p=-\infty}^{\infty}|\tilde{\lambda}(p\omega_0)|^2\,{\rm Re}\left[Z_{||}(p\omega_0)\right]\cdot\left[\frac{1-\cos(\frac{2\pi Mp}{h})}{1-\cos(\frac{2\pi p}{h})}\right]$$

Energy loss of a train of bunches

$$\Delta E_{\rm beam} = \frac{e^2\omega_0}{2\pi}\sum_{p=-\infty}^{\infty}|\tilde{\lambda}(p\omega_0)|^2\,{\rm Re}\left[Z_{||}(p\omega_0)\right]\cdot\left[\frac{1-\cos(\frac{2\pi Mp}{h})}{1-\cos(\frac{2\pi p}{h})}\right]$$

- The potential leading terms in the summation are those with $p = k \cdot h$, as the ratio in brackets tends to M^2.
- Narrow-band impedances peaked around multiples of the harmonic number of the accelerator are **the most efficient to drain energy** from the beam → beam induced heating, instabilities.
- This type of impedances, usually **associated to the RF systems** and their higher order modes (HOMs), **need mitigation** in the accelerator design (e.g. detuners, HOM absorbers).

Energy loss of a train of bunches

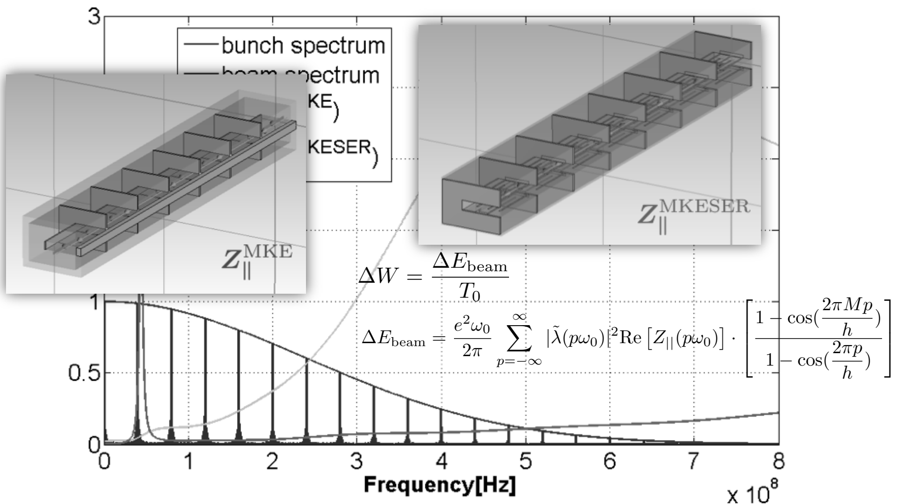
$$\Delta E_{\mathrm{beam}} = \frac{e^2\omega_0}{2\pi}\sum_{p=-\infty}^{\infty}|\tilde{\lambda}(p\omega_0)|^2\mathrm{Re}\left[Z_{||}(p\omega_0)\right]\cdot\left[\frac{1-\cos(\frac{2\pi Mp}{h})}{1-\cos(\frac{2\pi p}{h})}\right]$$

- The po s the ratio in brac
- Narrow ber of the acceler m induced heating
- This ty eir higher order r detuners, HOM a

Some examples include:

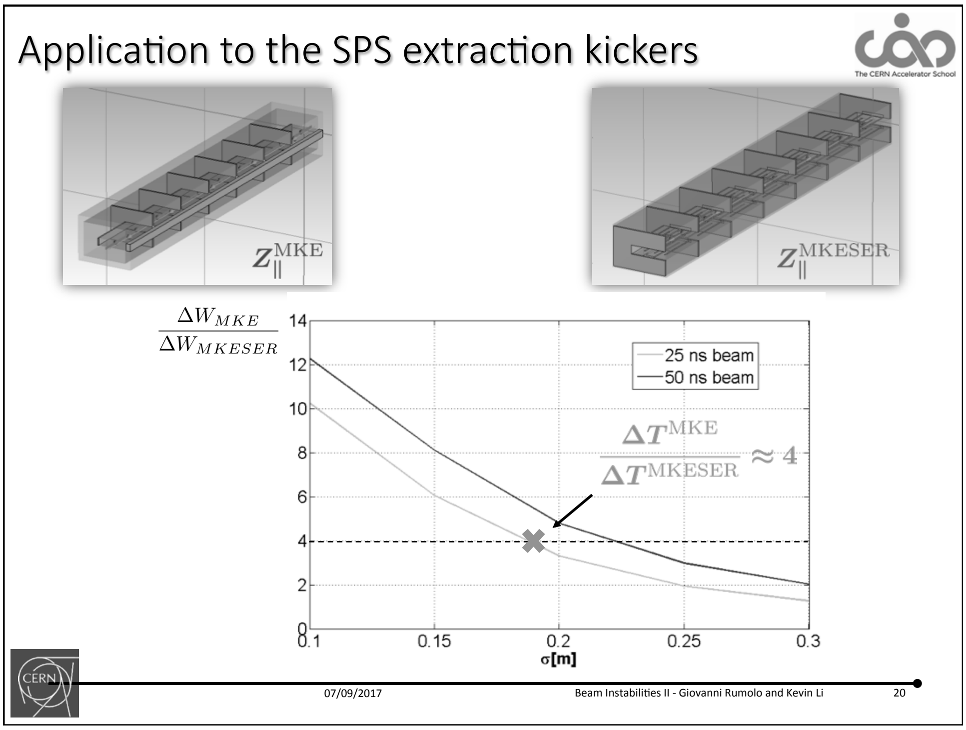
- Heating of the SPS extraction kickers
- Heating of the LHC beam screen
- Beams with exotic bunch spacing (see backup slides)
  - The case of "doublets"
  - Common chamber in a collider

Application to the SPS extraction kickers



$$\Delta W = \frac{\Delta E_{\mathrm{beam}}}{T_0}$$

$$\Delta E_{\mathrm{beam}} = \frac{e^2\omega_0}{2\pi}\sum_{p=-\infty}^{\infty}|\tilde{\lambda}(p\omega_0)|^2\mathrm{Re}\left[Z_{||}(p\omega_0)\right]\cdot\left[\frac{1-\cos(\frac{2\pi Mp}{h})}{1-\cos(\frac{2\pi p}{h})}\right]$$



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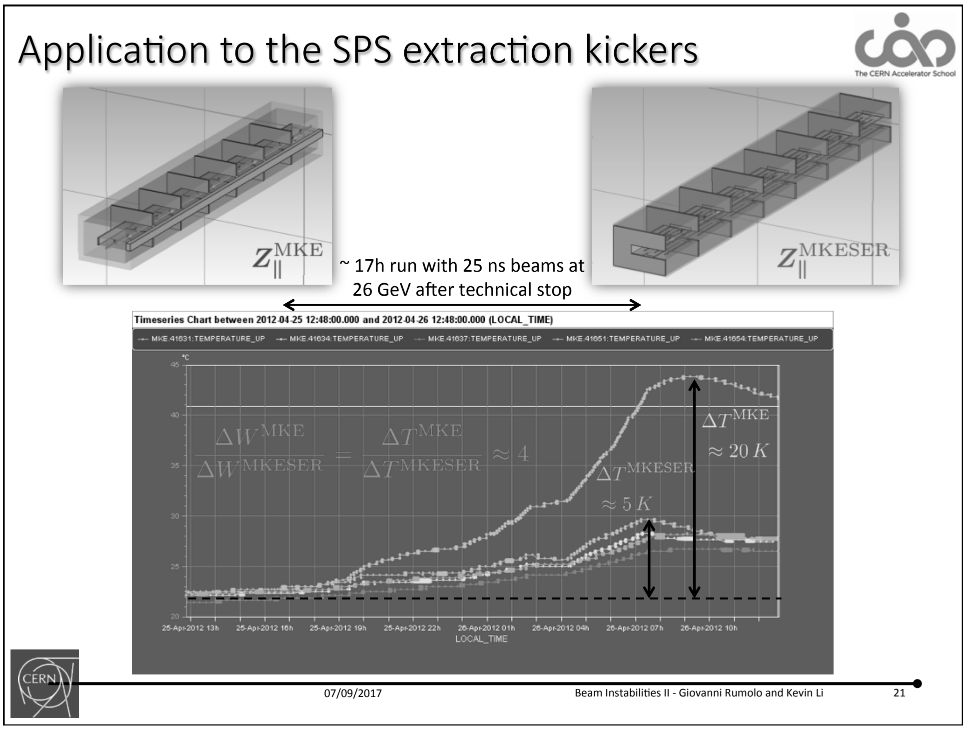

Application to the SPS extraction kickers

~ 17h run with 25 ns beams at 26 GeV after technical stop

Timeseries Chart between 2012-04-25 12:48:00.000 and 2012-04-26 12:48:00.000 (LOCAL_TIME)

Signpost

- We have further dived into the mechanism of energy loss and have seen the **impact of longitudinal impedances on machine elements** as these lead to **beam induced heating**.
- We have found that beam induced heating depends on the overlap of the **beam power spectrum** and the **impedance** of a given object.
- We have seen a **real world example** of the impact of an objects impedance on the beam induced heating.

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Recap: formal description of a beam instability

- The mode and thus the instability is fully characterized by a single number defined by an eigenvalue problem:

the complex tune shift Ω

- For the case of longitudinal wake fields, two regimes can be found:
  - Regime of potential well distortion (i.e. perturbations to **equilibrium solutions are damped**)
    - Stable phase shift
    - Synchrotron frequency shift
    - Different matching (→ bunch lengthening for lepton machines)
  - Regime of longitudinal instability (i.e. perturbations to **equilibrium solutions grow exponentially**):
    - Dipole mode instabilities
    - Coupled bunch instabilities
    - Microwave instability (longitudinal mode coupling)

Potential well distortion and Haissinki equation

- The **equilibrium distribution** in the presence of a longitudinal wake field can be found analytically. The (linearized) **longitudinal Hamiltonian** with longitudinal wake fields is given as:

$$H = -\frac{1}{2}\eta\,\delta^2 - \frac{1}{2\eta}\left(\frac{\omega_s}{\beta c}\right)^2 z^2 + \frac{e^2}{\beta^2 EC}\sum_k \int_0^z dz'' \int_{z''}^{\infty} dz'\,\lambda(z'+kC)W_{\parallel}(z''-z'-kC)$$

- We assume a Gaussian beam distribution:

$$\psi(z,\delta) = \exp\left(-\frac{\delta^2}{2\sigma_\delta^2}\right)\lambda(z)$$

- The equilibrium (matched) line charge density is then given by the self-consistency equation (**Haissinki equation**):

$$\lambda(z) = \exp\left(\left(-\frac{\omega_s z}{2\eta\sigma_\delta\beta c}\right)^2 + \frac{e^2}{\eta\sigma_\delta^2\beta^2 EC}\sum_k \int_0^z dz'' \int_{z''}^{\infty} dz'\,\lambda(z'+kC)W_{\parallel}(z''-z'-kC)\right)$$

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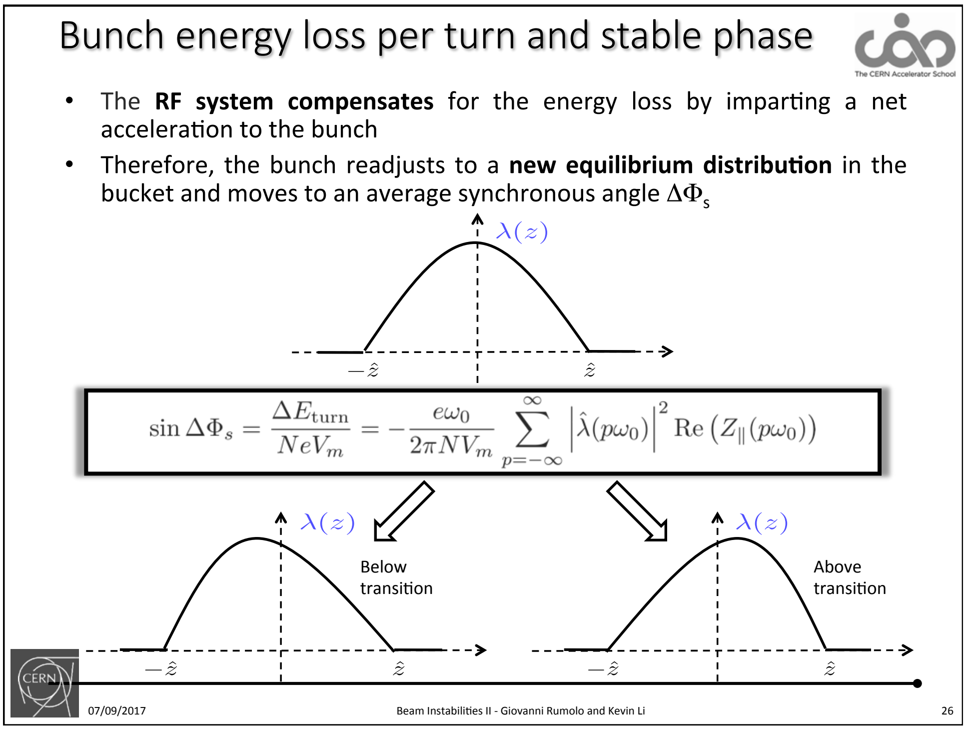
A simple Taylor expansion in z already qualitatively reveals some of the effects of the longitudinal wake fields onto the beam:

1. First order:
   shift in the mean position (**stable phase shift**)
2. Second order:
   change in bunch length accompanied by an (incoherent) **synchrotron tune shift**

$$\lambda(z) = \exp\left(\left(-\frac{\omega_s z}{2\eta\sigma_\delta\beta c}\right)^2 + \frac{e^2}{\eta\sigma_\delta^2\beta^2 EC}\sum_k \int_0^z dz'' \int_{z''}^{\infty} dz'\,\lambda(z'+kC)W_{\parallel}(z''-z'-kC)\right)$$



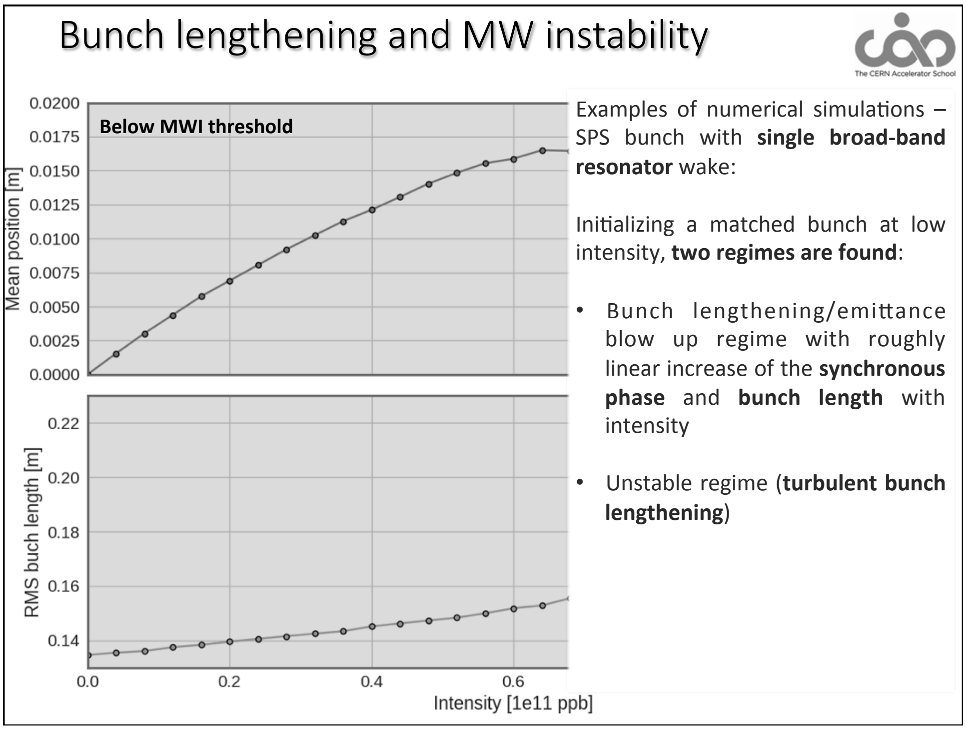
Bunch energy loss per turn and stable phase

- The **RF system compensates** for the energy loss by imparting a net acceleration to the bunch
- Therefore, the bunch readjusts to a **new equilibrium distribution** in the bucket and moves to an average synchronous angle $\Delta\Phi_s$

$$\sin\Delta\Phi_s = \frac{\Delta E_{\mathrm{turn}}}{NeV_m} = -\frac{e\omega_0}{2\pi N V_m}\sum_{p=-\infty}^{\infty}\left|\hat{\lambda}(p\omega_0)\right|^2 \mathrm{Re}\left(Z_{\parallel}(p\omega_0)\right)$$

Below transition

Above transition



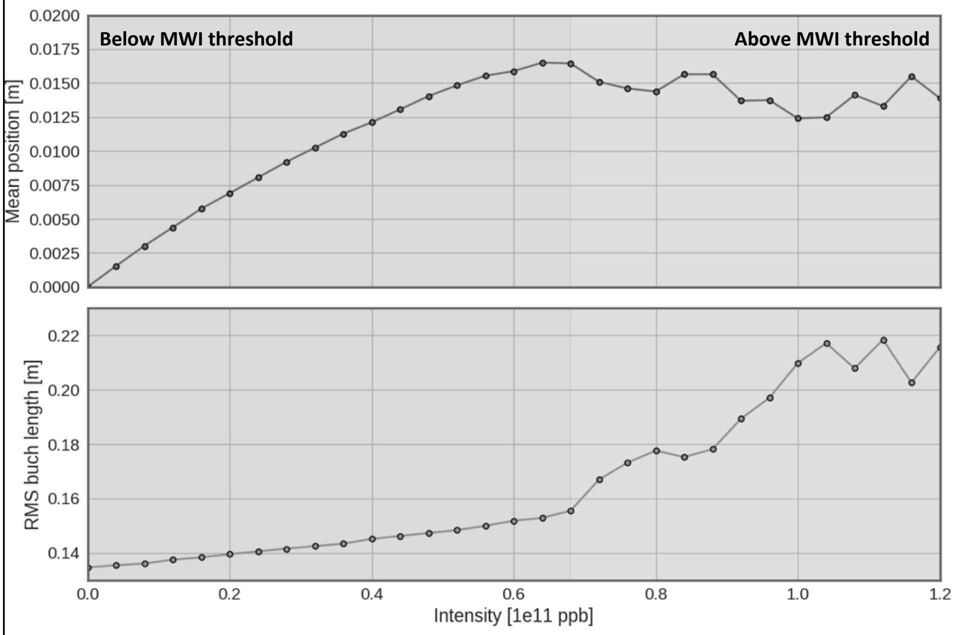
Bunch lengthening and MW instability

Examples of numerical simulations – SPS bunch with **single broad-band resonator** wake:

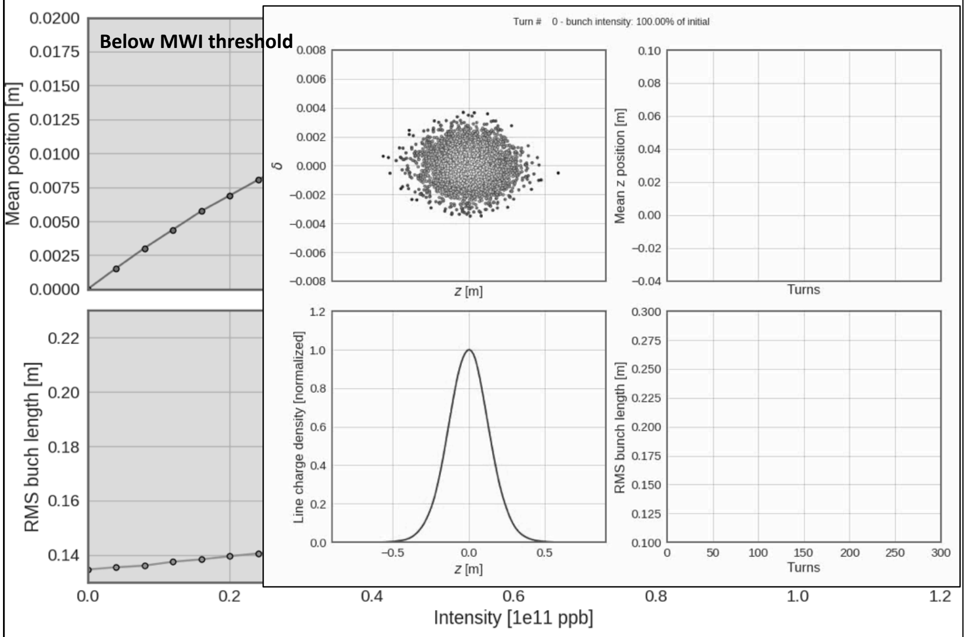
Initializing a matched bunch at low intensity, **two regimes are found**:

- Bunch lengthening/emittance blow up regime with roughly linear increase of the **synchronous phase** and **bunch length** with intensity
- Unstable regime (**turbulent bunch lengthening**)

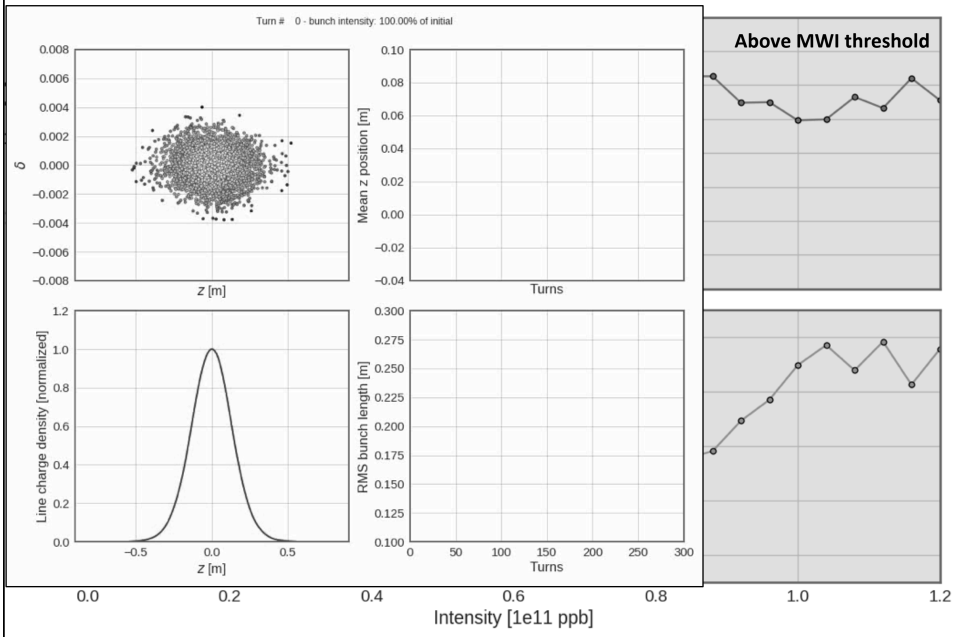
Bunch lengthening and MW instability



Bunch lengthening and MW instability



Bunch lengthening and MW instability



Signpost

- We have learned about the **impact of the longitudinal impedance on the beam**.
- We found the **Haissinki equation** and discussed the **potential well distortion** along with the **stable phase shift** and **synchrotron tune shift**.
- We looked at some generic wake fields and the **two regimes** of potential well distortion with **bunch lenthening** and its transition to the **microwave instability**.
- We will now look specifically at multi-turn wake fields and the phenomenon of **Robinson instability** and damping.

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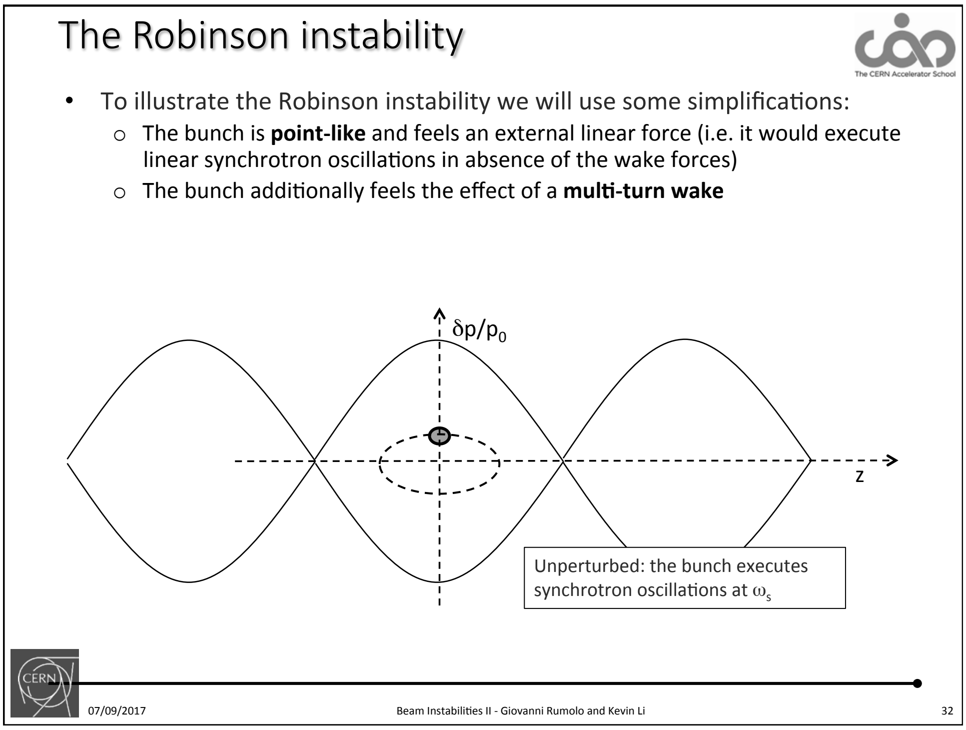
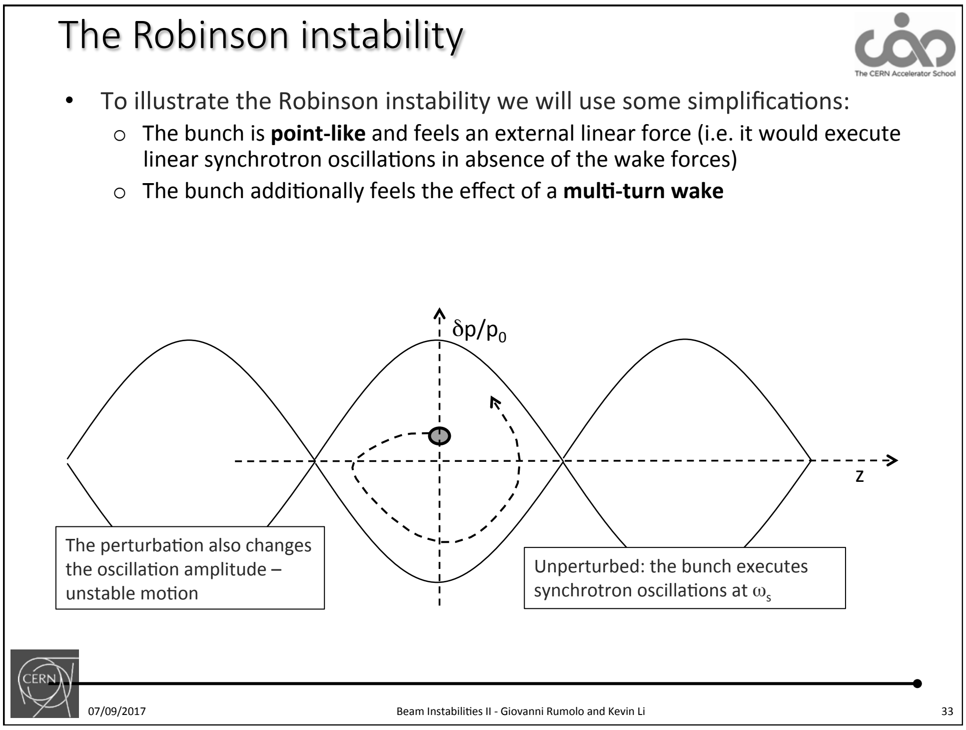


The Robinson instability

- To illustrate the Robinson instability we will use some simplifications:
  - The bunch is **point-like** and feels an external linear force (i.e. it would execute linear synchrotron oscillations in absence of the wake forces)
  - The bunch additionally feels the effect of a **multi-turn wake**

Unperturbed: the bunch executes synchrotron oscillations at ω_s



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The perturbation also changes the oscillation amplitude – unstable motion

Unperturbed: the bunch executes synchrotron oscillations at ω_s

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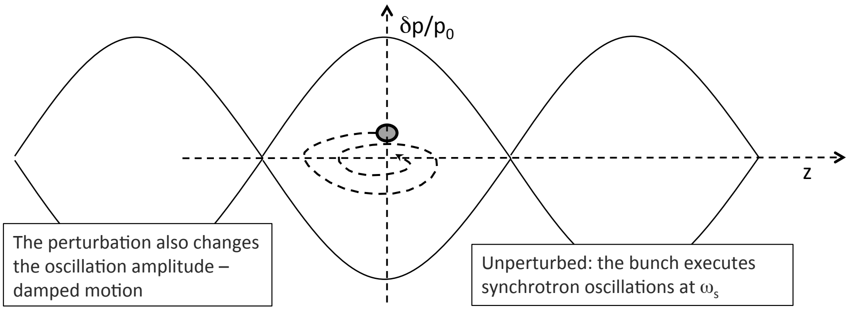

The perturbation also changes the oscillation amplitude – damped motion

Unperturbed: the bunch executes synchrotron oscillations at ω_s

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- Longitudinal Hamiltonian

$$\begin{aligned} H &= -\frac{1}{2}\eta\,\delta^2 - \frac{1}{2\eta}\left(\frac{\omega_s}{\beta c}\right)^2 z^2 + \frac{e^2}{\beta^2 EC}\sum_k \int_0^z dz'' \int_{z''}^{\infty} dz'\,\lambda(z'+kC)\,W_{\parallel}(z''-z'-kC) \\ &= -\frac{1}{2}\eta\,\delta^2 - \frac{1}{2\eta}\left(\frac{\omega_s}{\beta c}\right)^2 z^2 + \frac{Ne^2}{\beta^2 EC}\sum_k \int_0^z dz''\,W_{\parallel}\Big(z(t)-z(t-kT_0)-kC\Big) \end{aligned}$$

- Expansion of wake field (we assume that the wake can be linearized on the scale of a synchrotron oscillation)

$$\begin{aligned} W_{\parallel}(z(t)-z(t-kT_0)-kC) &\approx W_{\parallel}(kC) + W'_{\parallel}(kC)\Big(z(t)-z(t-kT_0)\Big) \\ &\approx W_{\parallel}(kC) + W'_{\parallel}(kC)\,kT_0\,\frac{dz(t)}{dt} \end{aligned}$$

The Robinson instability

- The **first term** only contributes as a constant term in the solution of the equation of motion, i.e. the synchrotron oscillation will be executed around a certain z0 and not around 0. This term represents the **stable phase shift** that compensates for the energy loss
- The **second term** is a dynamic term introduced as a **"friction" term** in the equation of the oscillator, which can **lead to instability**!

- Equations of motion

$$\frac{d^2z}{dt^2} + \omega_s^2\, z^2 = \frac{Ne^2\eta}{Cm_0\gamma}\sum_{k=-\infty}^{\infty} \cancel{W_\parallel(kC)} + \boxed{W'_\parallel(kC)\,kT_0\,\frac{dz}{dt}}$$

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- Ansatz

$$z(t) \propto \exp\left(-i\Omega t\right)$$

$$\boxed{\frac{i}{C}\sum_{p=-\infty}^{\infty}\Big(p\omega_0\, Z_\parallel\left(p\omega_0\right) - \left(p\omega_0+\Omega\right)\, Z_\parallel\left(p\omega_0+\Omega\right)\Big)}$$

Expressed in terms of impedance

- Solution

$$\left(\Omega^2-\omega_s^2\right) = -\frac{Ne^2\eta}{Cm_0\gamma}\sum_{k=-\infty}^{\infty}\boxed{\Big(1-\exp\left(-ik\Omega T_0\right)\Big)\, W'_\parallel(kC)}$$

The Robinson instability

- We assume a small deviation from the synchrotron tune:
  - Re(Ω - ω_s) → **Synchrotron tune shift**
  - Im(Ω - ω_s) → **Growth/damping rate**, only depends on the dynamic term, if it is positive there is an instability!

- **Solution:**

$$\left(\Omega^2-\omega_s^2\right)=-\frac{iNe^2\eta}{C^2 m_0\gamma}\sum_{p=-\infty}^{\infty}\Big(p\omega_0\,Z_\parallel\left(p\omega_0\right)-\left(p\omega_0+\Omega\right)\,Z_\parallel\left(p\omega_0+\Omega\right)\Big)$$
$$\approx 2\omega_s\,\left(\Omega-\omega_s\right)$$

- **Tune shift:**

$$\Delta\omega_s=\mathrm{Re}\left(\Omega-\omega_s\right)=\frac{e^2}{m_0c^2}\frac{N\eta}{2\omega_s\gamma T_0^2}$$
$$\sum_{p=-\infty}^{\infty}\Big(p\omega_0\,\mathrm{Im}\left[Z_\parallel\right]\left(p\omega_0\right)-\left(p\omega_0+\omega_s\right)\,\mathrm{Im}\left[Z_\parallel\right]\left(p\omega_0+\omega_s\right)\Big)$$

- **Growth rate:**

$$\tau^{-1}=\mathrm{Im}\left[\Omega-\omega_s\right]=\frac{e^2}{m_0c^2}\frac{N\eta}{2\omega_s\gamma T_0^2}\sum_{p=-\infty}^{\infty}\Big(\left(p\omega_0+\omega_s\right)\,\mathrm{Re}\left[Z_\parallel\right]\left(p\omega_0+\omega_s\right)\Big)$$

The Robinson instability

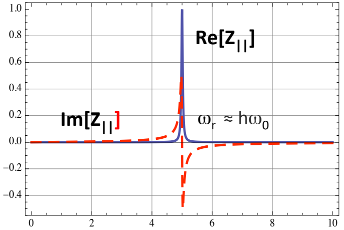
- We assume the impedance to be peaked at a frequency ω_r close to $h\omega_0 \gg \omega_s$ (e.g. RF cavity fundamental mode or HOM)
- Only two dominant terms are left in the summation at the RHS of the equation for the growth rate
- Stability requires that η and $\Delta\,\mathrm{Re}\left[Z_\parallel\right]\left(p\omega_0\right)$ have different signs

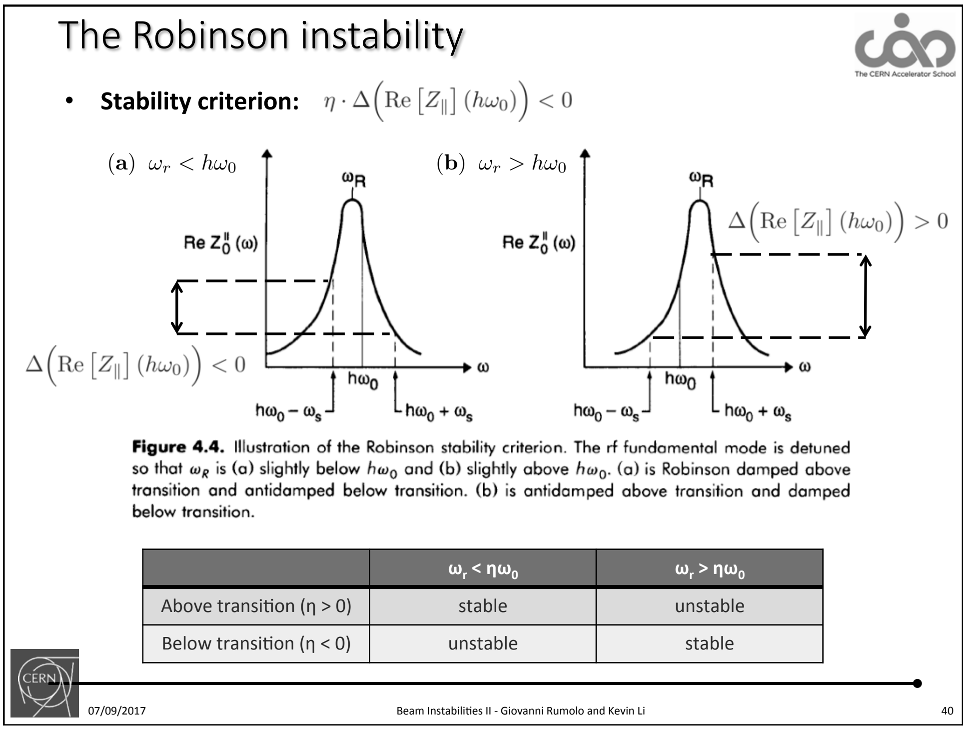
- **Solution:**

$$\tau^{-1}=\mathrm{Im}\left(\Omega-\omega_s\right)=\frac{e^2}{m_0c^2}\frac{N\eta}{2\omega_s\gamma T_0^2}\sum_{p=-\infty}^{\infty}\Big(\left(p\omega_0+\omega_s\right)\,\mathrm{Re}(Z)_\parallel\left(p\omega_0+\omega_s\right)\Big)$$
$$=\frac{e^2}{m_0c^2}\frac{N\eta h\omega_0}{2\omega_s\gamma T_0^2}\underbrace{\Big(\mathrm{Re}\left[Z_\parallel\right]\left(h\omega_0+\omega_s\right)-\mathrm{Re}\left[Z_\parallel\right]\left(h\omega_0-\omega_s\right)\Big)}_{\Delta\left(\mathrm{Re}\left[Z_\parallel\right]\left(h\omega_0\right)\right)}$$

- **Stability criterion:**

$$\eta\cdot\Delta\Big(\mathrm{Re}\left[Z_\parallel\right]\left(h\omega_0\right)\Big)<0$$



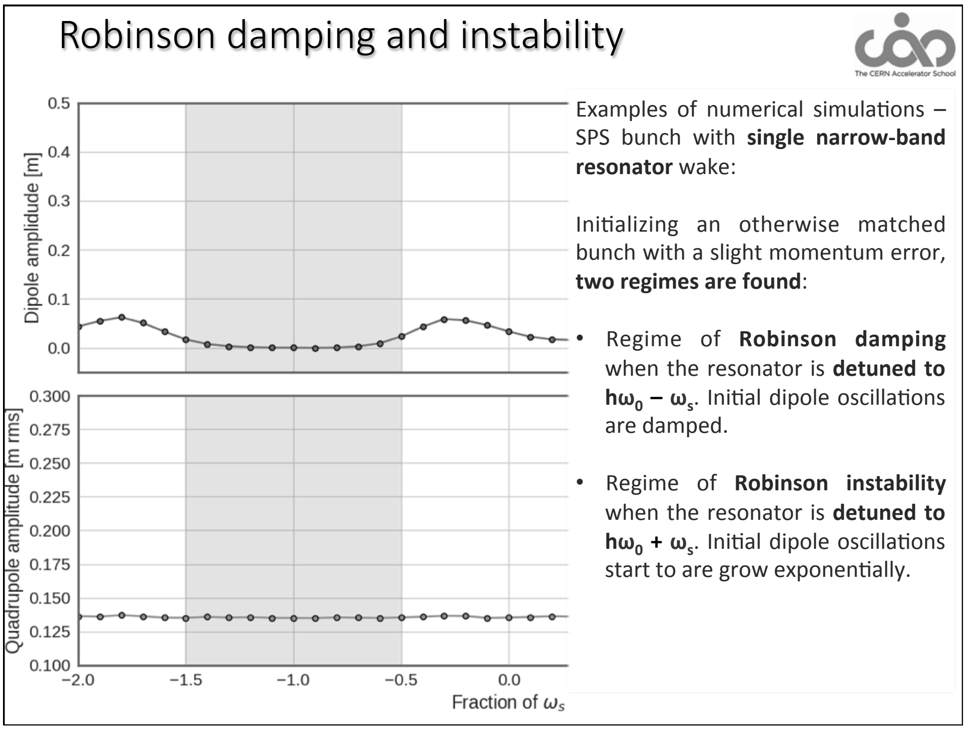


The Robinson instability

- **Stability criterion:** $\eta \cdot \Delta\Big(\mathrm{Re}\left[Z_{\|}\right](h\omega_0)\Big) < 0$

Figure 4.4. Illustration of the Robinson stability criterion. The rf fundamental mode is detuned so that ω_R is (a) slightly below $h\omega_0$ and (b) slightly above $h\omega_0$. (a) is Robinson damped above transition and antidamped below transition. (b) is antidamped above transition and damped below transition.

	$\omega_r < \eta\omega_0$	$\omega_r > \eta\omega_0$
Above transition (η > 0)	stable	unstable
Below transition (η < 0)	unstable	stable



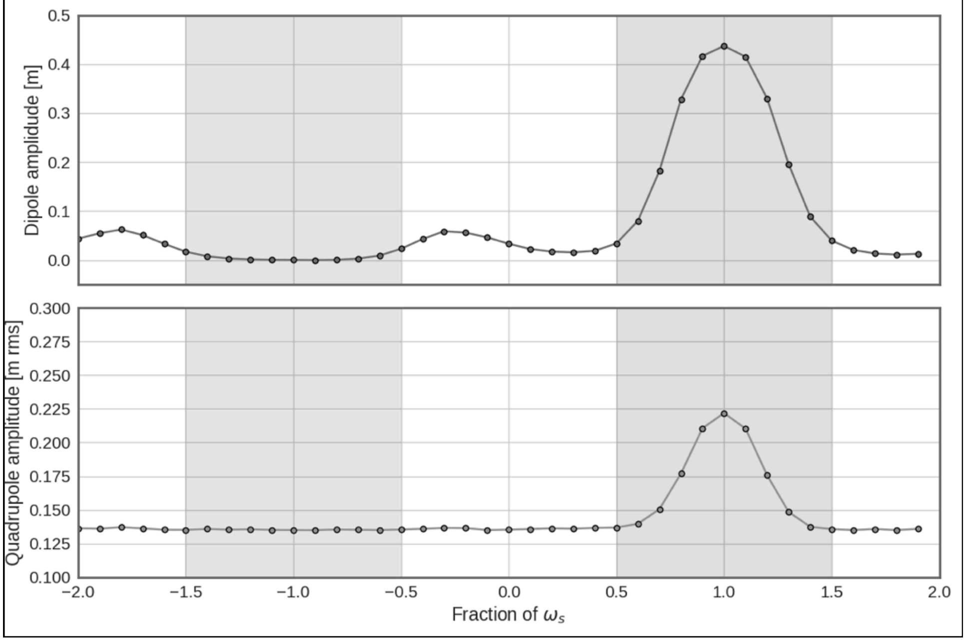
Robinson damping and instability

Examples of numerical simulations – SPS bunch with **single narrow-band resonator** wake:

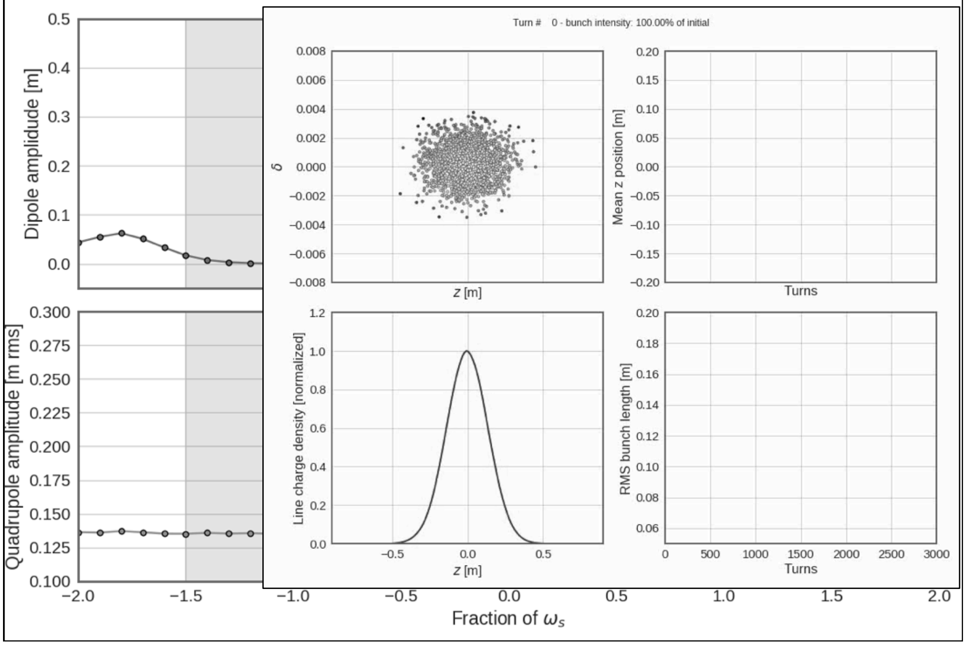
Initializing an otherwise matched bunch with a slight momentum error, **two regimes are found**:

- Regime of **Robinson damping** when the resonator is **detuned to $h\omega_0 - \omega_s$**. Initial dipole oscillations are damped.
- Regime of **Robinson instability** when the resonator is **detuned to $h\omega_0 + \omega_s$**. Initial dipole oscillations start to are grow exponentially.

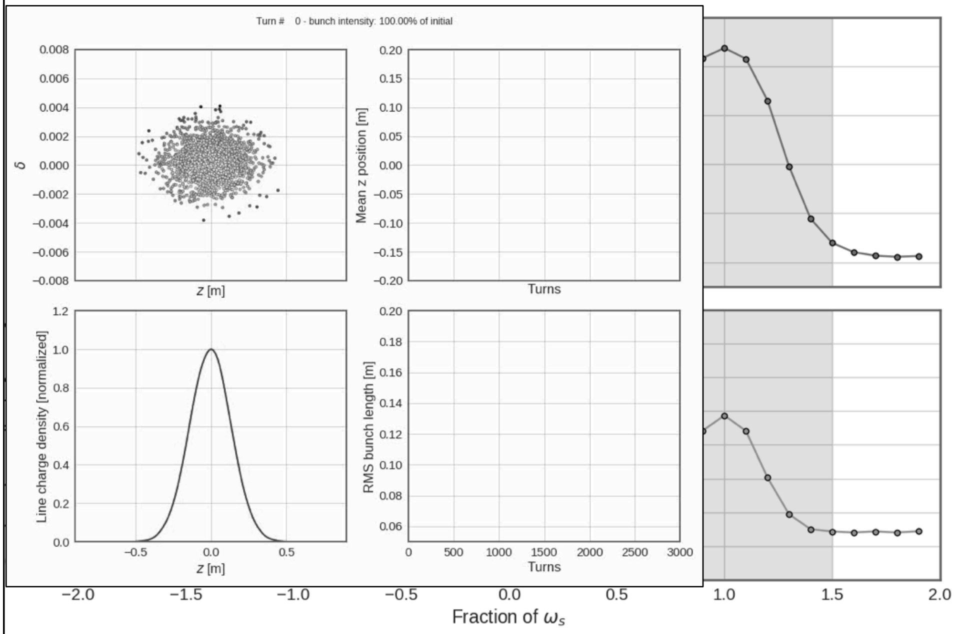
Robinson damping and instability



Robinson damping and instability



Robinson damping and instability



Other longitudinal instabilities

- The **Robinson instability** occurs for a single bunch under the action of a **multi-turn wake field**
  - It contains a term of coherent synchrotron tune shift
  - It results into an unstable rigid bunch dipole oscillation
  - It does not involve higher order moments of the bunch longitudinal phase space distribution

- Other **important collective effects** can affect a bunch in a beam – some of them of which we have also seen
  - Potential well distortion (resulting in synchronous phase shift, bunch lengthening or shortening, synchrotron tune shift/spread)
  - High intensity single bunch instabilities (e.g. microwave instability)
  - Coasting beam instabilities (e.g. negative mass instability)
  - Coupled bunch instabilities

- To be able to study these effects we would need to resort to a **more detailed description** of the bunch(es)
  - Vlasov equation (kinetic model)
  - Macroparticle simulations

Signpost

- We have **discussed longitudinal wake fields** and impedances and their impact on both the machine as well as the beam.
- We have learned about **beam induced heating** and how it is related to the beam power spectrum and the machine impedance.
- We have discussed the effects of **potential well distortion** (stable phase and synchrotron tune shifts, bunch lengthening and shortening).
- We have seen some examples of **longitudinal instabilities** (Microwave, Robinson).

Part 2: Longitudinal wakefields –
impact on machine elements and beam dynamics

- Longitudinal wake fields and the longitudinal wake function
- Energy loss – beam induced heating and stable phase shift
- Potential well distortion, bunch lengthning and microwave instability
- Robinson instability

End part 2