



Landau Damping

part 1

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Vladimir Kornilov, The CERN Accelerator School, London, UK, Sept 3-15, 2017



1



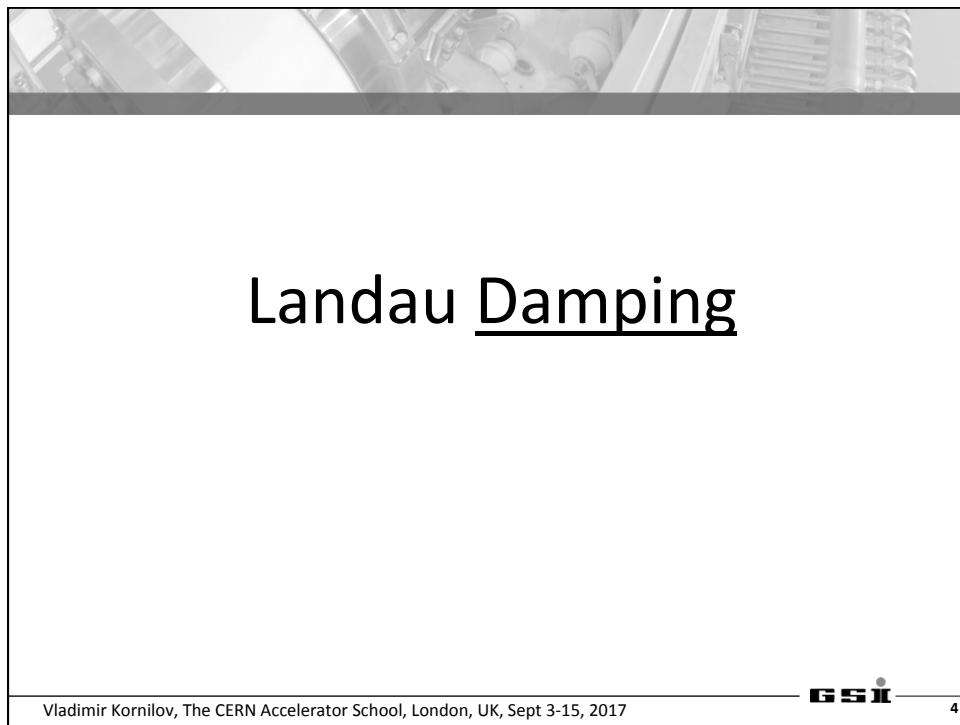
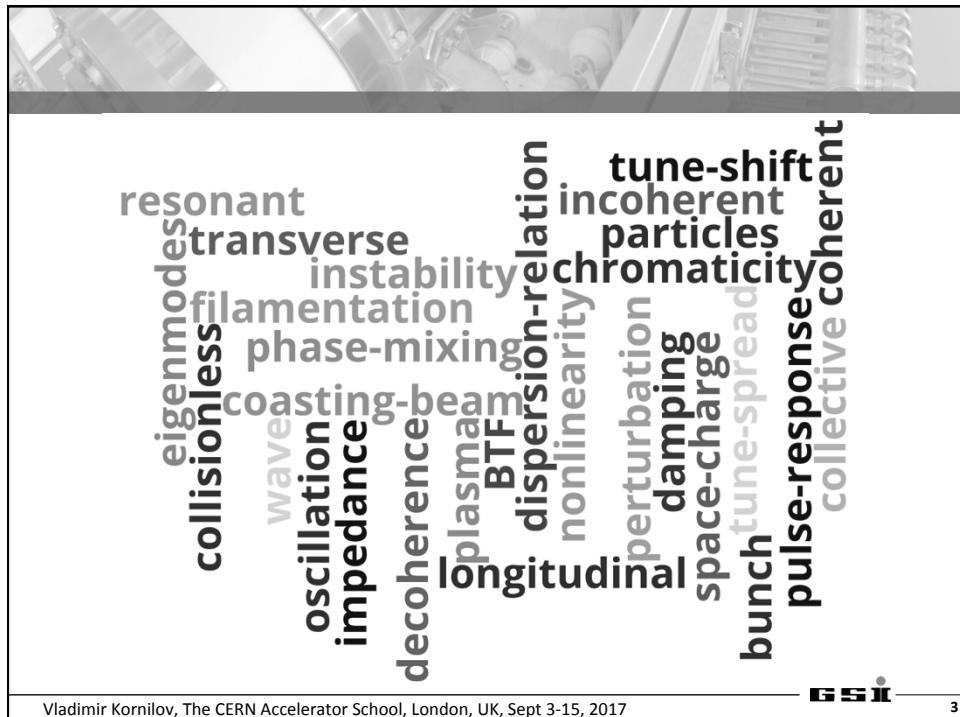
Landau Damping

a basic mechanism of beam dynamics
for all kinds of beams

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2



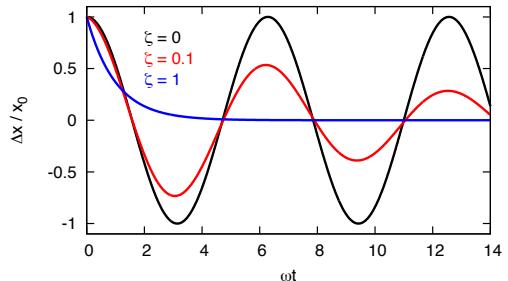
Landau Damping

Damping

A Harmonic Oscillator

With a damping (friction):

$$x'' + 2\zeta\omega_0 x' + \omega_0^2 x = 0$$



$\zeta = 0$
 $\zeta = 0.1$
 $\zeta = 1$

no damping
damped
critically damped

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Landau Damping

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Lev Landau (1908-1968)
Institute for Physical Problems, Moscow

Nobel Prize Physics 1962
“Theory of Superfluidity”

Discovery of Collisionless Damping:
L. Landau, *On the vibrations of the electronic plasma*, Journal of Physics **10**, 25-34 (1946)

Experimental confirmation:
J. Malmberg, C. Wharton,
Phys Rev Lett **13**, 184 (1964)

For our damping, “Landau”=“collisionless”=“frictionless”

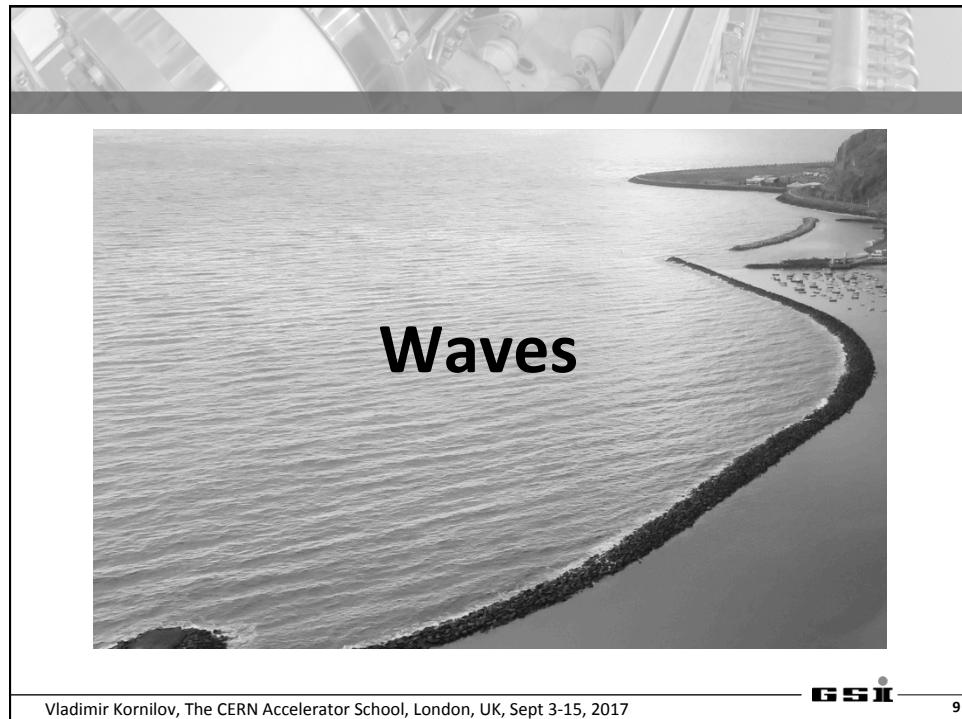
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What kind of oscillations?

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Oscillations: Waves

Water wave

Direction of travel →

A B

Calm sea level

Wave length

Wave height

crest

trough

motion of water molecules

Wave Frequency
The number of wave crests passing point A each second

Wave Period
The time required for the wave crest at point A to reach point B

Sound wave

Increased Pressure

Decreased Pressure

Atmospheric Pressure

Motion of air molecules associated with sound.

Propagation of sound

Traveling oscillation in a medium.
Very different from the medium particle motion.

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Oscillations: Waves

Water wave

Direction of travel →

Wavelength: Distance between two consecutive crests or troughs.

Wave height: Vertical distance from trough to crest.

Crest: Maximum elevation of the wave.

Trough: Minimum elevation of the wave.

motion of water molecules

Wave Frequency: The number of wave crests passing point A each second.

Wave Period: The time required for the wave crest at point A to reach point B.

Sound wave

Atmospheric Pressure

Increased Pressure

Decreased Pressure

Propagation of sound

Motion of air molecules associated with sound.

Landau damping:
wave ↔ particles collisionless interaction.

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Oscillations: Waves

Waves can be
unstable or damped

The wave frequency is complex:
 $\omega = \omega_r + i\omega_i$

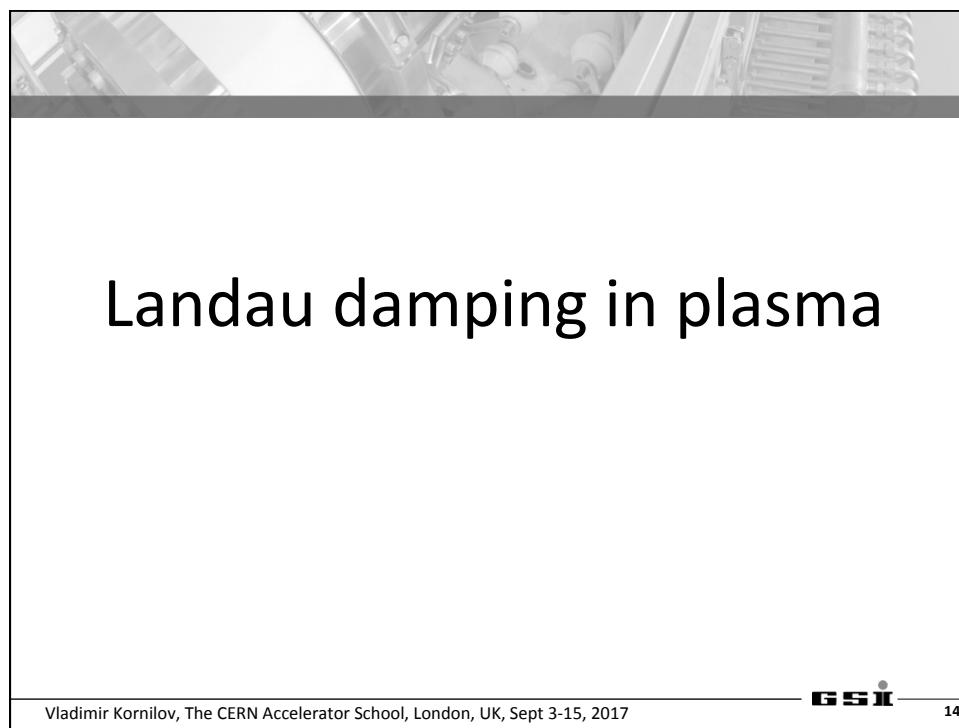
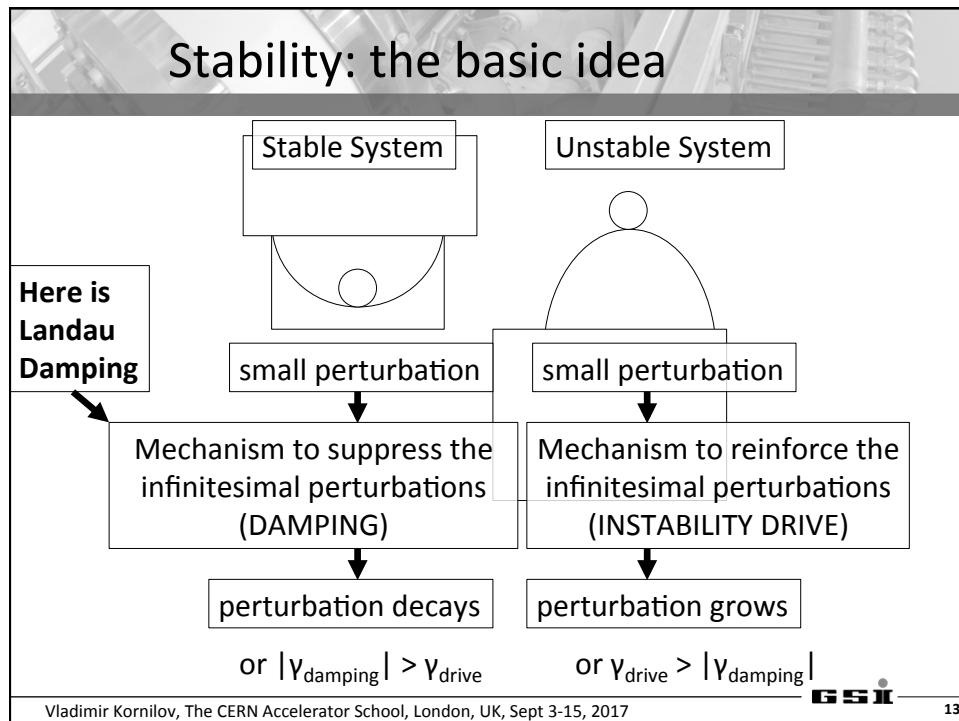
The wave physical parameter:
 $A(t) = A_0 \cos(\omega_r t) e^{i\omega_i t}$

unstable $\omega_i > 0$

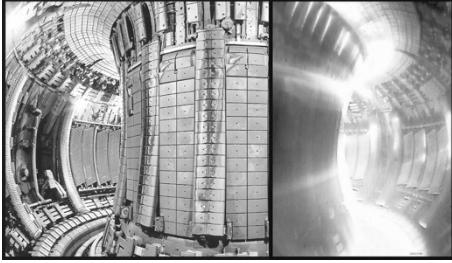
damped $\omega_i < 0$

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Plasma



Plasma in the JET tokamak

Plasma is a quasi-neutral gas of unbound ions and electrons.

Waves in plasma: collective propagating oscillations of particles and E-M fields.

Electrons are much lighter: oscillations of the electron density

Some waves can be damped.

“Friction” in plasma is collisions.

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Plasma Wave

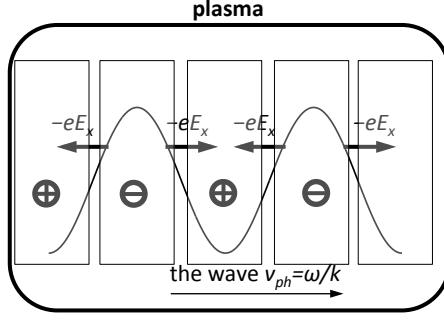
A basic plasma oscillation:
Langmuir wave

Wave number $k=2\pi/\lambda$

The phase velocity
 $v_{ph} = \omega/k$

There are resonant particles $v_x \approx v_{ph}$

The plasma frequency
 $\omega_p^2 = \frac{n_e e^2}{m_e \epsilon_0}$



The dispersion relation

$$\frac{\omega_p^2}{k^2} \int \frac{\partial \hat{f}_0 / \partial v_x}{v_x - \omega/k} dv_x = 1$$

has a singularity

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Landau Damping In Plasma

The wave frequency is complex
 $\omega = \omega_r + i\omega_i$

The dispersion relation can be solved,
the integral is calculated as PV + residue

$$\frac{\omega_p^2}{k^2} \left[\text{PV} \int \frac{\partial \hat{f}_0 / \partial v_x}{v_x - \omega/k} dv_x + i\pi \frac{\partial \hat{f}_0}{\partial v_x} \Big|_{v_x=\frac{\omega}{k}} \right] = 1$$

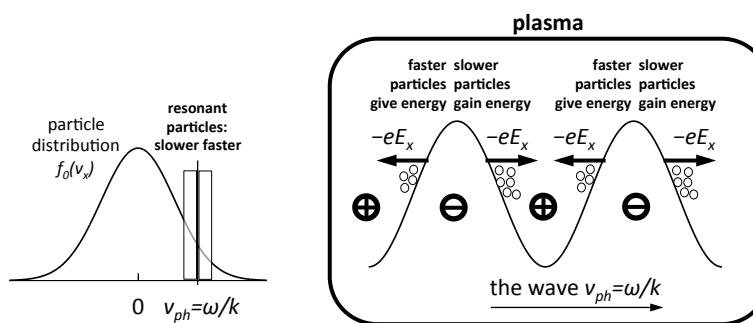
$$\begin{aligned}\omega_r^2 &= \omega_p^2 + 3k^2 v_{th}^2 \\ \omega_i &= -\frac{\pi \omega_r}{2} \frac{\omega_p^2}{k^2} \frac{\partial \hat{f}_0}{\partial v_x} \Big|_{v_x=\frac{\omega}{k}}\end{aligned}$$

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17

Landau Damping In Plasma



negative $f_0(v_x)$ slope: $N_{\text{gain}} > N_{\text{give}}$ → the wave decays, **damping**
positive $f_0(v_x)$ slope: $N_{\text{gain}} < N_{\text{give}}$ → the wave grows, **instability**

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18

Landau Damping In Plasma

THE WAVE

over electric field

slower resonant particles

faster resonant particles

$$\frac{\partial f_0}{\partial v} \Big|_{v=\frac{\omega}{k}}$$

Main ingredients of Landau damping:

- wave-particle collisionless interaction. Here this is the electric field.
- energy transfer: the wave \leftrightarrow the (few) resonant particles.

The result is the exponential decay of a small perturbation.

Landau damping is a fundamental mechanism in plasma physics.
Extensively studied in experiment, simulations and theory.

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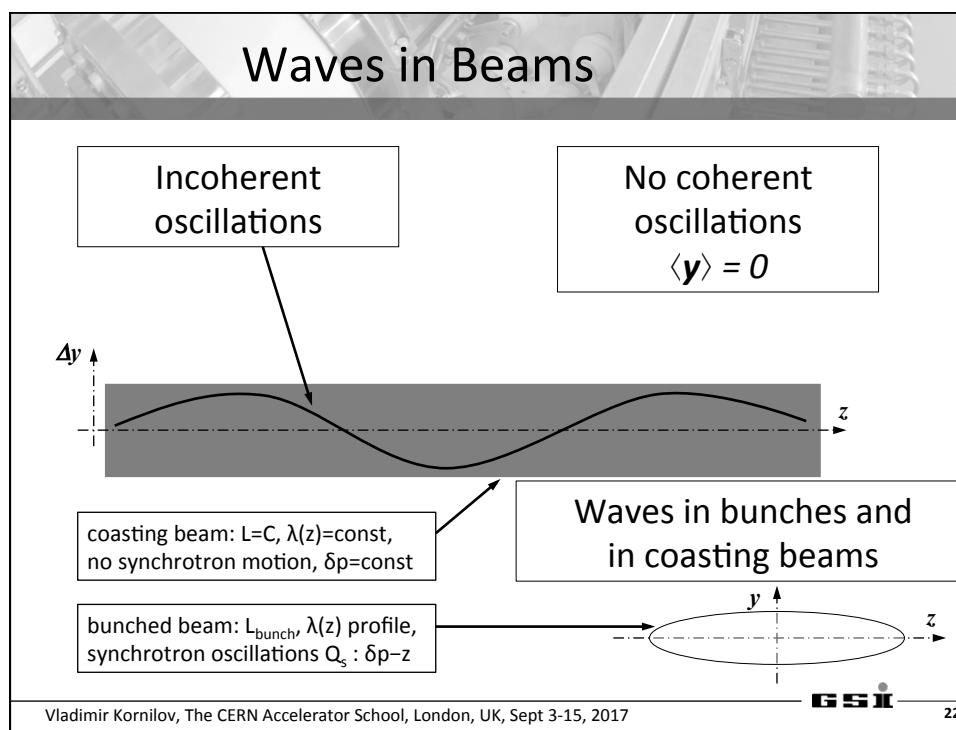
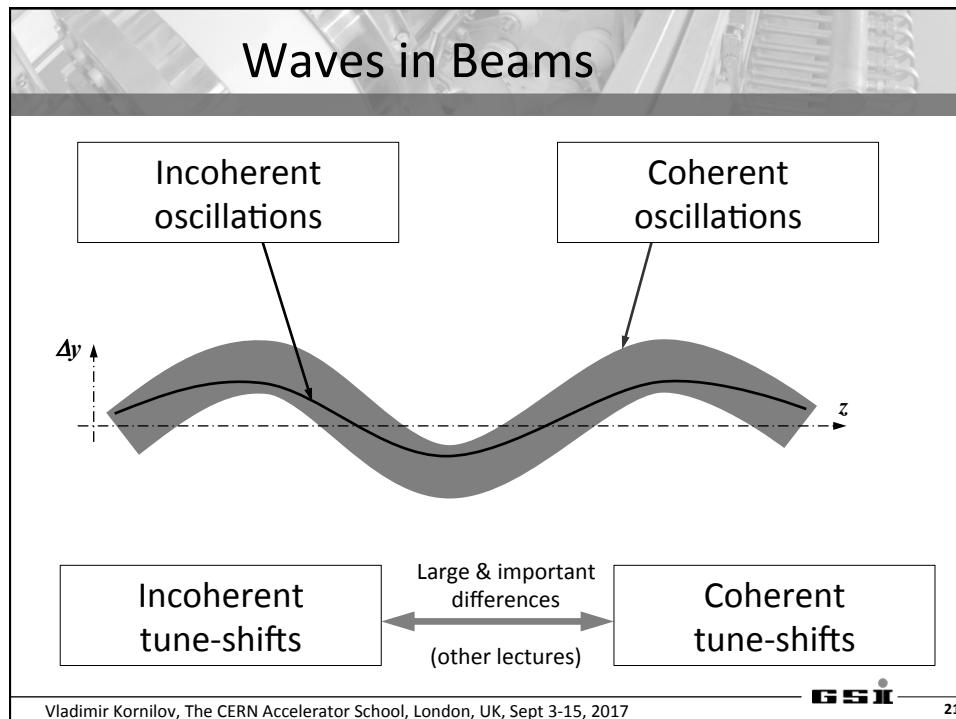
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Waves in particle beams in accelerators?

ISIS
Neutron and Muon source

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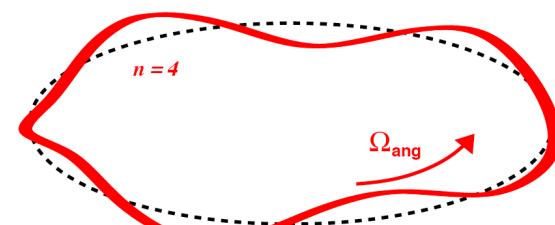


Waves in Beams

Transverse oscillations in a coasting beam

$x(s, t) = x_0 e^{ins/R - i\Omega t}$

n is the mode index.
 Wave length: C/n
 Frequencies:
 slow wave $\Omega_s = (n - Q_\beta)\omega_0$
 fast wave $\Omega_f = (n + Q_\beta)\omega_0$



Angular rotation (Ω_s):

$$\Omega_{\text{ang}} = \left(1 - \frac{Q_\beta}{n}\right)\omega_0$$

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Waves in Coasting Beams

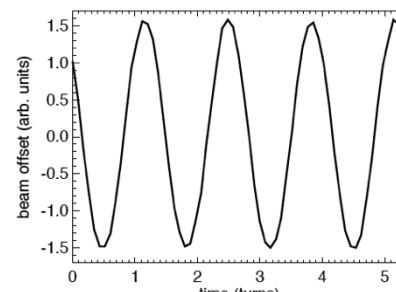
Experimental observations of the coasting-beam waves

A coasting beam in SIS18.
 $n=4$, as expected for $Q=3.25$,
 with correct Ω_s and Ω_{ang}



SIS18 synchrotron at GSI Darmstadt

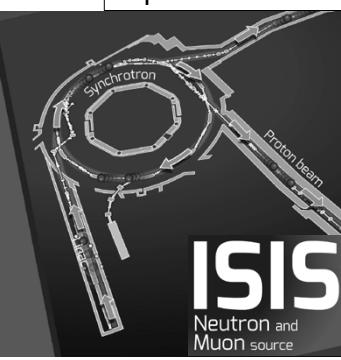
V. Kornilov, O. Boine-Frankenheim,
 GSI-Acc-Note-2009-008, GSI Darmstadt (2009)



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Waves in Bunched Beams

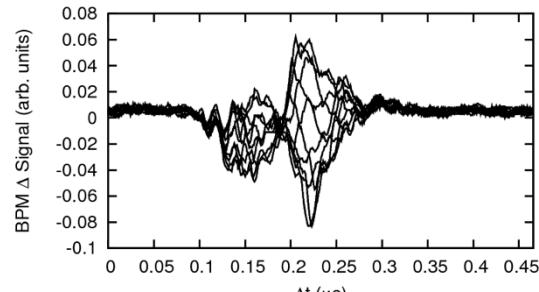
Experimental observations of the waves in bunches



ISIS
Neutron and Muon source

ISIS synchrotron at RAL, UK

Unstable head-tail modes in ISIS.
High-intensity beams, 2 bunches,
head-tail mode $k=1$, $\tau=0.1$ ms.



BPM Δ Signal (arb. units)

Δt (μ s)

V. Kornilov, et.al, HB2014 East Lansing, MI, USA, Nov 10-14, 2014

GSI 25

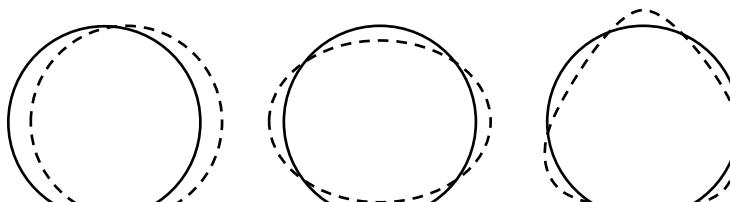
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Collective oscillations in beams

Different types of coherent oscillations

Transverse, Longitudinal

| | | |
|-------------------|-----------------------|----------------------|
| Dipolar ($m=1$) | Quadrupolar ($m=2$) | Sextupolar ($m=3$) |
|-------------------|-----------------------|----------------------|



Here we consider mostly the dipole transverse oscillations.
For the others: the physics and the formalism are similar.

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Special waves: Eigenmodes

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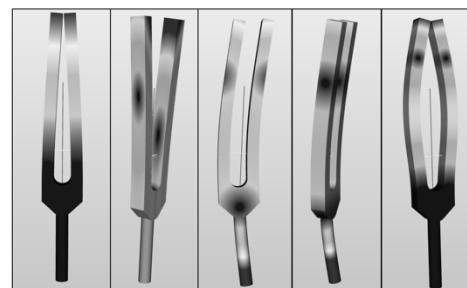
27

Eigenmodes

Eigenmodes: intrinsic orthogonal oscillations of the dynamical system,
with the fixed frequencies (eigenfrequencies)

$$A\vec{x} = \lambda\vec{x}$$

eigenvalue eigenmode



We often talk about the shift:

$$\Delta\Omega = \Omega - \Omega_{\text{eigenfrequency}}$$

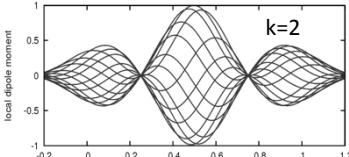
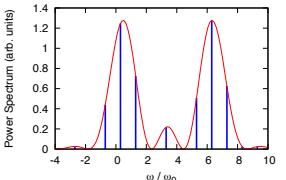
Eigenmodes of a tuning fork.
Pure tone at eigenfrequencies.

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28

Eigenmodes

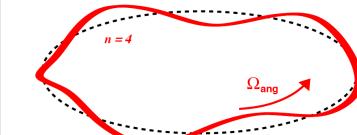
| | |
|---|--|
| <p>Transverse eigenmodes in a coasting beam</p> <p>Eigenmode: $x(s, t) = x_0 e^{ins/R - i\Omega t}$</p> <p>Eigenfrequency: $\Omega_s = (n - Q_\beta)\omega_0$</p>  | <p>Transverse eigenmodes in a bunched beam: Head-Tail Modes</p> <p>Eigenmode:</p>  <p>Eigenfrequencies:</p>  |
|---|--|

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Unstable Oscillations

small initial perturbation $\langle x \rangle$



produces

excitation force
 $G \propto Z_{\text{ext}}^\perp I_0 \langle x \rangle$

reinforcing mechanism

forced oscillations (eigenmode)

the perturbation is amplified
 $\langle x \rangle \times (1+\Delta)$

The result is ΔQ_{coh} and the exponential growth: instability

$$\langle x \rangle(t) = x_0 e^{\text{Im}(\Omega)t}$$

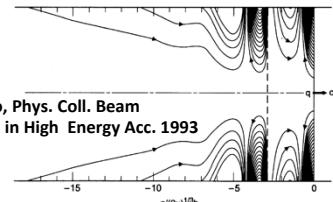
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Wake Fields, Impedances

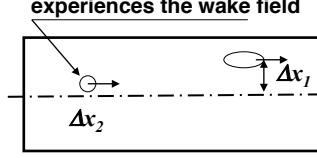
Dipolar wakes: $F_{x2} \sim \Delta x_1$
 (driving) the same for the whole trailing slice: coherent

Quadrupolar wakes: $F_{x2} \sim \Delta x_2$
 (detuning) different for individual particles: incoherent



A.Chao, Phys. Coll. Beam
Instab. in High Energy Acc. 1993

$z/(2\lambda)^{1/2} b$



experiences the wake field

Δx_1

trailing leading

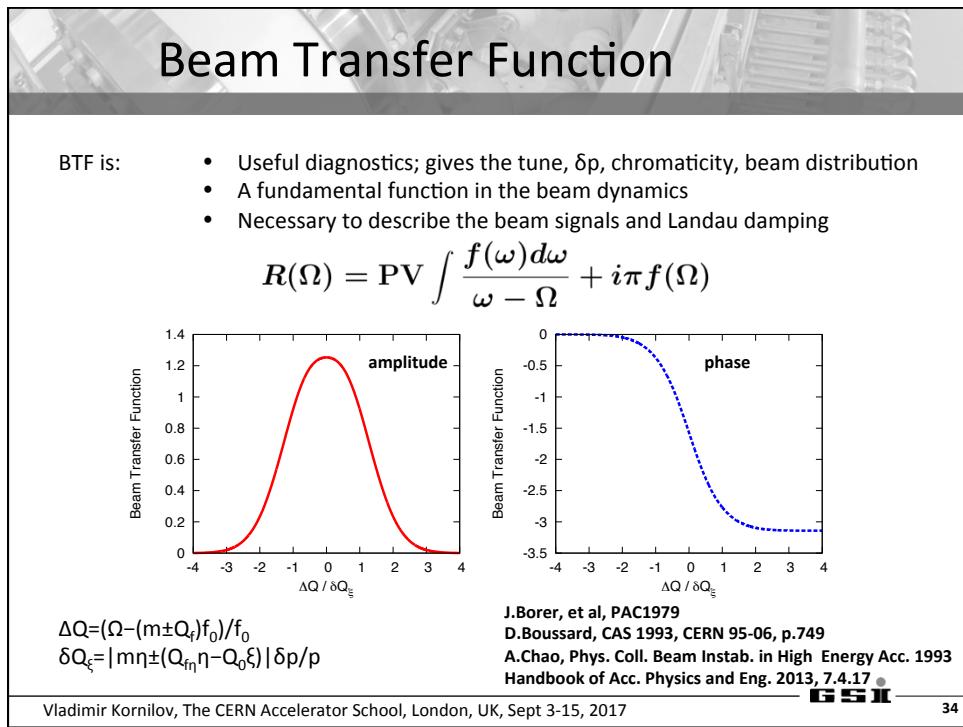
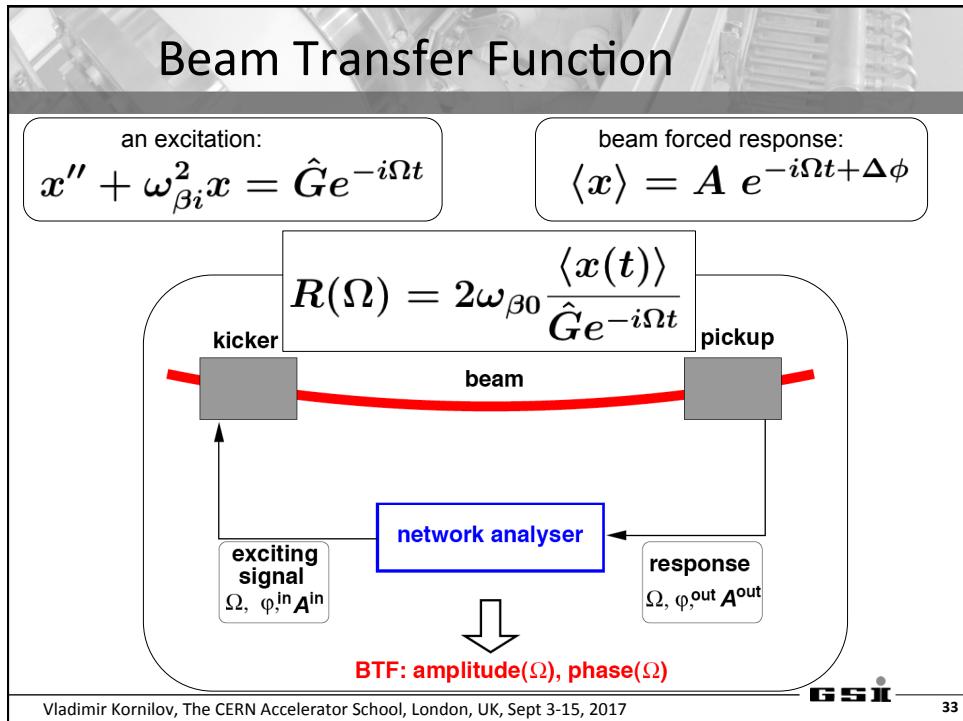
Transverse collective instabilities: Dipolar Wakes $W_1(z)$, Impedances $Z_1(\omega)$

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31

Beam Transfer Function (BTF)

K.Y.Ng, Physics of Intensity Dependent Beam Instabilities, 2006
 A.Hofmann, Proc. CAS 2003, CERN-2006-002
 A.Chao, Phys. Coll. Beam Instab. in High Energy Acc. 1993

Vladimir Kornilov, The CERN Accelerator School, London, UK, Sept 3-15, 2017
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32



Beam Transfer Function

BTF: a standard measurement with a network analyser

- Collective response to the excitation
- Observe the incoherent spectrum
- Still, the beam is stable: Landau Damping!

A coasting beam U^{73+} in SIS18.
Transverse signal.
Lower side-band of $m=24$

amplitude

phase

frequency, MHz

V.Kornilov, et al, GSI-Acc-Note-2006-12-001, GSI Darmstadt (2006)

Vladimir Kornilov, The CERN Accelerator School, London, UK, Sept 3-15, 2017

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Landau Damping:

Interaction wave \leftrightarrow resonant particles

V.K. Neil and A.M. Sessler, Rev. Sci. Instrum. 6, 429 (1965)
 L.J. Laslett, V.K. Neil, and A.M. Sessler, Rev. Sci. Instrum. 6, 46 (1965)
 H.G. Hereward, CERN Report 65-20 (1965)
 D. Möhl, H. Schönauer, Proc. IX Int. Conf. High Energy Acc., p. 380 (1974)
 A. Hofmann, Proc. CAS 2003, CERN-2006-002
 A. Chao, Phys. Coll. Beam Instab. in High Energy Acc. 1993
 K.Y. Ng, Physics of Intensity Dependent Beam Instabilities, 2006

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Driven Harmonic Oscillator

$\omega_i = 2\pi f$

$$x'' + \omega_i^2 x = \hat{G} e^{-i\Omega t}$$

| | | | | |
|--------------|---|--|---|--|
| The solution | = | homogeneous solution (pulse response) initial conditions | + | particular solution (forced oscillations) |
|--------------|---|--|---|--|

Off-resonance ($\Omega \neq \omega_i$) and at resonance ($\Omega = \omega_i$), different particular solutions.
Zero initial conditions.

$$x_G(t) = \frac{2\hat{G}}{\omega_i^2 - \Omega^2} \sin\left(\frac{\omega_i - \Omega}{2}t\right) \sin\left(\frac{\omega_i + \Omega}{2}t\right)$$

$$x_G(t) = \frac{\hat{G}}{2\Omega} t \sin(\Omega t)$$

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Driven Harmonic Oscillator

off-resonant beating solution $x_G(t) = \frac{2\hat{G}}{\omega_i^2 - \Omega^2} \sin\left(\frac{\omega_i - \Omega}{2}t\right) \sin\left(\frac{\omega_i + \Omega}{2}t\right)$

resonant solution $x_G(t) = \frac{\hat{G}}{2\Omega} t \sin(\Omega t)$

$\Delta\omega_i / \Omega = 0.03$
 $\Delta\omega_i / \Omega = 0.01$
 $\omega_i = \Omega$

gain energy give energy

wave particle energy transfer

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Landau Damping: Dispersion Relation

D. Möhl, H. Schönauer, Proc. IX Int. Conf. High Energy Acc., p. 380 (1974)
A.Hofmann, Proc. CAS 2003, CERN-2006-002
A.Chao, Phys. Coll. Beam Instab. in High Energy Acc. 1993
K.Y.Ng, Physics of Intensity Dependent Beam Instabilities, 2006
W.Herr, Introduction to Landau Damping, CAS2013, CERN-2014-009

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39

Coherent Oscillations

An easy derivation of the dispersion relation

the external drive is INTENSITY × IMPEDANCE × PERTURBATION

$$G = \frac{\langle F_x \rangle}{m\gamma} = \frac{q\beta}{m\gamma C} iZ_{\text{ext}}^\perp I_0 \langle x \rangle$$

the no-damping complex coherent tune shift is
INTENSITY × IMPEDANCE

$$\Delta Q_{\text{coh}} = \frac{I_0 q_{\text{ion}}}{4\pi\gamma mcQ_0\omega_0} iZ_{\text{ext}}^\perp$$

only the dipole
impedance here,
no incoherent effects

thus, the external drive is

$$G = 2\omega_{\beta 0}\omega_0 \Delta Q_{\text{coh}} \langle x \rangle$$

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40

Dispersion Relation

An easy derivation of the dispersion relation

the external drive is IMPEDANCE TUNE SHIFT × PERTURBATION

$$G = 2\omega_{\beta 0}\omega_0 \Delta Q_{coh} \langle x \rangle$$

the beam response is the BTF

$$\langle x \rangle = \frac{G}{2\omega_{\beta 0}\sigma_\omega} R(u)$$

combined: the DISPERSION RELATION

$$\Delta Q_{coh} R(\Omega) = 1$$

provides the resulting Ω for the given impedance and beam

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Stability Diagram

the resulting Ω for the given impedance and beam

$$\Delta Q_{coh} R(\Omega) = 1$$

$$\Delta Q_{coh} \omega_0 \int \frac{f(\omega) d\omega}{\omega - \Omega} = 1$$

$Re(Z) > 0$: the slow wave

$$\omega_s = (n - Q_0)\omega_0$$

$$\delta Q_\xi = |\eta(n - Q_0) + Q_0 \xi| \delta_p$$

Circle Criterion: E.Keil, W.Schnell, CERN ISR-TH-RF/69-48 (1969)

Circle Criterion

$$\frac{|\Delta Q_{coh}|}{\delta Q_\xi} = 1$$

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Stability Diagram

the resulting Ω for the given impedance and beam

$\Delta Q_{\text{coh}} R(\Omega) = 1$

$$\frac{|\Delta Q_{\text{coh}}|}{\delta Q_\xi} = 1$$

$$\delta Q_\xi = |\eta(n - Q_0) + Q_0 \xi| \delta_p$$

Strength of Landau Damping is proportional to the tune-spread

Tune spread provides Landau Damping

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GSI 43

Beam Transfer Function

BTF provides a direct measure of Landau Damping

$\Delta Q_{\text{coh}} R(\Omega) = 1$

| | |
|-----------------------|-----------------------------|
| Measured BTF in SIS18 | Resulting Stability Diagram |
|-----------------------|-----------------------------|

V.Kornilov, et al, GSI-Acc-Note-2006-12-001, GSI Darmstadt (2006)

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Longitudinal Stability

Coasting Beam:
Spread in the revolution frequency

$$\mathcal{A} I_0 \frac{Z_{\parallel}(\Omega_{\parallel})}{n} \int \frac{\partial f(\omega_0)/\partial\omega_0}{\omega_0 - \Omega_{\parallel}/n} d\omega_0 = 1$$

$$\left| \frac{Z}{n} \right| \leq 0.6 \frac{2\pi\beta^2 E_0 \eta (\Delta p/p)^2}{eI_0}$$

Bunched beams:

$$\Delta\omega_s^{\text{coh}} \int \frac{f(\omega_s)d\omega_s}{\Omega_{\parallel} - \omega_s} = 1$$

the physics and the formalism are similar

K.Y.Ng, Physics of Intensity Dependent Beam Instabilities, 2006
 A.Hofmann, Proc. CAS 2003, CERN-2006-002
 E.Keil, W.Schnell, CERN ISR-TH-RF/69-48 (1969)

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Landau Damping

Incomplete (!) mechanism of Landau Damping in beams for the end of the first part

PERTURBATION
the wave

driving field
 $G \propto Z_{\text{ext}}^{\perp} I_0 \langle x \rangle$

energy transfer to resonant particles
 $f(\omega)|_{\omega=\Omega_{\text{coh}}}$

G suppression

```

graph LR
    P[PERTURBATION<br/>the wave] --> DF[driving field<br/> $G \propto Z_{\text{ext}}^{\perp} I_0 \langle x \rangle$ ]
    DF --> ET[energy transfer to<br/>resonant particles<br/> $f(\omega)|_{\omega=\Omega_{\text{coh}}}$ ]
    ET --> G[G<br/>suppression]
    G -- feedback --> P
  
```

Main ingredients of Landau damping:

- ✓ wave-particle collisionless interaction: Impedance driving field
- ✓ energy transfer: the wave \leftrightarrow the (few) resonant particles

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Landau Damping

End of part 1

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47