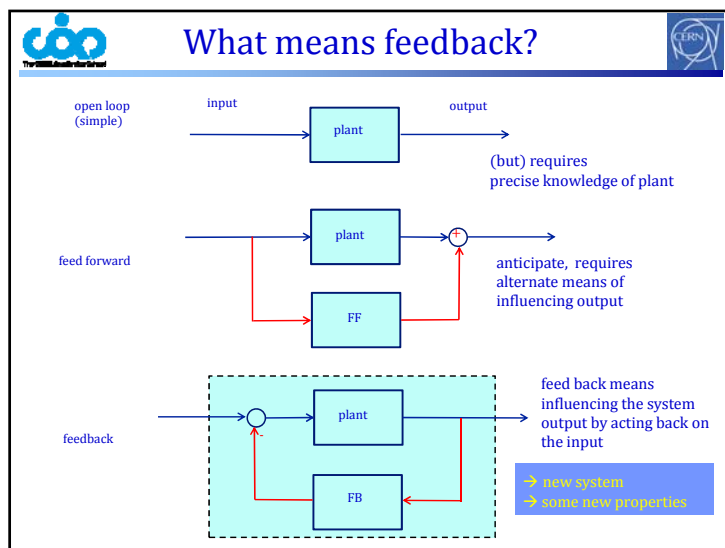




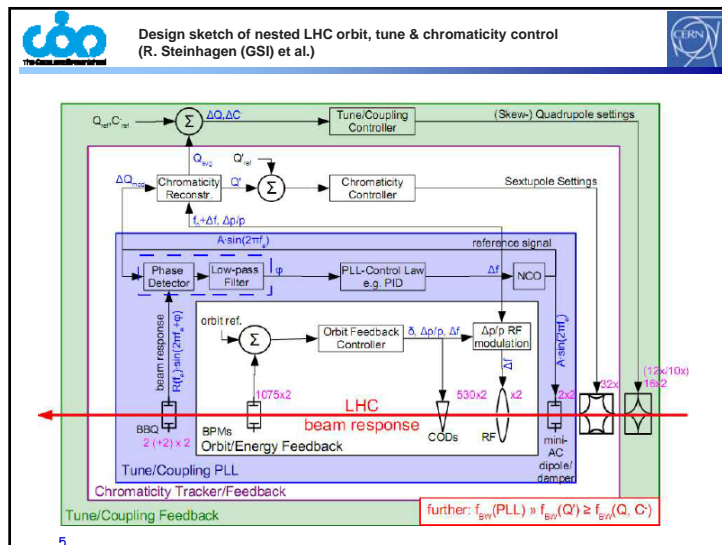
Outline

- What is feedback?
- What are the applications in accelerators?
- Coupled-bunch instabilities
- Basics of feedback systems
- Feedback system components
- Digital signal processing
- Using feedbacks for beam diagnostics



Feedback applications in accelerators

- An accelerator, which relies on active beam feedback to get basic performance, is based on a questionable concept. Feedbacks should not be used to fix equipment, that can be fixed or redesigned.
- Typically feedbacks are employed to achieve ultimate performance and long term stability.
- Feedbacks are used in the transverse and longitudinal plane.
- We concentrate on feedback systems based **on beam signals** (almost every technical equipment has internal feedback controllerspower converters, RF systems, instrumentation...)
- Beam feedbacks:
 - 1) Transverse and/or longitudinal damping against beam instabilities
 - 2) Injection damping
 - 3) Slow control of machine parameters (orbit, tune, chromaticity)
 1+2 have hard real time constraints (turn by turn), 3 has lower bandwidth
- Apart from showing one example, we focus on feedback **types 1** and **2**



beam instabilities /motivation for feedback

- Transverse (betatron) and longitudinal (synchrotron) oscillations
 - strongly damped by radiation damping in lepton accelerators (lightsources)
 - undamped in proton accelerators (disregarding 100 TeV designs)
- Interaction of the electromagnetic field with metallic surroundings ("wake fields")
- Wake fields act back on the beam and produces growth of oscillations
- If the growth rate is stronger than the natural damping the oscillation gets unstable
- Consequences are **emittance increase or particle loss**.
- Since wake fields are proportional to the bunch charge, the onset of instabilities and their amplitude are normally **current dependent**
- Another "instability", i.e. large beam oscillation is due to errors at the moment of injection:
 - rather uncritical for lepton machines (radiation damping)
 - vital for hadron machines (filamentation and emittance increase → loss in luminosity)
- People always aim at higher brightness beams or higher luminosity collisions, which means
 - maximum beam/bunch intensity
 - minimum beam emittance
- Sooner or later feedbacks are employed to gain the last factors of performance.

Sources of instabilities

Cavity High Order Modes (HOM)
High Q spurious resonances of the accelerating cavity excited by the bunched beam act back on the beam itself
Each bunch affects the following bunches through the wake fields excited in the cavity
The cavity HOM can couple with a beam oscillation mode having the same frequency and give rise to instability

Resistive wall impedance
Interaction of the beam with the vacuum chamber.
Particularly strong in low-gap chambers and in-vacuum insertion devices (undulators and wigglers)

Interaction of the beam with other objects
Discontinuities in the vacuum chamber, small cavity-like structures, ...
Ex. BPMs, vacuum pumps, bellows, ...

Ion instabilities
Gas molecules ionized by collision with the electron beam
Positive ions remains trapped in the negative electric potential
Produce electron-ion coherent oscillations

Passive cures

Cavity High Order Modes (HOM)
Thorough design of the RF cavity
Mode dampers with antennas and resistive loads
Tuning of HOMs frequencies through plungers or changing the cavity temperature

Resistive wall impedance
Usage of low resistivity materials for the vacuum pipe
Optimization of vacuum chamber geometry

Interaction of the beam with other objects
Proper design of the vacuum chamber and of the various installed objects

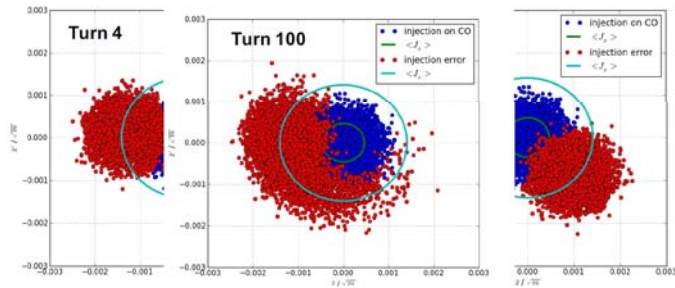
Ion instabilities
Ion cleaning with a gap in the bunch train

Landau damping by increasing the tune spread
Higher harmonic RF cavity (bunch lengthening)
Modulation of the RF
Octupole magnets (transverse)

Active Feedbacks

Steering error – non-linear machine

- What will happen to particle distribution and hence emittance?



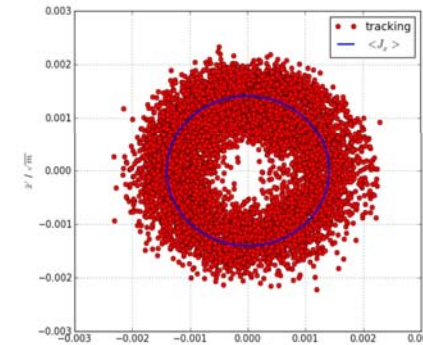
- The beam is filamenting....

H.Schmickler, CERN

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Steering error – non-linear machine

- Phase-space after an even longer time



→ Much shorter than the filamentation time, the injection oscillation must be damped.

Equation of motion of one particle: harmonic oscillator analogy

"x" is the oscillation coordinate (transverse or longitudinal displacement)

Natural damping

Betatron/Synchrotron frequency:
tune (ν) × revolution frequency (ω₀)

$$\ddot{x}(t) + 2D\dot{x}(t) + \omega^2 x(t) = 0$$

If $\omega \gg D$, an approximated solution of the differential equation is a damped sinusoidal oscillation:

$$x(t) = e^{-\frac{t}{\tau_D}} \sin(\omega t + \varphi)$$

where $\tau_D = 1/D$ is the "damping time constant" (D is called "damping rate")

Excited oscillations (ex. by quantum excitation) are damped by natural damping (ex. due to synchrotron radiation damping). The oscillation of individual particles is **uncorrelated** and shows up as an emittance growth

Damping of Coherent Bunch Oscillations

Coupling with other bunches through the interaction with surrounding metallic structures add a "driving force" term $F(t)$ to the equation of motion:

$$\ddot{x}(t) + 2D\dot{x}(t) + \omega^2 x(t) = F(t)$$

Under given conditions the oscillation of individual particles becomes correlated and the centroid of the bunch oscillates giving rise to **coherent bunch (coupled bunch) oscillations**

Each bunch oscillates according to the equation of motion:

$$\ddot{x}(t) + 2(D-G)\dot{x}(t) + \omega^2 x(t) = 0$$

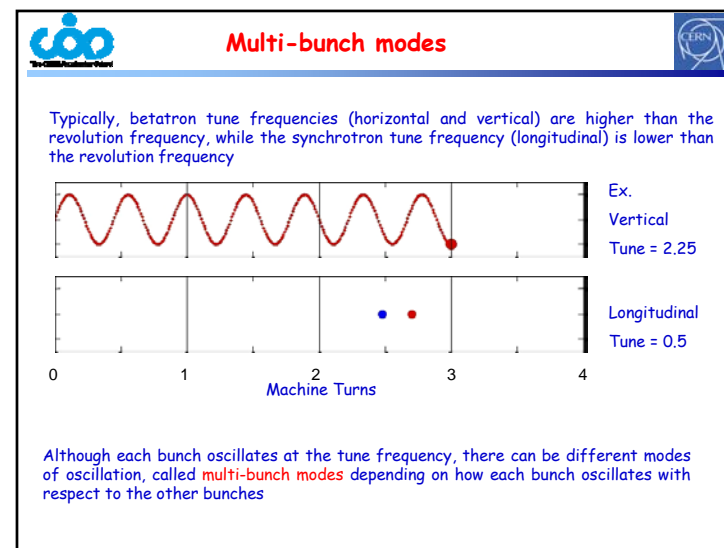
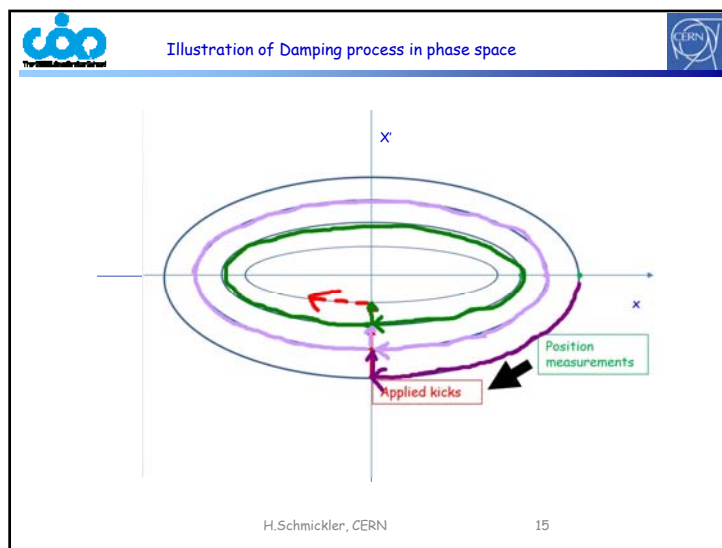
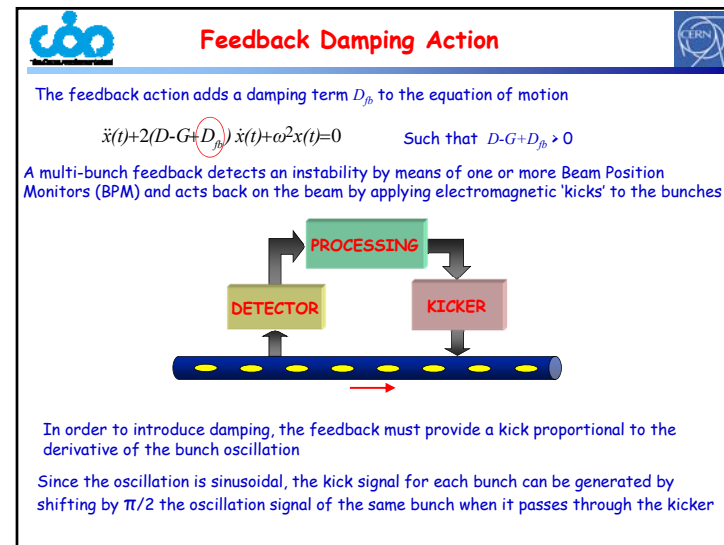
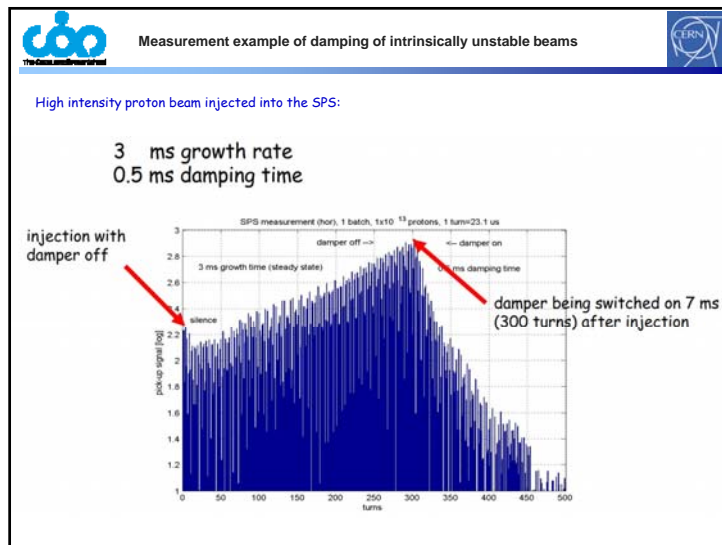
where $\tau_G = 1/G$ is the "growth time constant" (G is called "growth rate")

If $D > G$ the oscillation amplitude decays exponentially

If $D < G$ the oscillation amplitude grows exponentially

as: $x(t) = e^{-\frac{t}{\tau}} \sin(\omega t + \varphi)$ where $\frac{1}{\tau} = \frac{1}{\tau_D} - \frac{1}{\tau_G}$

Since G is proportional to the beam current, if the latter is lower than a given current threshold the beam remains stable, if higher a coupled bunch instability is excited



Multi-bunch modes

Let us consider M bunches equally spaced around the ring
 Each multi-bunch mode is characterized by a bunch-to-bunch phase difference of:

$$\Delta\Phi = m \frac{2\pi}{M} \quad m = \text{multi-bunch mode number } (0, 1, \dots, M-1)$$

Each multi-bunch mode is associated to a characteristic set of frequencies:

$$\omega = pM\omega_0 \pm (m+\nu)\omega_0$$

Where:

- p is and integer number $-\infty < p < \infty$
- ω_0 is the **revolution frequency**
- $M\omega_0 = \omega_{rf}$ is the RF frequency (bunch repetition frequency)
- ν is the **tune**

Two sidebands at $\pm(m+\nu)\omega_0$ for each multiple of the RF frequency

Multi-bunch modes

The spectrum is a repetition of frequency lines at multiples of the bunch repetition frequency with sidebands at $\pm\nu\omega_0$: $\omega = p\omega_{rf} \pm \nu\omega_0 \quad -\infty < p < \infty \quad (\nu = 0.25)$

Since the spectrum is periodic and each mode appears twice (upper and lower side band) in a ω_{rf} frequency span, we can limit the spectrum analysis to a $0-\omega_{rf}/2$ frequency range

The inverse statement is also true:

Since we 'sample' the continuous motion of the beam with only one pickup, any other frequency component above half the 'sampling frequency' (i.e the bunch frequency ω_{rf}) is not accessible (Nyquist or Shannon Theorem)

Multi-bunch modes: example1

Vertical plane. One single stable bunch

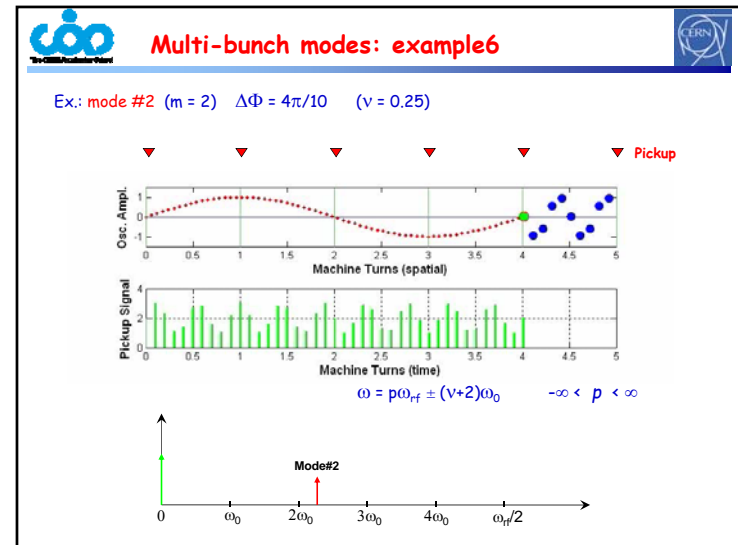
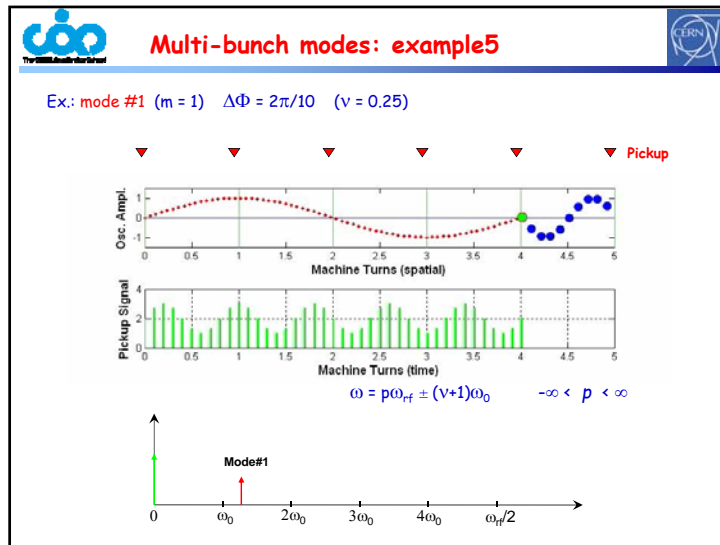
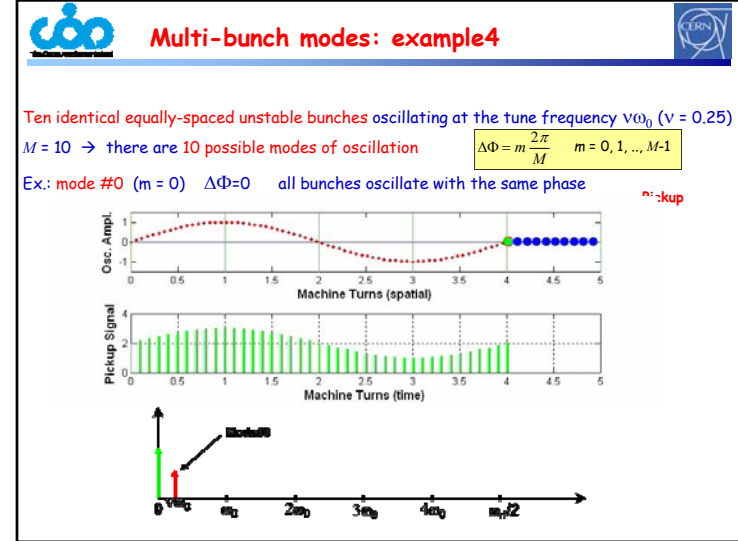
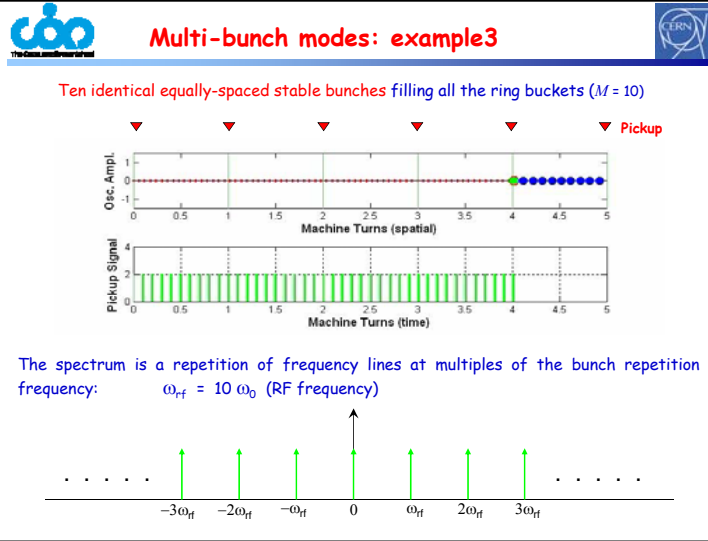
Every time the bunch passes through the pickup (▼) placed at coordinate 0, a pulse with constant amplitude is generated. If we think it as a Dirac impulse, the spectrum of the pickup signal is a repetition of frequency lines at multiple of the revolution frequency: $p\omega_0$ for $-\infty < p < \infty$

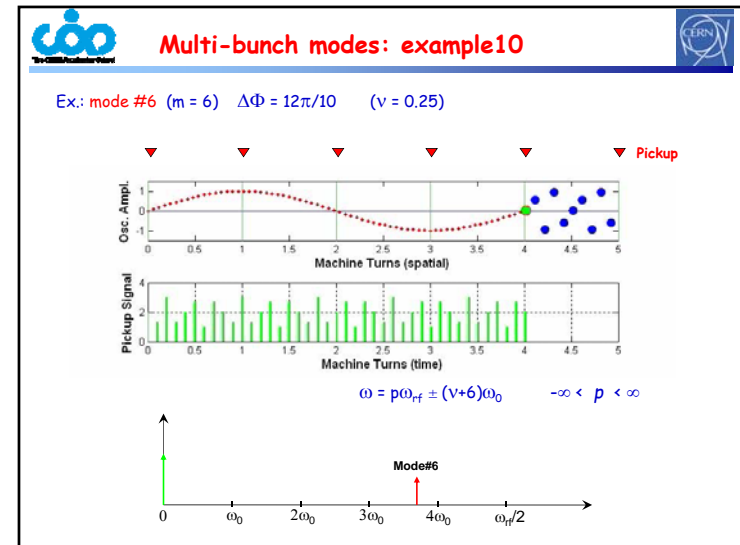
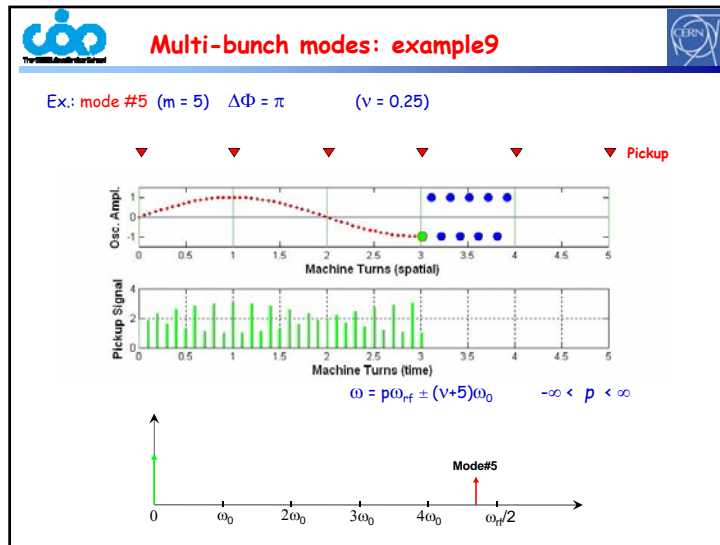
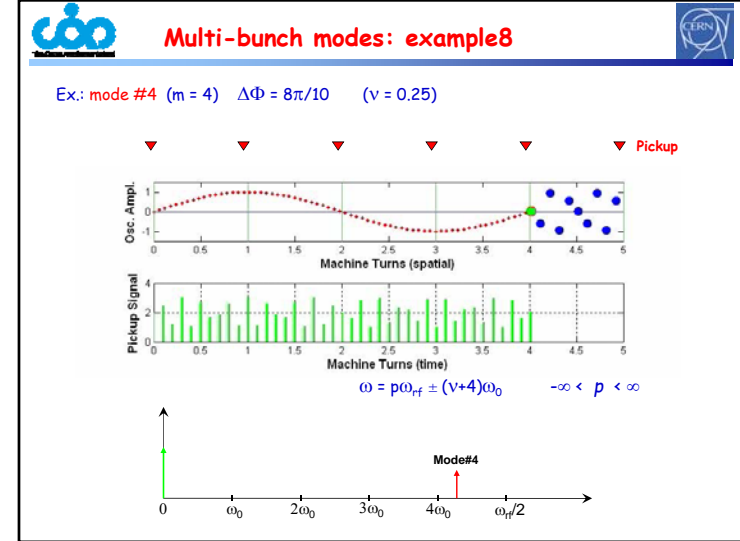
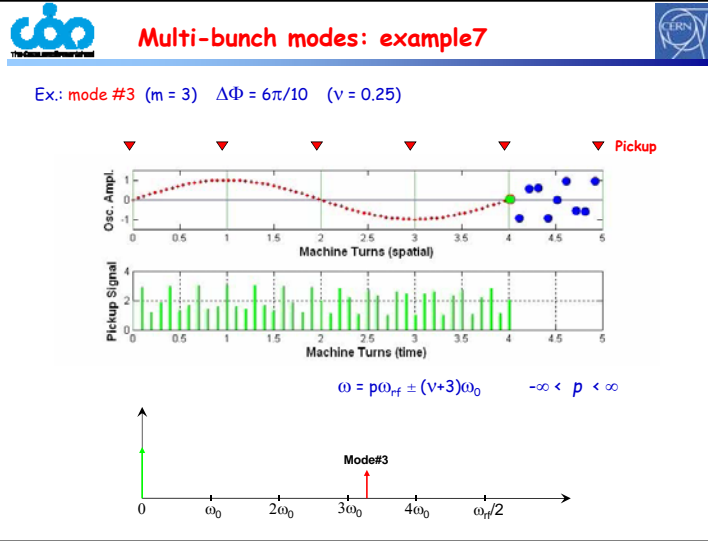
Multi-bunch modes: example2

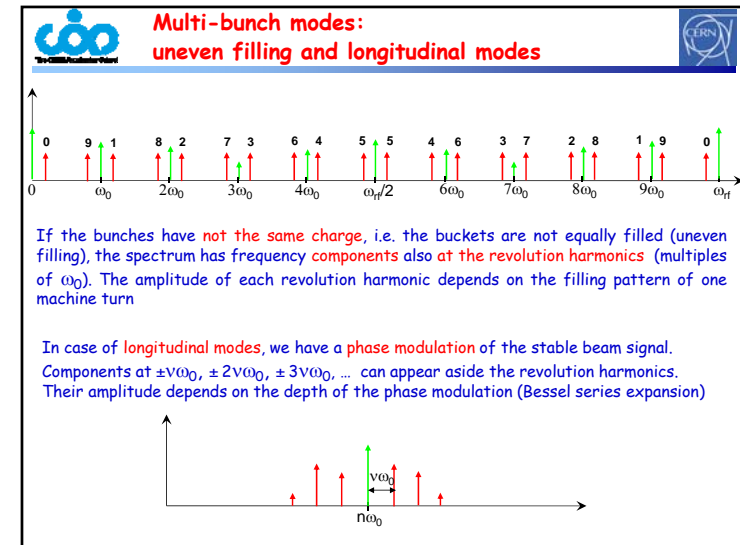
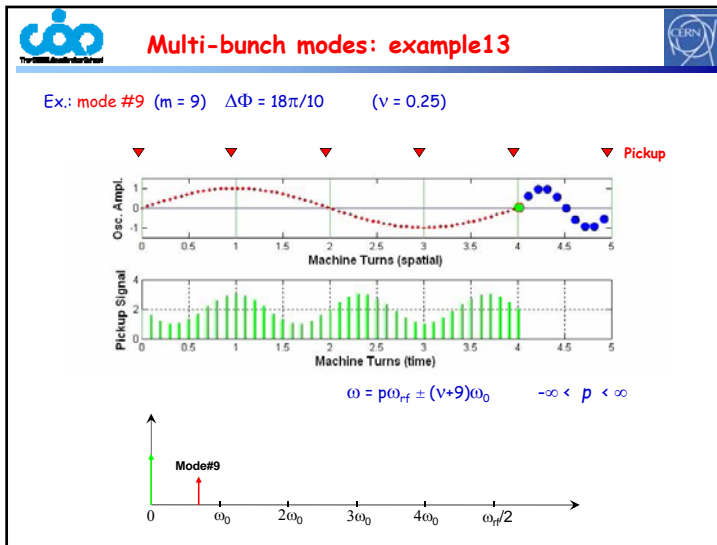
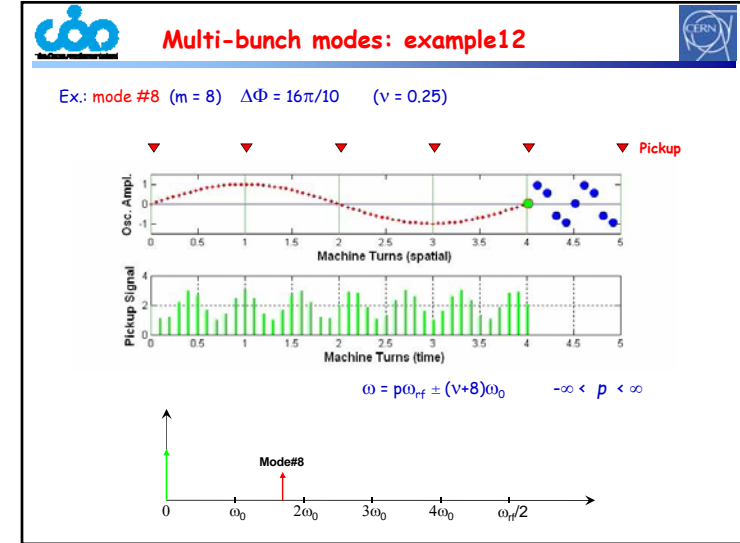
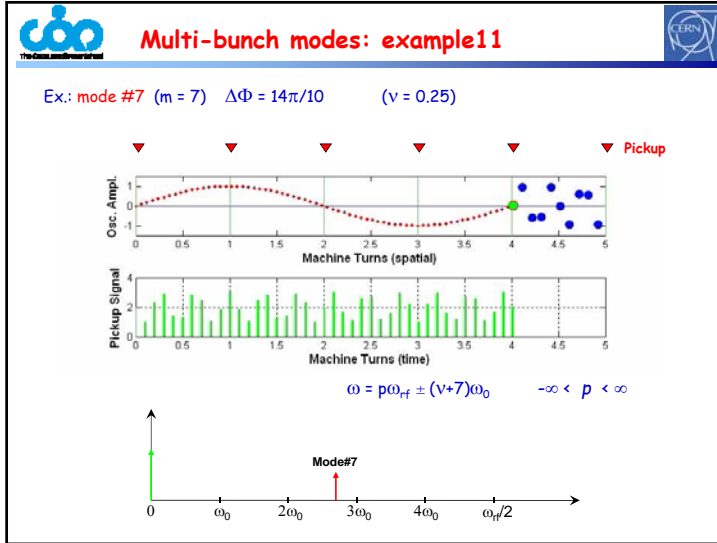
One single unstable bunch oscillating at the tune frequency $\nu\omega_0$: for simplicity we consider a vertical tune $\nu < 1$, ex. $\nu = 0.25$. $M = 1 \rightarrow$ only mode #0 exists

The pickup signal is a sequence of pulses modulated in amplitude with frequency $\nu\omega_0$

Two sidebands at $\pm\nu\omega_0$ appear at each of the revolution harmonics






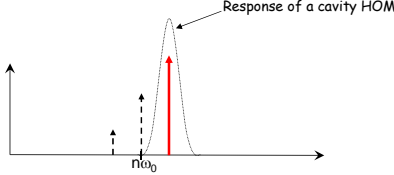


Multi-bunch modes: coupled-bunch instability

One multi-bunch mode can become unstable if one of its sidebands overlaps, for example, with the frequency response of a cavity high order mode (HOM). The HOM couples with the sideband giving rise to a **coupled-bunch instability**, with consequent increase of the sideband amplitude



Synchrotron Radiation Monitor showing the transverse beam shape

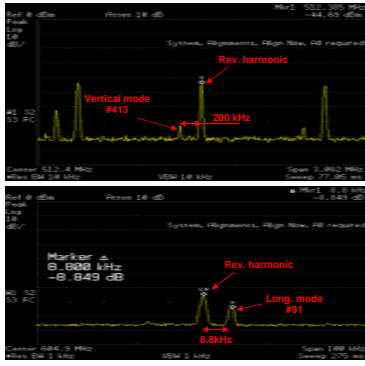


Effects of coupled-bunch instabilities:

- ☹ increase of the transverse beam dimensions
- ☹ increase of the effective emittance
- ☹ beam loss and max current limitation
- ☹ increase of lifetime due to decreased Touschek scattering (dilution of particles)

Real example of multi-bunch modes

ELETTRA Synchrotron: $f_{rf}=499.654$ MHz, bunch spacing ≈ 2 ns, 432 bunches, $f_0 = 1.15$ MHz
 $V_{hor} = 12.30$ (fractional tune frequency = 345 kHz), $V_{vert} = 8.17$ (fractional tune frequency = 200 kHz)
 $V_{long} = 0.0076$ (8.8 kHz)

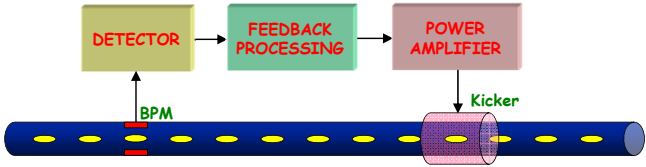
$$\omega = pM\omega_0 \pm (m+v)\omega_0$$


Spectral line at 512.185 MHz
 Lower sideband of $2f_{rf}$, 200 kHz apart from the 443rd revolution harmonic
 → vertical mode #413

Spectral line at 604.914 MHz
 Upper sideband of f_{rf} , 8.8 kHz apart from the 523rd revolution harmonic
 → longitudinal mode #91

Feedback systems

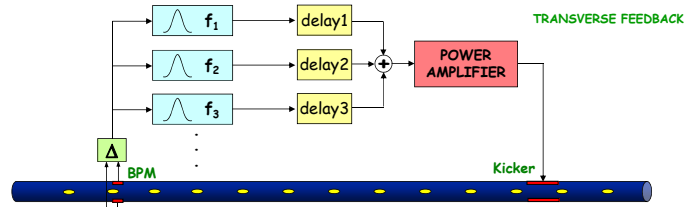
A multi-bunch feedback system detects the instability using one or more Beam Position Monitors (BPM) and acts back on the beam to damp the oscillation through an electromagnetic actuator called **kicker**



BPM and detector measure the beam oscillations
 The feedback processing unit generates the correction signal
 The RF power amplifier amplifies the signal
 The kicker generates the electromagnetic field

Mode-by-mode feedback

A **mode-by-mode** (frequency domain) feedback acts separately on each unstable mode



TRANSVERSE FEEDBACK

An analog electronics generates the position error signal from the BPM buttons
 A number of processing channels working in parallel each dedicated to one of the controlled modes
 The signals are band-pass filtered, phase shifted by an adjustable delay line to produce a negative feedback and recombined

Bunch-by-bunch feedback

A bunch-by-bunch (time domain) feedback individually steers each bunch by applying small electromagnetic kicks every time the bunch passes through the kicker: the result is a damped oscillation lasting several turns

The correction signal for a given bunch is generated based on the motion of the same bunch

Example of implementation using a time division scheme

Every bunch is measured and corrected at every machine turn but, due to the delay of the feedback chain, the correction kick corresponding to a given measurement is applied to the bunch one or more turns later

Damping the oscillation of each bunch is equivalent to damping all multi-bunch modes

Analog bunch-by-bunch feedback: one-BPM feedback

Transverse feedback

The correction signal applied to a given bunch must be proportional to the derivative of the bunch oscillation at the kicker, thus it must be a sampled sinusoid shifted $\pi/2$ with respect to the oscillation of the bunch when it passes through the kicker

The signal from a BPM with the appropriate betatron phase advance with respect to the kicker can be used to generate the correction signal

The detector down converts the high frequency (typically a multiple of the bunch frequency f_{rf}) BPM signal into base-band (range $0 - f_{rf}/2$)

The delay line assures that the signal of a given bunch passing through the feedback chain arrives at the kicker when, after one machine turn, the same bunch passes through it

Analog bunch-by-bunch feedback: two-BPM feedback

Transverse feedback case

The two BPMs can be placed in any ring position with respect to the kicker providing that they are separated by $\pi/2$ in betatron phase

Their signals are combined with variable attenuators in order to provide the required phase of the resulting signal

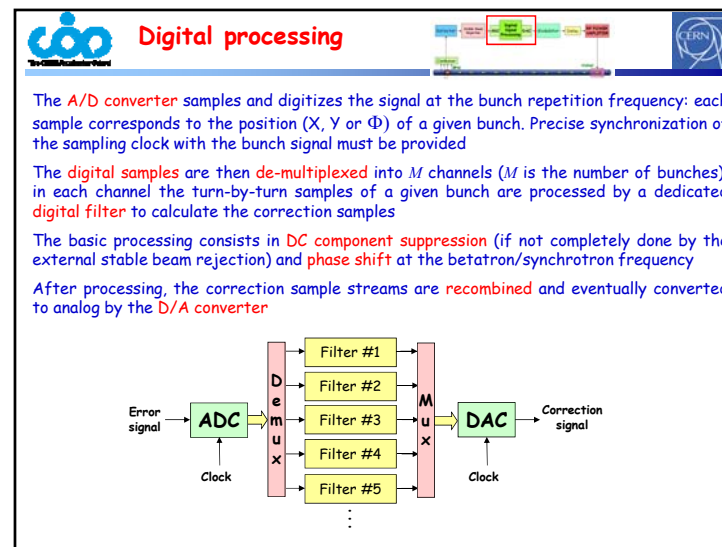
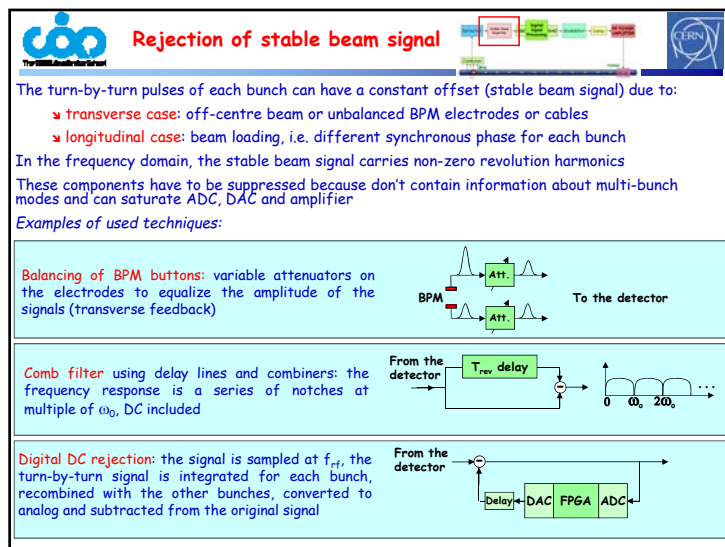
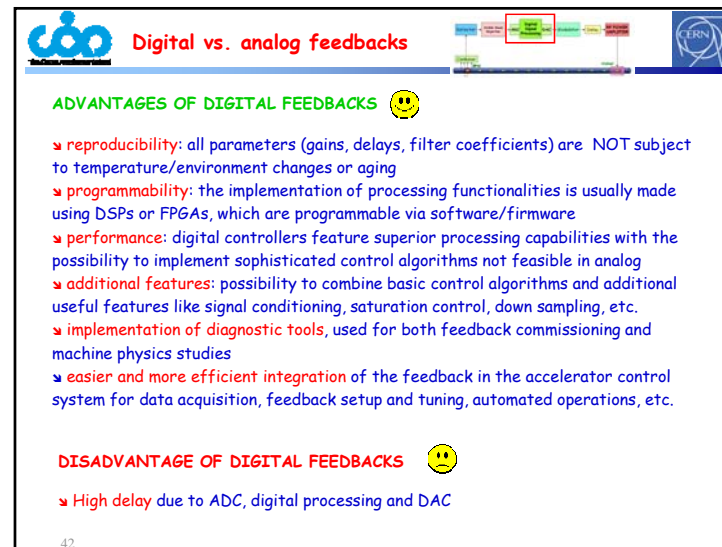
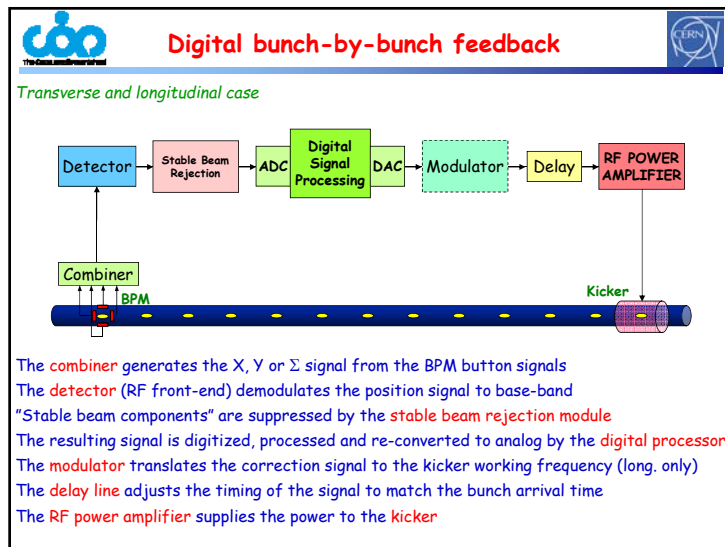
Analog feedback: revolution harmonics suppression

Transverse feedback case

The revolution harmonics (frequency components at multiples of ω_b) are useless components that have to be eliminated in order not to saturate the RF amplifier

This operation is also called "stable beam rejection"

Similar feedback architectures have been used to build the transverse multi-bunch feedback system of a number of light sources: ex. ALS, BessyII, PLS, ANKA, ...



Examples of digital processors

- **PETRA** transverse and longitudinal feedbacks: one ADC, a digital processing electronics made of discrete components (adders, multipliers, shift registers, ...) implementing a FIR filter, and a DAC
- **ALS/PEP-II/DAΦNE** longitudinal feedback (also adopted at **SPEAR**, **Bessy II** and **PLS**): A/D and D/A conversions performed by VXI boards, feedback processing made by DSP boards hosted in a number of VME crates
- **PEP-II** transverse feedback: the digital part, made of two ADCs, a FPGA and a DAC, features a digital delay and integrated diagnostics tools, while the rest of the signal processing is made analogically
- **KEKB** transverse and longitudinal feedbacks: the digital processing unit, made of discrete digital electronics and banks of memories, performs a two tap FIR filter featuring stable beam rejection, phase shift and delay
- **Elettra/SLS** transverse and longitudinal feedbacks: the digital processing unit is made of a VME crate equipped with one ADC, one DAC and six commercial DSP boards (Elettra only) with four microprocessors each

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Examples of digital processors

- **CESR** transverse and longitudinal feedbacks: they employ VME digital processing boards equipped with ADC, DAC, FIFOs and PLDs
- **HERA-p** longitudinal feedback: it is made of a processing chain with two ADCs (for I and Q components), a FPGA and two DACs
- **SPring-8** transverse feedback (also adopted at **TLS**, **KEK Photon Factory** and **Soleil**): fast analog de-multiplexer that distributes analog samples to a number of slower ADC FPGA channels. The correction samples are converted to analog by one DAC
- **ESRF** transverse/longitudinal and **Diamond** transverse feedbacks: commercial product 'Libera Bunch by Bunch' (by Instrumentation Technologies), which features four ADCs sampling the same analog signal opportunely delayed, one FPGA and one DAC
- **HLS** transverse feedback: the digital processor consists of two ADCs, one FPGA and two DACs
- **DAΦNE** transverse and **KEK-Photon-Factory** longitudinal feedbacks: commercial product called 'i6p' (by Dimtel), featuring an ADC-FPGA-DAC chain

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Amplifier and kicker

The **kicker** is the **feedback actuator**. It generates a transverse/longitudinal electromagnetic field that steers the bunches with small kicks as they pass through the kicker. The overall effect is damping of the betatron/synchrotron oscillations

The **amplifier** must provide the necessary RF power to the kicker by amplifying the signal from the DAC (or from the modulator in the case of longitudinal feedbacks)

A **bandwidth** of at least $f_{rf}/2$ is necessary: from $\sim DC$ (all kicks of the same sign) to $\sim f_{rf}/2$ (kicks of alternating signs)

The **bandwidth of amplifier-kicker** must be sufficient to correct each bunch with the appropriate kick without affecting the neighbour bunches. The amplifier-kicker design has to maximize the kick strength while minimizing the cross-talk between corrections given to adjacent bunches

Shunt impedance, ratio between the squared voltage seen by the bunch and twice the power at the kicker input:

$$R = \frac{V^2}{2P_{IN}}$$

Important issue: the group delay of the amplifier must be as constant as possible, i.e. the phase response must be linear, otherwise the feedback efficiency is reduced for some modes and the feedback can even become positive

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Kicker and Amplifier: transverse FB

For the transverse kicker a **stripline** geometry is usually employed

Amplifier and kicker work in the $\sim DC - \sim f_{rf}/2$ frequency range

The ELETTRA/SLS transverse kicker (by Micha Dehler-PSI)

Shunt impedance of the ELETTRA/SLS transverse kickers

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Kicker and Amplifier: longitudinal FB

A "cavity like" kicker is usually preferred
Higher shunt impedance and smaller size
The operating frequency range is typically $f_{rf}/2$ wide and placed on one side of a multiple of f_{rf} :
ex. from $3f_{rf}$ to $3f_{rf}+f_{rf}/2$

A "pass-band" instead of "base-band" device
The base-band signal from the DAC must be modulated, i.e. translated in frequency
A SSB (Single Side Band) amplitude modulation or similar techniques (ex. QPSK) can be adopted

The ELETTRA/SLS longitudinal kicker (by Micha Dehler-PSI)

RF power requirements

τ = feedback damping time
 ω_0 = revolution frequency
 ω_s = synchrotron frequency
 α = momentum compaction factor
 f_{rf} = RF frequency
 R_k = kicker shunt impedance
 E_0 = beam energy
 ϕ_{max} = maximum oscillation amplitude

Max oscillation amplitude

$$P_k = \frac{2}{R_k} \left(\frac{\omega_s E_0 \phi_{max}}{\omega_0 \alpha f_{rf} \tau} \right)^2$$

Longitudinal

$$P_k = \frac{2}{R_k \beta_k} \left(\frac{E_0}{e} \right)^2 \left(\frac{T_0}{\tau} \right)^2 \left(\frac{A_{Bmax}}{\sqrt{\beta_B}} \right)^2$$

Transverse

Required damping time

The required RF power depends on:
 - the strength of the instability
 - the maximum oscillation amplitude

If we switch the feedback on when the oscillation is small, the required power is lower

Obvious from formulae: parameter optimization required depending on application:

- injection damping: large initial amplitude, short "ON" time, then silence
- instability damping: in best case starts from "zero amplitude", always "ON"

→ difficult compromise between power bandwidth, max. amplitude, digital resolution
 (for beams without radiation damping the residual noise from limited detector/digital resolution leads to emittance "heating")

Digital signal processing

M channel/filters each dedicated to one bunch: M is the number of bunches

To damp the bunch oscillations the turn-by-turn kick signal must be the derivative of the bunch position at the kicker: for a given oscillation frequency a $\pi/2$ phase shifted signal must be generated

In determining the real phase shift to perform in each channel, the phase advance between BPM and kicker must be taken into account as well as any additional delay due to the feedback latency (multiple of one machine revolution period)

The digital processing must also reject any residual constant offset (stable beam component) from the bunch signal to avoid DAC saturation

Digital filters can be implemented with FIR (Finite Impulse Response) or IIR (Infinite Impulse Response) structures. Various techniques are used in the design: ex. frequency domain design and model based design

A filter on the full-rate data stream can compensate for amplifier/kicker not-ideal behaviour

Digital filter design: 3-tap FIR filter

The minimum requirements are:

1. DC rejection (coefficients sum = 0)
2. Given amplitude response at the tune frequency
3. Given phase response at the tune frequency

A 3-tap FIR filter can fulfil these requirements: the filter coefficients can be calculated analytically

Z transform of the FIR filter response

In order to have zero amplitude at DC, we must put a "zero" in $z=1$. Another zero in $z=c$ is added to fulfill the phase requirements.
 c can be calculated analytically:

$$H(z) = k(1 - z^{-1})(1 - cz^{-1})$$

$$H(z) = k(1 - (1+c)z^{-1} + cz^{-2}) \quad z = e^{j\omega}$$

$$H(\omega) = k(1 - (1+c)e^{-j\omega} + ce^{-2j\omega})$$

$$e^{-j\omega} = \cos \omega - j \sin \omega, \quad \alpha = \arg(H(\omega))$$

$$\tan(\alpha) = \frac{c(\sin(\omega) - \sin(2\omega)) + \sin(\omega)}{c(\cos(2\omega) - \cos(\omega)) + 1 - \cos(\omega)}$$

$$c = \frac{\tan(\alpha)(1 - \cos(\omega)) - \sin(\omega)}{(\sin(\omega) - \sin(2\omega)) - \tan(\alpha)(\cos(2\omega) - \cos(\omega))}$$

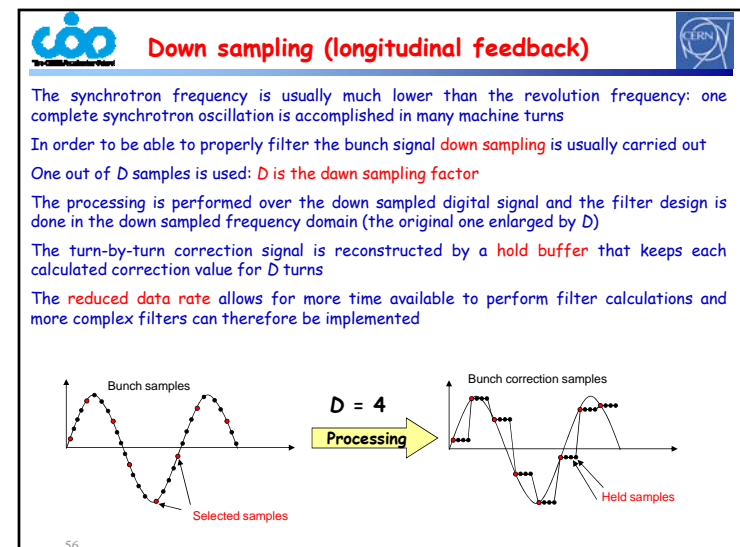
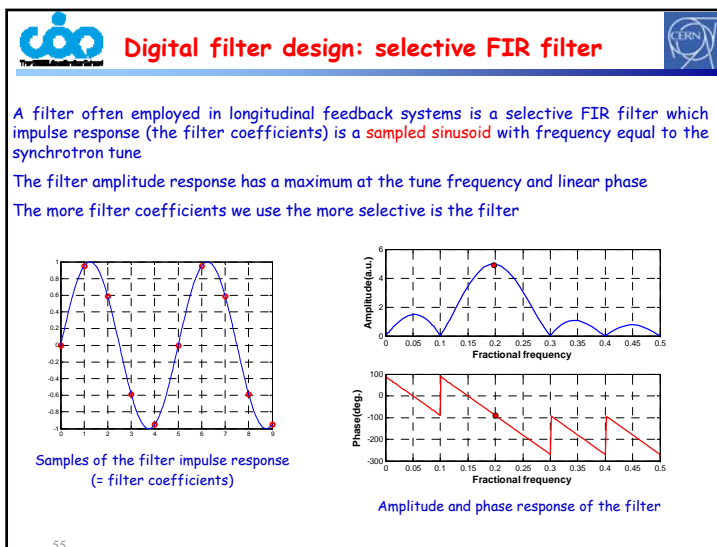
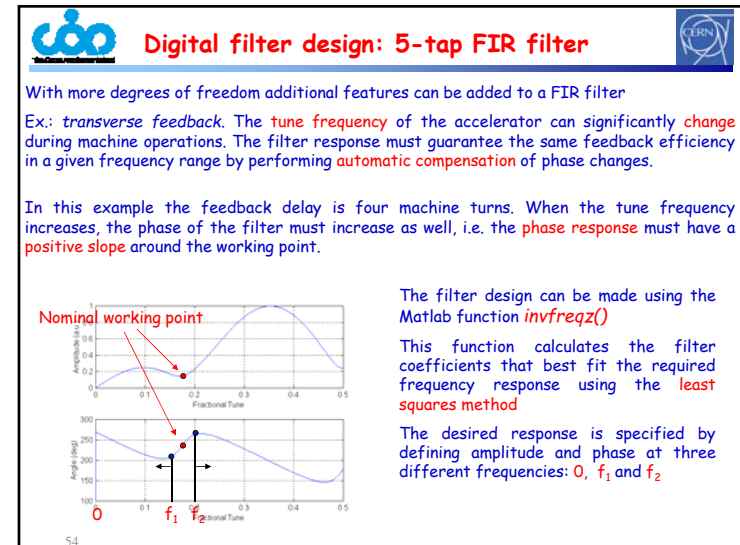
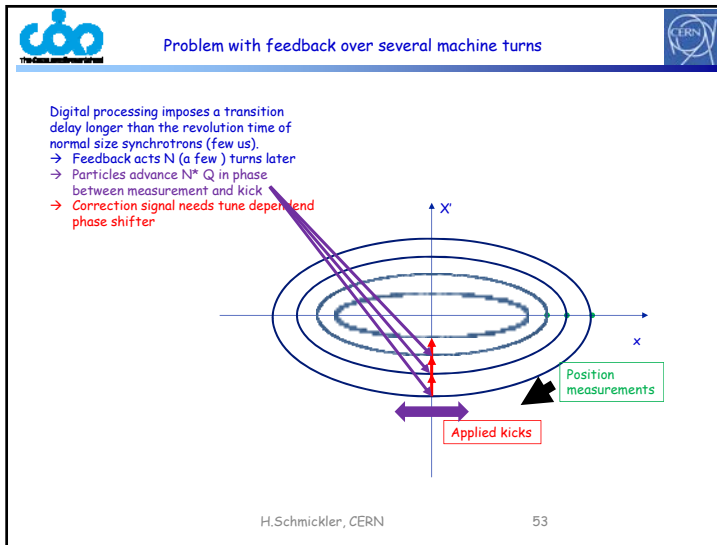
k is determined given the required amplitude response at tune $|H(\omega)|$:

$$k = \frac{|H(\omega)|}{\sqrt{(1 - (1+c)\cos(\omega) + c\cos(2\omega))^2 + ((1+c)\sin(\omega) - c\sin(2\omega))^2}}$$

Example:

- Tune $\omega/2\pi = 0.2$
- Amplitude response at tune $|H(\omega)| = 0.8$
- Phase response at tune $\alpha = 222^\circ$

$$H(z) = -0.63 + 0.49z^{-1} + 0.14z^{-2}$$



Integrated diagnostic tools

A feedback system can implement a number of diagnostic tools useful for commissioning and optimization of the feedback system as well as for machine physics studies:

1. **ADC data recording:** acquisition and recording, in parallel with the feedback operation, of a large number of samples for off-line data analysis
2. **Modification of filter parameters on the fly** with the required timing and even individually for each bunch: switching ON/OFF the feedback, generation of grow/damp transients, optimization of feedback performance, ...
3. **Injection of externally generated digital samples:** for the excitation of single/multi bunches

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Diagnostic tools: excitation of individual bunches

The feedback loop is switched off for one or more selected bunches and the excitation is injected in place of the correction signal. Excitations can be:

- white (or pink) noise
- sinusoids

In this example two bunches are vertically excited with pink noise in a range of frequencies centered around the tune, while the feedback is applied on the other bunches. The spectrum of one excited bunch reveals a peak at the tune frequency

This technique is used to measure the betatron tune with almost no deterioration of the beam quality

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Diagnostic tools: multi-bunch excitation

Interesting measurements can be performed by adding pre-defined signals in the output of the digital processor

1. By injecting a sinusoid at a given frequency, the corresponding beam multi-bunch mode can be excited to test the performance of the feedback in damping that mode
2. By injecting an appropriate signal and recording the ADC data with filter coefficients set to zero, the beam transfer function can be calculated
3. By injecting an appropriate signal and recording the ADC data with filter coefficients set to the nominal values, the closed loop transfer function can be determined

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Diagnostic tools: transient generation

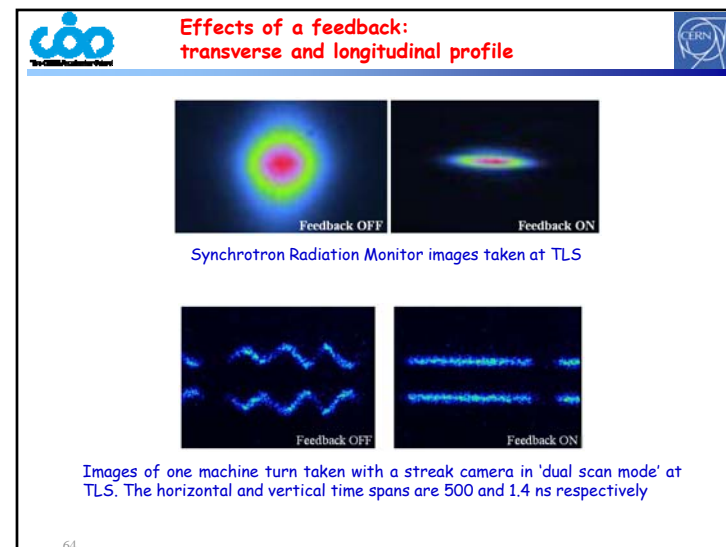
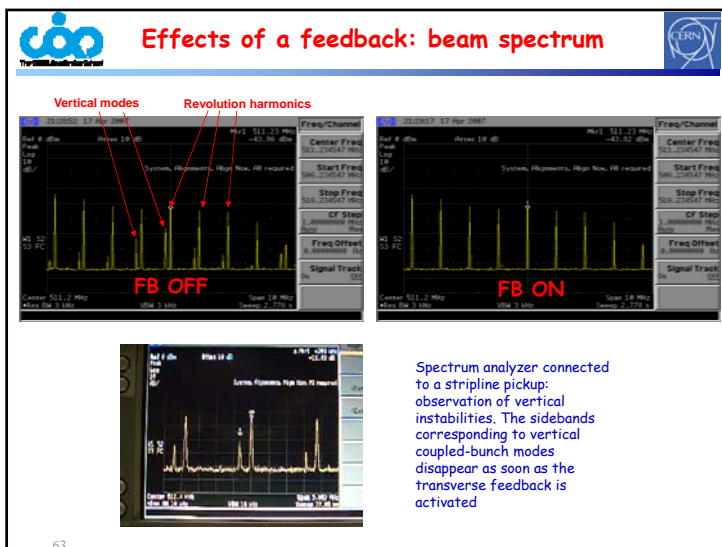
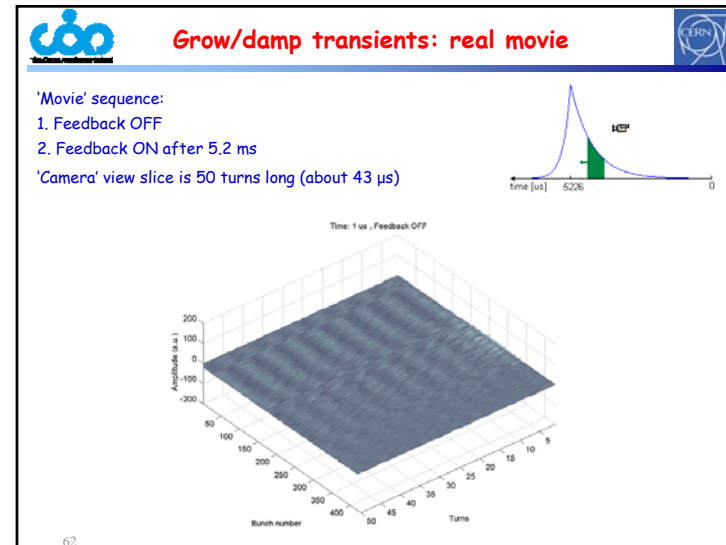
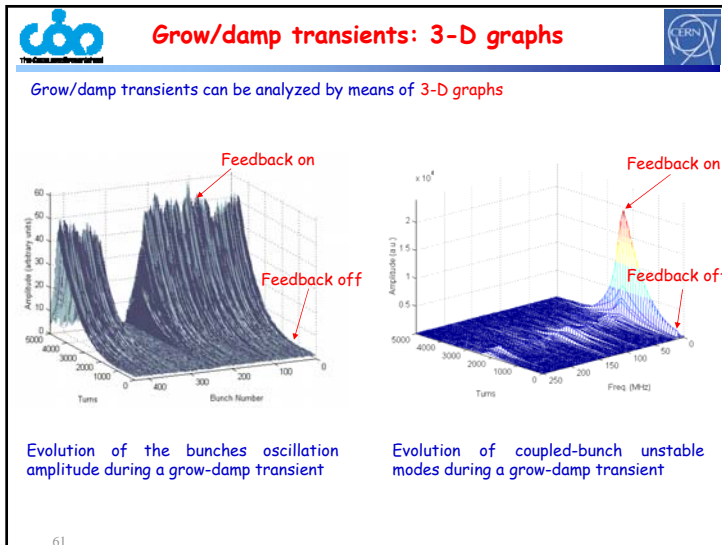
A powerful diagnostic application is the generation of transients. Transients can be generated by changing the filter coefficients accordingly to a predefined timing and by concurrently recording the oscillations of the bunches

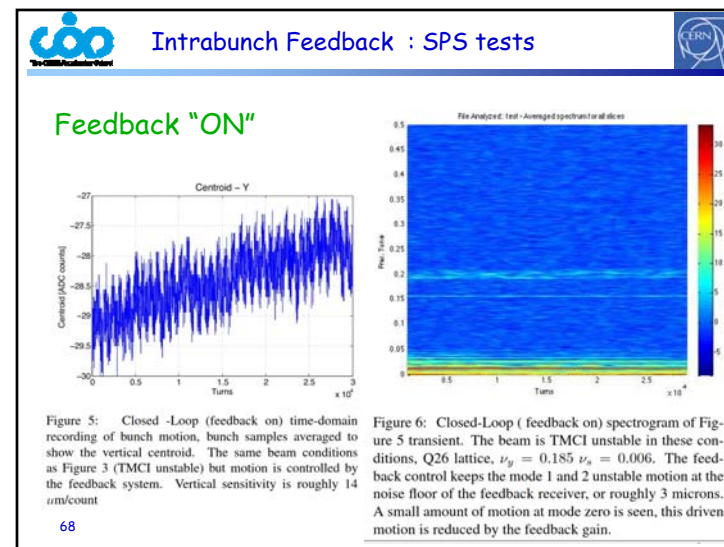
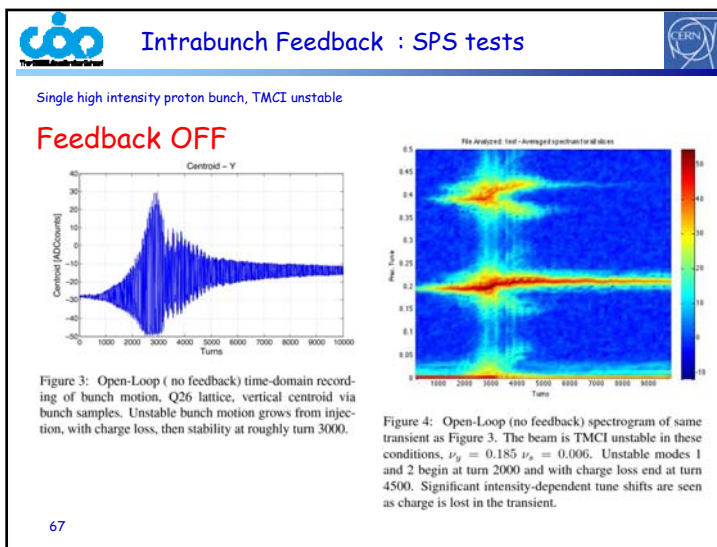
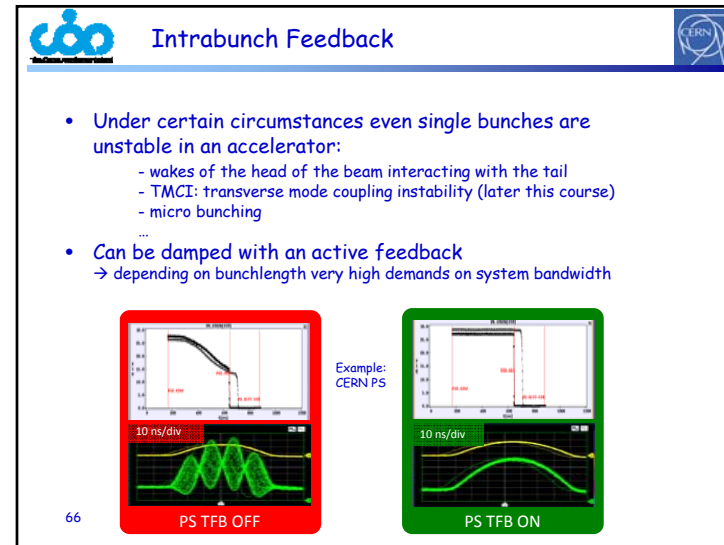
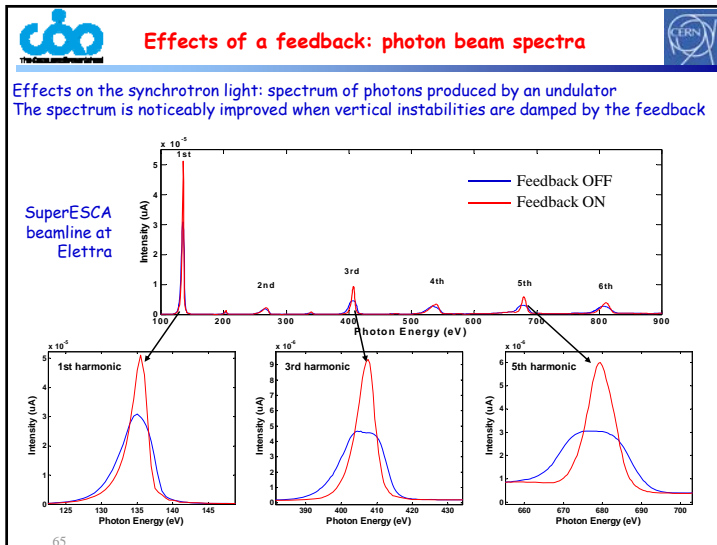
Different types of transients can be generated, damping times and growth rates can be calculated by exponential fitting of the transients:

1. Constant multi-bunch oscillation → FB on: damping transient
2. FB on → FB off → FB on: grow/damp transient
3. Stable beam → positive FB on (anti-damping) → FB off: natural damping transient

...

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Without derivation: emittance growth from injection errors

$$\frac{\varepsilon}{\varepsilon_0} = 1 + \frac{1}{2} \frac{\Delta x^2 + (\beta \Delta x' + \alpha \Delta x)^2}{\beta \varepsilon_0} \left(\frac{1}{1 + \tau_{DC} / \tau_d} \right)^2$$

ε_0 : beam emittance before injection
 ε : beam emittance after damped injection oscillation
 τ_{DC} : damping time of active feedback system
 τ_d : filamentation time
 Δx : position error at injection
 $\Delta x'$: angle error at injection
 α, β : twiss parameters at injection point

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Even after perfect beam steering not all bunches can be injected into the LHC without position and/or angle error due to pulse-shape of kicker magnets

→ unwanted emittance growth

→ loss in luminosity/bunch instabilities

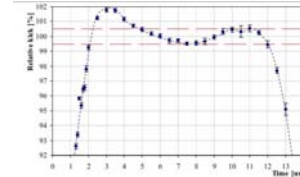


Figure 1: Ripple of SPS LSS6 extraction kickers (LHC beam 1) measured with beam.

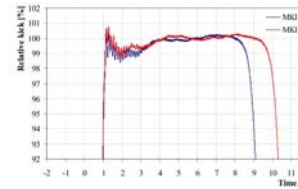


Figure 3: Measured LHC MKI injection kicker waveform, for different magnets and different pulse lengths.

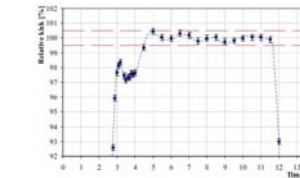


Figure 2: Ripple of SPS LSS4 extraction kickers (LHC beam 2) measured with beam.

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Simulated emittance growth on various bunches after injection into the LHC:

tolerance is 2.5% emittance growth

only a few bunches above 1% emittance growth

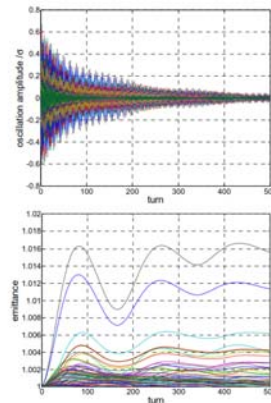


Figure 5: Evolution of single bunch oscillation amplitudes (top) and emittance increases (bottom) as a function of time after injection, using the measured MKI kick.

Figures on past three slides from:
 Emittance growth at the LHC injection from SPS and LHC kicker ripple, G. Kotzian et al, EPAC 2008

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→ Marco Lonza (Elettra) for his splendid animations

→ Many papers about coupled-bunch instabilities and multi-bunch feedback systems (PETRA, KEK, SPring-8, DaΦne, ALS, PEP-II, SPEAR, ESRF, Elettra, SLS, CESR, HERA, HLS, DESY, PLS, BessyII, SRRC, SPS, LHC, ...)

→ Intrabunch feedback at the SPS:
 Wideband vertical intra-bunch feedback at the SPS
 J. Fox (SLAC), W. Hofle (CERN) et al., proceedings of IPAC 2015, Richmond USA

→ Injection damping: Verena Kain; CAS in Erice 2017

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Appendix

1. 3 slides : Power requirements for transverse dampers

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power requirements: transverse feedback

The transverse motion of a bunch of particles not subject to damping or excitation can be described as a pseudo-harmonic oscillation with amplitude proportional to the square root of the β -function

$$x(s) = a \sqrt{\beta(s)} \cos \varphi(s), \quad \text{where} \quad \varphi(s) = \int_0^s \frac{ds}{\beta(s)}$$

The derivative of the position, i.e. the angle of the trajectory is:

$$x' = -\frac{a}{\sqrt{\beta}} \sin \varphi + \frac{a\beta'}{2\sqrt{\beta}} \cos \varphi, \quad \text{with} \quad \varphi' = \frac{1}{\beta}$$

By introducing $\alpha = -\frac{\beta'}{2}$ we can write: $x' = \frac{a}{\sqrt{\beta}} \sqrt{1+\alpha^2} \sin(\varphi + \arctan \alpha)$

At the coordinate s_k , the electromagnetic field of the kicker deflects the particle bunch which varies its angle by k : as a consequence the bunch starts another oscillation

$$\begin{cases} x(s_k) = x_i(s_k) \\ x'(s_k) = x'_i(s_k) + k \end{cases}$$

By introducing $A = a\sqrt{\beta}$, $A_i = a_i\sqrt{\beta}$ the two-equation two-unknown-variables system becomes:

$$\begin{cases} A \cos \varphi = A_i \cos \varphi_i \\ A \frac{\sqrt{1+\alpha^2}}{\beta} \sin(\varphi + \arctan \alpha) = A_i \frac{\sqrt{1+\alpha_i^2}}{\beta} \sin(\varphi_i + \arctan \alpha_i) + k \end{cases}$$

The solution of the system gives amplitude and phase of the new oscillation:

$$\begin{cases} A_i = \sqrt{(A \sin \varphi - k\beta)^2 + A^2 \cos^2 \varphi} \\ \varphi_i = \arccos\left(\frac{A}{A_i} \cos \varphi\right) \end{cases}$$

power requirements: transverse feedback

From $A_i = \sqrt{(A \sin \varphi - k\beta)^2 + A^2 \cos^2 \varphi}$ if the kick is small ($k \ll \frac{A}{\beta}$) then $\frac{\Delta A}{A} = \frac{A - A_i}{A} \approx \frac{\beta}{A} k \sin \varphi$

In the linear feedback case, i.e. when the turn-by-turn kick signal is a sampled sinusoid proportional to the bunch oscillation amplitude, in order to maximize the damping rate the kick signal must be in-phase with $\sin \varphi$, that is in quadrature with the bunch oscillation

$$k = g \frac{A}{\beta} \sin \varphi \quad \text{with} \quad 0 < g < 1$$

The optimal gain g_{opt} is determined by the maximum kick value k_{max} that the kicker is able to generate. The feedback gain must be set so that k_{max} is generated when the oscillation amplitude A at the kicker location is maximum:

$$g_{opt} = \frac{k_{max}}{A_{max} \beta} \quad \text{Therefore} \quad k = \frac{k_{max}}{A_{max}} A \sin \varphi$$

For small kicks $\frac{\Delta A}{A} \approx \frac{k_{max}}{A_{max} \beta} \beta \sin^2 \varphi$ the relative amplitude decrease is monotonic and its average is: $\left\langle \frac{\Delta A}{A} \right\rangle \approx \frac{\beta k_{max}}{2 A_{max}}$

The average relative decrease is therefore constant, which means that, in average, the amplitude decrease is exponential with time constant τ (damping time) given by:

$$\frac{1}{\tau} = \left\langle \frac{\Delta A}{A} \right\rangle \frac{1}{T_0} = \frac{\beta k_{max}}{2 A_{max} T_0} \quad \text{where } T_0 \text{ is the revolution period.}$$

By referring to the oscillation at the BPM location: $\frac{1}{\tau} = \frac{k_{max}}{2 T_0 A_{BPM}} \sqrt{\beta_s \beta_b}$ A_{BPM} is the max oscillation amplitude at the BPM

power requirements: transverse feedback

For relativistic particles, the change of the transverse momentum p of the bunch passing through the kicker can be expressed by:

$$\Delta p = \frac{e}{c} V_{\perp} \quad \text{where} \quad V_{\perp} = \int_0^L (\vec{E} + c \times \vec{B})_{\perp} dz \quad \text{is the kick voltage and} \quad p = \frac{E_b}{c}$$

e = electron charge, c = light speed, \vec{E}, \vec{B} = fields in the kicker, L = length of the kicker, E_b = beam energy

V_{\perp} can be derived from the definition of kicker shunt impedance: $R_s = \frac{V_{\perp}^2}{2 P_s}$

The max deflection angle in the kicker is given by:

$$k_{max} = \frac{\Delta p}{p} = e \frac{V_{\perp}}{E_b} = \left(\frac{e}{E_b} \right) \sqrt{2 P_s R_s}$$

From the previous equations we can obtain the power required to damp the bunch oscillation with time constant τ :

$$P_k = \frac{2}{R_s \beta_s} \left(\frac{E_b}{e} \right)^2 \left(\frac{T_0}{\tau} \right)^2 \left(\frac{A_{BPM}}{\sqrt{\beta_s}} \right)^2$$