

# Landau Damping

## part 2

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GSI Darmstadt, Germany

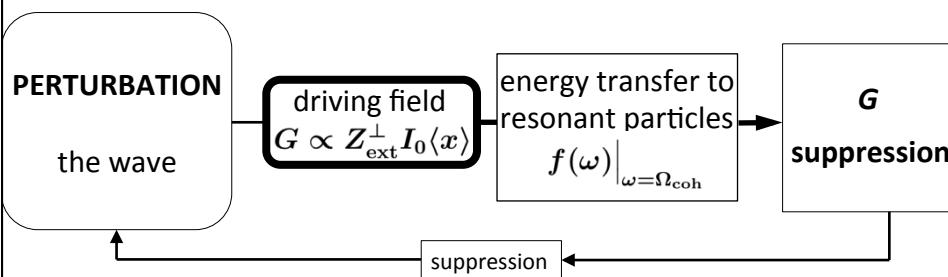
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## Landau Damping

Incomplete (!) mechanism of Landau Damping in beams  
for the end of the first part



Main ingredients of Landau damping:

- ✓ wave-particle collisionless interaction: Impedance driving field
- ✓ energy transfer: the wave  $\leftrightarrow$  the (few) resonant particles

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## Existence of Landau damping

In any accelerator, there are many  $\text{Re}(Z)$  sources

In any beam, there are many unsuppressed eigenmodes

$$\text{Im}(\Delta Q_{\text{coh}}) = \frac{\lambda_0 r_p}{\gamma Q_0} \frac{\text{Re}(Z^\perp)}{Z_0/R}$$

driving dipole impedance here

Still, the beams are often stable without an active mitigation

There must be a fundamental damping mechanism in beams

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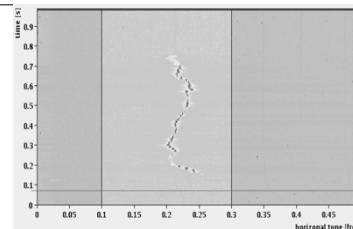


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## Existence of Landau damping

Additionally to  $\text{Re}(Z)$ , deliberate excitation is often applied  
(tune measurements, optics control, ...)

Energy is directly transferred to the beam,  
mostly at the beam resonant frequencies



Tune measurements at PS,  
kick every 10 ms,  
M.Gasior, et al, CERN

The beams are stable and absorb some energy

There must be a fundamental damping mechanism in beams

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# Damping

Basic consideration of a collective mode stability

$$\Delta\Omega = \Delta\Omega_{\text{Re}} + i\gamma_{\text{drive}} + i\gamma_{\text{damping}}$$

change the parameters and  
the source of the  
driving mechanism

use and enhance the  
intrinsic damping  
mechanism

|   |                              |
|---|------------------------------|
| $\gamma_{\text{drive}} + \gamma_{\text{damping}} > 0$ | Instability                  |
| $\gamma_{\text{drive}} + \gamma_{\text{damping}} < 0$ | Stabilized mode              |
| $\gamma_{\text{drive}} > 0$                           | Driven (unsuppressed) mode   |
| $\gamma_{\text{drive}} < 0$                           | Mode suppressed by its drive |

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# Another Leading Actor: Decoherence

K.Y.Ng, Physics of Intensity Dependent Beam Instabilities, 2006  
A.Hofmann, Proc. CAS 2003, CERN-2006-002  
A.Chao, Phys. Coll. Beam Instab. in High Energy Acc. 1993  
A.W. Chao, et al, SSC-N-360 (1987)

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## Decoherence

Collective beam oscillations after a short (one turn or shorter) kick

- Usually the beam displacement is comparable to the beam size
- In reality, signals can be very complicated
- A common diagnostics
- Decoherence is a process affected by many effects and mechanisms**

A bunched beam  $\text{Ar}^{18+}$  in SIS18. Transverse signal after a kick

V.Kornilov, O.Boine-F., PRSTAB 15, 114201 (2012)

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## Pulse Response

8 particles with different frequencies

Betatron oscillations: frequency spread

$$\delta\omega = Q_0 \xi \omega_0 \delta_p$$

$$g(t) = \frac{\langle x(t) \rangle}{x_0}$$

$$g(t) = \int f(\omega) \cos(\omega t) d\omega$$

$$g(t) = \text{Fourier}^{-1}\{R(\omega)\} = \frac{1}{2\pi} \int R(\omega) e^{-i\omega t} d\omega$$

The Pulse Response is the Fourier image of BTF

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## Phase-Mixing (Filamentation)

Phase-mixing of non-correlated particles with a tune spread

- Beam offset oscillations decay
- Beam size increases (blow-up)

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## Phase-Mixing, Coasting Beam

Gaussian Distribution

$Q_0=10.3, \xi=1, \delta_p=0.5 \times 10^{-3}$

Lorentz Distribution

$Q_0=10.3, \xi=1, \delta_p=0.5 \times 10^{-3}$

$$f(\omega_\beta) = \frac{1}{\sqrt{2\pi}\delta\omega} e^{-\omega_\beta^2/2\delta\omega^2}$$

$$g(t) = e^{-\delta\omega^2 t^2/2} \cos(\omega_\beta t)$$

$$f(\omega_\beta) = \frac{1}{\pi \delta\omega} \frac{1}{1 + \omega_\beta^2/\delta\omega^2}$$

$$g(t) = e^{-\delta\omega t} \cos(\omega_\beta t)$$

This is the case without any collective interactions:  
phase-mixing of non-correlated particles

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## Phase-Mixing, Bunched Beam

Due to the particle synchrotron motion, the initial positions return after a synchrotron period

$$A(N) = A_0 \exp\left\{-2\left(\frac{\xi Q_0 \delta_p}{Q_s} \sin(\pi Q_s N)\right)^2\right\}$$

In literature, named as "decoherence" and "recoherence"

$\xi=0.5$   
 $\xi=1.4$   
 $N_s=100$   
 $Q_s=0.01$

bunch offset /  $\sigma_0$

time (turns)

dashed lines: amplitude  
solid lines: offset

In reality, the decoherence is very different (other effects)

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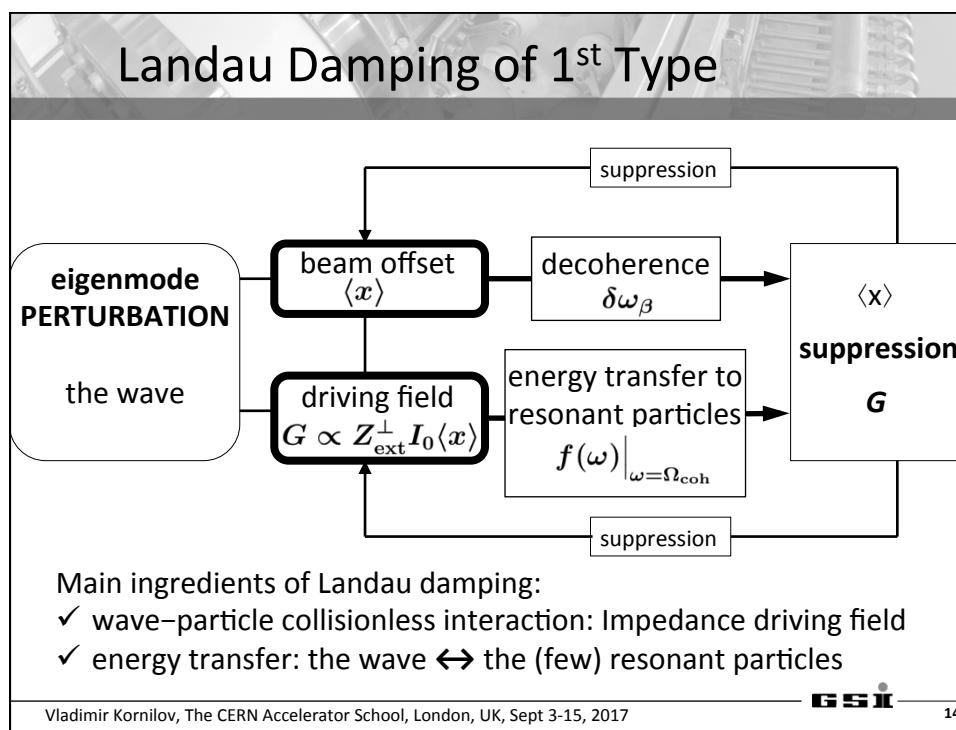
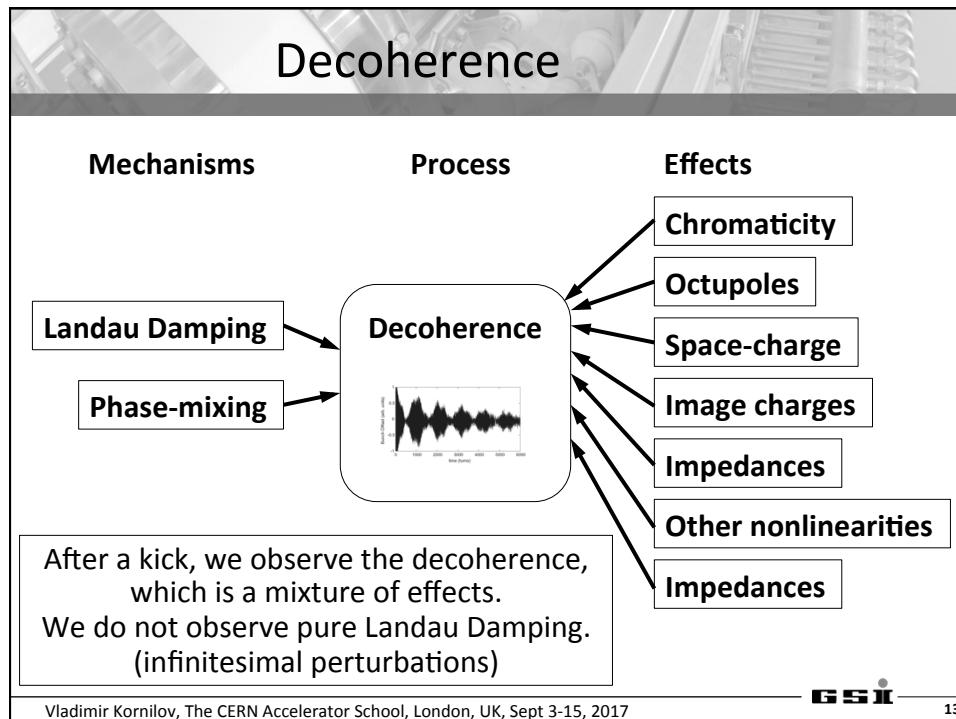
## Decoherence

| Mechanisms                                   | Process                | Effects   |
|--|------------------------|---|
| <b>Landau Damping</b><br><b>Phase-mixing</b> | <b>Decoherence</b><br> | <b>Chromaticity</b><br><b>Octupoles</b><br><b>Space-charge</b><br><b>Image charges</b><br><b>Impedances</b><br><b>Other nonlinearities</b><br><b>Impedances</b> |

K.Y.Ng, Physics of Intensity Dependent Beam Instabilities, 2006  
A.Hofmann, Proc. CAS 2003, CERN-2006-002  
A.W. Chao, et al, SSC-N-360 (1987)  
V.Kornilov, O.Boine-F., PRSTAB 15, 114201 (2012)  
I.Karpov, V.Kornilov, O.Boine-F., PRAB 19, 124201 (2016)

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# Landau Damping: the wave & the tune spread

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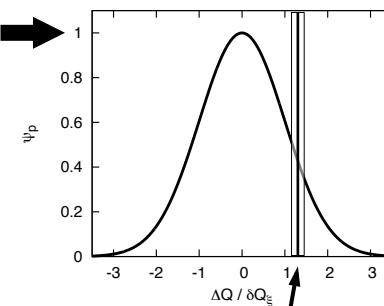
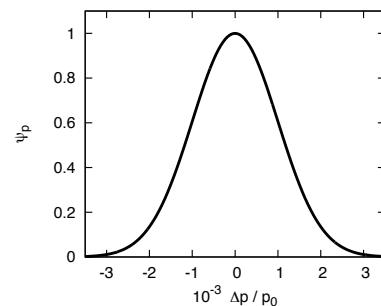


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## Landau Damping

Momentum spread translates into tune spread

$$\delta Q_\xi = |\eta(n - Q_0) + Q_0 \xi| \delta p$$



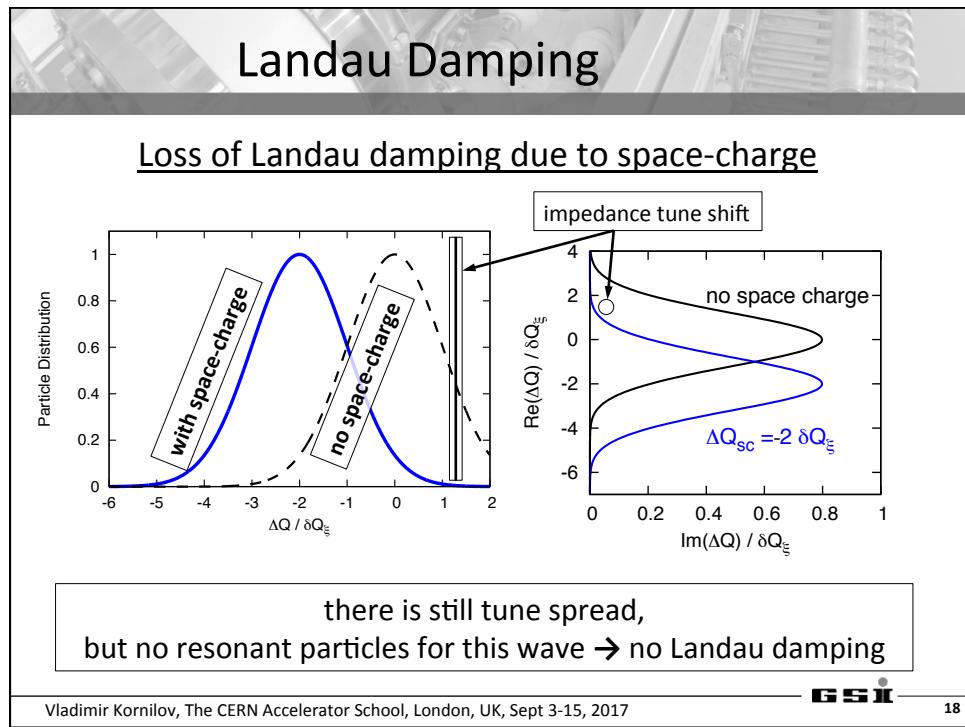
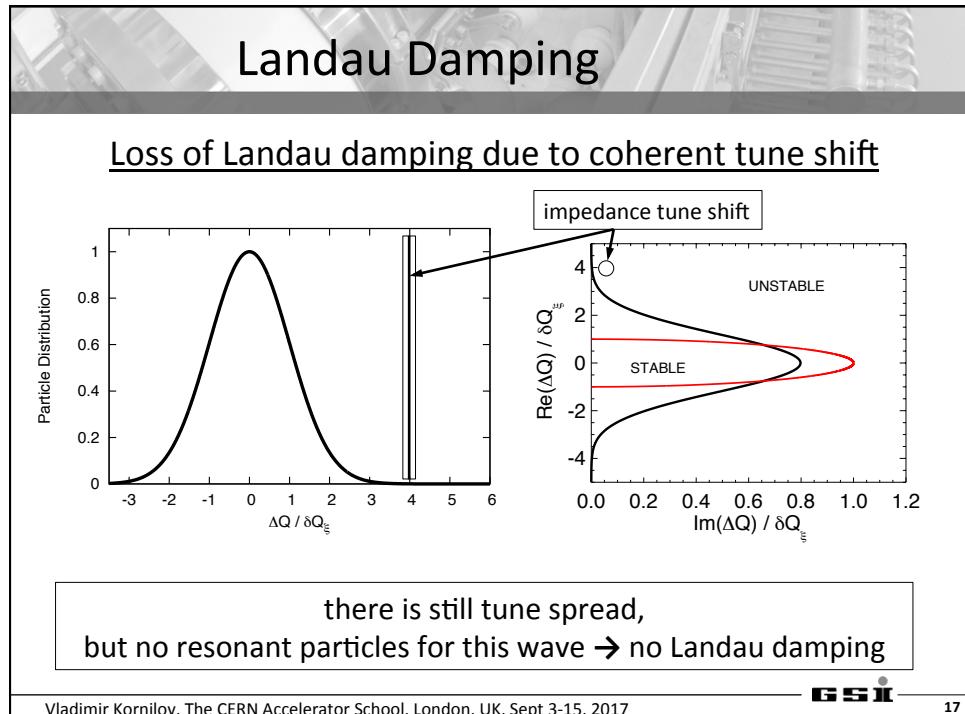
$$\Delta Q_{coh} \omega_0 \int \frac{f(\omega) d\omega}{\omega - \Omega} = 1$$

resonant particles from both sides contribute to the energy transfer, thus  $f(\omega)$

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# Landau Damping

## Loss of Landau damping due to space-charge

Coherent dipole oscillations of four O<sup>8+</sup> bunches in the SIS18 of GSI Darmstadt at the injection energy 11.4 MeV/u

O.Boine-F., T.Shukla, PRSTAB **8**, 034201 (2005)

there is still tune spread,  
but no resonant particles for this wave → no Landau damping

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# Landau Damping

**Remark 1**

Some analytical models or simulations predict  
**Landau antidamping:**  
 unstable oscillations for zero  
 or negative impedance

No antidamping for Gaussian-like distributions!

D.Pestrikov, NIM A 562, 65 (2006)  
 V.Kornilov, et al, PRSTAB 11, 014201 (2008)  
 A.Burov, V.Lebedev, PRSTAB 12, 034201 (2009)

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## Landau Damping

Remark 2

Head-tail modes in bunches:

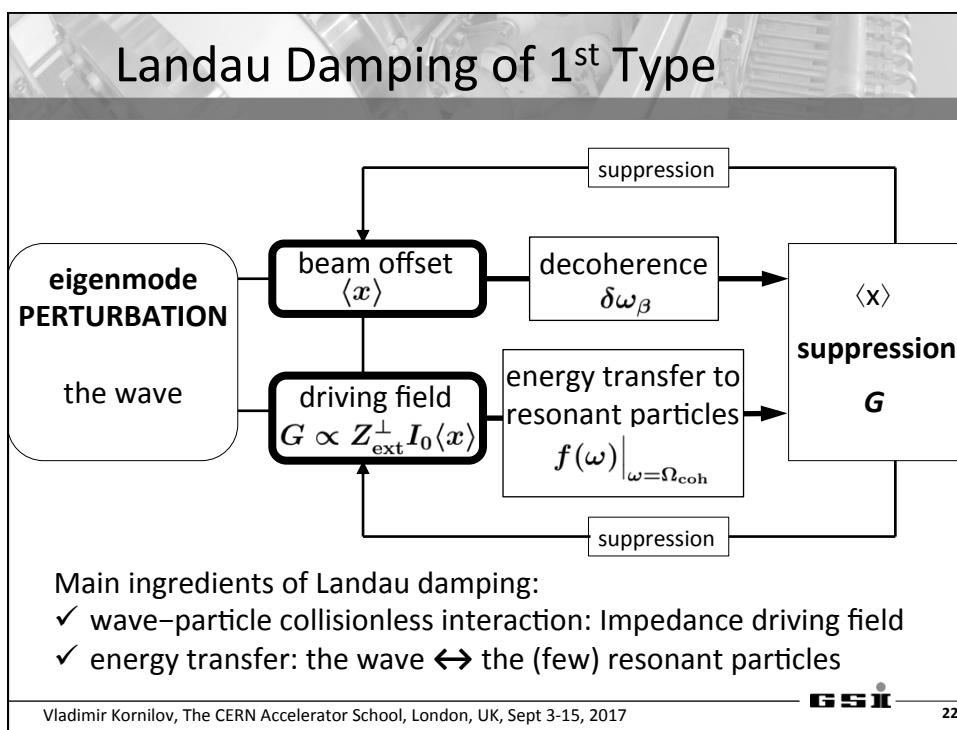
- there is a **tune spread**  
(due to chromaticity  $\xi$ )
- the coherent frequency overlaps  
with the incoherent spectrum

still, there is  
**NO Landau Damping!**

a tune spread does not automatically mean Landau Damping

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## Landau Damping in Beams of 2<sup>nd</sup> type

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## Landau Damping of 2<sup>nd</sup> type

Different situation from  $\xi + \delta p$ :  
Tune spread due to amplitude-dependent tune shifts

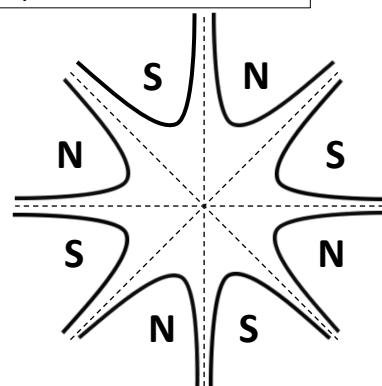
For example, an octupole magnet:

$$\begin{aligned} B_x &= O_3(3x^2y - y^3) \\ B_y &= O_3(x^3 - 3xy^2) \end{aligned}$$

First-order frequency shift of  
the anharmonic oscillations

$$\begin{aligned} \ddot{x} + \omega_0^2 x &= \varepsilon x^3 \\ \omega &\approx \omega_0 - \frac{3\varepsilon A^2}{8\omega_0} \end{aligned}$$

Amplitude-dependent betatron tune shifts



Schematic yoke profile  
of an octupole magnet

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## Decoherence

Phase-Mixing is very different from the chromaticity case

- No recoherence
- Kick amplitude dependent
- Analytical model for:

$$Q(a) = Q_0 - \mu \left( \frac{a}{\sigma_x} \right)^2$$

$$A(N) = \frac{A_0}{1 + \theta^2} \exp \left\{ -\frac{A_0^2}{2\sigma_x^2} \frac{\theta^2}{1 + \theta^2} \right\}$$

$$\theta = 2\pi\mu N$$

A.W. Chao, et al, SSC-N-360 (1987)  
V.Kornilov, O.Boine-F., PRSTAB 15, 114201 (2012)  
I.Karpov, V.Kornilov, O.Boine-F., PRAB 19, 124201 (2016)

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## Landau Damping of 2<sup>nd</sup> type

Tune spread due to amplitude-dependent tune shifts

Amplitude-dependent betatron tune shifts in a lattice:

$$\Delta Q_x^{\text{oct}} = \left( \int \frac{K_3 \beta_x^2}{16\pi} ds \right) J_x - \left( \int \frac{K_3 \beta_x \beta_y}{8\pi} ds \right) J_y$$

$$K_3 = \frac{6O_3}{B\rho}$$

$$x(s) = \sqrt{2J_x \beta_x(s)} \cos(\phi_x)$$

Every particle has a different amplitude ( $J_x, J_y$ )  
→ tune spread → Landau damping?

The resonant particles drift away in tune from the resonance as they get excited

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## Landau Damping of 2<sup>nd</sup> type

Particle excitation for amplitude-dependent tune shifts

Once the particle is driven away from the resonance, the energy is transferred back to the wave

We already guess: the distribution slope ( $df/da$ ) might be involved

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## Landau Damping of 2<sup>nd</sup> type

The dispersion relation

$$\Delta Q_{\text{coh}} \int \frac{1}{\Delta Q_{\text{ex}} - \Omega/\omega_0} J_x \frac{\partial f}{\partial J_x} dJ_x dJ_y = 1$$

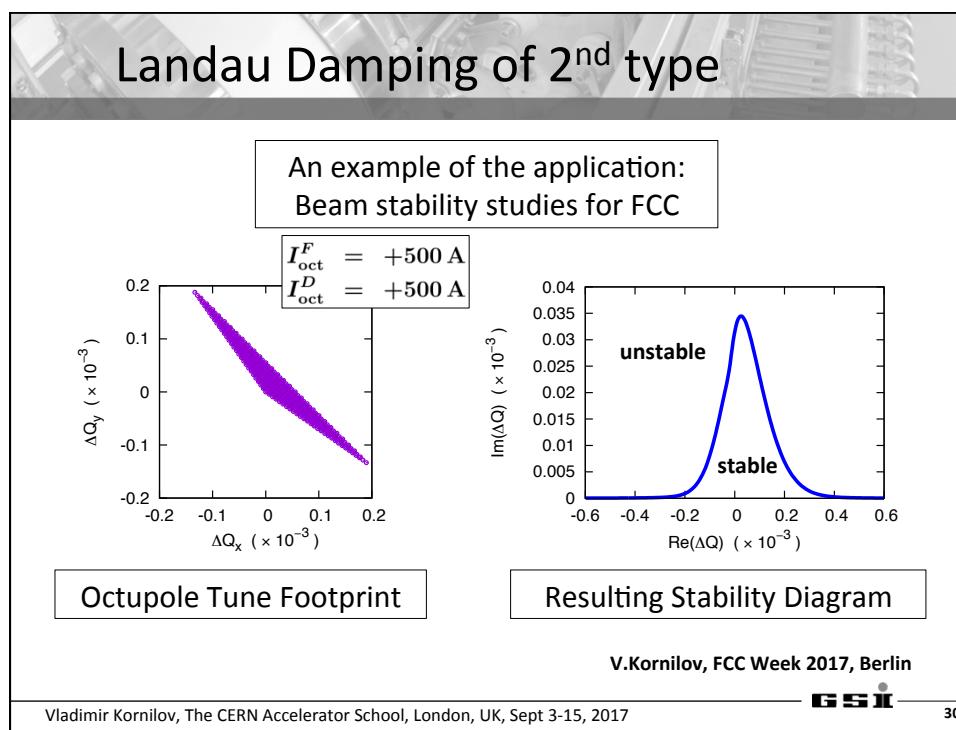
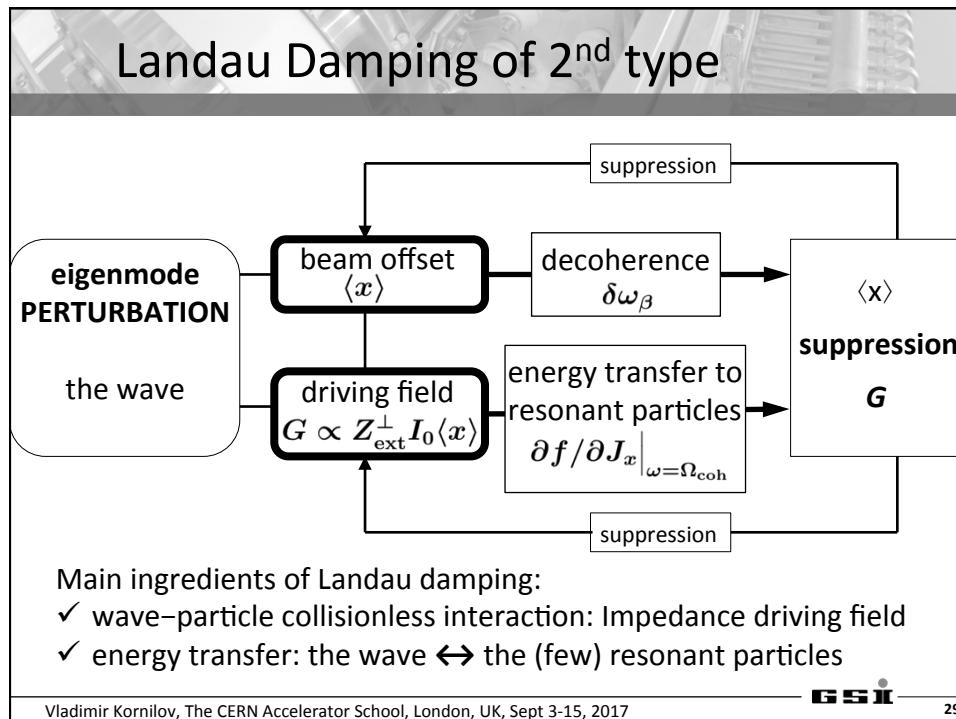
$\Delta Q_{\text{coh}}$ : coherent no-damping tune shift imposed by an impedance  
 $\Delta Q_{\text{ex}}(J_x, J_y)$ : external (lattice) incoherent tune shifts

L.Laslett, V.Neil, A.Sessler, 1965  
D.Möhl, H.Schönauer, 1974  
J.Berg, F.Ruggiero, CERN SL-96-71 AP 1996

The resulting damping is a complicated 2D convolution of the distribution  $\{df(J_x, J_y)/dJ_x\}$  and tune shifts  $\Delta Q_{\text{ext}}(J_x, J_y)$

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## Landau Damping of 2<sup>nd</sup> type

Drawback: octupoles, like other nonlinearities, can reduce Dynamic Aperture and Lifetime.

A balance is necessary

Octupoles are the essential part of the beam stability in many machines

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## Amplitude-Dependent Detuning

Another source of amplitude-dependent tune-shifts:  
Tune-spread due to Beam-Beam effects produces Landau Damping

Tune Footprints

Resulting Stability Diagrams

X.Buffat, et al, PRSTAB 17, 111002 (2014)

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# Landau Damping in Beams of 3<sup>rd</sup> type

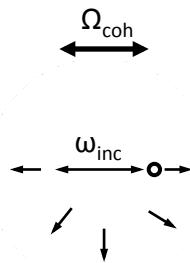
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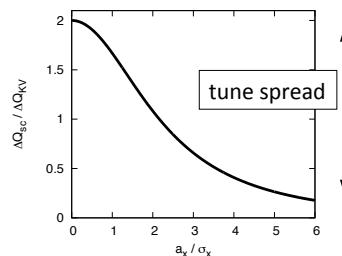
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## Landau Damping of 3<sup>rd</sup> type

Different situation:  
Wave-Particle interaction is due to space-charge



Electric field  
of the self-field  
space charge



For the resonant particles  $Q_{inc} \approx Q_{coh}$ ,  
wave ↔ particles energy transfer should be possible

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## Landau Damping of 3<sup>rd</sup> type

The dispersion relation

$$\int \frac{\Delta Q_{coh} - \Delta Q_{sc}}{\Delta Q_{ex} + \Delta Q_{sc} - \Omega/\omega_0} J_x \frac{\partial f}{\partial J_x} dJ_x dJ_y dp = 1$$

$f(J_x, J_y, p)$

$\Delta Q_{coh}$  : no-damping coherent tune shift imposed

$\Delta Q_{ex}(J_x, J_y, p)$  : external (lattice) incoherent tune shift

$\Delta Q_{sc}(J_x, J_y)$  : space-charge tune shift

L.Laslett, V.Neil, A.Sessler, 1965  
D.Möhl, H.Schönauer, 1974

The resulting damping is a complicated 2D convolution of the distribution  $\{df(J_x, J_y)/dJ_x\}$  and tune shifts  $\Delta Q_{sc}(J_x, J_y)$ ,  $\Delta Q_{ext}(J_x, J_y)$

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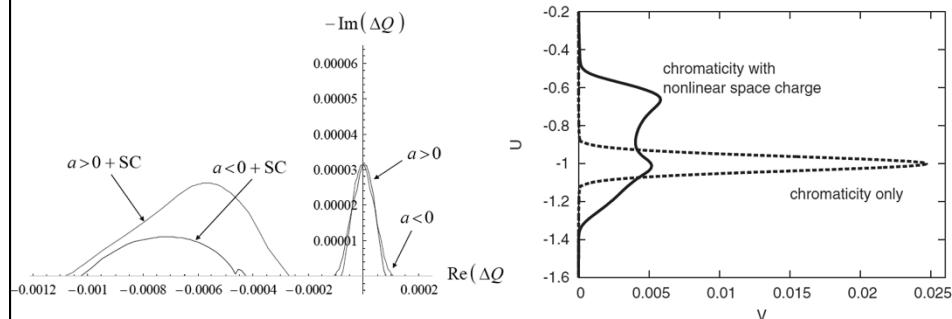


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## Landau Damping of 3<sup>rd</sup> type

$$\int \frac{\Delta Q_{coh} - \Delta Q_{sc}}{\Delta Q_{ex} + \Delta Q_{sc} - \Omega/\omega_0} J_x \frac{\partial f}{\partial J_x} dJ_x dJ_y dp = 1$$

Solution examples



E.Metral, F.Ruggiero, CERN-AB-2004-025 (2004)

V.Kornilov, O.Boine-F, I.Hofmann, PRSTAB 11, 014201 (2008)

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## Landau Damping of 3<sup>rd</sup> type

The dispersion relation

$$\int \frac{\Delta Q_{coh} - \Delta Q_{sc}}{\Delta Q_{ex} + \Delta Q_{sc} - \Omega/\omega_0} J_x \frac{\partial f}{\partial J_x} dJ_x dJ_y dp = 1$$

$\Delta Q_{ex}=0$ : no pole, no damping!

Momentum conservation in a closed system

Even if  $\Omega_{coh}$  is inside the spectrum,  
and there are resonant particles  $Q_{inc} \approx Q_{coh}$ ,  
there is no Landau damping in coasting beams only due to space-charge

L.Laslett, V.Neil, A.Sessler, 1965  
D.Möhl, H.Schönauer, 1974  
V.Kornilov, O.Boine-F, I.Hofmann, PRSTAB 11, 014201 (2008)  
A.Burov, V.Lebedev, PRSTAB 12, 034201 (2009)

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## Landau Damping

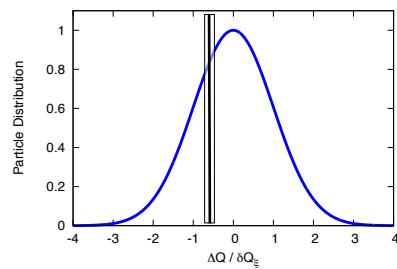
Remark

Thus, we discussed two examples:

1. Head-tail modes and the chromaticity tune-spread
2. Nonlinear space-charge in coasting beams

- there is a **tune spread**
- the coherent frequency overlaps with the incoherent spectrum

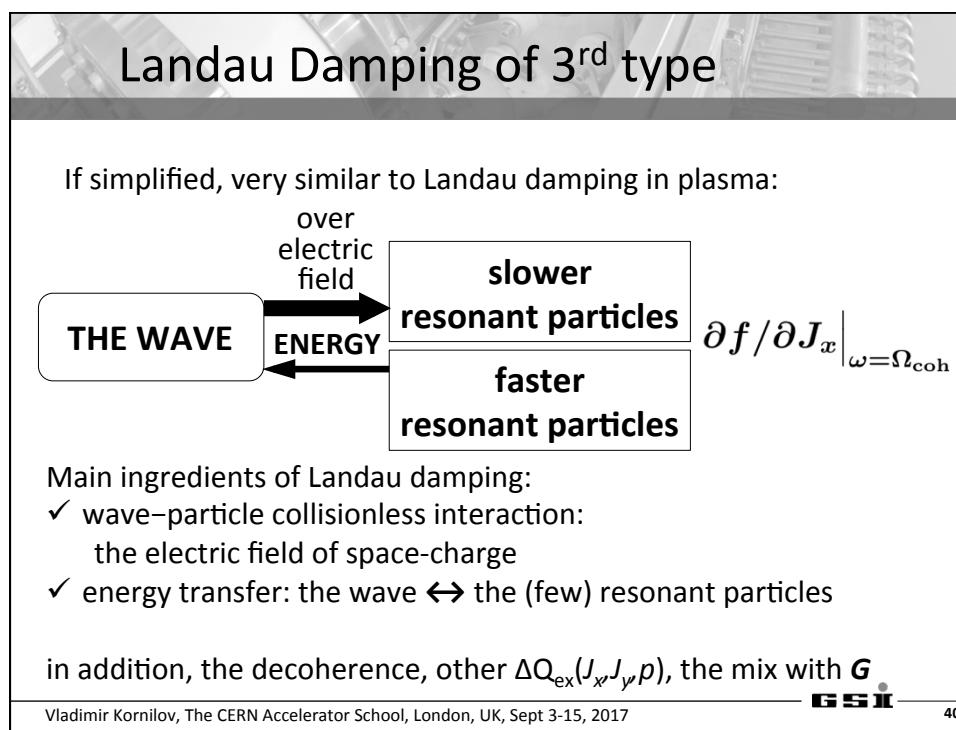
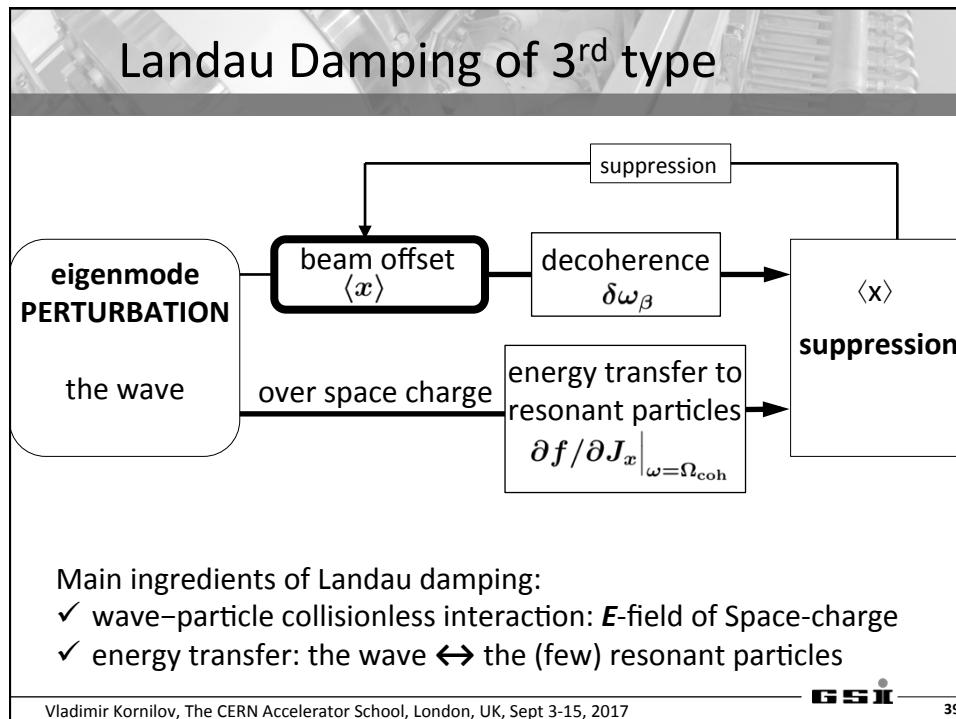
still, there is **NO Landau Damping!**



a tune spread does not automatically mean Landau Damping

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## Landau Damping of 3<sup>rd</sup> type

Landau damping in bunches

- There is damping due to only space charge
- Space-charge tune spread due to longitudinal bunch profile

A.Burov, PRSTAB 12, 044202 (2009)  
V.Balbekov, PRSTAB 12, 124402 (2009)  
V.Kornilov, O.Boine-F, PRSTAB 13, 114201 (2010)

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## Landau Damping in beams

- Waves: collective oscillations in a medium of particles
- Wave ↔ particles collisionless interaction
- Dispersion relation and Stability Diagram
- Decoherence is a result of Phase-Mixing, Landau damping, etc
- Collective Frequency ↔ Tune Spread
- Tune spreads of different nature produce different types of Landau damping
- A tune spread does not automatically mean Landau Damping
- A special role of Space-Charge as a provider of a tune spread and the wave↔particles interaction

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