EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

Werner.Herr@cern.ch

CAS 2015

Exercises for tutorial on "Non-linear Dynamics" at the CAS 2015 in Otwock

W. Herr, BE Department, CERN, 1211-Geneva 23

Abstract

Exercises complementing the lectures on "Tools for Non-Linear Dynamics".

Recommendation: pick a few of the exercises and try to solve them in detail

Geneva, Switzerland

1st September 2015

1 Exercise 1

1.1 Problem:

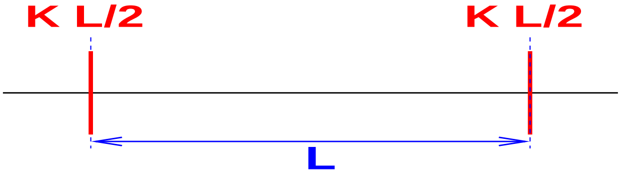
Assume a thin lens kick $f(x)$ and show that it is always symplectic (use 1D to keep it simple):

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} x_0 \\ x'_0 + f(x_0) \end{pmatrix}$$

2 Exercise 2

2.1 Problem:

Assume two kicks at the end and beginning of a drift space: Compute to lumped



matrix $\mathcal{M}_{lumped}$.

3 Exercise 3

3.1 Problem:

Use the function $f(x,p) \;=\; a \cdot x + b \cdot p$ (where a and b are constants) to get the maps:

$$e^{:f:}x = ?$$
$$e^{:f:}p = ?$$

What is the physical meaning ?

4 Exercise 4

4.1 Problem:

a)Assume a matrix M of the type:

$$M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$$

described by a generator f. Use the properties of Lie transforms to evaluate the effect of this matrix on the moments x^2, xp, p^2:

$$e^{:f:}x^2 = ?$$

$$e^{:f:}p^2 = ?$$

$$e^{:f:}xp = ?$$

b) Discuss the results, considering what you learnt in previous lectures.

5 Exercise 5

5.1 Problem:

Assume a function:

$$f(x) = \frac{1}{x + \frac{1}{x}}$$

Compute the first derivative $f'(x)$ fo $x = 2$ using the Automatic Differentiation algorithm.

6 Exercise 6

6.1 Problem:

a) Compute the map:

$$X(L) = ?$$

$$P(L) = X'(L) = ?$$

for a thick sextupole (1D) (length L, strength k) with the equation of motion:

$$x'' = -k \cdot x^2$$

up to order $\mathcal{O}(L^2)$, using the symplectic integration method.

b) Compute the map:

$$X(L) = ?$$

for a thick sextupole (2D) with the Hamiltonian (to give the equation of motion above):

$$H = \frac{1}{3}k(x^3 - 3xy^2) + \frac{1}{2}(p_x^2 + p_y^2)$$

using the Lie transformation method, compare with the solution from a).