Tutorial for the course of Beam Instabilities

(CAS, Advanced Accelerator Physics Course – Warsaw, Poland, 27 September – 9 October, 2015)

Parameter	Symbol	Unit	Value
Circumference	C	m	6911
Momentum	p_0	GeV/c	26
Momentum spread rms	$\delta p/p_0$		2.1×10^{-3}
Cavity voltage	V	MV	4
Harmonic number	h		4620
Momentum compaction	α		0.00192
Norm. emittance rms	$\epsilon_{x,y}$	μm	2
Tunes	$Q_{x,y}$		26.13/26.18

Consider the proton accelerator with the beam and machine parameters at injection defined in the table above.

1. If the maximum acceptable space charge tune spread is 0.21, what is the maximum population that a matched Gaussian bunch can have? (Assume beam radius $a = \sqrt{2}\ \sigma_{x,y}$)
2. The accelerator can be represented longitudinally with a broad-band impedance having $R_s = 230$ kΩ, $Q = 1$ and $f_r = \omega_r/2\pi = 1.4$ GHz. What is the energy loss per particle per turn due to this impedance, if one bunch with the intensity calculated at the point 1. is circulating in the machine? Would it be different if several bunches with the same intensity were circulating? What is the associated stable phase shift?
3. A 1 m long vertical collimator made of graphite ($\sigma = 5 \times 10^4$ S/m) is installed at a dispersion free location with $\beta_y = 20$ m. The collimator has a horizontal aperture of 10 cm and the vertical gap is closed to a half-height $g = 5\sigma_y$. What is the average orbit kick received by the bunch above, if it goes through the collimator: a) with a horizontal displacement $x_0 = 1$ cm; b) with a vertical displacement $y_0 = 2$ mm.
4. The accelerator can be modeled with a transverse wake function per unit length of 440 GV/(Cm^2). Please give an estimate of the TMCI threshold. By how much would this threshold change if a new optics is implemented with integer part of the tune of 20 and $\gamma_t = 18$?

Solutions

1. **If the maximum acceptable space charge tune spread is 0.21, what is the maximum population that a matched Gaussian bunch can have?**

$$\Delta Q = -\frac{r_0 R_0}{2\epsilon_{x,y}\beta\gamma^2} \cdot \frac{N_{\max}}{\sqrt{2\pi}\sigma_z} = -0.21$$

with $R_0 = C/(2\pi)$. We have used the formula given in Massimo's lecture with $a^2 = 2\,\sigma^2_{x,,y} = 2 <\beta_{x,y}> \varepsilon_{x,y}/\beta\gamma = 2R_0/Q_{x,y}\,\varepsilon_{x,y}/\beta\gamma$
In order to calculate the maximum bunch population N_{max}, we first need to calculate β, γ, σ_z
From the momentum one can easily calculate that the beam is basically ultrarelativistic with $\beta \cong 1$ and $\gamma = 27.7$.
The momentum spread is given, from the matching condition of the bunch to the stationary bucket, we have:

$$\sigma_z = \frac{C\eta(\delta p/p_0)}{Q_s}$$

with $\eta = |\alpha - 1/\gamma^2| = 6.2 \times 10^{-4}$ and Q_s, synchrotron tune, given by

$$Q_s = \sqrt{\frac{\eta e V h}{2\pi p_0 c}} = 0.0084$$

Using the relation above we have $\sigma_z = 0.17$ m.
The maximum bunch population is given by

$$N_{\max} = \frac{\sqrt{2\pi}\sigma_z \cdot 2\pi\gamma^2 \cdot 2\epsilon_{x,y} \cdot \Delta Q}{r_0 C} = 1.6 \times 10^{11}\ \mathrm{p}$$

2. **The accelerator can be represented longitudinally with a broad-band impedance having $R_s = 230$ kΩ, $Q = 1$ and $f_r = \omega_r/2\pi = 1.4$ GHz. What is the energy loss per particle per turn due to this impedance, if one bunch with the intensity calculated at the point 1. is circulating in the machine? Would it be different if several bunches with the**

same intensity were circulating? What is the associated stable phase shift?

The broad band impedance can be written as

$$Z_{||}(\omega) = \frac{R_s}{1 + iQ\left(\dfrac{\omega}{\omega_r} - \dfrac{\omega_r}{\omega}\right)} \approx \frac{R_s}{Q^2\omega_r^2}\omega^2$$

where the expansion is true for $\omega \ll \omega_r$. Since $(c/\sigma_z) \ll \omega_r$, we can write the integral of the energy loss in the following form

$$\Delta E = -\frac{e^2}{2\pi N}\int_{-\infty}^{\infty}|\tilde{\lambda}(\omega)|^2 Z_{||}(\omega)d\omega = -\frac{e^2}{2\pi N}\int_{-\infty}^{\infty}N^2\exp\left(-\frac{\omega^2\sigma_z^2}{c^2}\right)\frac{R_s\omega^2}{Q^2\omega_r^2}d\omega$$

$$= -\frac{Ne^2R_sc^3}{2\pi Q^2\omega_r^2\sigma_z^3}\int_{-\infty}^{\infty}\exp(-x^2)x^2dx = -\frac{Ne^2R_sc^3}{2\pi Q^2\omega_r^2\sigma_z^3}\frac{\sqrt{\pi}}{2} = 60\,\mathrm{keV/turn}$$

The energy loss per particle per turn does not change if there is more than one bunch in the machine, because the broad band impedance does not couple different bunches (decay time of the wake much lower than bunch length).
The stable phase shift is given by:

$$\Delta\phi_s = \arcsin\left(\frac{\Delta E}{eV}\right) = 1.5 \times 10^{-2}\ (\approx 0.85^o)$$

3 A 1 m long vertical collimator made of graphite ($\sigma = 5 \times 10^4$ S/m) is installed at a dispersion free location with $\beta_y = 20$ m. The collimator has a horizontal aperture of 10 cm and the vertical gap is closed to a half-height g = 5σ. What is the average orbit kick received by the bunch above, if it goes through the collimator: a) with a horizontal displacement $x_0 = 1$ cm; b) with a vertical displacement $y_0 = 2$ mm.

$$\sigma_y^{\mathrm{coll}} = \sqrt{\frac{\epsilon_y}{\gamma}\beta_y} = 1.2\ \mathrm{mm} \Rightarrow g = 6\ \mathrm{mm}$$

The collimator is a flat structure, for which $Z_{xd}(\omega) = -Z_{xq}(\omega)$, while $Z_{yd}(\omega) = \pi^2/12\, Z_{RW}(\omega)$, $Z_{yq}(\omega) = \pi^2/24\, Z_{RW}(\omega)$.

The expression of $Z_{RW}(\omega)$ is given in the lecture.

$$\frac{Z_{RW(x,y)}(\omega)}{L} = \frac{1}{2\pi b^3}\sqrt{\frac{2Z_0 c}{\sigma|\omega|}}\,[1+\mathrm{sgn}(\omega)\cdot i]$$

With horizontal offset

$$\langle \Delta x' \rangle = -\frac{e^2 x_0}{2\pi N E_0}\int |\tilde{\lambda}(\omega)|^2\,[\mathrm{Im}[Z_{xd}(\omega) + \mathrm{Im}[Z_{xq}(\omega)]\,d\omega = 0$$

With vertical offset

$$\langle \Delta y' \rangle = -\frac{e^2 y_0}{2\pi N E_0}\frac{\pi^2}{8}\int |\tilde{\lambda}(\omega)|^2 \mathrm{Im}[Z_{RW}(\omega)]d\omega =$$

$$= -\frac{\pi N e^2 y_0}{16 E_0}\frac{1}{2\pi b^3}\sqrt{\frac{2Z_0 c}{\sigma}}\cdot\sqrt{\frac{c}{\sigma_z}}\cdot\frac{1}{2}\Gamma\left(\frac{1}{4}\right) = 14.3\ \mathrm{nrad}$$

4 **The accelerator can be modeled with a transverse wake function per unit length of 440 GV/(Cm^2). Please give an estimate of the TMCI threshold. By how much would this threshold change if a new optics is implemented with integer part of the tune of 20 and γ_t = 18?**

The formula for the TMCI threshold calculated with a two-particle model reads (see lecture)

$$N_{\mathrm{thr}} = \frac{8}{\pi e^2}\frac{p_0 \omega_s}{\langle \beta_{x,y}\rangle}\left(\frac{C}{W_0}\right)$$

$<\beta_{x,y}> = R_0/Q_{x,y}$ = 42 m
$\omega_s = Q_s\, c/R_0$

Therefore, one can calculate N_{thr} = 1.7 x 10^{11} p
In the Q20 optics, we need to recompute η, $<\beta_{x,y}>$ and the synchrotron tune.

$\eta^{Q20} = |1/\gamma_t^2 - 1/\gamma^2|$ = 0.0018
$<\beta_{x,y}>^{Q20} = R_0/Q_{x,y}$ = 55 m

For the synchrotron tune, we can make two assumptions:

- The cavity voltage is unchanged:

$$Q_s^{\mathrm{Q20}} = \sqrt{\frac{\eta^{\mathrm{Q20}} eVh}{2\pi p_0 c}} = 0.014$$

This yields N_{thr}^{Q20} = 2.2 x 10^{11} p, therefore an improvement of the TMCI threshold by ~30%

- The synchrotron tune itself is scaled with η, such as to preserve the matching of the bunch with the previously given momentum spread and bunch length:

$$Q_s^{\mathrm{Q20}} = Q_s^{\mathrm{Q26}} \cdot \frac{\eta^{\mathrm{Q20}}}{\eta^{\mathrm{Q26}}} = 0.024$$

This yields N_{thr}^{Q20} = 3.7 x 10^{11} p, therefore an improvement of the TMCI threshold by a factor 2.2