Exclusive $b \to c\ell\overline{\nu}$ decays SM predictions and uncertainties

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Prospects and challenges for semitauonic decays at LHCb

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Preliminaries

- discussing mostly results that stem from hard work of other people
- giving credit wherever possible
- personal opinions will be highlighted as such

Ingredients

Aim and procedure

provide SM predictions for the (differential) rates of exclusive $b \to c\tau\overline{\nu}$ decays

how to do that?

– work in an effective field theory w/o dynamic W, Z and t
  – expansion of local operators in $1/M_W$ ($\sim \sqrt{G_F}$)
  – leading contributions from $V - A \times V - A$ interaction @ mass dimension 6

$$\mathcal{O}_{LL} \equiv [\overline{c}\gamma^\mu(1-\gamma_5)b]\left[\overline{\ell}\gamma_\mu(1-\gamma_5)\nu\right] \equiv J^\mu_{(h)} J^{(\ell)}_\mu$$

    – factorization into hadronic and leptonic currents in limit $\alpha_e \to 0$
  – leads to Lagrangian

$$\mathcal{L} = \frac{G_F V_{cb}}{\sqrt{2}} \mathcal{O}_{LL} + \cdots + \text{h.c.}$$

    – dots: operators of higher mass dimension, suppressed by powers of $M_B^2/M_W^2 \simeq 0.4\%$
– first sources of theory uncertainties: V_{cb}, α_e

Electroweak corrections

- G_F defined as matching coefficient in $\mu \to e\overline{\nu}_e\nu_\mu$
  - electroweak matchin correction for $b \to c\ell\overline{\nu}_\ell$ [A. Sirlin, Rev.Mod.Phys. 50 (1978) 573]
  - absorbable into universal correction $G_F \to G_F\eta_{\text{EW}}$

$$\eta_{\text{EW}} = 1 + \frac{\alpha_e}{\pi} \ln\left(\frac{M_Z}{m_b}\right) \simeq 1.007$$

- universal correction to all exclusive and inclusive $b \to c\ell\nu$ processes
- produces 1.4% upward shift in all observables (e.g. the decay rate)

Hadronic matrix elements

what is ⟨charm hadron $|\bar{c}\gamma^\mu(1-\gamma_5)b|$ beauty hadron⟩?

– parametrize matrix elements based on Lorentz symmetry and parity invariance of QCD

$$\langle k, s_c|\bar{c}\gamma^\mu(1-\gamma_5)b|p, s_b\rangle = \sum_i F_i(p\cdot k)L^\mu(p,k,s_b,s_c)$$

p, k	momenta of initial and final state hadrons
s_b, s_c	angular momentum configurations of initial and final state
L_i^μ	Lorentz structure (four vectors, spinors, ...)
F_i	form factors, scalar-valued functions of momentum transfer

– further source of uncertainties: form factors

$\overline{B} \to D^{(*)}$ form factors

[Caprini,Lellouch,Neubert Nucl. Phys. B 530, 153 (1998)]

$$\langle D(k)|\bar{c}\gamma^\mu(1-\gamma_5)b|\overline{B}(p)\rangle = f_+(q^2)\left[(p+k)^\mu - \frac{M_B^2 - M_D^2}{q^2}q^\mu\right] + f_0(q^2)\frac{M_B^2 - M_D^2}{q^2}q^\mu$$

$$\langle D^*(k,\eta)|\bar{c}\gamma^\mu(1-\gamma_5)b|\overline{B}(p)\rangle = -(M_B + M_{D^*})A_1(q^2)\left[\eta^{*\mu} - \frac{\eta^*\cdot q}{q^2}q^\mu\right]$$
$$+ A_2(q^2)\frac{\eta^*\cdot q}{M_B + M_{D^*}}\left[(p+k)^\mu - \frac{M_B^2 - M_{D^*}^2}{q^2}q^\mu\right]$$
$$- 2M_{D^*}A_0(q^2)\frac{\eta^*\cdot q}{q^2}q^\mu + \frac{2iV(q^2)}{M_B + M_{D^*}}\varepsilon^{\mu\eta^* pk}$$

- form factors $f_{+,0}$, V, $A_{0,1,2}$ only accessible through non-perturbative methods
  - lattice QCD [e.g. Heechang,Bouchard,Lepage,Monahan,Shigemitsu 1505.03925]
  - QCD sum rules on the light-cone with B-meson LCDAs [Faller,Khodjamirian,Klein,Mannel 0809.0222]
  - QCD sum rules at zero hadronic recoil [Bigi,Shifman,Uraltsev,Vainshtein hep-ph/9405410]

Heavy quark expansion / HQET

- large quark masses allow for Heavy Quark Effective field Theory (HQET)
  - introduces new fields h_v, where v is the velocity of the quark field
  - for heavy-to-heavy transitions

$$b(p) \mapsto h_v + O\left(1/m_b\right) \qquad \bar{c} \mapsto h_{v'} + O\left(1/m_c\right)$$

    with velocities $v \equiv p/M_B$, $v' \equiv k/M_{D^{(*)}}$
  - describe kinematics with boost $\omega \equiv v \cdot v'$
- matching of currents in QCD and HQET yields

$$\bar{c}\gamma^\mu b = \eta_V(\omega)\overline{h}_{v'}\gamma^\mu h_v + \dots$$

$$\bar{c}\gamma^\mu \gamma_5 b = \eta_A(\omega)\overline{h}_{v'}\gamma^\mu \gamma_5 h_v + \dots$$

- HQET Wilson coefficients $\eta_{V(A)}$ for the (axial)vector current

$$\eta_V(1) \simeq 1.02 + O\left(\alpha_s^2\right) \qquad \eta_A(1) \simeq 0.96 + O\left(\alpha_s^2\right)$$

  - differ starting from $O\left(\alpha_s\right)$

[see e.g. Manohar&Wise, Heavy Quark Physics]

Heavy quark limit / Isgur-Wise function

useful to consider the heavy quark limit $m_b \to \infty$, $m_c \to \infty$, $m_c/m_b =$ const.

[excellent review: M. Neubert, Heavy Quark Symmetry (SLAC-PUB-6263)]

– all $\overline{B} \to D$ and $\overline{B} \to D^*$ form factors reduce to one single function

$$f_+ \sim f_0 \sim V \sim A_1 \sim A_2 \sim A_0 = \xi$$

where $\xi(\omega)$: Isgur-Wise function, with normalization: $\xi(\omega = 1) = 1$

– systematic corrections from insertion of derivatives and non-local matrix elements

$$\text{e.g.} \quad \langle D^{(*)}|\overline{h}_c i\not{D}_\perp^\alpha \Gamma \gamma_\alpha h_b|\overline{B}\rangle\,, \quad \langle D^{(*)}|\mathcal{T}\left\{\mathcal{L}_{\text{HQET}}(x), [\overline{h}_c(0)\Gamma h_b(0)]\right\}|\overline{B}\rangle$$

– emergence of subleading Isgur-Wise functions $\xi_{1,2,3}$ at $O\left(1/m_b\right)$
– heavy quark symmetries protect $f_+(1)$, $A_1(1)$ from contributions of exactly order $1/m_b$ (Luke's theorem)

Putting things together

Ratios for different lepton flavours

- cancellation of uncertainties
  - full cancelation of V_{cb}
  - full cancelation of η_{EW}
  - partial cancelation of form factors

R_H particularly sensitive to form factor ratios of “time-like polarization“ over some reference form factor

- for R_D: f_0/f_+
- for R_{D^*}: A_0/A_1
- for R_{Λ_c}: $f_0/f_\perp$, $g_0/g_\perp$

R_D

[HPQCD Collaboration Phys.Rev. D92 (2015) no.5, 054510]

form factors f_+ and f_0

- lattice results for 4 momentum configurations
- interpolated across entire kinematics spectrum
- full correlation among form factor parameters published
- lattice results reliable since D is QCD-stable

HPQCD collaboration obtains

$$R_D = 0.300 \pm 0.008$$

Nota bene: form factors were also used to extract $|V_{cb}|$ from BaBar data

- they obtain $|V_{cb}| = (40.2 \pm 1.7|_{\text{latt+stat}} \pm 1.3|_{\text{syst}}) \cdot 10^{-3}$
- compare inclusive $|V_{cb}| = (42.2 \pm 0.8) \cdot 10^{-3}$

R_{D^*}

[Fajfer,Kamenik,Nisandzic, Phys.Rev. D85 (2012) 094025]

- situation more complicated than R_D
- lattice results only known at $\omega = 1$
- extract form factor ratios from shape of $\overline{B} \to D\mu\overline{\nu}$ spectrum
  - caveat: $\overline{B} \to D\mu\overline{\nu}$ is insensitive to A_0/A_1!
  - use HQET estimate for this ratio

$$R_{D^*} = 0.252 \pm 0.003|_{\text{param.}} + 0.???|_{\text{exp.syst}}$$

- parametric uncertainty dominantly from perturbative/power corrections to HQET estimate for A_0/A_1
- how large are systematic experimental uncertainties?

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Model-dependence of exclusive $\overline{B} \to D^* \mu \overline{\nu}$ measurements

Fit model, based on HQET & dispersive bounds [Caprini,Lellouch,Neubert Nucl. Phys. B 530, 153 (1998)]

$$A_1(\omega) = A_1(1)\left[1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^2\right]$$

$$\frac{V(\omega)}{A_1(\omega)} \propto R_1(1) - 0.12(\omega - 1) + 0.05(\omega - 1)^2$$

$$\frac{A_2(\omega)}{A_1(\omega)} \propto R_2(1) + 0.11(\omega - 1) - 0.06(\omega - 1)^2$$

where $z \equiv z(\omega) = (\sqrt{\omega + 1} - \sqrt{2})/(\sqrt{\omega + 1} + \sqrt{2})$

- q^2 spectrum of the decay is relevant to extraction of V_{cb} and form factor ratios ($\to R_{D^*}$ inputs)
- Belle analyses do not provide histograms of observables as functions of q^2 or ω [e.g. Belle hep-ex/0810.1657]
  - (personally) could not find BaBar analysis that do, either!
  - only fits of HQET-inspired parametrization to subleading power in $1/m_b$ are available!
- reanalysis of the Belle data is not possible
  - new Belle analyses do provide the “raw” q^2 spectrum [e.g. Belle (semi lep.) 1603.06711]

Impact of leptonic τ decays

Experimental PoV

- at LHCb, Belle (w/o hadronic tagging) no hard criterium to distinguish $\overline{B} \to D\mu\overline{\nu}$ from $\overline{B} \to D\tau(\to \mu\overline{\nu}\nu)\overline{\nu}$
  - disentangle both modes statistically
- experiment sees neutrino-inclusive decay rate

$$\frac{\mathrm{d}\Gamma(\overline{B} \to D\mu X_{\overline{\nu}})}{\mathrm{d}q^2\,\mathrm{d}\cos\theta_{[\mu]}} \equiv \frac{\mathrm{d}\Gamma(\overline{B} \to D\mu\overline{\nu}_\mu)}{\mathrm{d}q^2\,\mathrm{d}\cos\theta_{[\mu]}} + \frac{\mathrm{d}\Gamma(\overline{B} \to D\tau(\to \mu\overline{\nu}_\mu\nu_\tau)\overline{\nu}_\tau)}{\mathrm{d}q^2\,\mathrm{d}\cos\theta_{[\mu]}}$$

$$\equiv \frac{\mathrm{d}\Gamma_1}{\mathrm{d}q^2\,\mathrm{d}\cos\theta_{[\mu]}} + \frac{\mathrm{d}\Gamma_3}{\mathrm{d}q^2\,\mathrm{d}\cos\theta_{[\mu]}}$$

  - analytical results for Γ_3 not used in experimental analyses
  - first considered only recently

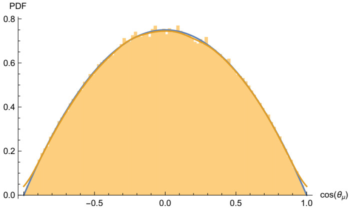
[Bordone,Isidori,DvD 1602.06143]

for NP contributions see also [Alonso,Kobach,Camalich 1602.07671]

- varying means to statistically disentangle both decays
  - Belle (II) uses NeuroBayes ($\to$ black box)

Signal 1: $\overline{B} \to D\mu\overline{\nu}$

$$\overline{B}(p) \;\to\; D(k)\; \mu(q_{[\mu]})\; \overline{\nu}(q_{[\overline{\nu}_\mu]})$$



- pseudo-scalar to pseudo-scalar transition
  - one hadronic form factors f_+
- simple kinematics:
  - momentum transfer $q^2 = (p-k)^2$
  - muon helicity angle $\cos\theta_{[\mu]}$ in $\mu\overline{\nu}$ RF
- distribution in $\theta_{[\mu]}$

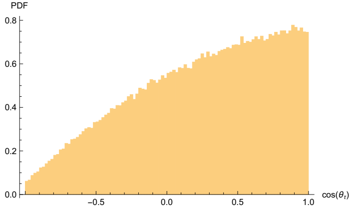
$$\frac{\mathrm{d}^2\Gamma}{\mathrm{d}q^2\,\mathrm{d}\cos\theta_{[\mu]}} = a + b\cos\theta_{[\mu]} + c\cos^2\theta_{[\mu]}$$
$$= a\sin^2\theta_{[\mu]} + O\left(m_\mu^2/q^2\right)$$

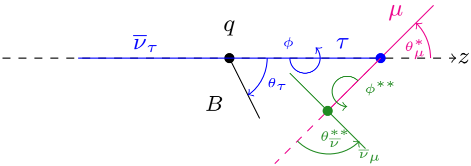
  - SM: $c = -a$, $b = 0 + O(\alpha_e)$
  - a, b, c: functions of q^2

Signal 2: $\overline{B} \to D\tau(\to \mu\overline{\nu}\nu)\overline{\nu}$

$$\overline{B}(p) \to D(k)\ \mu(q_{[\mu]})\ \overline{\nu}_\mu(q_{[\overline{\nu}_\mu]})\nu_\tau(q_{[\nu_\tau]})\overline{\nu}_\tau(q_{[\overline{\nu}_\tau]})$$

- hadronic matrix element
  - both form factors contribute
- complicated kinematics:
  - momentum transfer $q^2 = (p-k)^2$
  - $\overline{\nu}\nu$ mass $q^2_{[\nu_\tau\overline{\nu}_\mu]}$
  - five angles





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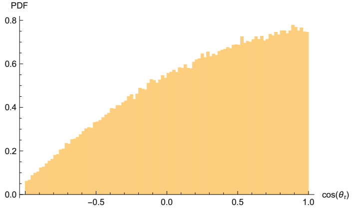
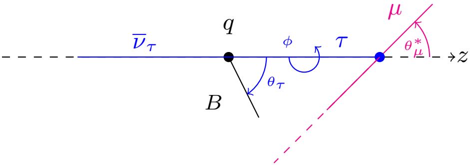
  - five angles, only 3 of which are needed for pheno studies





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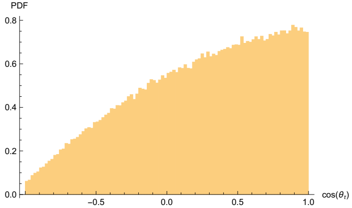
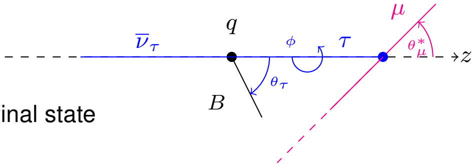


$$\frac{d^5\Gamma_3}{dq^2 dq^2_{[\nu_\tau\overline{\nu}_\mu]} d^2\Omega d\Omega^*} = \frac{\tilde{\Gamma}_3}{\pi m_\tau^8 q^6}\Big[A + B\cos\theta_{[\tau]} + C\cos^2\theta_{[\tau]} + \big(D\sin\theta_{[\tau]} + E\sin\theta_{[\tau]}\cos\theta_{[\tau]}\big)\cos\phi\Big]$$

$A, \ldots, E$ are functions of q^2, $q^2_{[\nu_\tau\overline{\nu}_\mu]}$ and $\cos\theta^*_{[\mu]}$

[Bordone,Isidori,DvD 1602.06143]

Observable



$\cos\theta_{[\mu]}$: muon helicity angle

- physical observable only in the 1ν final state
- defined in q rest frame
  - in terms of Lorentz invariants

$$\cos\theta_{[\mu]} \equiv 2\frac{\left(q - 2q_{[\mu]}\right)\cdot k}{\sqrt{\lambda}}$$

  - 3ν case: $\theta_{[\mu]} \neq \theta^*_{[\mu]}$

see backups for functional dep.

- boundaries

$$-1 \leq \cos\theta_{[\mu]} \leq 1 \qquad \text{for } 1\nu \text{ final state}$$

$$-1 \leq \cos\theta_{[\mu]} \lesssim 56.7 \qquad \text{for } 3\nu \text{ final state}$$

  - upper bounds very different from each other
  - not suitable for common parametrization

Neutrino-inclusive decay

aim: obtain normalized, differential decay widths for the neutrino-inclusive decay

– decay widths in terms of $\cos\theta_{[\mu]}$ and E_μ yield complicated expressions

[see Alonso,Kobach,Camalich 1602.07671]

– our approach: Monte Carlo simulations of pseudo events [Bordone,Isidori,DvD 1603.06143]

  – implement signal PDFs in EOS, including dependence on form factor parameters

  [DvD et al. http://github.com/eos/eos]

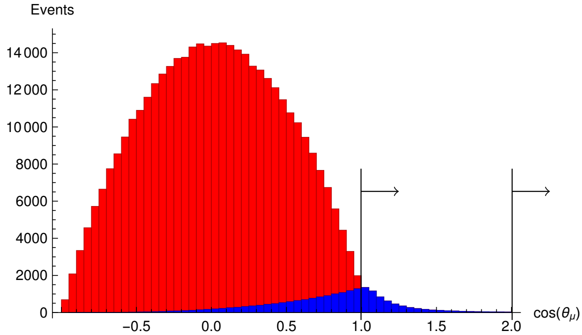
  – draw $\approx 4 \cdot 10^6$ pseudo events of $(q^2, \cos\theta_{[\mu]})$ for the 1ν final state
  – draw $\approx 4 \cdot 10^6$ pseudo events of $(q^2, q^2_{[\nu_\tau \overline{\nu}_\mu]}, \Omega, \Omega^*)$ for the 3ν final state

  Ω, Ω^*: solid angles

  – compute observable of interest for each set of pseudo events
  – combine sets with weights

$$\omega_1 = \frac{1}{1 + R_D\,\mathcal{B}(\tau \to \mu\nu\overline{\nu})} \qquad \omega_3 = 1 - \omega_1$$

Distribution in $\cos\theta_{[\mu]}$



neutrino-inclusive, 3ν final state

Heavy tails

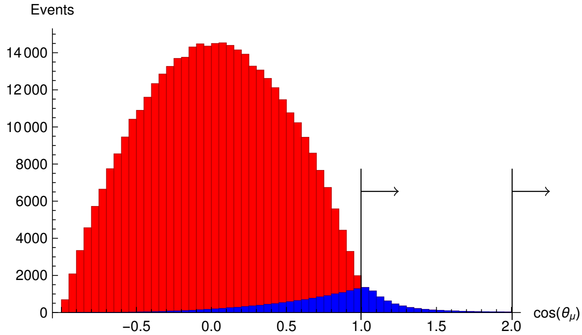
$\cos\theta_{[\mu]} > 1$ 32.3% of 3ν ev.

$\cos\theta_{[\mu]} > 2$ $\approx 2\%$ of 3ν ev.

University of Zurich[UZH]

Physik Institut

Distribution in $\cos\theta_{[\mu]}$



neutrino-inclusive, 3ν final state

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New method to extract $R_D \mathcal{B}(\tau \to \mu\bar{\nu}\nu)$

[Bordone,Isidori,DvD 1602.06143]

– cash in on heavy tail of $B \to D\mu X_{\bar{\nu}}$, and turn it into new method to extract R_D

– we suggest measurement of

$$\rho_D^{\text{exp}} \equiv \frac{\#\text{of } X_\nu \text{ events with } \cos\theta_\mu > 1}{\text{total \# of } X_\nu \text{ events}}$$

– precise calculation possible for

$$\rho_D^0 \equiv \frac{\#\text{of } 3\nu \text{ events with } \cos\theta_\mu > 1}{\text{total \# of } 3\nu \text{ events}} = 0.323 \pm 0.002 \quad (0.6\%)$$

  – uncertainty statistical
  – parametric uncertainty (form factors) within ± 0.002

– combine to extract

$$R_D \mathcal{B}(\tau \to \mu\bar{\nu}\nu) = \frac{\rho_D^{\text{exp}}}{\rho_D^0 - \rho_D^{\text{exp}}}$$

– will probably also work for other charmed hadrons: D^*, Λ_c

Conclusion

My personal conclusion

- presentation of form factors results in terms of (sub)leading Isgur-Wise functions is useful for theory prediction
  - early signs of “retirement” of this type of presentation with lattice results for $\overline{B} \to D$?
- however, assuming the validity of the heavy quark expansion for the extraction of experimental data is not a good idea
  - introduces model assumptions, which cannot be unfolded at a later point

[see e.g. BaBar/Belle measurements of $\overline{B} \to D^* \ell \overline{\nu}$ spectra]

- I think that the SM predictions
  - for R_D are very reliable [HPQCD Collaboration Phys.Rev. D92 (2015) no.5, 054510]
  - for R_{D^*} are state of the art, but suffer from the model-dependence of the experimental inputs [Fajfer,Kamenik,Nisandzic, Phys.Rev. D85 (2012) 094025]
- looking forward to lattice update of $\overline{B} \to D^*$ form factors
  - no estimates yet for R_{D_s}, $R_{D_s^*}$: form factors not yet known
- new way to extract $R_{D^{(*)},\Lambda_c}$ by means of distribution in $\cos\theta_\mu$
  - tail for $\cos\theta_\mu > 1$ very precisely predictable

for R_{Λ_c} see upcoming talk by Stefan Meinel

Appendix

R_{Λ_c}

[Detmold,Lehner,Meinel Phys.Rev. D92 (2015) no.3, 034503]

six form factors

- lattice results for 10 momentum configurations
- interpolated across entire kinematics spectrum
- full correlation among form factor parameters published
- lattice results reliable since Λ_c is QCD-stable

$$R_{\Lambda_c} = 0.3318 \pm 0.0074 \pm 0.0070 \qquad (\tau/e)$$
$$R_{\Lambda_c} = 0.3328 \pm 0.0074 \pm 0.0070 \qquad (\tau/\mu)$$

Observables

1ν

$$E_\mu\Big|_{1\nu} = \frac{1}{4M_B}\left[(M_B^2 - M_P^2 + q^2) - \sqrt{\lambda}\cos\theta_\mu\right] ,$$

3ν

$$\cos\theta_{[\mu]}\Big|_{3\nu} = 2\beta_{\nu\overline{\nu}}\Bigg\{ \left(\frac{(1-2\beta_{\nu\overline{\nu}})}{\beta_{\nu\overline{\nu}}} + 2\beta_\tau\right)\frac{M_B^2 - M_P^2 - q^2}{2\sqrt{\lambda}} + \beta_\tau\cos\theta_{[\tau]}$$

$$-\left(2\beta_\tau\frac{M_B^2 - M_P^2 - q^2}{2\sqrt{\lambda}} - (1-\beta_\tau)\cos\theta_{[\tau]}\right)\cos\theta^*_{[\mu]} - \sqrt{1-2\beta_\tau}\sin\theta^*_{[\mu]}\sin\theta_{[\tau]}\cos\phi\Bigg\}$$

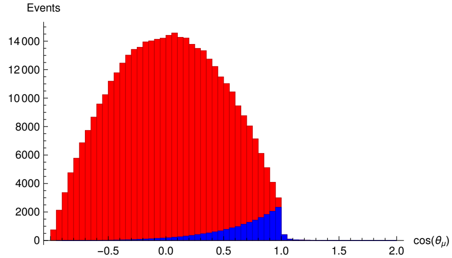
$$E_\mu\Big|_{3\nu} = \frac{\beta_{\nu\overline{\nu}}}{2M_B}\Big[(M_B^2 - M_P^2 + q^2)((1-\beta_\tau) + \beta_\tau\cos\theta^*_{[\mu]})$$

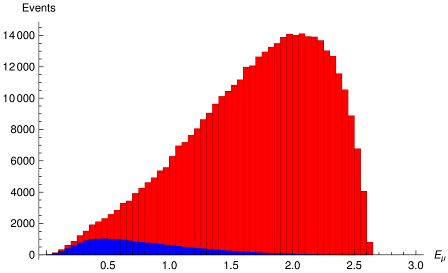
$$-\sqrt{\lambda}(\beta_\tau + (1-\beta_\tau)\cos\theta^*_{[\mu]})\cos\theta_{[\tau]} + \sqrt{1-2\beta_\tau}\sqrt{\lambda}\sin\theta^*_{[\mu]}\sin\theta_{[\tau]}\cos\phi\Big]$$

with $\lambda = \lambda(M_B^2, M_D^2, q^2)$ the Källèn function

The case of $\overline{B} \to \pi\tau\overline{\nu}$

- $D \to \pi$ easy enough
- however, small mass of π makes for some numerical changes
- tail $\cos\theta_{[\mu]} > 1$ very light: $\approx 3.3\%$
  new method will probably not work for pions
- distribution in E_μ broader
  - $R_\pi \approx 0.7$ larger, thus control of subtraction much more important!
  - background PDF parametrization should work as well as for D





Luke's theorem

[M.E. Luke, Phys.Lett. B252 (1990) 447-455]

– Luke considered first-order power correction to the form factors
– at zero recoil, heavy quark symmetries protect the form factor f_+ and A_1 from such first-order corrections