SPS DIS and Spin physics – Introduction

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Task: Provide background for

- Abstract 26: Transverse spin asymmetries
- Abstract 27: Generalized Parton Distributions
- Abstract 28: Longitudinal spin structure of the nucleon
- Abstract 29: Drell-Yan Physics with COMPASS

Aim: to understand in detail the structure of hadrons, for its own sake and as input for other physics, especially to reduce systematic uncertainties.

Bottom line: A lot was learned in recent years. All facts could fit together consistently, but crucial pieces are still missing. Whether our knowledge or ignorance dominates is a question of taste.

parton distribution functions

In the nucleon there exist three twist-2 parton distribution functions.

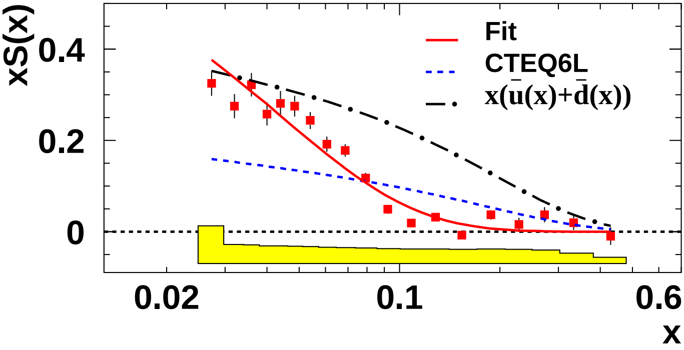
$$q(x, Q^2), \;\; \Delta q(x, Q^2) = q^{\uparrow}(x, Q^2) - q^{\downarrow}(x, Q^2), \;\; \delta q(x, Q^2)$$

subject to the constraints $\Delta q(x) \leq q(x)$ and $2|\delta q(x)| \leq q(x) + \Delta q(x)$

q(x): the main open problems are

- behavior at small-x $\Rightarrow$ collider experiments, LHeC
- the strange quark distribution $s(x, Q^2)$

Hermes 2008

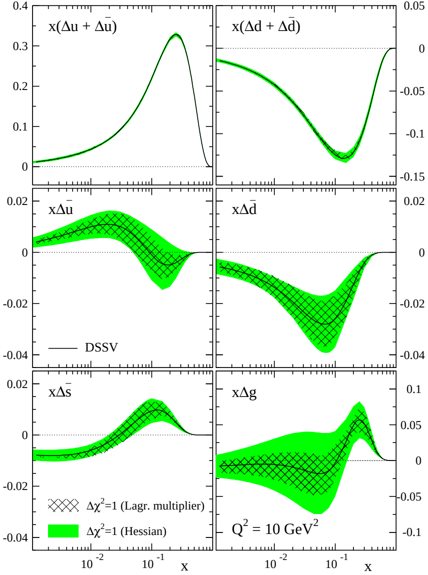


The strangeness content of the nucleon is still badly known. This might or might not affect the interpretation of some data relevant for electro-weak precision physics (NuTeV anomaly).

$\Delta q(x)$: the main open problems are (again)

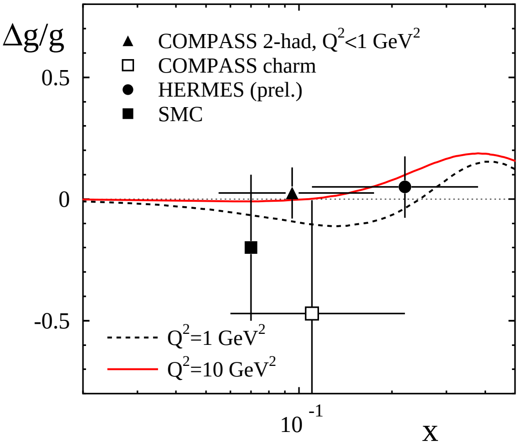
- behavior at small-x $\Rightarrow$ collider experiments like RHIC
- the flavor decomposition and $\Delta g(x, Q^2)$
- also fragmentation functions are not very well known, which affects the analysis of SIDIS data.

a very recent global fit:
Florian, Sassot, Stratmann, Vogelsang, arXiv:0904.3821

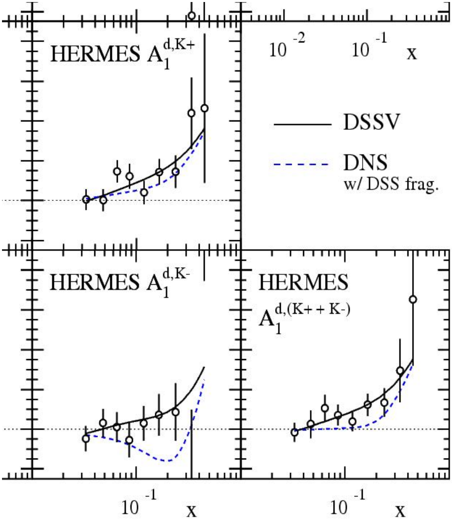


x(Δu + Δū)
x(Δd + Δd̄)
xΔū
xΔd̄
DSSV
xΔs̄
xΔg
Δχ²=1 (Lagr. multiplier)
Δχ²=1 (Hessian)
Q² = 10 GeV²
x

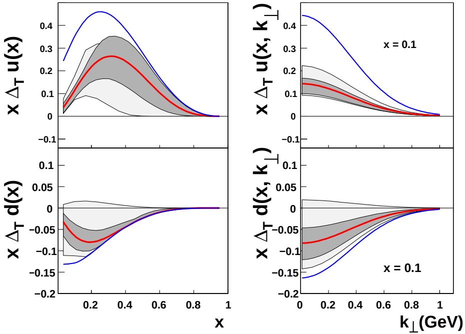
$\Delta g(x)$ from open charm – well understood but poor statistics
$\Delta g(x)$ from hadron pairs – better statistics but poorly understood



fragmentation functions affect the extracted $\Delta s(x, Q^2)$

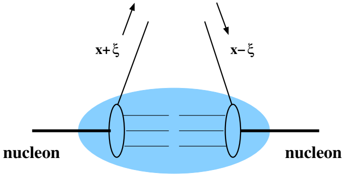


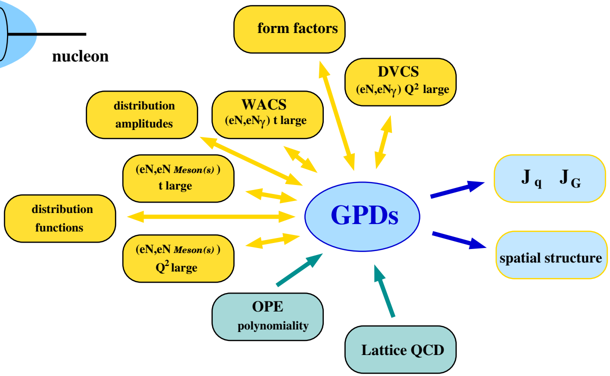
$\delta q(x) = \Delta_T(x)$: one is still at the level of models, e.g. Anselmino et al. arXiv:0812.4366



transverse momentum dependent physics has two aspects:
GPDs: theoretically rigorous
$q(x, k_\perp)$-physics: many open questions (initial and final state interactions, Glauber region, ...).

Generalized Parton Distributions





Definition of GPDs

$$h(P_1) + \Gamma^*(q_1) \to h(P_2) + \Gamma(q_2)$$

with $\Delta_\mu = q_{2\mu} - q_{1\mu}$, $t = \Delta^2$, $P_\mu = (P_{1\mu} + P_{2\mu})/2$
and $\xi = -Q^2/2P\cdot q_1$

Spin $\frac{1}{2}$ - the nucleon (modulo gauge links)

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}^+ z^-} \langle P_2|\, \bar{q}(-\frac{1}{2}z)\,\gamma^+ q(\frac{1}{2}z)\,|P_1\rangle\Big|_{z^+=0,\, z_\perp=0}$$

$$= \frac{1}{P^+}\left[H_q(x,\xi,t)\,\bar{N}(P_2)\gamma^+ N(P_1) + E_q(x,\xi,t)\,\bar{N}(P_2)\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}N(P_1)\right]$$

Some properties of GPDs:

- relation to form factors and distribution functions

$$H_q(x,0,0) = q(x) \qquad \int_{-1}^{1} dx H_q(x,\xi,t) = F_{1q}(t)$$

$$\tilde{H}_q(x,0,0) = \Delta q(x) \qquad \int_{-1}^{1} dx H_q(x,\xi,t) = g_{Aq}(t)$$

- OPE

$$\int_{-1}^{1} dx\, x^{n-1}\, H(x,\xi,t) = \sum_{\substack{k=0 \\ \text{even}}}^{n-1} (2\xi)^k\, A_{n,k}(t) + \mathrm{mod}(n+1,2)\,(2\xi)^n C_n(t)$$

$$\int_{-1}^{1} dx\, x^{n-1}\, E(x,\xi,t) = \sum_{\substack{k=0 \\ \text{even}}}^{n-1} (2\xi)^k\, B_{n,k}(t) - \mathrm{mod}(n+1,2)\,(2\xi)^n C_n(t)$$

The generalized form factors $A^q_{n,i}(t)$, $B^q_n(t)$, $C^q_n(t)$ are given by matrix elements of local operators.

$$\langle P_2|\textbf{Sym}\, \bar{q}\gamma^\mu \overleftrightarrow{D}^{\mu_1} \ldots \overleftrightarrow{D}^{\mu_n} q|P_1\rangle = \textbf{Sym}\, \bar{u}\gamma^\mu u \sum_{i \text{ even}}^{n} A^q_{n,i}(t)\Delta^{\mu_1}\ldots\Delta^{\mu_i}P^{\mu_{i+1}}\ldots P^{\mu_n}$$

$\Rightarrow$ They can be and have been calculated on the lattice.

A famous relation:

$$\left\langle J^3_q \right\rangle = \frac{1}{2}[A^q_{2,0}(0) + B^q_{2,0}(0)] \qquad \text{Ji's sumrule}$$

• GPDs give information on the transverse structure of hadrons in the impact parameter plane. The transverse mass is $\sqrt{q_{\parallel}^2 + m^2}$. Therefore a probabilistic interpretation makes sense.

$$H_q(x, 0, b_{\perp}^2) = \frac{1}{(2\pi)^2} \int d^2\Delta_{\perp}\, e^{ib_{\perp}\Delta_{\perp}} H_q(x, 0, \Delta_{\perp}^2)$$

This might be relevant for LHC due to multiple hard interactions in one $p + p$ collision.

The corresponding density, expressed in terms of GPDs

$$\frac{1}{(2\pi)^2}\int d^2\Delta_\perp e^{ib_\perp\cdot\Delta_\perp}\int\frac{dz^-}{2\pi}e^{ix\bar{P}^+z^-}\langle P_2|\,\bar{q}(-\frac{1}{2}z)\,\gamma^+[1+\vec{s}\cdot\vec{\gamma})\gamma_5]q(\frac{1}{2}z)\,|P_1\rangle\Big|_{z^+=0}^{z_\perp=0}$$

$$= \quad \frac{1}{2}\left[F + s^i F_T^i\right]$$

$$= \quad \frac{1}{2}\Bigg[H - S^i\epsilon^{ij}b^j\frac{1}{m}E' - s^i\epsilon^{ij}b^j\frac{1}{m}\left(E_T' + 2\tilde{H}_T'\right)$$

$$+ s^iS^i\left(H_T - \frac{1}{4m^2}\Delta_b\tilde{H}_T\right) + s^i(2b^ib^j - b^2\delta^{ij})S^j\frac{1}{m^2}\tilde{H}_T''\Bigg]$$

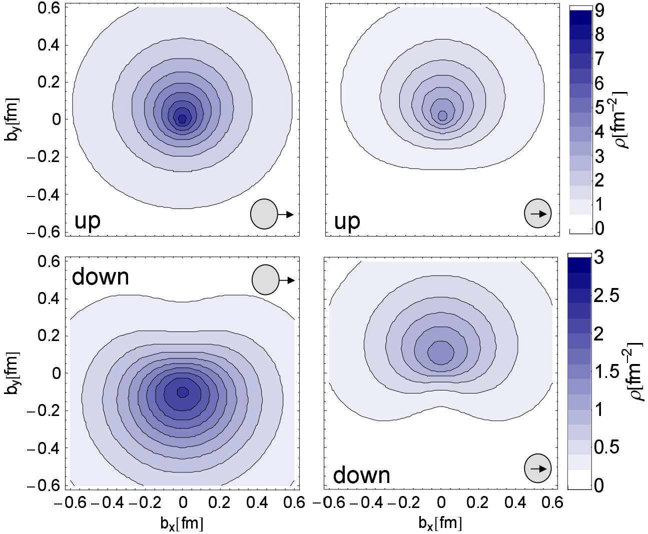
has a simple interpretation:

$S^i\epsilon^{ij}b^j$ coupling of proton spin to quark angular momentum

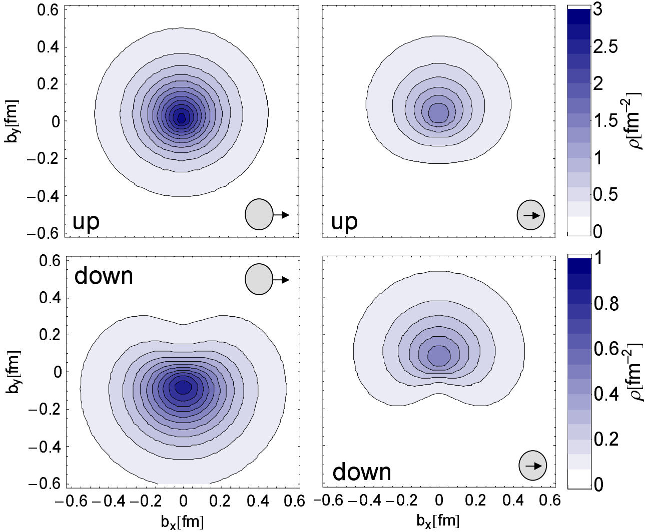
$s^i\epsilon^{ij}b^j$ coupling of quark spin to quark angular momentum

s^iS^i coupling of quark spin and proton spin

The nucleon, first moment ($n = 1$)

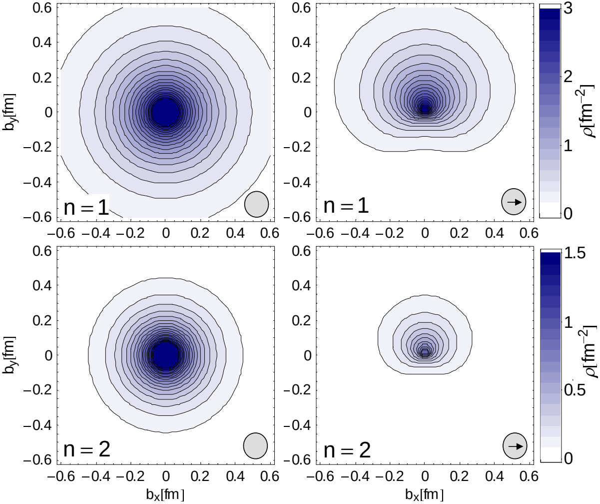


The nucleon, second moment ($n = 2$)

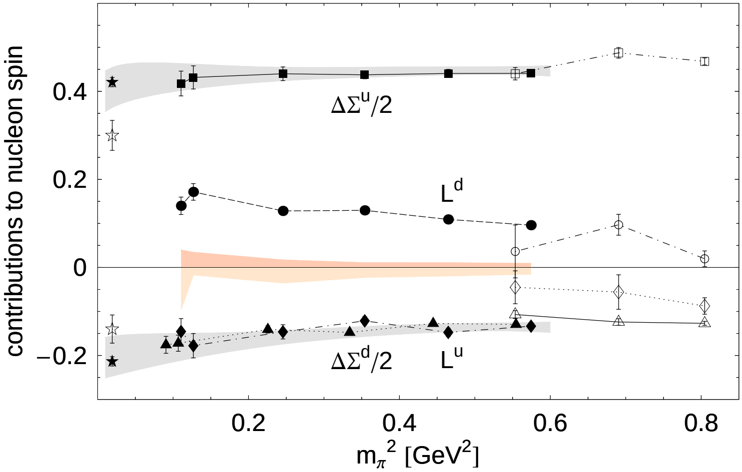


The pion first and second moment

Boer-Mulders function ⇒ F. Bradamantes talk.



LHPC results for the longitudinal spin structure

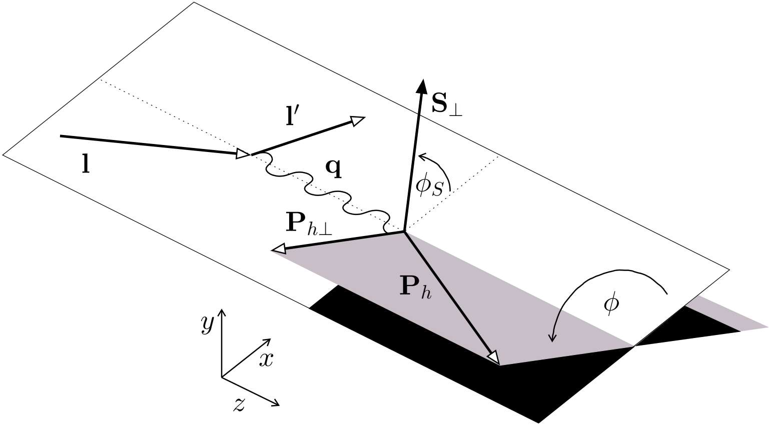


There are many technical problems:

- GPDs occur only within convolutions and there are many different ones, each depending on x, Q^2, ξ and t.
  $\Rightarrow$ you need all experimental input you can get (N. d'Hose's talk)
  $\Rightarrow$ their determination requires complementary lattice input
- present day lattice calculations have still substantial systematic and statistical uncertainties (but the situation improves rapidly)
- The concept of orbital angular momentum is still unsettled. It is not clear whether $L_q := J_q(GPD) - \frac{1}{2}\Sigma q$ is related to physically motivated suggestions for orbital angular momentum operators

but in principle GPD physics is rigorously defined.

SIDIS: The Boer-Mulders and Sivers asymmetries



The angles ϕ and ϕ_s, data from HERMES and COMPASS

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi\,dP_{h\perp}^2} = \ldots + \varepsilon \cos(2\phi)\, F_{UU}^{\cos 2\phi} + \ldots$$
$$+ \; |S_\perp| \sin(\phi - \phi_S)\, F_{UT}^{\sin(\phi-\phi_S)} + \ldots$$

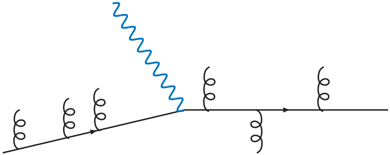
$F_{UU}^{\cos 2\phi}$: Boer-Mulders asymmetry, non-zero if:

- The transverse spatial quark distribution in an unpolarized nucleon depends on the transverse quark spin direction and
- The final state interaction depends on the transverse spatial quark distribution.

$F_{UT}^{\sin(\phi-\phi_S)}$: Sivers asymmetry, non-zero if:

- e.g. suitable final state interaction with remnant

The relevance of gauge links ~ initial and final state interactions



In collinear kinematics gauge links have usually no effect

$$\Big\langle p(P_i,S_i)\Big| \Big(\bar{q}(y)\Big)_\alpha \Big(q(0)\Big)_\beta \Big|p(P_i,S_i)\Big\rangle$$

$$\Rightarrow \quad \Big\langle p(P_i,S_i)\Big| \Big(\bar{q}(y)\Big)_\alpha \; \exp\Big\{-ig\int_{-\infty}^{y} dz\cdot A(z)\Big\}$$

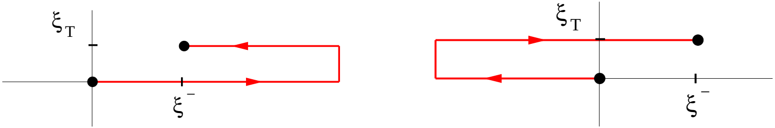
$$\exp\Big\{ig\int_{-\infty}^{0} dz\cdot A(z)\Big\}\Big(q(0)\Big)_\beta \Big|p(P_i,S_i)\Big\rangle$$

$$\Rightarrow \quad \Big\langle p(P_i,S_i)\Big| \Big(\bar{q}(y)\Big)_\alpha \; \exp\Big\{-ig\int_{0}^{y} dz\cdot A(z)\Big\}\Big(q(0)\Big)_\beta \Big|p(P_i,S_i)\Big\rangle$$

$$\overset{A^+=0 \text{ gauge}}{\Longrightarrow} \quad \Big\langle p(P_i,S_i)\Big| \Big(\bar{q}(y)\Big)_\alpha \Big(q(0)\Big)_\beta \Big|p(P_i,S_i)\Big\rangle$$

gauge links for SIDIS and DY

DY: $sin(\phi_{\ell^+\ell^-} - \phi_S)$ asymmetry in $p+p\Big|_{UT}$



for a pseudo T-odd quantity this leads to a minus sign

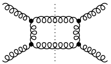
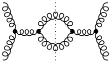
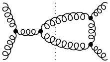
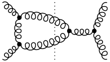
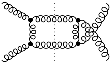
The Sivers function

$$f_{1T}^{\perp}(x, k_\perp, \zeta)\Big|_{DIS} \;=\; -\, f_{1T}^{\perp}(x, k_\perp, \zeta)\Big|_{DY}$$

The structure of gauge links was worked out in detail in

C.J. Bomhof, P.J. Mulders and F. Pijlman; EPJ C47 (2006) 147; arXiv hep-ph/0601171

'The construction of gauge-links in arbitrary hard processes'

	$\Phi_g \propto \langle \frac{1}{2N_c^2}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[+]\dagger}\,F(0)\,\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\,] + \frac{1}{2}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[+]\dagger}\,F(0)\,\mathcal{U}^{[+]}\,]\Big(\frac{\mathrm{Tr}\,[\mathcal{U}^{[\square]}]}{N_c}\,\frac{\mathrm{Tr}\,[\mathcal{U}^{[\square]\dagger}]}{N_c} - \frac{3}{N_c^2}\Big) + \frac{1}{2}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[+]\dagger}\,F(0)\Big\{\frac{1}{2}\,\frac{\mathrm{Tr}\,[\mathcal{U}^{[\square]}]}{N_c}\mathcal{U}^{[-]} + \frac{1}{N_c^2}\mathcal{U}^{[+]}\Big\}\,] + \frac{1}{2}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[-]\dagger}\,F(0)\Big\{\frac{1}{2}\,\frac{\mathrm{Tr}\,[\mathcal{U}^{[\square]\dagger}]}{N_c}\mathcal{U}^{[+]} + \frac{1}{N_c^2}\mathcal{U}^{[-]}\Big\}\,]\,\rangle$ $\Delta_g \propto \langle \frac{1}{2N_c^2}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[\square]}\mathcal{U}^{[-]\dagger}\,F(0)\,\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[-]}\,] + \frac{1}{2}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[-]\dagger}\,F(0)\,\mathcal{U}^{[-]}\,]\Big(\frac{\mathrm{Tr}\,[\mathcal{U}^{[\square]}]}{N_c}\,\frac{\mathrm{Tr}\,[\mathcal{U}^{[\square]\dagger}]}{N_c} - \frac{3}{N_c^2}\Big) + \frac{1}{2}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[+]\dagger}\,F(0)\Big\{\frac{1}{2}\,\frac{\mathrm{Tr}\,[\mathcal{U}^{[\square]}]}{N_c}\mathcal{U}^{[-]} + \frac{1}{N_c^2}\mathcal{U}^{[+]}\Big\}\,] + \frac{1}{2}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[-]\dagger}\,F(0)\Big\{\frac{1}{2}\,\frac{\mathrm{Tr}\,[\mathcal{U}^{[\square]\dagger}]}{N_c}\mathcal{U}^{[+]} + \frac{1}{N_c^2}\mathcal{U}^{[-]}\Big\}\,]\,\rangle$
  	$\Phi_g \propto \langle \frac{1}{2}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[+]\dagger}\,F(0)\,\mathcal{U}^{[-]}\,]\frac{\mathrm{Tr}\,[\mathcal{U}^{[\square]}]}{N_c} + \frac{1}{2}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[-]\dagger}\,F(0)\,\mathcal{U}^{[+]}\,]\frac{\mathrm{Tr}\,[\mathcal{U}^{[\square]\dagger}]}{N_c} - \frac{1}{2N_c}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[\square]}\,]\,\mathrm{Tr}[\,F(0)\,\mathcal{U}^{[\square]\dagger}\,] - \frac{1}{2N_c}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[\square]\dagger}\,]\,\mathrm{Tr}[\,F(0)\,\mathcal{U}^{[\square]}\,]\,\rangle$ $\Delta_g \propto \langle \frac{1}{2}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[+]\dagger}\,F(0)\,\mathcal{U}^{[-]}\,]\frac{\mathrm{Tr}\,[\mathcal{U}^{[\square]}]}{N_c} + \frac{1}{2}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[-]\dagger}\,F(0)\,\mathcal{U}^{[+]}\,]\frac{\mathrm{Tr}\,[\mathcal{U}^{[\square]\dagger}]}{N_c} - \frac{1}{2N_c}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[\square]}\,]\,\mathrm{Tr}[\,F(0)\,\mathcal{U}^{[\square]\dagger}\,] - \frac{1}{2N_c}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[\square]\dagger}\,]\,\mathrm{Tr}[\,F(0)\,\mathcal{U}^{[\square]}\,]\,\rangle$
	$\Phi_g \propto \langle \frac{1}{N_c^2}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[+]\dagger}\,F(0)\,\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\,] + \mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[+]\dagger}\,F(0)\,\mathcal{U}^{[+]}\,]\Big(\frac{\mathrm{Tr}\,[\mathcal{U}^{[\square]}]}{N_c}\,\frac{\mathrm{Tr}\,[\mathcal{U}^{[\square]\dagger}]}{N_c} - \frac{3}{N_c^2}\Big) + \frac{1}{N_c^2}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[+]\dagger}\,F(0)\,\mathcal{U}^{[+]}\,] + \frac{1}{N_c^2}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[-]\dagger}\,F(0)\,\mathcal{U}^{[-]}\,] + \frac{1}{2N_c}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[\square]}\,]\,\mathrm{Tr}[\,F(0)\,\mathcal{U}^{[\square]\dagger}\,] + \frac{1}{2N_c}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[\square]\dagger}\,]\,\mathrm{Tr}[\,F(0)\,\mathcal{U}^{[\square]}\,]\,\rangle$ $\Delta_g \propto \langle \frac{1}{N_c^2}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[\square]}\mathcal{U}^{[-]\dagger}\,F(0)\,\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[-]}\,] + \mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[-]\dagger}\,F(0)\,\mathcal{U}^{[-]}\,]\Big(\frac{\mathrm{Tr}\,[\mathcal{U}^{[\square]}]}{N_c}\,\frac{\mathrm{Tr}\,[\mathcal{U}^{[\square]\dagger}]}{N_c} - \frac{3}{N_c^2}\Big) + \frac{1}{N_c^2}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[+]\dagger}\,F(0)\,\mathcal{U}^{[+]}\,] + \frac{1}{N_c^2}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[-]\dagger}\,F(0)\,\mathcal{U}^{[-]}\,] + \frac{1}{2N_c}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[\square]}\,]\,\mathrm{Tr}[\,F(0)\,\mathcal{U}^{[\square]\dagger}\,] + \frac{1}{2N_c}\,\mathrm{Tr}[\,F(\xi)\,\mathcal{U}^{[\square]\dagger}\,]\,\mathrm{Tr}[\,F(0)\,\mathcal{U}^{[\square]}\,]\,\rangle$

It has to be decided experimentally whether

- the Sivers and Boer-Mulders- Asymmetries are large and whether observed asymmetries are polluted by higher twist effects
- there are sign changes between SIDIS and DY

This requires improved SIDIS and DY experiments in the valence quark region.

$\Rightarrow$ F. Bradamantes talk

Conceptual problems: The definition of TMDs typically requires a cut-off which spoils Ward-identities needed to guarantee the cancellation of divergencies

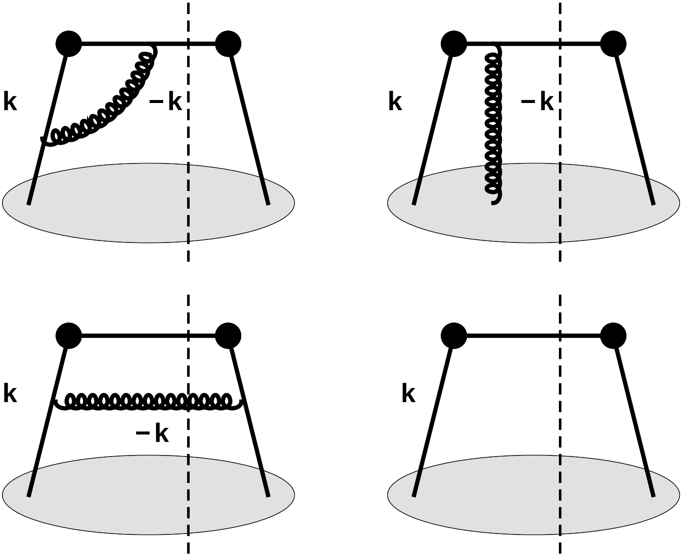
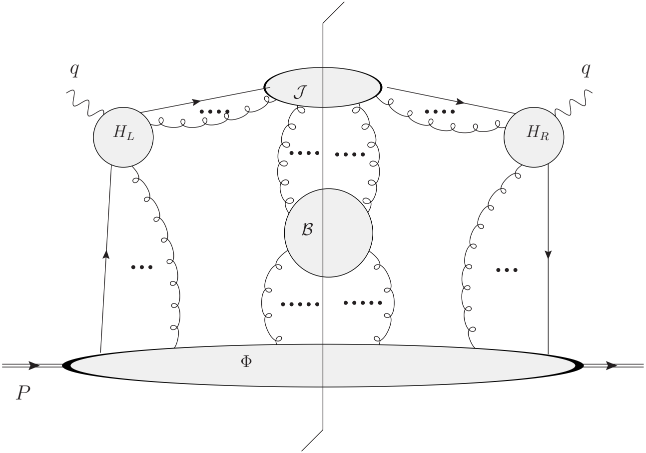
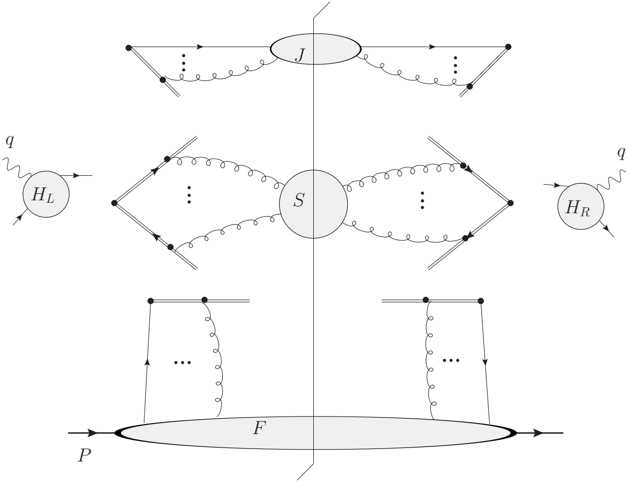


Illustration: A recent suggestion by Collins et al.



using Ward identities this can be factorized into



The proposed factorization formula

$$
\begin{aligned}
F_1 &= P_{\mu\nu} W^{\mu\nu} \\
&= \int \frac{d^4 k_{\mathrm{T}}}{(2\pi)^4} \frac{d^4 k_{\mathrm{J}}}{(2\pi)^4} \frac{d^4 k_S}{(2\pi)^4} (2\pi)^4 \delta^{(4)}(q + P - k_{\mathrm{T}} - k_{\mathrm{J}} - k_S) \\
&\times \left| H(Q, \mu) \right|^2 S(k_S, y_{\mathrm{T}}, y_{\mathrm{J}}, \mu)\, F(k_{\mathrm{T}}, y_p, y_{\mathrm{T}}, y_s, \mu)\, J(k_{\mathrm{J}}, y_{\mathrm{J}}, y_s, \mu)
\end{aligned}
$$

with

$$\tilde{F}_{\text{mod}}(w, y_p, y_{\text{T}}, y_s, \mu) = \frac{\langle p|\bar{\psi}(w)\, V_w^\dagger(n_s)\, I_{n_s;w,0}\, \frac{\gamma^+}{2}\, V_0(n_s)\psi(0)|p\rangle_R}{\langle 0|\, I^\dagger_{n_{\text{T}};w,0}\, V_w(n_{\text{T}})\, V_w^\dagger(n_s)\, I_{n_s;w,0}\, V_0(n_s)\, V_0^\dagger(n_{\text{T}})|0\rangle_R}$$

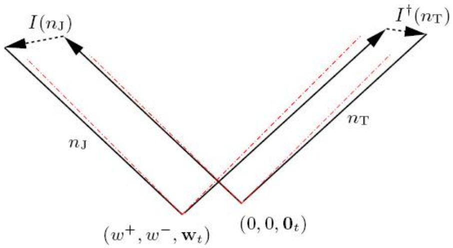


FIG. 8: Structure of the Wilson loop that appears in the definition of the soft factor (43). Dot-dashed lines indicate exactly light-like directions.

Conclusions

- Hadron structure physics has two motivations
  - To understand the phenomenology resulting from QCD
  - To provide input for e.g. the search for New Physics
- This combination of experiment, pQCD and lattice QCD can provide an unprecedentedly detailed picture
- Improvements in the kinematic region of fixed target physics is still needed for: $s(x, Q^2)$, $D_q^K(z, \mu^2)$, $\Delta\bar{u} - \Delta\bar{d}$, $\Delta g(x, Q^2)$, $\delta q(x, Q^2)$
- GPD-physics needs all the experimental input it can get.
- Naive T-odd asymmetries are sensitive to non-trivial gauge-link physics. They have to be investigated in SIDIS and DY experiments. Such studies could trigger fundamental theoretical developments going beyond collinear factorization.

- Many facilities aiming at this physics are under discussion/planning/construction but in times like these perspectives are unclear, and this is not a strong argument against a fixed-target program at CERN.