Scientific plans of the DIRAC experiment beyond 2010

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New Opportunities in the Physics Landscape at CERN
11 May 2009
Prospects of DIRAC

Creation of an intense source of $\pi\pi$, $\pi K$ and other exotic atoms at SPS proton beam and using them for accurate measurements of all S-wave $\pi\pi$ and $\pi K$ scattering length to check the precise low energy $QCD$ predictions.
DIRAC collaboration

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Geneva, Switzerland

Czech Technical University
Prague, Czech Republic

Institute of Physics ASCR
Prague, Czech Republic

Nuclear Physics Institute ASCR
Rez, Czech Republic

Trieste University and INFN-Trieste
Trieste, Italy

University of Messina
Messina, Italy

KEK
Tsukuba, Japan

Kyoto Sangyou University
Kyoto, Japan

Tokyo Metropolitan University
Tokyo, Japan

IFIN-HH
Bucharest, Romania

JINR
Dubna, Russia

SINP of Moscow State University
Moscow, Russia

IHEP
Protvino, Russia

Santiago de Compostela University
Santiago de Compostela, Spain

Bern University
Bern, Switzerland

Zurich University
Zurich, Switzerland
Outline

• Low-energy QCD precise predictions
• Method of $\pi\pi$ and $\pi K$ atoms lifetime measurement
• DIRAC setup
• Results on the $\pi\pi$ scattering lengths measurement
• Evidence for $\pi K$ atoms
• Request of 2010 run for observation of the long-lived states of $\pi\pi$ atoms. Prospects for the Lamb-shift measurement.
• New prospects of DIRAC at SPS CERN
Theoretical motivation

Standard Model

$L_{\text{weak}}$  
HIGH energy  
(small distance)

$L_{QCD}$  
$Q>>$

Low energy  
(large distance)

$L_{QED}$

$perturbative\ QCD:\ L_{QCD}(q,g)$  
Interaction→ „weak“  
(asympt. freedom):  
expansion in coupling.  
Check only $L_{\text{sym}}(m_q<<)$

$chiral\ sym.\ &\ breaking:\ L_{\text{eff}}(\text{GB}:\ \pi, K, \eta)$  
Interaction→ „strong“  
(confinement) - but:  
expansion in mom. & mass.  
Check $L_{\text{sym}}$ as well as $L_{\text{non-sym}}$

$L_{QCD} = L_{\text{sym}} + L_{\text{non-sym}}$  
(chiral symmetry)

spontaneously broken symmetry  
quark-condensate
Theoretical status

In ChPT the effective Lagrangian, which describes the $\pi\pi$ interaction, is an expansion in (even) terms:

$$L_{\text{eff}} = L^{(2)} + L^{(4)} + L^{(6)} + \cdots$$

(1-loop) (2-loop)

Colangelo et al. in 2001, using ChPT (2-loop) & Roy equations:

$$L_{\text{eff}}^{(2)} = 0.220 \pm 2.3\%$$
$$L_{\text{eff}}^{(4)} = -0.0444 \pm 2.3\%$$
$$L_{\text{eff}}^{(6)}$$

These results (precision) depend on the low-energy constants (LEC) $l_3$ and $l_4$:

Lattice gauge calculations from 2006 provided values for these $l_3$ and $l_4$.

$$a_0 = 0.220 \pm 2.3\%$$
$$a_2 = -0.0444 \pm 2.3\%$$

Because $l_3$ and $l_4$ are sensitive to the quark condensate, precision measurements of $a_0, a_2$ are a way to study the structure of the QCD vacuum.
The lifetime of $\pi^+\pi^-$ atoms is dominated by the annihilation process into $\pi^0\pi^0$:

$$\Gamma = \frac{1}{\tau} = \Gamma_{2\pi_0} + \Gamma_{2\gamma} \quad \text{with} \quad \frac{\Gamma_{2\gamma}}{\Gamma_{2\pi_0}} \approx 4 \times 10^{-3}$$

$$\Gamma_{1S,2\pi_0} = R |a_0 - a_2|^2 \quad \text{with} \quad \frac{\Delta R}{R} \approx 1.2\%$$

$$\tau = (2.9 \pm 0.1) \times 10^{-15} \text{ s}$$

$a_0$ and $a_2$ are the $\pi\pi$ S-wave scattering lengths for isospin I=0 and I=2.

If $$\frac{\Delta \tau}{\tau} = 4\% \quad \Rightarrow \quad \frac{\Delta |a_0 - a_2|}{|a_0 - a_2|} = 2\%$$
\[ \begin{align*} 
I. \text{ ChPT predicts s-wave scattering lengths:} \\
& a_0^{1/2} = 0.19 \pm 0.2 \quad a_0^{3/2} = -0.05 \pm 0.02 \quad \text{V. Bernard, N. Kaiser, U. Meissner. – 1991} \\
& \text{ and loop } L^{(2)}, L^{(4)} \\
& a_0^{1/2} - a_0^{3/2} = 0.23 \pm 0.01 \quad \text{A. Rossel. – 1999} \\
\end{align*} \]

**\( \pi K \) scattering lengths**

II. Roy-Steiner equations:

\[ a_0^{1/2} - a_0^{3/2} = 0.269 \pm 0.015 \quad \text{P.Büttiker et al. – 2004} \]
**$K^+\pi^-$ and $K^-\pi^+$ atoms lifetime**

$K\pi$-atom ($A_{K\pi}$) is a hydrogen-like atom consisting of $K^+$ and $\pi^-$ mesons:

$$E_B = -2.9 \text{ keV} \quad r_B = 248 \text{ fm} \quad p_B \approx 0.8 \text{ MeV}$$

The $K\pi$-atom lifetime (ground state 1S), $\tau = 1/\Gamma$ is dominated by the annihilation process into $K^0\pi^0$:

$$A_{K^+\pi^-} \rightarrow \pi^0 K^0 \quad A_{\pi^+K^-} \rightarrow \pi^0 \bar{K}^0$$

The lifetime is given by

$$\Gamma_{1S, K^0\pi^0} = R_K \left| a_{1/2} - a_{3/2} \right|^2 \quad \text{with} \quad \frac{\Delta R_K}{R_K} \approx 2\%$$

From Roy-Steiner equations:

$$a_{0}^{1/2} - a_{0}^{3/2} = 0.269 \pm 0.015$$

$$\tau = (3.7 \pm 0.4) \times 10^{-15} \text{ s}$$

If

$$\frac{\Delta \Gamma}{\Gamma} = 20\% \quad \Rightarrow \quad \frac{\Delta \left| a_{1/2} - a_{3/2} \right|}{\left| a_{1/2} - a_{3/2} \right|} = 10\%$$
The measurement of the $s$-wave $\pi K$ scattering lengths would test our understanding of the chiral $SU(3)_L \times SU(3)_R$ symmetry breaking of QCD ($u$, $d$ and $s$ quarks), while the measurement of $\pi\pi$ scattering lengths checks only the $SU(2)_L \times SU(2)_R$ symmetry breaking ($u$, $d$ quarks).

This is the principal difference between $\pi\pi$ and $\pi K$ scattering!

Experimental data on the $\pi K$ low-energy phases are absent.
Method of $A_{2\pi}$ observation and lifetime measurement

$\tau(A_{2\pi})$ is too small to be measured directly
e. m. interaction of $A_{2\pi}$ in the target

\[ A_{2\pi} \rightarrow \pi^+\pi^- \]

\[ Q < 3\text{MeV/c}, \Theta_{\text{lab}} < 3 \text{ mrad} \]

- Coulomb from short-lived sources
- non-Coulomb from long-lived sources

**Main features of the DIRAC set-up**

- Thin targets: $\sim 7 \times 10^{-3} X_0$
- Nuclear efficiency: $3 \times 10^{-4}$
- Vacuum magnetic spectrometer
- Proton beam $\sim 10^{11}$ proton/spill
- Resolution on $Q$: $Q_x \approx Q_y \approx 0.3 \text{ MeV/c}, Q_L \approx 0.5 \text{ MeV/c}$

**The same method for $A_{\pi K}$**

but: $p_K = \frac{m_K}{m_\pi} p_\pi$
Solution of the transport equations provides one-to-one dependence of the measured break-up probability ($P_{br}$) on pionium lifetime $\tau$.

All targets have the same thickness in radiation lengths $6.7\times10^{-3} X_0$.

There is an optimal target material for a given lifetime.
DIRAC preliminary results with GEM/MSGC

$Q_L$ distribution

$\leftarrow$ All events

$\leftarrow$ After background subtraction
DIRAC preliminary results with GEM/MSGC

\( Q_T \) distribution

\( Q_L < 2 \text{ MeV/c} \)

\( Q_L > 2 \text{ MeV/c} \)

← After background subtraction for \( Q_L < 2 \text{MeV/c} \)
DIRAC Experimental results

$A_{2\pi}$ lifetime

2005 DIRAC (PL B619, 50)  
$\tau = \begin{pmatrix} 2.91 & +0.45 \\ -0.38 & +0.19 \end{pmatrix}_{\text{stat}} \begin{pmatrix} 0.49 \\ -0.62 \end{pmatrix}_{\text{syst}}$  
$\text{fs} = \begin{pmatrix} \ldots & +0.49 \\ \ldots & -0.62 \end{pmatrix}_{\text{tot}}$  

...based on 2001 data (6530 observed atoms)  

$\Rightarrow |a_0 - a_2| = 0.264 \pm 7.2\%_{\text{stat}} \pm 10\%_{\text{syst}} = \pm 13\%_{\text{tot}}$

2008 DIRAC (SPSC 22/04/08)  
$\tau = \begin{pmatrix} 2.82 & +0.25 \\ -0.23 & +0.19 \end{pmatrix}_{\text{stat}} \begin{pmatrix} 0.49 \\ -0.30 \end{pmatrix}_{\text{syst}}$  
$\text{fs} = \begin{pmatrix} \ldots & +0.31 \\ \ldots & -0.30 \end{pmatrix}_{\text{tot}}$  

...major part 2001-03 data (13300 observed atoms)  

$\Rightarrow |a_0 - a_2| = 0.268 \pm 4.4\%_{\text{stat}} \pm 3.7\%_{\text{syst}} = \pm 5.5\%_{\text{tot}}$

Including GEM/MicroStripGasChambers $\Rightarrow$ number of reconstructed events is 17000  
$\Rightarrow$ the statistical error in $|a_0-a_2|$ is 3%, and the expected full error is <5%.
Comparison with other experimental results

$K \rightarrow 3\pi$:

2006 NA48/2 (PL B633, 173) ...with ChPT constraint between $a_0$ and $a_2$:

$$a_0 - a_2 = 0.264 \pm 2.3%_{\text{stat}} \pm 1.5%_{\text{syst}} \pm 4.9%_{\text{ext}} = \ldots \pm 5.6%_{\text{tot}}$$

2009 NA48/2 (seminar at CERN) ...without constraint ($a_2$ free):

$$a_0 - a_2 = 0.257 \pm 1.9%_{\text{stat}} \pm 0.8%_{\text{syst}} \pm 0.4%_{\text{ext}} \pm 3.5%_{\text{th}} = \ldots \pm 4.1%_{\text{tot}}$$

$$a_2 = -0.024 \pm 54%_{\text{stat}} \pm 38%_{\text{syst}} \pm 8.3%_{\text{ext}} \pm 63%_{\text{th}} = \ldots \pm 92%_{\text{tot}}$$

...with ChPT constraint between $a_0$ and $a_2$:

$$a_0 - a_2 = 0.263 \pm 0.8%_{\text{stat}} \pm 0.4%_{\text{syst}} \pm 0.8%_{\text{ext}} \pm 1.9%_{\text{th}} = \ldots \pm 2.2%_{\text{tot}}$$
Comparison with other experimental results

**Ke4:**

2008 **NA48/2** (EPJ C54, 411)

...without constraint (a₂ free):

\[
\Rightarrow a_2 = -0.0471 \pm 23\% \left| _{stat} \pm 8.5\% \right| _{syst} = \ldots \pm 25\% \left| _{tot} \right.
\]

\[
\Rightarrow a_0 = 0.233 \pm 6.9\% \left| _{stat} \pm 3.0\% \right| _{syst} = \ldots \pm 7.5\% \left| _{tot} \right.
\]

2009 **NA48/2** (seminar at CERN)

...without constraint (a₂ free):

\[
\Rightarrow a_2 = -0.0432 \pm 20\% \left| _{stat} \pm 7.9\% \right| _{syst} \pm 6.5\% \left| _{th} \right. = \ldots \pm 22\% \left| _{tot} \right.
\]

\[
\Rightarrow a_0 = 0.2220 \pm 5.8\% \left| _{stat} \pm 2.3\% \right| _{syst} \pm 1.7\% \left| _{th} \right. = \ldots \pm 6.5\% \left| _{tot} \right.
\]

...with ChPT constraint between a₀ and a₂:

\[
\Rightarrow a_0 = 0.2206 \pm 2.2\% \left| _{stat} \pm 0.8\% \right| _{syst} \pm 2.9\% \left| _{th} \right. = \ldots \pm 3.7\% \left| _{tot} \right.
\]
Trajectories of $\pi^-$ and $K^+$ from the $A_{K\pi}$ break-up

The numbers to the right of the tracks lines are the $\pi^-$ and $K^+$ momenta in GeV/c.

The $A_{K\pi}$, $\pi^-$ and $K^+$ momenta are shown in the following table:

<table>
<thead>
<tr>
<th>$P_{\text{atom}}$ (GeV/c)</th>
<th>$P_\pi$ (GeV/c)</th>
<th>$P_K$ (GeV/c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.13</td>
<td>1.13</td>
<td>4.0</td>
</tr>
<tr>
<td>5.77</td>
<td>1.27</td>
<td>4.5</td>
</tr>
<tr>
<td>6.41</td>
<td>1.41</td>
<td>5.0</td>
</tr>
<tr>
<td>10.26</td>
<td>2.26</td>
<td>8.0</td>
</tr>
</tbody>
</table>
single and multilayer targets

Upgraded DIRAC experimental setup

MDC, 18 planes
SFD
IH

π⁺,ρ,Κ⁺
π⁻,¯ρ,Κ⁻

shield1
vacuum

shield2
vacuum

MDC
IH

modified parts
Upgraded DIRAC experimental setup
Downstream detectors
In total:
173±54 πK-atomic pairs are observed with a significance of 3.2σ.

τ > 0.8 × 10^{-15} s at 90%CL

Beam request for 2010

Observation of the long-lived states of $A_{2\pi}$ is opening a possibility to measure the Lamb shift and to determine the new combination of $\pi\pi$ scattering lengths $2a_0 + a_2$.

For this observation, which was planed in our addendum, we need the run in 2010 during around 5 months in the same conditions as in 2009.
Energy splitting between np - ns states in $\pi^+\pi^-$ atom

$$\Delta E_n \equiv E_{ns} - E_{np}$$

$$\Delta E_n \approx \Delta E_{n\text{vac}} + \Delta E_{n\text{s}}$$

$$\Delta E_{n\text{vac}} \approx -0.107 \text{ eV from QED calculations}$$

$$\Delta E_{n\text{s}} \approx -0.45 \text{ eV numerical estimated value from ChPT}$$

$$a_0 = 0.220 \pm 0.005$$

$$a_2 = -0.0444 \pm 0.0010$$


$$\Rightarrow \Delta E_2 \approx -0.56 \text{ eV}$$

(1979) A. Karimkhodzhaev and R. Faustov

(1983) G. Austen and J. de Swart

(1986) G. Efimov et al.


(1999) A. Gashi et al.

(2000) D. Eiras and J. Soto

A. Rusetsky, priv. comm.
For $p_A = 5.6$ GeV/c and $\gamma = 20$

\[
\begin{align*}
\tau_{1s} &= 2.9 \times 10^{-15} \text{ s} , \\
\tau_{2s} &= 2.3 \times 10^{-14} \text{ s} , \\
\tau_{2p} &= 1.17 \times 10^{-11} \text{ s} , \\
\lambda_{1s} &= 1.7 \times 10^{-3} \text{ cm} , \\
\lambda_{2s} &= 1.4 \times 10^{-2} \text{ cm} , \\
\lambda_{2p} &= 7 \text{ cm} , \\
\lambda_{3p} &\approx 23 \text{ cm} , \\
\lambda_{4p} &\approx 54 \text{ cm}.
\end{align*}
\]

Metastable Atoms

Illustration for observation of the $A_{2\pi}$ long-lived states with breaking $^{25}$ foil.
Metastable Atoms

Probabilities of the $A_{2\pi}$ breakup (Br) and yields of the long-lived states for different targets provided the maximum yield of summed population of the long-lived states: $\Sigma(l \geq 1)$

<table>
<thead>
<tr>
<th>Target Z</th>
<th>Thickness $\mu$</th>
<th>Br</th>
<th>$\Sigma$ $(l \geq 1)$</th>
<th>2p$_0$</th>
<th>3p$_0$</th>
<th>4p$_0$</th>
<th>$\Sigma$ $(l =1, m = 0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>04</td>
<td>100</td>
<td>4.45%</td>
<td>5.86%</td>
<td>1.05%</td>
<td>0.46%</td>
<td>0.15%</td>
<td>1.90%</td>
</tr>
<tr>
<td>06</td>
<td>50</td>
<td>5.00%</td>
<td>6.92%</td>
<td>1.46%</td>
<td>0.51%</td>
<td>0.16%</td>
<td>2.52%</td>
</tr>
<tr>
<td>13</td>
<td>20</td>
<td>5.28%</td>
<td>7.84%</td>
<td>1.75%</td>
<td>0.57%</td>
<td>0.18%</td>
<td>2.63%</td>
</tr>
<tr>
<td>28</td>
<td>5</td>
<td>9.42%</td>
<td>9.69%</td>
<td>2.40%</td>
<td>0.58%</td>
<td>0.18%</td>
<td>3.29%</td>
</tr>
<tr>
<td>78</td>
<td>2</td>
<td>18.8%</td>
<td>10.5%</td>
<td>2.70%</td>
<td>0.54%</td>
<td>0.16%</td>
<td>3.53%</td>
</tr>
</tbody>
</table>
Metastable Atoms - Backgrounds

5 μ Ni → atomic pairs

100 μ Ni → Coulomb background

Q_T, MeV/c  Q_L, MeV/c  Q, MeV/c
\[ \frac{d\sigma_A^{nlm}}{d\vec{P}} = (2\pi)^3 \frac{E}{M} |\psi_{nlm}^{(C)}(0)|^2 \frac{d\sigma_s^0}{dp_1 dp_2} \cdot \frac{d\sigma}{dp_1} \cdot \frac{d\sigma}{dp_2} \]

for atoms \( \vec{v}_1 = \vec{v}_2 \) where \( \vec{v}_1, \vec{v}_2 \) – velocities of particles in the L.S. for all types of atoms

for \( A_{2\pi} \) production \( \vec{p}_1 = \vec{p}_2 \)

for \( A_{\pi K} \) production \( \vec{p}_\pi = \frac{m_\pi}{m_K} \vec{p}_K \)
Inclusive cross-sections for $\pi^+$, $\pi^-$ - mesons generation

$E_p = 450 GeV \quad \theta_L = 0^\circ$
Inclusive cross-sections for $K^+$, $K^-$ mesons generation

$E_p = 450\,\text{GeV}$ \hspace{1cm} $\theta_L = 0^\circ$
$A_{2\pi}$ momentum distributions (5.7°)

\[
\theta_L = 5.7^\circ \pm 1.3^\circ \quad E_p = 24\text{GeV}
\]

\[
\theta_L = 5.7^\circ \pm 1.3^\circ \quad E_p = 450\text{GeV}
\]
$A_{2\pi}$ momentum distributions (0-2°)

$\theta_L = 2^\circ \pm 1.3^\circ$ $E_p = 450 GeV$

$\theta_L = 0^\circ \pm 1.3^\circ$ $E_p = 450 GeV$
$A_{2\pi}$ and $A_{\pi K}$ momentum distributions

- red curve atom spectra in channel the aperture
- green curve atom spectra registered by the set-up
Yields of atoms at PS and SPS

Yield of dimeson atoms per one proton-Ni interaction, detectable by DIRAC upgrade setup at $\Theta_L=5.7^\circ$

<table>
<thead>
<tr>
<th>$E_p$</th>
<th>$A_{2\pi}$</th>
<th>$A_{K^+\pi^-}$</th>
<th>$A_{\pi^+K^-}$</th>
<th>$A_{2\pi}$</th>
<th>$A_{K^+\pi^-}$</th>
<th>$A_{\pi^+K^-}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_A$</td>
<td>$1.1\cdot10^{-9}$</td>
<td>$0.52\cdot10^{-10}$</td>
<td>$0.29\cdot10^{-10}$</td>
<td>$0.13\cdot10^{-7}$</td>
<td>$0.10\cdot10^{-8}$</td>
<td>$0.71\cdot10^{-9}$</td>
</tr>
<tr>
<td>$W_A/W_{\pi}$</td>
<td>$3.4\cdot10^{-8}$</td>
<td>$16.\cdot10^{-10}$</td>
<td>$9.\cdot10^{-10}$</td>
<td>$1.3\cdot10^{-7}$</td>
<td>$1.\cdot10^{-8}$</td>
<td>$7.1\cdot10^{-9}$</td>
</tr>
<tr>
<td>$W_A^N/W_{\pi}^N$</td>
<td>1.</td>
<td>1.</td>
<td>1.</td>
<td>3.8</td>
<td>6.2</td>
<td>8.</td>
</tr>
<tr>
<td>Total gain</td>
<td>1.</td>
<td>1.</td>
<td>1.</td>
<td>15.</td>
<td>25.</td>
<td>32.</td>
</tr>
</tbody>
</table>

A multiplier due to different spill duration $\sim 4$
**DIRAC prospects at SPS CERN**

Present low-energy QCD predictions for $\pi\pi$ and $\pi K$ scattering lengths

$\pi\pi$  $\delta a_0 = 2.3\%$  $\delta a_2 = 2.3\%$  $\delta (a_0 - a_2) = 1.5\%$

$\pi K$  $\delta a_{1/2} = 11\%$  $\delta a_{3/2} = 40\%$  $\delta a_{1/2} = 10\%$  $\delta a_{3/2} = 17\%$

...will be improved by Lattice calculations

...will be significantly improved by ChPT

Expected results of DIRAC ADDENDUM at PS CERN after 2008-2009

$\tau(A_{2\pi}) \rightarrow \delta(a_0 - a_2) = \pm 2\%(stat) \pm 1\%(syst) \pm 1\%(theor)$

$\tau(A_{\pi K}) \rightarrow \delta(a_{1/2} - a_{3/2}) = \pm 10\%(stat) \pm \ldots \pm 1.5\%(theor)$

2010-2011 Observation of metastable $\pi^+\pi^-$ atoms and study of a possibility to measure its Lamb shift.

Study of the possibility to observe $K^+K^-$ and $\pi^+\mu^-$ atoms using 2008-2009 data.

**DIRAC at SPS CERN beyond 2011**

$\tau(A_{2\pi}) \rightarrow \delta(a_0 - a_2) = \pm 0.5\%(stat)$

$\tau(A_{\pi K}) \rightarrow \delta(a_{1/2} - a_{3/2}) = \pm 2.5\%(stat)$

$(E_{np} - E_{ns})_{\pi\pi} \rightarrow \delta(2a_0 + a_2)$

$(E_{np} - E_{ns})_{\pi K} \rightarrow \delta(2a_{1/2} + a_{3/2})$
Thank you for your attention
Yields of the long-lived states $2p$ ($m = 0$) as a function of the $A_{2\pi}$ lifetime for Beryllium targets ($Z = 04$). Target thicknesses are given in microns on the right side of the picture.
External magnetic and electric fields

Atoms in a beam are influenced by external magnetic field and the relativistic Lorentz factor

\[ \vec{r} \equiv \text{relative distance between } \pi^+ \text{ and } \pi^- \text{ mesons in } A_{2\pi} \text{ atom} \]
\[ \vec{B}_{\text{Lab}} \equiv \text{laboratory magnetic field} \]

\[ \vec{F} \equiv \text{electric field in the CM system of an } A_{2\pi} \text{ atom} \]

\[ F = \beta \gamma \vec{B}_{\text{Lab}} \approx \gamma \vec{B}_{\text{Lab}} \]
The dependence of $A_{2\pi}$ life time in $2p$-states $\tau_{\text{eff}}$ from a strength of the electric field $F$

$$\tau_{\text{eff}} = \frac{\tau_{2p}}{1 + \frac{1}{4} \frac{\tau_{2p}}{\tau_{2s}} |\xi|^2} = \frac{\tau_{2p}}{1 + 120 |\xi|^2}$$

where:

$$|\xi|^2 \approx \frac{F^2}{(E_{2p} - E_{2s})^2}$$

$B_{\text{Lab}} = 4$ Tesla

$$\begin{cases} 
\gamma = 20, & |\xi| = 0.1 & \Rightarrow & \tau_{\text{eff}} = \frac{\tau_{2p}}{2.2} \\
\gamma = 40, & |\xi| = 0.2 & \Rightarrow & \tau_{\text{eff}} = \frac{\tau_{2p}}{6}
\end{cases}$$