Understanding tracking detectors that rely on ionisation

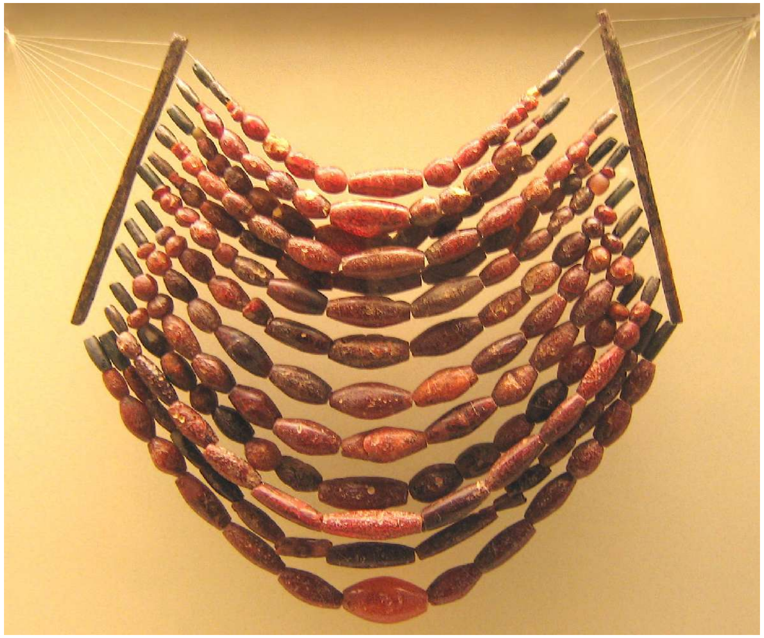
a few steps in the development of the ideas about detection processes, approximately in chronological sequence

Principles of ionisation-based tracking

- These devices work according to similar principles:
  - a charged particle passing through the gas ionises some of the gas molecules;
  - the electric field in the gas volume transports the ionisation electrons and, in some areas, also provokes multiplication;
  - the charge movements (of electrons and ions) lead to induced currents in electrodes, and these currents are recorded.

Austria ... long ago

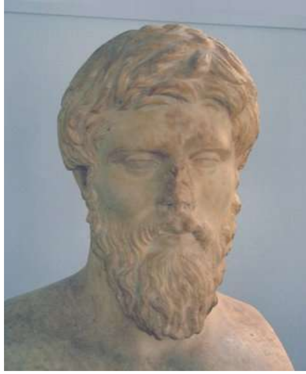
- 3300 BC: Similaun iceman



- 1200-500 BC: Hallstatt culture

  Magdalenenberg collier made of amber, Villingen, ~500 BC

Μέστριος Πλούταρχος
(46 – after 119)



2000–3000 years ago ...

- Herodotus (5th century BC) reports that Thales (6th century BC) knew about amber and magnetite.
- Plutarchos (1st century AD), like others long before him, was aware of static electricity and magnetism:

> Others on this occasion talked very much of antipathies, and produced a thousand instances of such strange effects; for example, the sight of a ram quiets an enraged elephant; a viper lies stock-still, if touched with a beechen leaf; a wild bull grows tame, if bound with the twigs of a fig-tree; and amber draws all light things to it, except basil and such as are dipped in oil; and a loadstone will not draw a piece of iron that is rubbed with onion. Now all these, as to matter of fact, are very evident; but it is hard, if not altogether impossible, to find the cause.

[Plutarchos, Morals, Symposiacs, book II, question VII. Translation: volume III of *The complete works of Plutarch: essays and miscellanies*, Crowell, New York, 1909.]

Magnets

- Magnetic materials were known in China and Greece by the 7th century BC, probably much earlier and elsewhere.
- The earth magnetic field could be detected by the Chinese "spoon" of the 2nd century BC:

- Application to navigation came by 10th-12th century AD.

[More: Smith College (virtual) Museum of Ancient Inventions.]

Giordano (Filippo) Bruno
(1548-1600)

In context 1600-1700

- Science & Mathematics
  - Indian science had just seen centuries of huge activity
  - 1600: Giordano Bruno burned
  - 1614: John Napier introduces logarithms
  - 1687: Isaac Newton publishes the "*Philosophiæ Naturalis Principia Mathematica*"
- Miscellaneous
  - 1533-1603: Elizabeth I
  - 1603: begin of the Tokugawa shogunate
  - 1564-1616: William Shakespeare
  - 1672-1725: Peter the Great, czar of Russia (1682–1725)

William Gilbert
(1544-1603)

1600: “Electric force” introduced

- 1544: William Gilbert born in Colchester
- 1600: De magnete, magneticisque corporibus,
  et de magno magnete tellure.
  - Concluded that the Earth is a magnet;
  - Credited with the first use of the term “electric force”.

> vim illam electricam nobis placet appella-
> re quæ ab humore prouenit

- 1601: Physician to Elizabeth I and James I.

[Guilielmi Gilberti, *De magnete* …, excudebat Petrus Short anno MDC, Londini, courtesy Universidad Complutense de Madrid and Google books]

GREAT has ever been the fame of the loadstone and of amber in the writings of the learned: many philosophers cite the loadstone and also amber whenever, in explaining mysteries, their minds become obfuscated and reason can no farther go.[1]

William Gilbert: origin of amber

For in other bodies is seen a considerable power of attraction, differing from that of the loadstone,—in amber, for example. The Greeks call this substance ἤλεκτρον, because, when heated by rubbing, it attracts to itself chaff; it is certain that amber comes for the most part from the sea: it is gathered on the coast after heavy storms, But it seems to be produced in the earth and at considerable depth below its surface, and to gain consistency under the action of the sea and the saltness of its waters. For at first it was a soft and viscous matter, and hence contains, buried in its mass forevermore (*æternis sepulchris relucentes*), but still (shining) visible, flies, grubs, midges, and ants.

Loadstone: magnet

Amber: fossilised tree resin, called "electron" in Greek

Chaff: husks of grain

Note how he insists on water & hardness

[Extracts from the translation by P. Fleury Mottelay of Book 2 of *De magnete* ..., first published in 1893, republished by Dover in 1958, scanned by Google books.]

such philosophy bears no fruit; for it rests simply on a few Greek or unusual terms—just as our barbers toss off a few Latin words in the hearing of the ignorant rabble in token of their learning, and thus win reputation

William Gilbert: role of *humors*

the earth's mass or rather the earth's framework and its crust consist of a twofold matter, a matter, to wit, that is fluid and humid, and a matter that is firm and dry.

Those that derive their growth mainly from humors,

if they possess sufficient firmness, and after being polished are rubbed, and shine after friction,—such substances attract all bodies presented to them in the air, unless the said bodies be too heavy. For amber and jet are concretions of water;

all bodies that derive their origin principally from humors,

attract all substances, whether humid or dry. Such as are parts of the true substance of the earth or differ but little from that, appear to attract also, but in a very different way, and, so to speak, magnetically:

neither metals, marbles, flints, woods, grasses, flesh, nor various other substances can attract or solicit a body, whether magnetically or electrically (for it pleases us to call electric force that force which has its origin* in humors).

Humor: electric charge

Water: electric

Earth: magnetic

First use of the expression "electric force"

William Gilbert: the *effluvium*

And now, at last, we have to see why corpuscles are drawn toward substances that derive their origin from water,

its force has to be awakened by friction till the substance attains a moderate heat, and gives out an effluvium, and its surface is made to shine.

Effluvium: electric field

All bodies are united and, as it were, cemented together by moisture,

But the peculiar effluvia of electrics, being the subtilest matter of solute moisture, attract corpuscles.'

Attraction

But those thinner effluvia lay hold of the bodies with which they unite, enfold them, as it were, in their arms, and bring them into union with the electrics; and the bodies are led to the electric source, the effluvia having greater force the nearer they are to that.

Relation with distance

such philosophy bears no fruit because few of the philosophers themselves are investigators, or have any first-hand acquaintance with things

William Gilbert

- Studied electrostatic attraction;
- apparently did numerous experiments himself;
- approximate concepts of charge (*humor*) & field (*effluvium*);
- forces decrease with distance;
- attraction is inhibited by intervening material;
- forces are transmitted fast.

- He did not observe repulsion, only attraction;
- problem to explain how an effluvium can at all attract;
- far-fetched relation between friction and attraction.

Francis Bacon,
1st viscount St Alban
(1561–1626)

Francis Bacon: triboluminescence

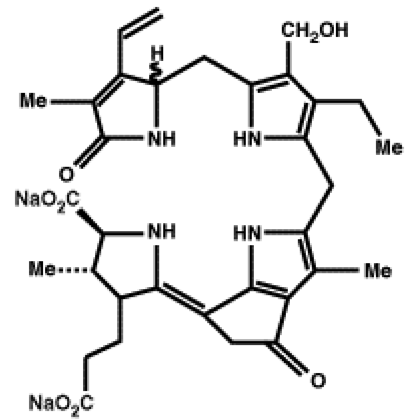
- Noted various mechanical processes that produce light:

> a girl's stomacher [= waistcoat], on being slightly shaken or rubbed, emitted sparks, which was caused perhaps by some alum or salts used in the dye, that stood somewhat thick and formed a crust, and were broken by the friction. It is also most certain that all sugar, whether refined or raw, provided only it be somewhat hard, sparkles when broken or scraped with a knife in the dark. In like manner sea and salt water is sometimes found to sparkle by night when struck violently by oars.

- Such effects became known as friction-light and friction-electricity (from the Greek τρίβος, rubbing).

[Franciscus de Verulamio, *Instauratio Magna* part 2, *Novum organum* book 2, aphorism XII, §11 (John Billius, London, 1620). Translation by James Spedding *et al.* of the works of Francis Bacon (Taggard & Thompson, Boston, 1863).]

Bioluminescence

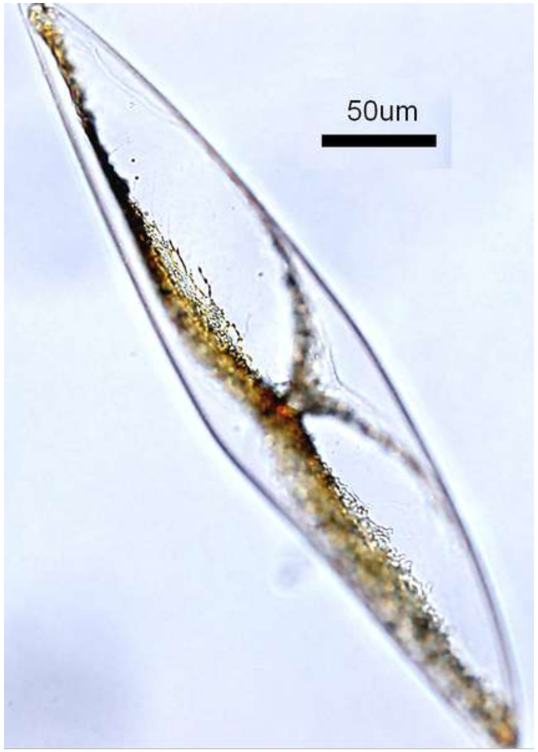


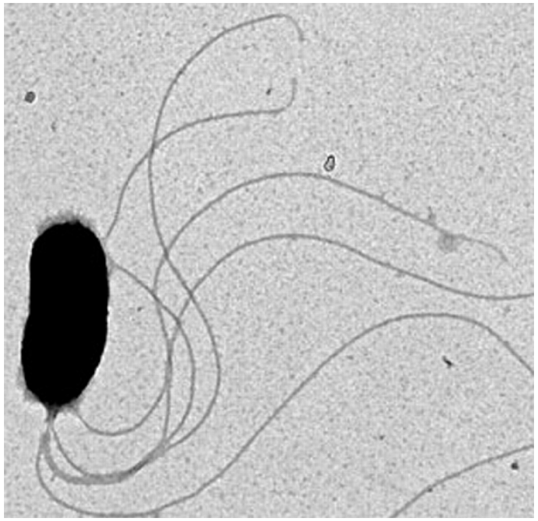
- Bioluminescence of the sea is caused by dinoflagellates (e.g. Pyrocystis Fusiformis) & symbiotic bacteria (e.g. Vibrio Fischeri).

- In dinoflagellates, mechanical deformations trigger a luciferase-catalysed reaction:

$$\text{luciferin} + O_2 \rightarrow \text{oxyluciferin} + \gamma \ (+ CO_2/H_2O)$$

- For further information:
`http://www.lifesci.ucsb.edu/~biolum`





Top: Pyrocystis Fusiformis
Bottom: Vibrio Fischeri
Left: dinoflagellate luciferin

Otto von Guericke
(1602-1686)

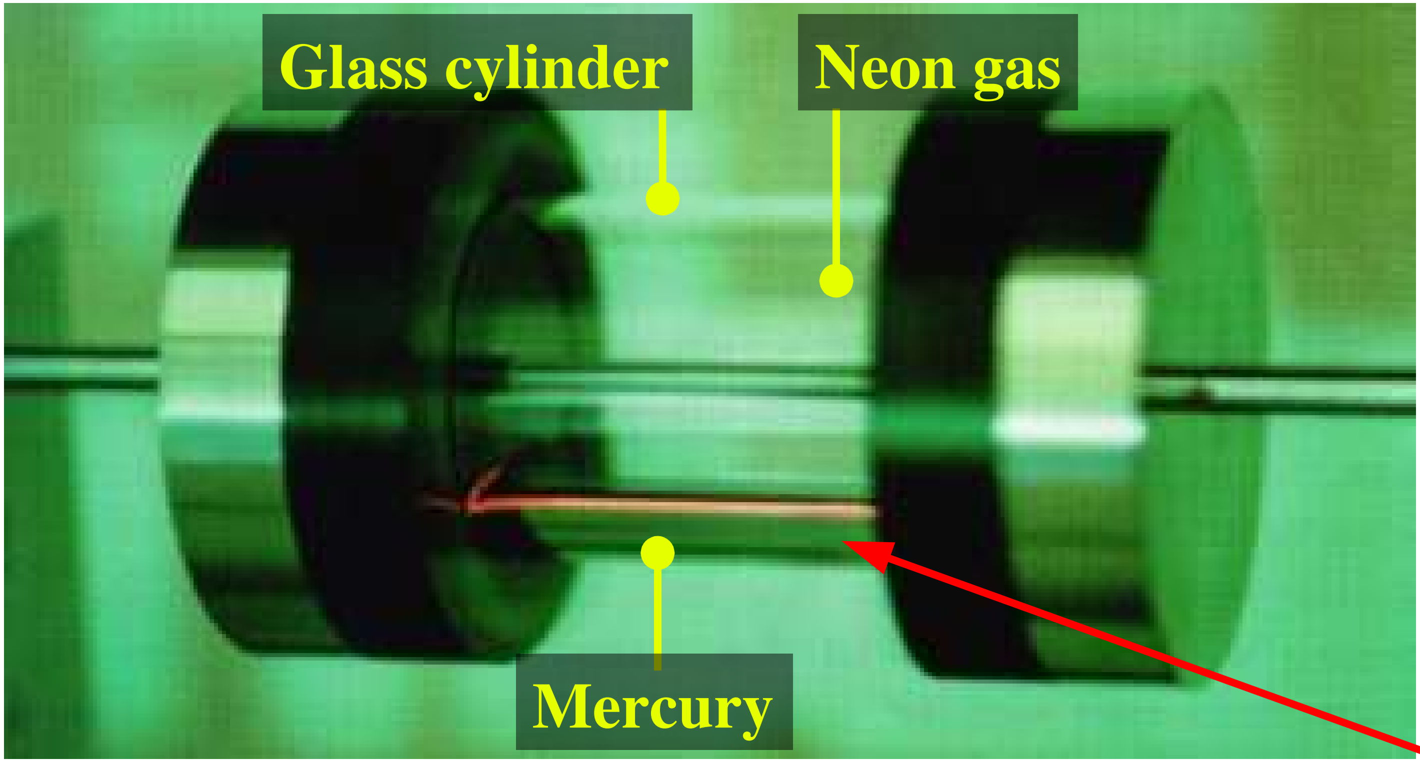
1672: Otto von Guericke

- 1650: Invented an air pump –
  light travels through vacuum, but sound does not
- 1654: Famous for the Magdeburg hemi-spheres
- 1663: Constructed an electric generator
- 1672: Electroluminescence on a glass/sulphur sphere

Jean Picard
(1620-1682)

1675: Jean Picard, barometric light

- “luminous glow appearing in the vacuum above the mercury in a barometer tube when the tube is shaken” [EB]



Axis of rotation

Why red ?

Prominent Hg lines:
435.835 nm (blue) &
546.074 nm (green).

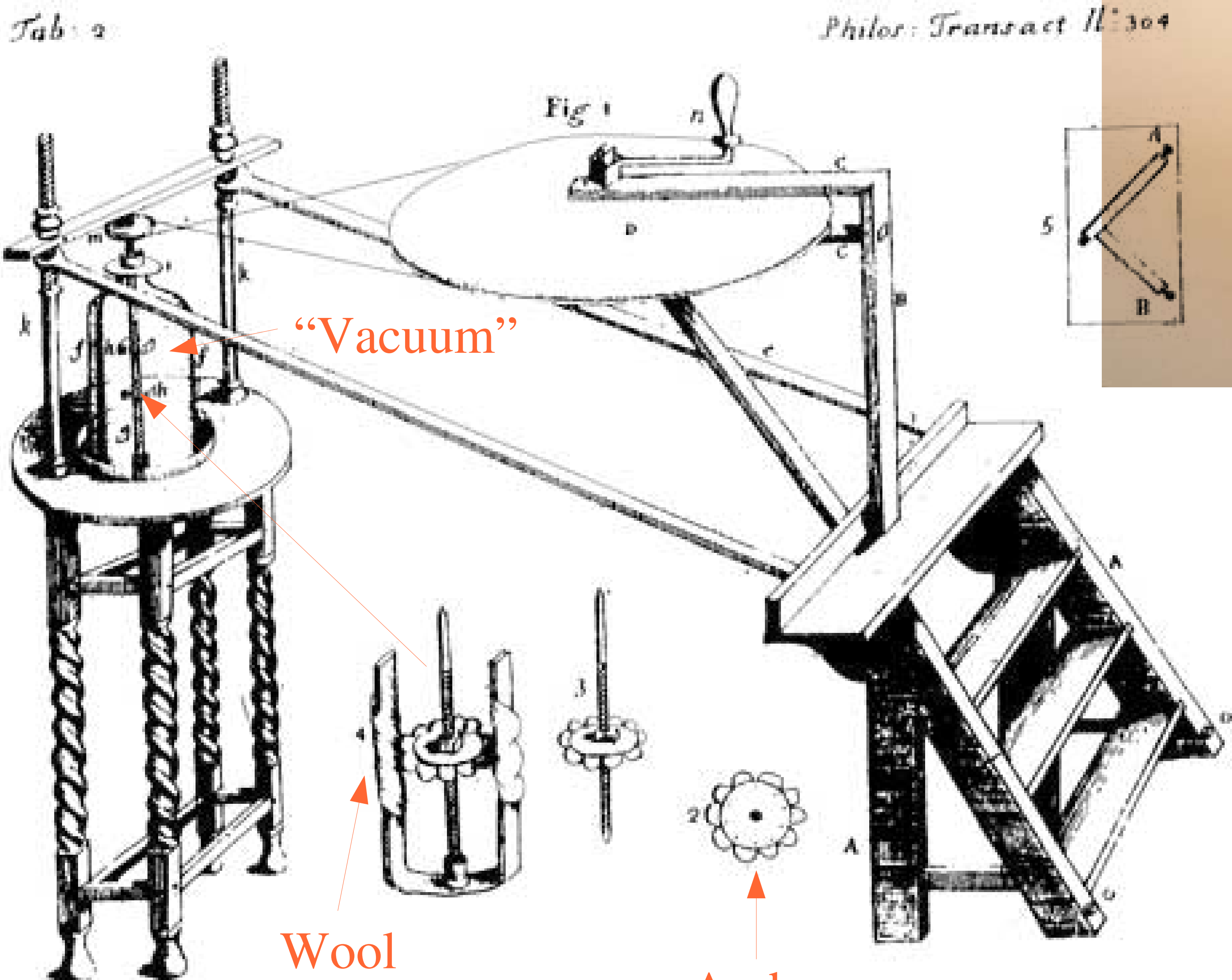
[R. Budakian *et al.*, Nature **391** (1998) 266-268.]

Francis Hauksbee

- Birth date, birth place and youth unknown:
- Dec 5th 1703: demonstrator Royal Society, Newton presiding
- 1703-1713: articles in the Philosophical Transactions
- 1705: Fellow of the Royal Society
- 1709: Physico-Mechanical Experiments on Various Subjects, containing an Account of several Surprizing Phenomena touching Light and Electricity, producible on the Attrition of Bodies. With many other Remarkable Appearances, not before observ'd. (London, R. Brugis)
- 1713: died
- 1719: 2nd edition of the "Physico-Mechanical Experiments"

Hauksbee's vacuum pump
[Science museum]

Francis Hauksbee – setup



Francis Hauksbee – some findings

- Repulsion identified as a genuine electric effect, not a mere mechanical rebound.

- Glass does *not*, while paper *does* interfere (try !);
- moist air & condensation destroy static electricity: moisture obstructs effluvia;
- rubbing in vacuum produces no effect.

- Link between attraction/repulsion and light;
- observed strong light at low gas pressure.

Pieter van Musschenbroeck
(1692-1761)

Other events in the 17th/18th century

- Stephen Gray 1666-1736
  - transport of charge
- Charles François de Cisternay Dufay 1698-1739
  - distinction of materials (dielectrics, metals ...)
- Jean-Antoine (Abbé) Nollet 1700-1770
  - symmetry in charge transport
- Benjamin Franklin 1706-1790
  - lightning shown to be electricity
- Pieter van Musschenbroeck 1692-1761
  - Leidse fles & discharges
- Luigi Galvani + 1737-1798
  Alessandro Giuseppe Antonio Anastasio Volta 1745-1827
  - Frog legs

- ... and many others ...

Giordano (Filippo) Bruno
(1548-1600)

In context 1700-1800

- Science
  - 1751: Publication of the *Encyclopédie* begins in France
- Miscellaneous
  - 1727: JS Bach composes the *Matthäuspassion* (BWV 244)
  - 1772-1782: *Siku quanshu* compiled
  - 1776: *Unanimous declaration of the 13 united states of America*
  - 1789: French Revolution begins
  - 1790: H.M.S. Bounty mutineers settle on Pitcairn Island
  - 1791: WA Mozart composes *Die Zauberflöte* (K 620)

Jean le Rond d'Alembert
(Nov 16th 1717 –Oct 29th 1783)

1749: 2d flow of liquids

- Jean le Rond d'Alembert takes part in a hydrodynamics contest in Berlin. Euler gives the price to Jaques Adami.
- d'Alembert and Euler don't speak for 10 years, but:

> 59. On peut encore trouver M & N par la méthode ſuivante qui eſt un peu plus ſimple. Puiſque $\frac{dp}{dz} = -\frac{dq}{dx}$ & $\frac{dp}{dx} = \frac{dq}{dz}$, donc $q\,dx + p\,dz$ & $p\,dx - q\,dz$ ſeront des différentielles complettes.

J. le Rond d'Alembert, "*Theoria resistentiae quam patitur corpus in fluido motum, ex principiis omnino novis et simplissimus deducta, habita ratione tum velocitatis, figurae, et massae corporis moti, tum densitatis compressionis partium fluidi*" (1749). Manuscript at the Berlin-Brandenburgische Akademie der Wissenschaften as document I-M478.

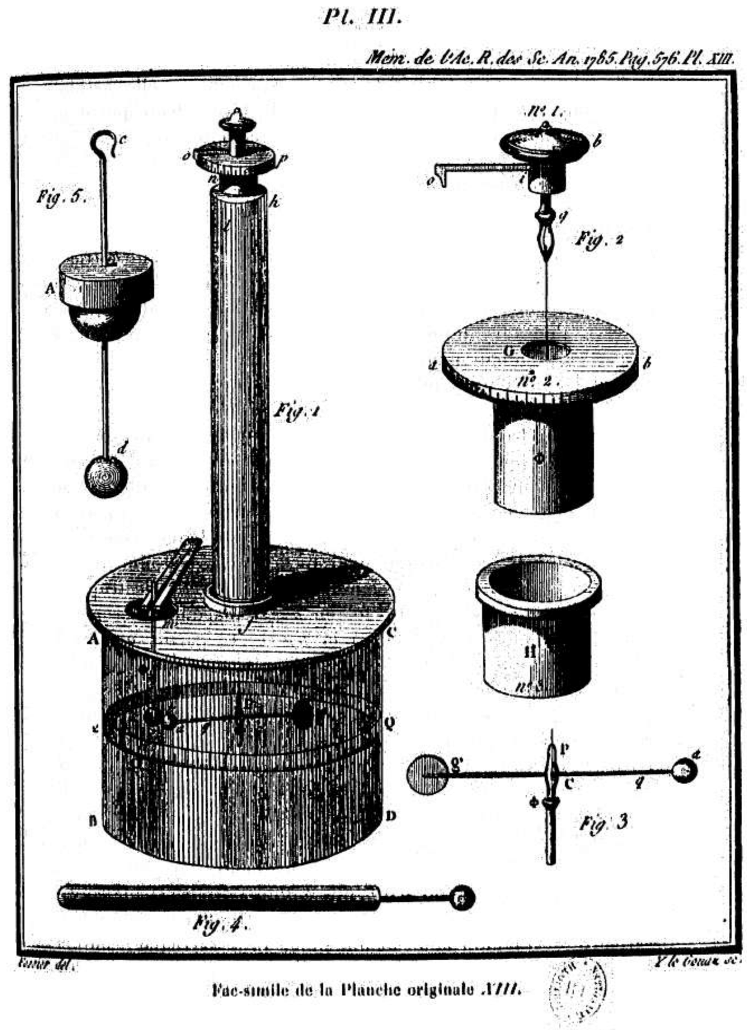
J. le Rond d'Alembert, "*Essai d'une nouvelle théorie de la résistance des fluides*" (1752) Paris.
Available from Gallica BnF.

1785: Inverse square law

- Coulomb quantified the forces using a torsion balance:

Loi fondamentale de l'électricité.

La force répulsive de deux petits globes électrisés de la même nature d'électricité est en raison inverse du carré de la distance du centre des deux globes.



[Charles-Augustin de Coulomb, *Construction et usage d'une balance électrique, fondée sur la propriété qu'ont les fils de métal d'avoir une force de torsion proportionnelle à l'angle de torsion*, Mémoires de l'Académie Royale des Sciences, Premier mémoire (1785). Reproduced in: *Mémoires de Coulomb*, edited by A. Potier (1884), available from the Gallica digital library, BnF Paris.]

Charles-Augustin de Coulomb

- Description:
  - lieutenant général du Génie;
  - physicien, membre de l'Académie des Sciences en 1785, de l'Institut en 1795 (1736-1806);
  - en uniforme d'officier du Génie, portant la croix de Saint-Louis;
  - tenant son invention, la balance éléctrique de torsion conçue en 1785 et un manuscrit portant la formule mathématique de la "force de torsion".
- Painted by: Louis Hierle (1856-1906)
- To be seen in Versailles.

[From: www.culture.fr]

Siméon-Denis Poisson
(1781-1840)

1811: Poisson equation

- Poisson introduced an electric potential by close analogy with Laplace's work on gravity.

On sait que les composantes de l'attraction ou de la répulsion qu'un corps exerce sur un point donné, sont exprimées par les différences partielles d'une certaine fonction des coordonnées de ce point, savoir, de la fonction qui représente la somme des molécules du corps, divisées par leurs distances respectives au point donné: désignons donc cette somme par V,

$$F \propto \nabla V$$
$$V = \int q / r$$

- His goal was reproducing Coulomb's experimental data. He didn't want to speculate on the nature of the fields.

[Siméon-Denis Poisson, *Sur la Distribution de l'Électricité à la surface des Corps conducteurs,* p 14, Mémoires de la classe des sciences mathématiques et physiques de l'institut impérial de France (Paris, 1811). Available from: Internet Archive.]

1814: Cauchy-Riemann equations

Augustin Louis Cauchy
(Aug 21st 1789 – May 23rd 1857)

Georg Friedrich Bernhard Riemann
(Sep 17st 1826 – Jul 20th 1866)

- Express the existence of a derivative of a complex analytic function $f = u + \mathrm{i}\, v$

$$f'(z) = \frac{\partial f}{\partial x} = \frac{\partial u}{\partial x} + \mathrm{i}\frac{\partial v}{\partial x} \qquad \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$= \frac{\partial f}{\partial \mathrm{i}\, y} = -\mathrm{i}\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} \qquad \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$$

- Imply that u is harmonic:

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 v}{\partial x \partial y} = \frac{\partial^2 v}{\partial y \partial x} = \frac{-\partial^2 u}{\partial y \partial y} \qquad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

- Reference: A.L. Cauchy, *Sur les intégrales définies* (1814). This *mémoire* was read in 1814, but only submitted to the printer in 1825.

Solutions for 2-dimensional fields

- As ansatz for the potential function ϕ, we use:

  $\phi = \mathrm{Re} \log F$

- Required properties of F:
  - $\log F$ analytic function in the problem domain;
  - F has simple zeroes at the wires;
  - leads to finite field energy.

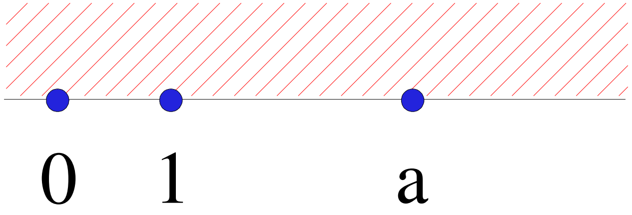
- Examples:
  - Single charge: $F = z$ hence: $\phi = \log(r)$
  - Row of charges: $F = \sin(\pi z/s)$, $F = \sinh(\pi z/s)$
  - Forest of charges: $F = \vartheta_1\left(\pi z/s_x, e^{-\pi s_y/s_x}\right) + \ldots$

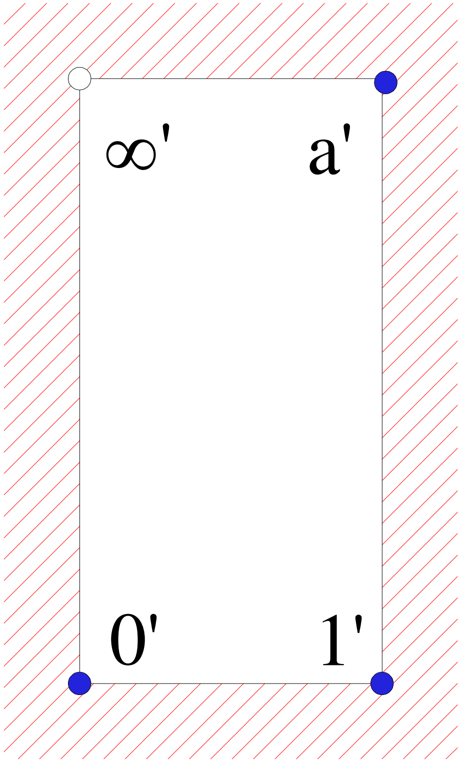
Conformal mappings – an example

- Schwartz-Christoffel transformation of a half-plane to the external part of a rectangle:

$$z \to \int_0^z \frac{\mathrm{d}\xi}{\sqrt{\xi(\xi-1)(\xi-a)}}$$

$$= \frac{2}{\sqrt{a}} \mathrm{sn}^{-1}\left(\sqrt{z}, \frac{1}{\sqrt{a}}\right)$$





Hans Christian Ørsted
(1777–1851)

1820: Link of current and B

- And electric current moves a magnet needle:

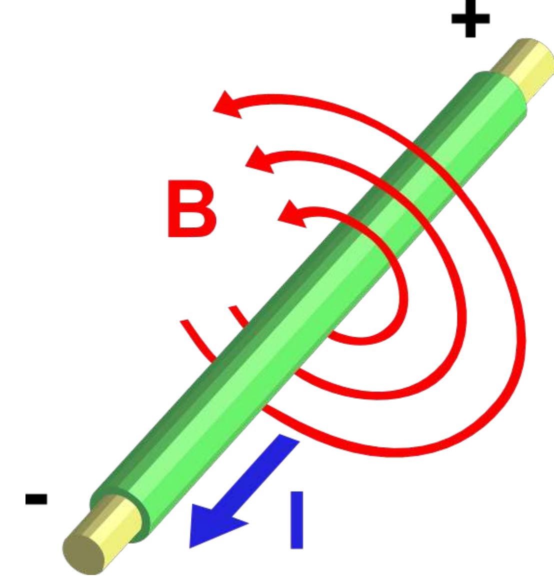
Conjungantur termini oppositi apparatus galvanici per filum metallicum, quod brevitatis causa in posterum conductorem conjungentem vel etiam filum conjungens appellabimus. Effectui autem, qui in hoc conductore et in spatio circumjacente locum habet, conflictus electrici nomen tribuemus.

Ponatur pars rectilinea hujus fili in situ horizontali super acum magneticam rite suspensam, eique parallela. Si opus fuerit, filum conjungens ita flecti potest, ut pars eius idonea situm ad experimentum necessarium obtineat. His ita comparatis, acus magnetica movebitur, et quidem sub ea fili conjungentis parte, quæ electricitatem proxime a termino negativo apparatus galvanici accipit, occidentem versus declinabit.

Current,
B-field

[Johannis Christianus Örsted, *Experimenta circa effectum conflictus electrici in acum magneticam,* Hafnia, 21de Julii 1820. Available from: www.ampere.cnrs.fr]

Hans Christian Ørsted



- Electric currents produce magnetic fields:

The opposite ends of the galvanic battery were joined by a metallic wire (...) To the effect which takes place in this conductor and in the surrounding space, we shall give the name of the conflict of electricity. Let the (...) wire be placed horizontally above the magnetic needle (...) parallel to it. (...) the needle will be moved, and the end of it next the negative side of the battery will go westward. (...) the galvanic circle must be complete, and not open (...) the effect cannot be ascribed to attraction (...) The electric conflict acts only on the magnetic particles of matter. (...) All non-magnetic bodies appear penetrable by the electric conflict, while magnetic bodies, or rather their magnetic particles, resist the passage of this conflict. (...) we may likewise infer that this conflict performs circles

current,
B-field

[John Christian Oersted, *Experiments on the effect of a current of electricity on the magnetic needle*, Translation by the author, transmitted to the editor of the Annals of Philosophy (1820). Available from: www.ampere.cnrs.fr]

André-Marie Ampère
(1775-1836)

1826: Mathematical expression

- The force acting between 2 currents was derived by André-Marie Ampère and summarised in his work: "Théorie des phénomènes électro-dynamiques, uniquement déduite de l'expérience".
- After a long introduction on methodology and an overview of the measurements, an ansatz is derived on page 29: $\left(\frac{k\,i\,i'\,\mathrm{d}s\,\mathrm{d}s'}{r^n}\right)$

j'obtins $\frac{k i i' \mathrm{d}s \mathrm{d}s'}{r^n}$ pour l'expression de l'action

- On page 60 (!), after an ingenious, argument the constants are established to be:

il faut donc que $n=2$, et, en vertu de l'équation $1-n-2k=0$, que $k=-\frac{1}{2}$.

$$\left(\frac{i\,i'\,\mathrm{d}s\,\mathrm{d}s'}{2\,r^2}\right)$$

Michael Faraday
(1791-1867)

Using the link of current and B

- Applications of the link between current and B field:
  - 1820s: first electromotor, on learning from François Arago about Hans Christian Ørsted's work, later also the dynamo;
  - 1830: induction between 2 coils around a common yoke.
  - before 1855: first to speak of a “line of magnetic force”

> 3071. A line of magnetic force may be defined as that line which is described by a very smallmagnetic needle, when it is so moved in either direction correspondent to its length, that the needle is constantly a tangent to the line of motion; or it is that line along which, if a transverse wire be moved in either direction, there is no tendency to the formation of any current in the wire, whilst if moved in any other direction there is such a tendency; or it is that line which coincides with the direction of the magnecrystallic axis of a crystal of bismuth, which is carried in either direction along it.

[Michael Faraday, *Experimental researches in electricity*, vol III, 1855. Available in Google books]

- A pure experimentalist, Faraday reputedly never wrote an equation in his life.

George Green's father's mill (Nottingham)

1828: George Green's work

- The basic techniques to solve electrostatics problems, still used today, were published by George Green in: " An Essay on the Application of Mathematical Analysis to the Theories of Electricity and Magnetism".

"(...) it was written by a young man, who has been obliged to obtain the little knowledge he possesses, at such intervals and by such means, as other indispensable avocations which offer but few opportunities of mental improvement, afforded."

- Now available from http://arxiv.org/pdf/0807.0088v1, originally only 53 copies were printed, only for the subscribers.

[Original printed for the author by T Wheelhouse, Nottingham (1828). Facsimile Mayer & Müller, Berlin (1889), scanned by Google books.]

George Green
(1783-1841)

Green's identities

- Green starts from what is now known as his 2nd identity:

> Let U and V be two continuous functions of the rectangular co-ordinates x, y, z, whose differential co-efficients do not become infinite at any point within a solid body of any form whatever; then will
>
> $$\int dx\,dy\,dz\, U\delta V + \int d\sigma\, U\left(\frac{dV}{dw}\right) = \int dx\,dy\,dz\, V\delta U + \int d\sigma\, V\left(\frac{dU}{dw}\right);$$
>
> dw points *in*wards

- In current notation, using the divergence theorem, proven by Михайло Васильович Остроградський in 1826, stated by Gauss in 1813, known to Lagrange in 1762:

$$\int_S (U\nabla^2 V - V\nabla^2 U)\,\mathrm{d}x = \int_S \nabla\cdot(U\nabla V - V\nabla U)$$

$$= \oint_{\partial S} (U\nabla V - V\nabla U)\cdot \mathrm{d}n$$

dn points *out*wards

- Serves as basis for:
  - reciprocity theorem, and thus signal calculations;
  - Green's function method.

Green's function technique

- The embryo of the Green's function technique is found in article 7 (p 21):

> Substituting for ρ', the value which results from this equation, in that immediately preceding we obtain
>
> $$V' = -\int \frac{dx\,dy\,dz\,\delta V}{4\pi r'} + \int \frac{\rho\, d\sigma}{r},$$
>
> which, by means of the equation (3) art. 3, becomes
>
> $$\int \frac{\rho\, d\sigma}{r} = \frac{1}{4\pi}\left\{\int d\sigma \overline{V} \left(\frac{d\frac{1}{r}}{dw}\right) - \int \frac{d\sigma}{r}\left(\overline{\frac{dV}{dw}}\right)\right\};$$
>
> the horizontal lines over the quantities, indicating that they belong to the surface itself.

- In this expression, one recognises the Green's function as $G = 1/r$:

$$V = \int_S \rho \frac{1}{r} \mathrm{d}x + \oint_{\partial S} \left(V \nabla \frac{1}{r} - \frac{1}{r} \nabla V\right) \cdot \mathrm{d}n$$

Green's reciprocity equation

- Reciprocity is a direct consequence of the Green identities if the potentials U and V at infinity are 0:

$$\int_S (U\nabla^2 V - V\nabla^2 U)\,\mathrm{d}x = \oint_{\partial S} (U\nabla V - V\nabla U)\cdot \mathrm{d}n = 0$$

$$\int_S V\rho_U = \int_S U\rho_V\,\mathrm{d}x$$

- The discrete version is used to calculate the current on electrodes, by comparison of 2 configurations:

$$\sum_i V_i q_i^U = \sum_i U_i q_i^V$$

Green reciprocity

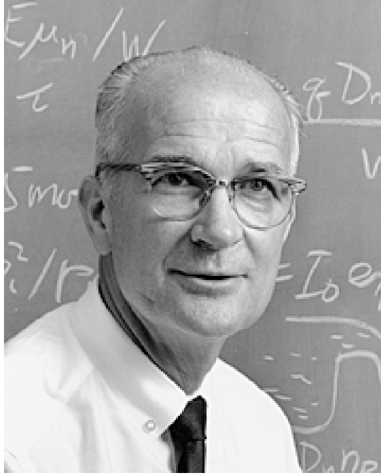
- Recall the capacitance equation: $\sum C_{ij} V_j = q_i$, written in matrix form: $C\,V = q$

- Consider 2 sets of charges and potentials that satisfy the capacitance equation: (V,q) and (V',q').

- Transform $C\,V = q$ into $V = C^{-1} q$ and $V^{T} = q^{T} C^{-1}$, multiply with the capacitance equation for the other configuration, $q' = C\,V'$ and you find the Green reciprocity equation: $V^{T} q' = q^{T} V'$.

William Bradford Shockley

(1910–1989)



Configurations

- Let's consider the following 2 configurations:

Wire j:

$V = V_j$, $q = q_j + \lambda_j$

Wire i≠j:

$V = V_i$, $q = q_i + \lambda_i$

Charge:

$V = ?$, $q = Q$

Wire j:

$V = q_j \phi(z_j - z_j)$, $q = q_j$

Wire i≠j:

$V = q_j \phi(z_i - z_j)$, $q = 0$

Charge:

$V = q_j \phi(z_Q - z_j)$, $q = 0$

[W. Shockley, *Currents to Conductors Induced by a Moving Point Charge*, J. Appl. Phys. **9** (1938) 635-636. Affiliation: Bell Telephone Laboratories, NY. A closely related argument can already be found in Maxwell's *Treatise* (1873).]

Deriving the weighting field

- Applying reciprocity gives:

$$\sum_{i=1}^{n_{\text{wires}}} \lambda_i \phi(z_j - z_i) = -Q\,\phi(z_{\text{charge}} - z_j)$$

- Differentiating to time:

$$\sum_{i=1}^{n_{\text{wires}}} I_i \phi(z_j - z_i) = Q\,\vec{\epsilon}(z_{\text{charge}} - z_j)\cdot\vec{v}_{\text{charge}}$$

- Identify $C_{ij}^{-1} = \phi(z_i - z_j)$ and solve for the currents:

$$\begin{aligned} I_i &= Q \sum_i \left(C_{ij}\,\vec{\epsilon}(z_{\text{charge}} - z_j) \right) \cdot \vec{v}_{\text{charge}} \\ &= Q\,\vec{E}^{W}(z_{\text{charge}} - z_j)\cdot\vec{v}_{\text{charge}} \end{aligned}$$

- Thus, E^{W} is computed using columns of the capacitance matrix elements as charges.

James Clerk Maxwell and his mother (1831-1879)

1873: Maxwell equations

- "A treatise on electricity and magnetism": a synthesis of the developments since 1600, which owes more to a pure experimentalist than to the theoreticians ...

There is also a considerable mass of mathematical memoirs which are of great importance in electrical science, but they lie concealed in the bulky Transactions of learned societies; they do not form a connected system; they are of very unequal merit, and they are for the most part beyond the comprehension of any but professed mathematicians.

For instance, Faraday, in his mind's eye, saw lines of force traversing all space where the mathematicians saw centres of force attracting at a distance: Faraday saw a medium where they saw nothing but distance: Faraday sought the seat of the phenomena in real actions going on in the medium, they were satisfied that they had found it in a power of action at a distance impressed on the electric fluids.

[James Clerk Maxwell, *A treatise on electricity and magnetism* (1st edition 1873, several later editions exist), Clarendon Press Series, published by Macmillan.]

Discovery of charged particles

- 1895: cathode rays → γ (Wilhelm Konrad Röntgen)
- 1896: U radioactivity (α, β, γ) (Henri Becquerel)
- 1897: cathode rays → e^- (Joseph John Thomson)
- 1898-1920: proton
- 1932: neutron (James Chadwick)
- 1937: $\mu^\pm$

[Four Curies: Pierre, Marie, Irène and Pierre's father, around 1904 at the BIPM]

1896: Ionisation by radiation

- Early in the study of radioactivity, ionisation by radiation was recognised:

” Becquerel discovered in 1896 the special radiating properties of uranium and its compounds. Uranium emits very weak rays which leave an impression on photographic plates. These rays pass through black paper and metals; they make air electrically conductive. “

[Pierre Curie, Nobel Lecture, June 6th 1905]

“A sphere of charged uranium, which discharges spontaneously in the air under the influence of its own radiation, retains its charge in an absolute vacuum. The exchanges of electrical charges that take place between charged bodies under the influence of the new rays, are the result of a special conductivity imparted to the surrounding gases, a conductivity that persists for several moments after the radiation has ceased to act.”

[Antoine Henri Becquerel, Nobel Lecture, December 11th 1903]

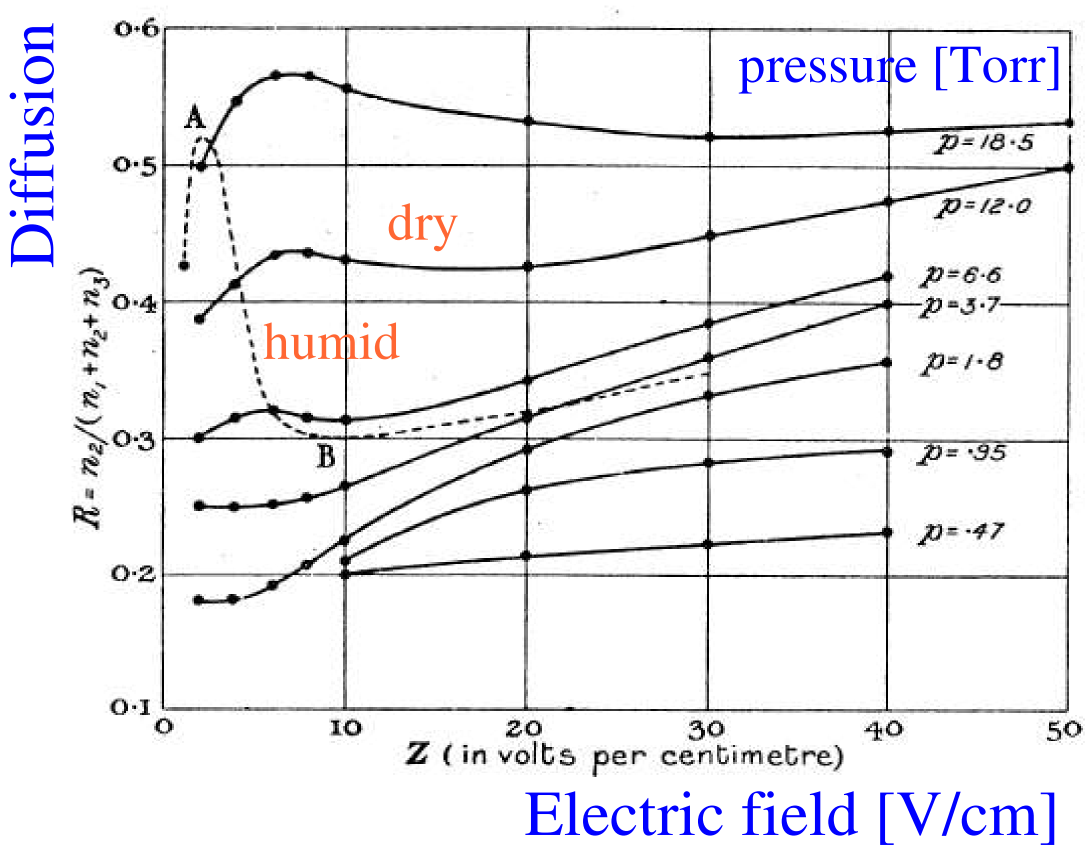
1900: Electron transport in gases

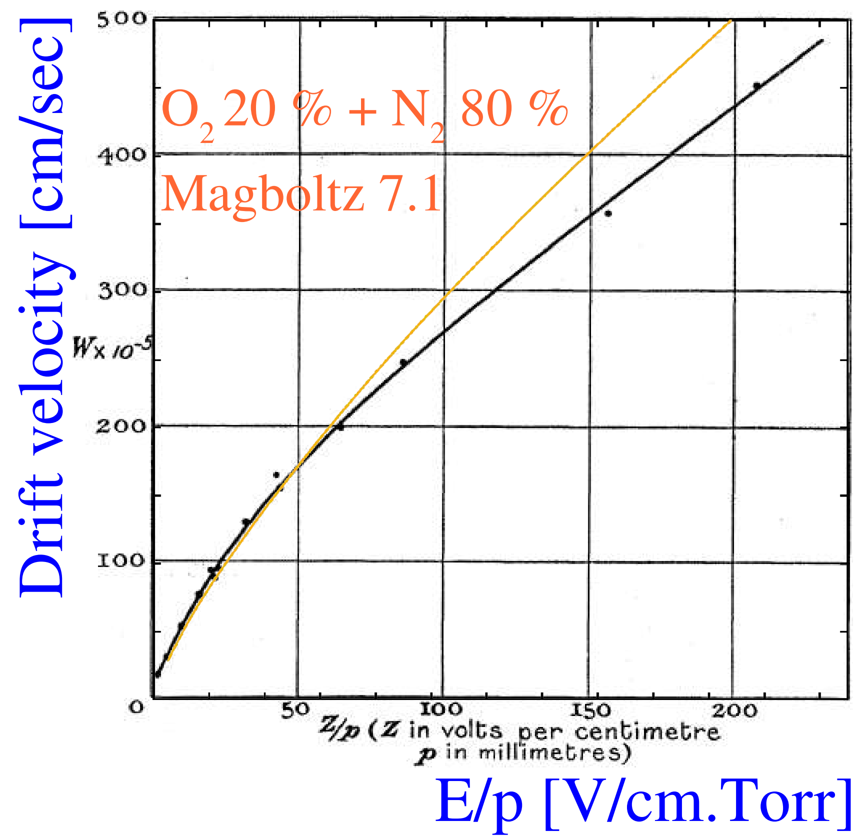
- The velocity of positive and negative "ions" produced in gases by Röntgen rays was extensively studied at the turn of the century, as shown by the abundant literature, *e.g.*
  - E. Rutherford, Phil. Mag. **44** (1897) 422;
  - J. Zeleny, Phil. Trans. R. Soc. Lond. A **195** (1900) 193-234;
  - J.S. Townsend, Nature **62** (1900) 340-341;
  - J.S. Townsend, Phil. Mag. **6-1** (1901) 198-227.

- Drift velocities, diffusion coefficients and Townsend coefficients were measured and interpreted.

Measurements

- Examples of measurements in air from 1913:





J. S. Townsend and H. T. Tizard, *The Motion of Electrons in Gases*, Proc. R. Soc. Lond. A **88** (1913) 336-347.

Sir John Sealy Edward Townsend
(1868-1957)

1901: Gas multiplication

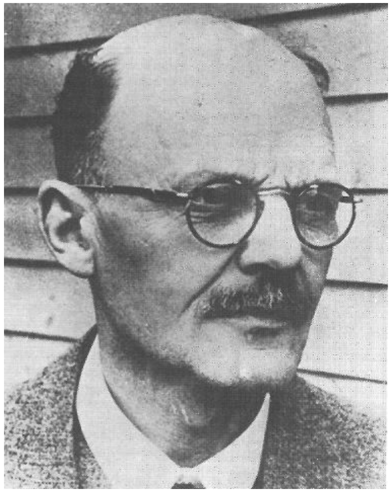
- John Townsend "*The conductivity produced in gases by the motion of negatively charged ions*":

> Negative Ionen erzeugen bei ihrer Wanderung durch Berührung neue Ionen. Die Anzahl der negativen Ionen in einer Entfernung x vom Punkte der Entstehung ist deshalb durch die Exponentialfunktion in der Form $N = N_0 e^{\alpha x}$ gegeben. Diese Formel wird durch Versuche bestätigt. Dabei ist α eine Größe, die außer von der Temperatur noch von der Größe der die Ionen bewegenden Kraft X und dem Druck p abhängt, und zwar ergibt sich diese Abhängigkeit in der Form $= pf(X/p)$.

[JS Townsend, "*The conductivity produced in gases by the motion of negatively charged ions*", Phil. Mag. **6-1** (1901) 198-227. Access to Phil. Mag is restricted. The above German-language abstract is available at `http://jfm.sub.uni-goettingen.de/`.]

1908: Geiger counter

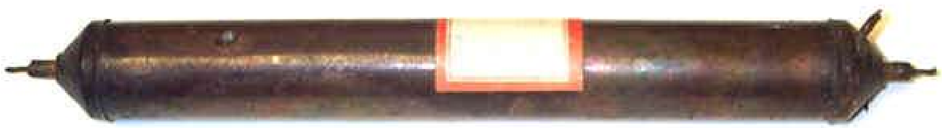
- Detects radiation by discharge.
- Can count α and β particles (at low rates).
- No tracking capability.
- First models in 1908 by Hans Geiger, further developed from 1928 with Walther Müller.



Hans Geiger
(1882-1945)

?

Walther Müller
(1905-1979)



A Geiger-Muller counter built in 1939 and used in the 1947-1950 for cosmic ray studies in balloons and on board B29 aircraft by Robert Millikan et al.

Made of copper, 30 cm long

Motivation for the Geiger counter

In considering a possible method of counting the number of α-particles, their well-known property of producing scintillations in a preparation of phosphorescent zinc sulphide at once suggests itself.

Efficiency losses of visual detection

The doubt, however, at once arises whether every α-particle produces a scintillation, for it is difficult to be certain that the zinc sulphide is homogeneous throughout. No confidence can be placed in such a method of counting the total number of α-particles (except as a minimum estimate),

It has been recognised for several years that it should be possible by refined methods to detect a single α-particle by measuring the ionisation it produces in its path.

Ionisation signal usable but small

We then had recourse to a method of automatically magnifying the electrical effect due to a single α-particle. For this purpose we employed the principle of production of fresh ions by collision. In a series of papers, Townsend* has worked out the conditions under which ions can be produced by collisions with the neutral gas molecules in a strong electric field. The effect is best shown in gases at a pressure of several millimetres of mercury.

Use multiplication at low pressure as discovered in 1901 by JS Townsend

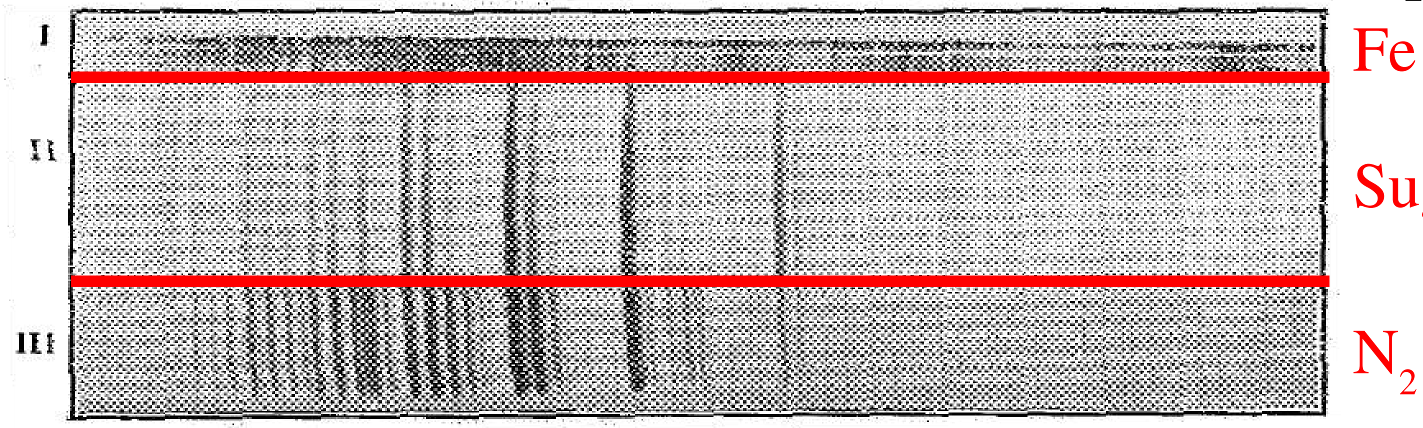
* 'Phil. Mag.,' February, 1901; June, 1902; April, 1903; September and November, 1903.

[E. Rutherford and H. Geiger, *An Electrical Method of Counting the Number of α-Particles from Radio-Active Substances,* Proc. R. Soc. Lond. A **81** (1908) 141-161]

Henri Longchambon
(1896-1969)

1922: Light from breaking sugar

- Henri Longchambon's observations:
  - triboluminescence spectrum of sugar matches N_2:



  - light intensity highest at p = 1-40 mmHg, no light in vacuum.
- He concludes:

> La triboluminescence du sucre serait donc due à une effluve s'effectuant *dans l'air entre deux* particules solides qui viennent d'être séparées brusquement et se trouvent chargées électriquement.

[Henri Longchambon, Comptes rendus hebdomadaires des séances de l'Académie des sciences, **174** (1922) 1633-1634, scan courtesy Gallica digital library, BnF Paris.]

Triboluminescence these days

- Current consensus pictures triboluminescence as follows:
  - separation of, preferably non-symmetric, crystals leads to charge separation;
  - discharge between the separated fragments;
  - the surrounding gas is ionised and/or excited.
- Other processes can contribute to light production:
  - discharge between material and grinding tool;
  - phosphorescence, fluorescence.

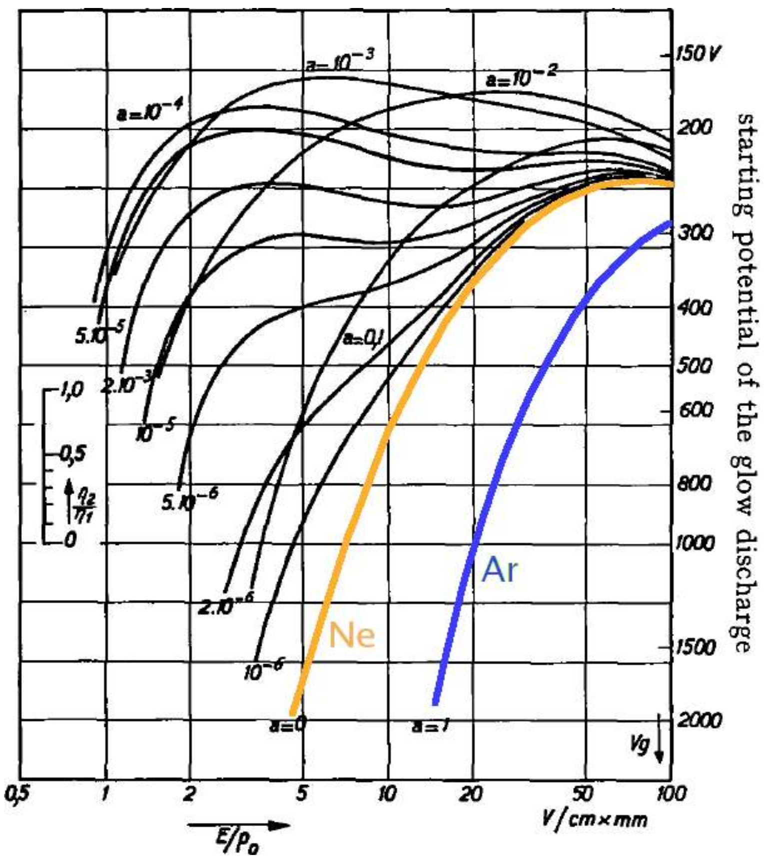
- Little studied these days, but relevant when wrapping electronic circuitry – also used for diagnostic purposes.

Frans Michel Penning,
photo taken around 1921
(1894-1953)

1928: Ar-Ne-Hg Penning effects

- Frans Michel Penning worked from 1924 on gas discharges at the Philips Natuurkundig Laboratorium.

Der Einfluß sehr geringer Beimischung von Hg und Ar auf die Zündspannung V_z des Neons wurde quantitativ bestimmt. Die Erklärung wurde gefunden in der Ionisierung der Fremdatome durch die metastabilen Atome des Neons. Die notwendige Bedingung für diesen Vorgang: $V_i' < V_{\text{met.}}$ wurde geprüft bei Ne, Ar und He als Hauptgasen mit verschiedenen anderen Gasen als Beimischung und stets bestätigt gefunden. Andererseits erniedrigten immer Beimischungen, wobei $V_i' < V_{\text{met.}}$ die Zündspannung; nur NO in Ar machte eine Ausnahme von dieser Regel, was aber auf Grund des Termschemas von NO nicht zu verwundern braucht.



F. M. Penning, *Über den Einfluß sehr geringer Beimischungen auf die Zündspannung der Edelgase*, *Z. Phys.* **46** (1928) 334-348.

F.M. Penning, *The starting potential of the glow discharge in neon argon mixtures between large parallel plates: II. Discussion of the ionisation and excitation by electrons and metastable atoms*, Physica **1** (1934) 1028-1044.

F.M. Penning, Electrische gasontladingen, Philips Technische Bibliotheek (posthumous, 1955). Translated in various languages.

1935: Electron energy distribution

- Calculation of the electron energy distribution
  - allowing for energy loss in elastic collisions;
  - detailed balancing of energy and momentum gain (E-field, diffusion) and loss (elastic collision);
  - velocity dependent cross section;
  - use of Legendre expansion (crediting H.A. Lorentz):

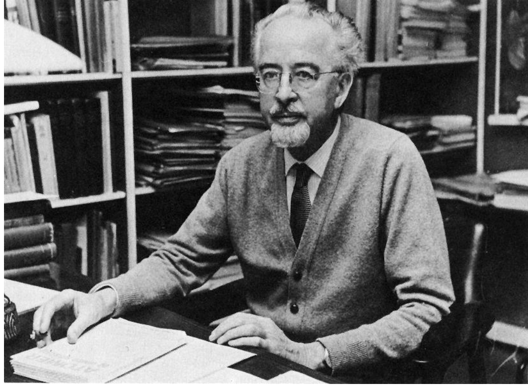
$$
\begin{aligned}
f(x, v, \omega) &= f_0(x, v) + P_1(\cos\omega) f_1(x, v) \\
&\quad + P_2(\cos\omega) f_2(x, v) + \cdots \\
&= f_0(x, v) + (\xi/v) f_1(x, v) + \cdots .
\end{aligned}
$$

(P_1, P_2: Legendre polynomials)

The function f_0 determines the random distribution in velocity, and f_1 determines the electron drift. The higher terms in the series are nearly always very small and do not correspond to any simple physical property of the distribution, but serve simply to improve the form of the distribution function.

[Philip M. Morse, W.P. Allis and E.S. Lamar, *Velocity Distributions for Elastically Colliding Electrons*, Phys. Rev. **48** (1935) 412–419]

Christian Møller
(1904-1980)



1930-1933: EM energy loss

- 1930 - Hans Bethe, non-relativistic quantum calculation:

The loss in kinetic energy per centimeter path is

$$-\frac{dT}{dx} = N E = \frac{4\pi e^4 z^2 N}{m v^2} \ln \frac{(2)\, m v^2}{c R h} .$$

Z: "hidden in N"
(2): only for electrons
R: Rydberg constant

- 1931 - Christian Møller solves relativistic e^- scattering.
- 1932 - Hans Bethe, relativistic quantum calculation:

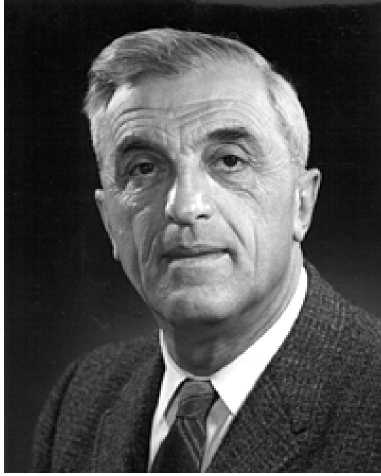
Ein Teilchen der Ladung ez möge sich mit der Geschwindigkeit v durch eine Substanz hindurchbewegen, welche in der Volumeneinheit N Atome der Ordnungszahl Z enthält. Dann verliert das Teilchen pro Zentimeter Weg die Energie

$$-\frac{dT}{dx} = \frac{2\pi e^4 N Z z^2}{m v^2} \left(\lg \frac{2 m v^2 W}{\bar{E}^2 \left(1 - \frac{v^2}{c^2}\right)} - \frac{v^2}{c^2} \right),$$

falls wir nur den Energieverlust durch solche Stöße ins Auge fassen, bei denen im einzelnen höchstens die Energie W auf das Atom übertragen wird[3]).

$\bar{E}$: average atomic ionisation energy
W: largest energy transfer per collision

Felix Bloch
(1905-1983)



1930-1933: EM energy loss

- 1933: Felix Bloch: better description for heavier atoms

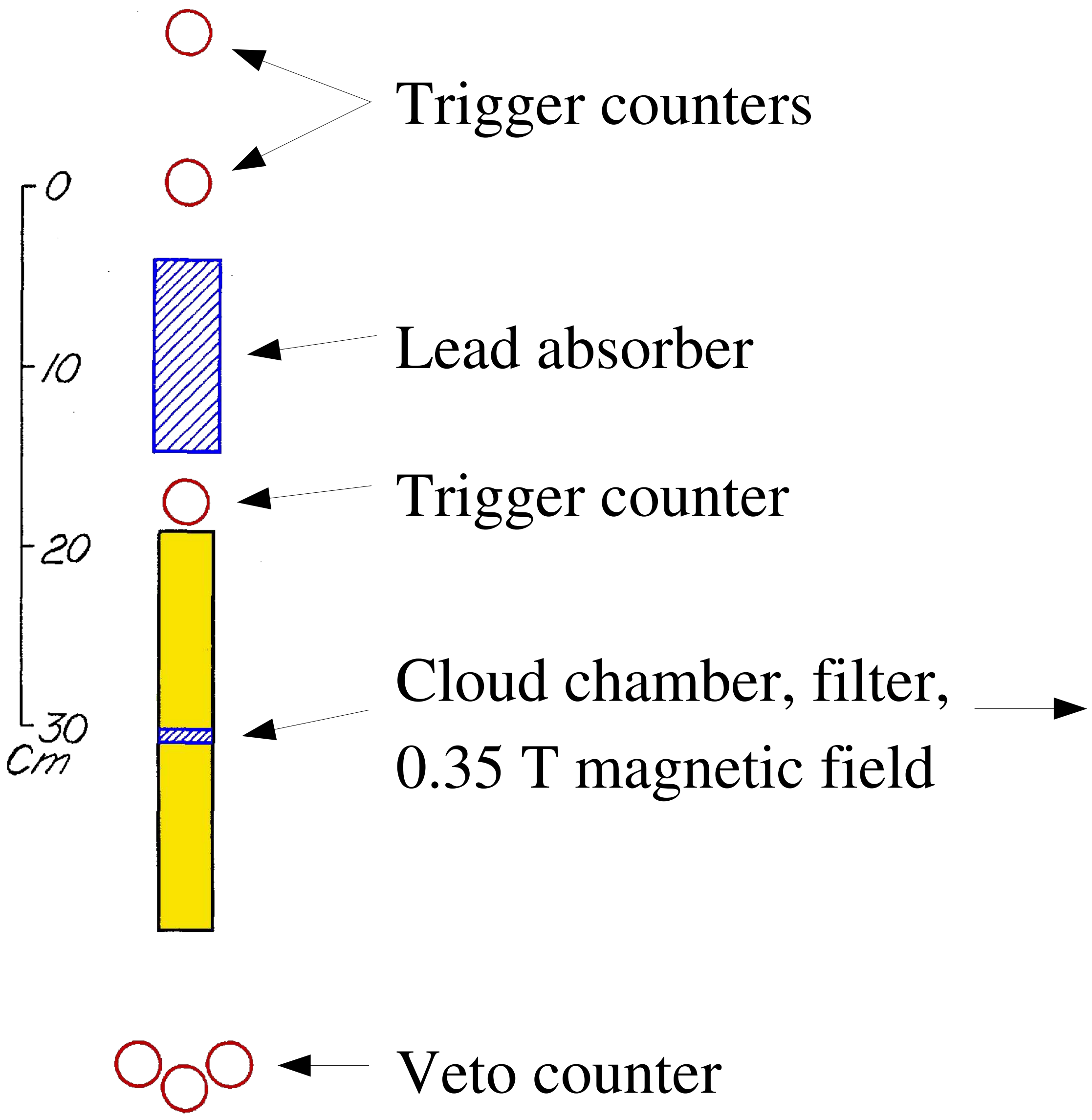
das Potentialfeld, das auf ein bestimmtes Elektron wirkt, wird angenähert beschrieben durch ein Coulombfeld mit abgeschirmter Kernladungszahl,

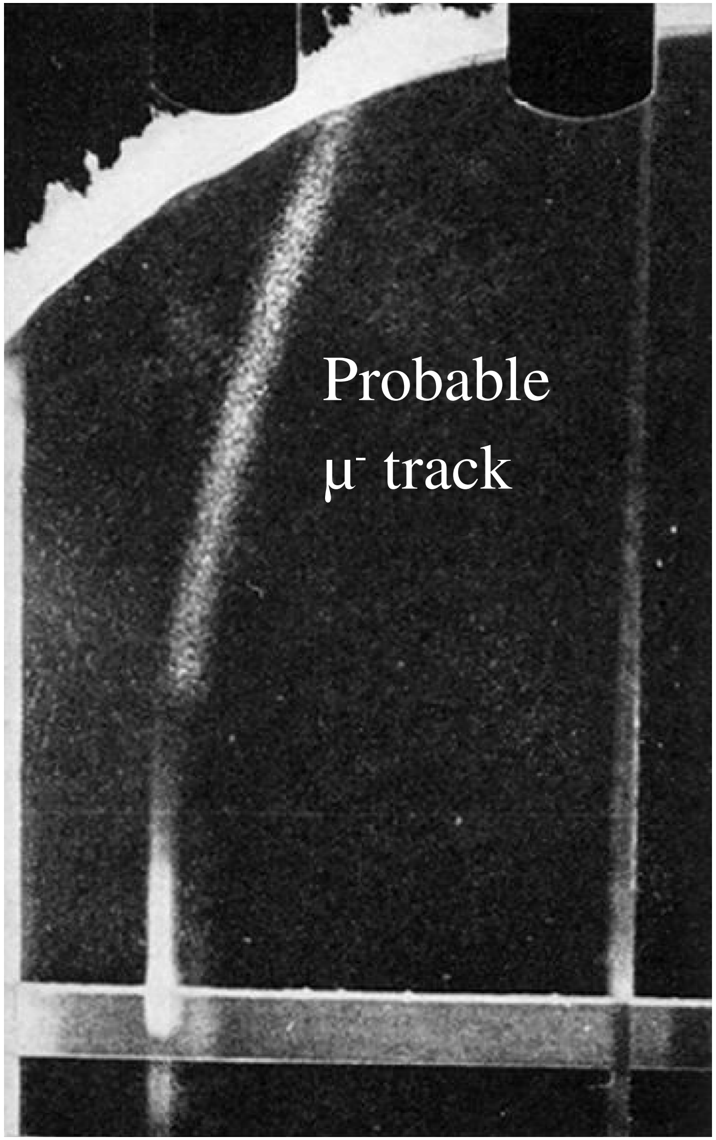
Diese Rechnungsweise ist nun deshalb bedenklich, weil auf ein Elektron keineswegs immer dasselbe Feld wirkt, sondern für die verschiedenen Anregungszustände des Elektrons mit *verschiedenen* Abschirmungszahlen zu rechnen wäre[1]).

Eine mehr befriedigende, weil konsequentere Annäherung

Nun ist von Thomas[2]) und Fermi[3]) gezeigt worden, daß sich, sobald die Ordnungszahl des Atoms nicht allzu klein ist, das Verhalten seiner Elektronen im Mittel mit guter Annäherung beschreiben läßt durch das eines vollkommen entarteten Fermigases, das an jeder Stelle unter dem Einfluß des von der Ladungsdichte der Elektronen sowohl wie vom Kern herrührenden Potentials steht.

1937: First $\mu^{\pm}$ experiment





Track "B"

1937: First $\mu^{\pm}$ experiment

- Collected 4000 events, made 1000 photos, only 2 were singled out ... "A" is most likely a proton, but from the curvature of track "B" is a negatively charged particle.
- Ionisation density 6 $\times$ density of "usual thin tracks", *i.e.* high energy charged particles.
- Assuming ionisation $\propto 1/v^2$ and using the curvature, the estimated mass was 130 ± 25 % m_e or 66 ± 17 MeV (*cf.* PDG 2008 value: 105.658367 ± 0.000004 MeV).

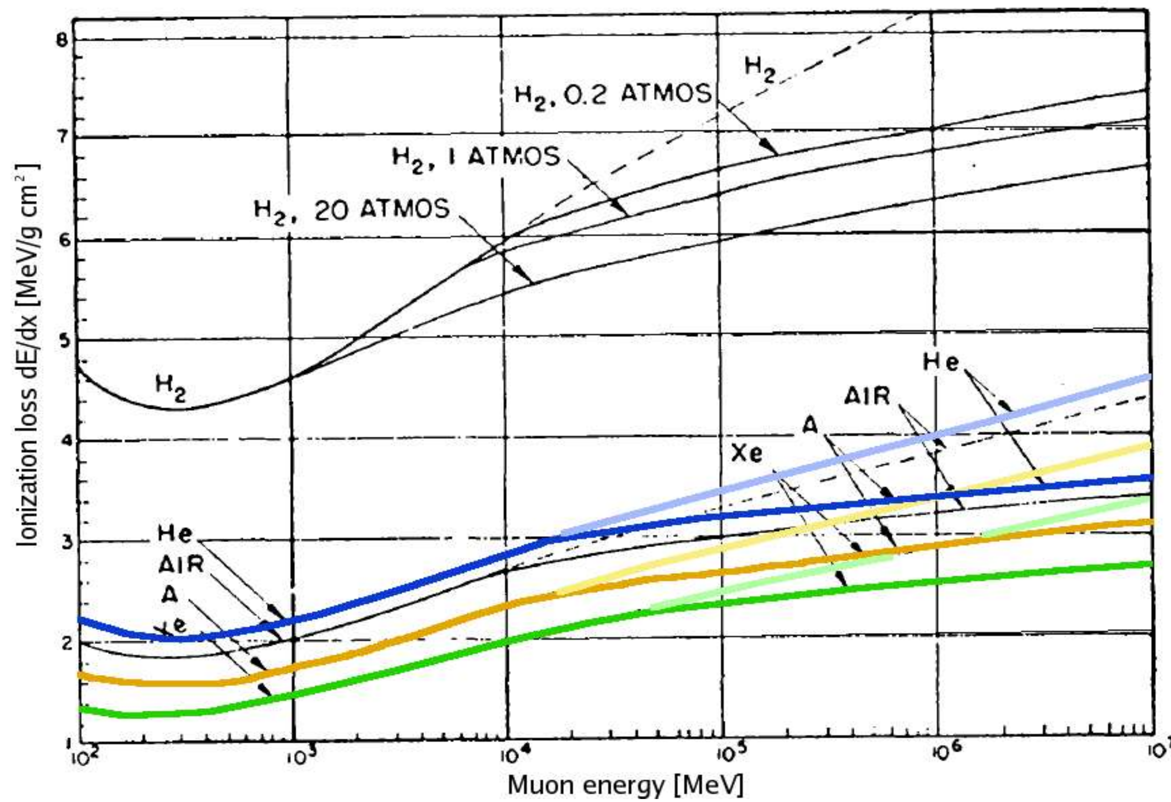
- Ref: J. C. Street and E. C. Stevenson, Phys. Rev **52** (1937) 1003.

1940: EM energy loss – density effect

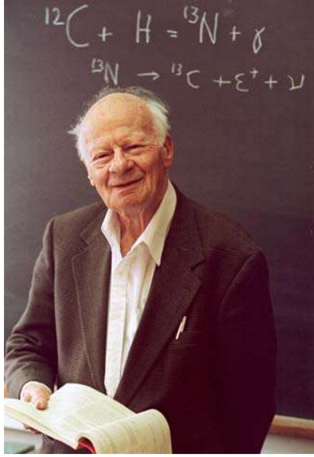
- 1940 - Enrico Fermi, density effect:

> It is shown that the loss of energy of a fast charged particle due to the ionization of the material through which it is passing is considerably affected by the density of the material. The effect is due to the alteration of the electric field of the passing particle by the electric polarization of the medium.

- 1952 - Sternheimer, parametrisation.



Hans Bethe
(1906-2005)



Bethe formula

- If we make the assumptions:
  - projectile mass $M \gg m$, the e$^-$ mass,
  - only Coulomb energy transfer to free e$^-$, not to the nuclei;
  - *effective* ionisation energy I < energy transfer < kinematics.

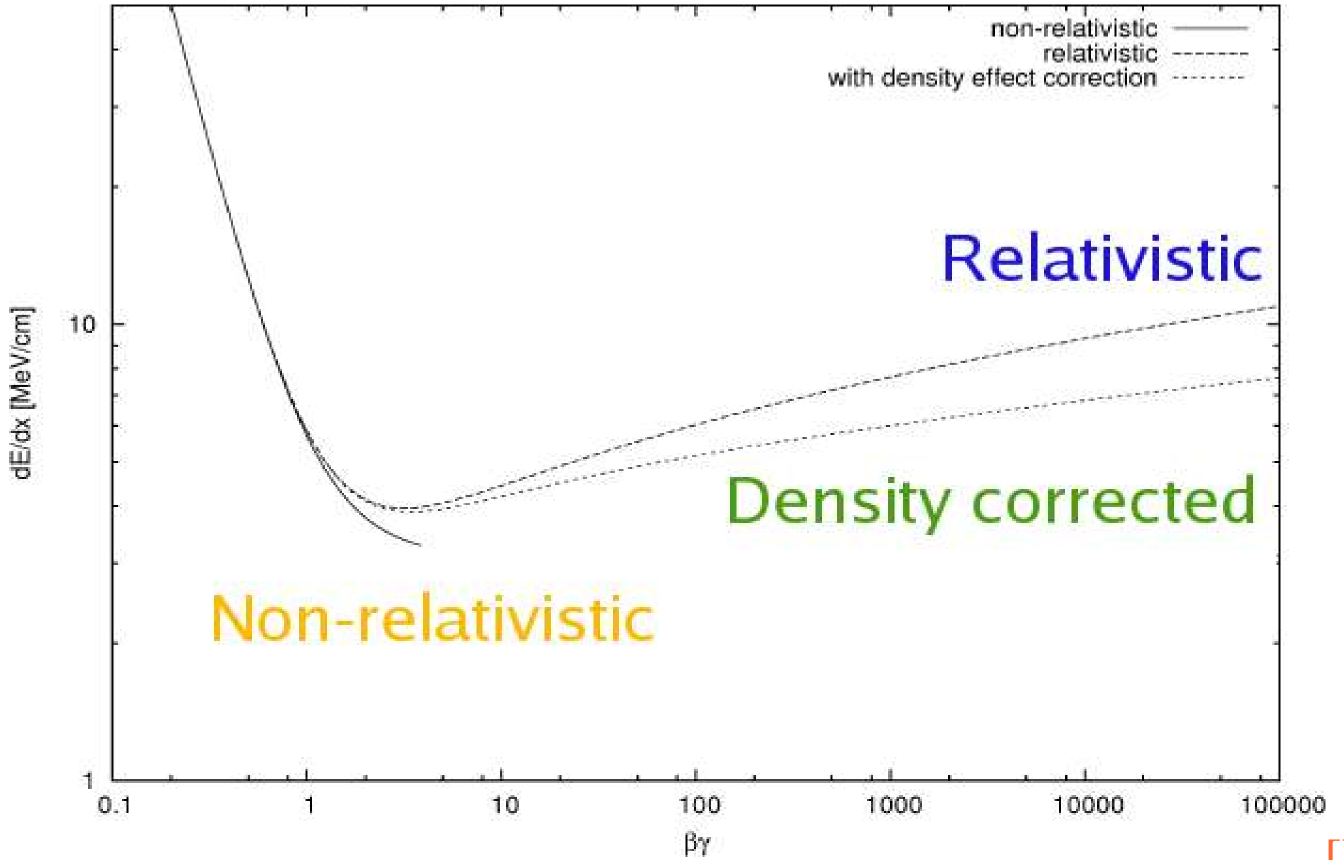
- The ionisation losses are given by (PDG version):

$$\frac{dE}{dx} \propto -\frac{Z^2 z}{\beta^2 a}\left|\frac{1}{2}\log\left(\frac{2mc^2\beta^2\gamma^2 T_{max}}{I^2}\right)-\beta^2-\frac{\delta}{2}+\textit{further corrections}\right|$$

  - β, γ: velocity of projectile;
  - Z^2: projectile charge (squared: d$p \propto Z$, d$E \propto$ dp^2);
  - z: target atomic number (linear: number of e$^-$ encountered).

Bethe formula

- Example for Si, assuming $I = 173$ eV (!), and using Sternheimer's parametrisation for the density effect.



[Diagram: Heinrich Schindler]

Лев Давидович Ландау
(1908-1968)

1944: Energy loss fluctuations

- Given a single-collision energy loss distribution $w(\epsilon)$, the distribution $f(s)$ of the energy loss s after many collisions is *schematically* given by the Laplace transform:

$$L\,f(x,s)=\bar{f}(x,p)=e^{-x\int_0^\infty (1-e^{-p\epsilon})w(\epsilon)\,d\epsilon}$$

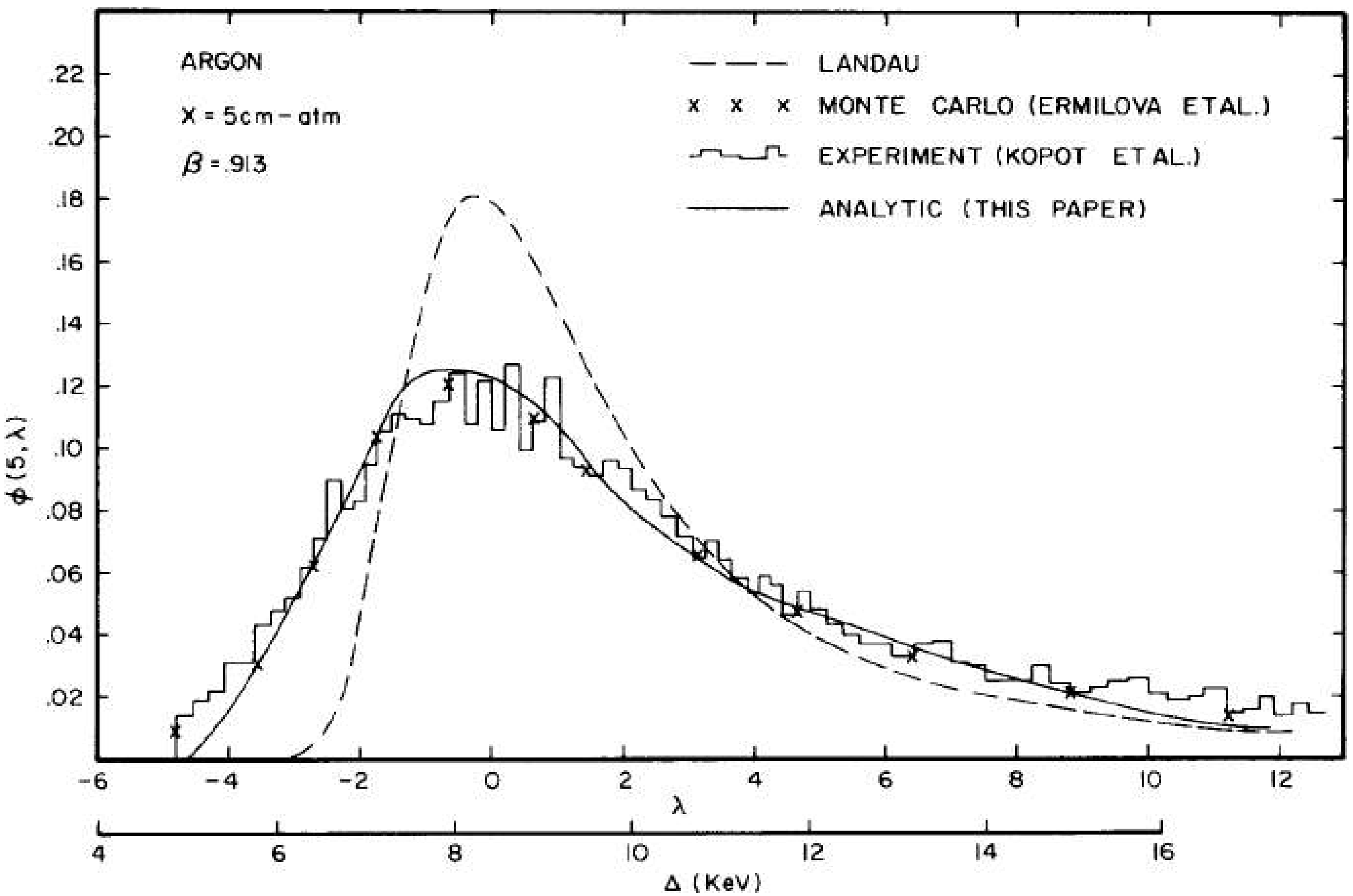
x: direction of travel
$s \leftrightarrow p$: total energy loss

- Landau showed (1944), assuming in particular:
  - thick layers: numerous small energy losses;
  - Rutherford-inspired energy loss distribution $w(\epsilon) \sim 1/\epsilon^2$;
  - neglect of the atomic structure:

$$L\,f(s)\approx s^s$$

Landau: an example

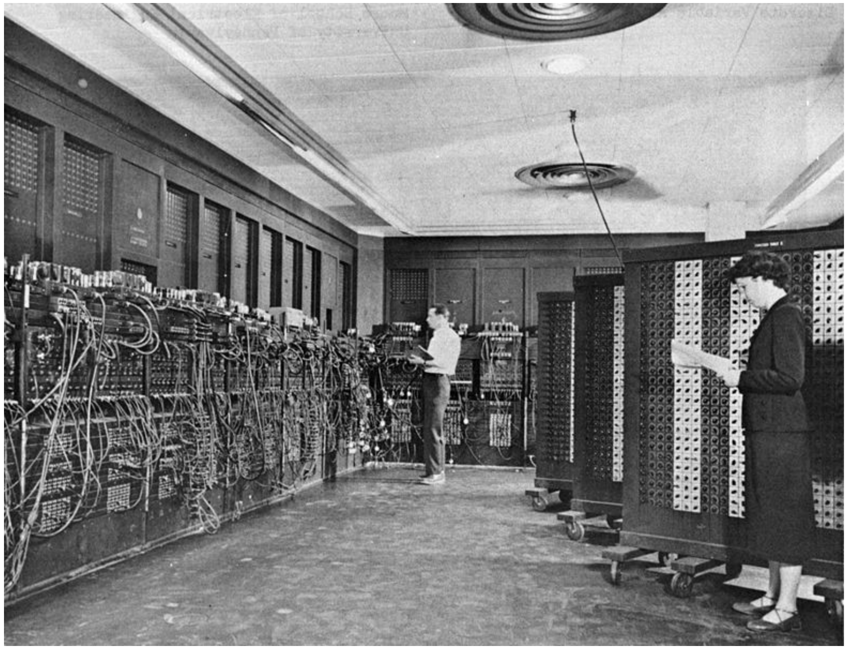
- 2 GeV protons on an (only !) 5 cm thick Ar gas layer:

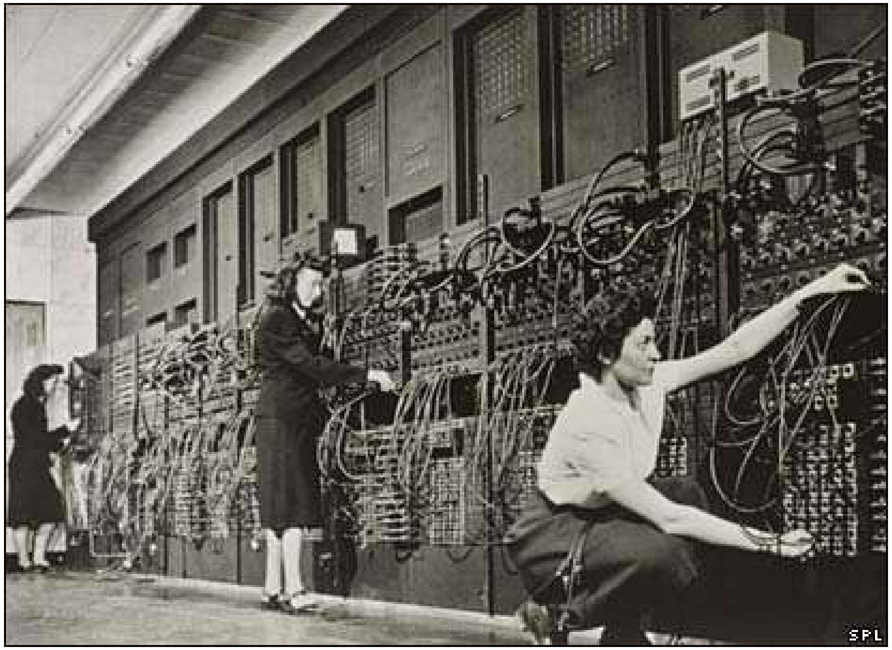


[Diagram: Richard Talman, NIM A **159** (1979) 189-211]

1946-1955: ENIAC

- Originally meant for ballistic trajectory calculations, the device was also suitable for atomic energy work.



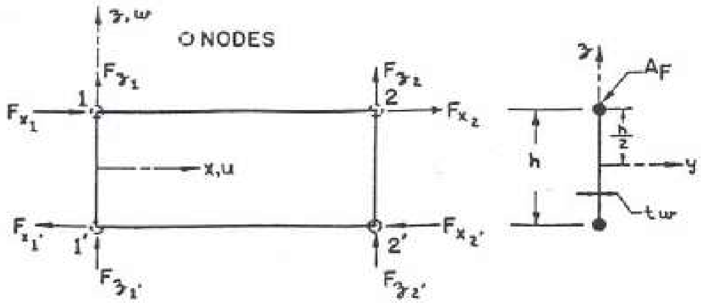


Ιωα'ννης Αργυ'ρης

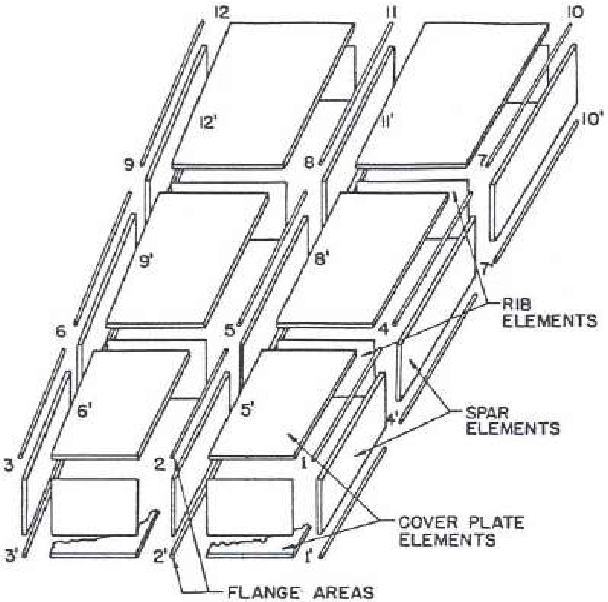
(1913-2004)

1956: Finite elements

- “*Stiffness and Deflection Analysis of Complex Structures*”, a study in the use of the finite element technique (then called “direct stiffness method”) for aircraft wing design.



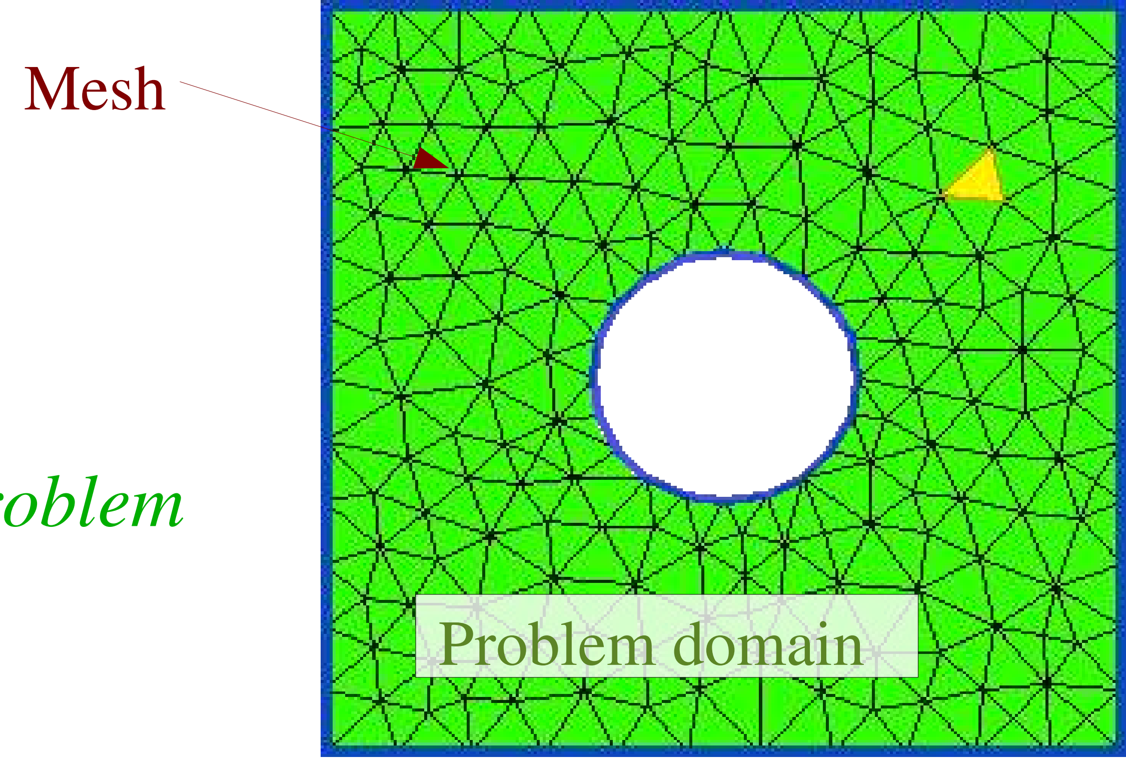
$$
\begin{array}{c}
\qquad\qquad\qquad\quad u_1 \qquad\quad v_1 \qquad w_1 \qquad\quad u_2 \qquad\quad v_2 \quad w_2 \\
[K] = \dfrac{6EI}{Lh^2(1+4n)} \begin{bmatrix}
(4/3)(1+n) & & & & & \\
0 & 0 & & & & \\
-(h/L) & 0 & h^2/L^2 & & & \\
(2/3)(1-2n) & 0 & -(h/L) & (4/3)(1+n) & & \\
0 & 0 & 0 & 0 & 0 & \\
h/L & 0 & -(h^2/L^2) & h/L & 0 & h^2/L^2
\end{bmatrix}
\end{array}
$$

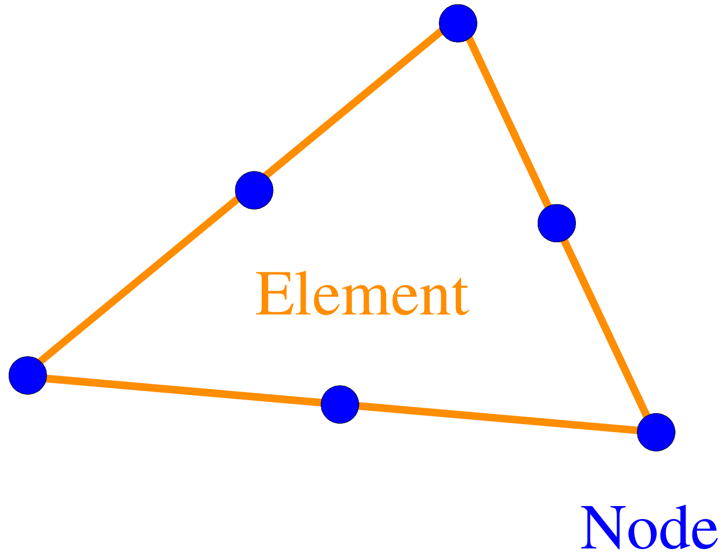


[M.J. Turner, R.W. Clough, H.C. Martin and L.J. Topp, *Stiffness and Deflection Analysis of Complex Structures*, J. Aero. Sc. **23** (1956), 805-824. MJT & LJT with Boeing.]

Terminology

- A *mesh* subdivides the *problem domain* into *elements*.
- *Elements* are simple geometric shapes: triangles, squares, tetrahedra, hexahedra etc.
- Important points of *elements* are called *nodes*. It is usual that several *elements* have a *node* at one and the same location.



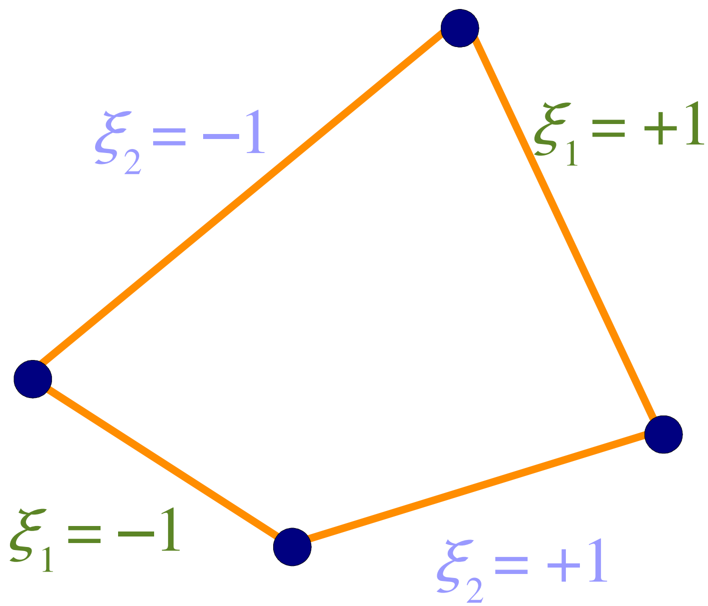


Shape functions - interpolation

- Each node has its own *shape function* $N_i(r)$:
  - continuous functions (usually polynomial),
  - defined only throughout the body of the element,
  - $N_i(r) = 1$ when $r = r_i$ i.e. on node i,
  - $N_i(r) = 0$ when $r = r_j$, $i \neq j$ i.e. on all other nodes.
- The solution of a finite element problem is given in the form of potential values at each of the nodes of each of the elements: v_i.
- At interior points of an element: $V(r) = \sum v_i N_i(r)$

Quadrilaterals

- The shape functions are chosen to be:
  - $N_1 = \frac{1}{4}\,(1 - \xi_1)\,(1 - \xi_2)$
  - $N_2 = \frac{1}{4}\,(1 + \xi_1)\,(1 - \xi_2)$
  - $N_3 = \frac{1}{4}\,(1 - \xi_1)\,(1 + \xi_2)$
  - $N_4 = \frac{1}{4}\,(1 + \xi_1)\,(1 + \xi_2)$
- Hexahedral shape functions are analogous.



- Quadrilateral and hexahedral elements are isoparametric by construction: the coordinates *follow* from the shape functions !

Solving the problem

- We have to solve Poisson with boundary conditions:

$$-\nabla\cdot\epsilon\nabla\phi=\rho \quad \text{(Poisson)} \qquad \begin{array}{ll} \phi|_{\partial D}=V & \text{(Dirichlet)} \\ \epsilon\nabla\phi\cdot n|_{\partial D}=h & \text{(Neumann)} \end{array}$$

- As ansatz, we sum the unknown potential at each node v_i multiplied by the combination α_i of all shape functions that contain this node: $\phi=\sum_{\text{nodes}} v_i\alpha_i$
- And we find a (linear) matrix equation:

$$\int_D \rho\alpha_j=-\int_D(\nabla\cdot\epsilon\nabla\phi)\alpha_j \qquad \text{(A trick: Poisson integrated)}$$

$$\int_D \rho\alpha_j=-\oint_{\partial D}\alpha_j(\epsilon\nabla\phi\cdot n)+\int_D\nabla\alpha_j\cdot\epsilon\nabla\phi \qquad \text{(Integrating by parts)}$$

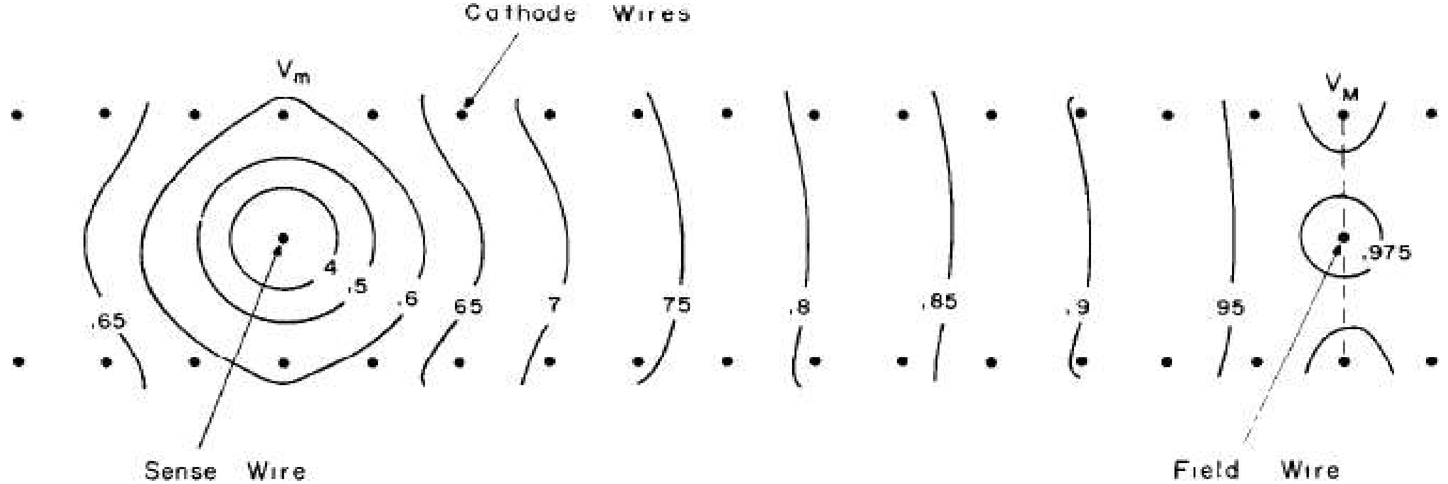
$$\int_D \rho\alpha_j+\oint_{\partial D}h\alpha_j=\sum v_i\epsilon_i\int_D\nabla\alpha_i\cdot\nabla\alpha_j \qquad \text{(Using boundary conditions)}$$

The price to pay for finite elements

- Finite elements focus (for us) on the wrong thing:
  - they solve V well, but we do not really need it:
    - Quadratic shape functions can do a fair job at approximating $V \approx \log(r)$ potentials.
    - Potentials are continuous.
  - E is what we use, but:
    - Gradients of quadratic shape functions are linear *i.e.* unsuitable to approximate our $E \approx 1/r$ fields with, left alone $E \approx 1/r^2$ fields.
    - Electric fields are discontinuous.
    - A local accuracy of ~50 % in high-field areas is not unheard of.
- In exchange, we get a lot of flexibility.

1970s: Analog method for 2d potentials

- Equivalent problem approach, still alive in the 1970s:
  - sheets of carbon paper of ~constant surface resistivity,
  - electrodes of any shape made with silver paint,
  - apply suitable boundary potentials,
  - measure the voltages.



[G. Charpak, F. Sauli and W. Duinker, *High-accuracy drift chambers and their use in strong magnetic fields*, Nuclear Instruments and Methods, **108** (1973) 413-426.]

Arthur V. Phelps

1962: Numerical e^- transport

- Iterative approach, allowing for inelastic cross section terms:
  - educated guess of cross sections (elastic & inelastic);
  - numerically solve the Boltzmann equation (no moments);
  - compare calculated and measured mobility and diffusion;
  - adjust cross sections.

"... more than 50,000 transistors plus extremely fast magnetic core storage. The new system can simultaneously read and write electronically at the rate of 3,000,000 bits of information a second, when eight data channels are in use. In 2.18 millionths of a second, it can locate and make ready for use any of 32,768 data or instruction numbers (each of 10 digits) in the magnetic core storage. The 7090 can perform any of the following operations in one second: 229,000 additions or subtractions, 39,500 multiplications, or 32,700 divisions. " (IBM 7090 documentation)

[L.S. Frost and A.V. Phelps, *Rotational Excitation and Momentum Transfer Cross Sections for Electrons in* H_2 *and* N_2 *from Transport Coefficients*, Phys. Rev. **127** (1962) 1621–1633.]

1980s: Higher moments, high precision

- Expansion in spherical harmonics;
- An accuracy of 1 % (and better) becomes routine.

> The starting point for most theoretical work is the Boltzmann equation for the electron velocity distribution function, $f(\mathbf{r}, \mathbf{v}, t)$. The latter is formally expanded in a series of spherical harmonics,
>
> $$f(\mathbf{r}, \mathbf{v}, t) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} f_{lm}(\mathbf{r}, v, t) Y^{\dagger}_{lm}(\hat{\mathbf{v}}) , \qquad (1)$$
>
> where $Y_{lm}(\hat{\mathbf{v}}) \equiv Y_{lm}(\theta, \phi) = P_l^{|m|}(\cos\theta)\, e^{im\phi}$, and θ, ϕ denote the polar angles of the unit velocity vector $\hat{\mathbf{v}}$ in some frame of reference.

S.L. Lin, R.E. Robson and E.A. Mason, *Moment theory of electron drift and diffusion in neutral gases in an electrostatic field,* J. Chem. Phys. **71** (1979) 3483-3498 (the “LRM” paper).

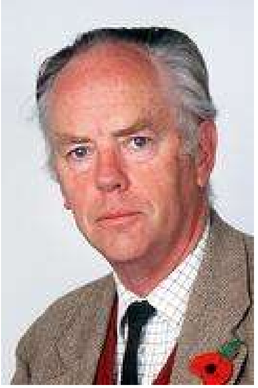
R.E. Robson and K.F. Ness, *Velocity distribution function and transport coefficients of electron swarms in gases: Spherical-harmonics decomposition of Boltzmann’s equation*, Phys. Rev. A **33** (1986) 2068–2077.

K.F. Ness and R.E. Robson, *Velocity distribution function and transport coefficients of electron swarms in gases. II. Moment equations and applications*, Phys. Rev. A **34** (1986) 2185–2209.

1970-1980: PAI model

- The photo-absorption and ionisation model was introduced by W.W.M. Allison & J.H. Cobb
  - Ann. Rev. Nucl. Part. Sci. **30** (1980) 253-298.
- Other often cited papers:
  - earlier papers from the 1960s
  - V.A. Chechin *et al.*, NIM **98** (1972) 577.
  - V.A. Chechin *et al.*, NIM **136** (1976) 551.
  - V.C. Ermilova *et al.*, NIM **145** (1977) 555.
  - F. Lapique and F. Piuz, NIM **175** (1980) 297.
  - ...
- First complete, widely available, computer program:
  - Heed: Igor Smirnov, *Modeling of ionization produced by fast charged particles in gases*, accepted for publication in NIM (Aug 2005).

Basics of the PAI model



Wade Allison

John Cobb

- Key ingredient: photo-absorption cross section $\sigma_\gamma(E)$

$$\frac{\beta^2\pi}{\alpha}\frac{\mathrm{d}\sigma}{\mathrm{d}E}=\frac{\sigma_\gamma(E)}{E}\log\left(\frac{1}{\sqrt{(1-\beta^2\epsilon_1)^2+\beta^4\epsilon_2^2}}\right)+$$

Relativistic rise

Cross section to transfer energy E

$$\frac{1}{N\hbar c}\left(\beta^2-\frac{\epsilon_1}{|\epsilon|^2}\right)\theta+$$

Čerenkov radiation

$$\frac{\sigma_\gamma(E)}{E}\log\left(\frac{2m_e c^2\beta^2}{E}\right)+$$

Resonance region

$$\frac{1}{E^2}\int_0^E\sigma_\gamma(E_1)\,\mathrm{d}E_1$$

Rutherford scattering

With:

$$\epsilon_2(E)=\frac{N_e\hbar c}{EZ}\sigma_\gamma(E)$$

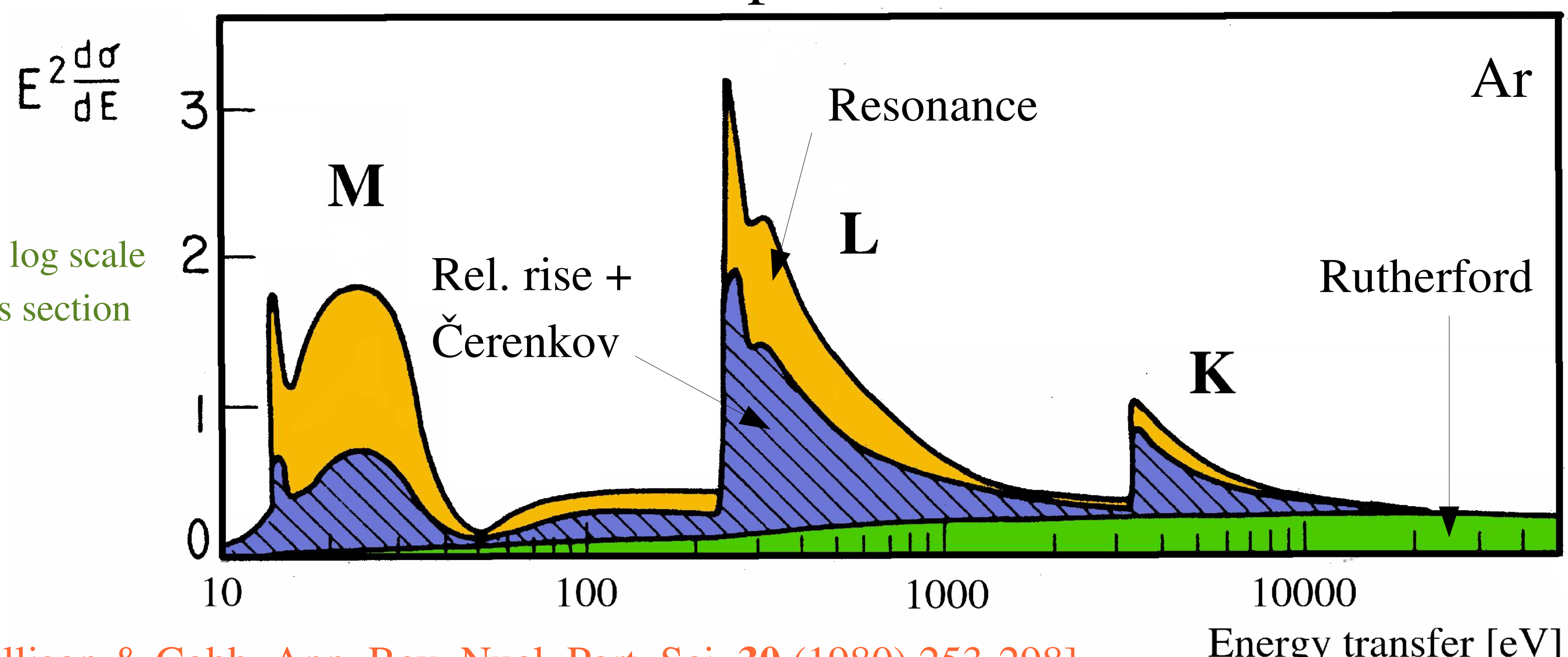
$$\epsilon_1(E)=1+\frac{2}{\pi}\mathrm{P}\int_0^\infty\frac{x\epsilon_2(x)}{x^2-E^2}\mathrm{d}x$$

$$\theta=\arg(1-\epsilon_1\beta^2+i\epsilon_2\beta^2)=\frac{\pi}{2}-\arctan\frac{1-\epsilon_1\beta^2}{\epsilon_2\beta^2}$$

Importance of the PAI model terms

- All electron orbitals (shells) participate:
  - outer shells: frequent interactions, few electrons;
  - inner shells: few interactions, many electrons.
- All terms in the formula are important.

Scaling with E^2:
equal areas on log scale
weighing cross section



[Adapted from Allison & Cobb, Ann. Rev. Nucl. Part. Sci. **30** (1980) 253-298]

2005: Ionisation process in detail

Igor Smirnov

- PAI model or absorption of real photons:

  Atom + π^- ➡ Ion^{+*} + π^- + e$^-$ (photo-electric effect)

  Atom + γ ➡ Ion^{+*} + e$^-$

- Decay of excited states:

  Ion^{+*} ➡ Ion^{++} + e$^-$ (Auger)

  Ion^{+*} ➡ Ion^{+} + γ (fluorescence)

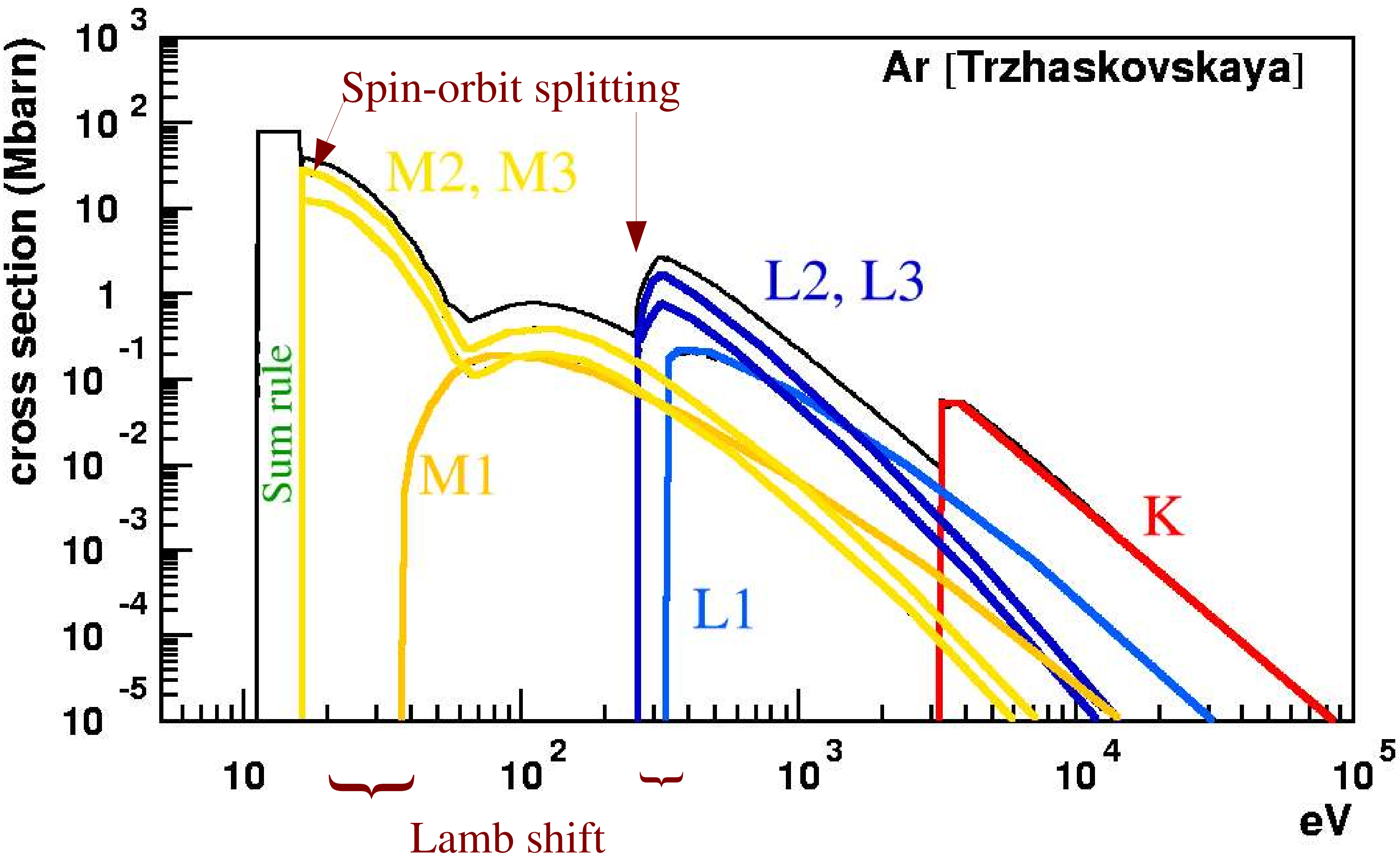
  Ion^{+*} ➡ Ion^{+*} (Coster-Kronig)

- Treatment of:
  - secondary photons, returning to the PAI model,
  - ionising photo-electrons and Auger-electrons, collectively known as δ-electrons:

  e$^-$ + Atom ➡ Ion^{+} + 2 e$^-$ (absorption of high-energy electrons)

Photo-absorption in argon

- Argon has 3 shells, hence 3 groups of lines:



K = 1s

L1 = 2s
L2 = 2p 1/2
L3 = 2p 3/2

M1 = 3s
M2 = 3p 1/2
M3 = 3p 3/2

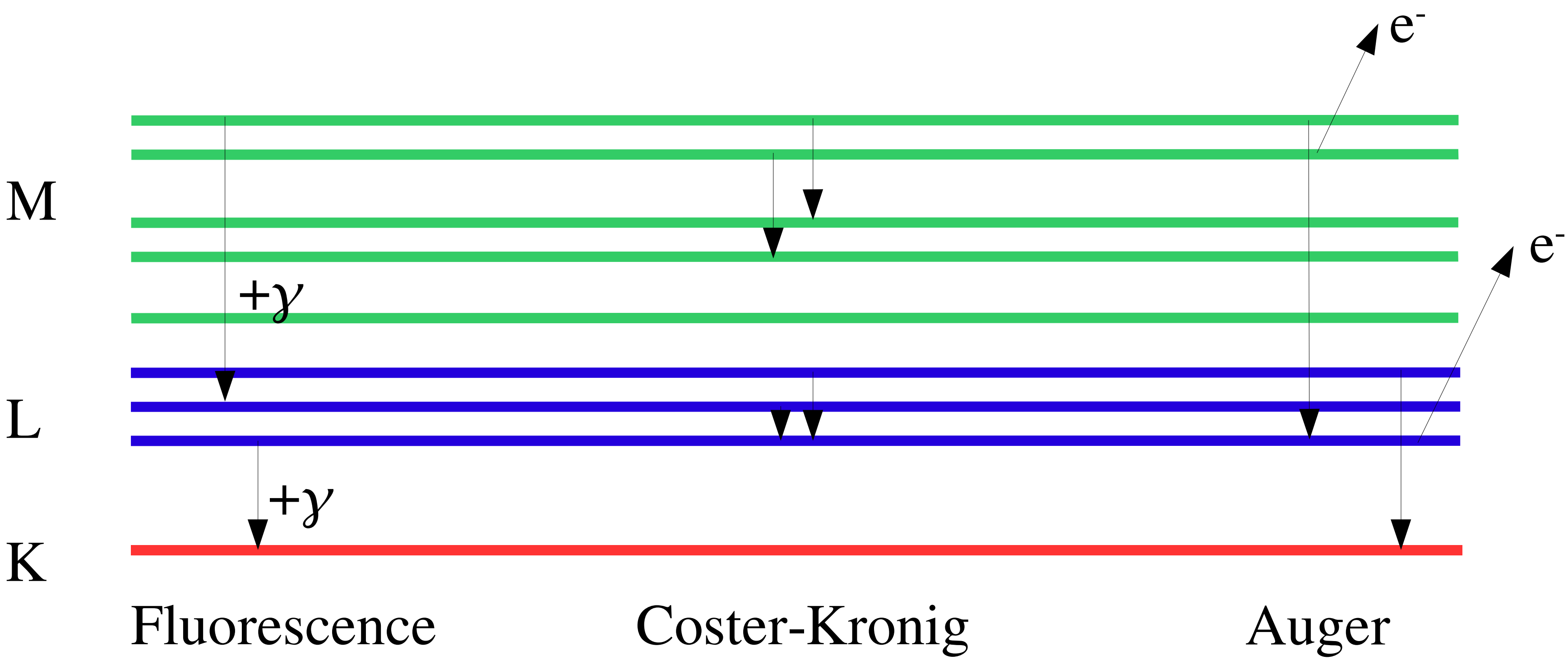
[Plot from Igor Smirnov]

De-excitation

Ralph de Laer Kronig
(1904-1995)

Liese Meitner
(1878-1968)

Pierre Victor Auger
(1899-1993)



References:

D. Coster and R. de L. Kronig, Physica **2** (1935) 1, 13.

L. Meitner, *Das beta-Strahlenspektrum von UX1 und seine Deutung*, Z. Phys. **17** (1923) 54-66.

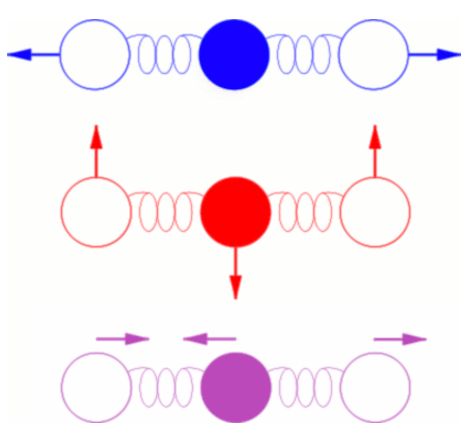
P. Auger, J. Phys. Radium **6** (1925) 205.

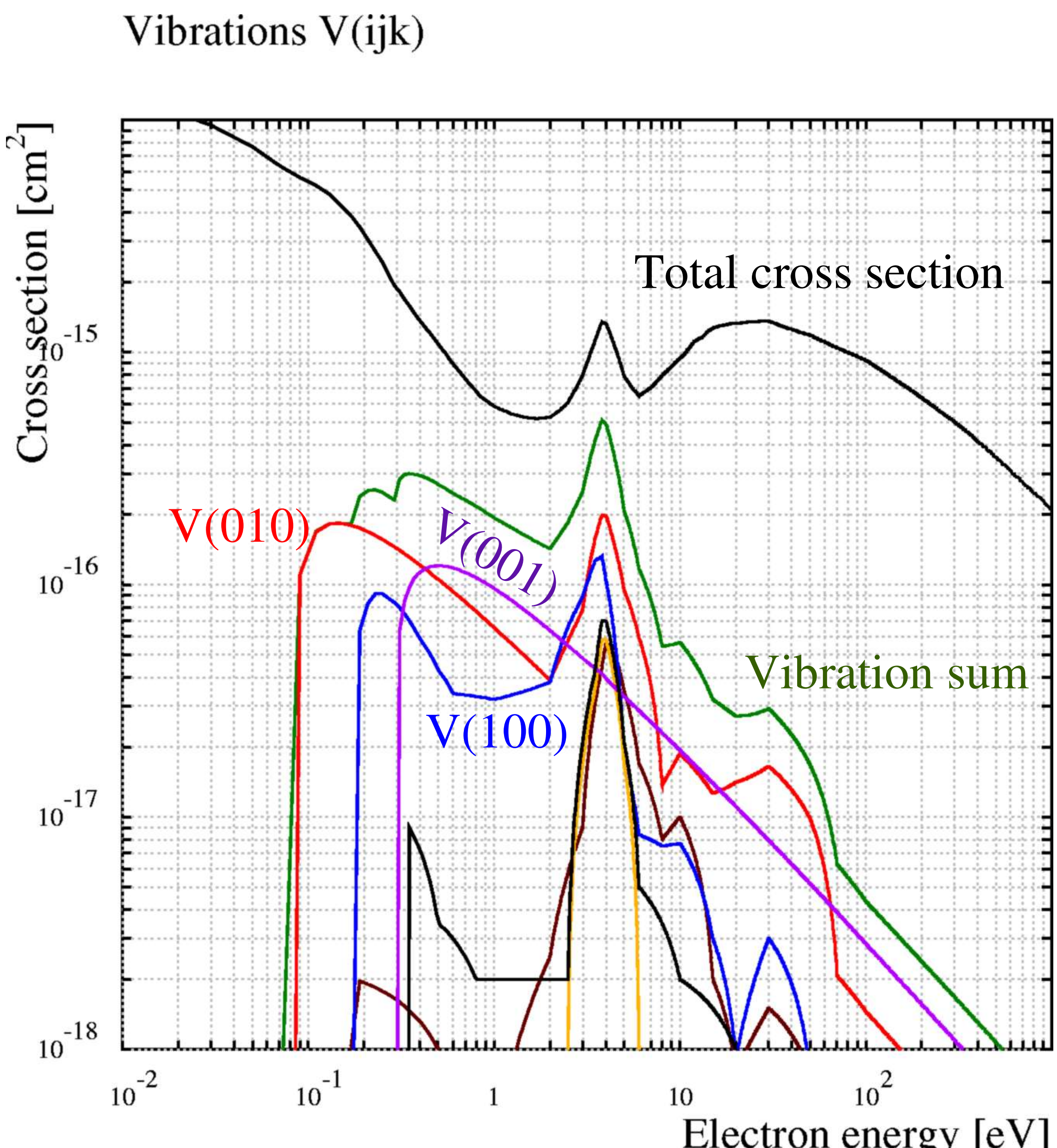
2000: Monte Carlo, Magboltz

- A large number of cross sections for 60 molecules...
  - All noble gases, *e.g.* argon:
    - elastic scattering,
    - 3 excited states and
    - ionisation.
  - Numerous organic gases, additives, *e.g.* CO_2:
    - elastic scattering,
    - 44 inelastic cross sections (vibrations, rotations, polyads)
    - 35 super-elastic cross sections,
    - 6 excited states,
    - attachment and
    - ionisation.

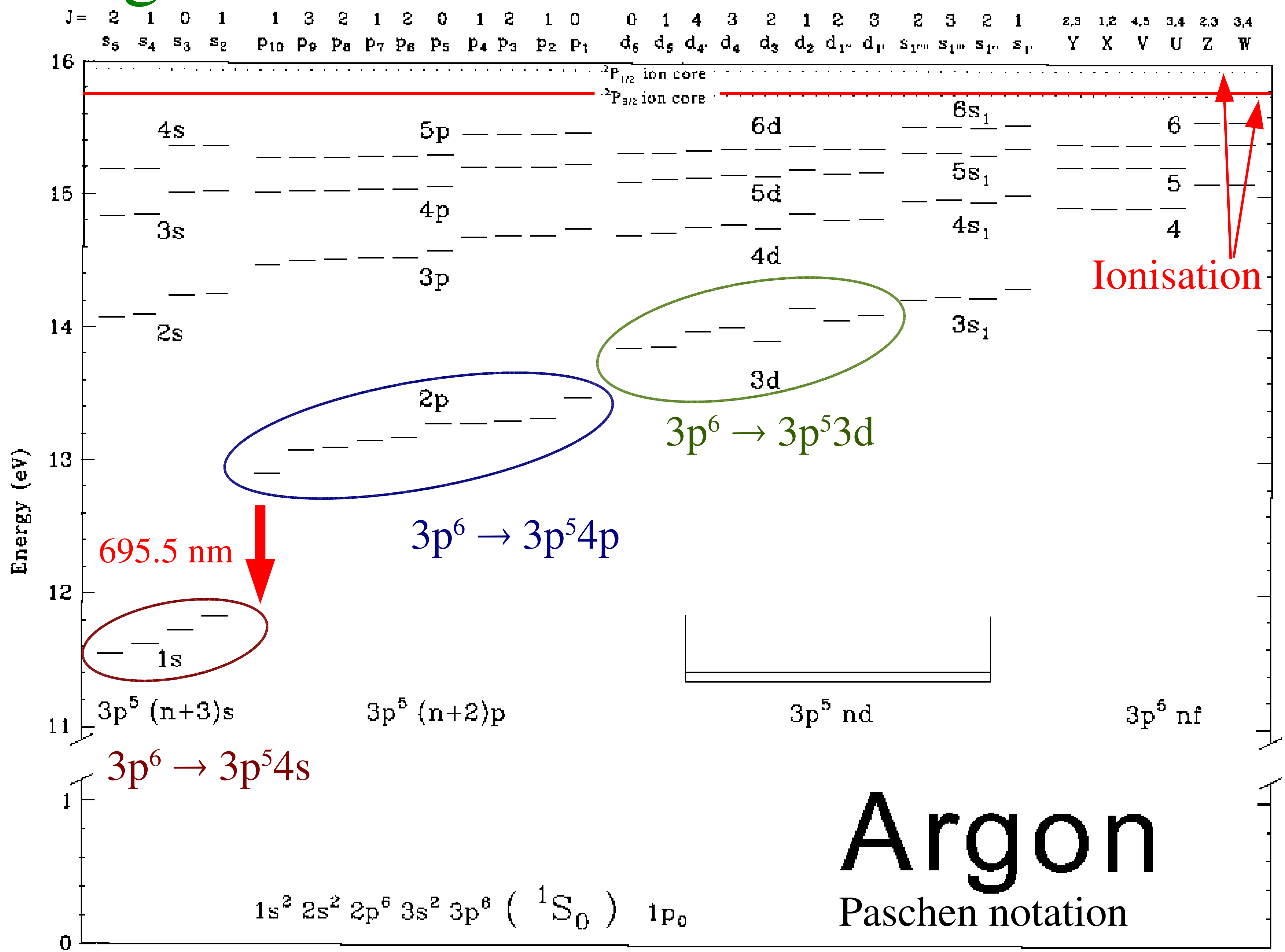
CO_2 – vibration modes

- CO_2 is linear:
  - O – C – O
- Vibration modes are numbered V(*ijk*)
  - *i*: symmetric,
  - *j*: bending,
  - *k*: anti-symmetric.





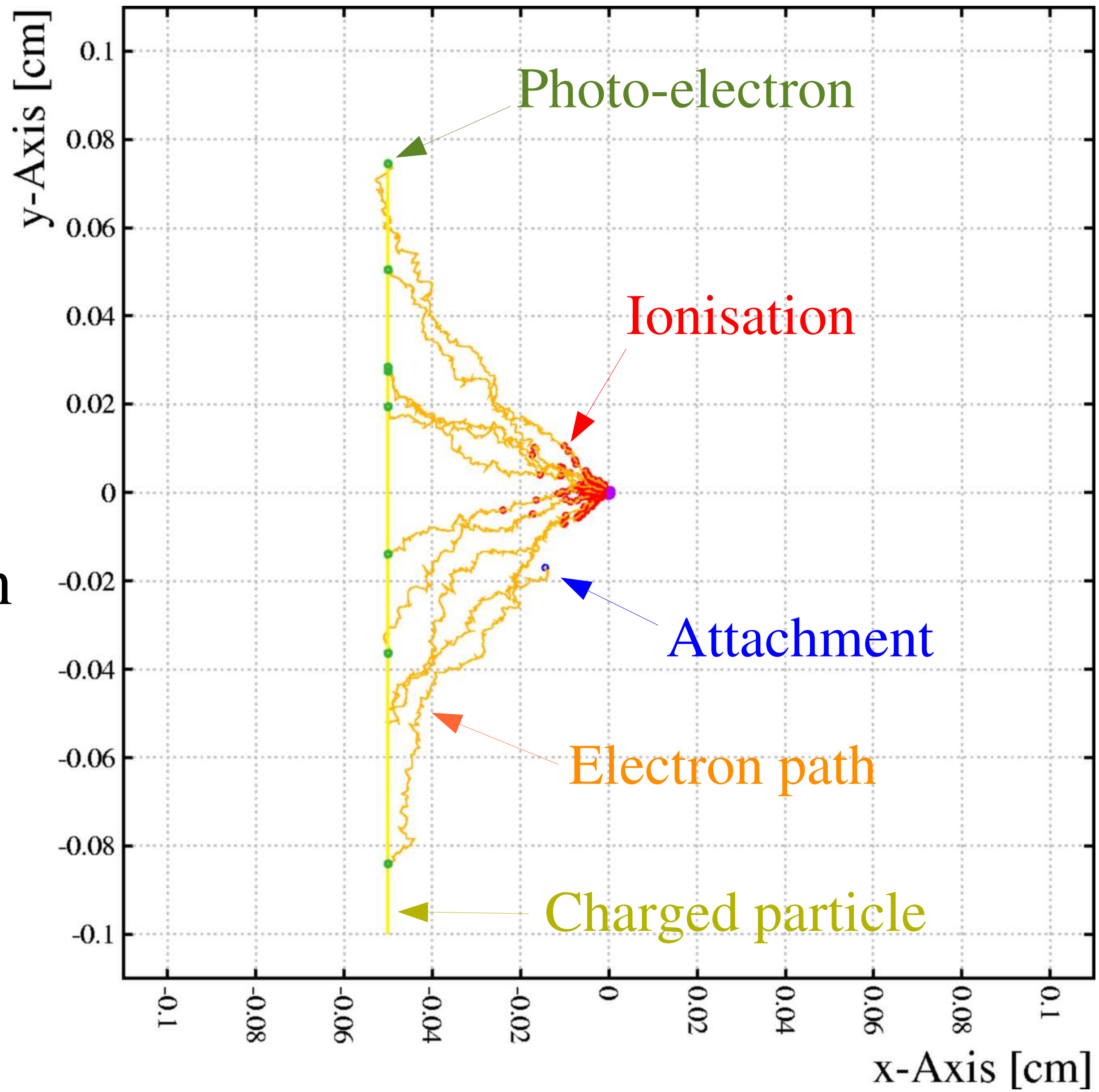
Argon levels



Molecular tracking: example

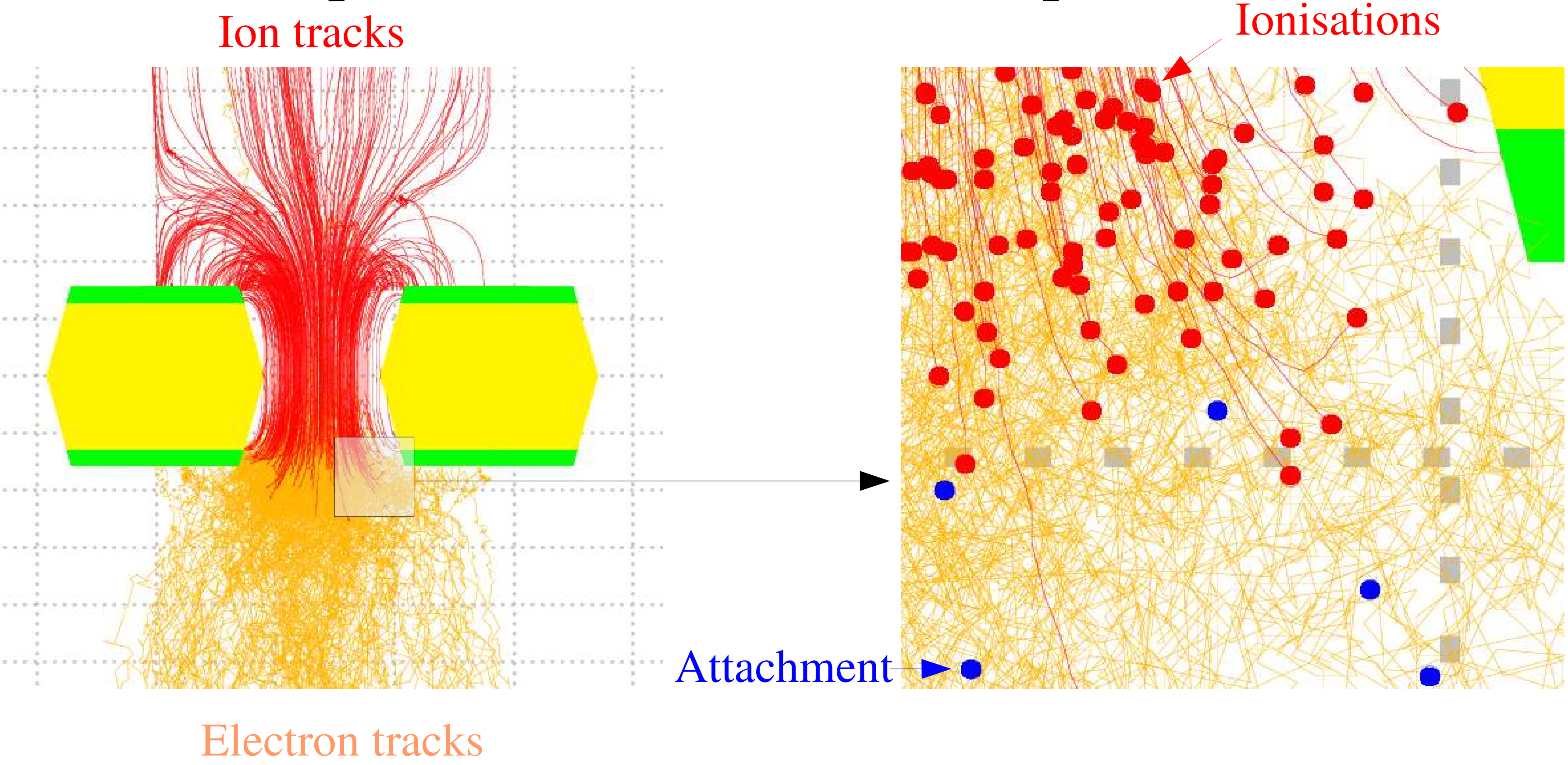
- Example:
  - CSC-like structure,
  - Ar 80 % CO_2 20 %,
  - 10 GeV μ.

- The electron is shown every 100 collisions, but has been tracked rigourously.



Simulation of micropattern devices

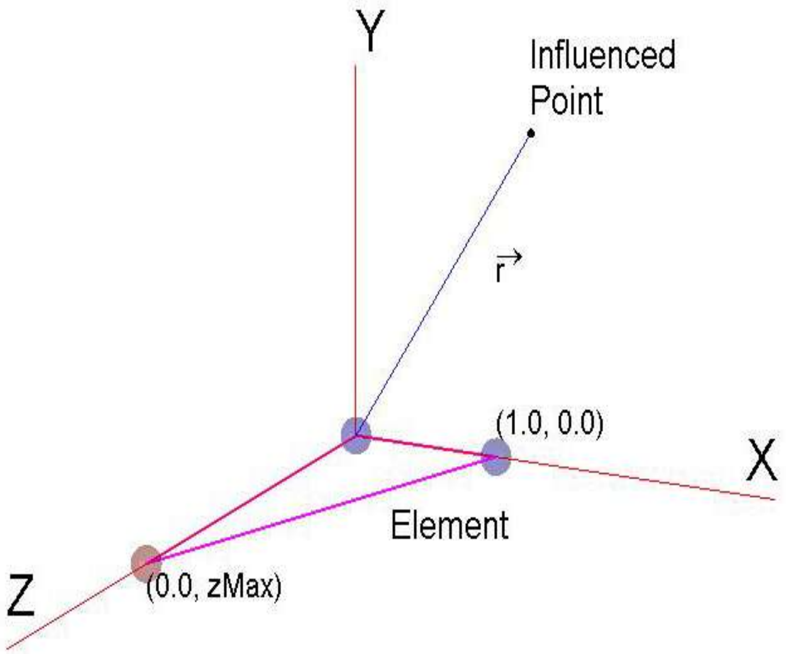
- Micropattern devices have characteristic dimensions that are comparable with the mean free path.



[Plot by Gabriele Croci and Matteo Alfonsi]

2009: neBEM – nodal vs distributed

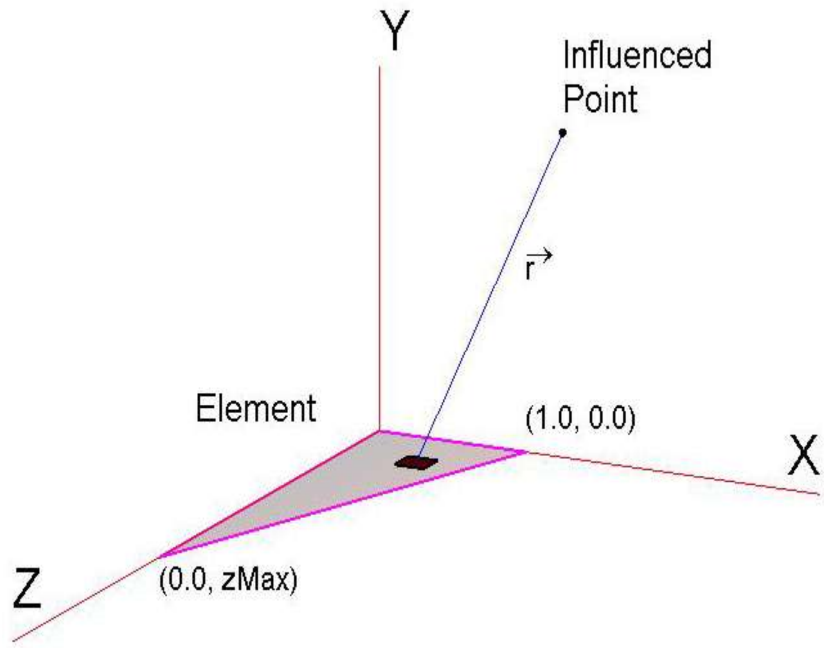
Influence of a flat triangular element in Usual BEM



Conventionally, charges are assumed to be concentrated at *nodes*. This is convenient since the preceding integration is avoided. Introduces large errors in the near field.

We have derived exact expressions for the integration of G and its derivative for uniform charge *distributions* over triangular and rectangular elements

Influence of a flat triangular element in ISLES



BEM approach to 3d Poisson equation

- Numerical implementation of boundary integral equations (BIE) based on Green's function by discretization of boundary.
- Boundary elements endowed with distribution of sources, doublets, dipoles, vortices.

Electrostatics BIE

$$\phi(\vec{r})=\int_S G(\vec{r},\vec{r}\,')\rho(\vec{r}\,')\,dS'$$

discretization

$$[A]\{\rho\}=\{\phi\}$$

$$\{\rho\}=[A]^{-1}\{\phi\}$$

Potential at *r*

Charge density at *r'*

Influence Coefficient Matrix

Green's function

$$G(\vec{r},\vec{r}\,')=\frac{1}{4\pi\epsilon|\vec{r}-\vec{r}\,'|}$$

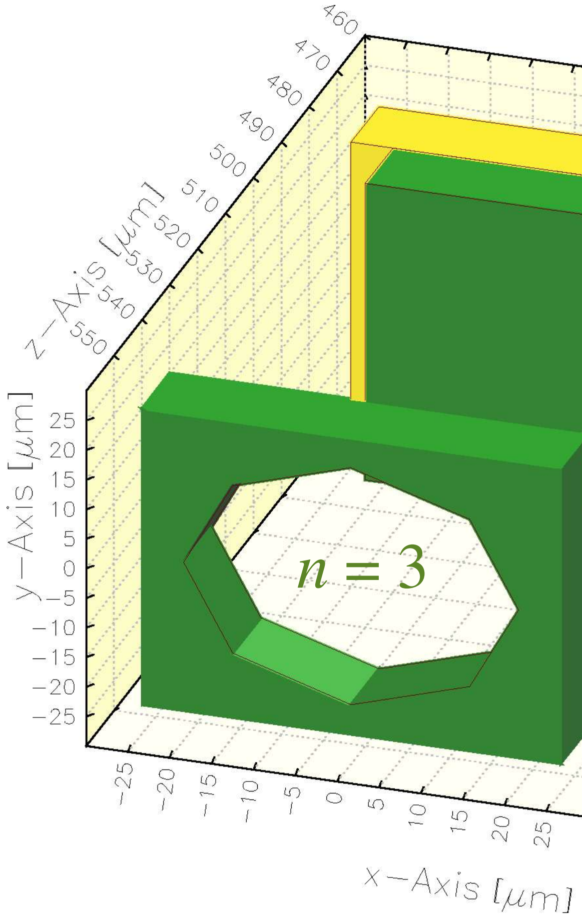
ε - permittivity of medium

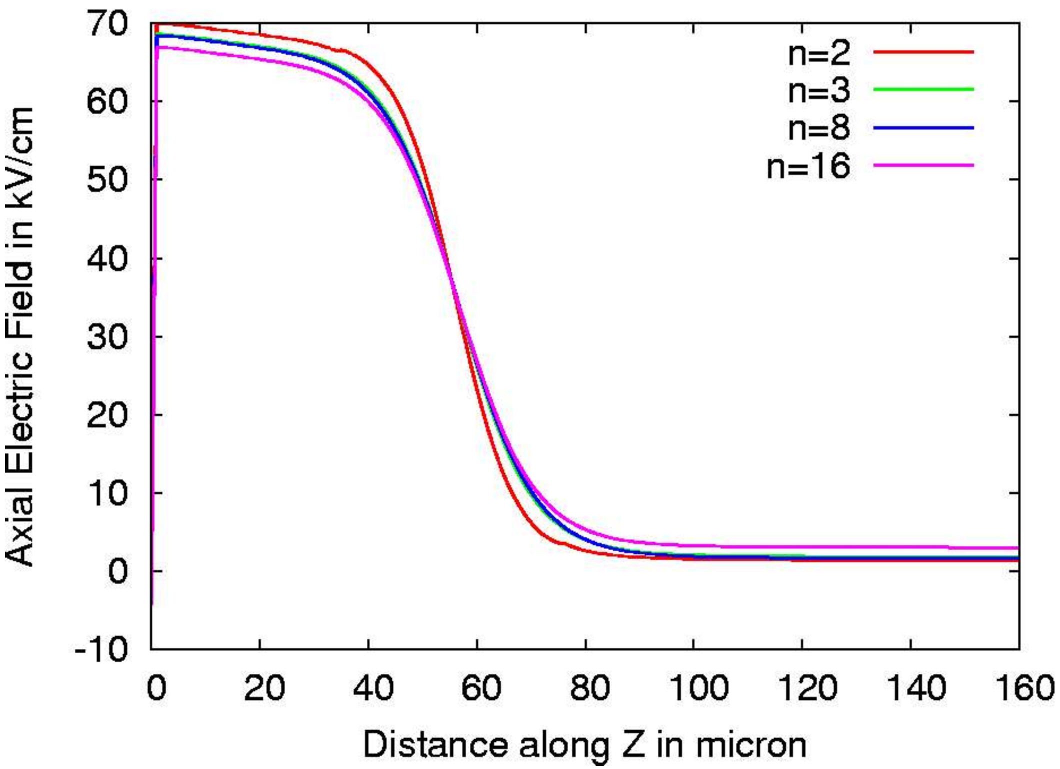
Accuracy depends critically on the estimation of [A], in turn, the integration of G, which involves singularities when r → r'.

Most BEM solvers fail here.

neBEM – example

- Effect on the axial field of the hole shape: poster B285 by Purba Bhattacharya

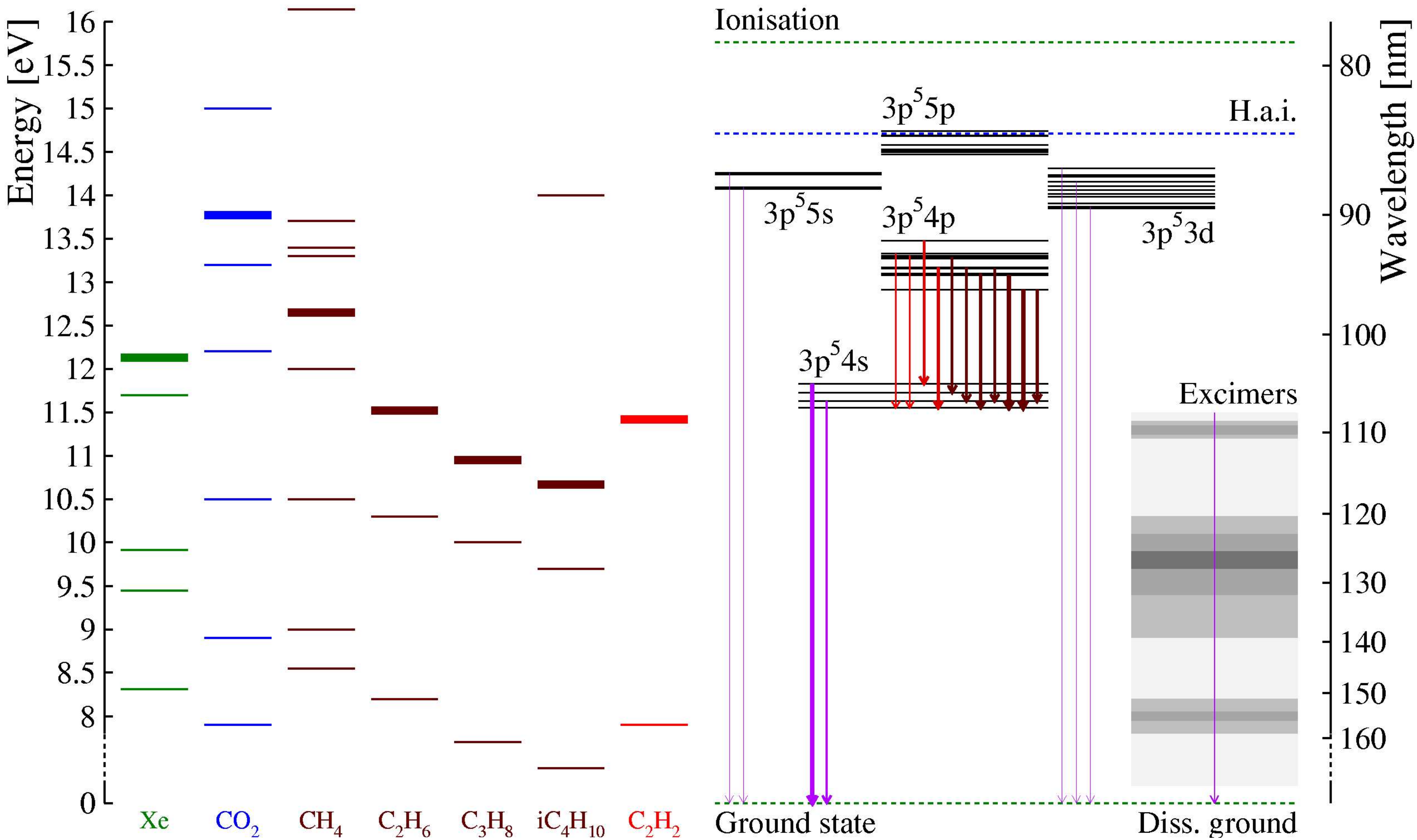




2010: Penning transfer processes

- Usually, it is the noble gas A^* which is excited and the admixture B which is ionised via one of several processes:
  - dipole-dipole coupling: $A^* \rightarrow A\,\gamma, \quad \gamma\,B \rightarrow B^+\,\mathrm{e}^-$
  - e^- exchange (“Auger”): $A^*\,B \rightarrow A^{*-}\,B^+, \quad A^{*-} \rightarrow A\,\mathrm{e}^-$
  - associative ionisation: $A^*\,A \rightarrow A^+_2\,\mathrm{e}^-$
  - excimers in some gases
- A^* can also undergo natural decay. In case of radiative decay, the photon sometimes ionises.
- Each process has its characteristic time dependence (decay time, collision frequency) which translates into a partial pressure dependence.

Level diagram argon and admixtures



Approach

- From gain curves, determine the Penning transfer rate *r*:

$$G = \exp \int \alpha \frac{\sum \nu_{\text{ion}} + \sum r \nu_{\text{exc}}}{\sum \nu_{\text{ion}}}$$

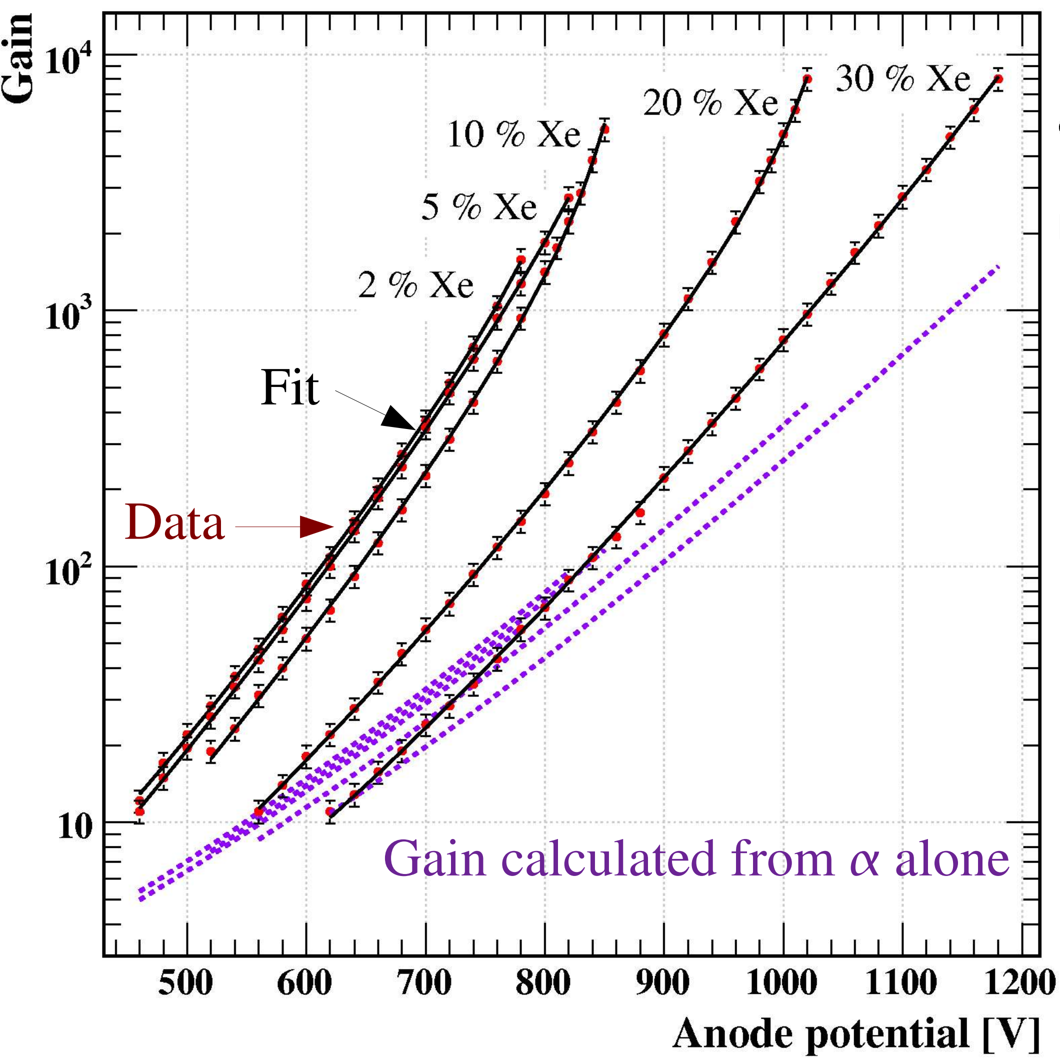
- From the transfer rates as function of pressure *p* and concentration *c*, determine the model parameters:

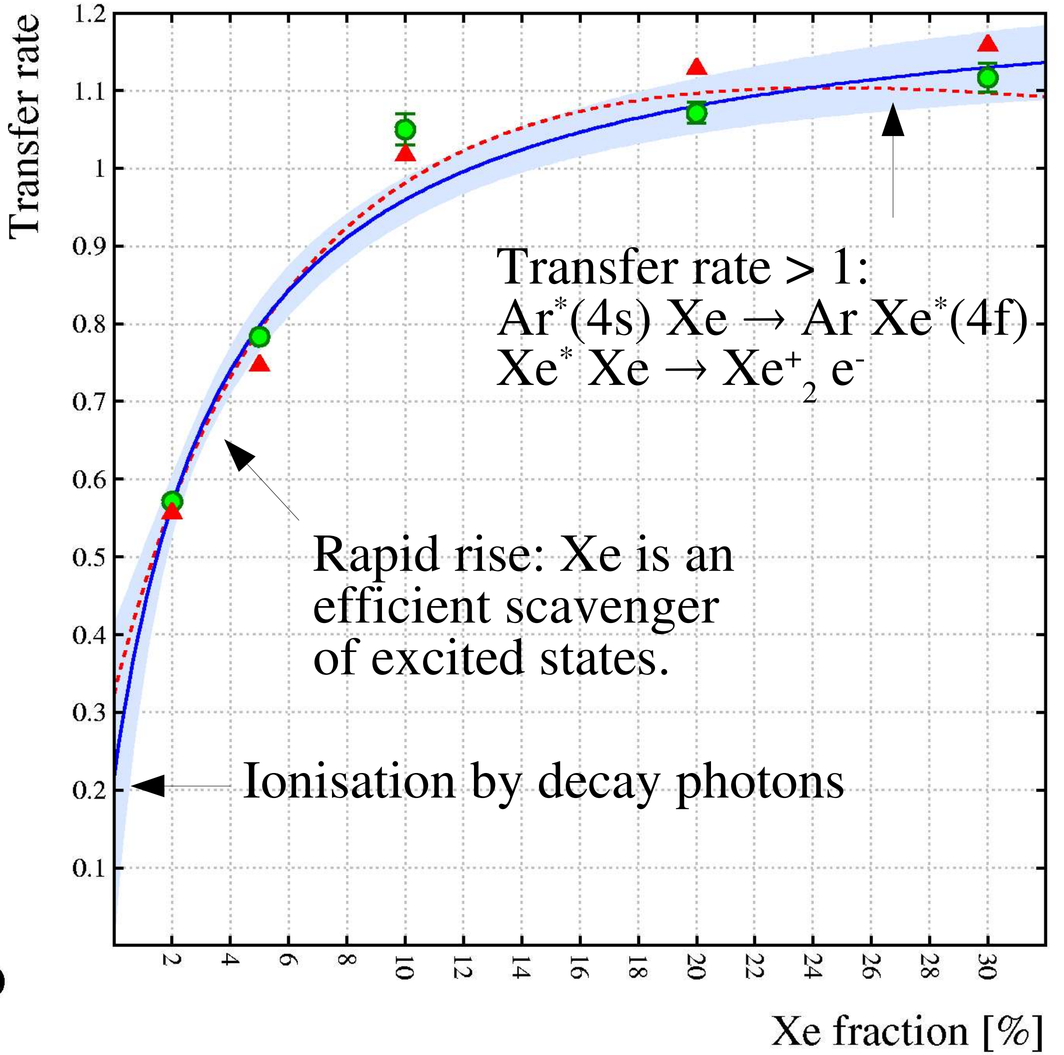
$$r = \frac{p\,c\,f_{B^+}/\tau_{AB} + p(1-c)\,f_{A^+}/\tau_{AA} + f_{\text{rad}}/\tau_{A^*}}{p\,c\,(f_{B^+} + f_{\bar{B}})/\tau_{AB} + p(1-c)(f_{A^+} + f_{\bar{A}})/\tau_{AA} + 1/\tau_{A^*}}$$

$A^* \rightarrow B^+$ $\quad$ $A^* \rightarrow A^+$ $\quad$ $A^* \rightarrow A\,\gamma$

Example 1: Ar-Xe

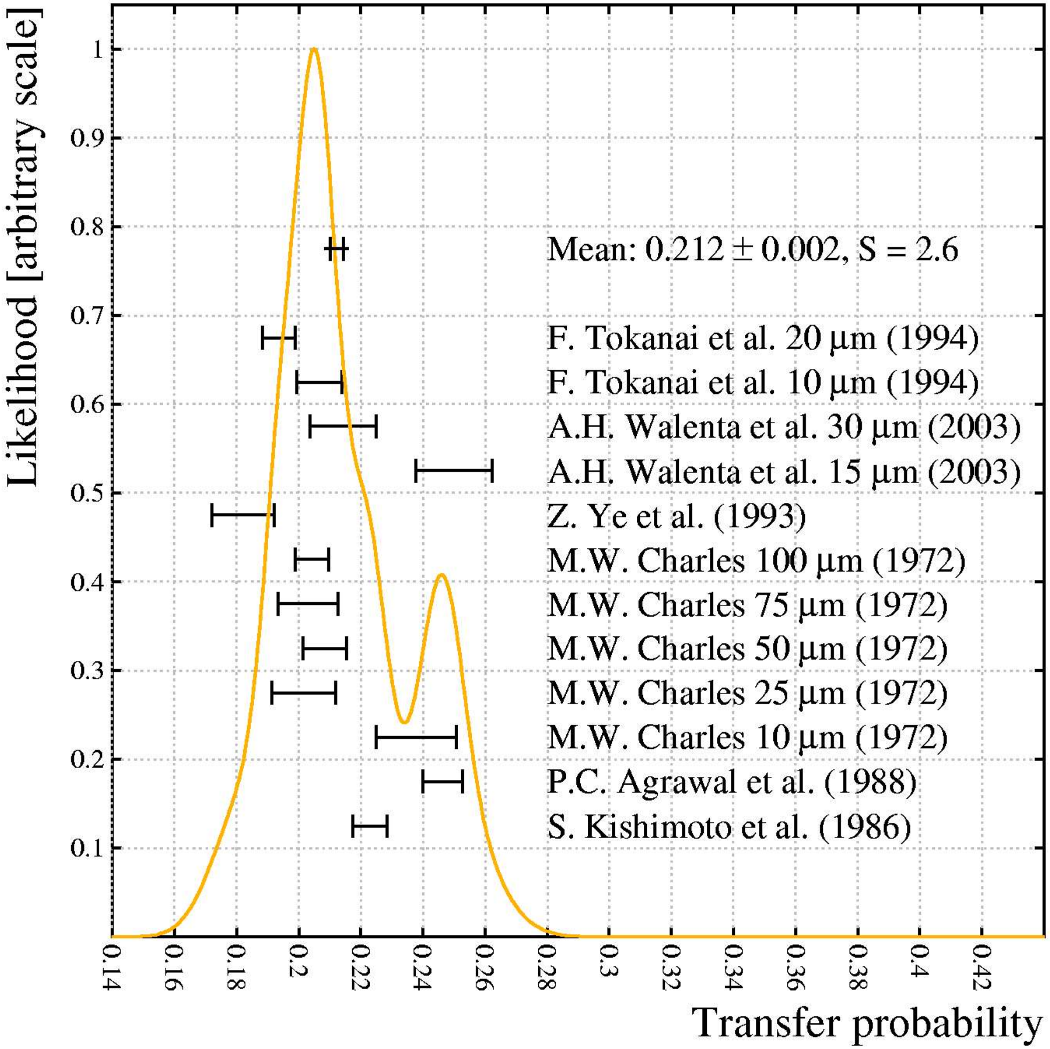
- Ar 4*p*, 3*d* and higher above the Xe ionisation threshold.

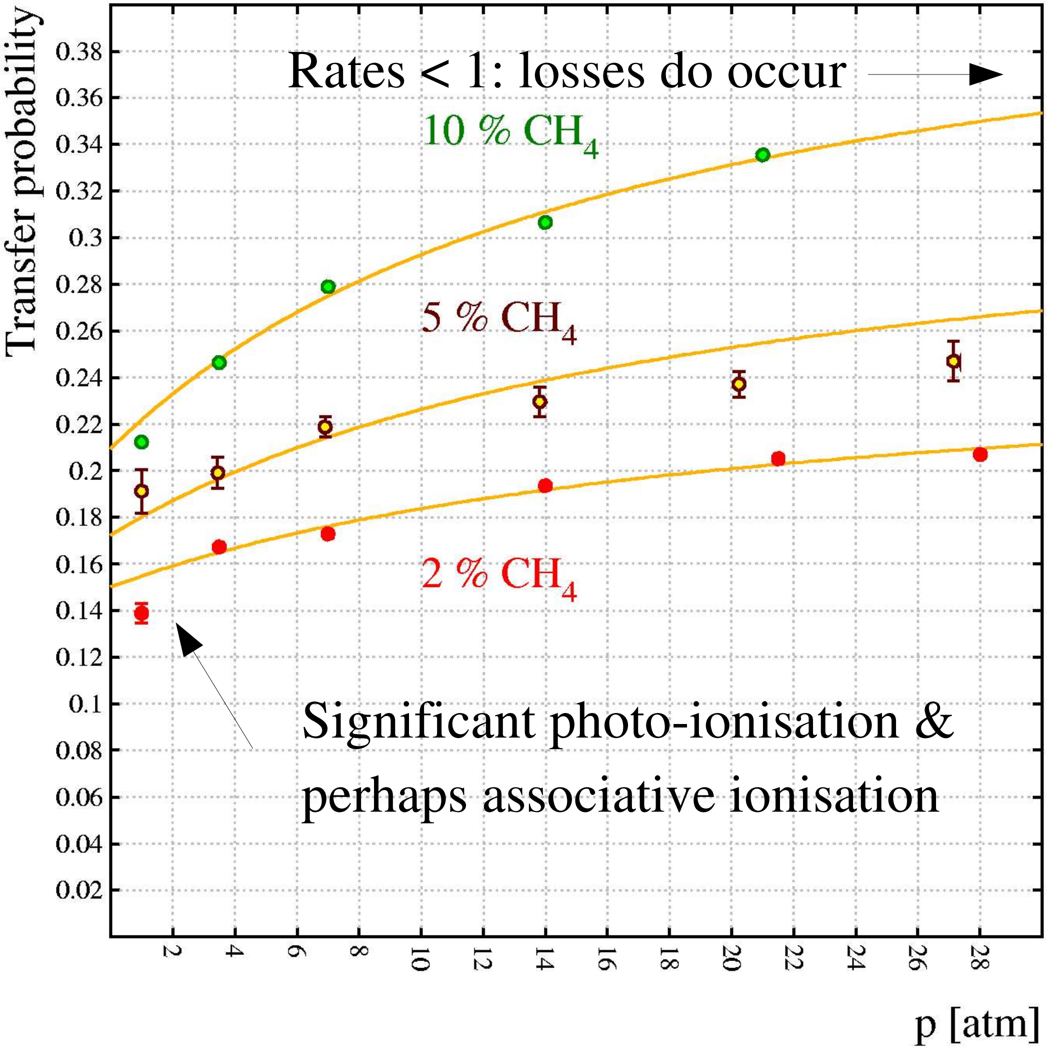




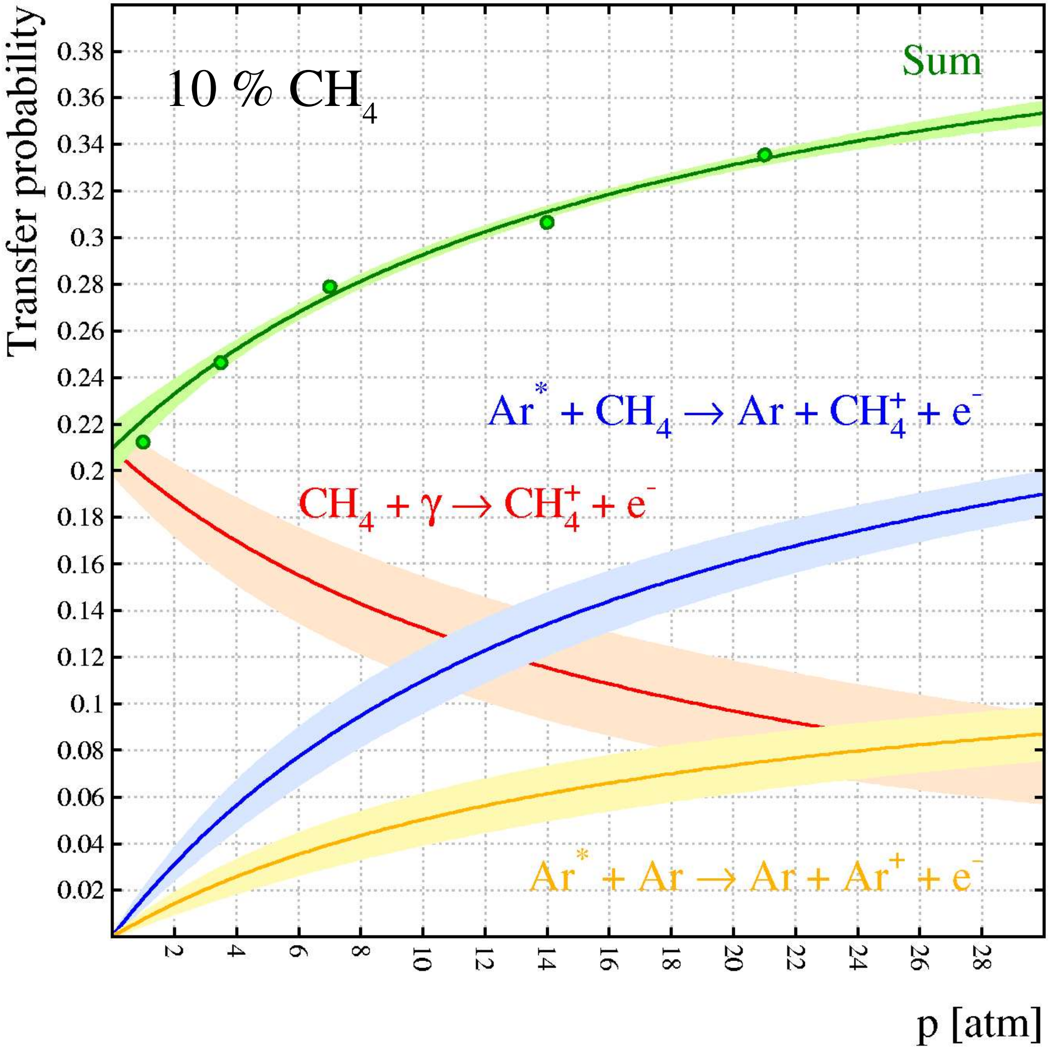
Example 2: Ar-CH_4

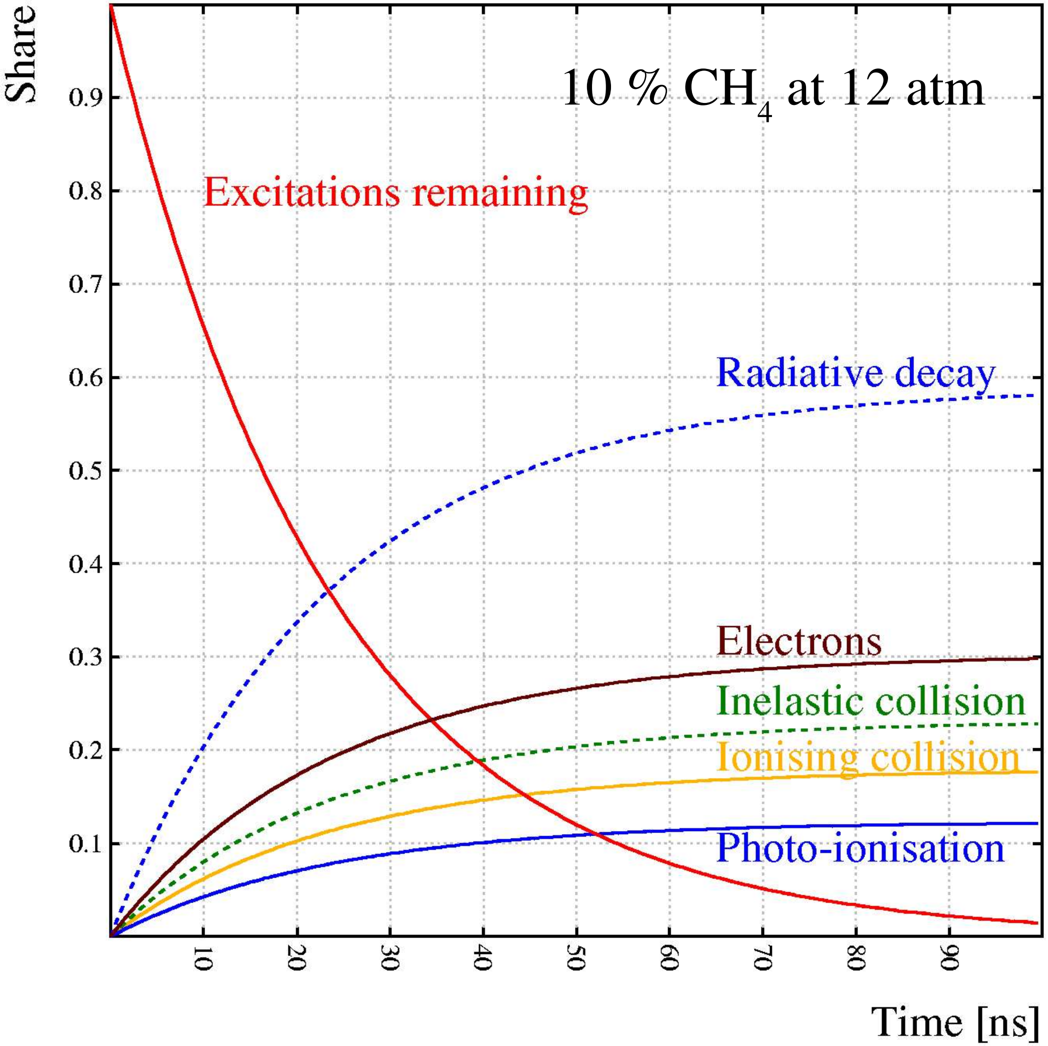
- Large numbers of gain curves available.





Ar-CH_4: processes and timing





En cuisine ...

Paul Bocuse “*En cuisine, on n'invente rien: on interprète.*” (7/3/2005)

Roger Vergé “*D’ailleurs, en matière de cuisine, on interprète, on accommode, on adapte… mais on n’invente pas !*” (1978)

Hélène Darroze “*Cette recette est sortie de ma tête. Je ne dirais pas 'inventée', car en cuisine, on n'invente rien... On prend des produits, on arrange, on accommode, on mélange et on teste.”* (2008)