Mapping the densities of exotic nuclei

Steven Karataglidis
University of Johannesburg, South Africa

UNIVERSITY OF JOHANNESBURG

Collaborators….

- K. Amos, P. Dortmans University of Melbourne
- C. Bennhold The George Washington University
- **T. Suda Tohoku University**
- H. Simon ELISe Project, GSI

UNIVERSITY OF JOHANNESBURG

Introduction

Much information on the neutron density of neutron-rich nuclei is known:

- ➡️ Halos
- ➡️ Skins
- ➡️ Nucleus-nucleus collisions
- ➡️ Proton scattering in inverse kinematics

Not much is known of the proton density; reactions tend to concentrate on probing the neutron density only.

UNIVERSITY OF JOHANNESBURG

Mapping the density

Usually, we (should) rely on more than one reaction to obtain information on the overall structures of nuclei:

- ➡️ Proton scattering pn part of interaction dominates, probes neutron density
- ➡️ Electron scattering Purely electromagnetic interaction. Longitudinal form factors probe directly the charge (proton) density.

Utilising both, self-consistently, provides a complete mapping of the matter density of the nucleus.

UNIVERSITY OF JOHANNESBURG

This requires, *a priori*, a self-consistent approach to both electron and proton (nucleon) scattering.

Requirements of both scatterings necessitates use of microscopic theories. The Shell Model is utilised to produce the underlying **one-body density matrix elements** (OBDME) which are used in both analyses of electron and nucleon scattering.

Electron scattering: de Forest and Walecka approach. (Adv. Phys. **15**, 1 (1966)); SK, *et al.* (PRC **51**, 2494 (1995)).

Nucleon scattering: Melbourne *g*-folding model. (K. Amos, *et al.*, Adv. Nucl. Phys. **25**, 275 (2000)).

UNIVERSITY OF JOHANNESBURG

Other predictions…

- Antonov *et al.* [PRC **72**, 044307 (2005)]: Form factors of He isotopes as obtained from densities calculated using large space shell model. Did not consider directly the effect of the halo in ^{6}He;
- Bertulani [JPG **34**, 315 (2007)]: Form factors of ^{6}He and ^{11}Li using densities obtained from a potential model. Did not find any effect of the halo on the elastic scattering charge form factors. Did find a significant effect of the proton halo on the charge form factor for ^{8}B.

Current (published) work: (SK and K. Amos, PLB **650**, 148 (2007))

- Elastic scattering form factors of He and Li isotopes to investigate whether charge density changes with addition of neutrons and introduction of the halo;
- Inelastic scattering form factors of ^{6}He to see whether the halo has an effect therein.

Electron scattering form factors

Form factors:

$$|F_J^\eta|^2 = \frac{1}{2J+1}\frac{4\pi}{Z^2}\left|\langle J_f \,\| T_J^\eta(q) \|\, J_i\rangle\right|^2$$

η is the type. Assuming one-body operators:

$$\langle J_f \,\| T_J^\eta(q) \|\, J_i\rangle = \sqrt{\frac{1}{2J+1}}\mathrm{Tr}(SM).$$

- *S* is the matrix of one-body transitions densities;
- *M* contains the matrix elements of one-body longitudinal or transverse electromagnetic operators.

MEC included via Siegert's theorem in long-wavelength limit;
Darwin term included in the longitudinal operator.

UNIVERSITY OF JOHANNESBURG

Formal theory of the optical potential

Optical potential: term associated with *elastic scattering only.* Split the Hilbert Space: P projects onto the elastic scattering channel; Q projects onto everything else. Thus:

$$P + Q = 1,\ PQ = QP = 0,\ Q\left|\Psi_{\mathrm{gs}}\right\rangle = 0$$

Schrödinger equation for the scattering state becomes:

$$\begin{aligned}(E - H_{PP})\, P\left|\Psi^{+}\right\rangle &= H_{PQ} Q \left|\Psi^{+}\right\rangle \\ (E - H_{QQ})\, Q\left|\Psi^{+}\right\rangle &= H_{QP} P \left|\Psi^{+}\right\rangle\end{aligned}$$

Recoupling, the S.E. for the projectile wave function is

$$\left\{E - H_0 - \left\langle \Phi_{\mathrm{gs}}\right| V \left|\Phi_{\mathrm{gs}}\right\rangle - \left\langle \Phi_{\mathrm{gs}}\left| V G^{(+)}_{QQ} V \right| \Phi_{\mathrm{gs}}\right\rangle\right\}\left|\chi^{+}\right\rangle = 0$$

where

$$G^{(+)} = \left[E - H_{QQ} + i\varepsilon\right]^{-1}.$$

UNIVERSITY OF JOHANNESBURG

Melbourne *g*-folding model

A refresher: for intermediate energy nucleon-nucleus scattering.

Effective NN potential - Melbourne model [K. Amos et al., Adv. Nucl. Phys. 25, 275 (2000)].

- Effective NN interaction obtained from g matrices.
- Bonn-B interaction used for the current examples.
- Momentum-space effective interaction mapped to coordinate space.
- Densities obtained from credible models of structure.

The g matrix is a solution of the Bethe-Goldstone equation:

$$g\left(\mathbf{q},\mathbf{q}';\mathbf{K}\right) = V\left(\mathbf{q}',\mathbf{q}\right) + \int V\left(\mathbf{q}',\mathbf{k}'\right) \frac{Q\left(\mathbf{k}',\mathbf{K};k_f\right)}{E\left(\mathbf{k},\mathbf{K}\right) - E\left(\mathbf{k}',\mathbf{K}\right)} g\left(\mathbf{k}',\mathbf{q};\mathbf{K}\right)\, d\mathbf{k}'$$

where Q is a Pauli operator and the energy denominator is dependent on auxiliary potentials (eg. effective mass operators).

Construction of the optical potential

In coordinate space, the OMP for elastic scattering is

$$\begin{aligned} U\left(\mathbf{r},\mathbf{r}';E\right) &= \delta(\mathbf{r}-\mathbf{r}')\sum_i n_i \int \varphi_i^*\left(\mathbf{s}\right) g_D\left(\mathbf{r},\mathbf{s};E\right)\varphi_i\left(\mathbf{s}\right) d\mathbf{s} \\ &+ \sum_i n_i \varphi_i^*\left(\mathbf{r}'\right) g_E\left(\mathbf{r},\mathbf{r}';E\right)\varphi_i\left(\mathbf{r}\right) \\ &= U_D\left(\mathbf{r},E\right)\delta(\mathbf{r}-\mathbf{r}') - U_E\left(\mathbf{r},\mathbf{r}';E\right) \end{aligned}$$

- First term is the "$g\varrho$" direct form of the optical potential.
- The nonlocality arises primarily and explicitly out of the exchange terms.
- Structure enters through the s.p. wave functions and occupations numbers.
- For nonzero spin targets, terms with nonzero spin coupling may be included via the DWA.

Structure of the target is critical.

UNIVERSITY OF JOHANNESBURG

Inelastic scattering

… is calculated within a distorted wave approximation.

$$T_{J_f J_i}^{M_f M_i \nu' \nu}(\theta) = \left\langle \chi_{\nu'}^{(-)} \right| \left\langle \Psi_{J_f M_f} \right| A g_{\text{eff}}(0,1) \mathcal{A}_{0,1} \left\{ \left| \chi_{\nu}^{(+)} \right\rangle \left| \Psi_{J_i M_i} \right\rangle \right\}$$

UNIVERSITY OF JOHANNESBURG

Nuclear Structure

One-body density matrix elements (OBDME), obtained using the Shell Model.

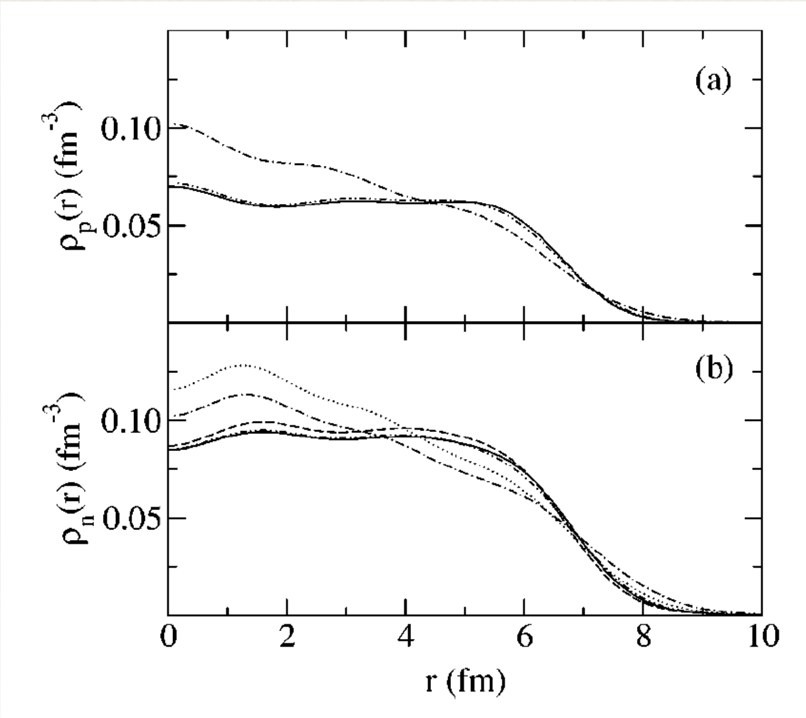
$$S_{j_1 j_2 J} = \left\langle J_f \left\| \left[a^{\dagger}_{j_2} \times \tilde{a}_{j_1} \right]^J \right\| J_i \right\rangle$$

Single-particle wave functions. Either:

- HO: (naive shell model) automatically gives the skin attributes;
- WS: with binding energy set to the single-nucleon separation energy gives the appropriate extension of the density consistent with a halo.

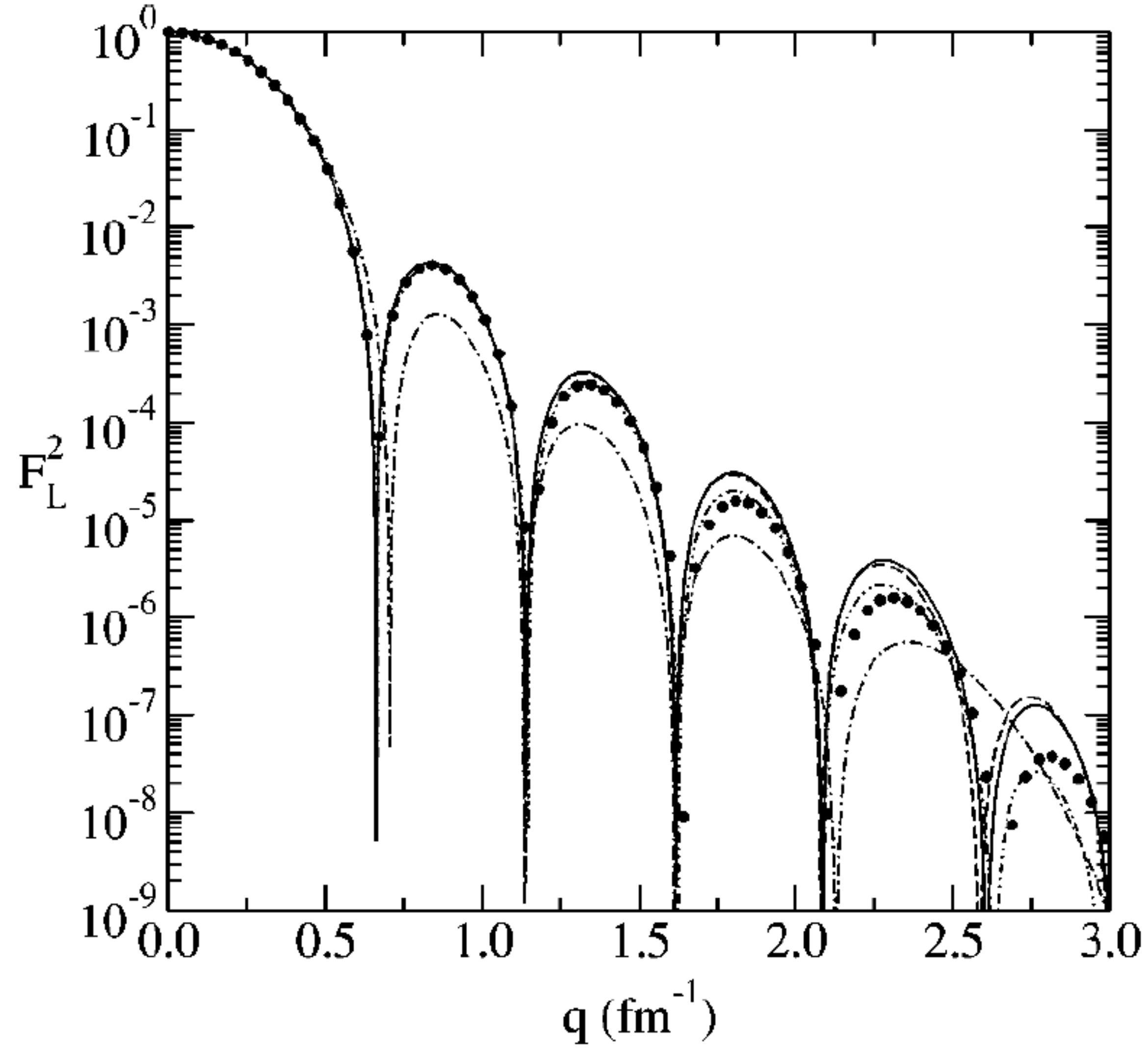
Do not fit…!

Statistical and systematic errors in cross-section magnitudes prevent accurate determination of densities by unfolding data. Instead, we use *complementary* data to determine the best model, and then use the model to predict the densities. Example… ^{208}Pb skin thickness (SK, *et al.*, PRC **65**, 044306 (2002)).



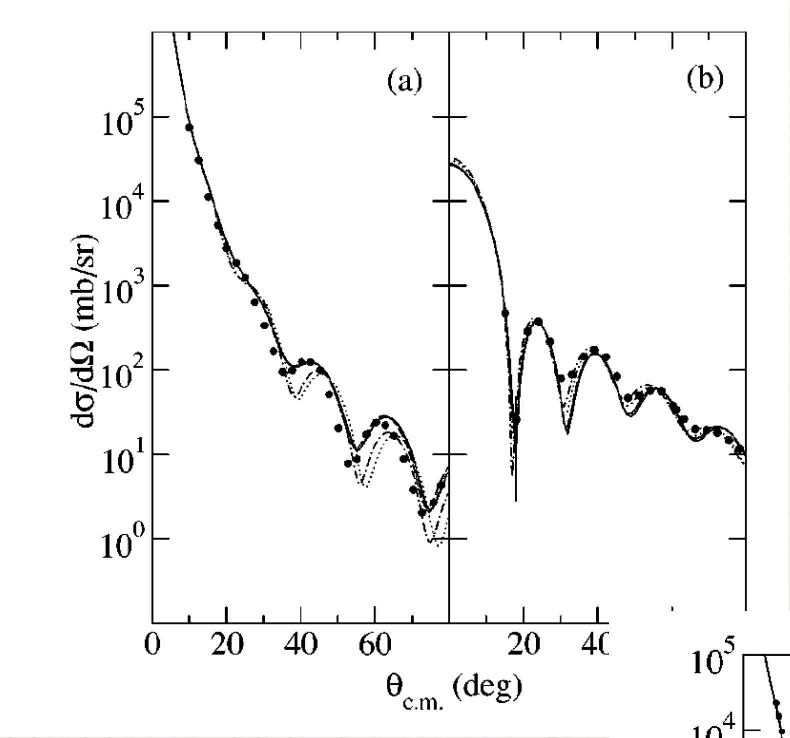
5 models: 2 oscillator, 3 Skyrme.

Electron scattering:

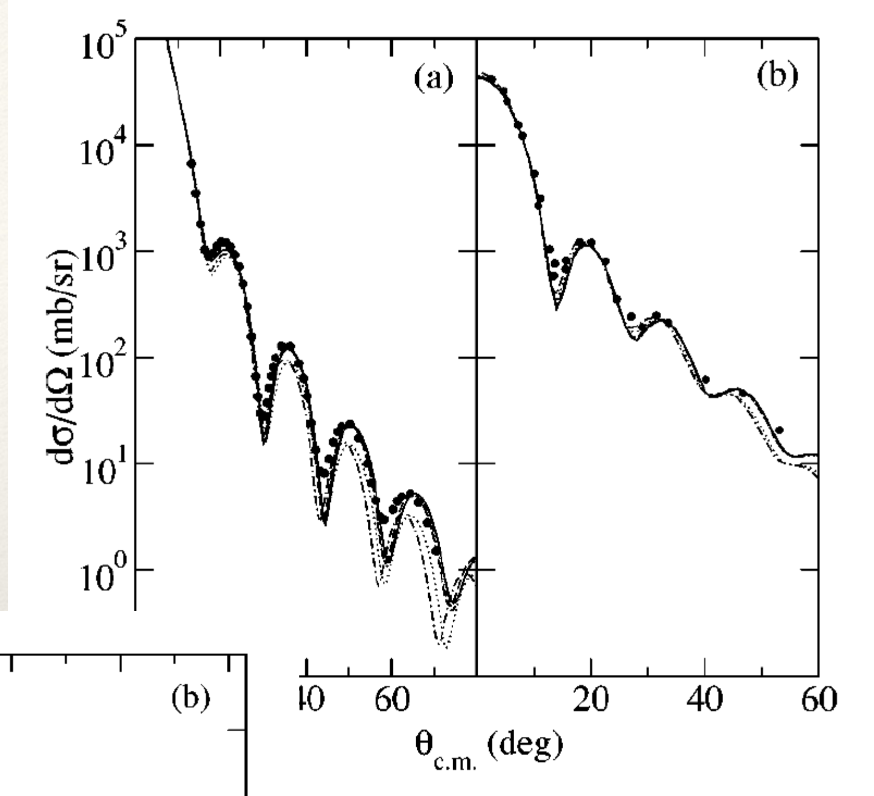


UNIVERSITY OF JOHANNESBURG

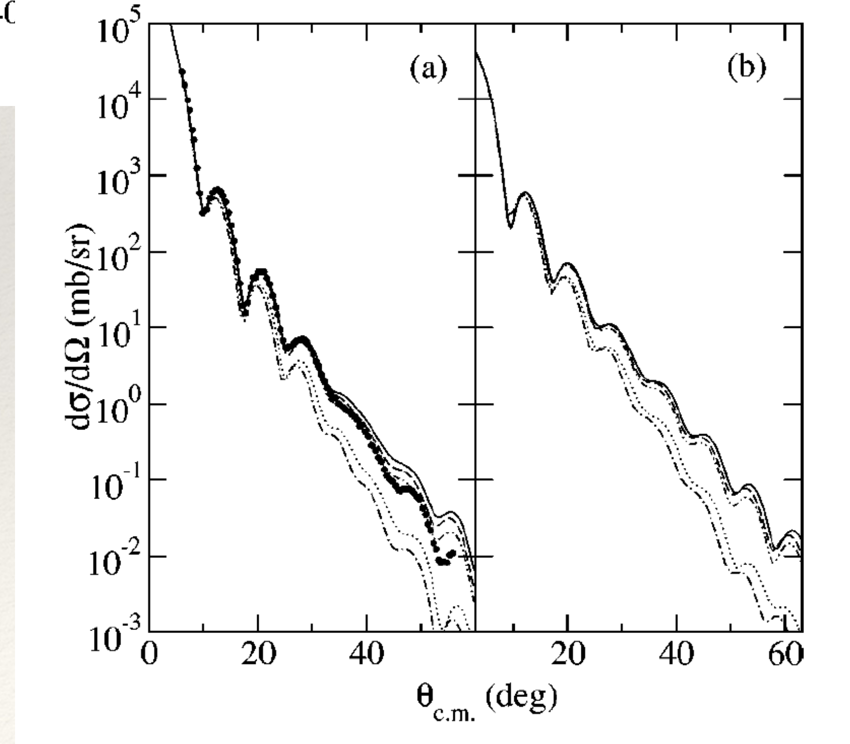
Nucleon scattering:



40 MeV



65 MeV



200 MeV

UNIVERSITY OF JOHANNESBURG

Skin thickness, ^{208}Pb

Best result, SKM* model: 0.17 fm

PRL **112,** 242502 (2014) PHYSICAL REVIEW LETTERS week ending 20 JUNE 2014

Neutron Skin of ^{208}Pb from Coherent Pion Photoproduction

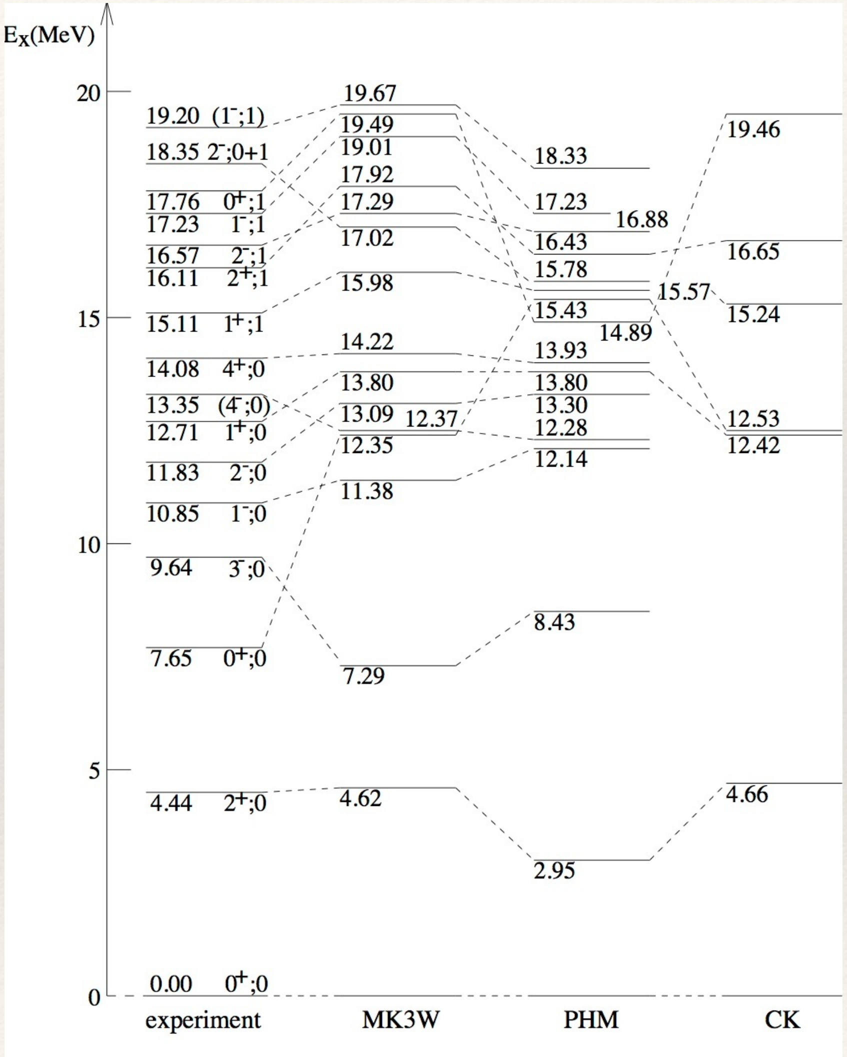
C. M. Tarbert,[1] D. P. Watts,[1,*] D. I. Glazier,[1] P. Aguar,[2] J. Ahrens,[2] J. R. M. Annand,[3] H. J. Arends,[2] R. Beck,[2,4] V. Bekrenev,[5] B. Boillat,[6] A. Braghieri,[7] D. Branford,[1] W. J. Briscoe,[8] J. Brudvik,[9] S. Cherepnya,[10] R. Codling,[3] E. J. Downie,[3] K. Foehl,[1] P. Grabmayr,[11] R. Gregor,[12] E. Heid,[2] D. Hornidge,[13] O. Jahn,[2] V. L. Kashevarov,[10] A. Knezevic,[14] R. Kondratiev,[15] M. Korolija,[14] M. Kotulla,[6] D. Krambrich,[2,4] B. Krusche,[6] M. Lang,[2,4] V. Lisin,[15] K. Livingston,[3] S. Lugert,[12] I. J. D. MacGregor,[3] D. M. Manley,[16] M. Martinez,[2] J. C. McGeorge,[3] D. Mekterovic,[14] V. Metag,[12] B. M. K. Nefkens,[9] A. Nikolaev,[2,4] R. Novotny,[12] R. O. Owens,[3] P. Pedroni,[7] A. Polonski,[15] S. N. Prakhov,[9] J. W. Price,[9] G. Rosner,[3] M. Rost,[2] T. Rostomyan,[7] S. Schadmand,[12] S. Schumann,[2,4] D. Sober,[17] A. Starostin,[9] I. Supek,[14] A. Thomas,[2] M. Unverzagt,[2,4] Th. Walcher,[2] L. Zana,[1] and F. Zehr[6]
(Crystal Ball at MAMI and A2 Collaboration)

Measured value: 0.15±0.03 fm.

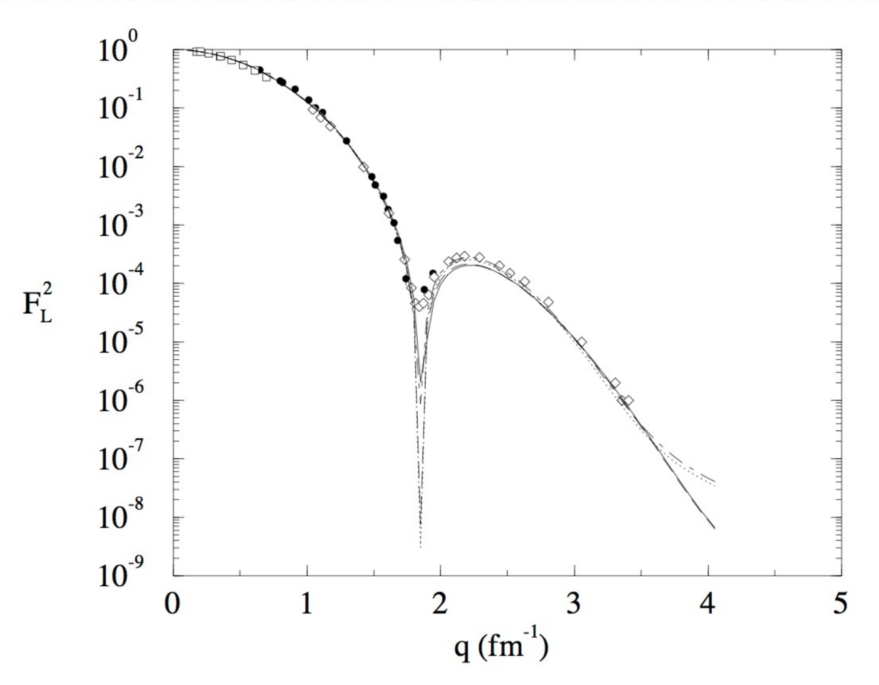
UNIVERSITY OF JOHANNESBURG

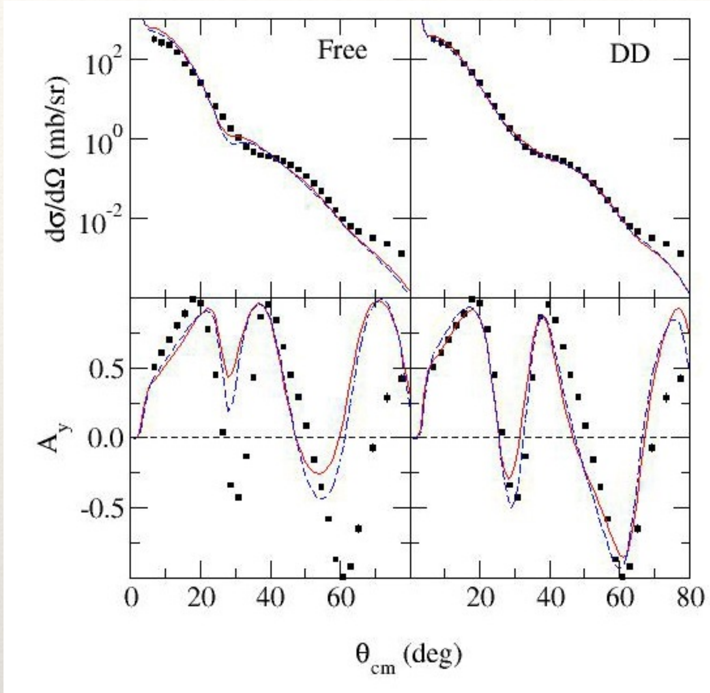
Example: ^{12}C

Spectrum
(SK, *et al.*, PRC **52**, 861 (1995)):



Elastic scattering (SK, *et al.*, PRC **52**, 861 (1995)):



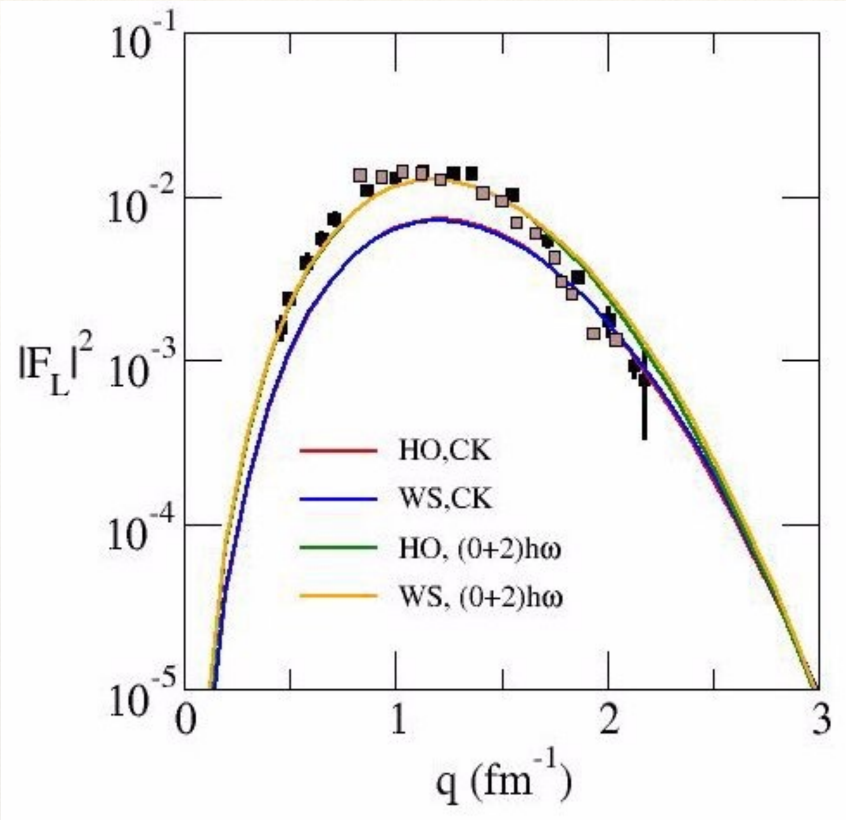


Elastic electron scattering

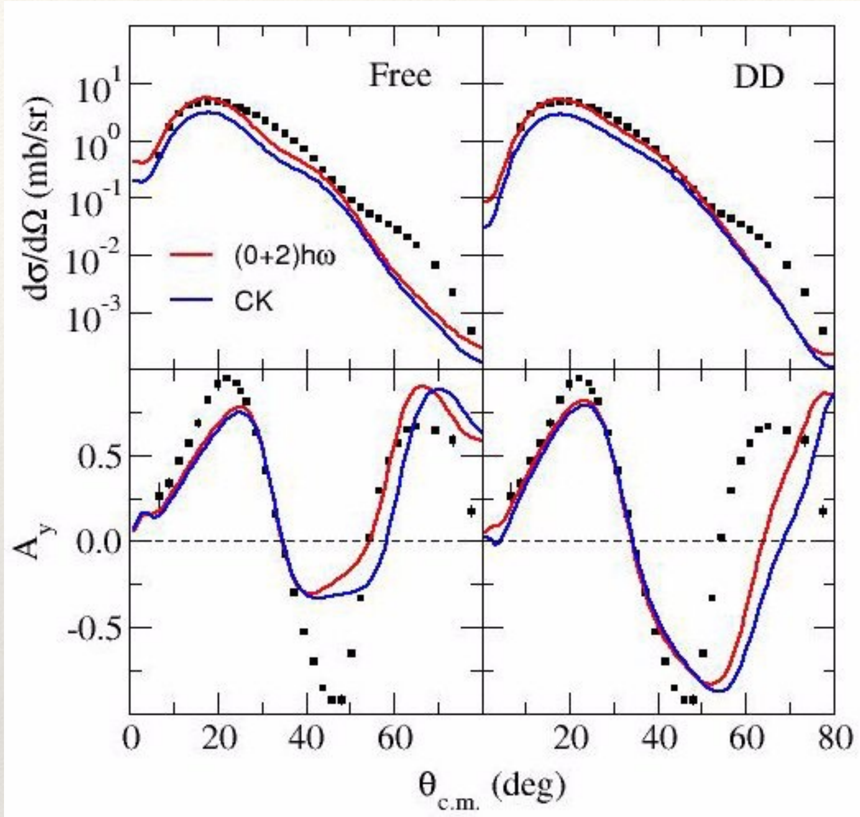
Elastic 200 MeV proton scattering

UNIVERSITY OF JOHANNESBURG

Inelastic scattering (2^+ state, 4.44 MeV)



Longitudinal form factor



200 MeV proton scattering

Densities, exotic nuclei

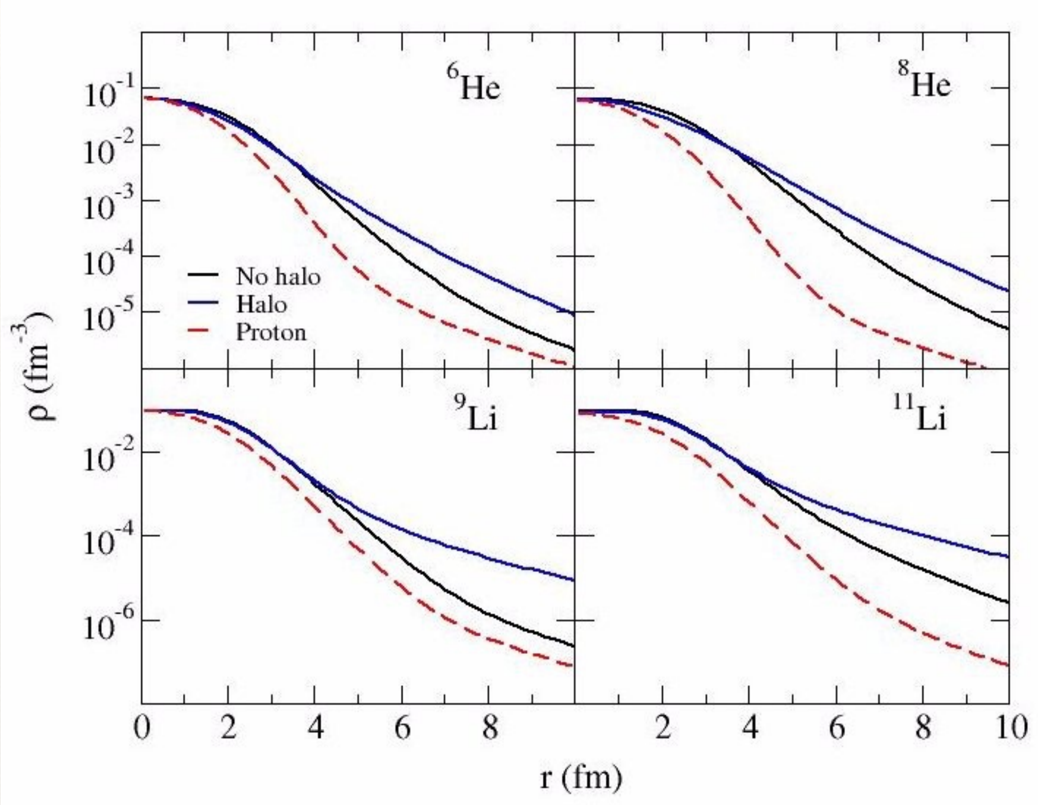
$4\hbar\omega$ shell model, Zheng interaction

(SK., *et al.*, PRC **61**, 024319 (2000)).

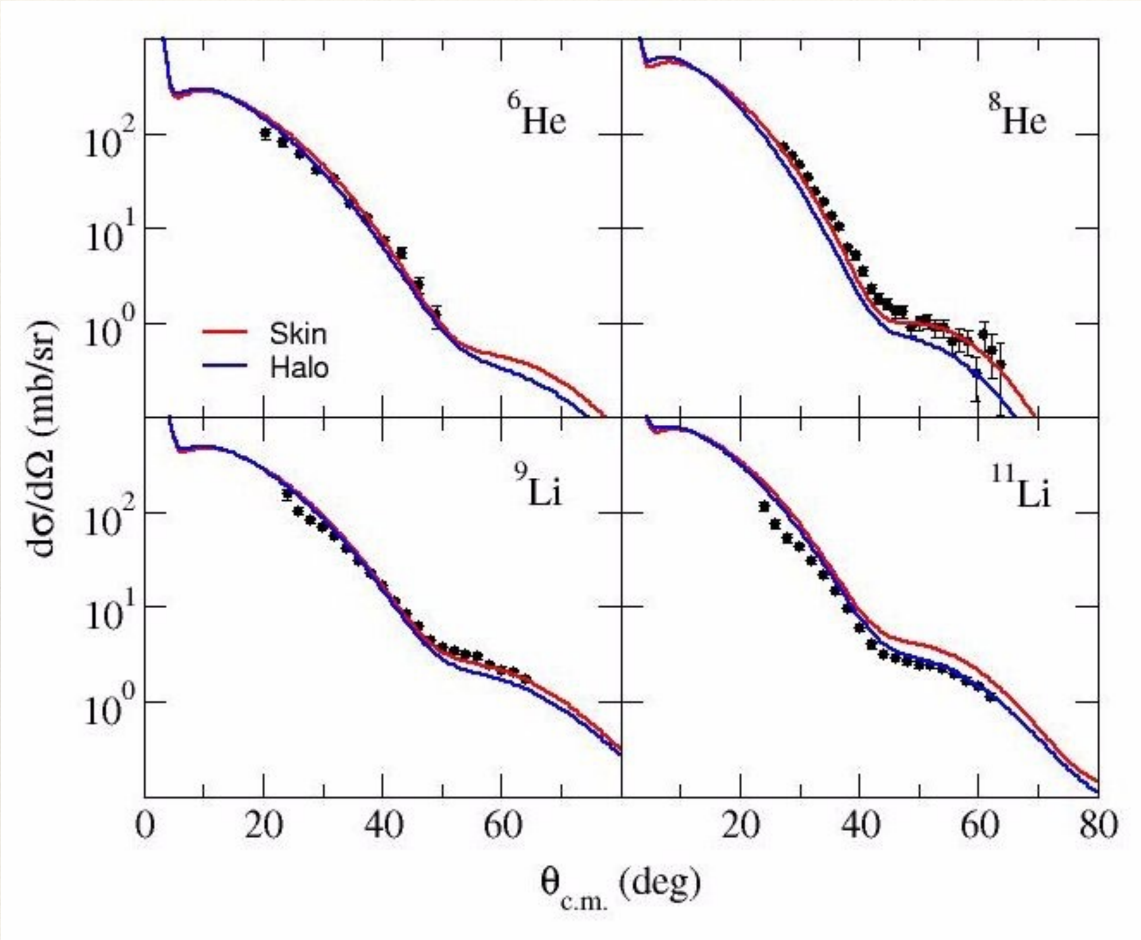
TABLE I. Root-mean-square (rms) radii in fm for ^{6}He, ^{8}He, ^{9}Li, and ^{11}Li. The results of our shell model calculations are compared to those obtained from a Glauber model analysis of the reaction cross sections [26,25], and also from a few-body model analysis of scattering data from hydrogen [2].

	r_{rms}		
Nucleus	non-halo	halo	Glauber model
^{6}He	2.301	2.586	2.54 ± 0.04
^{8}He	2.627	2.946	2.60 [a]
^{9}Li	2.238	2.579	2.30 ± 0.02
^{11}Li	2.447	2.964	3.53 ± 0.10

[a]Taken from Ref. [2].



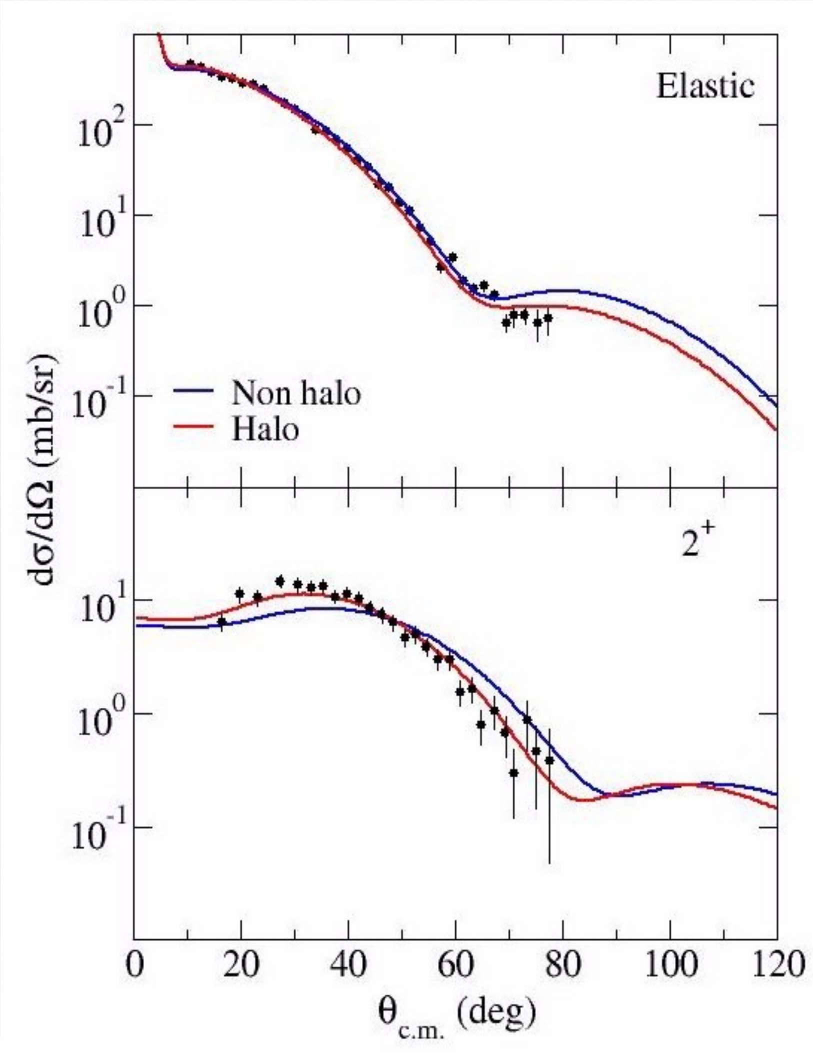
Elastic scattering



71A MeV

62A MeV

p-^{6}He scattering



Reaction cross section

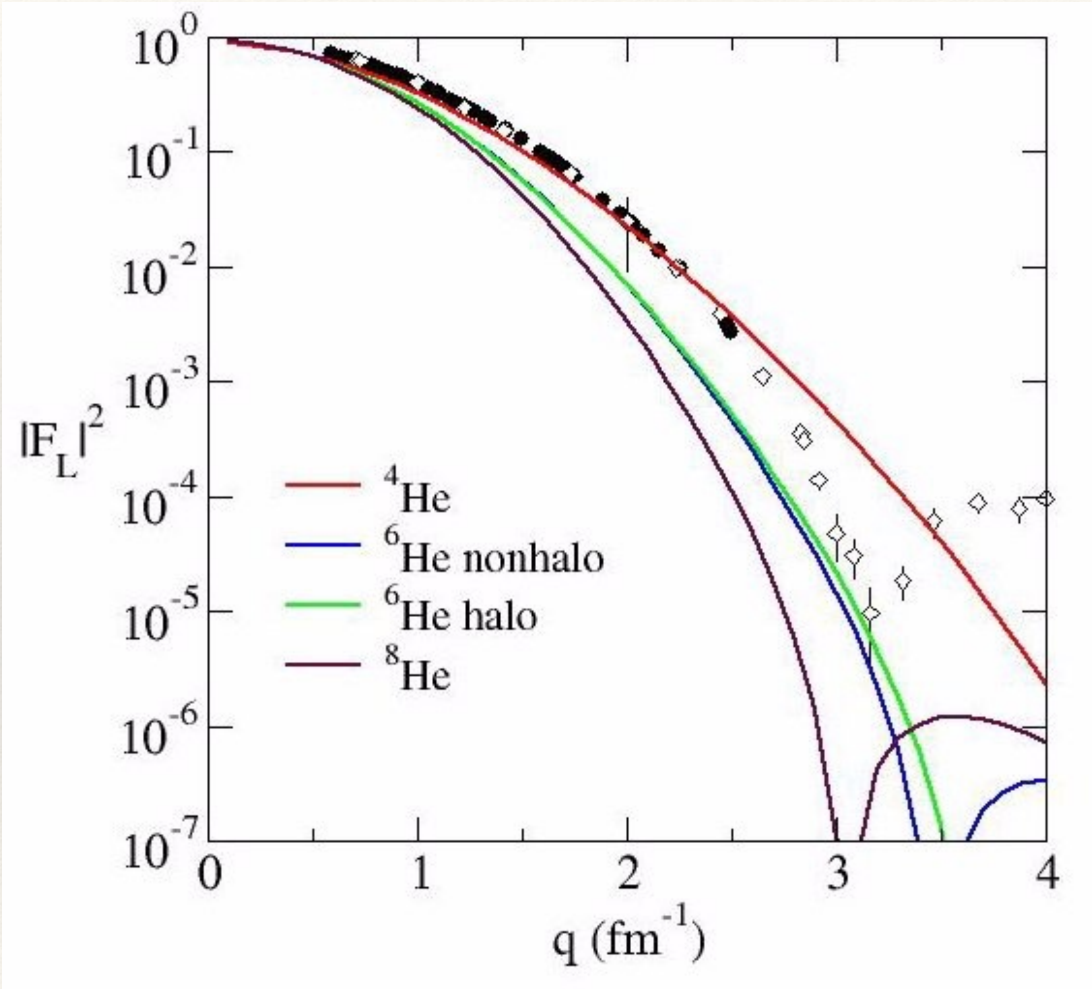
Predicted

$$\sigma_R = 353 \text{ mb (nonhalo)}$$
$$= 406 \text{ mb (halo)}$$

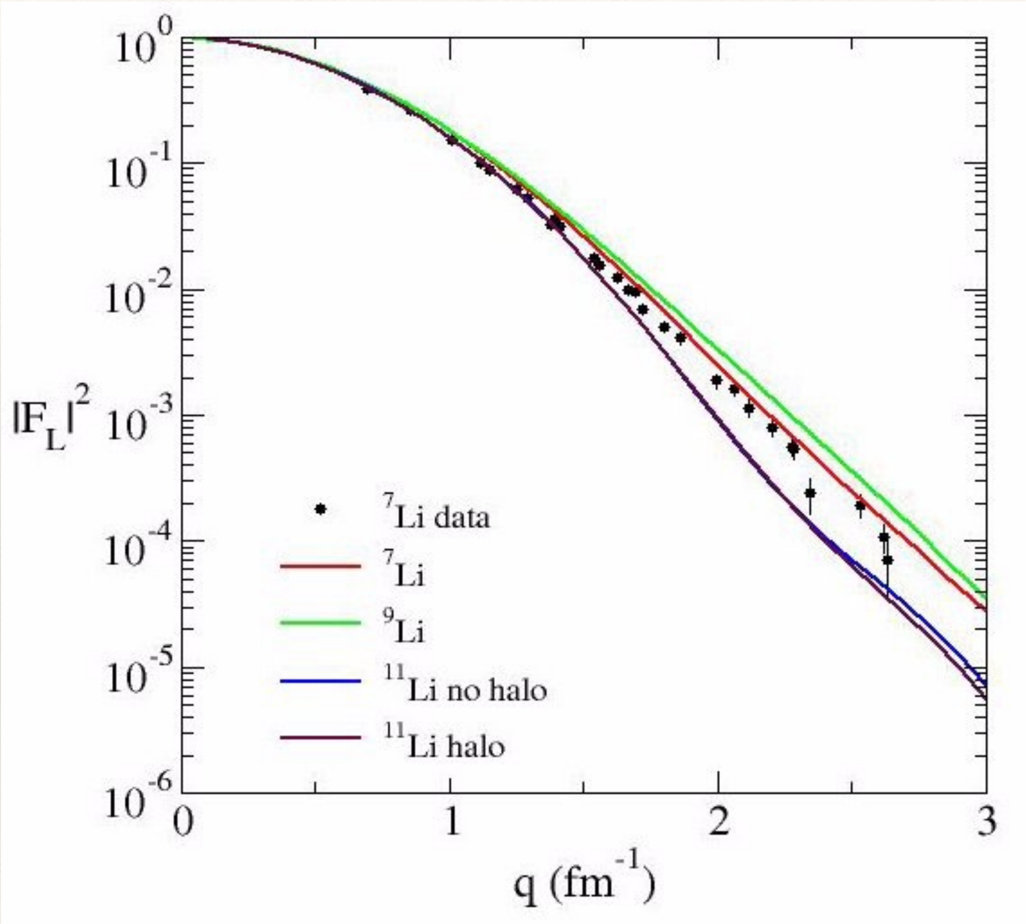
Measured

$$\sigma_R = 409 \pm 22 \text{ mb}$$

Electron scattering, He isotopes

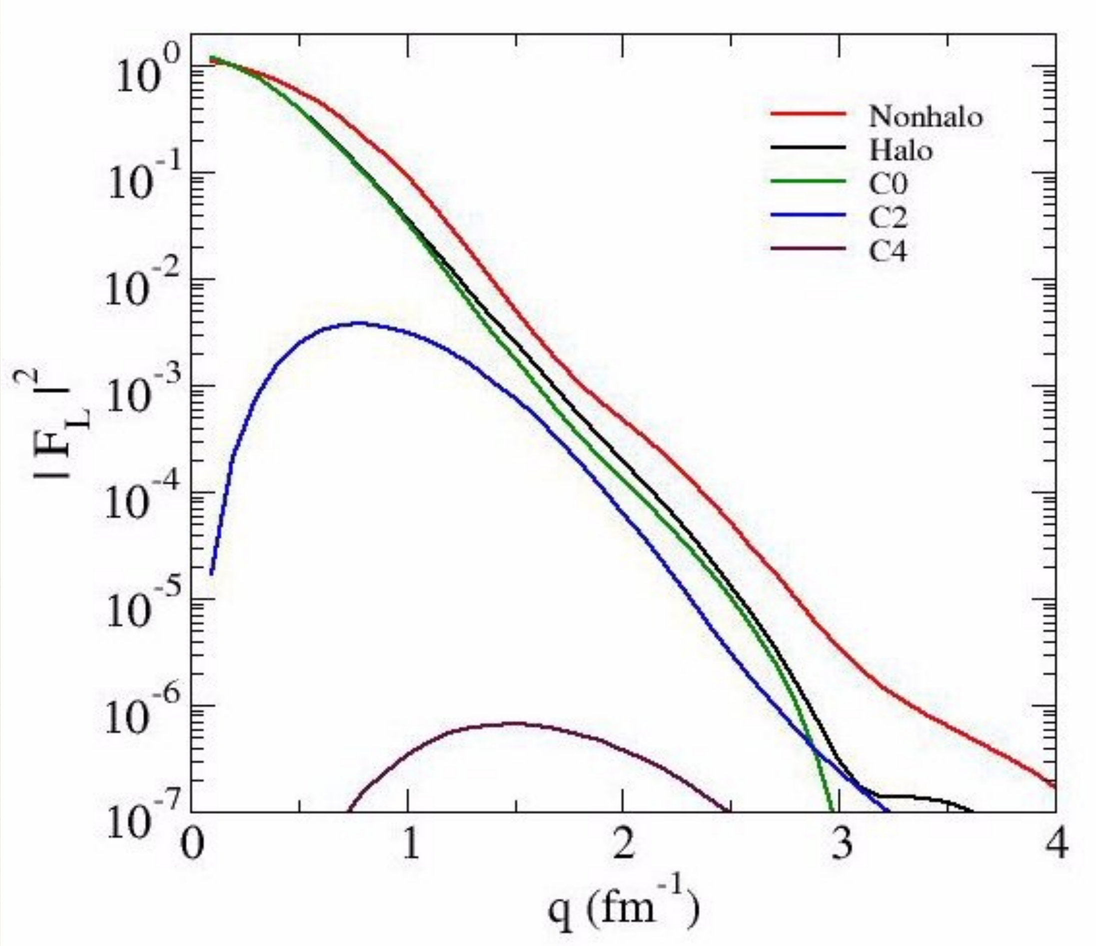


Electron scattering, Li isotopes

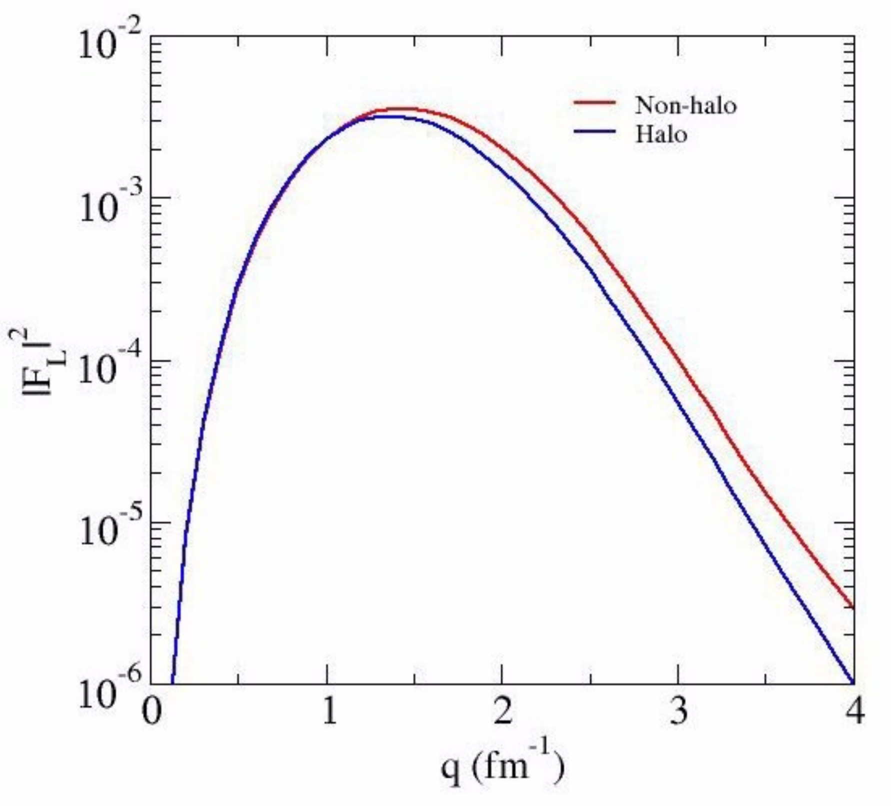


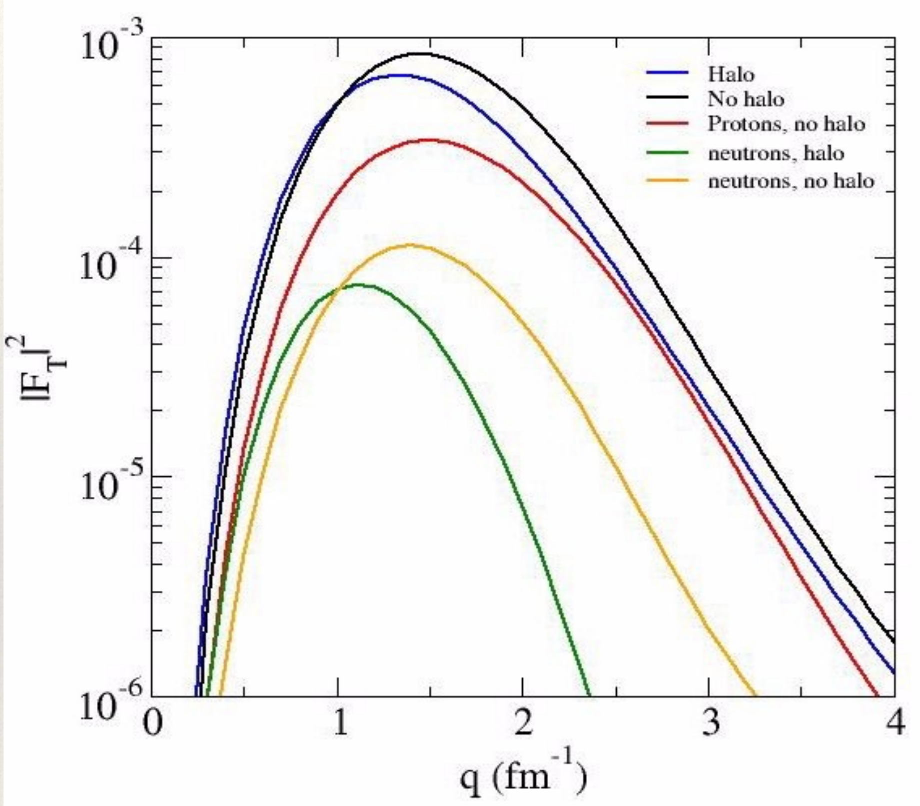
UNIVERSITY OF JOHANNESBURG

Electron scattering, ^8B

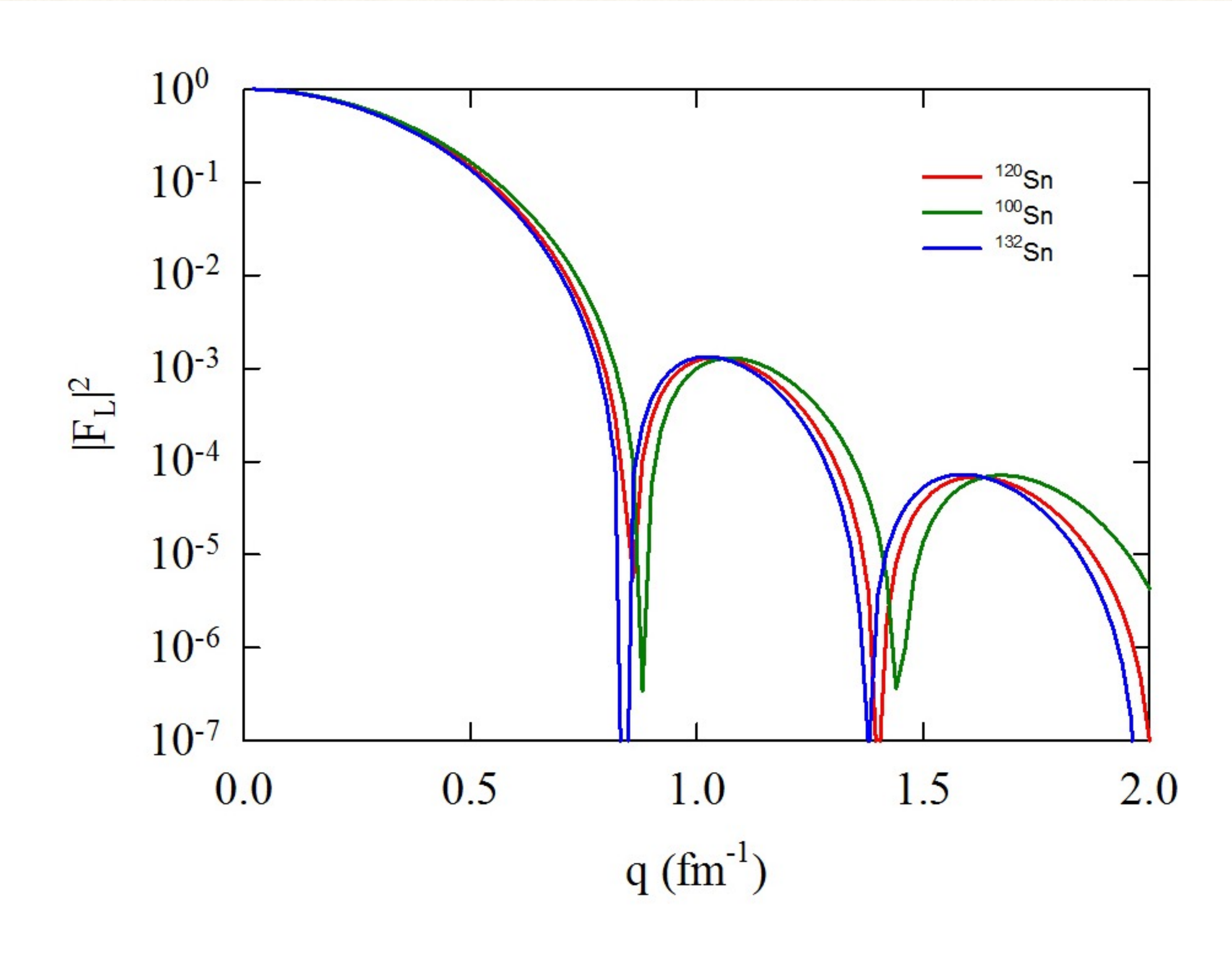


Inelastic electron scattering, ^{6}He



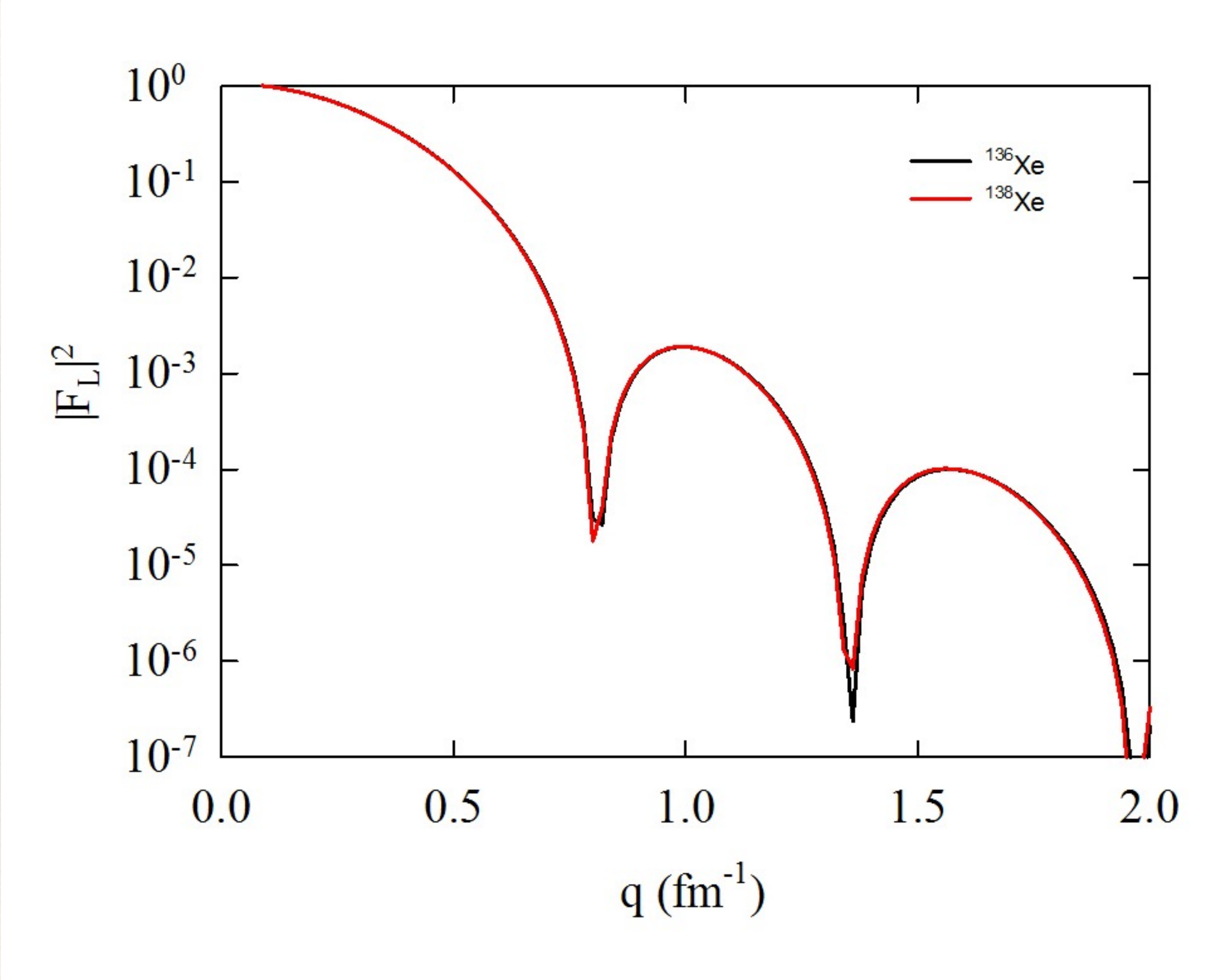


Heavier nuclei - isotopes of Sn



PRELIMINARY

Heavier nuclei - isotopes of Xe



PRELIMINARY

Conclusions

- Presented results for calculations of electron and proton scattering from light stable and exotic nuclei.
- For the He isotopes, the results of the calculations for the elastic longitudinal form factors follow a natural mass dependence.
- For the Li isotopes, the results of the calculations also follow a natural mass dependence.
- For ^{8}B, the proton halo **does** significantly affect the form factor.
- The inelastic scattering form factor for ^{6}He does show some effect due to the neutron halo, as consistent with the results of proton scattering.
- These complementary scatterings allow for as complete mapping of the matter densities as possible. Where possible, both elastic electron and intermediate energy elastic nucleon scattering should be measured.

UNIVERSITY OF JOHANNESBURG