QCD: fixed order results

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Many thanks to K. Melnikov and G. Salam for discussions on these topics

Disclaimer

- In this talk, I will focus on recent progress in higher order SM computations for LHC processes, especially on the ones appeared after QCD@LHC2015
- Nevertheless, many other interesting ''fixed order'' progress, mostly relevant to precise extraction of input parameters
- Five-loop running of α_s [Baikov, Chetyrkin, Kühn (2016)]
- DIS (→PDFs):
  - Heavy flavor → *see Johannes' talk tomorrow*
  - Di-jet production in DIS [Currie, Gehrmann, Niehues (2016)]
- Implications of the $\overline{\text{MS}}$ — on-shell 4-loop relation for m_t
  - Comparison with all-order estimates / renormalons and its implication for the top-mass extraction [Beneke, Marquard, Nason, Steinhauser (2016)]
- Also, NLO BSM analysis are more and more frequent

Why fixed order calculations?

Today: many ``tools'' for hadron collider physics. Yet, fixed-order calculations have a crucial role for LHC precision phenomenology

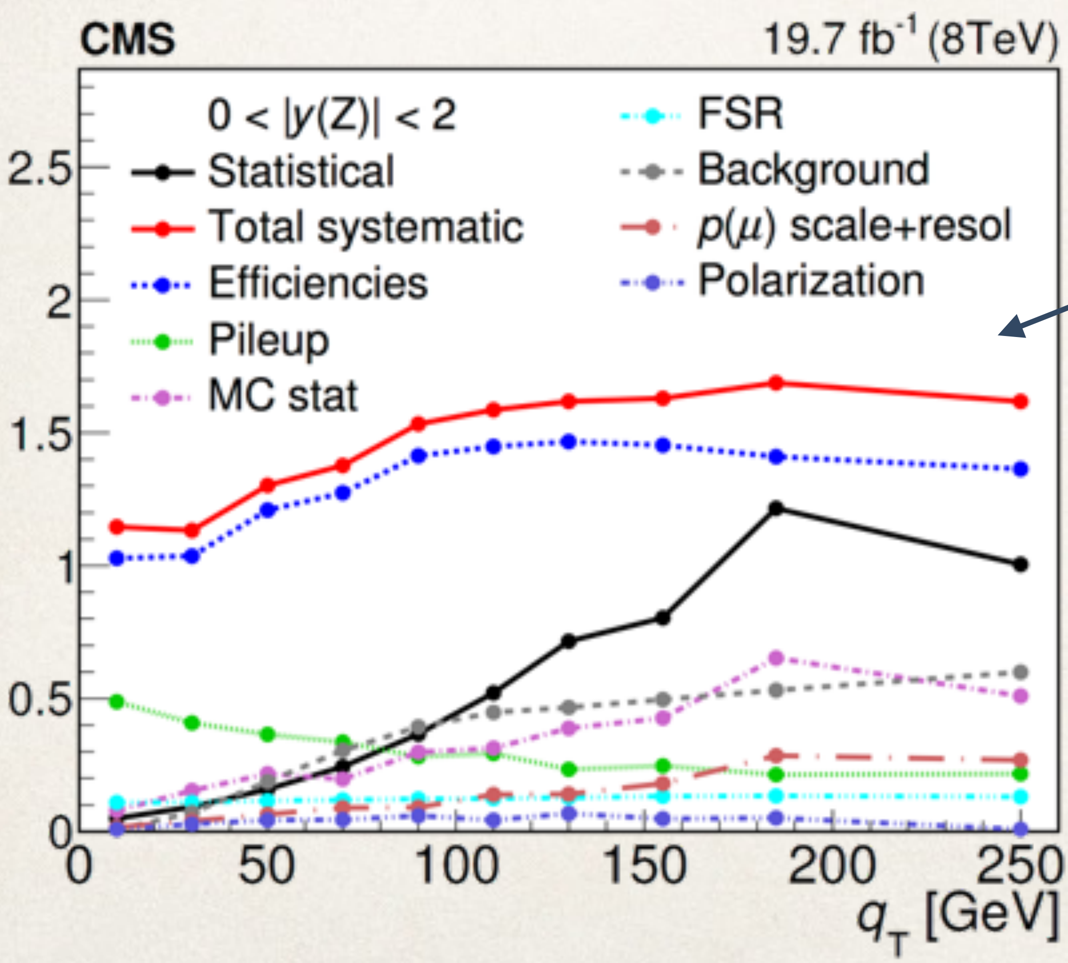
- WELL-DEFINED, VERY SOLID FRAMEWORK
  - Minimal assumptions, error estimate under reasonable control
- QCD IS NOW (MOSTLY) A BEAST WE NEED TO TAME IF WE WANT TO PROFITABLY SEARCH FOR NEW PHYSICS AT THE LHC
  - Whenever possible: focus on high-scale observables (minimal NP contamination), simple analysis (clean exp./th. comparison)
  - In this regime, typically process is a multi-scale problem. However, no huge scale hierarchies → fixed (high enough) order predictions correctly capture all the relevant logs
  - F.O. can deal with REALISTIC OBSERVABLES/CUTS. Minimize (hidden) extrapolation errors

Fixed-order predictions: accuracy goals

A poster-child for precision phenomenology: the (high p_t) Z transverse momentum distribution (no jets, no missing energy…)

$m_{\ell\ell}$ [GeV]	12–20	20–30	30–46	46–66	66–116	116–150
$\sigma(Z/\gamma^* \to \ell^+\ell^-)$ [pb]	1.45	1.03	0.97	14.96	537.10	5.59
Statistical uncertainty [%]	0.63	0.75	0.83	0.17	0.03	0.31
Detector uncertainty [%]	0.84	0.99	0.87	1.05	0.40	0.56
Background uncertainty [%]	0.18	0.85	1.42	1.28	0.06	0.77
Model uncertainty [%]	1.84	2.24	2.27	0.89	0.19	0.50
Total systematic uncertainty [%]	2.06	2.44	2.38	1.82	0.45	1.03

ATLAS 8 TeV
(+2.8% lumi)



CMS 8 TeV
(+2.6% lumi)

PRECISION MEASUREMENTS AT THE LHC:
FEW PERCENT (VERY HARD…)

``Few percent'': the theory side

$$d\sigma = \int dx_1 dx_2 f(x_1) f(x_2) d\sigma_{\text{part}}(x_1, x_2) F_J (1 + \mathcal{O}(\Lambda_{\text{QCD}}/Q))$$

Input parameters: ~few percent.
In principle improvable

NP effects: ~ few percent
No good control/understanding of them at this level. LIMITING FACTOR FOR FUTURE DEVELOPMENT

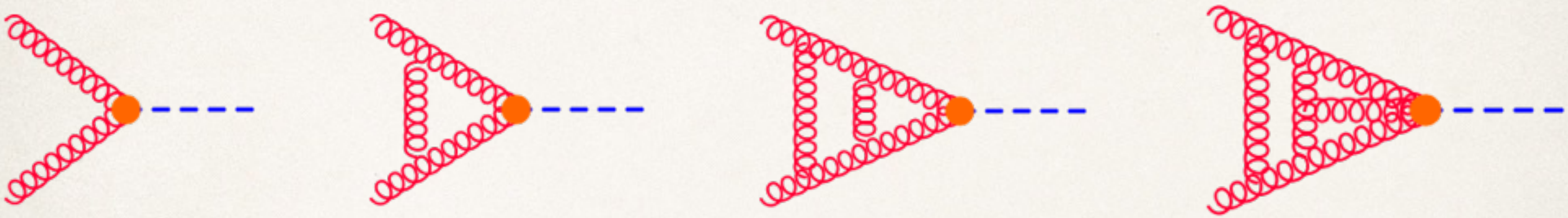
HARD SCATTERING MATRIX ELEMENT

- α_s ~ 0.1 → For TYPICAL PROCESSES, we need NLO for ~ 10% and NNLO for ~ 1 % accuracy
- Going beyond that is neither particularly useful (exp. precision) NOR POSSIBLE GIVEN OUR CURRENT UNDERSTANDING OF QCD, even if we knew how to compute multi-loop amplitudes and had N^KLO subtraction schemes (NP effects)

The elephant in the room

The obvious exception is HIGGS BOSON PRODUCTION

(gluon fusion: large color charge, typical correction ~ $\alpha_S C_A$ ~ 0.3)



K~2, ~100% uncertainty

K~1.2, ~10% uncertainty

K~1.02, ~percent - level uncertainty

[Anastasiou et al., PRL (2015)]

- The calculation of N^3LO corrections to Higgs boson production is truly one of the most amazing achievements in perturbative QCD in the recent past
- The (big) challenge is now to promote the fully inclusive N^3LO result to a fully exclusive calculation → realistic theory / experiment comparison at unprecedented level

see Bernhard's talk on Friday (also for $N^{(2,3)}LO$ VBF) and Marco's talk this afternoon

NLO computations:
status and recent progress

NLO computations: where we stand

Thanks to a *very good understanding of one-loop amplitudes* and to *significant development in MC tools* now

NLO IS THE STANDARD FOR LHC ANALYSIS

- Many publicly available codes allow anyone to perform NLO analysis for reasonably arbitrary [~ 4 particles (~ 3 colored) in the final state] LHC processes: MADGRAPH5_AMC@NLO, OPENLOOPS(+SHERPA), GOSAM(+SHERPA), RECOLA, HELAC…
- By default, they employ both unitarity-based (CUTTOOLS, SAMURAI, NINJA…) and tensor reduction (COLLIER, GOLEM95, PJFRY, IREGI…)
- Some surprises from OPENLOOPS
  - Tensor reduction (COLLIER) is competitive with unitarity methods
  - Amplitudes are fast and stable in degenerate kinematics → NNLO [so far tested with color-singlet final states, would be interesting to study other cases]
- The next step for automation: NLO EW (basically there), arbitrary BSM

NLO computations: where we stand

Thanks to a *very good understanding of one-loop amplitudes* and to *significant development in MC tools* now

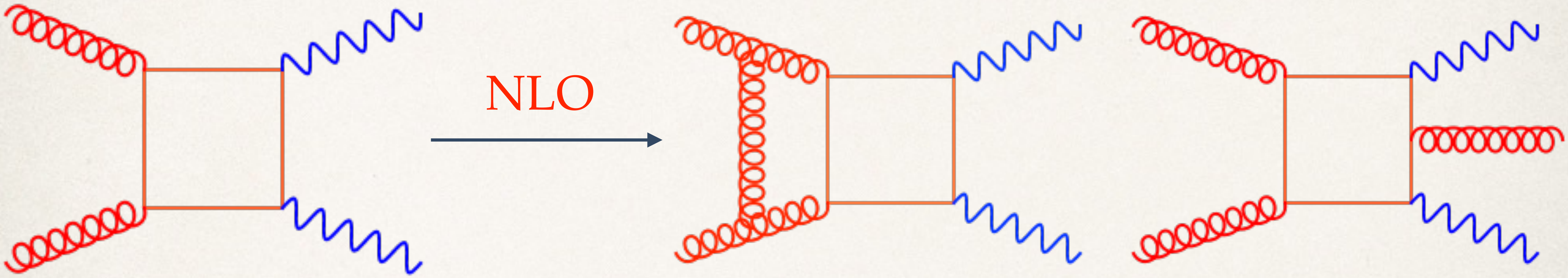
NLO IS THE STANDARD FOR LHC ANALYSIS

Dedicated codes allow for complicated final states, e.g.:

- V(V)+jets [BLACKHAT+SHERPA], jets [NJET+SHERPA], tt+jets [Höche et al. (2016)] → also allow for interesting theoretical analysis (mult. ratios predictions…)
- H+jets [GOSAM+SHERPA]. Recently: up to 3-jets at LO with full top-mass dependence [Greiner et al. (2016)] → investigate the high-p_t Higgs spectrum
- Off-shell effects in ttX processes: ttH [Denner and Feger (2015)], ttj [Bevilacqua et al. (2015) → see Heriberto's talk this afternoon]
  - These results, together with earlier results on single-top [Pittau (1996), Papanastasiou et al. (2013)] allow to test the NWA
  - So far, NWA works exactly as expected: Γ_t/m_t suppression in inclusive observables, large corrections only after kinematics edges and for M_{Wb} sensitive observables → important consequences for NNLO

NLO: loop-induced processes

In the past year, significant progress for loop-induced processes



- Relevant examples: Higgs p_t, $gg \rightarrow VV$ (especially after $qq \rightarrow VV$@NNLO), $gg \rightarrow VH$ (especially after qq@NNLO), di-Higgs…
- Despite being loop-suppressed, the large gluon flux makes the yield for these processes sizable
- gluon-fusion processes → expect large corrections
- At NLO simple infrared structure, but virtual corrections require complicated two-loop amplitudes
- Real emission: one-loop multi-leg, in principle achievable with 1-loop tools

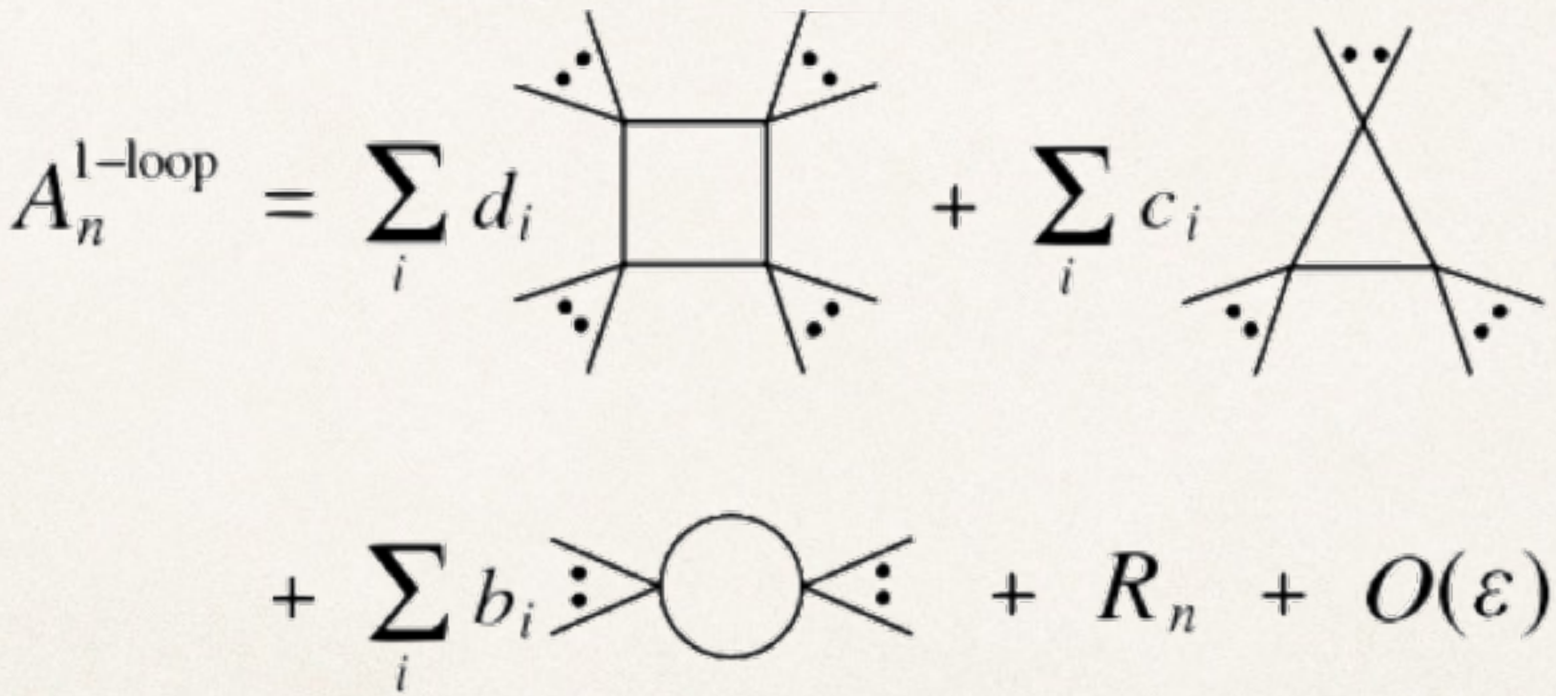
A small detour: loop amplitudes

Computation of loop-amplitudes in two steps:

1. reduce all the integrals of your amplitudes to a minimal set of independent \`master' integrals
2. compute the independent integrals

At one-loop:

- independent integrals are always the same (box, tri., bub., tadpoles)
- only (1) is an issue. Very well-understood (tensor reduction, unitarity…)



Beyond one-loop: reduction not well understood, MI many and process-dependent (and difficult to compute…)

Two-loop: reduction

- So far: based on traditional IBP-LI RELATIONS [Tkachov; Chetyrkin and Tkachov (1981); Gehrmann and Remiddi (2000)] / LAPORTA ALGORITHM [Laporta (2000)]

- State of the art for phenomenologically relevant amplitudes
  - $2 \to 2$ with massless internal particles (di-jet, H/V+jet, VV)
  - $2 \to 2$ with one mass scale (ttbar), significant progress towards top-induced H+J

- Going beyond: significant improvements of tools, NEW IDEAS

- Motivated by the one-loop success, many interesting attempts to generalize unitarity ideas / OPP approach to two-loop case
- We are still not there, but a lot of progress → *see Tiziano's talk on Thursday*
- Interesting proof-of-concept for unitarity-based approaches: 5/6-gluon all-plus amplitudes at two-loops [Badger, Frellesvig, Zhang (2013); Badger, Mogull, Ochiruv, O'Connell (2015); Badger, Mogull, Peraro (2016)]

Two-loop: master integrals

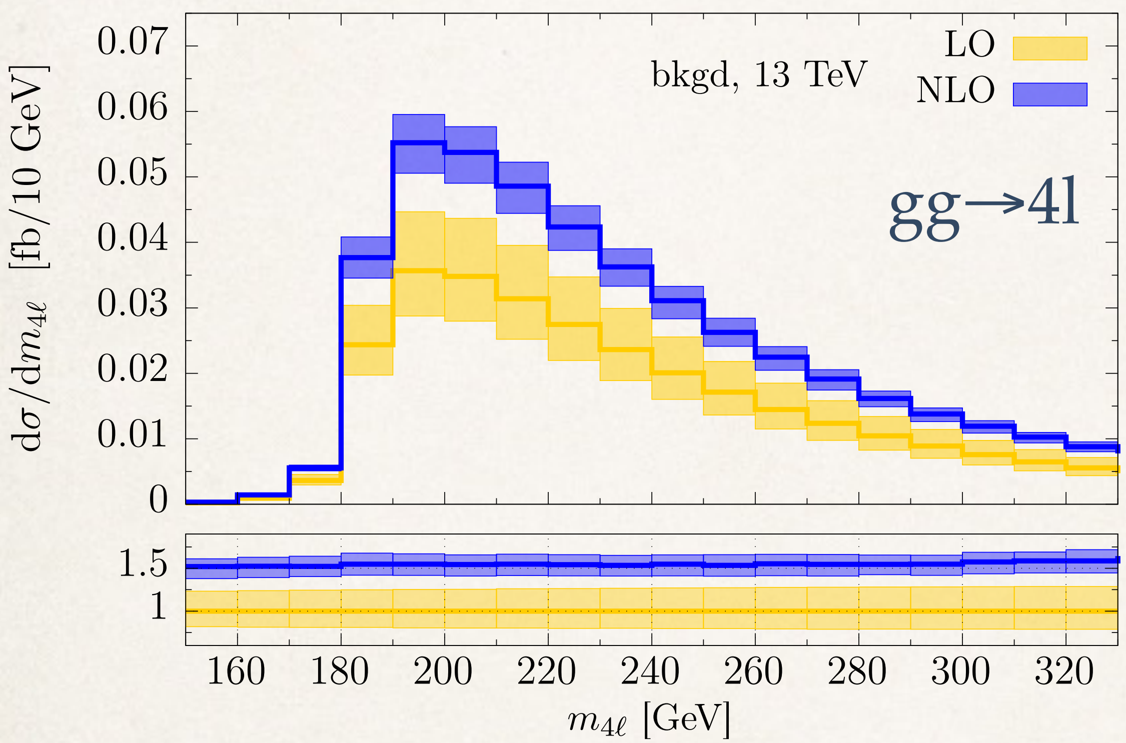
- For a large class of processes (~ phenomenologically relevant scattering amplitudes with massless internal lines) we think we know (at least in principle) how to compute the (very complicated) MI. E.g.: differential equations [Kotikov (1991); Remiddi (1997); Henn (2013); Papadopoulos (2014) → see Kosta's talk on Thursday]. Recent results for very complicated processes: planar 3-jet [Gehrmann, Henn, Lo Presti (2015)], towards planar Vjj/Hjj [Papadopoulos, Tommasini, Wever (2016) → see Kosta's talk]
- In these cases, the basis function for the result is very well-known (Goncharov PolyLogs) and several techniques allow to efficiently handle the result (symbol, co-products…) and numerically evaluate it
- Unfortunately, we know that GPL are not the end of the story. For pheno-relevant processes, we typically exit from this class when we consider amplitudes with internal massive particles (e.g. ttbar, H+J?)
- Progress in this cases as well (e.g. [Tancredi and Remiddi (2016); Adams, Bogner, Weinzierl (2015-16)]) but we are still far from a satisfactory solution → real conceptual bottleneck for further development

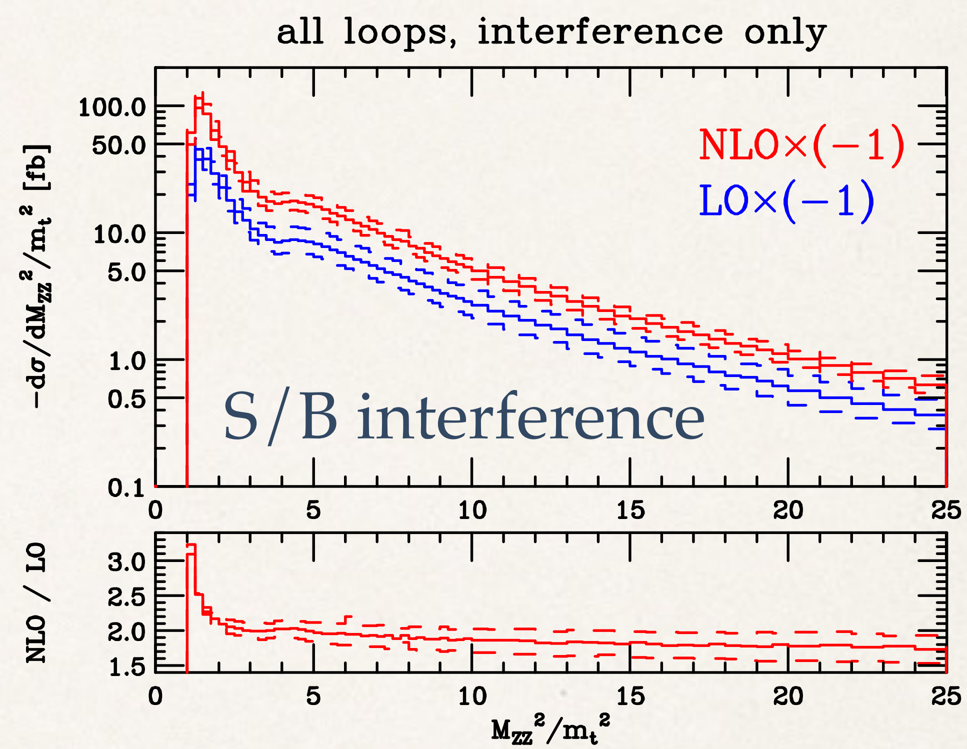
Back to loop induced: NLO for gg → VV

Thanks to the progress in loop-amplitude computations, NLO corrections to gg→WW/ZZ and to gg→(H)→VV signal/background interference

[FC, Melnikov, Röntsch, Tancredi (2015-16); Campbell, Ellis, Czakon, Kirchner (2015)]

→ *see Lorenzo's talk tomorrow*



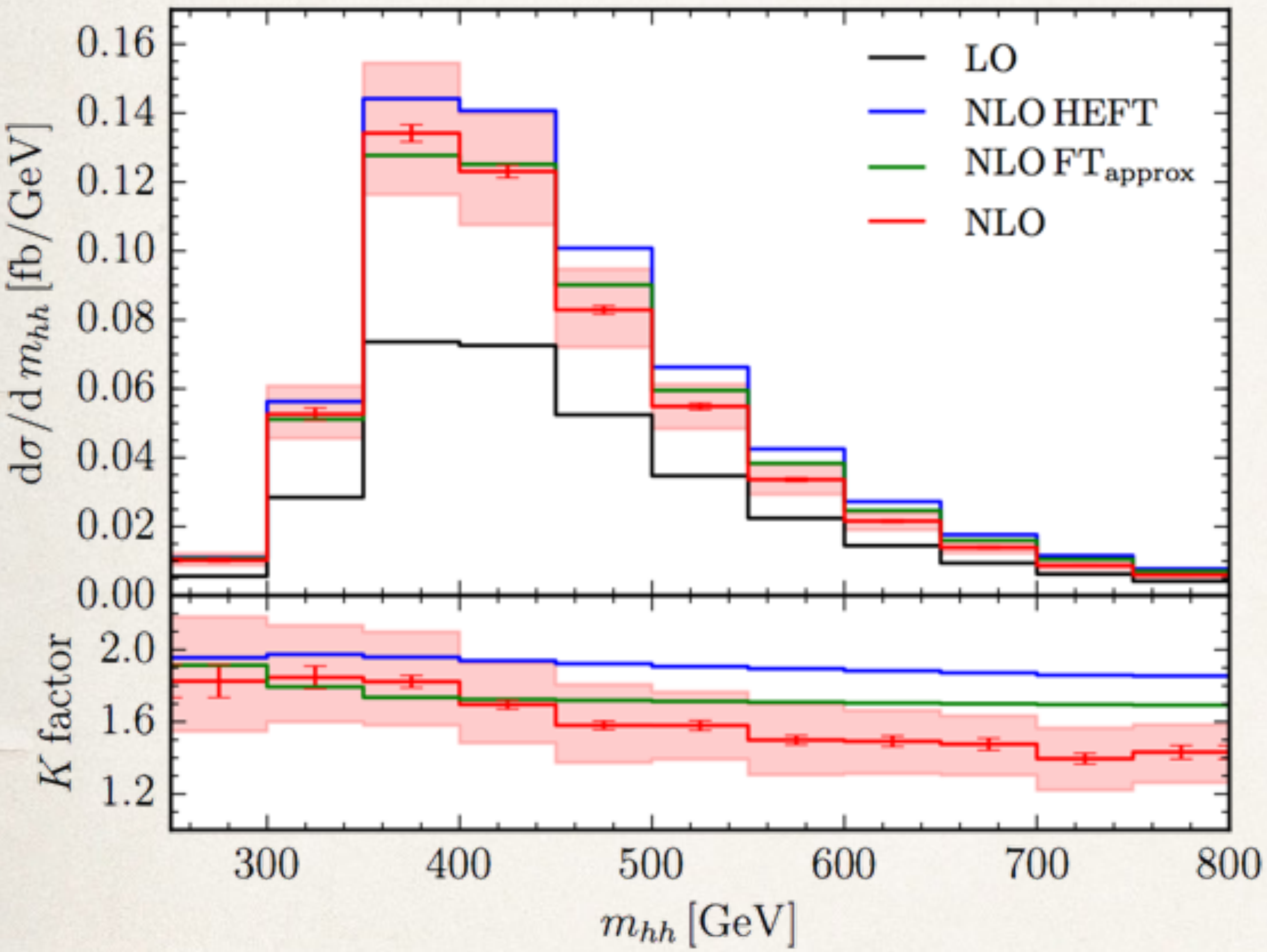


- Large corrections (relevant especially for precision pp→ZZ cross-section)
- Higgs interference: large, but as expected (K_{sig}~K_{bkg}~K_{int})
- Top mass effects (important for interference) through $1/m_t$ expansion → reliable only below threshold (although some hope for past-threshold extension via Padé approximations)

Loop induced: di-Higgs@NLO

[Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke (2016)]

→ *see Stephen's talk on Thursday*



- 2-loop amplitude beyond current reach (reduction and for MI)
- Completely different approach: *FULLY NUMERICAL INTEGRATION OF EACH INDIVIDUAL INTEGRAL WITH SECDEC*
- Table of 665 phase-space points
- Highly non-trivial computer-science component (GPUs, very delicate numerical integration…)

- Reasonable approximations to extend $1/m_t$ result beyond the top threshold (rescaled Born, exact real radiation) can fail quite significantly
- Exact K-factor much less flat than for m_t approximations

Loop induced: di-Higgs@NLO

[Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke (2016)]

Now that we know the exact result, many interesting questions:

- do we understand why the approximate m_t result fails so miserably (high energy matching, genuinely large two-loop components…)?
  - ideal playground for approximation testing. Can we find something which works? Can we study e.g. the Padé approximation used to extend the $1/m_t$ expansion in $gg \to VV$?
  - especially relevant because we now know FULLY DIFFERENTIAL NNLO CORRECTIONS IN THE $M_T \to \infty$ LIMIT ([de Florian et al (2016), see Jonas' talk on Thursday) → Would like to know best way to combine the results
- CAN THIS FULLY NUMERICAL APPROACH BE APPLIED TO MORE GENERAL CASES?
  - processes with more than two (m_{HH}, y_{HH}) variables ($gg \to 4l$)
  - processes with a more complicated tensor structure (H+J)

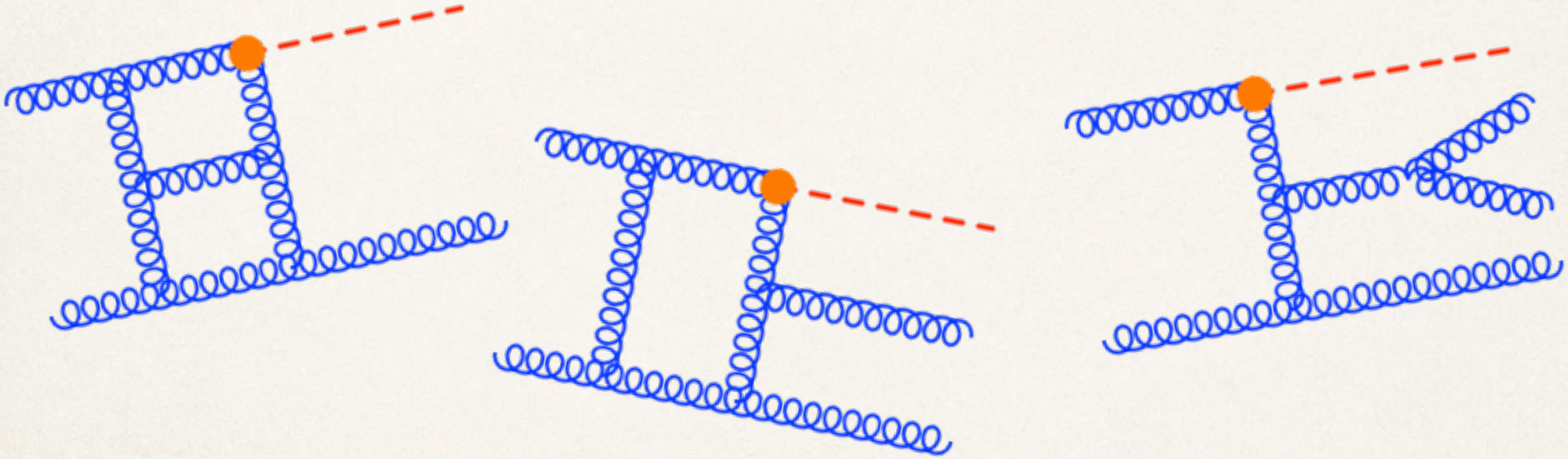
Beyond NLO:
progress in fully differential NNLO computations

Few percent accuracy

$\alpha_s \sim 0.1$ → few percent accuracy requires NNLO

- less dependence on unphysical variation ($\mu_{R,F}$) → dynamical scales and `art' of scale choice become less of an issue
- in several cases important test of perturbative stability (Higgs, VV…)

Different ingredients: two-loop (VV), one-loop+j (RV), tree+jj (RR)

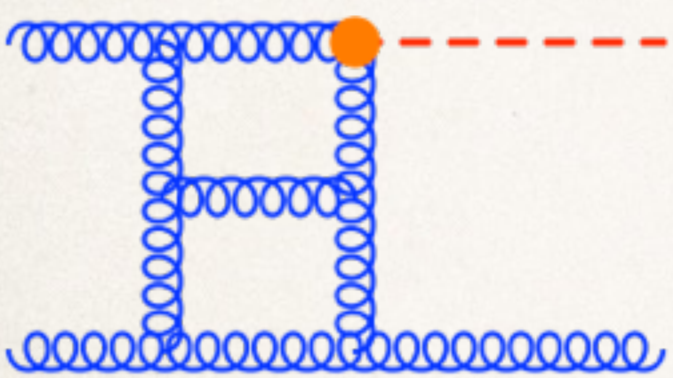


So NNLO for pp→X gives you for free `merged' results for pp→X (NNLO), pp→Xj (NLO) and pp→Xjj (LO)

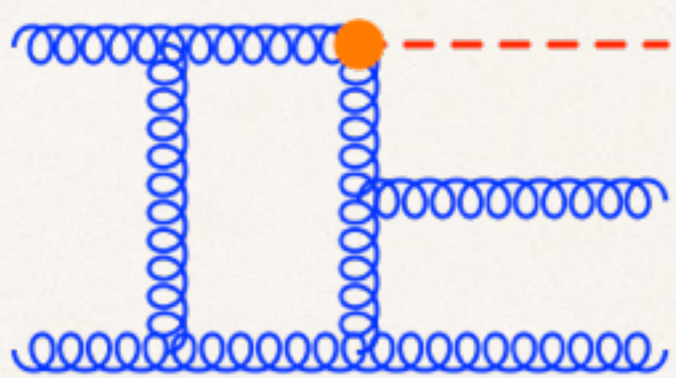
The problems with NNLO computations

Apart from complicated two-loop amplitudes, the big problem of NNLO computations is how to consistently handle IR singularities

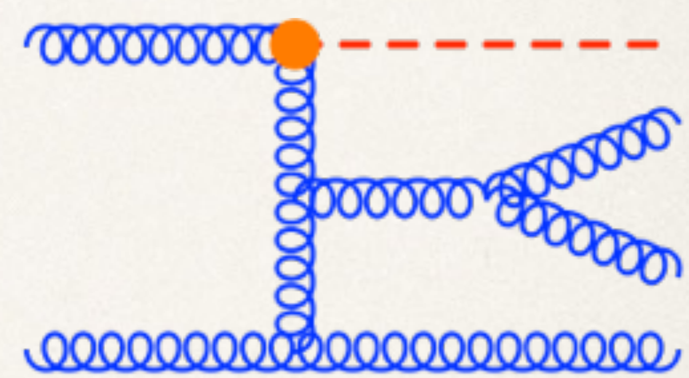
VV



RV



RR



$$\int \left[\frac{\mathrm{vv}_4}{\epsilon^4} + \frac{\mathrm{vv}_3}{\epsilon^3} + \frac{\mathrm{vv}_2}{\epsilon^2} + \frac{\mathrm{vv}_1}{\epsilon} + \mathrm{vv}_0\right] d\phi_2$$

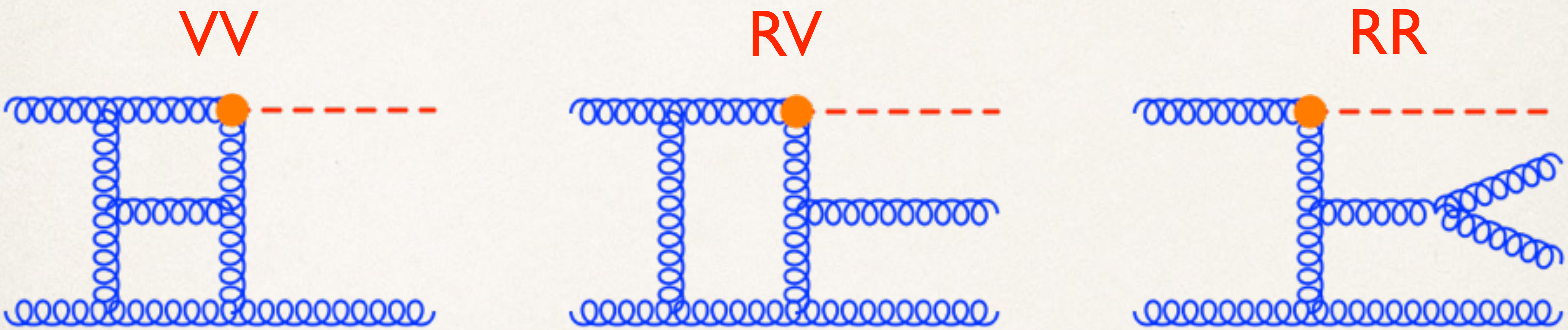
$$\int \left[\mathrm{rr}_0\right] d\phi_4$$

$$\int \left[\frac{\mathrm{rv}_2}{\epsilon^2} + \frac{\mathrm{rv}_1}{\epsilon} + \mathrm{rv}_0\right] d\phi_3$$

COMPLICATED IR STRUCTURE HIDDEN IN THE PHASE SPACE INTEGRATION

The problems with NNLO computations

Apart from complicated two-loop amplitudes, the big problem of NNLO computations is how to consistently handle IR singularities



- IR divergences hidden in PS integrations
- After integrations, all singularities are manifest and cancel (KLN)
- We are interested in realistic setup (arbitrary cuts, arbitrary observables) → we need fully differential results, we are not allowed to integrate over the PS
- The challenge is to Extract PS-Integration Singularities Without Actually Performing the PS-Integration

The solution: two philosophies

Same problem at NLO. Two different approaches have been developed

Phase space slicing

$$\int |M|^2 F_J d\phi_d = \int_0^\delta \left[|M|^2 F_J d\phi_d\right]_{s.c.} + \int_\delta^1 |M|^2 F_J \phi_4 + \mathcal{O}(\delta)$$

- conceptually simple, straightforward implementation
- must be very careful with residual δ dependence (esp. in diff. distr.)
- highly non-local → severe numerical cancellations

Subtraction

$$\int |M|^2 F_J d\phi_d = \int (|M|^2 F_J - \mathcal{S}) d\phi_4 + \int \mathcal{S} d\phi_d$$

- in principle can be made fully local → less severe numerical problems
- requires the knowledge of subtraction terms, and their integration

The solution: two philosophies

Both methods have proven useful for 2→ 2 computations

Phase space slicing

$$\int |M|^2 F_J d\phi_d = \int_0^\delta \left[|M|^2 F_J d\phi_d\right]_{s.c.} + \int_\delta^1 |M|^2 F_J \phi_4 + \mathcal{O}(\delta)$$

- q_t subtraction [Catani, Grazzini] → H, V, VH, VV, HH
- N-jettiness [Boughezal et al; Gaunt et al] → H, V, γγ, VH, Vj, Hj, single-top

Subtraction

$$\int |M|^2 F_J d\phi_d = \int (|M|^2 F_J - \mathcal{S}) d\phi_4 + \int \mathcal{S} d\phi_d$$

- antenna [Gehrmann-de Ridder, Gehrmann, Glover] → jj, Hj, Vj
- Sector-decomposition+FKS [Czakon; Boughezal, Melnikov, Petriello; Czakon, Heymes] → ttbar, single-top, Hj
- P2B [Cacciari, Dreyer, Karlberg, Salam, Zanderighi] → VBF_H, single-top
- *Colorful NNLO [Del Duca, Somogyi, Tocsanyi, Duhr, Kardos]: only e^+e^- so far*

The solution: two philosophies

Some of these techniques are quite generic

IN PRINCIPLE, they allow for ARBITRARY COMPUTATIONS

IN PRACTICE: `genuine' 2→2 REACTIONS, with big computer farms

Slicing: a closer look

Due to its highly non-local character, slicing leads to large numerical cancellations → abandoned at NLO

Why can we use it at NNLO?

- huge increase in computing power
- significant progress in NLO computations (speed / stability) → the CPU-intensive '+J' part is highly optimized for free (fully inherited by NLO)
- NNLO corrections smaller than NLO ones: can allow for larger uncertainty on them, without affecting the final result → δ_{cut} can be chosen not too prohibitively small (although careful if extreme precision is required, see m_W determinations)
- So far, relatively `simple' kinematics configurations tested. It would be interesting to stress-test slicing on e.g. 2→3 (impossible right now) or with intricate IR configurations (di-jet)
- Interesting theoretical development: towards leading power corrections in δ (would allow for larger δ_{cut}). Non trivial for generic processes

Subtraction: a closer look

Very different approaches, each with its own merits/problems

- antenna: almost fully local subtraction, fully analytic. Entirely worked out only for massless processes (technical problems, difficult integrated subtractions)
- sector-decomposition+FKS: fully local, numerical integration of integrated subtractions. As a consequence, massive processes are not a problem
- projection to Born: local, very nice trick to get integrated subtraction for free, but requires prior knowledge of $d\sigma^{NNLO}/d\Phi^{Born} \rightarrow$ limited applicability, small room for checks

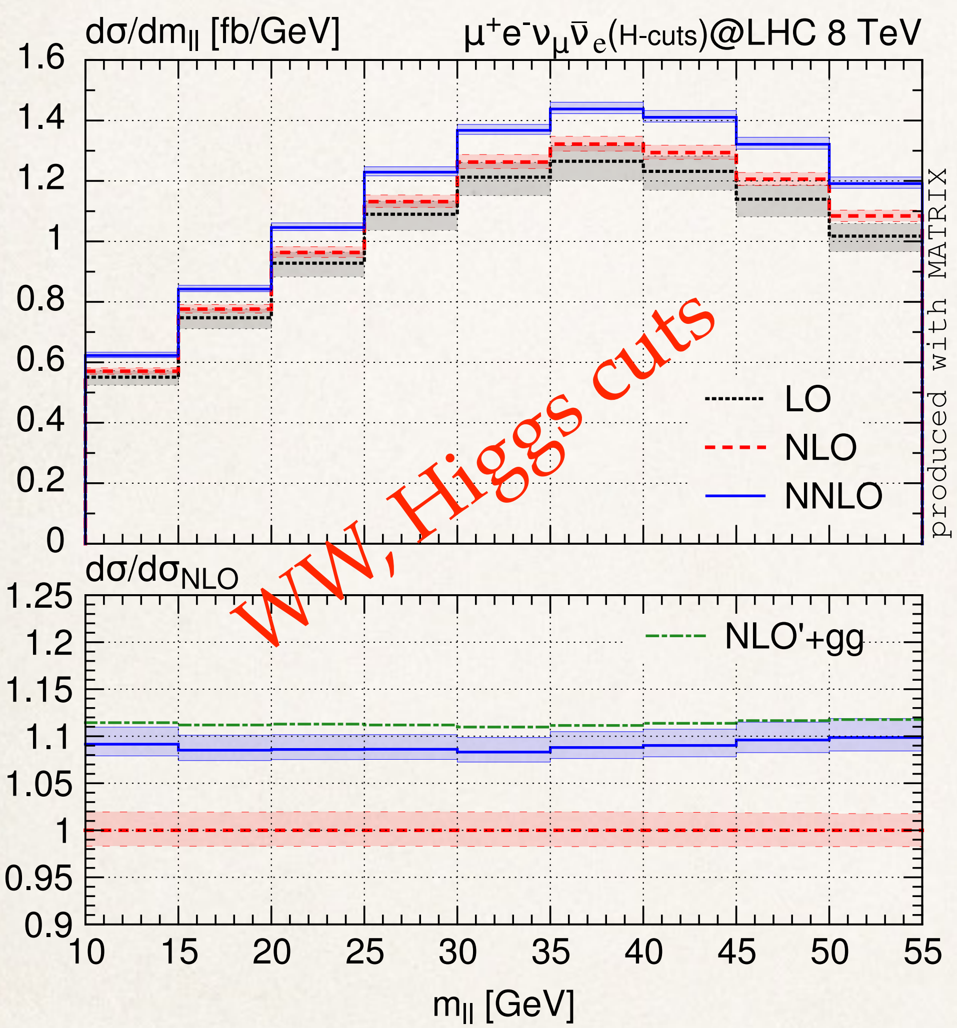
Many results, but still in `proof-of-concept' phase

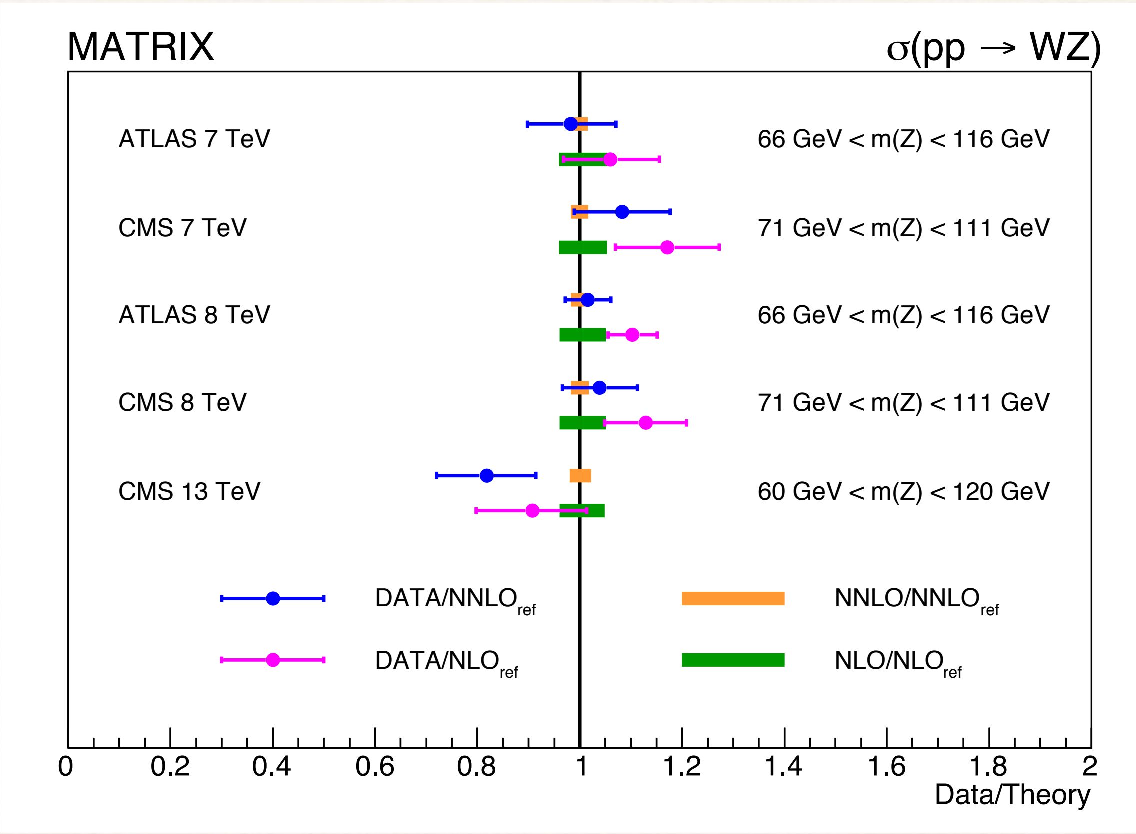
- an obviously optimal framework has not appeared yet
- despite flood of results, (a lot of) theoretical work still needed
- all the `latest technologies' in NLO not present here
- large room for improvement

Recent NNLO results: di-bosons

In the last few months, the PROGRAM OF COMPUTING FULLY DIFFERENTIAL NNLO CORRECTION TO DI-BOSON PROCESSES HAS BEEN COMPLETED

→ *see Marius' talk on Thursday*



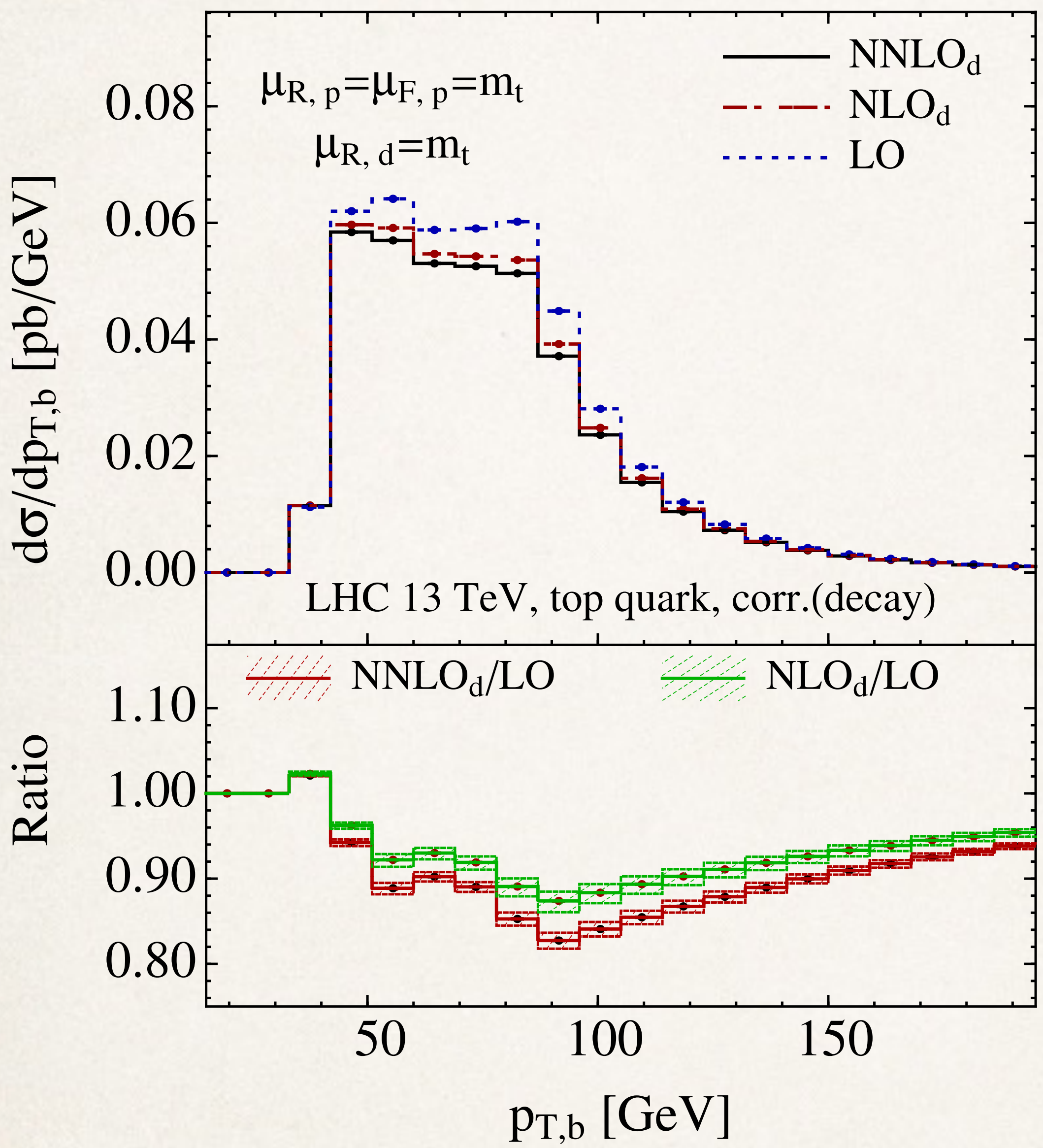


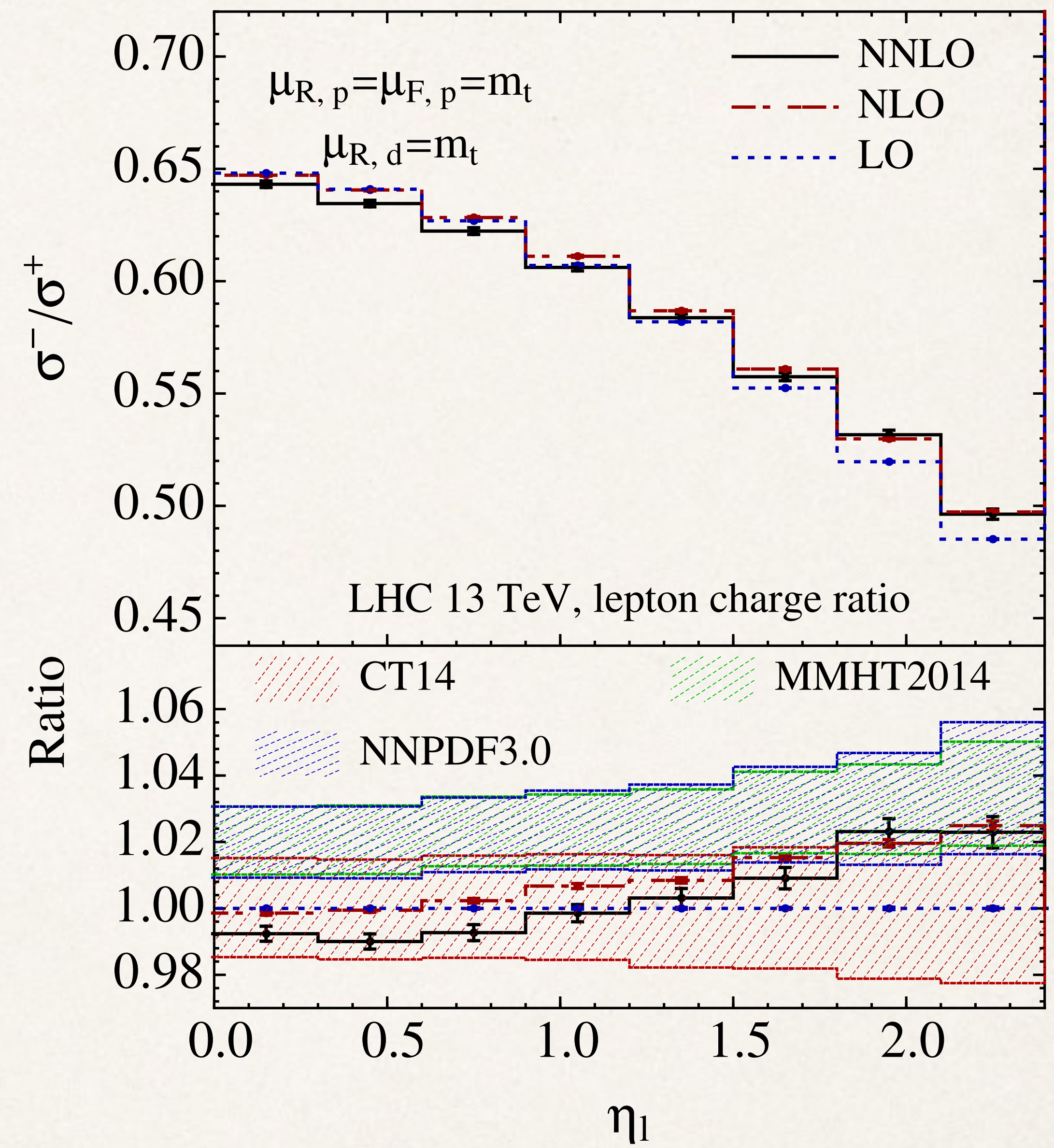
WZ vs data

- q_t subtraction. δ-indep. FULLY DEMONSTRATED at the differential level
- General picture: GOOD AGREEMENT DATA / NNLO (with some possible room for discussion for WW jet-veto, see [Dawson et al (2016)])

Recent NNLO results: single-top

t-channel single-top plus top-decay [Berger, Gao, Yuan, Zhu (2016)]



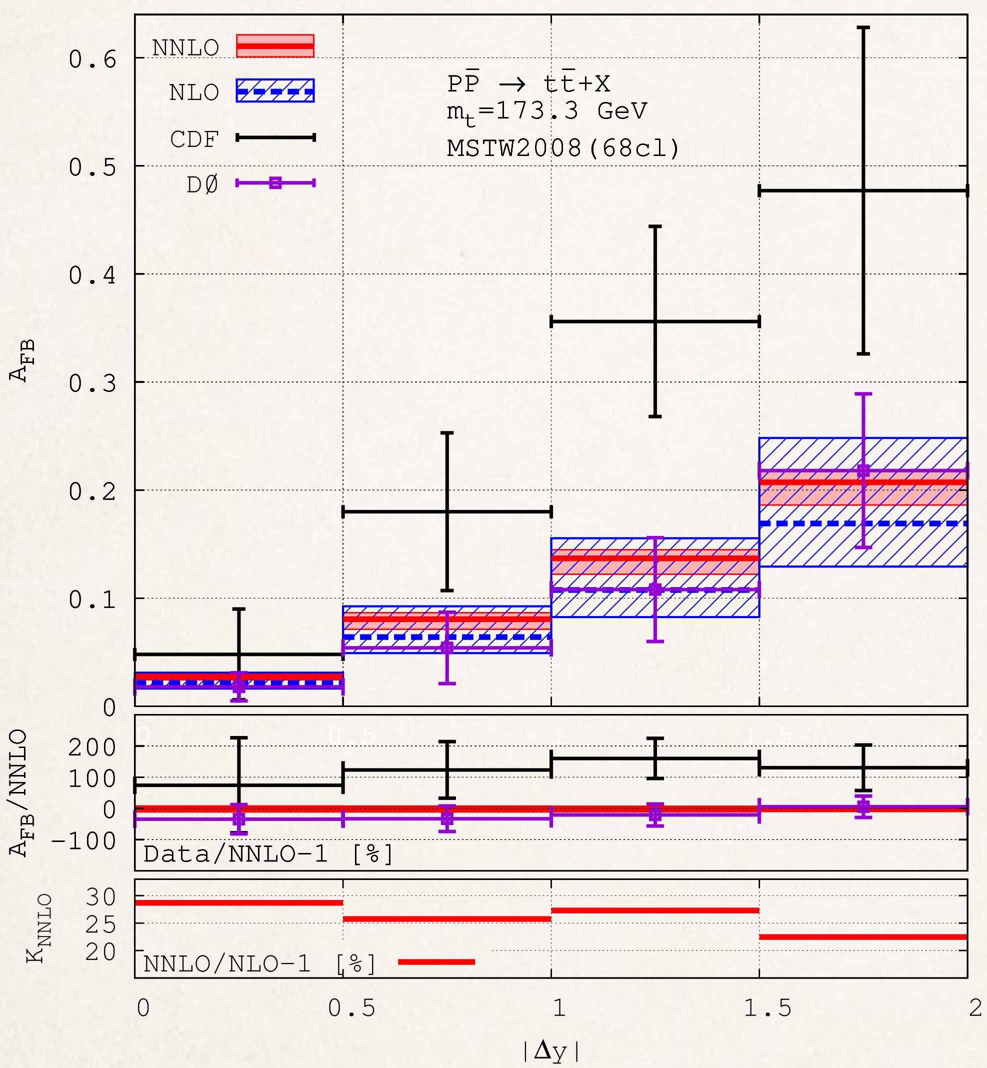


- Mixture of slicing and subtraction (P2B)
- $\mathrm{NNLO_{prod}} \otimes \mathrm{NNLO_{dec}}$ (in the NWA approximation) → very clean data/theory comparison possible

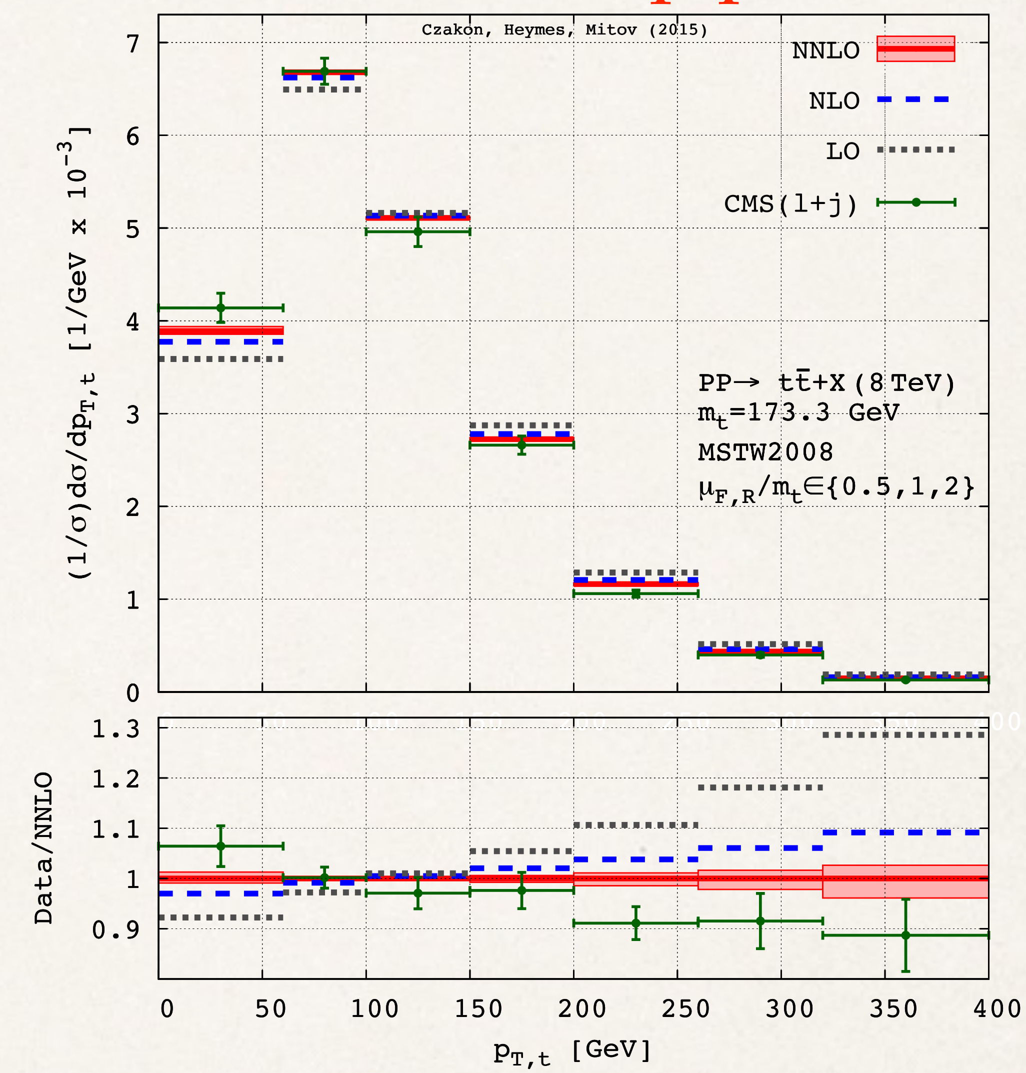
Recent NNLO results: ttbar

Fully differential ttbar results [Czakon, Heymes, Mitov (2015-16)]

Tevatron, A_{FB}



LHC, top p_t



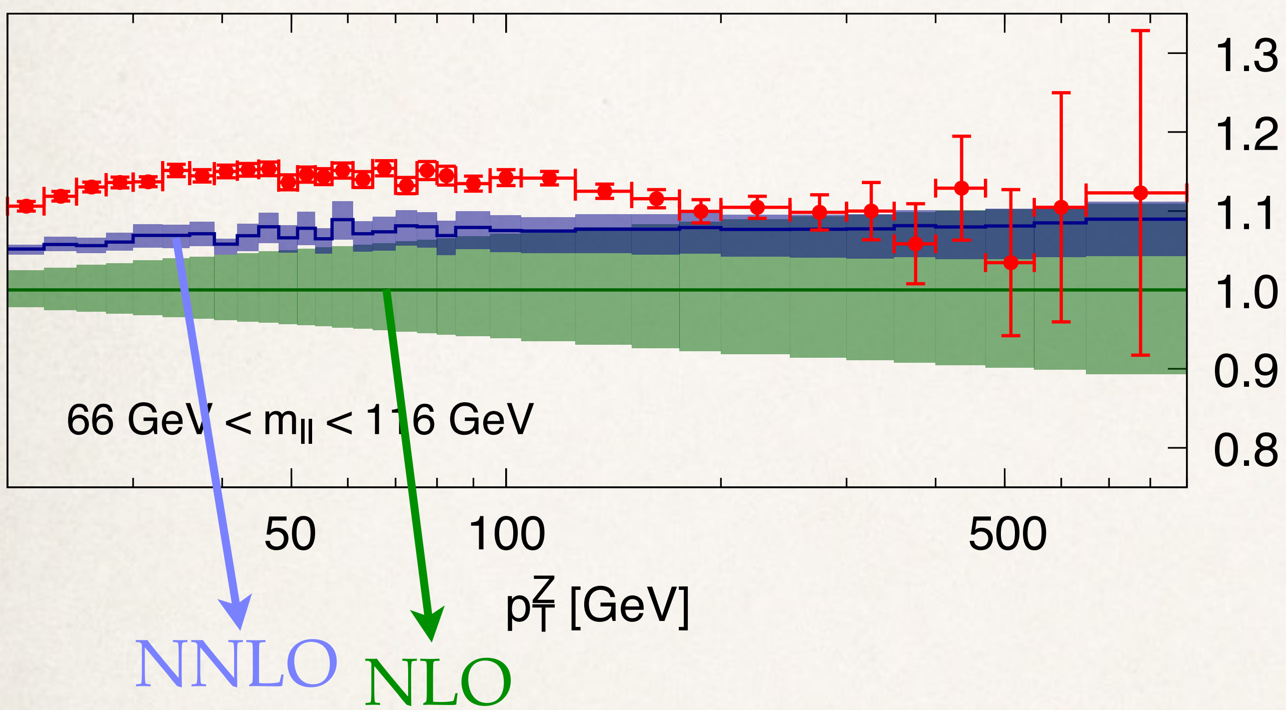
- Sector-decomposition + FKS: STRIPPER-4D
- Stable top, exhaustive differential studies, scale-dependence study
- Alleviated data/theory tension for $p_{t,top}$ at the LHC

Recent NNLO results: V+J phenomenology

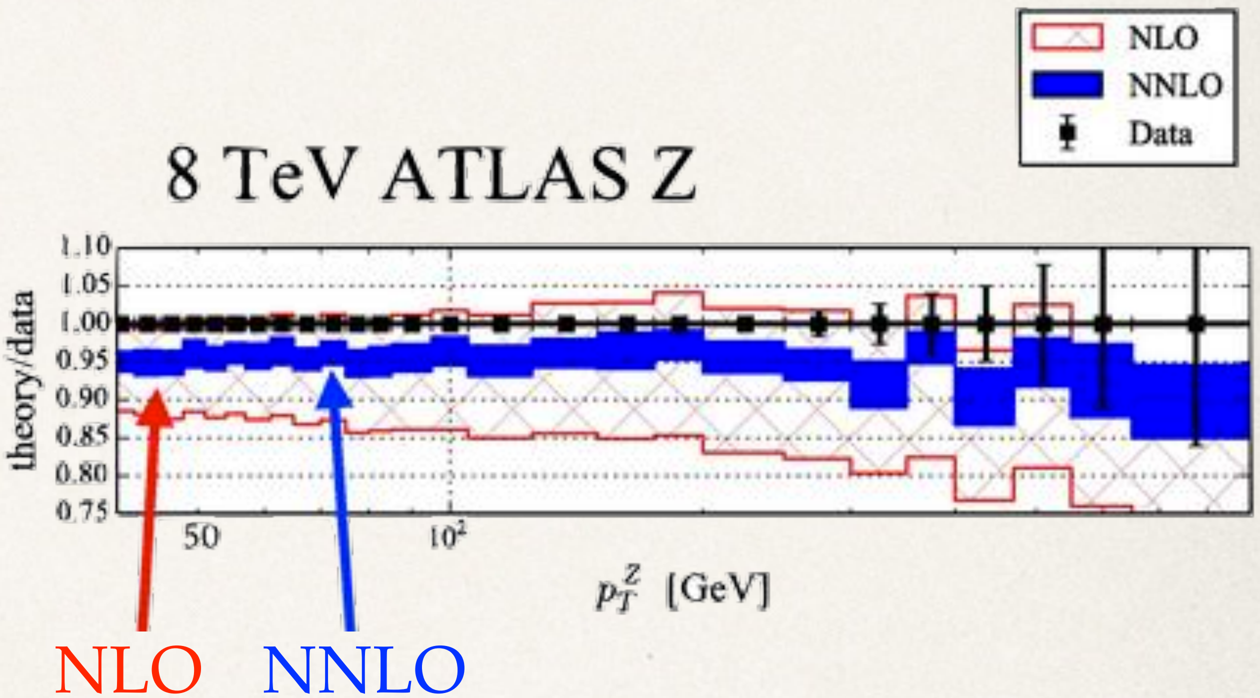
see Alexander's talk tomorrow

Data / theory ratio, Z+jet

Antenna [Gehrmann-de Ridder et al (2016)]



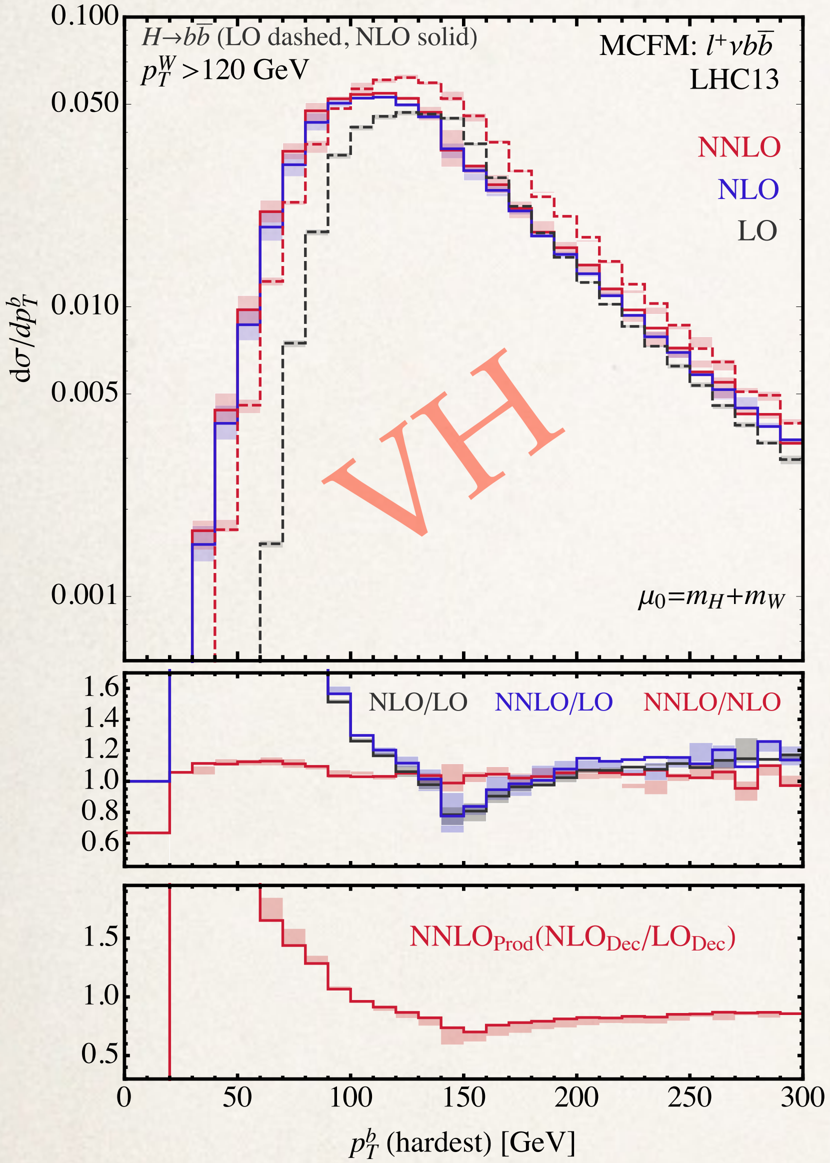
N-Jettiness [Boughezal et al (2016)]

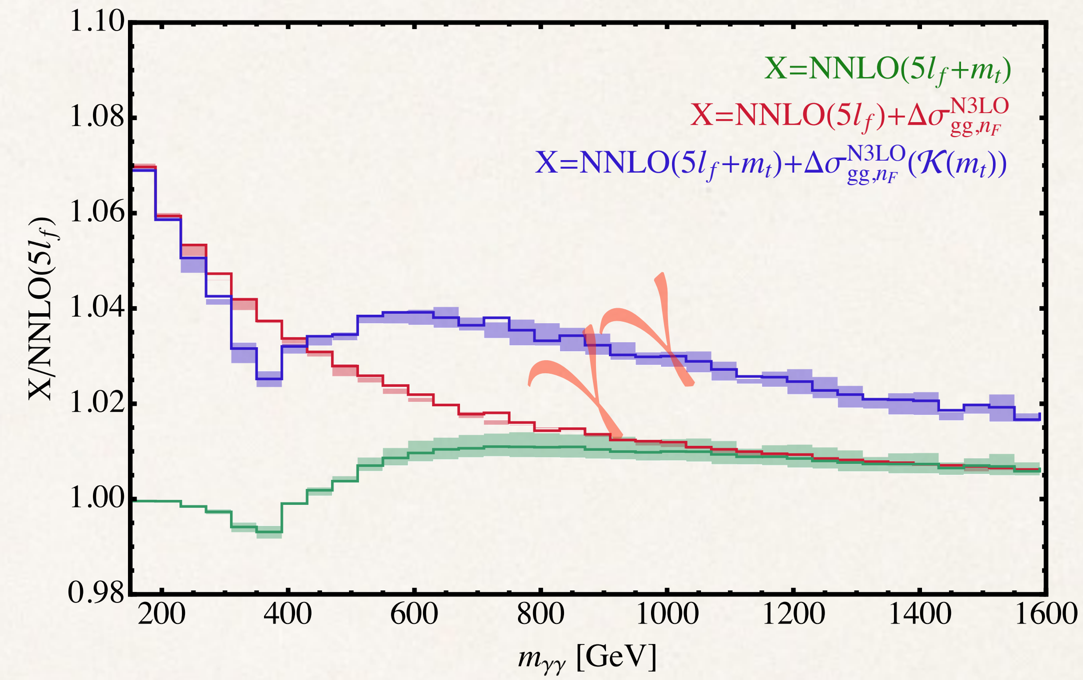


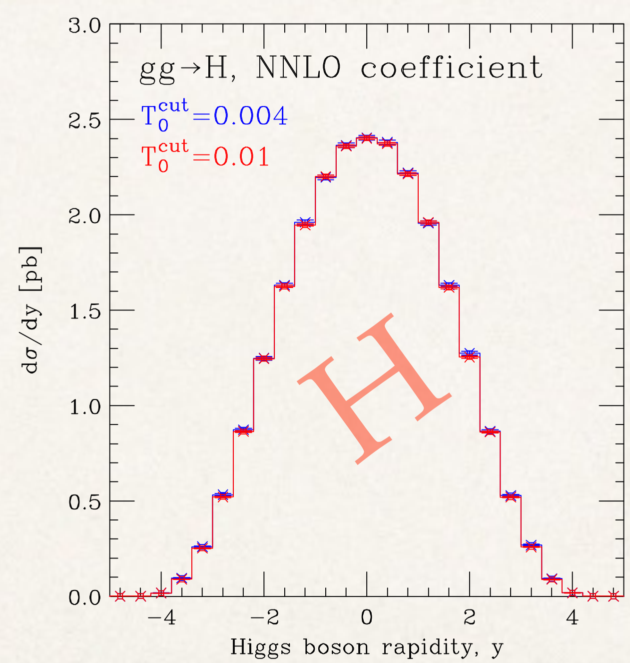
- Also at NNLO, slight data/theory tension
- Disappears for normalized ratios, but not accounted for systematics / luminosity uncertainties
- The cleanest possible measurement… SHOULD WE BE WORRIED?

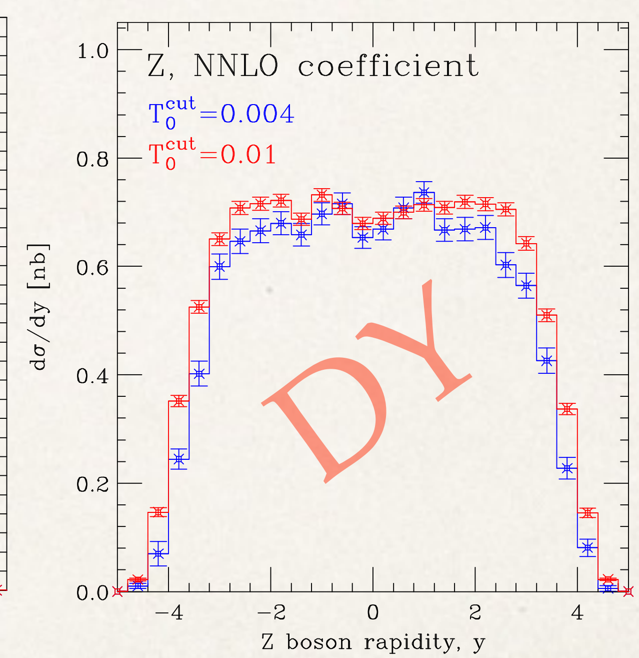
Recent NNLO results: MCFM@NNLO

[Campbell, Ellis, Williams (2016); Campbell et al (2016); Boughezal et al (2016)]





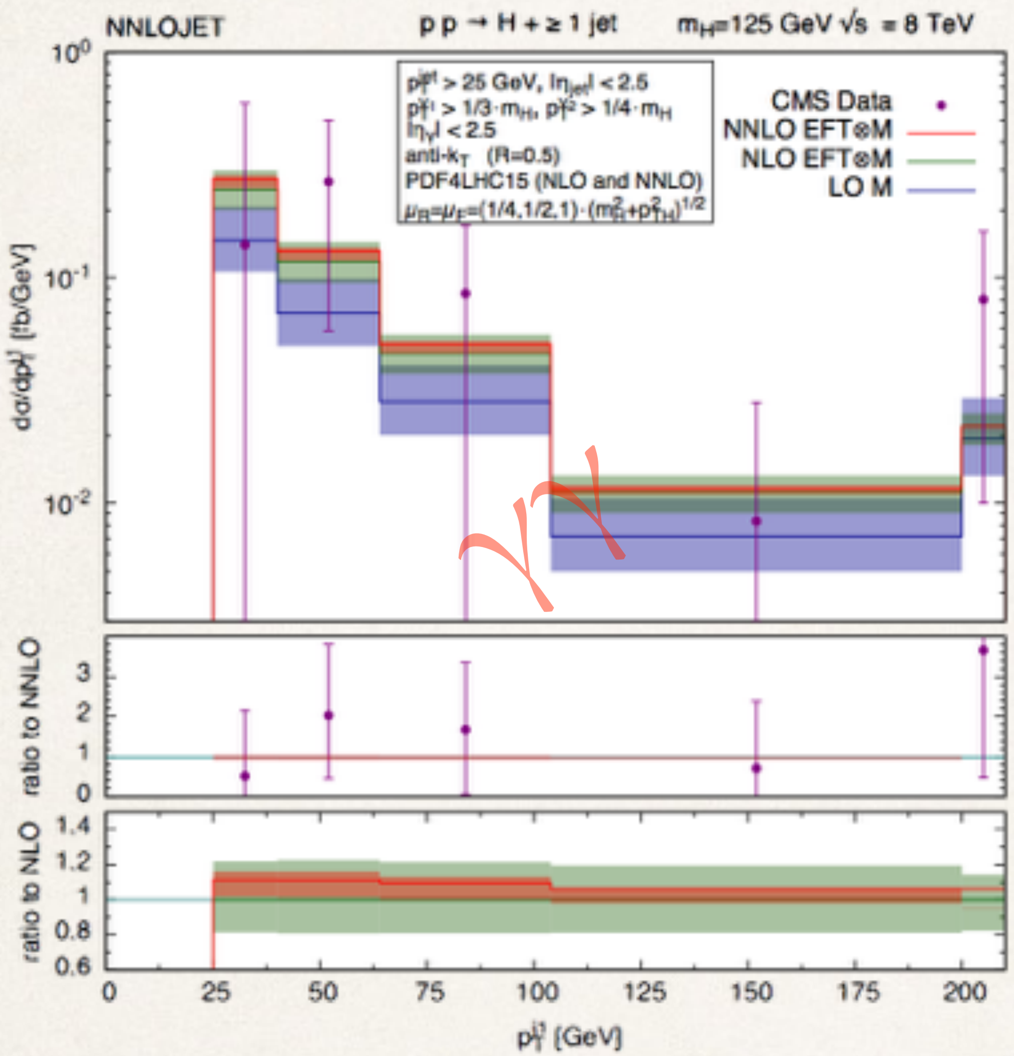




- NNLO slicing available for some color-singlet processes in MCFM
- V/H+J will be next?

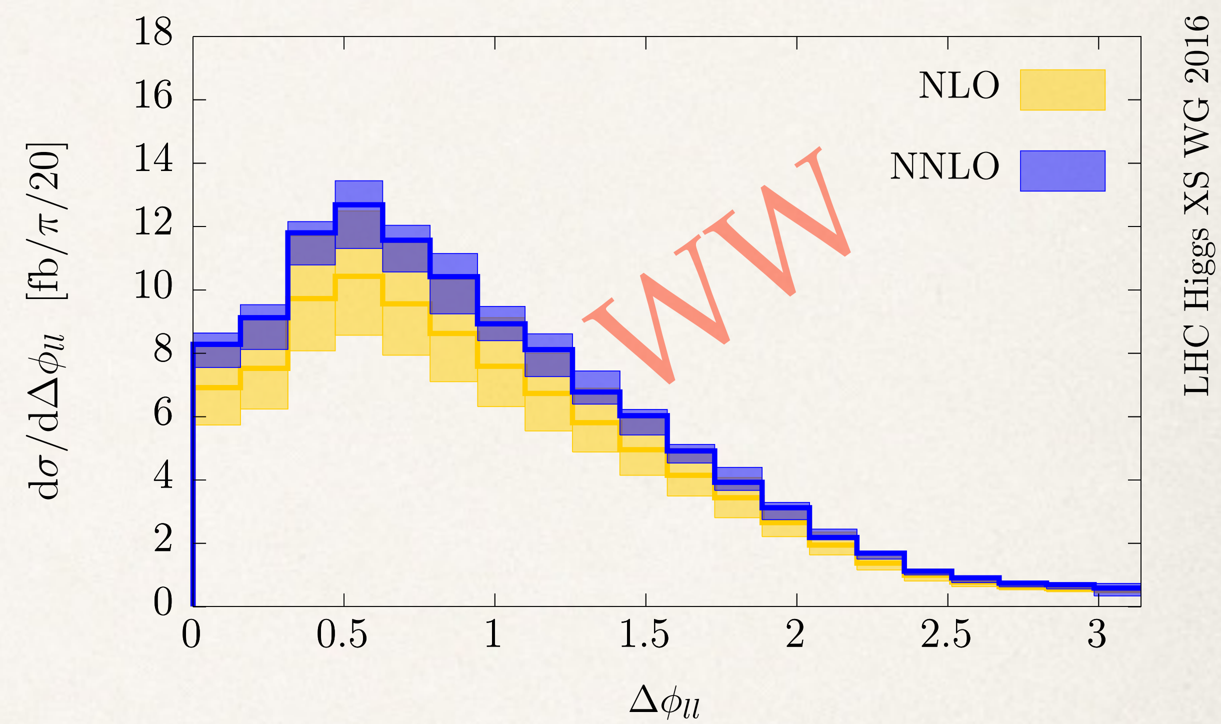
Recent NNLO results: H+J phenomenology

Antenna [Chen et al (2016)]



FKS+Sector Decomposition

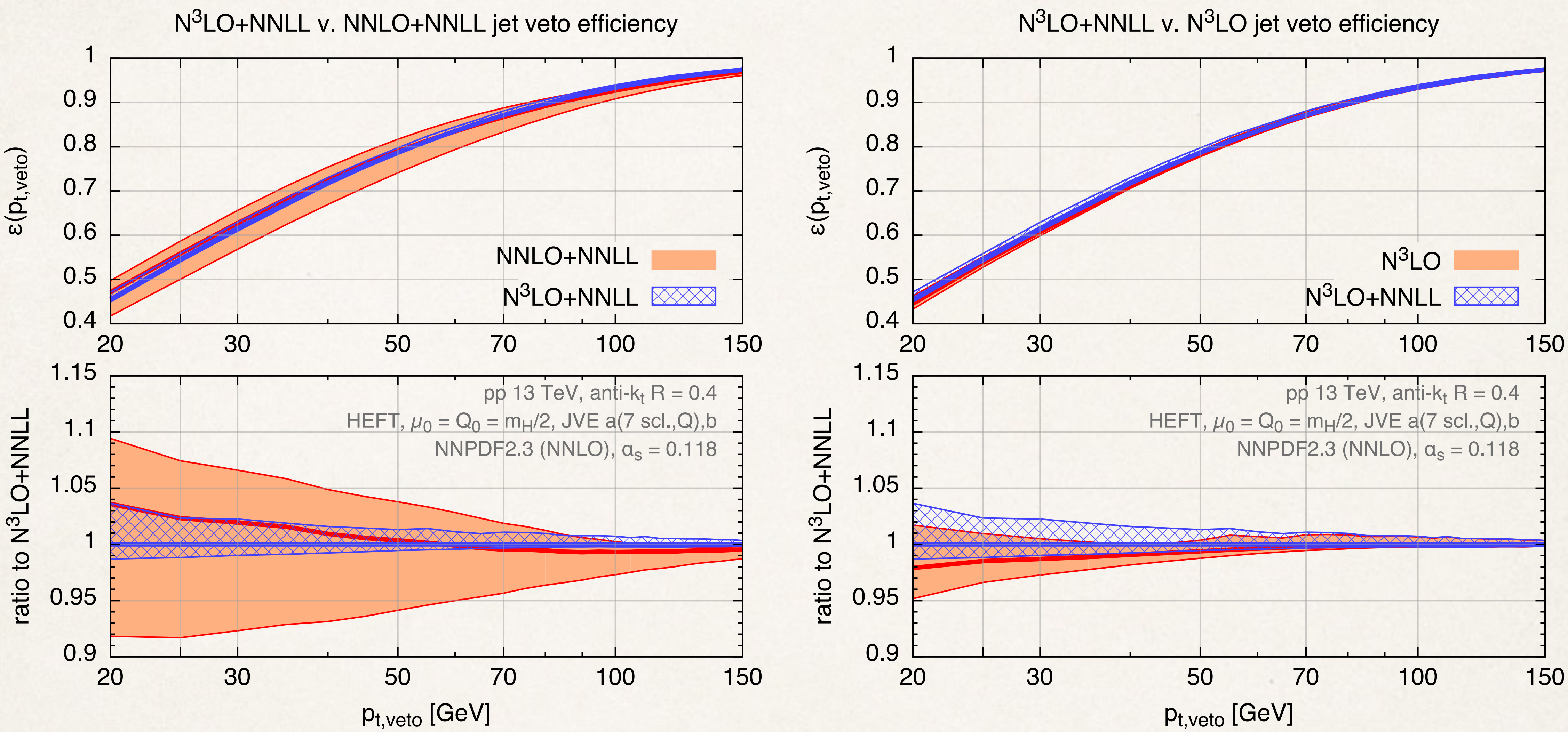
[FC, Melnikov, Schulze (2015+YR4)]



- Realistic final states → fiducial region
- Important benchmarking between different computations
- Non-trivial final states possible

Application of f.o. results: H and jet vetoes

[Banfi, FC, Dreyer, Monni, Salam, Zanderighi, Dulat (2015)]

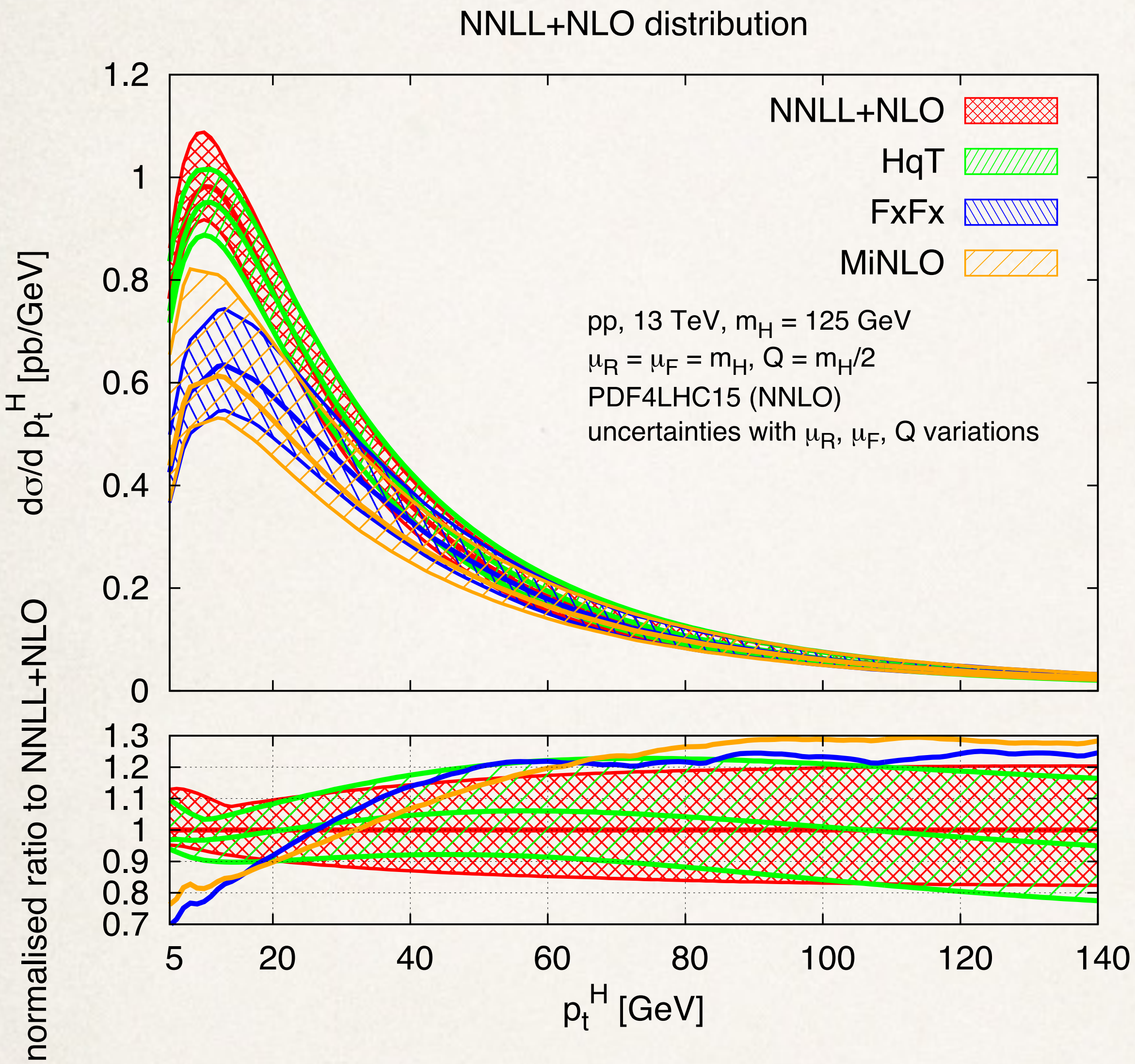


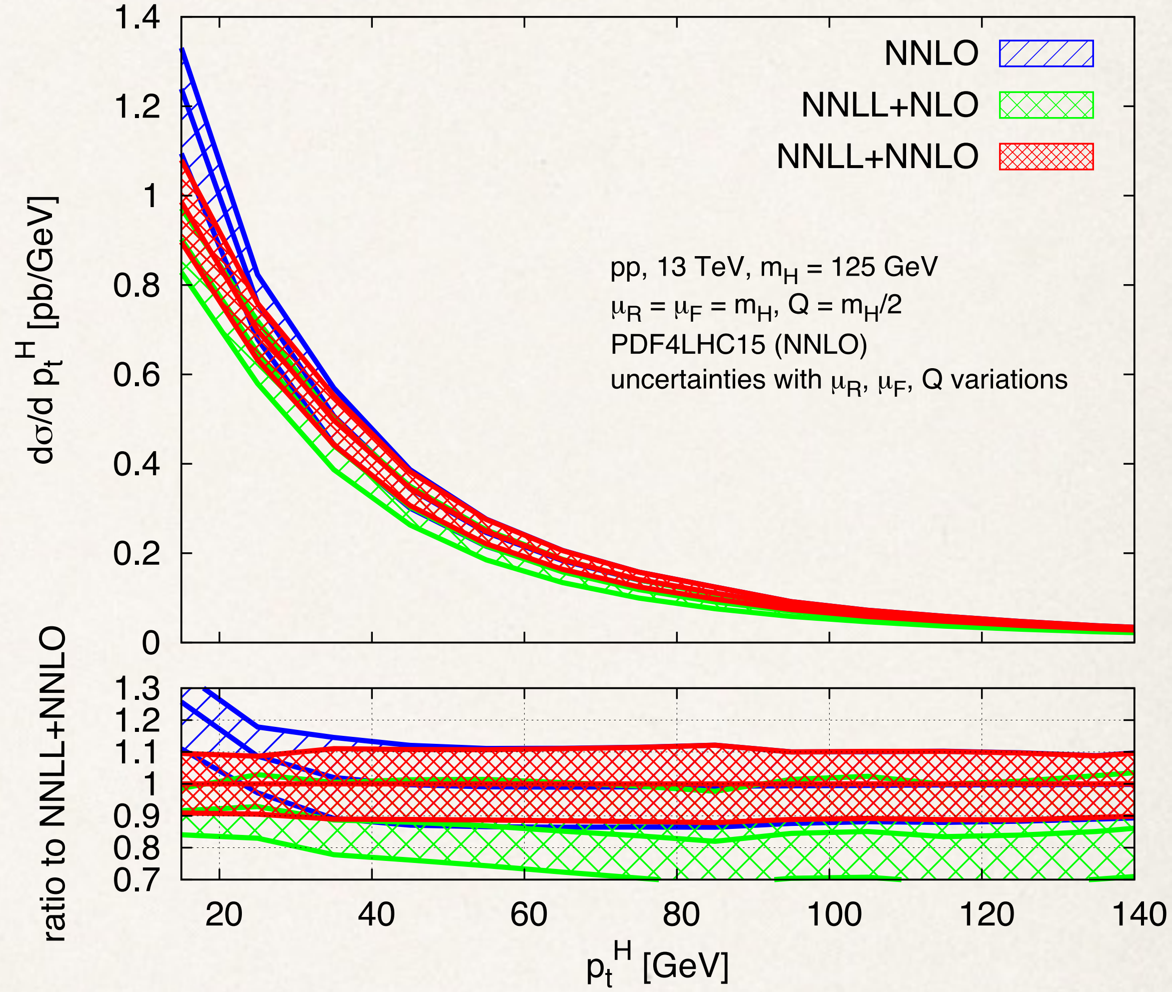
- Combination of f.o. N^{3}LO (Higgs inclusive) and NNLO (H+J exclusive) with NNLL resummation, LL$_R$ resummation, mass effects…
- No breakdown of fixed (high) order till very low scales

see Pier's talk tomorrow

Application of NNLO results: H p_T

[Monni, Re, Torrielli (2016)]





- Matching of NNLO H+J with NNLL Higgs p_T resummation
- Significant reduction of perturbative uncertainties
- Again, no breakdown of perturbation theory (resummation effects: 25% at p_T = 15 GeV, ~0% at p_T = 40 GeV)

see Pier's talk tomorrow

Conclusions and outlook

- Fixed order computation at the heart of LHC precision program
- Thanks to a lot of progress in the past, now NLO predictions are standard, even for complicated problem
- Recent breakthrough in NNLO conceptual problems lead to flood of new phenomenological results for genuinely 2→2 processes
- First genuine hadron-collider N^3LO computation

Great situation, but going beyond will require significant development

- multi-leg two-loop amplitudes (3-jet, H/V+jj)
- loop integrals with internal massive particles (Higgs p_T)
- improvements on NNLO subtraction schemes (both purely technical/ implementation-level and hopefully conceptual)
- Higgs@N^3LO differential

A LOT OF THEORETICAL FUN AHEAD, DIRECTLY RELEVANT FOR LHC PHENOMENOLOGY!

Thank you
very much for
your attention!