

QCD resummation in the precision era

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QCD in the IRC limit

- Gauge theories' perturbative predictions feature a logarithmic sensitivity to the Infrared and Collinear (IRC) dynamics
- Logarithms can appear both in amplitudes (e.g. large scale gaps) and at the cross section level whenever the measured observable probes kinematic regimes where the imbalance between real and virtual corrections affects the cancellation of IRC divergences (i.e. in the singular limit the cancellation only occurs when all perturbative orders are considered)
- e.g.
 - production of heavy systems near threshold in hadronic collisions
-> Talks by M. Bonvini, C. Wever
 - high-energy scattering
-> Talk by J. Smillie
 - limits of IRC-safe observables that constrain real radiation to soft and collinear configurations (e.g. exclusive regimes, shoulders,...)

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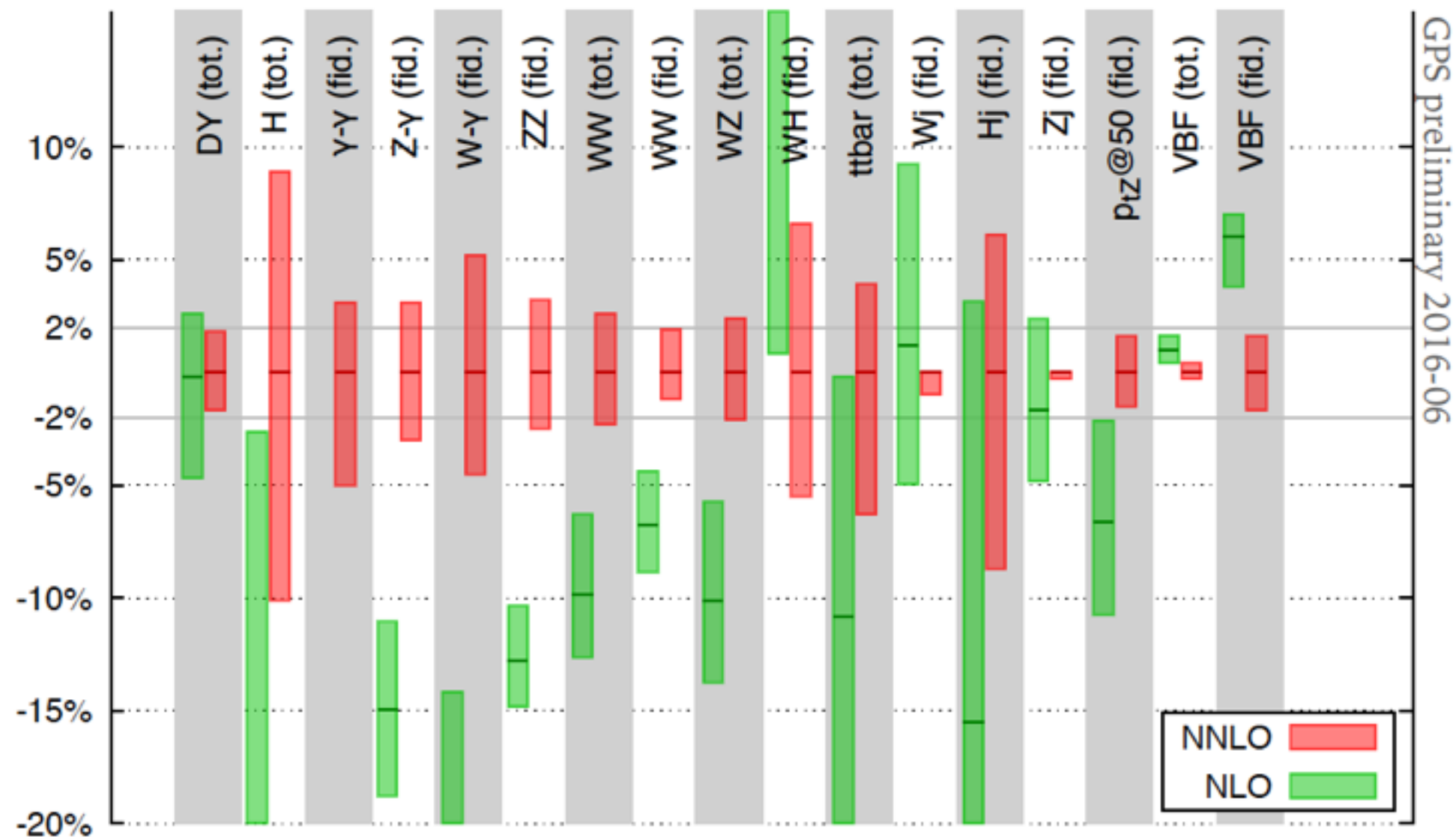
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Resummation: LEP vs LHC

- The two topics would require, in principle, different talks given the different nature of the two experiments
- LEP was a precision machine, largely used to test QCD and its properties (radiation coherence, non-abelian structure, strong coupling, non-perturbative (NP) corrections,...)
 - this often required probing regimes in which all-order effects are important. Resummation is of primary relevance for these studies
- LHC so far has been, primarily, a discovery machine. Collisions are QCD driven and sensitivity to long-distance physics comes together with contamination from background, pile-up (PU) and underlying events (UE). Observables computable precisely for a single collision are smeared, sometimes completely changed, by the huge amount of uncontrolled (theoretically) radiation
- Very often is necessary to cut out radiation effects without depleting too much the underlying signal. Resummation effects often small

Precision at the LHC

- The increase in luminosity improves discovery reach and precision
- Several measurements are being (or will be) performed with few-% accuracy - theory precision can become the limiting factor
- So far theory progress met this challenge (fixed-order, generators, EW,...)



Precision at the LHC

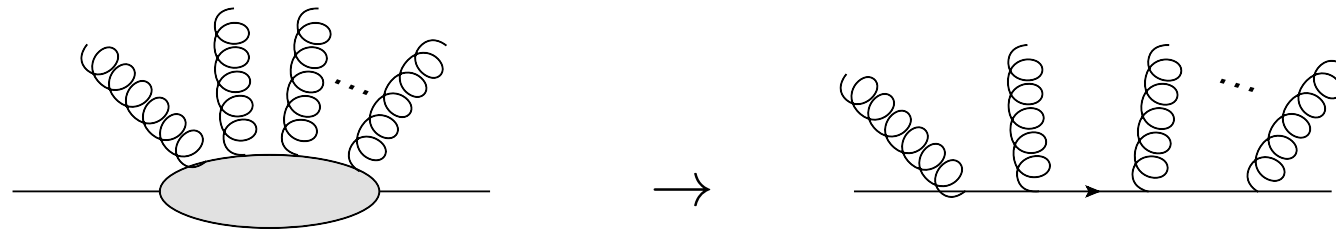
- At this level of accuracy many effects compete (higher-order/all-order QCD corrections, EW effects, hadronisation+NP corrections, PDFs + parameters), it's challenging to improve further.
- As far as resummation is concerned, a few questions come up:
 - Are all sources of all-order corrections under control (multiscale problems) ?
 - How's the contamination from radiation excluded/subtracted (e.g. what cuts) ? Is the resulting observable predictable (resummable) at all orders ?
 - Can we start from our knowledge of the radiation dynamics to devise new observables which are predictable at all orders and free of PU+UE contamination ? i.e. can we exploit more data ?

What does “resummable” even mean ?

- The all-order treatment for an observable $V(\{\tilde{p}\}, k_1, \dots, k_n)$ that satisfies the Sterman-Weinberg’s criteria relies on the concept of factorisation ...
 - ... of the QCD amplitudes in the IRC limits
 - ... **of the observable**. This implies that hard and singular IRC modes are not mixed when radiative corrections are considered
- The latter is often interpreted as the existence of factorised formula for the cross section in some conjugate space **Talks by T. Becher, L. Rothen**
 - powerful tool: systematic, observable dependent
- Ultimately we want to use what we learn from the theory to design precise generators that accurately predict many observables for a given process
- **We need to understand what hides behind factorisation to see if we can formulate an approach to resummation that applies to several observables**

What does “resummable” even mean ?

- Consider e.g. n soft & collinear independent emissions off a quark line



- Consider limit (most singular) of gluons strongly ordered in energy + angle

$$\sigma = \sum_n \int \frac{1}{n!} \prod_{i=1}^n [dk_i] |M(k_i)|^2 [\Theta(v - V(\{\tilde{p}\}, k_1, \dots, k_n)) - 1]$$

Amplitudes exponentiate
in this limit

What if it doesn't
scale this way ?

Exponentiation of the XS
is preserved only if
(in the considered limit)

$$V(\{\tilde{p}\}, k_1, \dots, k_n) \sim \max\{V(\{\tilde{p}\}, k_i)\}$$

- Restrictions become a little more subtle when the primary emissions branch
- Formulate conditions for an observable to be resummed -> **recursively IRC safety**

- if V is rIRC safe it can be systematically (and automatically) resummed

[@ NLL Banfi, Salam, Zanderighi '01-'04] [@ NNLL Banfi, McAslan, PM, Zanderighi '14 - '16; PM, Re, Torrielli '16]

for now only for global obs. with 2 coloured legs at LO

- one can formulate a recipe with the corrections to be computed (and how to compute them) at a given logarithmic order

What does “resummable” even mean ?

- Consider e.g. n soft & collinear independent emissions off a quark line



This is not a no-go theorem !

some non-rIRC safe observable could still be resumable,
although
no general structure emerges (yet)
No full resummations beyond LL exist today

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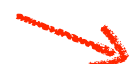
(in the considered limit)

$(\{ \tilde{p} \}, k_i)$

- Keeping in mind this property when thinking of new observables might come in handy
- Formulate conditions for an observable to be resummed → recursively into safety

- if V is rIRC safe it can be systematically (and automatically) resummed

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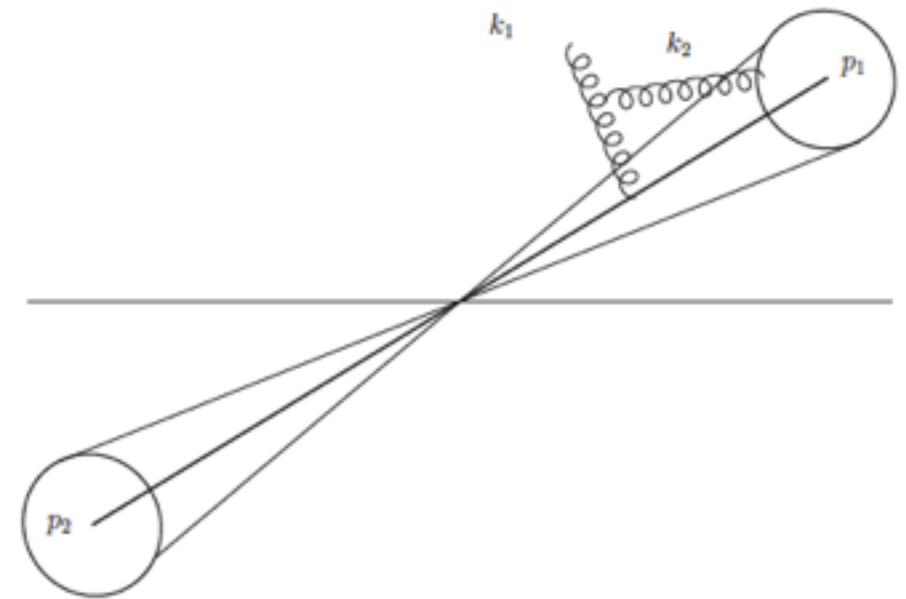
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Globalness

- Often observables weigh differently the radiation in different regions of the detector (jets, angular cuts, ...)

Additional logarithms appear
due to the RV imbalance
near the transition region

[Dasgupta, Salam '01]



- Two viable solutions:
 - non-global logarithms can be resummed at LL (general case in the planar limit) [Dasgupta, Salam '01; Banfi, Marchesini, Smye '02; Hatta, Ueda'14]
 - Recently fervent activity to understand NLL -> Talk by T. Becher [Caron-Huot '15; Larkoski, Mout, Neill '15; Becher, Neubert, Rothen, Shao '15-'16]
 - Possible to devise a general approach beyond LL ?
 - Define observables PU/UE-robust and *non-global free* -> substructure

Resummation in jet substructure

- Can these problems be modulated by grooming jets ?

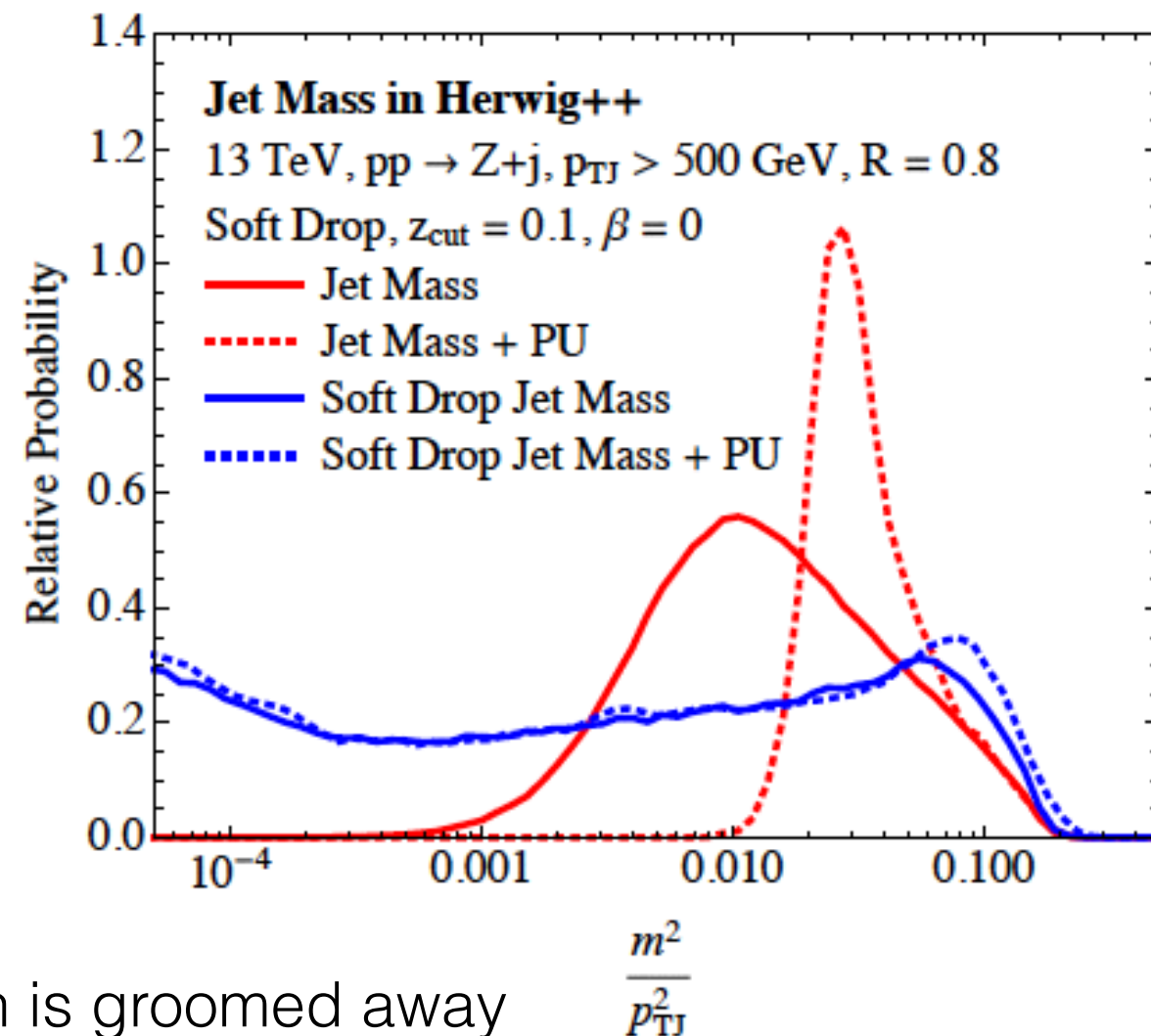
Proliferation of substructure technology in recent years

First analytic understanding at LL helped develop better-behaved observables

e.g. mMDT/Soft drop groomed jet mass:
[Dasgupta, Fregoso, Marzani, Salam '13]
[Larkoski, Marzani, Soyez, Thaler '14]

Recursive declustering of a C/A jet until

$$\frac{\min(p_{T1}, p_{T2})}{p_{T1} + p_{T2}} > z_{\text{cut}} \left(\frac{\Delta R_{12}}{R_0} \right)^\beta$$



In the regime $m_J^2/p_{t,J}^2 \ll z_{\text{cut}}$ the soft radiation is groomed away in a rIRC-safe way:

- $\beta = 0$: soft logarithms (wide-angle NG, interference effects) become $\ln(z_{\text{cut}})$
jet mass logarithms $\ln(m_J^2/p_{t,J}^2)$ are exclusively of collinear origin
- $\beta \neq 0$: additional NG logs of the jet mass are power suppressed

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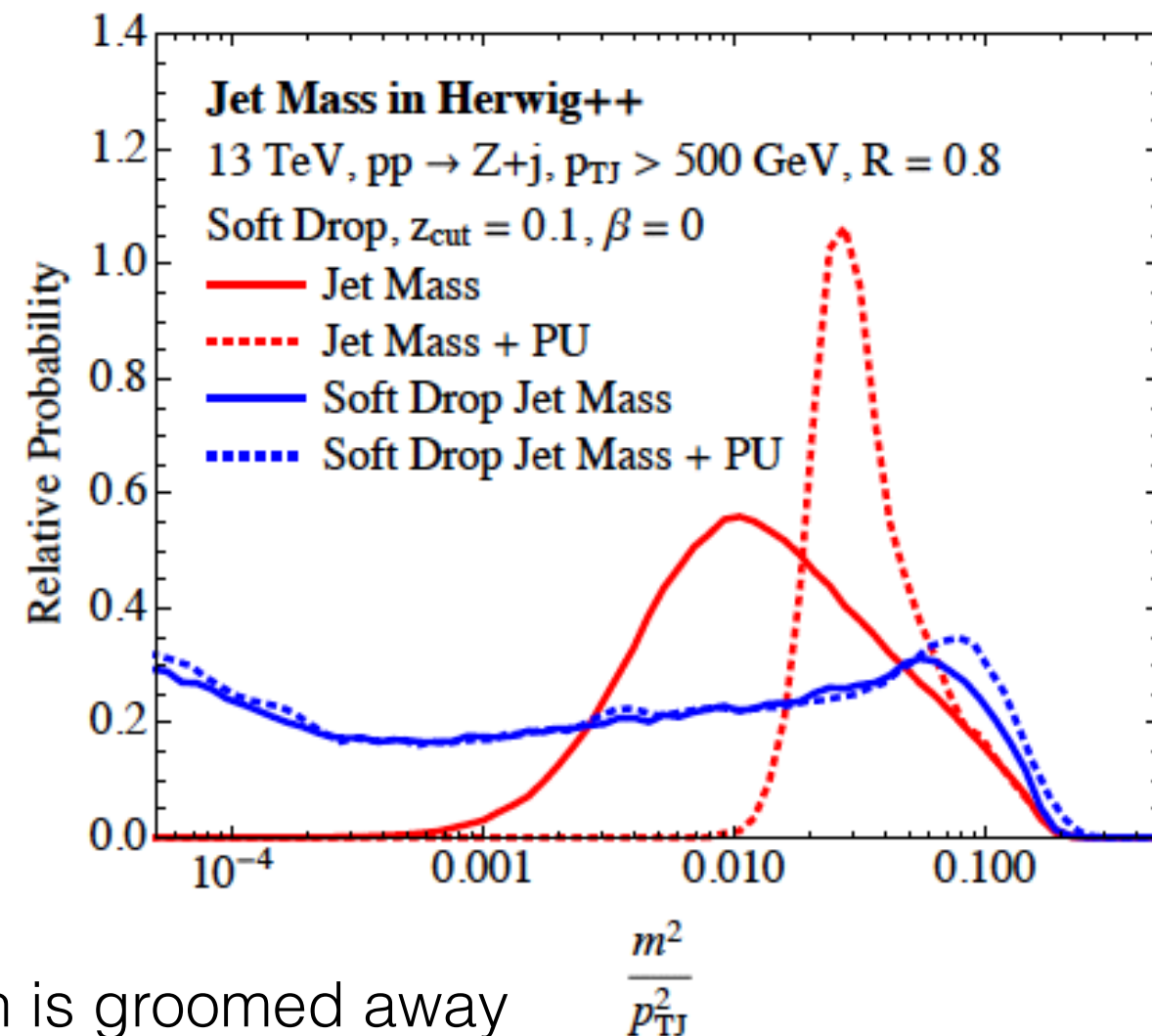
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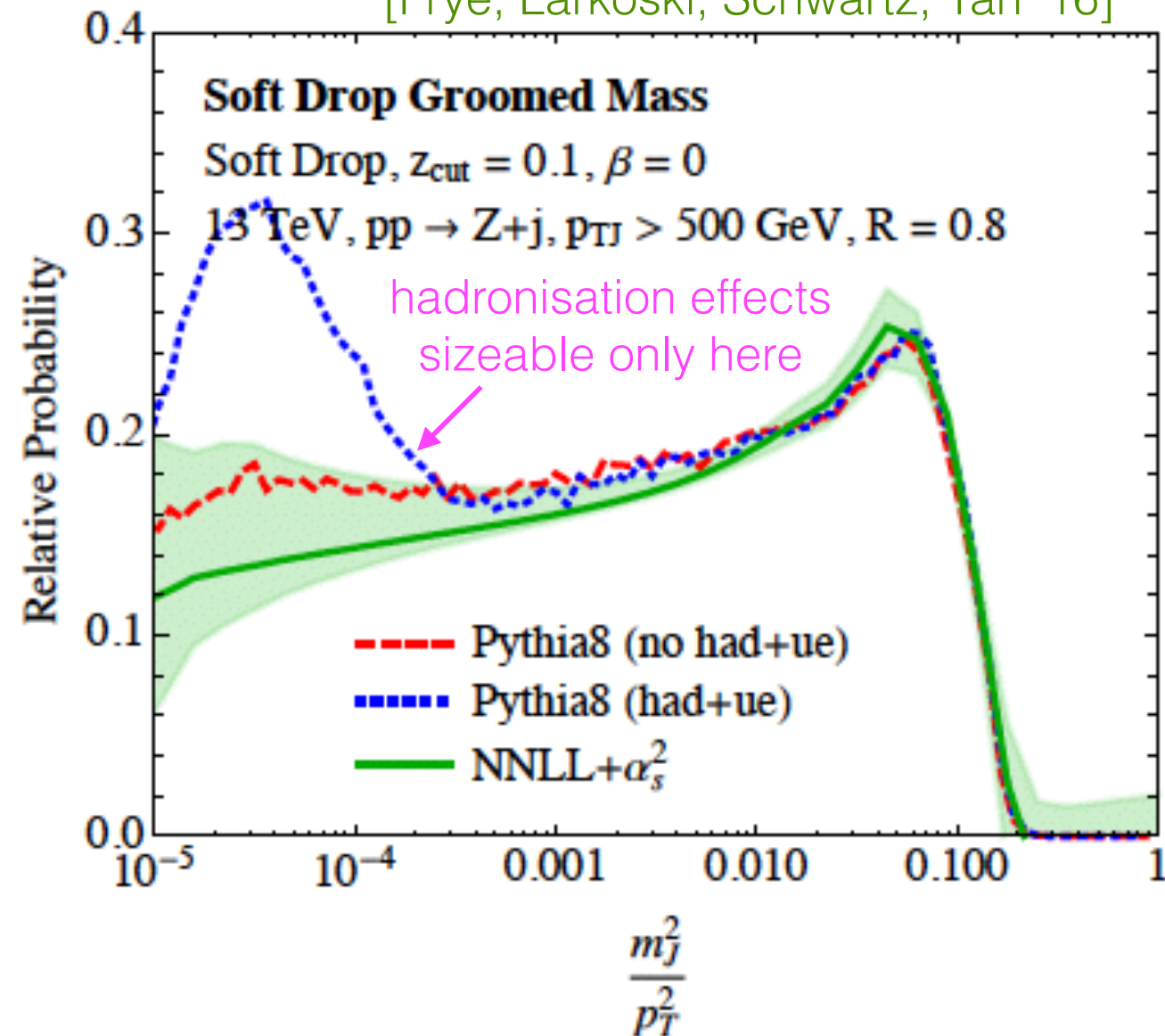
- All order treatment of the jet-mass logarithms reduces to a **process-independent** collinear jet function

[Frye, Larkoski, Schwartz, Yan '16]

Computed at N(N)LL in the regime

$$m_J^2/p_{t,J}^2 \ll z_{\text{cut}} \ll 1$$

Nice example of how an observable can undergo surgery to improve its PT behaviour



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- This can be used to resurrect some observables overwhelmed by PU/UE effects. Useful for extractions of the strong coupling ?

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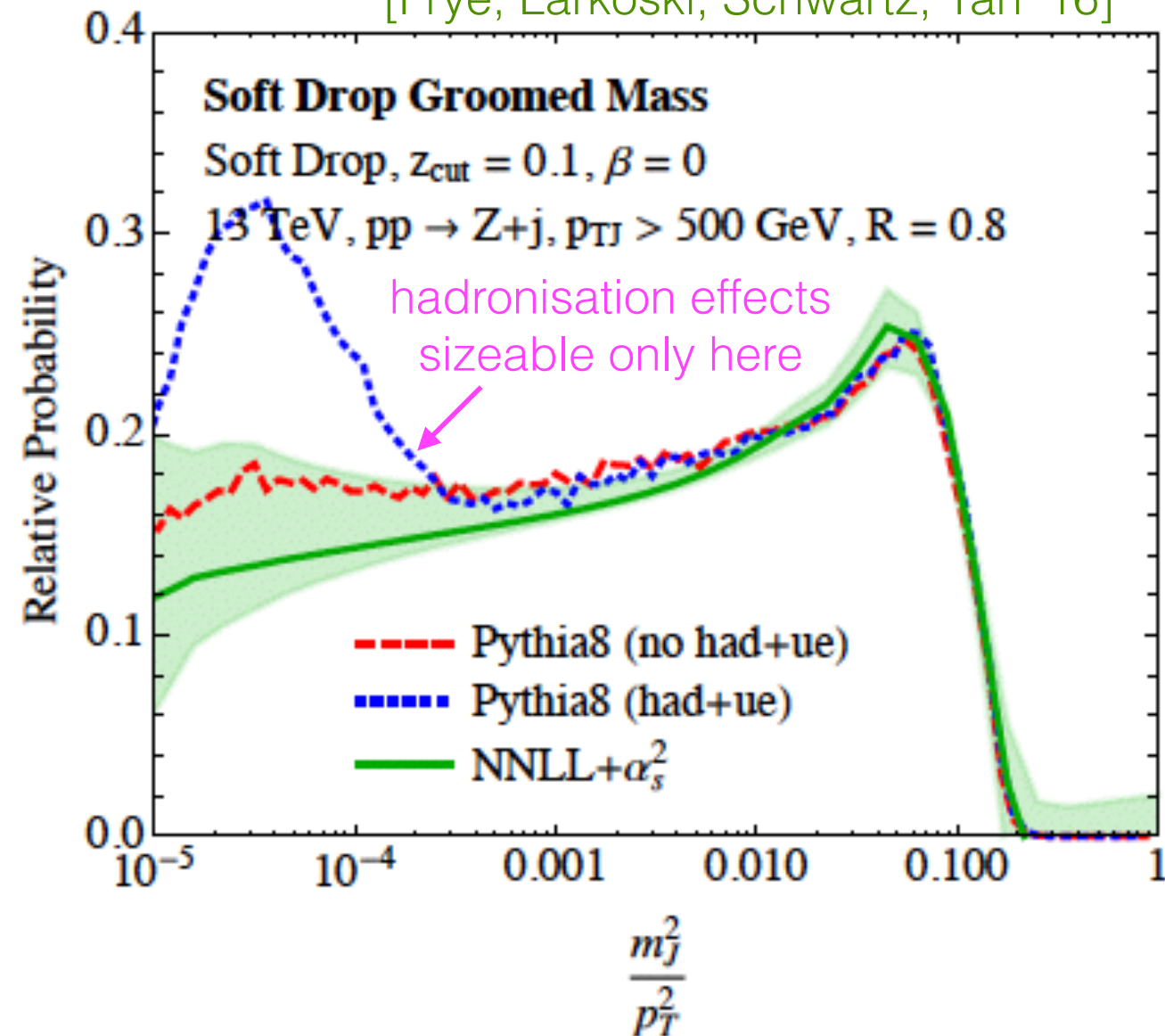
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Jet-radius logarithms

- small-R jets have received some attention lately

LL resummation with generating functionals

[Dasgupta, Dreyer, Salam, Soyez '14 - '16]

Formulation in SCET: [Chien, Hornig, Lee '15]

[Kolodrubetz, Pietrulewicz, Stewart, Tackmann, Waalewijn '16]

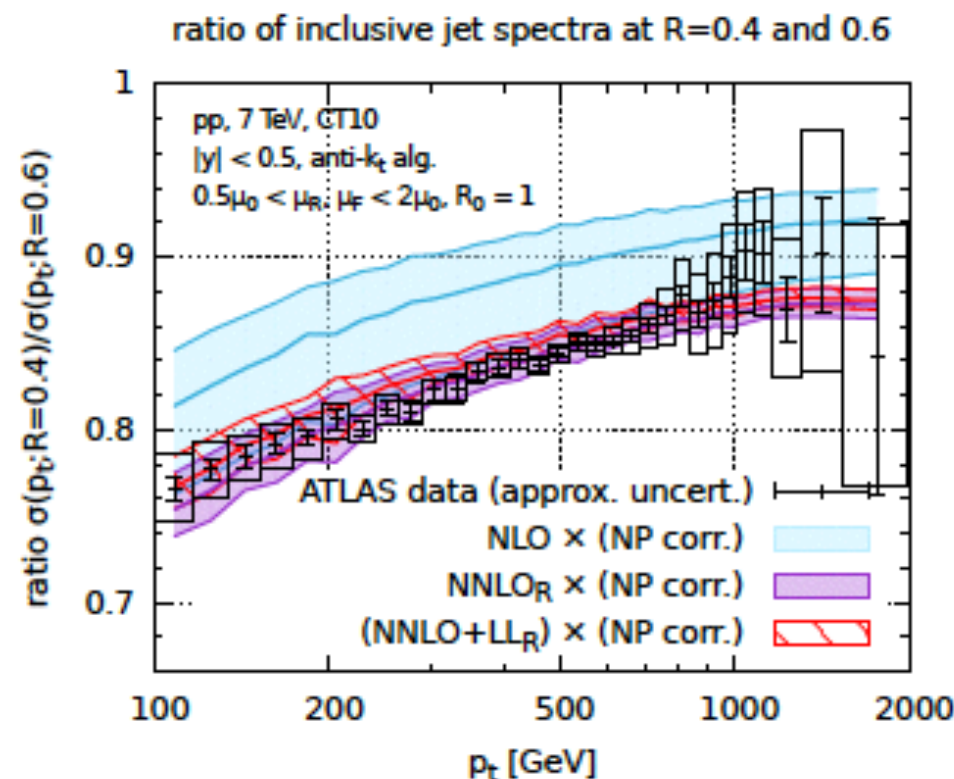
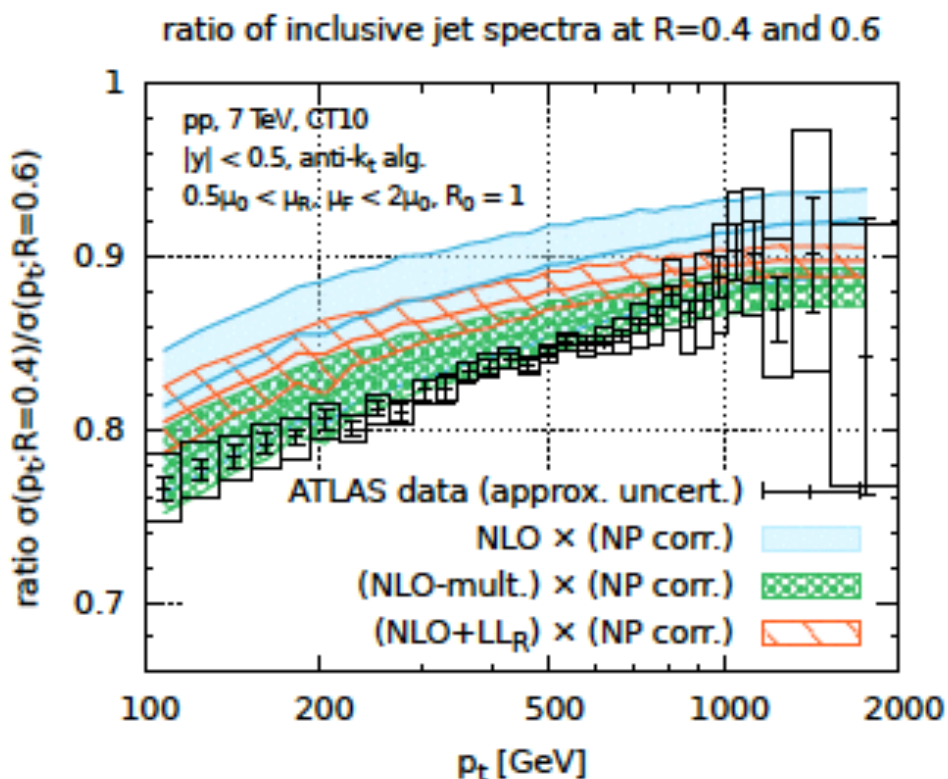
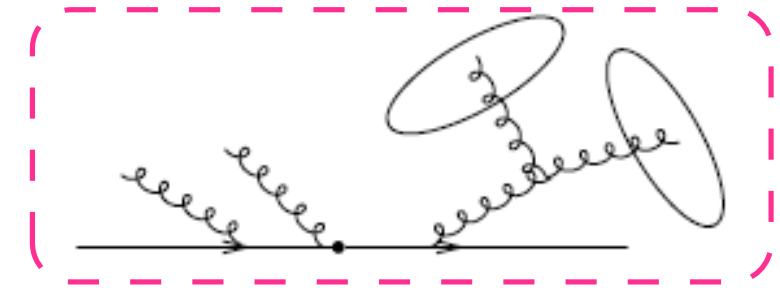
[Kang, Ringer, Vitev '16]

- All-order effects can become relevant when $R \sim 0.2-0.3$ or smaller are employed (heavy ions, substructure, jet-rates studies,...)

e.g. inclusive jet spectrum

[Dasgupta, Dreyer, Salam, Soyez '16]

[Kang, Ringer, Vitev '16]



Full NNLO
 corrections necessary

[Currie, Gehrmann-De Ridder,
 Gehrmann, Glover, Pires '13]

-> Talk by J. Currie

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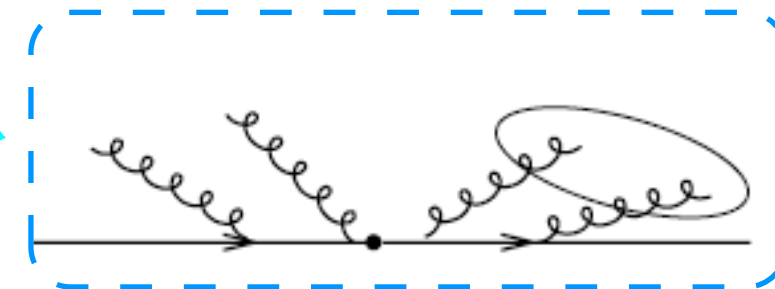
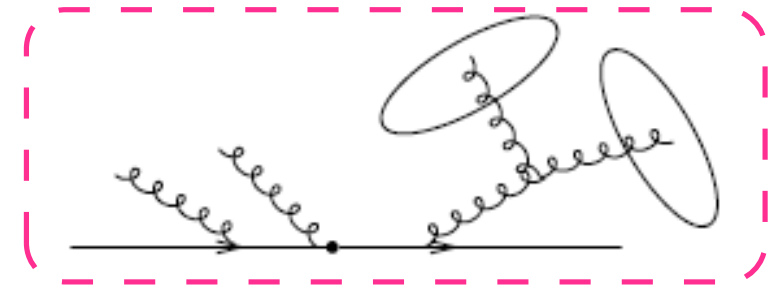
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- • Measurements at multiple R values powerful handle to modulate hadronisation/PT effects

[Dasgupta, Magnea, Salam '08]

$$\text{had} \sim -\frac{1}{R}, \quad \text{UE} \sim R^2, \quad \text{PT} \sim \ln \frac{1}{R}$$

PT also has a R^2 contribution



Higgs physics - 0 jet cross section

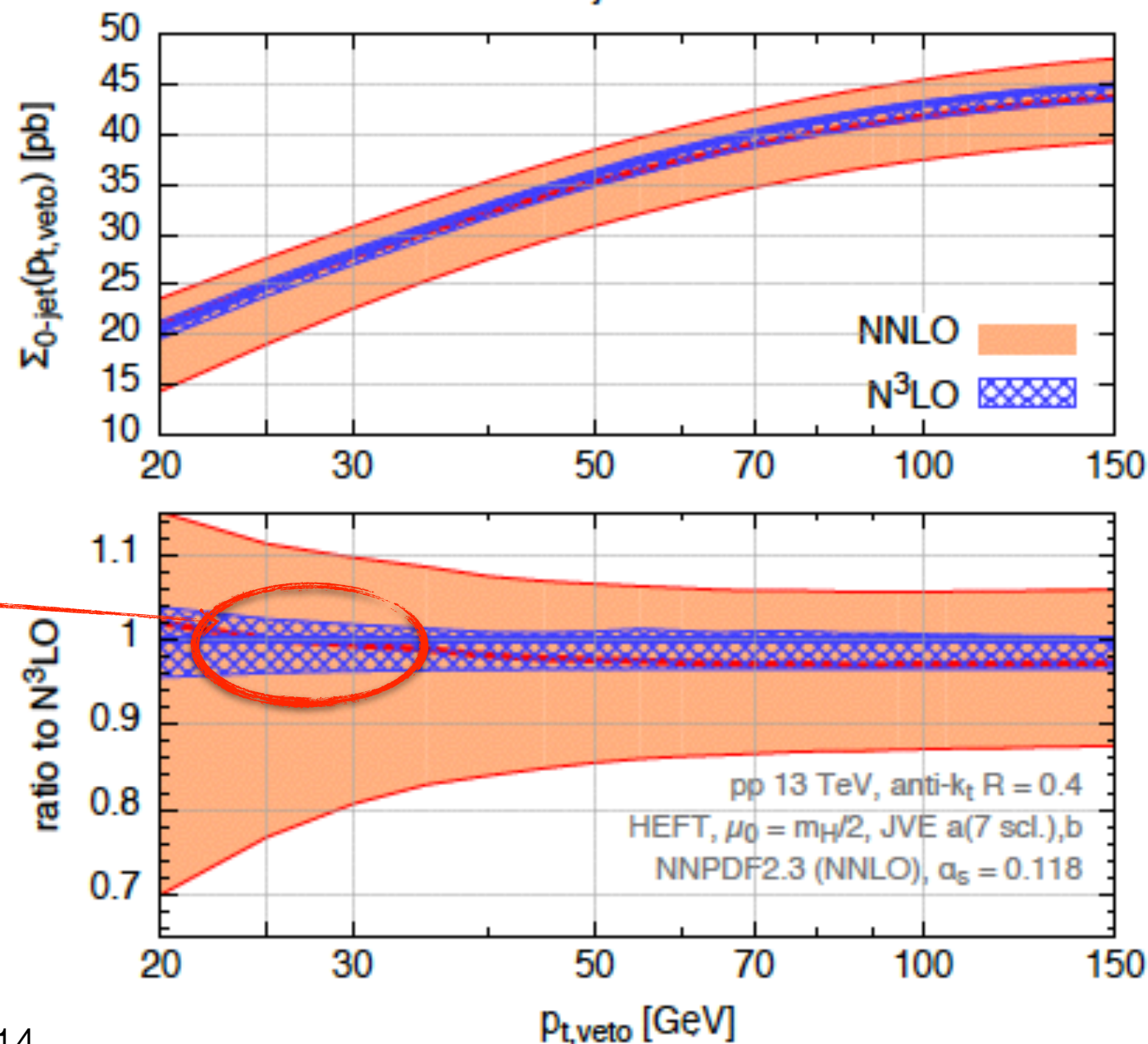
-> Talk by L. Rothen

- Important to understand (a priori) where exactly resummation and fixed-order are reliable (and estimate the matching uncertainty)

e.g. 0-jet cross section at N3LO+NNLL+LLR

[Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger '16]
[Boughezal, Caola, Melnikov, Petriello, Schulze '15]
[Banfi, Caola, Dreyer, PM, Salam, Zanderighi, Dulat '15]

N³LO v. NNLO jet veto cross section



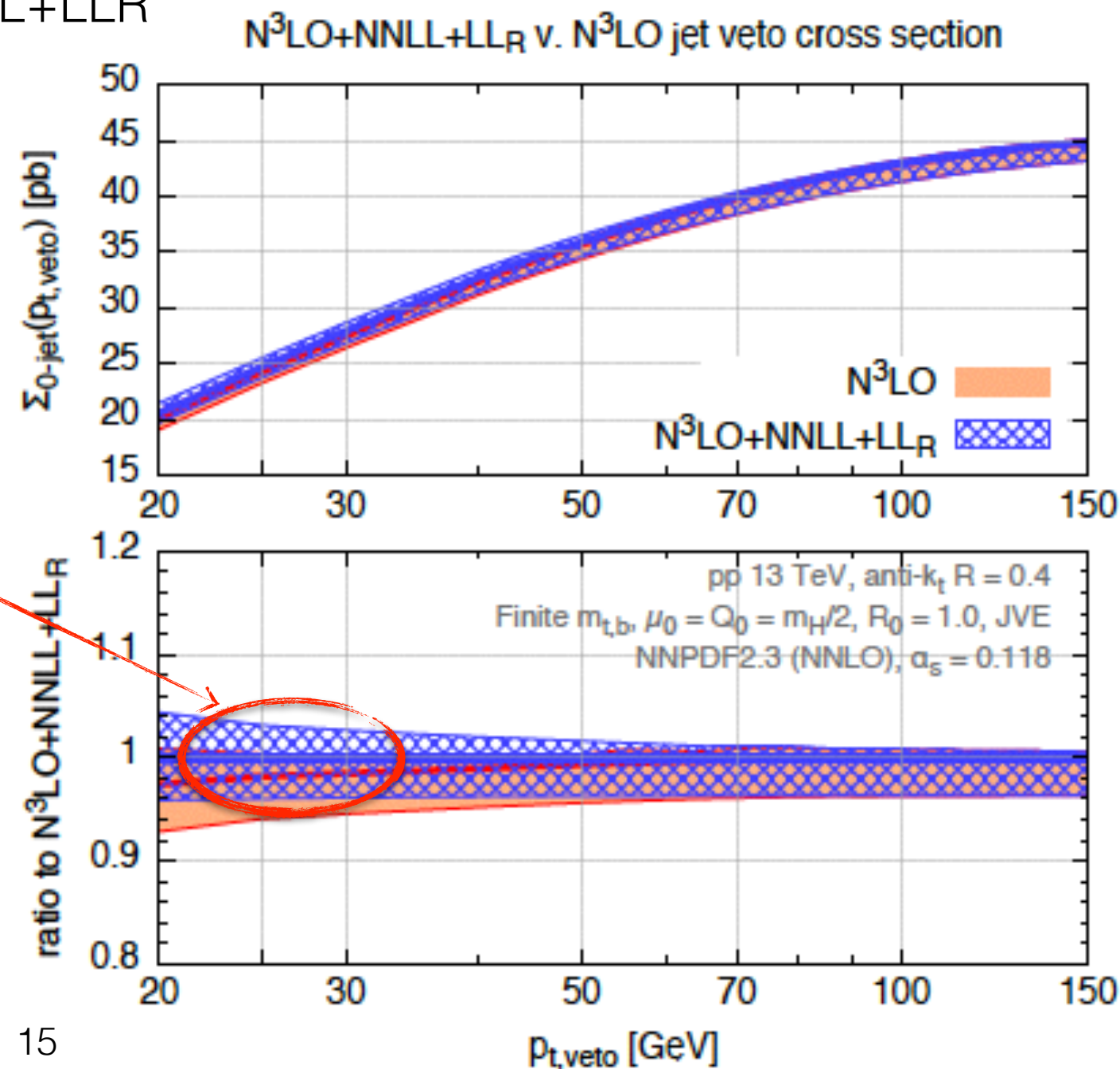
at $p_t \sim 25\text{-}30$ GeV
N3LO (pure fixed order) corrections
have a 1-2% impact
(this varies with central scales)

Higgs physics - 0 jet cross section

- Important to understand (a priori) where exactly resummation and fixed-order are reliable (and estimate the matching uncertainty)

e.g. 0-jet cross section at N3LO+NNLL+LLR

Impact of the resummation
is of the same order ($\sim 2\%$).
How accurate is this statement?



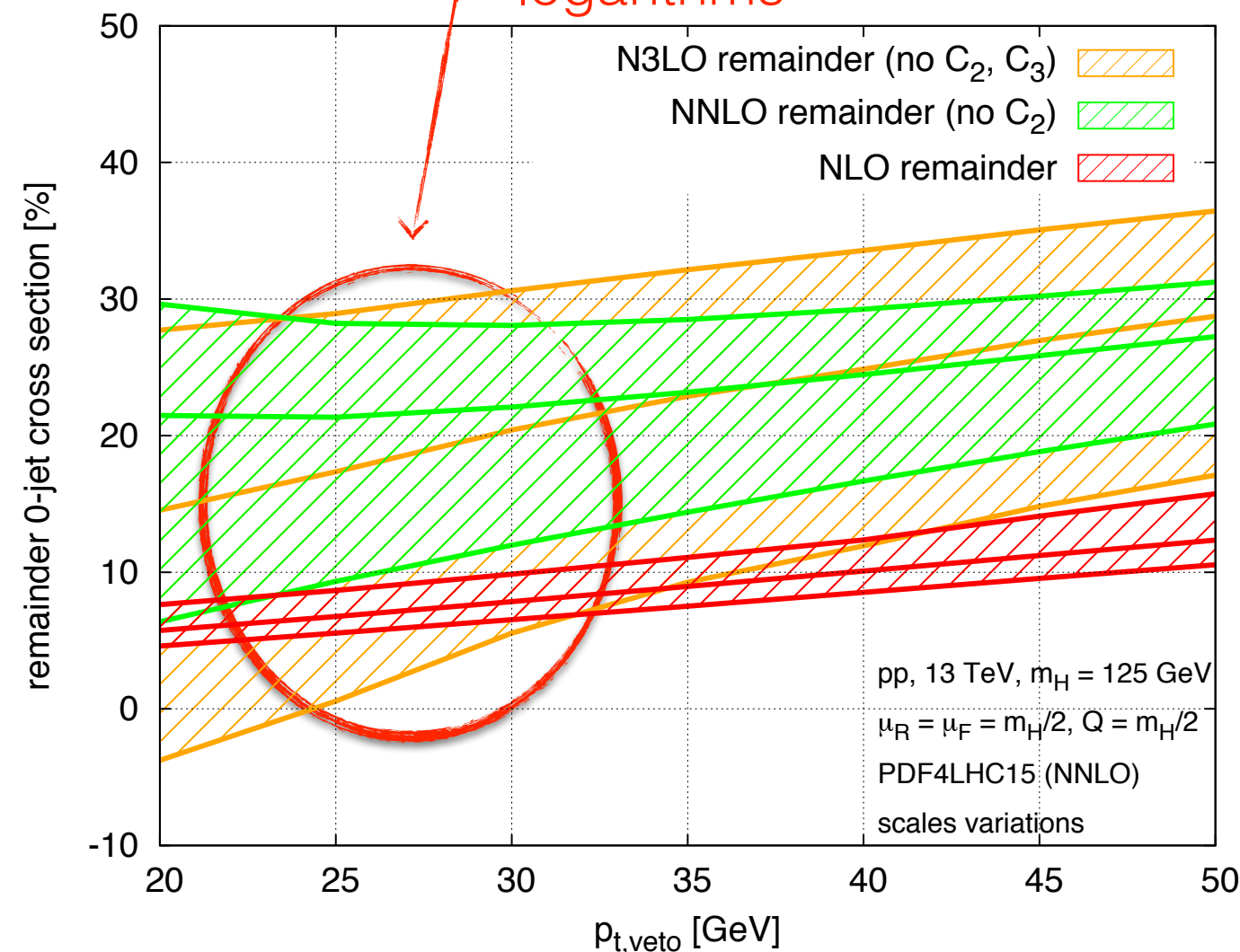
Higgs physics - 0 jet cross section

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e.g. 0-jet cross section at N3LO+NNLL+LLR

$p_t \sim 25\text{-}30$ GeV is a transition region where logarithms are the dominant part of the perturbative expansion, although fixed-order still works fine (i.e. the coupling suppression is still effective)
Resummation effects seem physical.

$\sim 70\text{-}80\%$ of the perturbative expansion is made of logarithms



Higgs physics - p_T distribution

-> see also talks by H. Sargsyan and M. Wiesemann

Currently known at NNLL+NNLO in HEFT:

N3LO tot: [Anastasiou, Duhr, Dulat, Herzog, Mistlberger '15 + Furlan, Gehrmann, Lazopoulos '16]

NNLO: [Boughezal, Caola, Melnikov, Petriello, Schulze '15; Caola, Melnikov, Schulze '15

Boughezal, Focke, Giele, Liu, Petriello '15; Chen, Cruz-Martinez, Gehrmann, Glover, Jaquier '16]

NNLL: [Grazzini, de Florian '01; Bozzi, Catani, de Florian, Grazzini '03; Becher, Neubert '10;

Neill, Rothstein, Vaidya '15; PM, Re, Torrielli '16]

Higgs physics - p_T distribution

Currently known at NNLL+NNLO in HEFT:

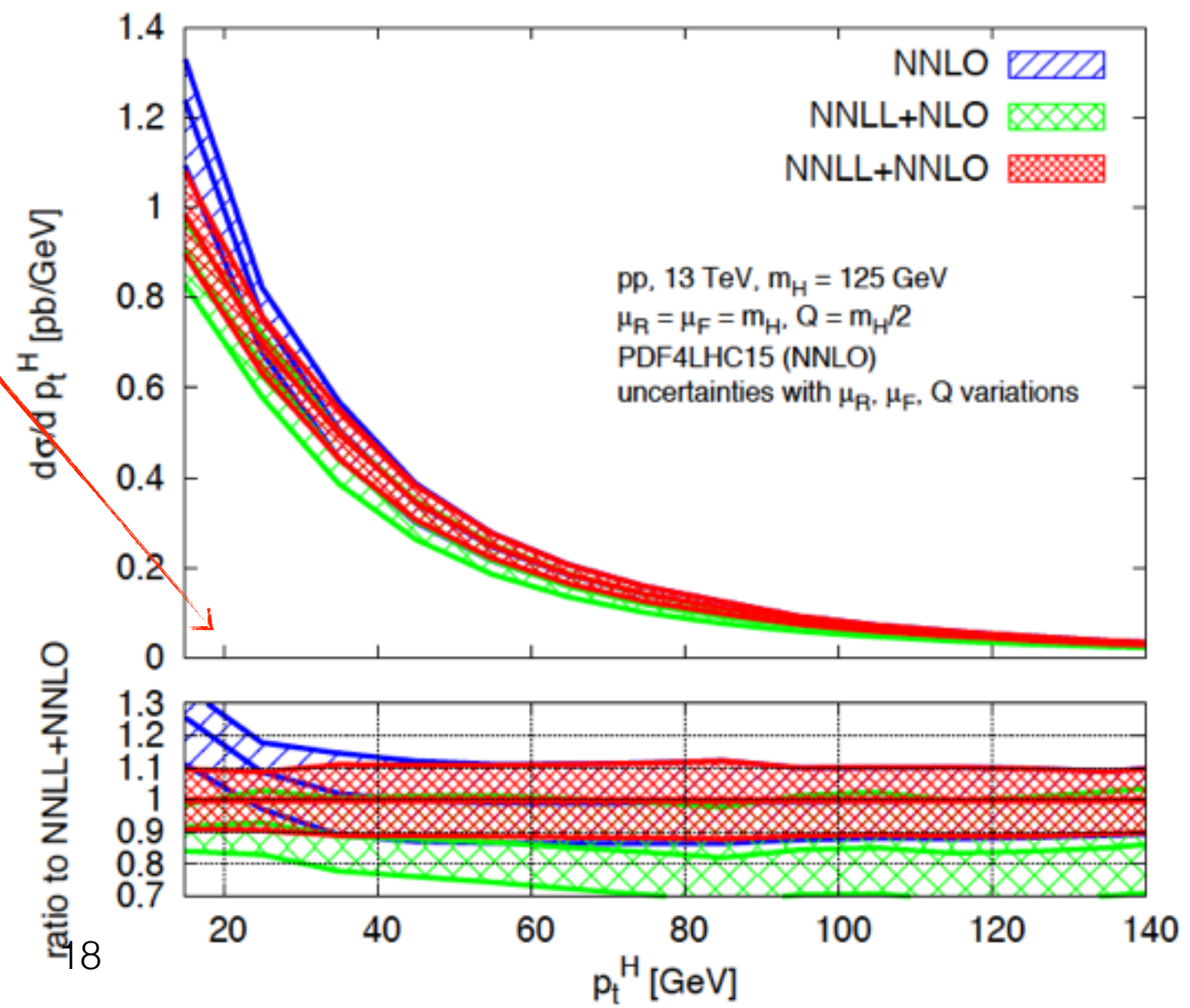
- Interesting example (one of many) of observable with zeros away from the Sudakov limit (two kinematic mechanisms competing in the limit $p_T \rightarrow 0$)
 - resummation if rIRC safe [PM, Re, Torrielli '16]
 - new handle on joint resummations and Sudakov shoulders

Resummation relevant below
 $p_T \sim 40$ GeV - th. uncertainties $\sim 10\%$

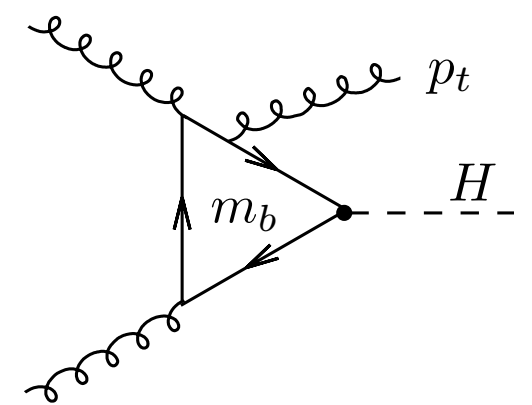
Many effects beyond this point:

- Luminosity uncertainties estimated to be $\sim 3\%$
- strong coupling unc. $\sim 2\%$
- quark masses (known at LO) $\sim 5\text{-}6\%$ in this region
- hadronisation $\sim 2\text{-}4\%$
- N3LL+NNLO on its way : little effect on central values, but theory uncertainties halved ($\sim 5\%$). We should start worrying about other effects

[Li, Zhu '16]



Quark masses



When the full theory is considered the bottom-quark amplitudes are enhanced by (regular) logarithms of the ratio p_t/m_b in the region $m_b^2 \ll p_t^2 \ll m_H^2$

- Subject of discussion in the past years: what's their impact at HO ? Should they be resummed ?
- Amplitude DL resummed in the abelian limit $\sim C f^n \alpha_s^n L^{2n}$ [Melnikov, Penin '16]
e.g.

$$\rightarrow A_{++\pm} = \pm 2 \ln \frac{m_b^2}{m_H^2} \int_0^{1-\tau_t} \frac{1 - e^{-x\eta(1-\eta)}}{x\eta} d\eta, \quad \tau_t \sim \ln \frac{m_b^2}{p_t^2} / \ln \frac{m_b^2}{m_H^2}$$

Corrections in the abelian limit beyond LO are moderate: at two loops $\sim 2\%$ of which only 0.2% is p_t dependent (strong cancellations) - an order of magnitude smaller at 3 loops...

→ Full NLO result important for %-level theory,
all-order corrections expected to remain moderate

Quark masses

The problem can be approached also from the high-energy limit (small-x)

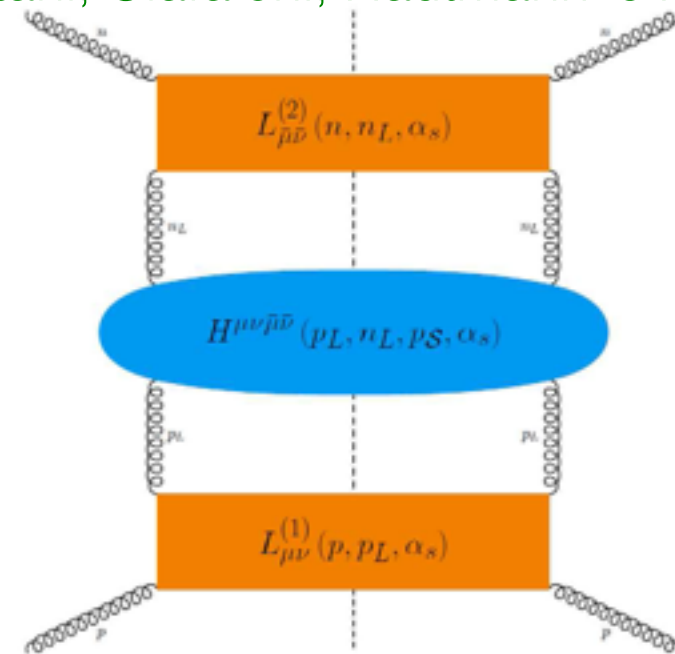
[Catani, Ciafaloni, Hautmann '91]

- LLx -> ladder of gluons strongly ordered in $k_t \sim \alpha_s^n$
 $z \ll 1, k_t^2 \ll s$

- One can be differential in the higgs rap. and p_t
 [Caola, Forte, Marzani '11] [Forte, Muselli '15]

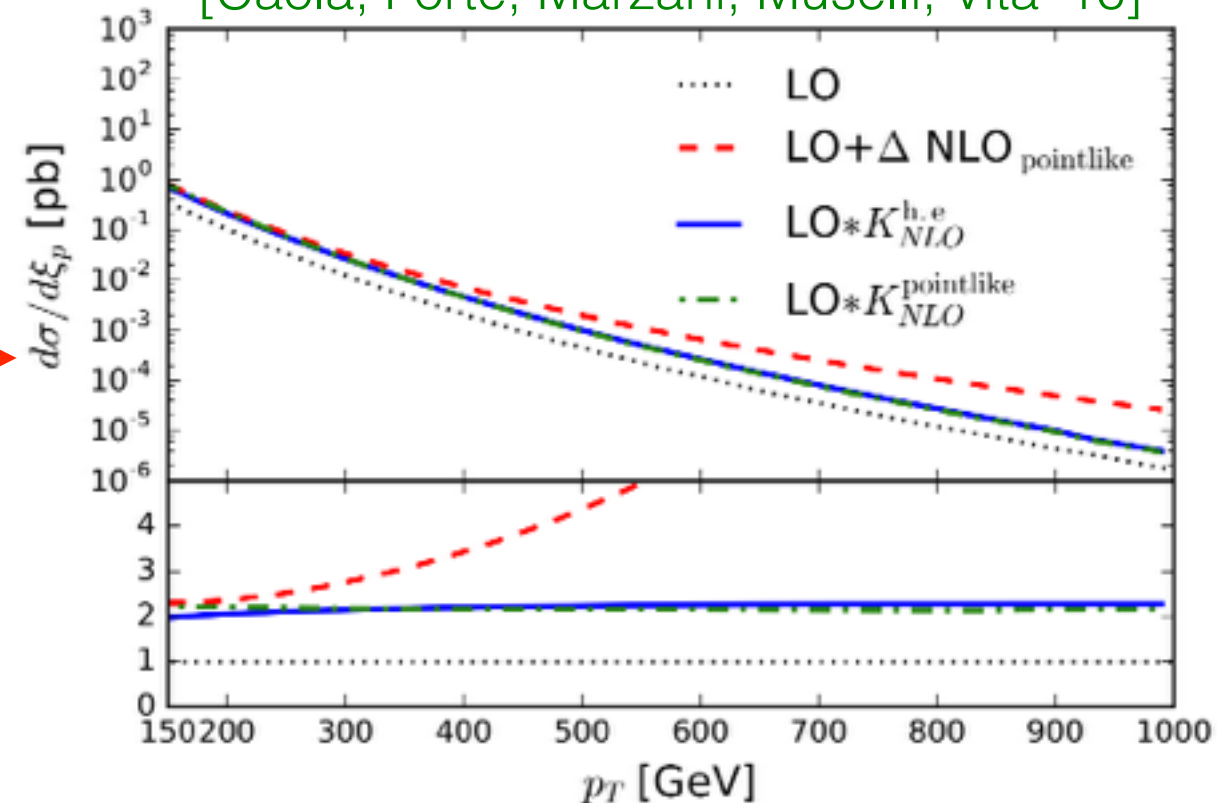
Due to strong k_t ordering only one gluon at a time probes the bottom loop, so it's only sensitive to $\ln^2(p_t/m_b)$ (amplitude level) at all orders. Plausible that subleading small-x terms carry more information

→ impact small within this approximation



[Caola, Forte, Marzani, Muselli, Vita '16]

HE approx works well for $x < 0.5 - 0.6$
 This allows one to derive an estimate for the NLO corrections in the high- p_t tail (top loop) →



Formulation for joint small-x/ p_t resummation
 also obtained recently [Marzani '16]

Quark masses - Yukawa couplings

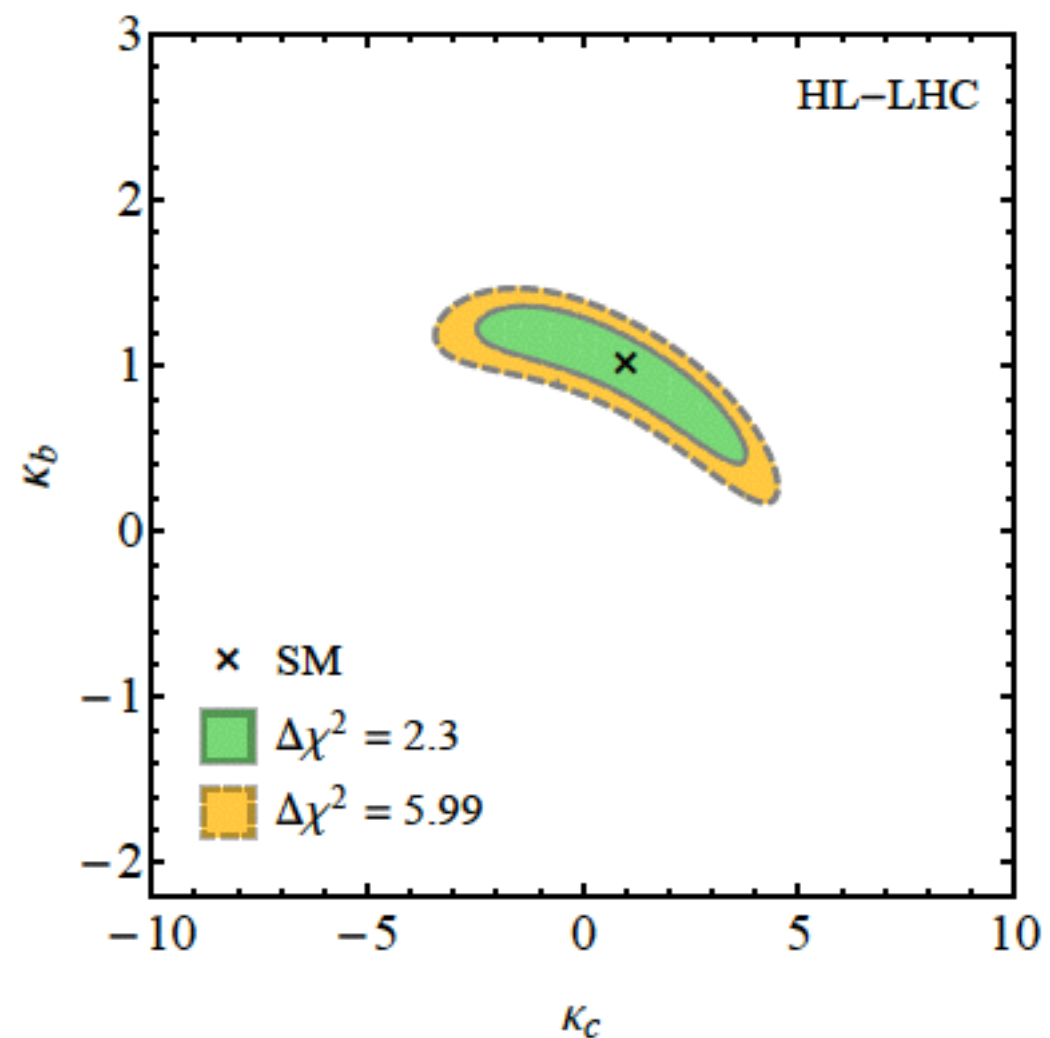
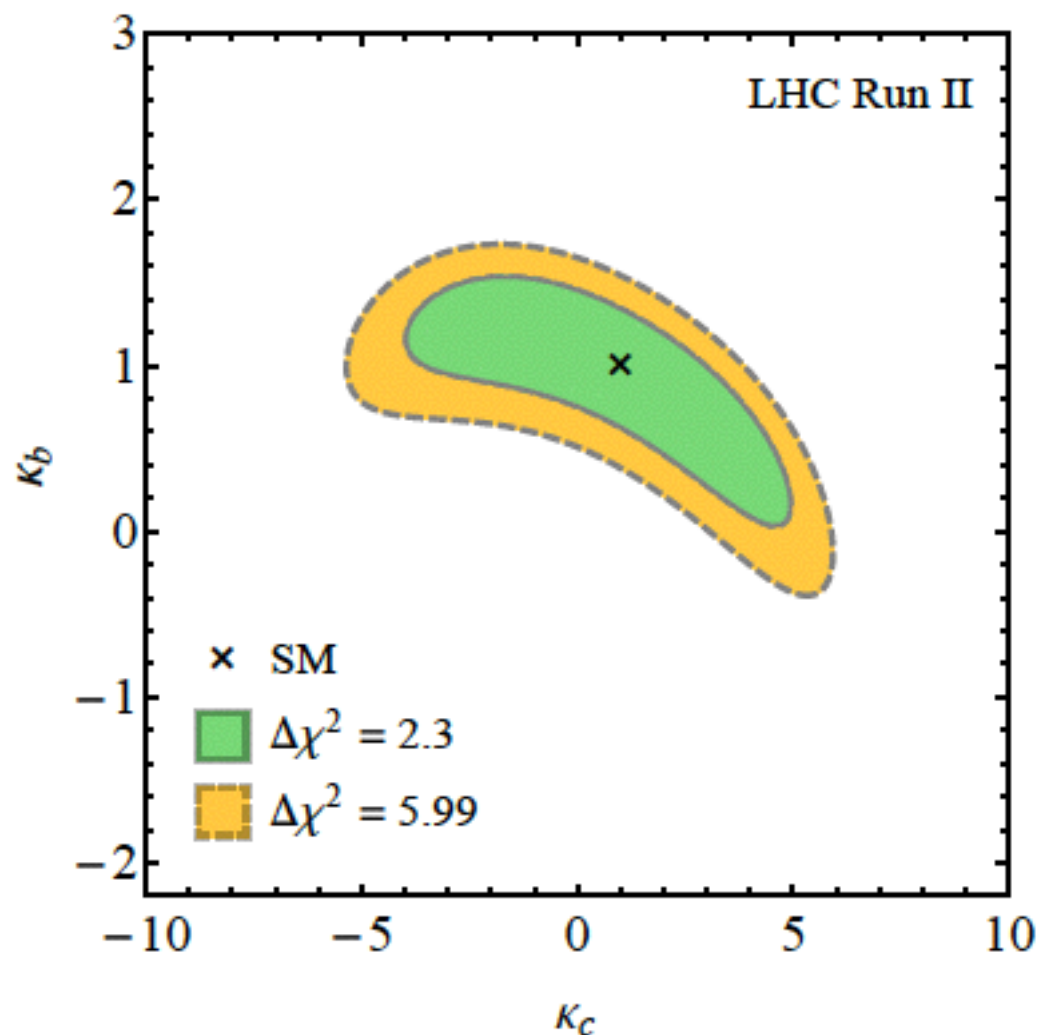
Although moderate in the SM, all-order effects can be useful when studying constraints on the light-quark Yukawa couplings using transverse momentum distributions of the Higgs and recoiling jets \rightarrow few % precision required

[Bishara, Haisch, PM, Re '16]

[Soreq, Zhu, Zupan '16]

[Bonner, Logan '16]

e.g. bottom and charm Yukawas



Z+jet

Astonishing precision in the Z pt spectrum (and related observables)

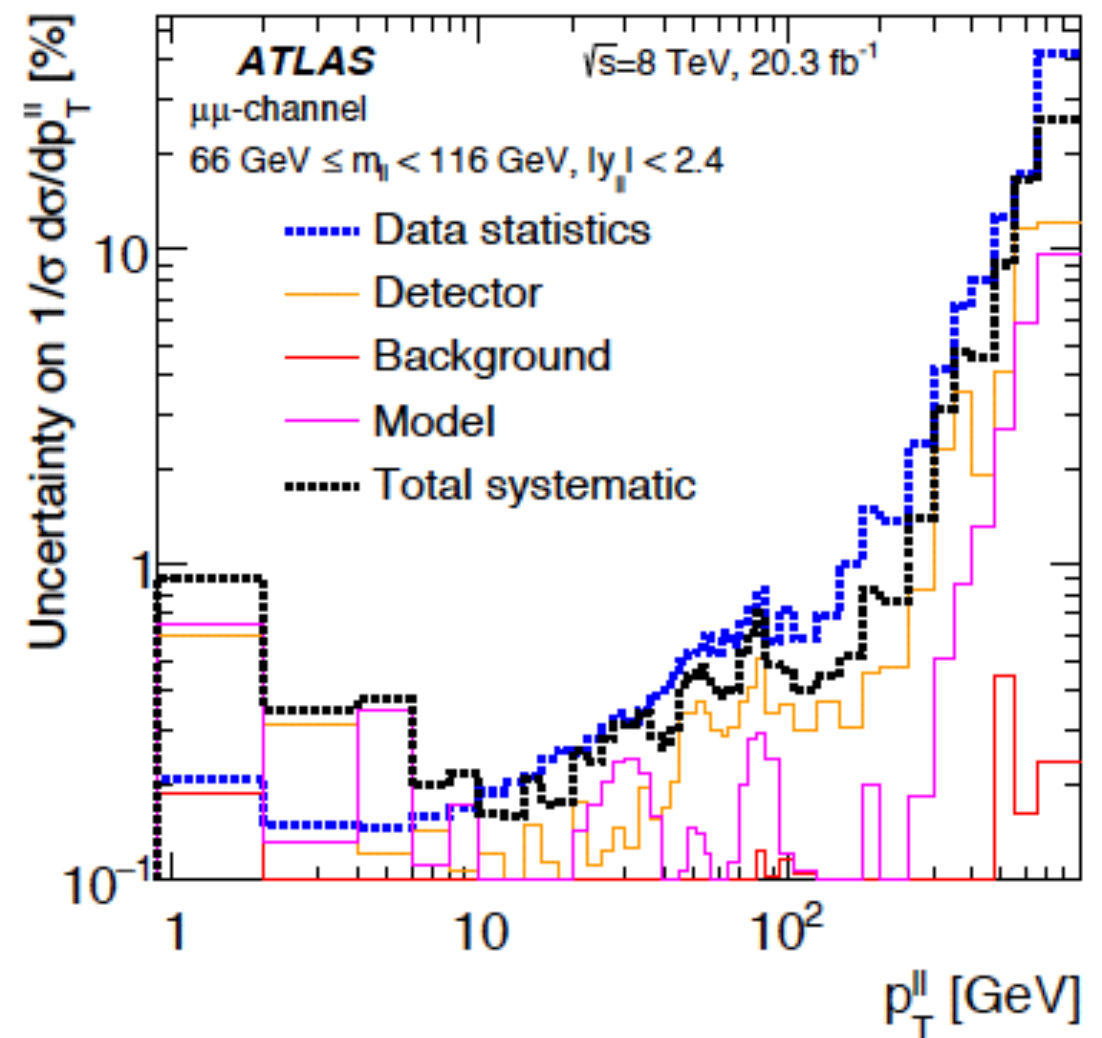
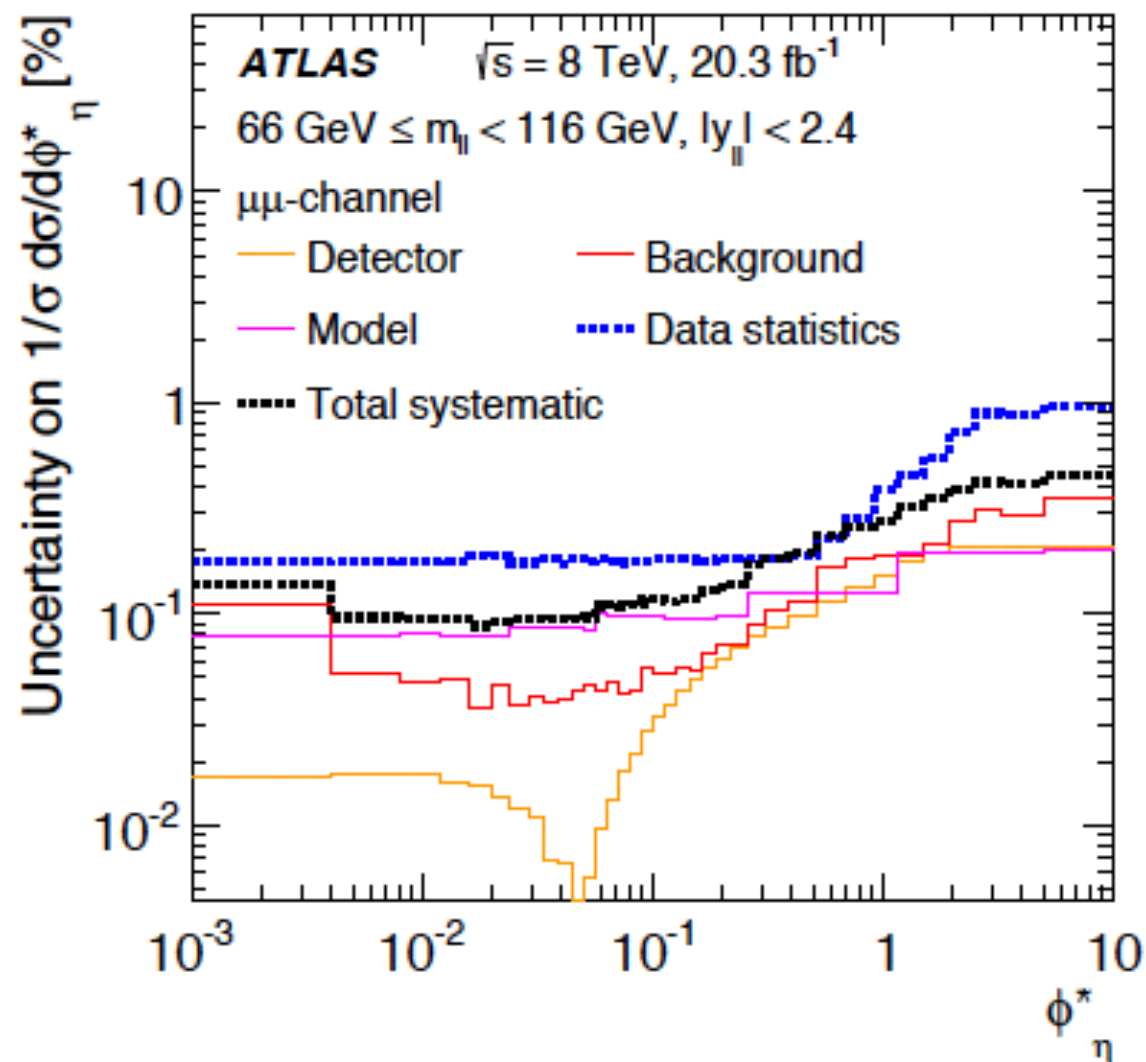
- $\sim 1\%$ uncertainty down to $p_T \sim 1$ GeV - an order of magnitude smaller for ϕ^* !
- modest discrepancy at $p_T > 30$ GeV with NNLO
- consider normalised fiducial distribution

-> Talk by A. Huss

[Gehrmann-De Ridder et al. '16]

[Boughezal et al. '16]

[ATLAS 1512.02192]



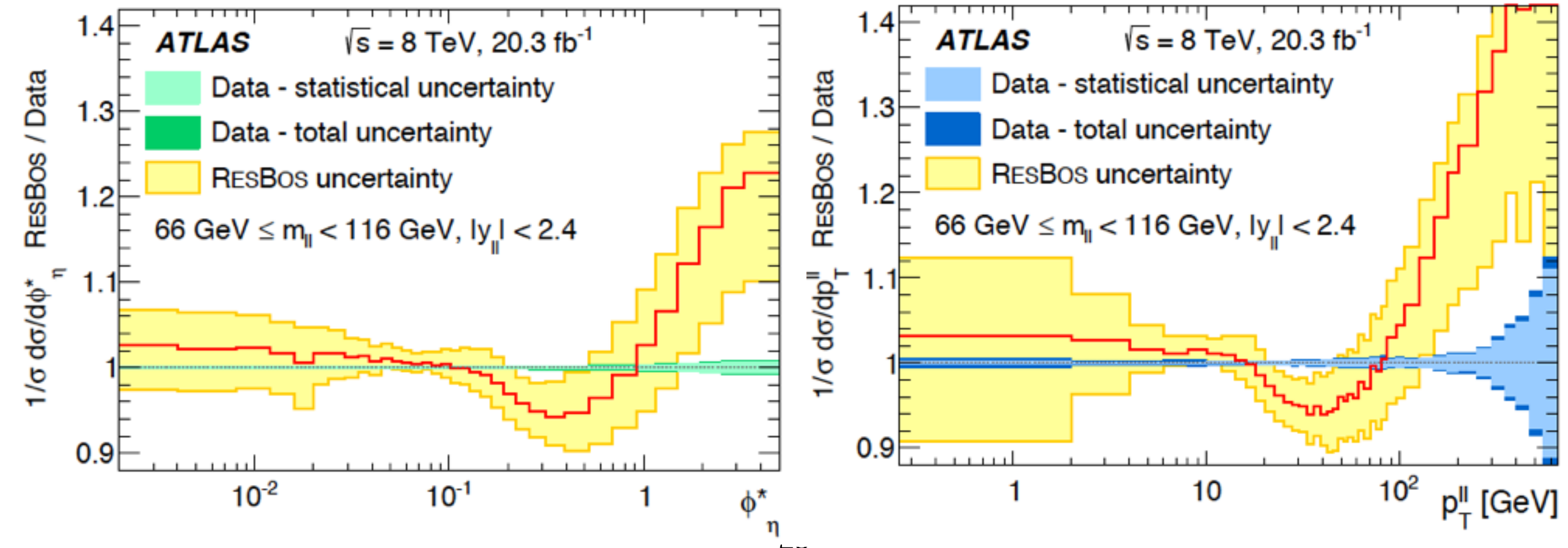
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- lack of theory at small p_T - N3LL already necessary below 10 GeV

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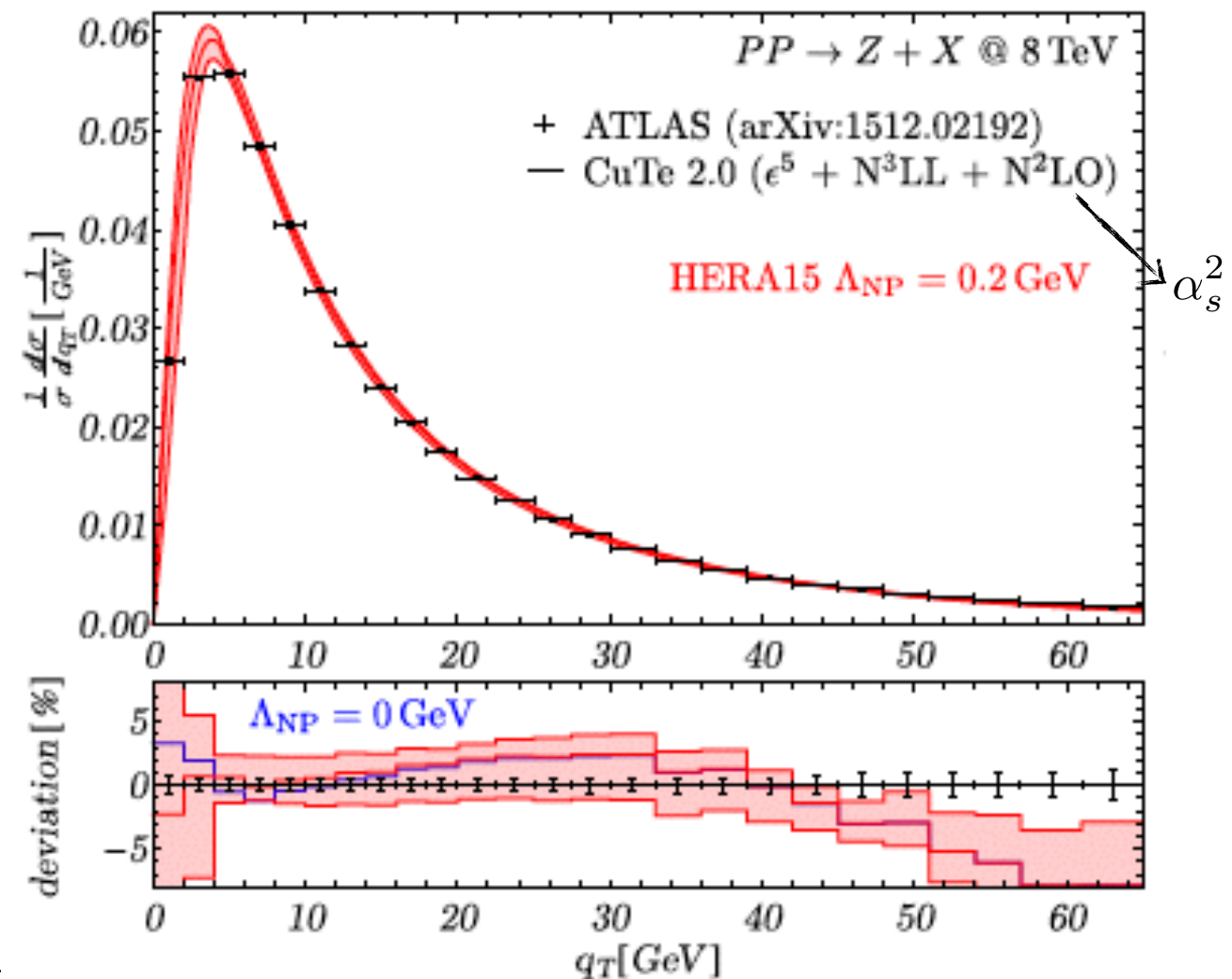
T. Luebbert's talk at SCET 2016

e.g. approx N3LL+NLO p_T distribution give a residual 2-3% uncertainty down to ~ 4 GeV (potentially better for ϕ^*)

Can we use this data to learn something on running coupling or NP effects (some discrepancies between MCs and analytic models) ?

$$F^{(3,0)} = f_F(4\pi)F^{(2,0)}$$

now computed exactly [Li, Zhu '16]



Strong coupling constant

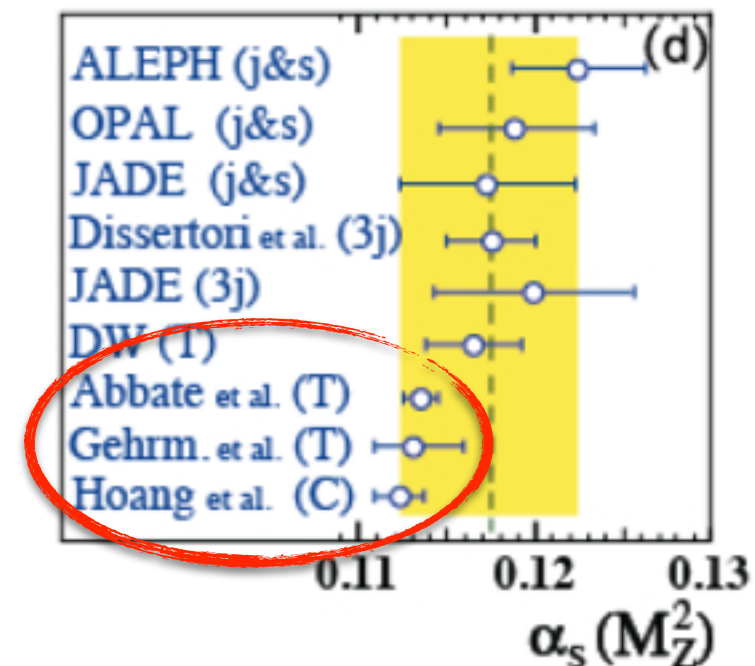
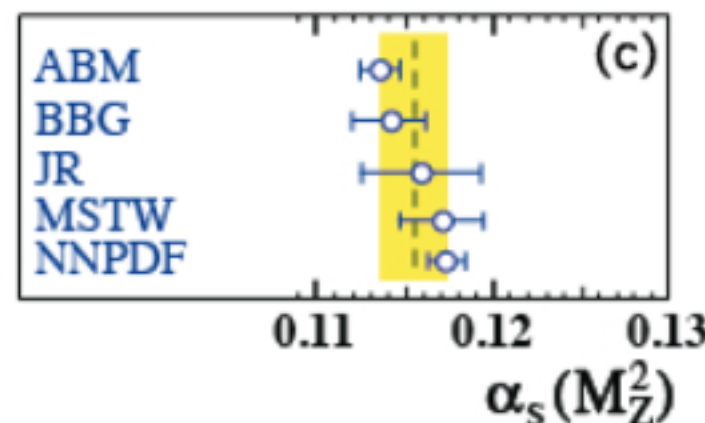
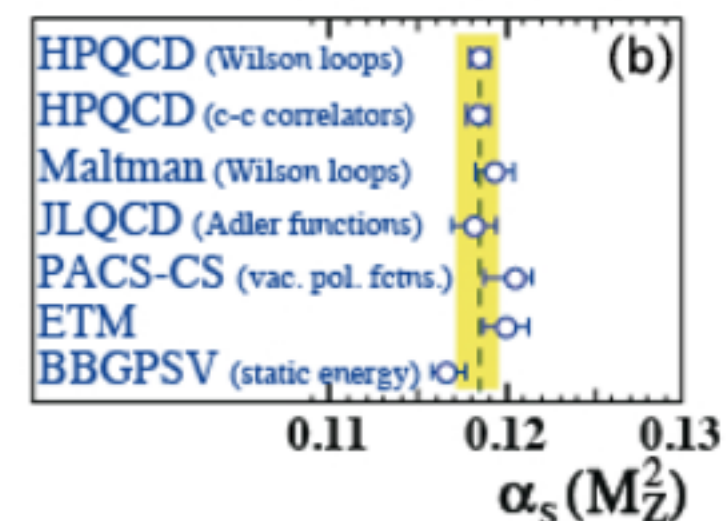
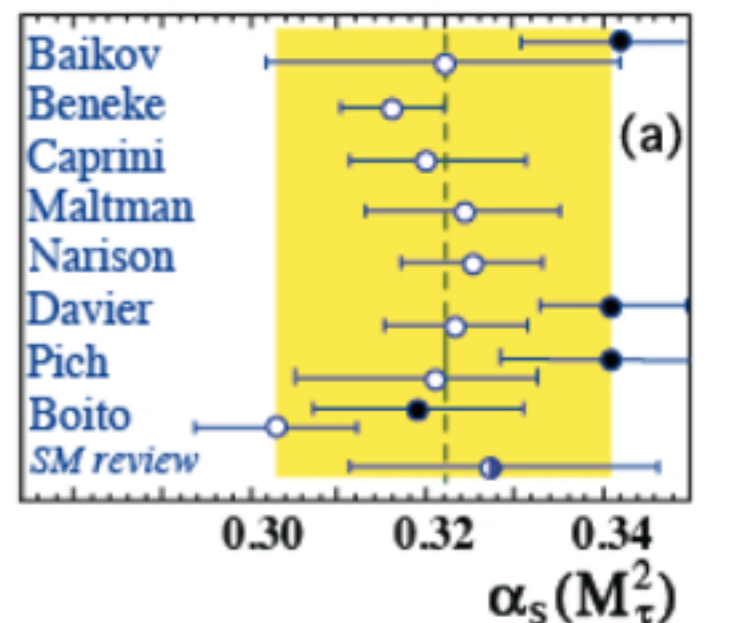
Tension between NNLL (N3LL)+NNLO extractions event-shape

World average: [Bethke, Salam, Dissertori '15]

$$\alpha_s(M_Z) = 0.1177 \pm 0.0013(1.1\%) \text{ weighted}$$

$$\alpha_s(M_Z) = 0.1181 \pm 0.0013(1.1\%) \text{ unweighted}$$

- Large tension between extractions from NNLL (N3LL)+NNLO event shapes and lattice calculations
- At LEP energies issues with high correlation between perturbative and hadronisation corrections from analytic models
- Thrust and C-parameter very similar (correlated) observables, with very same NP behaviour
- Low values of α_s are disfavoured by some LHC measurements



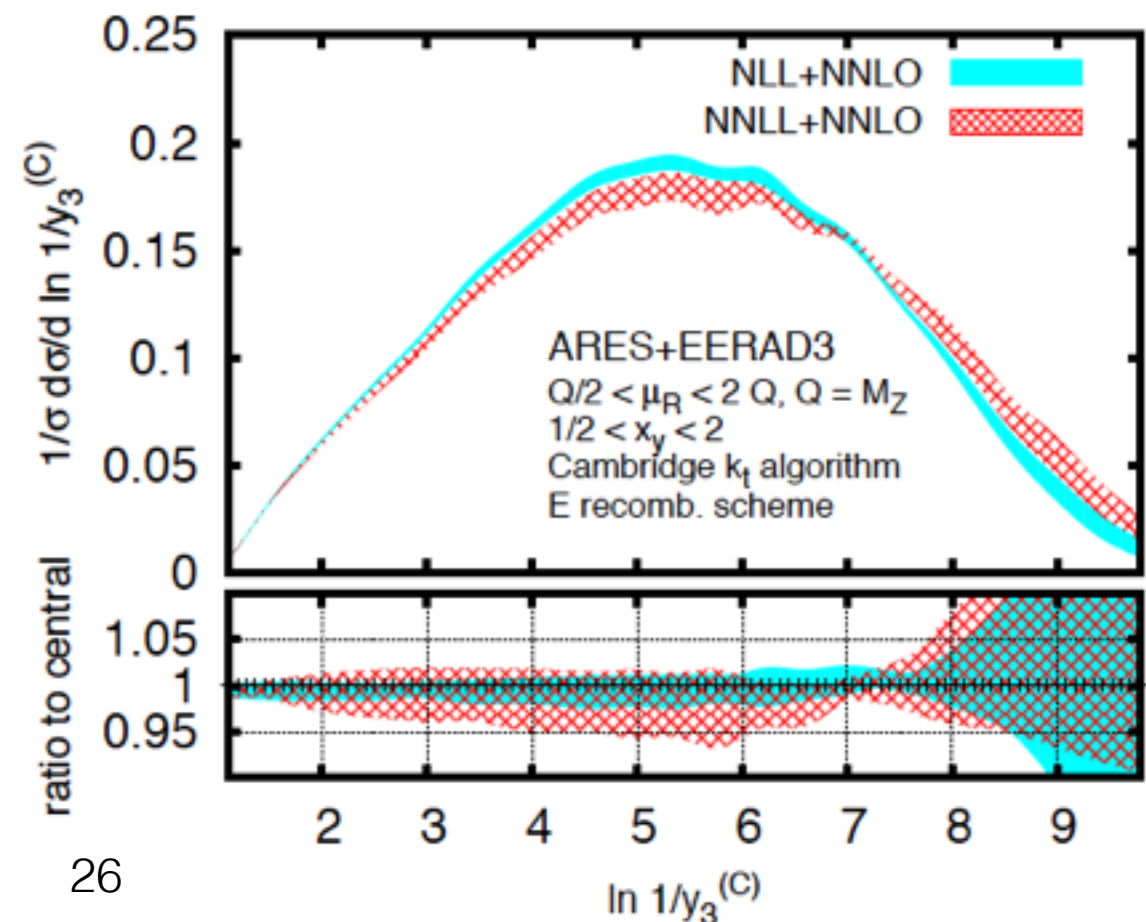
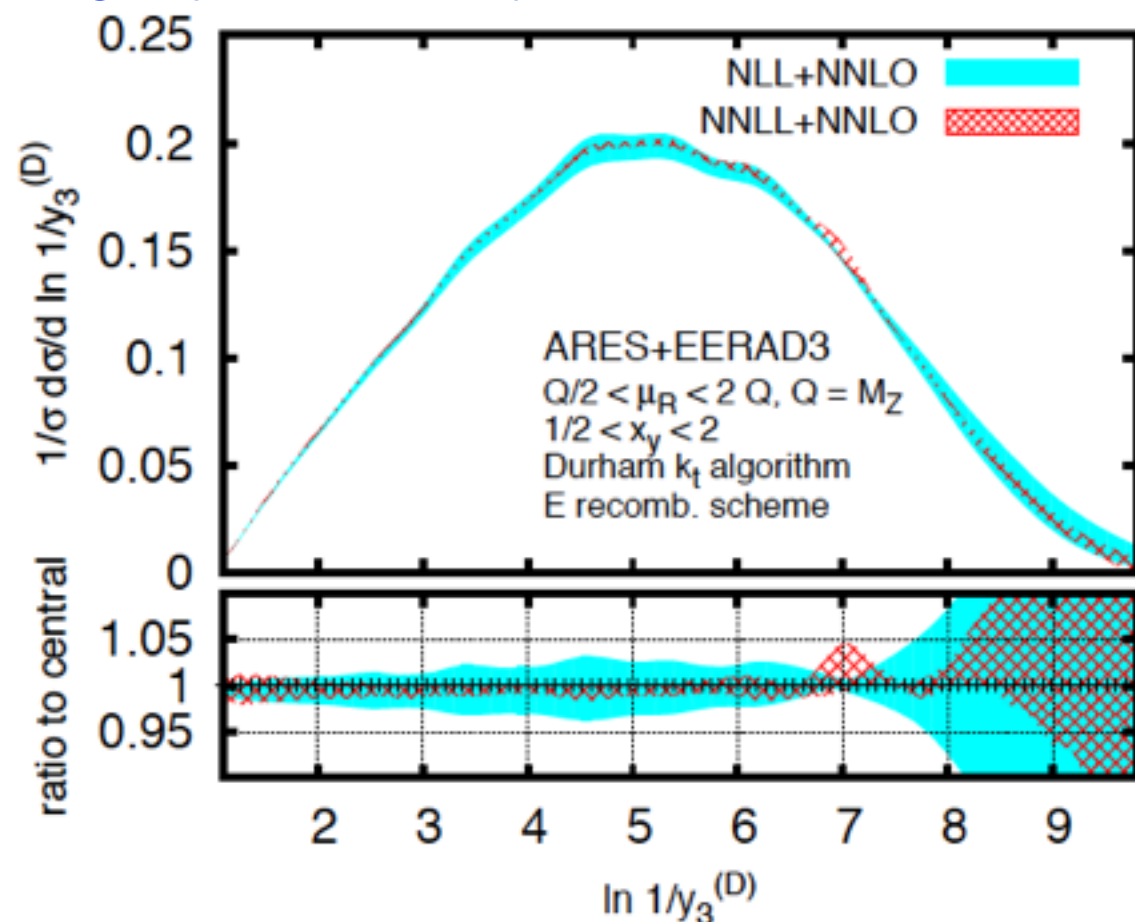
Strong coupling constant

- Important to perform a fit (possibly global) using observables with different (lower if possible) sensitivity to hadronisation effects (e.g. jet broadening, 3-jet resolution) at NNLL+NNLO
- Now this is possible given the advances in the field of the past years - **all LEP 2-jet-like observables are known to this accuracy**

[Gehrmann-De Ridder, Gehrmann, Glover, Heinrich '07]
 [Weinzierl '09] [Del Duca, Duhr, Kardos, Somogyi, Szor,
 Trocsanyi, Tullipant '16]

[Becher, Schwartz '08] [Chien, Schwartz '10]
 [Becher, Bell '12] [Hoang et al. '14]
 [Banfi, McAslan, PM, Zanderighi '14-'16]
 [Bell, Hornig, Lee, Talbert '16 (in progress)]

e.g. 3-jet resolution parameter



Strong coupling constant

- Important to perform a fit (possibly global) using observables with different (lower if possible) sensitivity to hadronisation effects (e.g. jet broadening, 3-jet resolution) at NNLL+NNLO
- Now this is a topic of discussion for the next few years - **all**

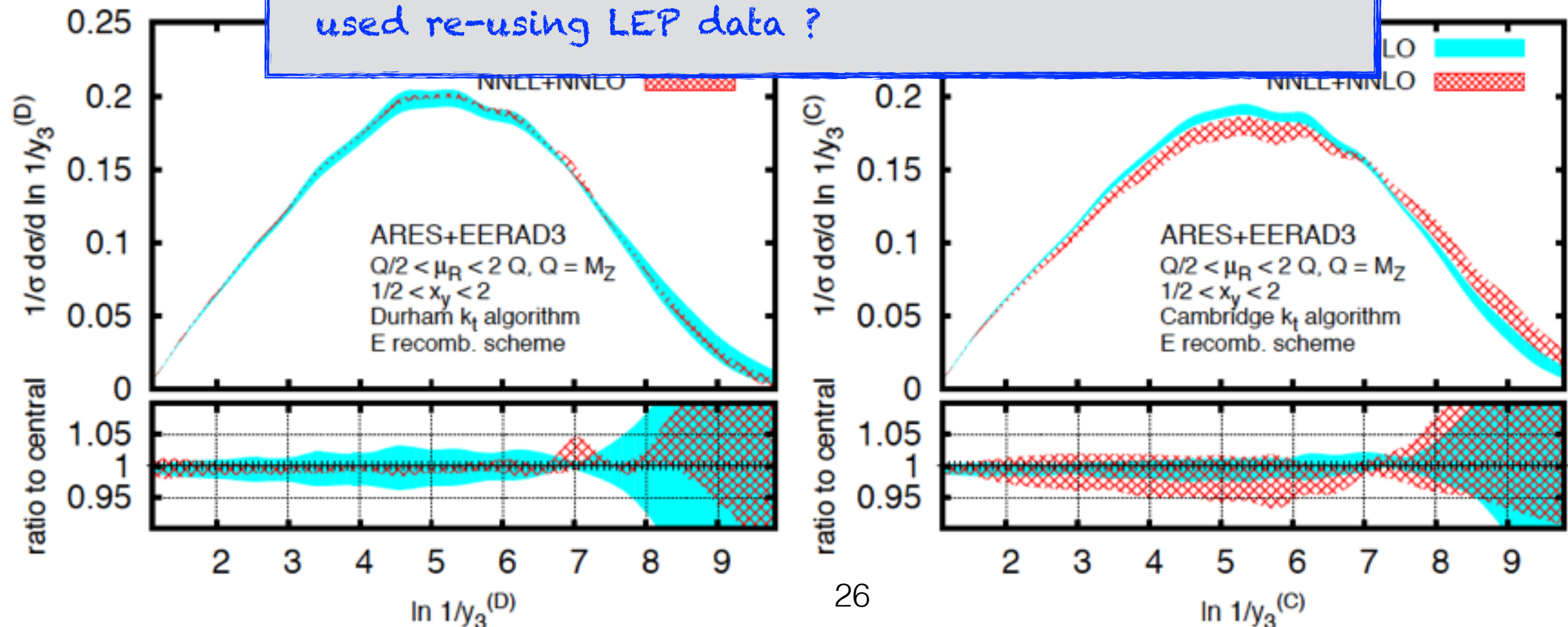
LEP 2-j

- Would a global fit with all observables lead to any improvement? \rightarrow new modelling of hadronisation (new tunes?)/limit ourselves to high-energy data?
- New observables (jet algorithms, substructure) to be used re-using LEP data?

[Gehrmann-De Ridder
[Weinzierl '09] [Del D
Trocsanyi, Tullipant '1

Chien, Schwartz '10]
g et al. '14]
nderighi '14-'16]
t '16 (in progress)]

e.g. 3-jet resolution



Conclusions

- With the precision foreseen at the LHC we often need control over few-% effects
 - With higher precision all-order effects can often be relevant even when cuts on radiation are applied to mitigate them
 - Understanding their dynamics would improve on PT control, and on future PS generators (PT and some NP effects)
 - In parallel with future (often motivated by theoretical interest) progress towards more complicated processes (multilegs, multiscale, non-global,...), it is important to figure out ways to exploit data using our knowledge of perturbative QCD beyond fixed order

Thank you for your attention