

## Timeline of grid technology - from QCD@LHC 2015

| 2000 | First implementation of grid <br> technique for DIS at H1- Markus Wobisch DESY-THESIS-2000-049 <br> First basic fit in DIS using jets H1 -EPJ C19, 289 (2001) |
| :--- | :--- |
| 2001 | First full fit in DIS using jets from ZEUS - PRD 67 0120071 (2003) <br> APPLgrid for jets in hadron-hadron collisions - Carli, Salam, Siegert <br> - C+,, fully open source <br> - user code for grid generation available <br> - arbitrary scale variation <br> fastNLO implementation for DIS and jets in hadron-hadron with NLOjet++ <br> and threshold corrections fro Kidonakis et al, Kluge Rabbertz, Wobisch <br> - Separate Fortran routines for only precomputed grids <br> - limited precomputed scale choices <br> - No user grid generation <br> APPLgrid for jets in hadron-hadron - first full release <br> - custom sparse memory structure for more efficient storage |
| - arbitrary beam energy scaling |  |

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## Timeline of grid technology (and associated items)

| Oct 2015 | Initial discussions for the APPLfast - NNLO project (of which more later) <br> March 2016 |
| ---: | :--- |
| May 2016 | Implementation of the photon density within the photon <br> Implementation with APPLgrid and aMCFast - Stefano Carrazza |
| July 2016 | APFELgrid - modified grids using APFEL evolution <br> Valerio Bertone, Stefano Carrazza, Nathan P. Hartland |
| InLeJET z+jets cross section 2016 | A. Gehrmann-De Ridder, T. Gehrmann, E.W.N. Glover, A. Huss, T. A. Morgan <br> top pair production at NNLO interfaced to fastNLO <br> Michael Czakon, David Heymes and Alexander Mitov |
| August 2016 First public APPLfast status report (today !) |  |

## Recap of the Numerical Technique

- For a calculation of a cross section from $m=1 \ldots N$ weights, $w_{m}$, from a Monte Carlo integration with momentum fraction $x_{m}$, form the product

$$
\sum_{m} w\left(x_{m}\right) q\left(x_{m}\right)
$$

- Can interpolate the function $q\left(x_{m}\right)$

$$
q\left(x_{m}\right) \approx \sum_{i} q^{(i)} I^{(i)}\left(x_{m}-x^{(i)}\right)
$$

- such that $\sum_{m} w\left(x_{m}\right) q\left(x_{m}\right) \approx \sum_{i} q^{(i)} \sum_{m} w\left(x_{m}\right) I^{(i)}\left(x_{m}-x^{(i)}\right)$

$$
\approx \sum_{i} q^{(i)} W^{(i)}
$$

- For a calculation of a cross section with $m=1$.. $N$ weights, from a Monte Carlo integration with momentum transfer $Q^{2}$

$$
\begin{aligned}
d \sigma & =\sum_{p} \sum_{m=1}^{N} w_{m}^{(p)}\left(\frac{\alpha_{s}\left(Q_{m}^{2}\right)}{2 \pi}\right)^{p} q\left(x_{m}, Q_{m}^{2}\right) \\
& =\sum_{p} \sum_{i j} q\left(x_{(i)}, Q_{(j)}^{2}\right)\left(\frac{\alpha_{s}\left(Q_{(j)}^{2}\right)}{2 \pi}\right)^{p} \sum_{m}^{N} w_{m}^{(p)} I_{i}^{x}\left(x_{m}\right) I_{j}^{Q^{2}}\left(Q_{m}^{2}\right) \\
& =\sum_{p} \sum_{i j} q\left(x_{(i)}, Q_{(j)}^{2}\right)\left(\frac{\alpha_{s}\left(Q_{(j)}^{2}\right)}{2 \pi}\right)^{p} W_{i j}^{(p)}
\end{aligned}
$$

## Proton-Proton Collisions

- For $p p$ collisions need an extra dimension for the PDF of the second colliding hadron

$$
d \sigma=\sum_{p} \sum_{m=1}^{N} w_{m}^{(p)}\left(\frac{\alpha_{s}\left(Q_{m}^{2}\right)}{2 \pi}\right)^{p} q_{1}\left(x_{1 m}, Q_{m}^{2}\right) q_{2}\left(x_{2 m}, Q_{m}^{2}\right)
$$

- But there is an implicit summation over parton flavours. Make use of symmetries in the matrix elements to use a vector of $k=1 \ldots M$ independent weights such that

$$
\sum_{i j=q, \bar{q}, g} w_{i j} q_{1 i}\left(x_{1}\right) q_{2 j}\left(x_{2}\right)=\sum_{k=1}^{M} w^{(k)} F^{(k)}\left(x_{1}, x_{2}\right)
$$

- so that

$$
d \sigma=\sum_{p} \sum_{k=1}^{M} \sum_{m=1}^{N} w_{m}^{(p)(k)}\left(\frac{\alpha_{s}\left(Q_{m}^{2}\right)}{2 \pi}\right)^{p} F_{m}^{(k)}\left(x_{1 m}, x_{2 m}, Q_{m}^{2}\right)
$$

- Which can be placed on a grid in the same way as for DIS
- So from the summation, everything is down to the quality of the interpolation of the pdf at the grid nodes
- It is s pure quadrature technique and is not, in principle subject to statistical fluctuation, or put another way ..
- Each individual weight gets added to the grid, and should be well approximated individually


## appl(e)tise <br> /'ap(e)|tлız/

noun trademark



1. a sparkling fruit juice based drink created by blending apple juice with carbonated water (now appeltiser)
verb
2. to modify or add code (to an existing calculation) for the purposes of storing information (from the calculation) such that the approximate result (of the calculation) with different input parameters can be quickly determined

## NNLOJET (and APPLfast-NNLO)

- Semi-automated calculation of cross sections at NNLO from the IPPP, Zurich, ETH and others
- Gehrmann-De Ridder et al arXiv: 1607.01749
- See talk from Alex Huss tomorrow
- APPLfast-NNLO
- Developers from NNLOJET, APPLgrid and fastNLO
- A single, combined interface for NNLOJET with both APPLgrid and fastNLO

- Many processes implemented in NNLOJET
- Developing a generic interface for all available processes
- Concentrating on $Z+$ jets at NNLO for the initial development and proof-of-concept



## Grid generation

- When generating a grid, need to know the optimal phase space in $x_{1}, x_{2}$ and $Q^{2}$
- Use a warmup run to determine the phase space - fill with dummy weights, interpolation isn't performed,
- Optimise the phase space to that determined by the warmup run
- If inadequate statistics are generated during the warmup run, the phase space will be incomplete
- Weights during the production run can fall outside the grid and will cause large deviations from the expected cross section
- To avoid this can

1. Run with more weights during the warmup run
2. Extend the phase space by some additional buffer zone into which the grid can overflow if required
3. Allow the grid to dynamically add additional buffers

- The first two have been implemented :
- Since neither the interpolation nor the weights are needed during the warmup, the option to run NNLOJET generating events with unit phase space has been implemented on the NNLOJET side
- Much faster at LO, and significantly greater than 100 times faster at NNLO
- Option 3 is being developed - but no concrete time scale at the moment


## Z + jets at Leading order



- Note the $\pm 0.1 \%$ maximum range in ratio plots
- As always, Leading Order component well reproduced - usual issues near the edges of the phase space


## Z + jets at NLO

- NNLOJET calculates using many (150) distinct internal processes, many with the same input partons
- Automatically reduce down to 33 parton luminosities for the NLO process - combine for Real and Virtual contributions
- Keep the internal mapping of the internal process ID to the parton luminosity

| 0 | 132741426573103133 | $(d, \bar{d})+(s, \bar{s})+(b, \bar{b})$ |
| :---: | :---: | :---: |
| 1 | 142843446674104134 | $(u, \bar{u})+(c, \bar{c})$ |
| 2 | 152945466775105135 | $(\bar{d}, d)+(\bar{s}, s)+(\bar{b}, b)$ |
| 3 | 163047486876106136 | $(\bar{u}, u)+(\bar{c}, c)$ |
| 4 | 173177107137 | $(d, g)+(s, g)+(b, g)$ |
| 5 | 183278108138 | $(u, g)+(c, g)$ |
| 6 | 193379109139 | $(g, d)+(g, s)+(g, b)$ |
| 7 | 203480110140 | $(g, u)+(g, c)$ |
| 8 | 212235368586115116145146 | $(g, g)$ |
| 9 | 233781111141 | $(g, \bar{d})+(g, \bar{s})+(g, \bar{b})$ |
| 10 | 243882112142 | $(g, \bar{u})+(g, \bar{c})$ |
| 11 | 253983113143 | $(\bar{d}, g)+(\bar{s}, g)+(\bar{b}, g)$ |
| 12 | 264084114144 | $(\bar{u}, g)+(\bar{c}, g)$ |
| 13 | 4991121151 | $(d, \bar{d})+(d, \bar{s})+(d, \bar{b})+(s, \bar{d})+(s, \bar{s})+(s, \bar{b})+(b, \bar{d})+(b, \bar{s})+(b, \bar{b})$ |
| 14 | 5092122152 | $(d, \bar{u})+(d, \bar{c})+(s, \bar{u})+(s, \bar{c})+(b, \bar{u})+(b, \bar{c})$ |
| 15 | 5193123153 | $(u, \bar{d})+(u, \bar{s})+(u, \bar{b})+(c, \bar{d})+(c, \bar{s})+(c, \bar{b})$ |
| 16 | 5294124154 | $(u, \bar{u})+(u, \bar{c})+(c, \bar{u})+(c, \bar{c})$ |
| 17 | 5387117147 | $(d, d)+(d, s)+(d, b)+(s, d)+(s, s)+(s, b)+(b, d)+(b, s)+(b, b)$ |
| 18 | 5488118148 | $(d, u)+(d, c)+(s, u)+(s, c)+(b, u)+(b, c)$ |
| 19 | 5589119149 | $(u, d)+(u, s)+(u, b)+(c, d)+(c, s)+(c, b)$ |
| 20 | 5690120150 | $(u, u)+(u, c)+(\bar{c}, u)+(c, c)$ |
| 21 | 5799129159 | $(\bar{d}, \bar{d})+(\bar{d}, \bar{s})+(\bar{d}, \bar{b})+(\bar{s}, \bar{d})+(\bar{s}, \bar{s})+(\bar{s}, \bar{b})+(\bar{b}, \bar{d})+(\bar{b}, \bar{s})+(\bar{b}, \bar{b})$ |
| 22 | 58100130160 | $(\bar{d}, \bar{u})+(\bar{d}, \bar{c})+(\bar{s}, \bar{u})+(\bar{s}, \bar{c})+(\bar{b}, \bar{u})+(\bar{b}, \bar{c})$ |
| 23 | 59101131161 | $(\bar{u}, \bar{d})+(\bar{u}, \bar{s})+(\bar{u}, \bar{b})+(\bar{c}, \bar{d})+(\bar{c}, \bar{s})+(\bar{c}, \bar{b})$ |
| 24 | 60102132162 | $(\bar{u}, \bar{u})+(\bar{u}, \bar{c})+(\bar{c}, \bar{u})+(\bar{c}, \bar{c})$ |
| 25 | 6195125155 | $(\bar{d}, d)+(\bar{d}, s)+(\bar{d}, b)+(\bar{s}, d)+(\bar{s}, s)+(\bar{s}, b)+(\bar{b}, d)+(\bar{b}, s)+(\bar{b}, b)$ |
| 26 | 6296126156 | $(\bar{d}, u)+(\bar{d}, c)+(\bar{s}, u)+(\bar{s}, c)+(\bar{b}, u)+(\bar{b}, c)$ |
| 27 | 6397127157 | $(\bar{u}, d)+(\bar{u}, s)+(\bar{u}, b)+(\bar{c}, d)+(\bar{c}, s)+(\bar{c}, b)$ |
| 28 | 6498128158 | $(\bar{u}, u)+(\bar{u}, c)+(\bar{c}, u)+(\bar{c}, c)$ |
| 29 | 69 | $(d, d)+(s, s)+(b, b)$ |
| 30 | 70 | $(\underline{u}, \underline{u})+(c, c)$ |
| 31 | 71 | $(\bar{d}, \bar{d})+(\bar{s}, \bar{s})+(\bar{b}, \bar{b})$ |
| 32 | 72 | $(\bar{u}, \bar{u})+(\bar{c}, \bar{c})$ |

## Z + jets at NNLO

## - Again the same 33 input parton luminosities as in the NNLO case, however, many more (794) individual internal processes

$0 \quad 163177191205206245246285301317347377391405435449450489490529530569593617647677707737$ 767899905906935941942

164178192207208247248286302318348378392406436451452491492531532570594618648678708738 768900907908936943944
165179193209210249250287303319349379393407437453454493494533534571595619649679709739 769909917918945953954

## 166180194211212251252

## 770910919920946955956

167181195229230269270289305321351381395409439481482521522561562585586609610621651681 711741771817818861862901902937938
168182196231232271272290306322352382396410440483484523524563564587588611612622652682 712742772819820863864903904939940
169183197233234273274293309323353383397411441473474513514553554577578601602623653683 713743773801802845846889890925926
170184198235236275276294310324354384398412442475476515516555556579580603604624654684 714744774803804847848891892927928
171172185186199200325326355356385386399400413414443444629630659660689690719720749750 779780797798799800841842843844885886887888921922923924
173187201237238277278295311327357387401415445477478517518557558581582605606625655685 715745775805806849850893894929930
174188202239240279280296312328358388402416446479480519520559560583584607608626656686 716746776807808851852895896931932
175189203241242281282297313329359389403417447485486525526565566589590613614627657687 717747777833834877878913914949950
176190204243244283284298314330360390404418448487488527528567568591592615616628658688 718748778835836879880915916951952
213253335365423457497537635665695725755785813821822857865866 214254336366424458498538636666696726756786814858 215255337367425459499539637667697727757787815859 216256338368426460500540638668698728758788816823824860867868 217257331361419461501541631661691721751781809853 218258332362420462502542632662692722752782810854 219259333363421463503543633663693723753783811855 220260334364422464504544634664694724754784812856 221261343373431465505545643673703733763793829873 222262344374432466506546644674704734764794830874 223263345375433467507547645675705735765795831875 224264346376434468508548646676706736766796832876 225265339369427469509549639669699729759789825837838869881882 226266340370428470510550640670700730760790826870
227267341371429471511551641671701731761791827871 228268342372430472512552642672702732762792828839840872883884 291307573597897933 292308574598898934 299315575599911947 300316576600912948
$(d, \bar{d})+(s, \bar{s})+(b, \bar{b})$
$(u, \bar{u})+(c, \bar{c})$
$(\bar{d}, d)+(\bar{s}, s)+(\bar{b}, b)$
$(\bar{u}, u)+(\bar{c}, c)$
$(d, g)+(s, g)+(b, g)$
$(u, g)+(c, g)$
$(g, d)+(g, s)+(g, b)$
$(g, u)+(g, c)$
$(g, g)$
$(g, \bar{d})+(g, \bar{s})+(g, \bar{b})$
$(g, \bar{u})+(g, \bar{c})$
$(\bar{d}, g)+(\bar{s}, g)+(\bar{b}, g)$
$(\bar{u}, g)+(\bar{c}, g)$
$(d, \bar{d})+(d, \bar{s})+(d, \bar{b})+(s, \bar{d})+(s, \bar{s})+(s, \bar{b})+(b, \bar{d})+(b, \bar{s})+(b, \bar{b})$
$(d, \bar{u})+(d, \bar{c})+(s, \bar{u})+(s, \bar{c})+(b, \bar{u})+(b, \bar{c})$
$(u, d)+(u, \bar{s})+(u, \bar{b})+(c, \bar{d})+(c, \bar{s})+(c, \bar{b})$
$(u, \bar{u})+(u, \bar{c})+(c, \bar{u})+(c, \bar{c})$
$(d, d)+(d, s)+(d, b)+(s, d)+(s, s)+(s, b)+(b, d)+(b, s)+(b, b)$
$(d, u)+(d, c)+(s, u)+(s, c)+(b, u)+(b, c)$
$(u, d)+(u, s)+(u, b)+(c, d)+(c, s)+(c, b)$
$(u, u)+(u, c)+(c, u)+(c, c)$
$(\bar{d}, \bar{d})+(\bar{d}, \bar{s})+(\bar{d}, \bar{b})+(\bar{s}, \bar{d})+(\bar{s}, \bar{s})+(\bar{s}, \bar{b})+(\bar{b}, \bar{d})+(\bar{b}, \bar{s})+(\bar{b}, \bar{b})$
$(\bar{d}, \bar{u})+(\bar{d}, \bar{c})+(\bar{s}, \bar{u})+(\bar{s}, \bar{c})+(\bar{b}, \bar{u})+(\bar{b}, \bar{c})$
$(\bar{u}, \bar{d})+(\bar{u}, \bar{s})+(\bar{u}, \bar{b})+(\bar{c}, \bar{d})+(\bar{c}, \bar{s})+(\bar{c}, \bar{b})$
$(\bar{u}, \bar{u})+(\bar{u}, \bar{c})+(\bar{c}, \bar{u})+(\bar{c}, \bar{c})$
$(\bar{d}, d)+(\bar{d}, s)+(\bar{d}, b)+(\bar{s}, d)+(\bar{s}, s)+(\bar{s}, b)+(\bar{b}, d)+(\bar{b}, s)+(\bar{b}, b)$
$(\bar{d}, u)+(\bar{d}, c)+(\bar{s}, u)+(\bar{s}, c)+(\bar{b}, u)+(\bar{b}, c)$
$(\bar{u}, d)+(\bar{u}, s)+(\bar{u}, b)+(\bar{c}, d)+(\bar{c}, s)+(\bar{c}, b)$
$(\bar{u}, u)+(\bar{u}, c)+(\bar{c}, u)+(\bar{c}, c)$
$(d, d)+(s, s)+(b, b)$
$(\bar{u}, \bar{u})+(c, c)$
$(\bar{d}, \bar{d})+(\bar{s}, \bar{s})+(\bar{b}, \bar{b})$
$(\bar{u}, \bar{u})+(\bar{c}, \bar{c})$

## Developments for grid filling

- For the filling call the fill methods for each process generated
- At NNLO have over 700 separate processes, each called many times for each phase space point - the Real emission components may have several different but related phase space points
- Calling the fill methods for each weight is maximally time consuming
- Implemented a filling cache
- Each weight is added to a weight vector corresponding to a unique phase space point and observable value
- When the phase space point changes, flush the cache to the grid
- Significantly reduces the number of calls to the grid filling, by factors of between 20 and 200 depending on the process


## Z + jets at NLO

- Early tests of grid closure of the NLO components are promising
- Only low statistics runs so far
- Even with short warmup, agreement to well within 0.1\%


M Sutton - Recent developments with fast calculations beyond Leading Order


## Z + jets at NNLO

- Early proof-of-concept convolutions ratios
- Note large excursions for some bins due to inadequate statistics used for the determination of the optimised phase space
- Working on improving the optimisation strategy


## Grid closure - LO

- Need to check proper closure of the grids
- Besides verification that grids generated with PDF " $X$ " can reproduce the cross section ..
- ... need to verify that the convolution with PDF " Y " with a grid generated with PDF " X " actually reproduces the expected cross section generated with PDF " $Y$ "
- Essentially trivial proof of concept for the LO component only
- Just a taster of things to come ...
- Calculations with different PDFs are statistically independent so need many events
- For NLO, NNLO comparison need the uncertainty on the calculation to be small
- Lots of events needed - will take a while to check properly ...



## Grid closure - NLO virtual component



- Checking the closure of the NLO components:
- Grid generated with NNPDF 2.3 nnlo
- Convoluted with CT14 compared to NNLOJET calculation with CT14
- Agreement to well within $0.1 \%$ - extremely encouraging
- Work ongoing testing the NNLO components



## The other things ... photon density in the proton

- Photon contribution to the proton
- First implementation of the addition of the photon PDF weights in the applgrid framework - Stefano Carrazza
- Code not yet available in the stable release
- Initial developments to integrate this feature in aMC@NLO LO DY applgrid convolution with and without an initial-state photon PDF (from NNPDF2.3QED)
- Ongoing developments to extend the store EW corrections in applgrid




## The other things (contd) ...

## APFELgrid/FK timings

gcc-5.2.1 on i7-6500U CPU @ 2.50GHz

- APFELgrid, Bertone, Carrazza, and Hartland
- arxiv.org/abs/1605.02070
- Manipulation of the full grids to reduce size for faster convolution
- Top pair production - Czakon, Heymes,and Mitov
- arxiv.org/abs/1606.03350v1
- fastNLO interface being developed by calculation authors
- NLO component agrees well, still developing the interface with the NNLO component
- DiffTop - Guzzi, Lipka, and Moch
- difftop.hepforge.org
- Approximate NNLO calculation using threshold resummation
- All of these are possible because of the open source nature of the grid code (APPLgrid and fastNLO)
- It is worth noting that appletising a calculation is not difficult if you know the code well
- If you are the author of a calculation it may be worth thinking about the structure of your code in light of such an interface



## Great expectations

- Over the last year there have been many significant developments for fast grid techniques
- Photon density, NNLO calculations for top, APPLfast-NNLO interface to NNLOJET
- The community of folk implementing grid interfaces in growing - no longer just the APPLgrid or fastNLO developers
- Interfacing new calculations with grid technology - either APPLgrid or fastNLO - should be reasonably straightforward if you know the calculation code - perhaps we need a forum to exchange experience
- The more generic interfaces being implemented now will no doubt help these efforts
- The APPLfast-NNLO proof of concept development is starting to mature
- Several stages still need to be checked - full combination of grids from multiple runs for large statistics, combination in to single cross section grid
- Still significant code development required, but starting to look as if we will be able to reproduce well the cross section at NNLO
- As the APPLfast-NNLO project comes to fruition, will hopefully be able to immediately take full advantage of the future developments of the NNLOJET code.
- The next few years will be an extremely interesting time for grid technologies and may start to allow genuine NNLO fits using the LHC data, and then, What larks!


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