Resummation in PDF fits

Marco Bonvini

Rudolf Peierls Centre for Theoretical Physics, University of Oxford

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Related to work with the NNPDF collaboration

Logarithmic enhancement: resummation

Single (double) logarithmic enhancements

$$\alpha_s^k \log^j \qquad 0 \le j \le (2)k$$

If/when

$$\alpha_s \log^{(2)} \sim 1$$

all such terms in the perturbative series are equally important:

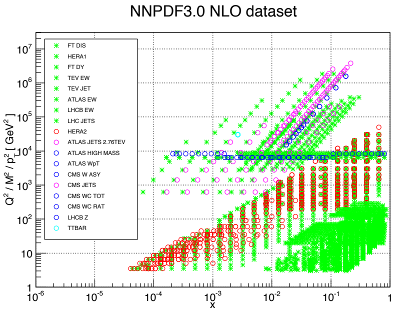
all-order RESUMMATION

Goals of resummations in PDF fits:

- provide PDFs consistent with resummed computations
- improve the quality of PDF fits
- investigate the impact of higher orders (and thus estimate the uncertainty from missing higher orders)
- getting closer to "all-order PDFs"

Large-x: threshold resummation

- $x \to 1$
- due to soft gluon emissions
- resums double logs $\left(\frac{\log^k(1-x)}{1-x}\right)_+$
- in Mellin space, $\log N$ at $N \to \infty$
- [MB,Marzani,Rojo,Rottoli,Ubiali,Ball,Bertone, Carrazza,Hartland 1507.01006]



Small-x: high-energy (BFKL) resummation

- $x \to 0$
- due to high-energy gluon emissions
- resums single logs $\frac{1}{x}\log^k x$
- in Mellin space, poles $1/(N-1)$ in the limit $N \to 1$
- [MB,Marzani,Peraro 1607.02153]
  [NNPDF (in preparation)]

Resum what?

Observable: $$\sigma = \sigma_0 \, C(\alpha_s(\mu)) \otimes f(\mu) \left[\otimes f(\mu) \right]$$

Evolution: $$\mu^2 \frac{d}{d\mu^2} f(\mu) = P(\alpha_s(\mu)) \otimes f(\mu)$$

Any object with a perturbative expansion and a log enhancement:

- coefficient functions $C(\alpha_s(\mu))$ (observable)
- splitting functions $P(\alpha_s(\mu))$ (evolution)

	observable coefficient functions $C(\alpha_s(\mu))$	evolution splitting functions $P(\alpha_s(\mu))$
large-x	(N)NNLL	—
small-x	LLx	NLLx

Threshold resummation of inclusive cross sections

Dressing the Born with soft gluon emissions leads to double log enhancement

$$C(N) = C_{\rm LO}(N)\left[1+\sum_{n=1}^{\infty}\alpha_s^n\sum_{k=0}^{2n}c_{nk}\log^k N\right]\times\left[1+\mathcal{O}\left(\frac{1}{N}\right)\right]$$

Known to N^{3}LL for DIS, DY, Higgs: $k = 2n, 2n-1, \ldots, 2n-6$
and to NNLL for many others: $k = 2n, 2n-1, \ldots, 2n-4$

Well known formalism, can be derived in several ways (diagrammatic approach, factorization methods, path-integral approach, SCET)

$$\frac{C(N)}{C_{\rm LO}(N)} = g_0(\alpha_s)\exp\left[\frac{1}{\alpha_s}g_1(\alpha_s L)+g_2(\alpha_s L)+\alpha_s g_3(\alpha_s L)+\alpha_s^2 g_4(\alpha_s L)+\ldots\right]$$

$L = \log N$

Available for

- total cross sections σ
- invariant mass distributions $d\sigma/dM^2$
- double-differential invariant mass + rapidity distributions $d\sigma/dM^2/dY$

Processes in a global (NNPDF) PDF fits (arXiv:1507.01006)

Process	observable	resummation available
DIS	$d\sigma/dx/dQ^2$ (NC, CC, charm, ...)	YES
DY Z/γ	$d\sigma/dM^2/dY$	YES
DY W	differential in the lepton kinematics	NO
$t\bar{t}$	total σ	YES
jets	inclusive $d\sigma/dp_t/dY$	YES/NO

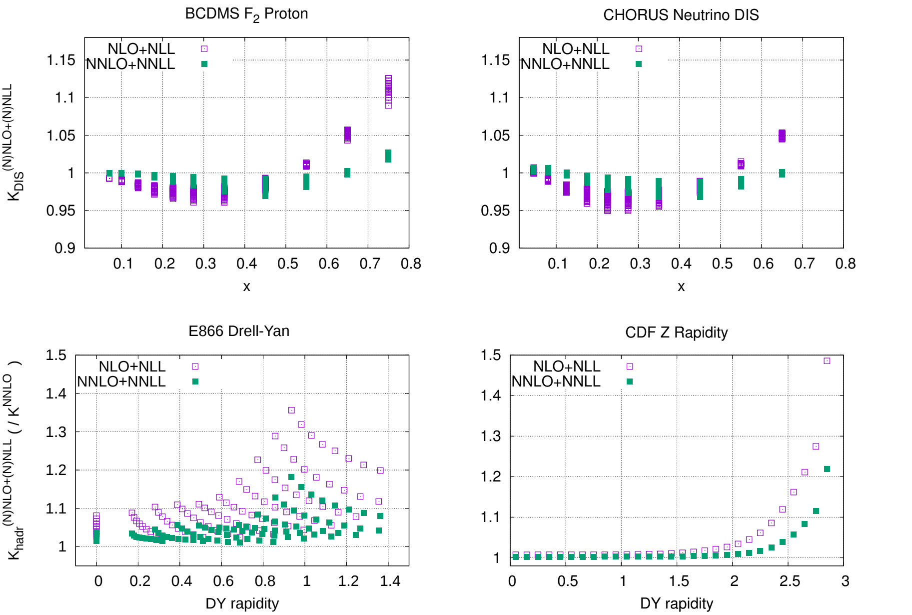
Including DY W requires threshold resummation at fully differential level: no public code available (yet?)

Jets are currently available at NLO and NLL, but partial NNLO results indicate that NLL is very poor: we excluded them (gluon poorly determined!)

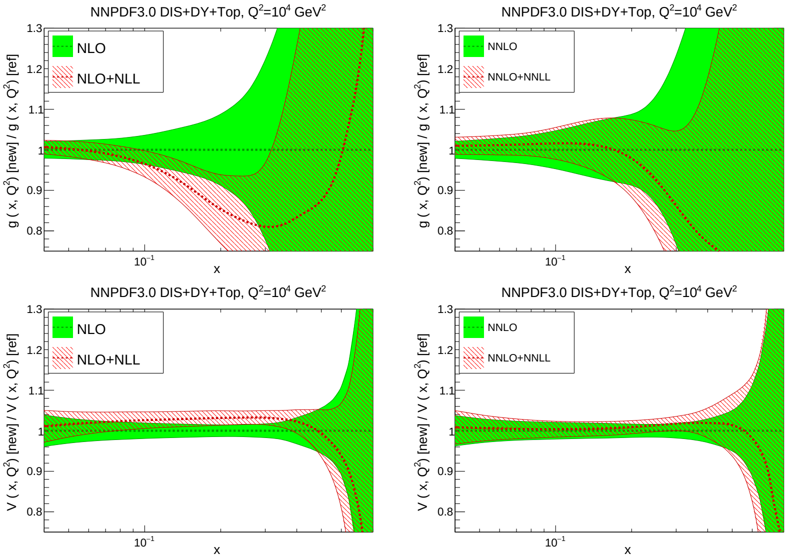
DIS, DY available from `TROLL (TROLL Resums Only Large-x Logarithms)`

`www.ge.infn.it/∼bonvini/troll`

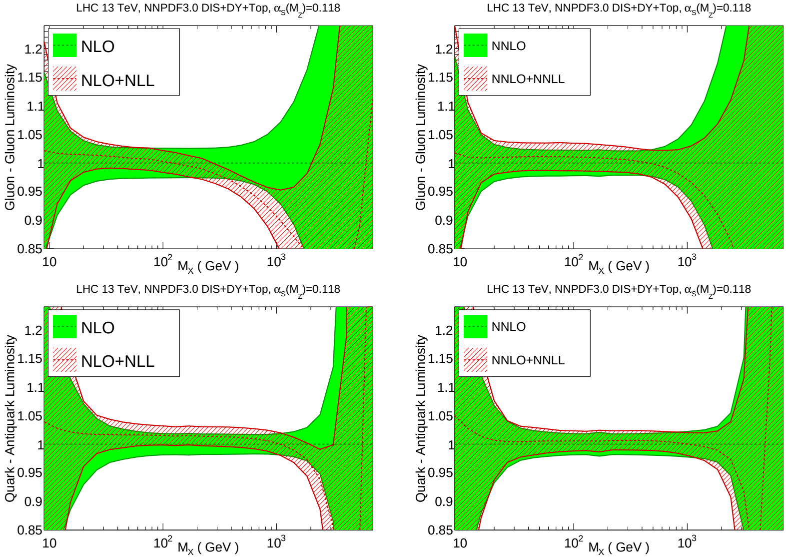
$t\bar{t}$ available from `top++` `www.alexandermitov.com/software`


BCDMS F2 Proton
NLO+NLL
NNLO+NNLL
K_DIS^(N)NLO+(N)NLL
x
CHORUS Neutrino DIS
NLO+NLL
NNLO+NNLL
x
E866 Drell-Yan
NLO+NLL
NNLO+NNLL
K_hadr^(N)NLO+(N)NLL (/ K^NNLO)
DY rapidity
CDF Z Rapidity
NLO+NLL
NNLO+NNLL
DY rapidity

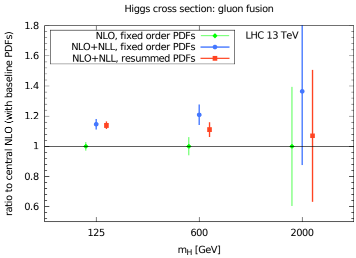
Impact on PDF fits: PDFs

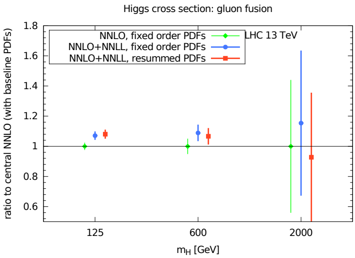


Impact on PDF fits: luminosities

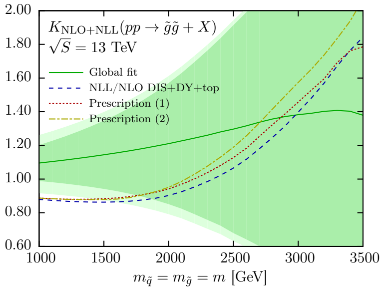


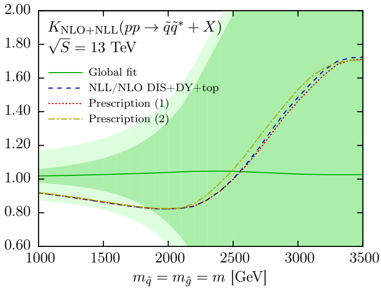
Higgs:





SUSY particles: [Beenakker,Borschensky,Krämer,Kulesza,Laenen,Marzani,Rojo 1510.00375]





Impact on PDF fits: χ^2

Experiment	NNPDF3.0 DIS+DY+top			
	NLO	NNLO	NLO+NLL	NNLO+NNLL
NMC	1.39	1.34	1.36	1.30
SLAC	1.17	0.91	1.02	0.92
BCDMS	1.20	1.25	1.23	1.28
CHORUS	1.13	1.11	1.10	1.09
NuTeV	0.52	0.52	0.54	0.44
HERA-I	1.05	1.06	1.06	1.06
ZEUS HERA-II	1.42	1.46	1.45	1.48
H1 HERA-II	1.70	1.79	1.70	1.78
HERA charm	1.26	1.28	1.30	1.28
DY E866	1.08	1.39	1.68	1.68
DY E605	0.92	1.14	1.12	1.21
CDF Z rap	1.21	1.38	1.10	1.33
D0 Z rap	0.57	0.62	0.67	0.66
ATLAS Z 2010	0.98	1.21	1.02	1.28
ATLAS high-mass DY	1.85	1.27	1.59	1.21
CMS 2D DY 2011	1.22	1.39	1.22	1.41
LHCb Z rapidity	0.83	1.30	0.51	1.25
ATLAS CMS top prod	1.23	0.55	0.61	0.40
Total	1.233	1.264	1.246	1.269

Resummed χ^2 slightly worse
DY fixed-target experiment are the origin of the problem

Small-x resummation: overview

Small-x resummation based on k_t-factorization

Developed in the 90s-00s [Catani,Ciafaloni,Colferai,Hautmann,Salam,Stasto] [Altarelli,Ball,Forte]

Affects both evolution (known to LLx and NLLx) and coefficient functions (known only at lowest logarithmic order, which is often NLLx) in the singlet sector

We follow the ABF [Altarelli,Ball,Forte 1995,...,2008] procedure to resum splitting functions and develop a new formalism for coefficient functions [MB,Marzani,Peraro 1607.02153]

We published (and keep developing) a public code

`HELL: High-Energy Large Logarithms` www.ge.infn.it/∼bonvini/hell

which delivers resummed splitting functions and coefficient functions

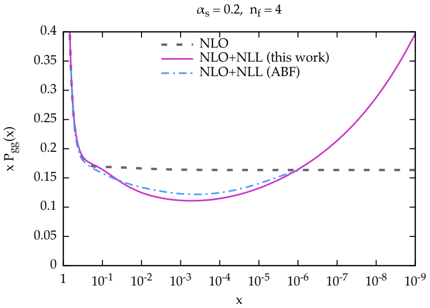
`HELL` has been interfaced to `APFEL` apfel.hepforge.org

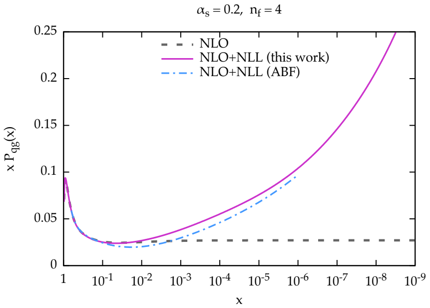
opening the door to its usage for PDF fitting

Small-x resummation in DGLAP evolution

Ingredients (ABF):

- duality with BFKL evolution
- symmetry of the BFKL kernel
- momentum conservation
- resummation of (subleading, but fundamental) running coupling effects





[MB,Marzani,Peraro 1607.02153]

At the moment resummation matched only to NLO
NNLO+NLLx is practically complicated, but will be done

Small-x resummation in the coefficient functions

High-energy (k_T) factorization:

$$\sigma \propto \int \frac{dz}{z} \int d^2\boldsymbol{k}\, \hat{\sigma}_g\left(\frac{x}{z}, \frac{Q^2}{\boldsymbol{k}^2}, \alpha_s(Q^2)\right) \mathcal{F}_g(z, \boldsymbol{k})$$

$$\begin{cases} \mathcal{F}_g(x, \boldsymbol{k}) : \text{unintegrated PDF} \\ \hat{\sigma}_g\left(z, \frac{Q^2}{\boldsymbol{k}^2}, \alpha_s\right) : \text{off-shell xs} \end{cases}$$

Defining

$$\mathcal{F}_g(N, \boldsymbol{k}) = U\left(N, \frac{\boldsymbol{k}^2}{\mu^2}\right) f_g(N, \mu^2)$$

we get [MB,Marzani,Peraro 1607.02153]

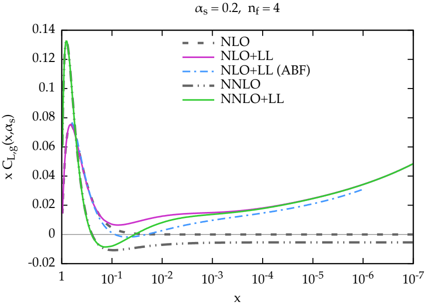
$$C_g(N, \alpha_s) = \int d^2\boldsymbol{k}\, \hat{\sigma}_g\left(N, \frac{Q^2}{\boldsymbol{k}^2}, \alpha_s\right) U\left(N, \frac{\boldsymbol{k}^2}{\mu^2}\right)$$

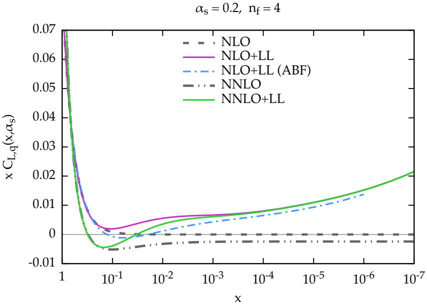
At LLx accuracy, U has a simple form, in terms of small-x resummed anom dim γ

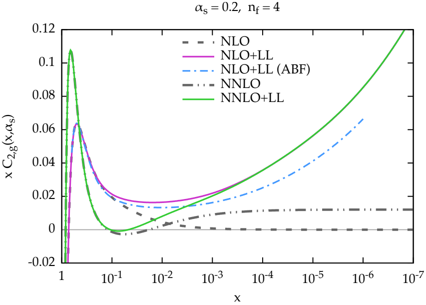
$$U\left(N, \frac{\boldsymbol{k}^2}{\mu^2}\right) \approx \boldsymbol{k}^2 \frac{d}{d\boldsymbol{k}^2} \exp \int_{\mu^2}^{\boldsymbol{k}^2} \frac{d\nu^2}{\nu^2} \gamma(N, \alpha_s(\nu^2))$$

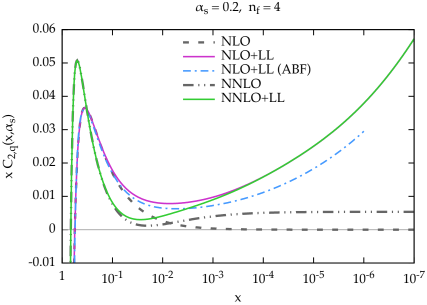
- Only known at LLx
- Just uses the off-shell cross sections $\hat{\sigma}(N, Q^2/\boldsymbol{k}^2, \alpha_s)$ (one for each process)
- Can be included directly in `HELL`
- Formally equivalent to ABF (practically easier and numerically stabler)

Small-x resummation in massless DIS









Towards a small-x resummed fit

For a fit we need additional process:

- massive DIS coefficient functions
- Drell-Yan
- jets

The first is the most urgent (allows a DIS-only fit)

The off-shell coefficients for heavy-quark production in DIS are available [Catani,Hautmann 1994], implementation in `HELL` in progress

Need also to update the VFNS! (FONLL in the NNPDF case)

Rather easy, however it turns out that matching conditions also resum

[MB,Marzani,.... (work in progress)]

Drell-Yan rapidity distributions have never been resummed in the ABF formalism due to technical difficulties $\rightarrow$ should be simpler and doable with the new formalism

Do we need small-x resummation for jets?

Small-x resummation: results

Conclusions

PDF fit with threshold resummation

- DIS + DY (Z/γ) + $t\bar{t}$ ✓
- sizeable effect at NLO+NLL, small effect at NNLO+NNLL
- to be done:
  - include missing processes (DY W, jets)
  - understand (or exclude?) fixed-target DY
  - consider other choices for resummation (different subleading terms, better description of the not-too-large-x region) [MB,Marzani 1405.3654]

PDF fit with high-energy resummation

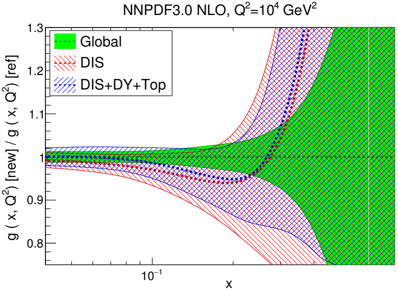
- NLO+NLLx evolution ✓
- resummed coefficient functions: massless DIS ✓, massive DIS ✓, DY ✗, jets ✗
- resummed matching conditions ✓
- preliminary NLO+NLLx fits are in progress
- to be done: evolution at NNLO+NNLLx

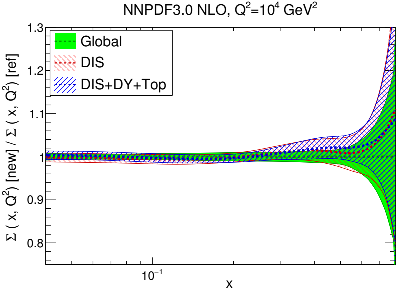
Outlook:

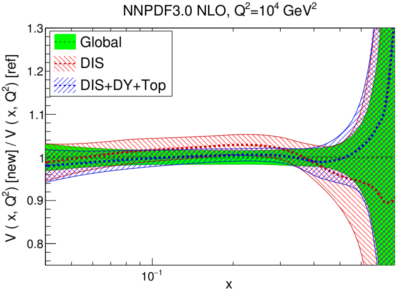
- PDF fit with joint (threshold + high-energy) resummation?
- other soft resummations?

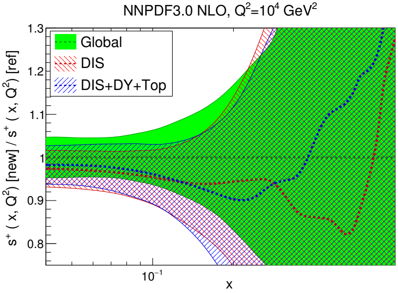
Backup slides

Reduced dataset

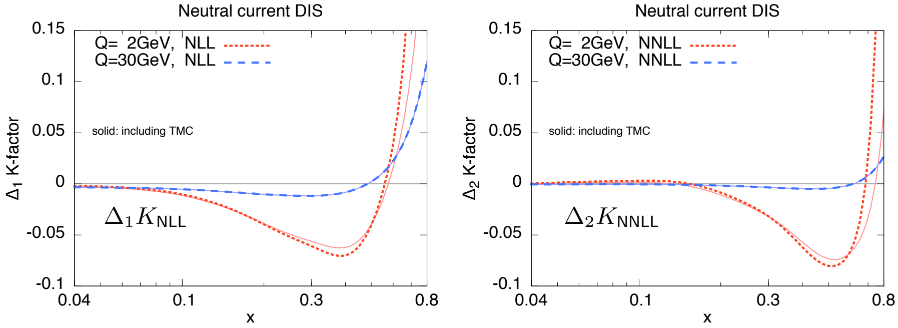






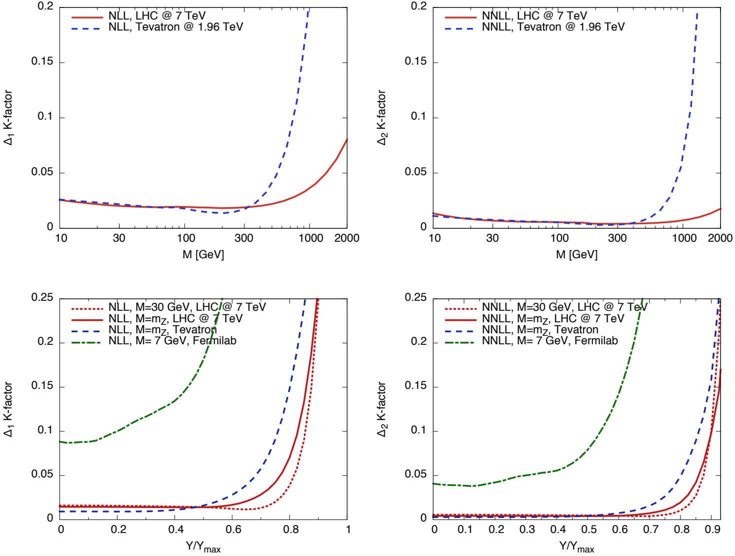


TROLL delivers $\Delta_j K_{\mathrm{N}^n\mathrm{LL}}$ to be used as $\sigma_{\rm res} = \sigma_{\mathrm{N}^j\mathrm{LO}} + \sigma_{\rm LO} \times \Delta_j K_{\mathrm{N}^n\mathrm{LL}}$



$$\frac{d\sigma_{\rm DY}}{dM^2}:$$

$$\frac{d\sigma_{\rm DY}}{dM^2dY}:$$



Improved threshold resummation

$$\frac{C(N)}{C_{\rm LO}(N)} = g_0(\alpha_s)\exp\left[\frac{1}{\alpha_s}g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \dots\right] \times \left[1 + \mathcal{O}\left(\frac{1}{N}\right)\right]$$

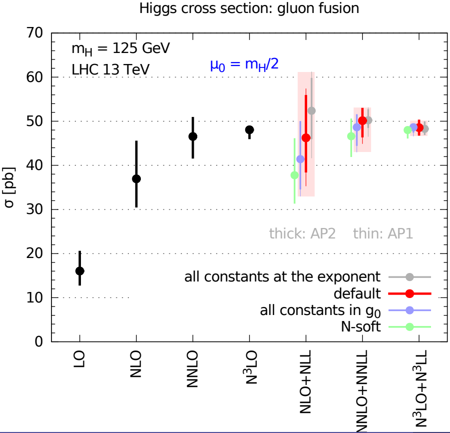
N-soft: standard resummation considered in our fit, neglects all $1/N$ terms

ψ-soft: improved resummation, includes some $1/N$ terms

ψ-soft is more predictive than N-soft

[MB,Marzani 1405.3654]

[MB,Marzani,Muselli,Rottoli 1603.08000]



Small-x resummation: brief overview

DGLAP: $$\mu^2 \frac{d}{d\mu^2} f(x,\mu^2) = \int \frac{dz}{z} P\left(\frac{x}{z}, \alpha_s(\mu^2)\right) f(z,\mu^2)$$

BFKL: $$x \frac{d}{dx} f(x,\mu^2) = \int \frac{d\nu^2}{\nu^2} K\left(x, \frac{\mu^2}{\nu^2}, \alpha_s(\cdot)\right) f(x,\nu^2)$$

double Mellin transform $f(N,M) = \int dx\, x^N \int \frac{d\mu^2}{\mu^2} \left(\frac{\mu^2}{\mu_0^2}\right)^{-M} f(x,\mu^2)$

DGLAP: $M\, f(N,M) = \gamma(N, \alpha_s(\cdot)) f(N,M)$ + boundary

BFKL : $N\, f(N,M) = \chi(M, \alpha_s(\cdot)) f(N,M)$ + boundary

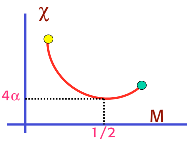
When both are valid (small x, large μ^2), consistency between the solutions gives (at fixed coupling)

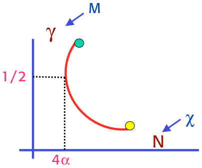
$$\chi(\gamma(N,\alpha_s),\alpha_s) = N$$

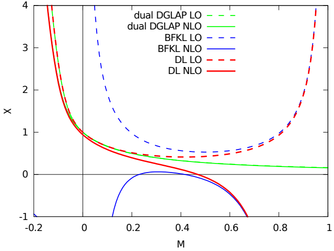
duality relation

For $\chi(M,\alpha_s) = \alpha_s \chi_0(M)$

the dual γ contains all orders in α_s/N







What do we get?

- LL: strong growth at small x (not observed)
- NLL: no enhancement at small x (!!)

Totally unstable,
due to perturbative instability of the BFKL kernel

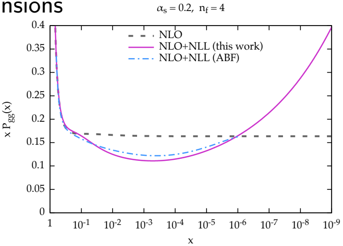
ABF solution [Altarelli,Ball,Forte 1995,...,2008]

- use duality to resum BFKL kernel
- exploit symmetry $M \to 1 - M$ of χ
- impose momentum conservation
- reuse duality to get resummed anomalous dimensions

The result is perturbatively stable!

Finally

- resum running coupling contributions (changes the nature of the small-N singularity: branch-cut to pole)



Resummation in the evolution: large x

Singlet diagonal (P_{qq}, P_{gg}) and non-singlet ($P^{\pm}_{\rm ns}$):

$$P(x,\alpha_s) = \frac{A(\alpha_s)}{(1-x)_+} + B(\alpha_s)\delta(1-x) + C(\alpha_s)\log(1-x) + \dots$$

$$\gamma(N,\alpha_s) = -A(\alpha_s)\log N + [B(\alpha_s) - \gamma A(\alpha_s)] - C(\alpha_s)\frac{\log N}{N} + \dots$$

no log enhancement!

Singlet off-diagonal (P_{qg}, P_{gq}):

$$P(x,\alpha_s) = \sum_{n=0}^{\infty} \alpha_s^{n+1}\left[\sum_{k=0}^{2n} d_{nk}\log^k(1-x) + \dots\right]$$

$$\gamma(N,\alpha_s) = \sum_{n=0}^{\infty} \alpha_s^{n+1}\left[\sum_{k=0}^{2n} \tilde{d}_{nk}\frac{\log^k N}{N} + \dots\right]$$

Double log enhancement of the next-to-soft (NS) contributions [Vogt 1005.1606]
Can be resummed up to NNLL ($k=0,1,2$) [Almasy,Soar,Vogt 1012.3352]
Expected effect: negligible

Resummation in the evolution: small x

Singlet:

$$P(x,\alpha_s) = \sum_{n=0}^{\infty} \alpha_s^{n+1} \left[\sum_{k=0}^{n} a_{nk} \frac{\log^k x}{x} + \sum_{k=0}^{2n} b_{nk} \log^{2k} x + \dots \right]$$

$$\gamma(N,\alpha_s) = \sum_{n=0}^{\infty} \alpha_s^{n+1} \left[\sum_{k=0}^{n} \frac{a_{nk}}{(N-1)^{k+1}} + \sum_{k=0}^{2n} \frac{b_{nk}}{N^{k+1}} + \dots \right]$$

Single log enhancement at leading small x, in the singlet sector

$$P_{\text{singlet}} = \begin{pmatrix} P_{gg} & P_{gq} \\ P_{qg} & P_{qq} \end{pmatrix} = \begin{pmatrix} \text{LL} & \text{LL} \\ \text{NLL} & \text{NLL} \end{pmatrix}$$

Non-singlet:

$$P(x,\alpha_s) = \sum_{n=0}^{\infty} \alpha_s^{n+1} \left[\sum_{k=0}^{2n} b_{nk} \log^k x + \dots \right]$$

is double log enhanced but subleading.