# Transverse-momentum resummation for top-quark pair production at hadron colliders 

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## Outline

- Motivation
- $q_{T}$-resummation
- Preliminary results
- $q_{T}$-subtraction
- Results
- Summary


## Top quark

Mass of the top quark obtained through combining the measurements at the Tevatron and LHC colliders is $m_{t}=173.34 \pm 0.27$ (stat) $\pm 0.71$ (syst) GeV
[ATLAS and CDF and CMS and D0 Collaborations (2014)].

- Strong coupling to the Higgs boson
- Crucial to the hierarchy problem


## Top quark pair production

- The top quark pair production is the main source of the top quark events in the Standard Model (SM).
- Many New Physics models involve heavy top partners which then decay into a top quark pair.

The study of the $t \bar{t}$ pair production at hadron colliders can

- shed light on the electroweak symmetry breaking mechanism.
- provide information on the backgrounds of many NP models.


## Top quark pair production

- Because of its large mass the top quark decay before hadronization, allowing for a better experimental


More precise calculations are needed from the theory side

## QCD corrections

Theoretical efforts for obtaining precision predictions for the $t \bar{t}$ production at hadron colliders started almost 3 decades ago

- NLO QCD corrections are calculated by [Nason, Dawson and Ellis (1988), Beenakker, Kuijf, van Neerven and Smith (1989), Beenakker, van Neerven, Meng, Schuler and Smith (1989)].
- NNLO corrections in the threshold region is worked out in [Ahrens, Ferroglia, Neubert, Pecjak, Yang (2010)].
- The calculation of the full NNLO QCD corrections was completed for the total cross section and for the $t \bar{t}$ asymmetry. [Barnreuther, Czakon, Mitov (2012), Czakon, Mitov (2012), Czakon, Mitov (2013), Czakon, Fiedler, Mitov (2013), Czakon, Fiedler, Mitov (2014)]. The first NNLO results for differential distributions at the Tevatron and LHC have been computed [Czakon, Heymes, Mitov (2016), Czakon, Fiedler, Heymes, Mitov (2016)].
- Other computations of differential distributions are underway [Abelof, Gehrmann-De Ridder, Maierhofer (2014), Abelof, Gerhrmann-de Ridder (2014), Abelof, Gerhrmann-de Ridder, Majer (2016)].
- Last year the NNLO corrections for all off-diagonal channels have been computed by our group [Bonciani, Catani, Grazzini, HS, Torre].


## $q_{T}$ distribution

- When $q_{T}^{2} \sim M^{2}, \alpha_{S}\left(M^{2}\right)$ is small, and the standard fixed order expansion is theoretically justified.
- When $q_{T}^{2} \ll M^{2}$ large logarithms of the form $\alpha_{S}^{n} \log \left(M^{2} / q_{T}^{2}\right)$ appear, due to soft and collinear gluon emissions. Effective expansion variable is the $\alpha_{S}^{n} \log \left(M^{2} / q_{T}^{2}\right)$, which can be $\sim 1$ even for small $\alpha_{S}$. These large logarithms need to be resummed to all orders in $\alpha_{S}$, in order to get reliable predictions over the whole range of the transverse momenta.

The resummation of large logs results in exponentiating these large logarithmic terms
$\sigma^{(r e s)} \sim \sigma^{(0)} C\left(\alpha_{S}\right) \exp \left\{L g_{1}\left(\alpha_{S} L\right)+g_{2}\left(\alpha_{S} L\right)+\alpha_{S} g_{3}\left(\alpha_{S} L\right)+\ldots\right\}$.

## Resummation for the $t \bar{t}$ production

Production of coloured particles imposes additional complications compared to the production of a colourless system.

- Soft and collinear QCD radiation from the final state particles
- Colour flow between initial and final state particles leading to non-trivial colour correlations

The top quark is massive

- The collinear limit is not singular $\longrightarrow$ LL structure unaffected
- Additional NLL from large-angle soft radiation


## Resummation for the $t \bar{t}$ production

- The first attempt to develop a $q_{T}$-resummation formalism at next-to-leading logarithmic (NLL) accuracy for $t \bar{t}$ production was done in [Berger, Meng (1994), Mrenna, Yuan (1997)]. However, they did not consider color mixing between singlet and oktet final states and missed the initial-final gluon exchange.
- The resummation for the $t \bar{t} q_{T}$ spectrum, based on soft collinear effective theory (SCET), was performed at NNLL+NLO. [Zhu, Li, Li, Shao, Yang (2013)]. This work is limited to the study of the $q_{T}$ cross section after integration over the azimuthal angles of the produced heavy quarks.
- The $q_{T}$-resummation in QCD was performed at the fully-differential level with respect to the kinematics of the produced heavy quarks. [Catani, Grazzini, Torre (2014)].


## The resummation procedure at small $q_{T}$

 $h_{1}\left(P_{1}\right)+h_{2}\left(P_{2}\right) \rightarrow Q\left(p_{3}\right)+\bar{Q}\left(p_{4}\right)+X$.- Consider the most general fully-differential cross section

$$
\frac{d \sigma\left(P_{1}, P_{2} ; \mathbf{q}_{\mathbf{T}}, M, y, \Omega\right)}{d^{2} \mathbf{q}_{\mathbf{T}} d M^{2} d y d \Omega},
$$

where $P_{1}$ and $P_{2}$ are the momenta of incoming hadrons, $\mathbf{q}_{\mathbf{T}}, M$ and $y$ are the transverse momentum vector, invariant mass and rapidity of the $Q \bar{Q}$ pair, $\Omega$ is a set of two additional independent kinematical variables that specify the angular distribution of heavy quarks with respect to the momentum $q$ of the $Q \bar{Q}$ pair. For instance $\Omega=\left\{y_{3}, \phi_{3}\right\}$.

- Decompose the cross section in a singular and a regular part

$$
d \sigma=d \sigma^{(\mathrm{sing})}+d \sigma^{(\mathrm{reg})}
$$

- $d \sigma^{\text {(sing) }}$ embodies all the singular terms in the limit $q_{T} \rightarrow 0$.
- $d \sigma^{(\mathrm{reg})}$ includes the remaining non-singular terms.


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- Decompose the cross section in a singular and a regular should be part replaced by $d \sigma^{\text {res }} d \sigma=d \sigma^{(\text {sing })}+d \sigma^{(\mathrm{reg})}$.
- $d \sigma^{\text {(sing })}$ embodies all the singular terms in the limit $q_{T} \rightarrow 0$.
- $d \sigma^{(\mathrm{reg})}$ includes the remaining non-singular terms.


## The all-order resummation formula

- Is obtained by working in impact parameter b space.

$$
\begin{aligned}
\frac{d \sigma^{(\mathrm{res})}}{d^{2} \mathbf{q}_{\mathbf{T}} d M^{2} d y d \Omega} & =\frac{M^{2}}{s} \sum_{c=q, \bar{q}, g}\left[d \sigma_{c c}^{(0)}\right] \int \frac{d^{2} \mathbf{b}}{(2 \pi)^{2}} e^{i \mathbf{b} \mathbf{q}_{\mathbf{T}}} S_{c}(M, b) \\
& \times \sum_{a_{1}, a_{2}} \int_{x_{1}}^{1} \frac{d z_{1}}{z_{1}} \int_{x_{2}}^{1} \frac{d z_{2}}{z_{2}}\left[(\boldsymbol{H} \Delta) C_{1} C_{2}\right]_{c \bar{c} ; a_{1} a_{2}} \times \\
& f_{a_{1} / h_{1}}\left(x_{1} / z_{1}, b_{0}^{2} / b^{2}\right) f_{a_{2} / h_{2}}\left(x_{2} / z_{2}, b_{0}^{2} / b^{2}\right)
\end{aligned}
$$

$b_{0}=2 e^{-\gamma_{E}}\left(\gamma_{E}\right.$ is the Euler number).

$$
x_{1}=\frac{M}{\sqrt{s}} e^{+y} \quad x_{2}=\frac{M}{\sqrt{s}} e^{-y}
$$

$\ln S_{c}(M, b)=\int_{M^{2}}^{b_{0}^{2} / b^{2}} \frac{d q^{2}}{q^{2}}\left[A_{c}\left(\alpha_{S}\left(q^{2}\right)\right) \ln \frac{M^{2}}{q^{2}}+B_{c}\left(\alpha_{S}\left(q^{2}\right)\right)\right]$.
$\mathrm{LL}: A_{c}^{(1)}, \quad \mathrm{NLL}: A_{c}^{(2)}, B_{c}^{(1)}$.

## The all-order resummation formula

$$
\begin{aligned}
\frac{d \sigma^{(\text {res })}}{d^{2} \mathbf{q}_{\mathbf{T}} d M^{2} d y d \Omega} & =\frac{M^{2}}{s} \sum_{c=q, \bar{q}, g}\left[d \sigma_{c c}^{(0)}\right] \int \frac{d^{2} \mathbf{b}}{(2 \pi)^{2}} e^{i \mathbf{b q _ { \mathbf { T } }}} S_{c}(M, b) \\
& \times \sum_{a_{1}, a_{2}} \int_{x_{1}}^{1} \frac{d z_{1}}{z_{1}} \int_{x_{2}}^{1} \frac{d z_{2}}{z_{2}}\left[(\mathbf{H} \Delta) C_{1} C_{2}\right]_{c \bar{c} ; a_{1} a_{2}} \times \\
& f_{a_{1} / h_{1}}\left(x_{1} / z_{1}, b_{0}^{2} / b^{2}\right) f_{a_{2} / h_{2}}\left(x_{2} / z_{2}, b_{0}^{2} / b^{2}\right)
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$$



## The soft-parton factor $\boldsymbol{\Delta}$

$\left[(\boldsymbol{H} \boldsymbol{\Delta}) C_{1} C_{2}\right]_{q \bar{q} ; a_{1} a_{2}}=\boldsymbol{H} \boldsymbol{\Delta}_{q \bar{q}} C_{q a_{1}}\left(\alpha_{S}\left(b_{0}^{2} / b^{2}\right)\right) C_{\bar{q} a_{2}}\left(\alpha_{S}\left(b_{0}^{2} / b^{2}\right)\right)$.
$\left[(\boldsymbol{H} \boldsymbol{\Delta}) C_{1} C_{2}\right]_{g g ; a_{1} a_{2}}=\boldsymbol{H} \boldsymbol{\Delta}_{g g} C_{g a_{1}}^{\mu_{1} \nu_{1}}\left(\mathbf{b} ; \alpha_{S}\left(b_{0}^{2} / b^{2}\right)\right) \cdot C_{g a_{2}}^{\mu_{2} \nu_{2}}\left(\mathbf{b} ; \alpha_{S}\left(b_{0}^{2} / b^{2}\right)\right)$.

$$
\begin{gathered}
\boldsymbol{H} \boldsymbol{\Delta}_{q \bar{q}}=\frac{\left\langle\tilde{\mathcal{M}}_{q \bar{q} \rightarrow Q \bar{Q}}\right| \Delta\left|\tilde{\mathcal{M}}_{q \bar{q} \rightarrow Q \bar{Q}}\right\rangle}{\alpha_{S}^{2}\left(M^{2}\right)\left|\mathcal{M}_{q \bar{q} \rightarrow Q \bar{Q}}^{(0)}\left(p_{1}, p_{2}, p_{3}, p_{4}\right)\right|^{2}} . \\
\left|\tilde{\mathcal{M}}_{c \bar{c} \rightarrow Q \bar{Q}}\right\rangle=\left[1-\tilde{I}_{c \bar{c} \rightarrow Q \bar{Q}}\right]\left|\mathcal{M}_{c \bar{c} \rightarrow Q \bar{Q}}\right\rangle .
\end{gathered}
$$

$\boldsymbol{\Delta}\left(b, M ; y_{34}, \phi_{3}\right)=\mathbf{V}^{\dagger}\left(b, M ; y_{34}\right) \mathbf{D}\left(\alpha_{S}\left(b_{0}^{2} / b^{2}\right) ; \phi_{3 b}, y_{34}\right) \mathbf{V}\left(b, M ; y_{34}\right)$.

$$
\mathbf{V}\left(b, M ; y_{34}\right)=\bar{P}_{q} \exp \left\{-\int_{b_{0}^{2} / b^{2}}^{M^{2}} \frac{d q^{2}}{q^{2}} \boldsymbol{\Gamma}_{t}\left(\alpha_{S}\left(q^{2}\right) ; y 34\right)\right\} .
$$

- $\Gamma_{t}$ is the soft anomalous dimension matrix.
- $\mathbf{D}$ is the azimuthal-correlation matrix.
$\left\langle\mathbf{D}\left(\alpha_{S} ; \phi_{3 b}, y_{34}\right)\right\rangle_{\mathrm{av} .}=1$.
- All the perturbative coefficients are computed at NLO QCD.


## The soft anomalous dimension operator

$$
\boldsymbol{\Gamma}_{t}\left(\alpha_{S} ; y_{34}\right)=\frac{\alpha_{S}}{\pi} \boldsymbol{\Gamma}_{t}^{(1)}\left(y_{34}\right)+\left(\frac{\alpha_{S}}{\pi}\right)^{2} \boldsymbol{\Gamma}_{t}^{(2)}\left(y_{34}\right)+\sum_{n=3}^{\infty}\left(\frac{\alpha_{S}}{\pi}\right)^{n} \boldsymbol{\Gamma}_{t}^{(n)}\left(y_{34}\right)
$$

- The colour basis: s-channel singlet-oktet exchange tensors
- $c_{1}^{q \bar{q}}=\delta_{i j} \delta_{k l}, \quad c_{2}^{q \bar{q}}=t_{j i}^{c} t_{k l}^{c}$,
- $c_{1}^{g g}=\delta_{a b} \delta_{k l}, \quad c_{2}^{g g}=i f^{a b c} t_{k l}^{c}, \quad c_{3}^{g g}=d^{a b c} t_{k l}^{c}$.

$$
\Gamma_{i j}=\frac{1}{\left\langle c_{i} \mid c_{i}\right\rangle}\left\langle c_{i}\right| \Gamma_{t}\left|c_{j}\right\rangle .
$$

The soft anomalous dimension matrix is non-diagonal $\longrightarrow$ needs to be diagonalized!
Transform the original basis $|I\rangle=R_{C l}|c\rangle$ by the diagonalization matrix

$$
R^{-1} \Gamma R=\Gamma^{\mathrm{diag}}
$$

In the new basis the matrix element of soft anomalous dimension operator is given by

$$
\Gamma_{I J}^{\text {diag }}=\left\langle\mathcal{X}_{l}\right| \Gamma_{t}|J\rangle,\left|\mathcal{X}_{l}\right\rangle=\sum_{J}\left(S^{-1}\right)_{J l}|J\rangle, S_{I J}=\langle I \mid J\rangle
$$

## RG evolution of the soft evolution operator

$$
\mathbf{V}\left(b, M ; y_{34}\right)=\bar{P}_{q} \exp \left\{-\int_{b_{0}^{2} / b^{2}}^{M^{2}} \frac{d q^{2}}{q^{2}} \Gamma_{t}\left(\alpha_{S}\left(q^{2}\right) ; y 34\right)\right\}
$$

V fulfils the following evolution equation

$$
\frac{d \mathbf{V}\left(b, Q ; y_{34}\right)}{d \ln \left(b_{0}^{2} / b^{2}\right)}=\Gamma_{t}\left(\alpha_{S}\left(b_{0}^{2} / b^{2}\right) ; y_{34}\right) \mathbf{V}\left(b, Q ; y_{34}\right)
$$

The solution to this equation can be written as

$$
\begin{gathered}
\mathbf{V}(b, Q)=\mathbf{K}\left(\alpha_{S}\left(b_{0}^{2} / b^{2}\right)\right) \mathbf{V}^{(\mathrm{LO})}\left(\alpha_{S}\left(b_{0}^{2} / b^{2}\right), \alpha_{S}\left(Q^{2}\right)\right) \mathbf{K}^{-1}\left(\alpha_{S}\left(Q^{2}\right)\right), \\
\mathbf{K}\left(\alpha_{S}\right)=\mathbf{1}+\sum_{n=1}^{\infty}\left(\frac{\alpha_{S}}{\pi}\right)^{n} \mathbf{K}^{(n)} . \\
\frac{d \mathbf{V}^{(\mathrm{LO})}\left(\alpha_{S}, \alpha_{S}^{\prime}\right)}{d \ln \alpha_{S}}=-\frac{1}{\beta_{0}} \boldsymbol{\Gamma}_{t}^{(1)} \mathbf{V}^{(\mathrm{LO})}\left(\alpha_{S}, \alpha_{S}^{\prime}\right), \\
\mathbf{V}^{(\mathrm{LO})}\left(\alpha_{S}\left(b_{0}^{2} / b^{2}\right), \alpha_{S}\left(Q^{2}\right)\right)=\exp \left\{\frac{1}{\beta_{0}} \boldsymbol{\Gamma}_{t}^{(1)} \ln \left(\frac{\alpha_{S}\left(Q^{2}\right)}{\alpha_{S}\left(b_{0}^{2} / b^{2}\right)}\right)\right\} . \\
V_{c C^{\prime}}^{(\mathrm{LO})}=\frac{1}{\langle c \mid c\rangle}\langle c| \mathbf{V}^{(\mathrm{LO})}\left|c^{\prime}\right\rangle=\sum_{l} R_{c l} \exp \left\{\frac{\lambda_{l}^{(1)}}{\beta_{0}} \int_{b_{0}^{2} / b^{2}}^{Q^{2}} \frac{d q^{2}}{q^{2}} \beta\left(\alpha_{S}\left(q^{2}\right)\right)\right\} R_{l c^{\prime}}^{-1},
\end{gathered}
$$

$\lambda_{\curlywedge}^{(1)}$ are the eigenvalues of first-order soft anomalous dimension matrix $=$

## The NLO+NLL structure of the final-state radiation

The $\boldsymbol{H} \boldsymbol{\Delta}$ factor can be organized as follows to all orders

$$
\begin{aligned}
& \boldsymbol{H} \boldsymbol{\Delta} \sim \sum_{\{C\}} \mathcal{H}^{\{C\}}\left(\alpha_{S}\left(\mu_{R}^{2}\right)\right) \exp \left\{\mathcal{G}_{\{C\}}\left(\alpha_{S}\left(\mu_{R}^{2}\right)\right\},\right. \\
& \mathcal{H}^{\{C\}}\left(\alpha_{S}\right)=\alpha_{S}^{2}\left(\mathcal{H}^{\{C\},(0)}+\frac{\alpha_{S}}{\pi} \mathcal{H}^{\{C\},(1)}+\mathcal{O}\left(\alpha_{S}\left(\mu_{R}^{2}\right)\right)\right), \\
& \underset{\text { GLL }}{\mathcal{G}_{\{C\}}\left(\alpha_{S}\right)}=g_{\{C\}}^{(2)}\left(\alpha_{S}\right)+\frac{\alpha_{S}}{\pi} g_{\{C N\}}^{(3)}\left(\alpha_{S}\right)+\cdots
\end{aligned}
$$

- At NLO + NLL $(Q=M)$

$$
g_{C}^{(2)}\left(\alpha_{S} L\right)=\frac{\lambda_{l}^{(1) *}+\lambda_{J}^{(1)}}{\beta_{0}} \ln (1-\lambda),
$$

with $\lambda=\frac{1}{\pi} \beta_{0} \alpha_{S}\left(\mu_{R}^{2}\right) L, L=\ln \frac{Q^{2} b^{2}}{b_{0}^{2}}$.

$$
\begin{aligned}
& \boldsymbol{H} \boldsymbol{\Delta} \sim \sum_{\{C\}} \alpha_{S}^{2}\left(\mathcal{H}^{\{C\},(0)} \exp \left\{g_{C}^{(2)}\right\}+\frac{\alpha_{S}}{\pi} \mathcal{H}^{\{C\},(1)}\right), \\
& \mathcal{H}^{\{C\},(0)}=\left\langle\widetilde{\mathcal{M}}^{(0)} \mid \mathcal{X}_{l}\right\rangle S_{I J}\left\langle\mathcal{X}_{J} \mid \widetilde{\mathcal{M}}^{(0)}\right\rangle \\
& \mathcal{H}^{\{C\},(1)}=\left(\left\langle\widetilde{\mathcal{M}}^{(1)} \mid \mathcal{X}_{\boldsymbol{X}}\right\rangle\left\langle\mathcal{X}_{J} \mid \widetilde{\mathcal{M}}^{(0)}\right\rangle+\left\langle\widetilde{\mathcal{M}}^{(0)} \mid \mathcal{X}_{\boldsymbol{X}}\right\rangle\left\langle\mathcal{X}_{J} \mid \widetilde{\mathcal{M}}^{(1)}\right\rangle\right) S_{I J} \equiv
\end{aligned}
$$

## Preliminary results at NLO+NLL

- Large distortion of the spectrum due to soft radiation off the top quarks.



## Resummation scale variation

- The resummation scale variation is of the order of $15 \%$ up to $q_{T}=100 \mathrm{GeV}$. Increases at large transverse momenta.



## Renormalisation and factorisation scale variation

- The renormalisation and factorisation scale variation is of the order of $15 \%$ in the low- $q_{T}$ region.
- shrinks a bit in the intermediate region, and increases at large transverse momenta, reaching to $100 \%$.



## Comparison to the data

- Both CMS and ATLAS experiments measured the $q_{T}$ distribution of the $t \bar{t}$ pair at the LHC at $\sqrt{s}=7 \mathrm{TeV}$. CMS-PAS-TOP-11-013, G. Aad et al. [ATLAS Collaboration], 2013.
- Very recently ATLAS has also published the measurements of normalized differential cross sections at $\sqrt{s}=8 \mathrm{TeV}$ M. Aaboud et al. [ATLAS Collaboration], 2016.

| $p_{T, t \bar{t}}[\mathrm{GeV}]$ | $\frac{1}{\sigma} \frac{d \sigma}{d p_{T, t t}}\left[\mathrm{TeV}^{-1}\right]$ ATLAS | $\frac{1}{\sigma} \frac{d \sigma}{d p_{T, t t}}\left[\mathrm{TeV}^{-1}\right] \mathrm{NLO}+\mathrm{NLL}$ |
| :---: | :---: | :---: |
| $0-30$ | $14.3 \pm 1.0$ | $14.96 \pm 0.99$ |
| $30-70$ | $7.60 \pm 0.16$ | $7.81 \pm 0.36$ |
| $70-120$ | $2.94 \pm 0.28$ | $2.84 \pm 0.28$ |
| $120-180$ | $1.14 \pm 0.12$ | $0.99 \pm 0.08$ |
| $180-250$ | $0.42 \pm 0.04$ | $0.34 \pm 0.02$ |
| $250-350$ | $0.143 \pm 0.018$ | $0.096 \pm 0.020$ |
| $350-1000$ | $0.0099 \pm 0.0015$ | $0.0062 \pm 0.0040$ |

Table : Normalized $p_{T, t t}$ distribution at $\sqrt{s}=8 \mathrm{TeV}$.

- Good agreement with the experimental results along the whole range of transverse momenta.
- The theoretical uncertainties are of the order of experimental uncertainties.


## $q_{T}$-subtraction

Knowledge of the low $q_{T}$ limit is essential also for the fixed order calculation in the $q_{T}$-subtraction formalism. $q_{T}$-subtraction formalism has been originally proposed for the production of colourless high-mass systems in hadron collisions. [Catani, Grazzini (2007)].
This subtraction formalism has been successfully applied to number of important processes of this class.

- pp $\rightarrow H$ [Catani, Grazzini (2007)].
- $p p \rightarrow V$. [Catani, Cieri, Ferrera, de Florian, Grazzini (2009)].
- pp $\rightarrow \gamma \gamma$. [Catani, Cieri, Ferrera, de Florian, Grazzini (2011)].
- pp $\rightarrow$ WH. [Ferrera, Grazzini, Tramontano (2011)].
- pp $\rightarrow \boldsymbol{Z} \gamma$. [Grazzini, Kallweit, Rathlev, Torre (2013)].
- pp $\rightarrow$ ZZ. [Cascioli, Gehrmann, Grazzini, Kallweit, Maierhöfer, von Manteuffel, Pozzorini, Rathlev, Tancredi, Weihs (2014)].
- $p p \rightarrow W^{+} W^{-}$. [Gehrmann, Grazzini, Kallweit, Maierhöfer, von Manteuffel, Pozzorini, Rathlev, Tancredi (2014)].
- pp $\rightarrow$ ZH. [Ferrera, Grazzini, Tramontano (2014)].


## $q_{T}$-subtraction for $t \bar{t}$

- The fully differential cross section at $\mathrm{N}(\mathrm{NLO})$ :

$$
\begin{array}{r}
d \sigma_{\mathrm{N}(\mathrm{NLO})}^{t \bar{t}}=\mathcal{H}_{\mathrm{N}(\mathrm{NLO})}^{t \bar{t}} \otimes d \sigma_{\mathrm{LO}}^{t \bar{t}}+\left[d \sigma_{\mathrm{N}(\mathrm{LO})}^{t \bar{t}+\mathrm{jet}}-d \sigma_{\mathrm{N}(\mathrm{LO})}^{\mathrm{CT}}\right] . \\
\text { Regular as } q_{T} \rightarrow 0
\end{array}
$$

- $\mathcal{H}_{\mathrm{N}(\mathrm{NLO})}^{t \bar{t}}$ is the hard factor, which contains information on the virtual corrections to the LO process.
- $d \sigma_{\mathrm{LO}}^{t \bar{t}}$ is the Born cross section.
- $d \sigma_{\mathrm{N}(\mathrm{LO})}^{t \bar{t}+\mathrm{jet}}$ is the $\mathrm{N}(\mathrm{LO})$ cross section of $t \bar{t}+\mathrm{jet}(\mathrm{s})$ process.
$-d \sigma_{\mathrm{N}(\mathrm{LO})}^{\mathrm{CT}}$ is the counterterm, which can be derived by expanding the resummation formula.


## Results at NLO

- Distributions for the $t \bar{t}$ system.



## Results at NLO

- Distributions for the top quark.

- Very good agreement!


## Results at NNLO

| Cross section $[\mathrm{pb}]$ | $\mathcal{O}\left(\alpha_{S}^{4}\right)_{q g}$ | $\mathcal{O}\left(\alpha_{S}^{4}\right)_{q(\bar{q}) q^{\prime}}$ |
| :---: | :---: | :---: |
| $q_{T}$ subtraction | $-2.25(5)$ | $0.151(3)$ |
| Top++ | -2.253 | 0.148 |

Table : $\mathcal{O}\left(\alpha_{S}^{4}\right)$ contribution to the total cross section for $t \bar{t}$ production at the LHC at $\sqrt{s}=8 \mathrm{TeV}$.

| Cross section [pb] | $\mathcal{O}\left(\alpha_{S}^{4}\right)_{q g}$ | $\mathcal{O}\left(\alpha_{S}^{4}\right)_{q(\bar{q}) q^{\prime}}$ |
| :---: | :---: | :---: |
| $q_{T}$ subtraction | $-61.5(5)$ | $1.33(1)$ |
| Top++ | -61.53 | 1.33 |

Table : $\mathcal{O}\left(\alpha_{S}^{4}\right)$ contribution to the total cross section for $t \bar{t}$ production at the LHC at $\sqrt{s}=2 \mathrm{TeV}$.
$q g=q g+\bar{q} g, \quad q(\bar{q}) q^{\prime}=q q+\bar{q} \bar{q}+q q^{\prime}+\bar{q} \bar{q}^{\prime}+q \bar{q}^{\prime}+\bar{q} q^{\prime}$

## Summary

- I have presented the computation of the $q_{T}$-resummed cross section for the $t \bar{t}$ production at hadron colliders at NLO+NLL in QCD.
- The calculation is more complicated with respect to hadroproduction of colourless systems due to the additional radiation of soft gluons off the top quarks.
- The soft radiation contribution is sizeable and makes the spectrum harder.
- Within the uncertainties our results agree with the most up-to-date ATLAS measurement of the $q_{T}$ distribution.
- The uncertainties due to the scale variations are large, being of the order of experimental uncertainties.
- We have used the knowledge of the low $q_{T}$ behaviour of the amplitudes to extend the $q_{T}$ subtraction method for the $t \bar{t}$ production at hadron colliders at NLO and NNLO in all non-diagonal channels.
- We have implemented the calculation in a fully-differential Monte Carlo program and found good agreement with the known results.


## Backup Slides

## Comparision to the fixed order

- The spectrum for $Q=\frac{m_{t}}{2}$ matches very well the fixed order curve at large transverse momenta.
- For the scales $Q=m_{t}$ and $Q=2 m_{t}$ the need of a switching procedure at intermediate values is evident.



## Azimuthal correlations

- Production of a colorless system
- The gluonic collinear functions are the only source of azimuthal correlations

$$
C_{g a}^{\mu \nu}\left(z ; p_{1}, p_{2}, \mathbf{b}\right)=d^{\mu, \nu}\left(p_{1}, p_{2}\right) C_{g a}(z)+D^{\mu, \nu}\left(p_{1}, p_{2} ; \mathbf{b}\right) G_{g a}(z)
$$

- Top-quark pair production
$\Delta\left(b, M ; y_{34}, \phi_{3}\right)=\mathbf{V}^{\dagger}\left(b, M ; y_{34}\right) \mathbf{D}\left(\alpha_{S} ; \phi_{3 b}, y_{34}\right) \mathbf{V}\left(b, M ; y_{34}\right)$.
- Additional azimuthal correlations produced by the dynamics of soft-parton radiation, embodied in D.
$\mathbf{D}\left(\alpha_{S} ; \phi_{3 b}, y_{34}\right)=1+\frac{\alpha_{S}}{\pi} \mathbf{D}^{(1)}\left(\phi_{3 b}, y_{34}\right)+\mathcal{O}\left(\alpha_{S}^{2}\right)$
$\left\langle\mathbf{D}\left(\alpha_{S}\left(b_{0}^{2} / b^{2}\right) ; \phi_{3 b}, y_{34}\right)\right\rangle_{\mathrm{av} .}=1 \rightarrow$ vanishing contribution to $\langle\sigma\rangle_{\mathrm{av} .}$ at $\mathcal{O}\left(\alpha_{S}\right)$
But contributes at $\mathcal{O}\left(\alpha_{S}^{2}\right)$ due to the interference of the initial-final state azimuthal correlations
$\longrightarrow$ non-trivial integration over the azimuthal angle (computed analytically!)


## Our fixed-order implementation

Up to NLO our implementation is based on

- The scattering amplitudes and phase space generation of MCFM program.
- We use the corresponding routines of Higgs boson production code HNNLO and vector boson production code DYNNLO, suitably modified for the $t \bar{t}$ production.
At NNLO accuracy the $t \bar{t}+$ jet cross section is evaluated by using the MUNICH code which provides:
- Fully automatic implementation of the NLO dipole subtraction formalism.
- Interface to the OPENLOOPS one-loop generator.

