Theory precision for the qqH inclusive XS

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The inclusive Higgs cross section

 $\sim 90\%$ of the inclusive Higgs cross section comes from gluon fusion

The new LHCHXSWG recommendation for the ggH XS is based on the Zurich result: (LHC 13 TeV, $m_H=125~{\rm GeV}$)

```
\sigma = 48.58 \,\mathrm{pb} = 16.00 \,\mathrm{pb} \\ + 20.84 \,\mathrm{pb} \\ + 9.56 \,\mathrm{pb} \\ + 1.49 \,\mathrm{pb} \\ - 2.05 \,\mathrm{pb} \\ + 0.34 \,\mathrm{pb} \\ + 2.40 \,\mathrm{pb}  (NNLO, rEFT)

((t, b, c), \,\mathrm{exact \, NLO}) \\ + (NNLO, \,1/m_t) \\ + (NNLO, \,1/m_t) \\ + (NNLO, \,1/m_t)
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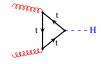
[Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger 1602.00695]

A long story 1977....2016...

rEFT: Born-rescaled Effective Field Theory

Leading order

[Wilczek 1977] [Georgi, Glashow, Machacek, Nanopoulos 1977]



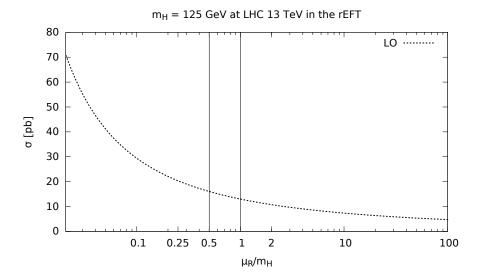
The limit $m_t \gg m_H$ is finite, and leads to an EFT

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4v} \frac{C_1}{C_1} G^a_{\mu\nu} G^{a\mu\nu} H$$

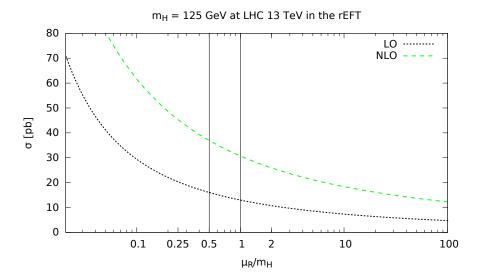
with effective Hgg vertex



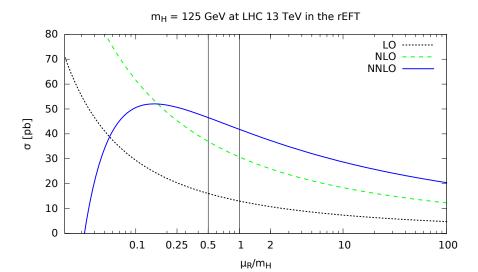
rescaled EFT (rEFT):
$$\sigma^{\mathrm{rEFT}} = \frac{\sigma_{\mathrm{LO}}^{\mathrm{exact}}(m_t)}{\sigma_{\mathrm{LO}}^{\mathrm{EFT}}} \sigma^{\mathrm{EFT}}$$



[Wilczek 1977] [Georgi, Glashow, Machacek, Nanopoulos 1977]



[Dawson 1991] [Djouadi, Spira, Zerwas 1991]



[Harlander, Kilgore 2002] [Anastasiou, Melnikov 2002]

Beyond NNLO

- Approximate N³LO
 - soft approximation (only log terms)
 - soft + high-energy approximation
 - soft + next-to-soft approximation

[Moch,Vogt 2005] [Ball,MB,Forte,Marzani,Ridolfi 2013] [deFlorian,Mazzitelli,Moch,Vogt 2014]

Gluon luminosity, peaked at small x, enhances the partonic coefficient at large z

$$\sigma_{gg} = \tau \int_{\tau}^{1} \frac{dz}{z} \, \mathscr{L}_{gg} \left(\frac{\tau}{z}\right) C_{gg}(z), \qquad \tau = \frac{m_H^2}{s}$$

The soft $z \to 1$ region dominates

[Becher, Neubert, Xu 2007] [MB, Forte, Ridolfi 2012]

Approximations based on the knowledge of this limit

Beyond NNLO

Approximate N³LO

- soft approximation (only log terms)
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[Moch,Vogt 2005] [Ball,MB,Forte,Marzani,Ridolfi 2013] [deFlorian,Mazzitelli,Moch,Vogt 2014]

Full N³LO

Wilson coefficient at N³LO [Chetyrkin,Kniehl,Steinhauser 1997] three loops [Baikov, Chetyrkin,Smirnov,Smirnov,Steinhauser 2009] [Lee,Smirnov,Smirnov 2010] [Gehrmann, Glover,Huber,Ikizlerli,Studerus 2010] one emission at two loops [Gehrmann,Jaquier,Glover, Koukoutsakis 2012] [Duhr,Gehrmann 2013] [Li,Zhu 2013] one emission at one loop [Anastasiou,Duhr,Dulat,Herzog,Mistlberger 2013] [Kilgore 2013] three emissions (soft expansion) [Anastasiou,Duhr,Dulat,Mistlberger 2013] scale dependent terms [Anastasiou, Bühler,Duhr,Herzog 2012] [Höschele,Hoff,Pak,Steinhauser,Ueda 2012] [Bühler,Lazopoulos 2013] two emissions at one loop [Li,vonManteuffel,Schabinger,Zhu 2014] all soft and next-to-soft terms at N³LO [Anastasiou,Duhr,Dulat,Furlan,Gehrmann,Herzog,Mistlberger 2014] 37 terms in the soft expansion [Anastasiou,Duhr,Dulat,Herzog,Mistlberger 2015] exact qq' [Anzai,Hasselhuhn,Höschele,Hoff,Kilgore,Steinhauser,Ueda 2015]

$$C^{(3)}(z) = C_{\text{soft}}^{(3)}(z) + \sum_{k=0}^{5} \log^k (1-z) \sum_{j=0}^{\infty \to 37} c_{kj} (1-z)^j$$

Beyond NNLO

- Approximate N³LO
 - soft approximation (only log terms)
 - soft + high-energy approximation
 - soft + next-to-soft approximation

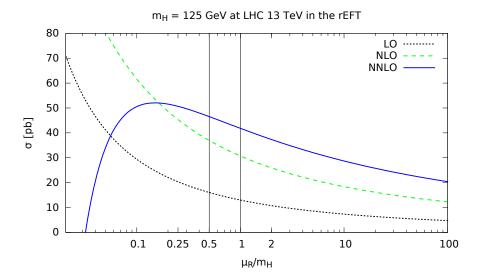
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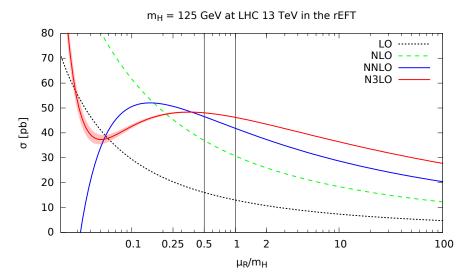
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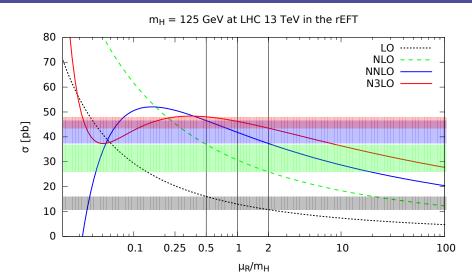
threshold resummation

[Catani,deFlorian,Grazzini,Nason 2003] [MB,Marzani 2014]
[Ahrens,Becher,Neubert,Yang 2008] [MB,Rottoli 2014]
[Schmidt,Spira 2015] [MB,Marzani,Muselli,Rottoli 2016]

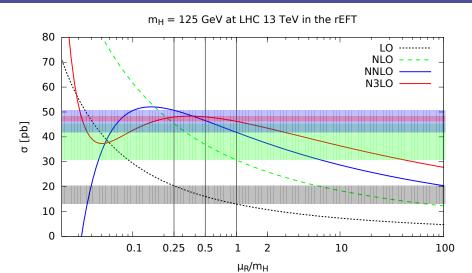




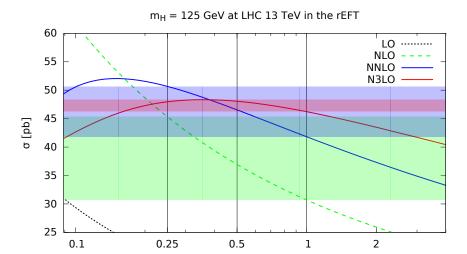
Red band: error from the truncation to 37 terms



$$1/2 < \mu_{\rm R}/m_{\rm H} < 2$$



$$1/4 < \mu_{\rm R}/m_{\rm H} < 1$$



N³LO scale variation error: very small and very asymmetric

 μ_R/m_H

Interpretation and reliability of scale variation error

LHCHXSWG interpretation: 100% c.l. flat interval LHCHXSWG alternative interpretation: 68% c.l. gaussian interval

Either interpretation is arbitrary — no statistical foundation

Criticisms:

- ullet close to stationary point o symmetrize error
- the pattern at lower orders suggests that scale variation is not a good estimator
- sensitivity to the choice of the central scale

Alternatives?

- Cacciari-Houdeau
- probe higher orders with different contributions
- revert to a different perturbative expansion

[MB, Marzani, Muselli, Rottoli 2016]

Threshold resummation

Resums to all orders in α_s logarithmic terms $\log(1-z)$ in the partonic coefficient

$$C_{gg}(N, \alpha_s) \stackrel{N \to \infty}{=} g_0\left(\alpha_s, \frac{m_H}{m_t}\right) \times \exp S(\alpha_s, \ln N)$$

$$\alpha_s S(\alpha_s, \ln N) = g_1(\alpha_s \ln N) + \alpha_s g_2(\alpha_s \ln N) + \alpha_s^2 g_3(\alpha_s \ln N) + \alpha_s^3 g_4(\alpha_s \ln N) + \dots$$

For years the LHCHXSWG recommendation was based on NNLO+NNLL

[Catani,deFlorian,Grazzini,Nason 2003] [deFlorian,Grazzini 2012]

NNLO+N³LL also available

dQCD: [MB,Marzani 2014] [Schmidt,Spira 2015]

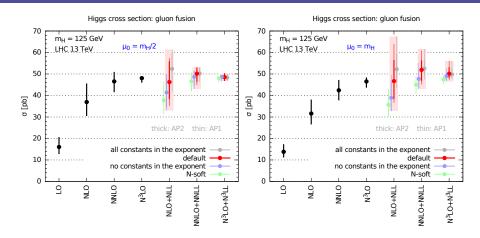
SCET: [Ahrens, Becher, Neubert, Yang 2008] [MB, Rottoli 2014]

 N^3LO+N^3LL recently released

[MB,Marzani,Muselli,Rottoli 2016]

42 variations

Threshold resummed perturbative expansion



Perturbative convergence sped up! Reduction of theory error increasing the order Less sensitivity to central scale More robust error estimate (statistical interpretation still missing...)

[MB, Marzani, Muselli, Rottoli 2016]

All Things Considered

slide by Luca Rottoli

	$\mu_0 = m_{\rm H}/4$	$\mu_0 = m_{\rm H}/2$	$\mu_0 = m_H$	$\mu_0 = 2m_{\rm H}$
LO	18.6+5.8	$16.0^{+4.3}_{-3.1}$	$13.8^{+3.2}_{-2.4}$	$11.9^{+2.5}_{-1.9}$
NLO	$44.2^{+12.0}_{-8.5}$	$36.9^{+8.4}_{-6.2}$	$31.6^{+6.3}_{-4.8}$	$27.5^{+4.9}_{-3.9}$
NNLO	$50.7^{+3.4}_{-4.6}$	$46.5^{+4.2}_{-4.7}$	$42.4_{-4.4}^{+4.6}$	$38.6^{+4.4}_{-4.0}$
N^3LO	$48.1^{+0.0}_{-7.5}$	$48.1_{-1.8}^{+0.1}$	$46.5_{-2.6}^{+1.6}$	$44.3^{+2.5}_{-2.9}$

Scale Variations

The answer to the ultimate question of life, the universe and everything

	$\mu_0 = m_{\rm H}/4$	$\mu_0 = m_{\scriptscriptstyle \mathrm{H}}/2$	$\mu_0 = m_{\rm H}$	$\mu_0 = 2m_{\mathrm{H}}$
LO+LL	$24.0^{+8.9}_{-6.8}$	$20.1^{+6.2}_{-5.0}$	$16.9^{+4.5}_{-3.7}$	$14.3^{+3.3}_{-2.8}$
$_{\rm NLO+NLL}$	$46.9^{+15.1}_{-12.6}$	$46.2^{+15.0}_{-13.2}$	$46.7^{+20.8}_{-13.8}$	$47.3^{+26.1}_{-15.8}$
${\rm NNLO+NNLL}$	$50.2^{+5.5}_{-5.3}$	$50.1^{+3.0}_{-7.1}$	$51.9^{+9.6}_{-8.9}$	$54.9^{+17.6}_{-11.5}$
$\rm N^3LO{+}N^3LL$	$47.7^{+1.0}_{-6.8}$	$48.5^{+1.5}_{-1.9}$	$50.1^{+5.9}_{-3.5}$	$52.9^{+13.1}_{-5.3}$

	$\mu_0 = m_{\scriptscriptstyle \mathrm{H}}/4$	$\mu_0=m_{\rm H}/2$	$\mu_0=m_{\rm H}$	$\mu_0=2m_{\rm H}$
Fixed-order expansion				
Resummed expansion	48.9 ± 0.5	48.9 ± 0.6	50.2 ± 1.0	52.6 ± 1.6
	1			

Acceleration

Cacciari-Hodeau

	$\mu_0=m_{\rm H}/4$	$\mu_0=m_{\rm H}/2$	$\mu_0=m_{\rm h}$	$\mu_0=2m_{\scriptscriptstyle \mathrm{H}}$
СН	$48.1 \pm 0.7 (1.2)$	$48.1 \pm 0.6 (1.0)$	$46.5 \pm 2.1 (3.5)$	$44.3 \pm 3.5 (5.8)$
$\overline{\mathrm{CH}}$	$48.1 \pm 1.2 (1.9)$	$48.1 \pm 1.2 (2.0)$	$46.5 \pm 4.2 (7.0)$	$44.3 \pm 6.9 (11.5)$

PSR 2016, Paris, July 4-6, 2016



$$\sigma = 48.58 \,\mathrm{pb} = 16.00 \,\mathrm{pb} \\ + 20.84 \,\mathrm{pb} \\ + 9.56 \,\mathrm{pb} \\ + 1.49 \,\mathrm{pb} \\ - 2.05 \,\mathrm{pb} \\ + 0.34 \,\mathrm{pb} \\ + 2.40 \,\mathrm{pb}$$
 (CO, rEFT)
(NLO, rEFT)
(NNLO, rEFT)
((t, b, c), exact NLO)
(NNLO, 1/m_t)
(EW, QCD-EW)

Quark mass effects

top quark beyond LO and light quarks:

- NLO: exact result for any quark [Spira,Djouadi,Graudenz,Zerwas 1995]
- ullet NNLO: top quark mass effects as an expansion in $1/m_t$ [Harlander,(Mantler,Marzani),Ozeren 2009(10)] [Pak,Rogal,Steinhauser 2009]

Numerical impact:

top mass corrections (corrections to rEFT)

$$\begin{split} \sigma_{\text{exact, only top}} - \sigma_{\text{rEFT}} = & 0 & \text{LO} \\ -0.24 \, \text{pb} & \text{NLO} \\ +0.34 \, \text{pb} & \text{NNLO (1}/m_t \text{ corrections)} \end{split}$$

for comparison, the corrections to the pure EFT are

$$\sigma_{\text{exact, only top}} - \sigma_{\text{EFT}} = \begin{array}{ccc} +0.95\,\mathrm{pb} & \text{LO} \\ +0.99\,\mathrm{pb} & \text{NLO} \\ +0.90\,\mathrm{pb} & \text{NNLO (1/}m_t \text{ corrections)} \end{array}$$

rEFT much closer to exact than pure EFT!

Quark mass effects

bottom and charm corrections

$$\begin{array}{lll} \sigma_{\rm exact, \ t+b+c} - \sigma_{\rm exact, \ only \ top} = & -1.17 \, {\rm pb} & \quad \ \ \, {\rm LO} \\ & -0.66 \, {\rm pb} & \quad \ \ \, {\rm NLO} \end{array}$$

Uncertainty due to missing b, c effects beyond NLO:

• option 1 (Zurich): take the relative effect at $\mathcal{O}(\alpha_s)$ (NLO) and apply to the $\mathcal{O}(\alpha_s^2)$

$$\frac{|-0.66\,\mathrm{pb}|}{\sigma_{\mathrm{NLO}} - \sigma_{\mathrm{LO}}} \times (\sigma_{\mathrm{NNLO}} - \sigma_{\mathrm{NLO}}) = 0.32\,\mathrm{pb}$$

Enlarge by factor 1.3 to take into account differences between $\overline{\text{MS}}$ and pole masses: $\pm 0.40\,\mathrm{pb}$

• option 2: take the NLO effect $(\pm 0.66 \, \mathrm{pb})$ as the uncertainty

Uncertainty due to missing $1/m_t$ corrections: $\pm 1\% = \pm 0.49 \,\mathrm{pb}$

Parametric uncertainties due to quark masses are negligible

Quark mass effects at resummed level

Threshold resummation

$$C_{gg}(N, \alpha_s) \stackrel{N \to \infty}{=} g_0\left(\alpha_s, \frac{m_{\rm H}}{m_t}, \frac{m_{\rm H}}{m_b}, \ldots\right) \times \exp S(\alpha_s, \ln N)$$

quark mass dependence appears only in g_0 , and is determined by matching to fixed order.

- ullet include in g_0 all known mass dependent terms $\hbox{[deFlorian,Grazzini~2012]~[MB,Marzani~2014]}$
- include only the exact top at NLL only [Schmidt,Spira 2015] Motivation: bottom quarks generate additional logarithms in g_0 that are not resummed \rightarrow fixed order treatment is preferred

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\sigma = 48.58 \,\mathrm{pb} = 16.00 \,\mathrm{pb} \\ + 20.84 \,\mathrm{pb} \\ + 9.56 \,\mathrm{pb} \\ + 1.49 \,\mathrm{pb} \\ - 2.05 \,\mathrm{pb} \\ + 0.34 \,\mathrm{pb} \\ + 2.40 \,\mathrm{pb}  (LO, rEFT)

(NLO, rEFT)

(NNLO, rEFT)

((t, b, c), exact NLO)

(NNLO, 1/m_t)

(EW, QCD-EW)
```

Electroweak corrections

Cross section gets EW corrections as well:

$$\sigma = \sigma_0 \left[1 + \alpha_s \sigma_1 + \alpha_s^2 \sigma_2 + \alpha_s^3 \sigma_3 + \ldots + \alpha \lambda_{\text{EW}} (1 + \alpha_s s_1 + \ldots) + \alpha^2 \cdots \right]$$

- Additive approach + mixed QCD-EW (Zurich): Estimate s_1 from an EFT $(m_H \ll m_{Z,W})$ [Anastasiou,Boughezal,Petriello 2008] Gives a +4.9% effect Uncertainty estimated by varying s_1 : $\pm 1\%$
- Complete factorization:

[Actis,Passarino,Sturm,Uccirati 2008]

$$\sigma = \sigma_0 (1 + \alpha \lambda_{\text{EW}}) \left[1 + \alpha_s \sigma_1 + \alpha_s^2 \sigma_2 + \alpha_s^3 \sigma_3 + \dots \right]$$

Gives a +5.1% effect

Uncertainty estimated by comparing the complete factorized result to the additive one: $\pm 2.5\%$

Missing N³LO PDFs

All results are computed with NNLO PDFs.

What's the expected impact of N³LO PDFs?

Compare at NNLO the difference between NNLO and NLO PDFs

$$\frac{\sigma_{\mathrm{NNLO}}(\mathrm{NNLO\;PDFs}) - \sigma_{\mathrm{NNLO}}(\mathrm{NLO\;PDFs})}{\sigma_{\mathrm{NNLO}}(\mathrm{NNLO\;PDFs})}$$

and use half this value (Zurich) Gives $\pm 0.56 \text{ pb}$

 Cacciari-Houdeau approach Sequence:

[Forte, Isgrò, Vita 2013]

 $\sigma_{\mathrm{N^3LO}}(\mathrm{LO~PDFs}),~\sigma_{\mathrm{N^3LO}}(\mathrm{NLO~PDFs}),~\sigma_{\mathrm{N^3LO}}(\mathrm{NNLO~PDFs})$

Gives a ± 1 pb at 68% DoB

PDF + α_s uncertainty

PDF4LHC 15 prescription (using the hessian set PDF4LHC15_nnlo_100)

Gives $\pm 1.56 \, \mathrm{pb}$

Sum in quadrature of $\pm 0.90\,\mathrm{pb}$ from PDFs and $\pm 1.26\,\mathrm{pb}$ from $\alpha_s=0.1180\pm0.0015$

Conclusions

The inclusive ggH cross section is a hot topic

State of the art (after 40 years):

- N 3 LO QCD large- m_t EFT
- NLO QCD exact
- NNLO QCD top mass corrections
- NLO EW + mixed NLO QCD-EW in the EFT
- N³LL threshold resummation (QCD)

codes

ggHiggs SusHi ihixs

HIGLUE

TROLL

RGHiggs

LHCHXSWG recommendation:

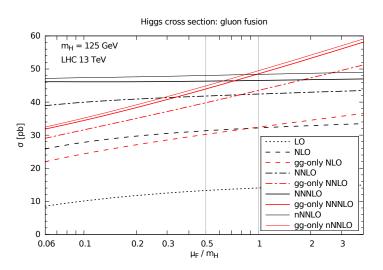
$$\sigma = 48.6^{\,+2.2}_{\,-3.3}\,\mathrm{pb}\,\mathrm{(theory)} \pm 1.56\,\mathrm{pb}\,\mathrm{(PDF} + \alpha_s)$$

Some theory uncertainties are subject to debate

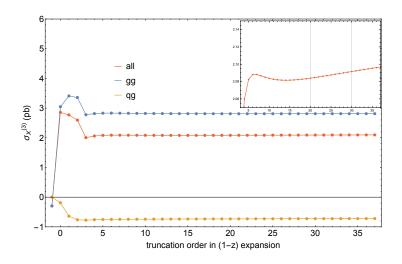
General (personal) comment: a robust and statistically sound way of assessing theory uncertainties is called for

Backup slides

Factorization scale dependence



Soft expansion



Improved threshold resummation

Standard dQCD resummation gives

$$C_{gg}(N, \alpha_s) \stackrel{N \to \infty}{=} g_0\left(\alpha_s, \frac{m_H}{m_t}\right) \times \exp S(\alpha_s, \ln N)$$

$$\alpha_s S(\alpha_s, \ln N) = g_1(\alpha_s \ln N) + \frac{\alpha_s}{\alpha_s} g_2(\alpha_s \ln N) + \frac{\alpha_s^2}{\alpha_s^2} g_3(\alpha_s \ln N) + \frac{\alpha_s^3}{\alpha_s^3} g_4(\alpha_s \ln N) + \dots$$

[Sterman 1987] [Catani, Trentadue 1989] [Forte, Ridolfi 2003]

Resums $\ln^j N$, contains constants (corresponding to $\delta(1-z)$), and nothing else.

Resummation doesn't fix subleading contributions suppressed by 1/N!

Can be improved taking into account

[MB,Marzani 2014]

• exact single gluon emission kinematics

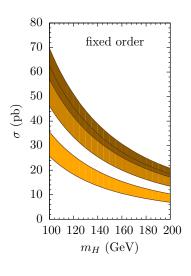
$$\ln N \to \psi_0(N)$$

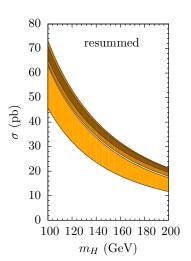
ullet collinear contributions from the full splitting function P_{qq}

$$N \to N+1$$
 (in its simplest form)

exponentiation of (some) constants

[Ahrens, Becher, Neubert, Yang 2008]





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Sequence transformations according to Weniger

Idea (Stirling, Euler): speed up convergence by applying a **transformation** to the sequence s_n

$$\lim_{n \to \infty} \frac{s_n' - s}{s_n - s} = 0$$

slide by Luca Rottoli

Very wide class of sequence transformation

$$\mathcal{G}_{k}^{(n)}(q_{m},s_{n},\omega_{n}) = \frac{\sum_{j=0}^{k} (-1)^{j} \binom{k}{j} \prod_{m=1}^{k-1} \frac{n+j+q_{m}}{n+k+q_{m}} \frac{s_{n+j}}{\omega_{n+j}}}{\sum_{j=0}^{k} (-1)^{j} \binom{k}{j} \prod_{m=1}^{k-1} \frac{n+j+q_{m}}{n+k+q_{m}} \frac{1}{\omega_{n+j}}}$$

Application to the inclusive Higgs cross section

Choose some good algorithms and compute some guesses David, Passarino 2013

Choose **many** 0(100) algorithms and compute **many** guesses Bonvini, Marzani, Muselli, LR 2016

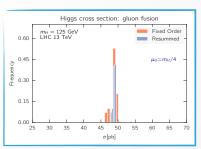
- No information on the asymptotic behaviour of the series, so it is not clear how to prefer an algorithm rather than another
- Result should not depend on the scale

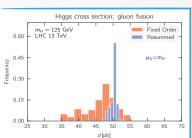


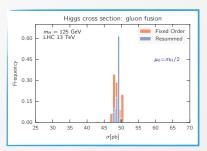
Theory precision for the qqH inclusive XS

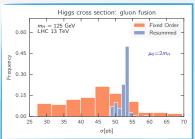
Higgs cross section results

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I believe that we do not know anything for certain, but everything probably (Christiaan Huygens)

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Statistical model for the interpretation of theory errors, from which one can compute the uncertainty on the truncated perturbative series for a given degree of belief (DoB) given the first terms in the expansion. Cacciari, Houdeau (2011)

Probability density for σ

CH
$$\sigma = \sigma_{LO} \sum_{k=0}^{\infty} c_k(\lambda) \left(\frac{\alpha_s}{\lambda}\right)^k$$

$$\overline{\text{CH}}$$
 $\sigma = \sigma_{\text{LO}} \sum_{k=0}^{\infty} b_k(\lambda, k_0) (k + k_0)! \left(\frac{\alpha_s}{\lambda}\right)^k$

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Possible power growth

Possible factorial growth Bagnaschi, Cacciari, Guffanti, Jenniches (2014)

Determination of λ

- Survey over several observables (assumes λ is process-independent)
 - Bagnaschi, Cacciari, Guffanti, Jenniches (2014
- fit λ requiring the first known coefficients are of the same size

Forte, Isgrò, Vita (2013)

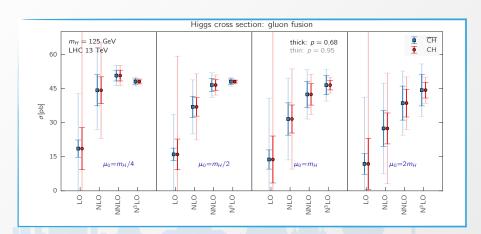


Theory precision for the qqH inclusive XS

Higgs cross section results

slide by Luca Rottoli

Pascal bet on the existence of God basing on calculation of probabilities, we use calculation of probabilities to bet on the value of the cross section of the God's particle Higgs



Scale dependence of resummed result

