

Theory status of bottom-quark associated Higgs production



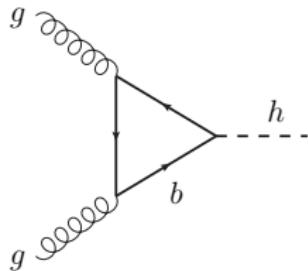
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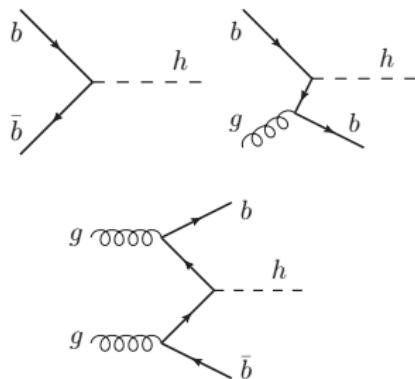


Production modes providing access to Y_b



Gluon fusion

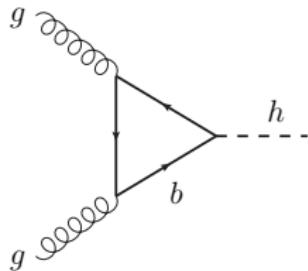
- ▶ bottom-top loop interference
- ▶ few % of total Higgs production xs



Bottom - associated production

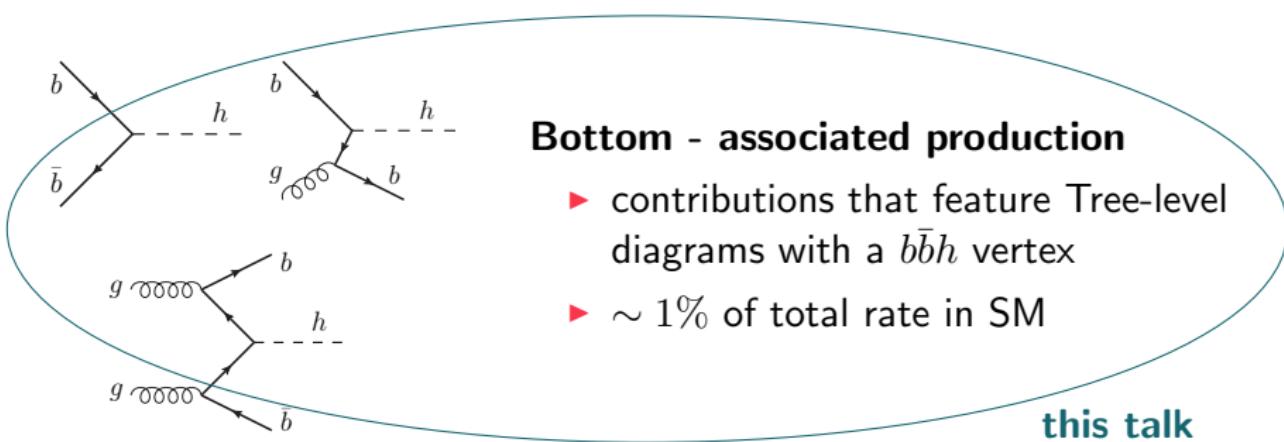
- ▶ contributions that feature Tree-level diagrams with a $b\bar{b}h$ vertex
- ▶ $\sim 1\%$ of total rate in SM

Production modes providing access to Y_b



Gluon fusion

- ▶ bottom-top loop interference
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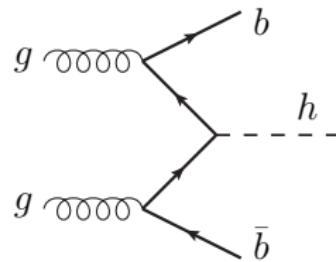
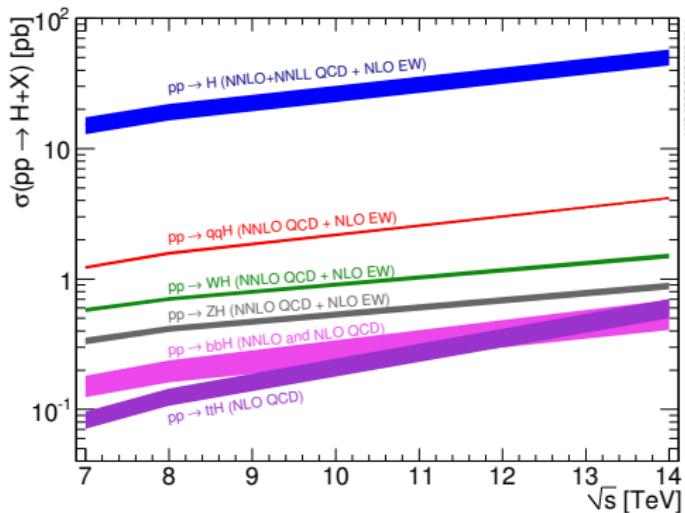


Bottom - associated production

- ▶ contributions that feature Tree-level diagrams with a $b\bar{b}h$ vertex
- ▶ $\sim 1\%$ of total rate in SM

this talk

$b\bar{b}H$ @ LHC is small but non-negligible



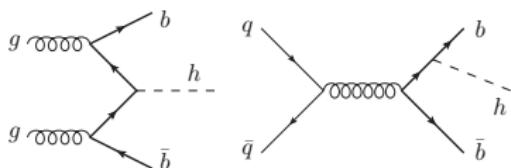
- ▶ total cross section small, but similar to $t\bar{t}h$ in SM
- ▶ enhanced in BSM scenarios, e.g. large $\tan\beta$ 2HDM, SUSY, ...
- ▶ common features/issues with other processes involving “initial-state” heavy quarks (e.g. single-top, $b\bar{b}Z$, ...)

4F/5F schemes are valid in opposite limits

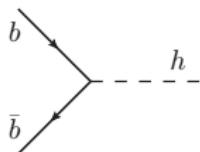
$$m_b \sim m_H$$

$$m_b \ll m_H$$

4-flavour scheme



5-flavour scheme



- ▶ finite- m_b effects ✓
- ▶ collinear logs $\sim \log(m_b/m_H)$ may spoil perturbative behaviour of cross section ✗
- ▶ $\log(m_b/m_H)$ -terms resummed via DGLAP evolution in effective b -quark PDF ✓
- ▶ no mass power-corrections ✗

Aside from power-corrections m_b^2/m_H^2 , the two methods of computing the cross section agree at all orders.

$b\bar{b}h$ inclusive cross section

4F NLO [Dittmaier et al '03; Dawson et al '03, Wiesemann et al. '14 (`mg5_aMC`)]

5F NNLO [Harlander, Kilgore '03 (`bbh@nnlo`, `Sushi`); Bühler et al '12]

Santander Matching [Harlander, Krämer, Schumacher '11]

NLO+NLL(NNLL_{partial}) matched results [Bonvini, AP,Tackmann '15,'16] } focus
FONLL-A,B results [Forte, Napoletano, Ubiali '15,'16]

Fixed order region: $m_b \sim Q \sim m_H$

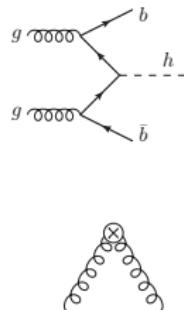
Single matching step required

$$\begin{array}{c} \text{---} \mu_H \sim m_b \sim Q \\ \uparrow \\ \text{DGLAP } n_f = 4 \\ \downarrow \\ \text{---} \mu_\Lambda \end{array}$$

full QCD



theory of collinear gluons & light
quarks
(up to corrections of $\Lambda_{\text{QCD}}^2/Q^2$)
[standard QCD factorization]

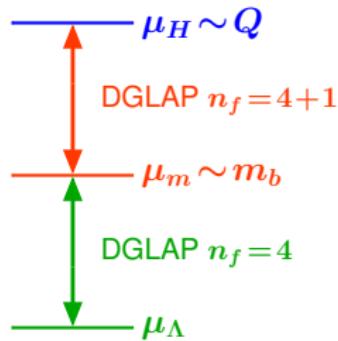


$$\sigma^{\text{FO}} = \sum_{i,j=g,q,\bar{q}} \mathcal{D}_{ij}(m_H, m_b, \mu_F) f_i^{[4]}(\mu_F) f_j^{[4]}(\mu_F)$$

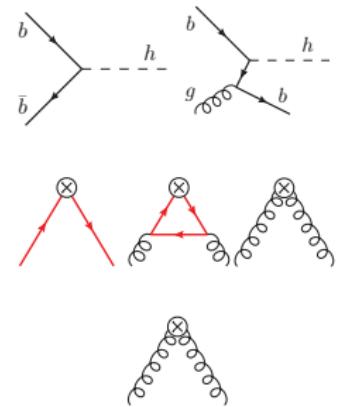
$$f_i^{[4]}(\mu_F) = \mathcal{U}_{ik}^{[4]}(\mu_F, \mu_0) f_k^{[4]}(\mu_0)$$

Resummation region: $m_b \ll Q \sim m_H$

Two matching steps required



full QCD
↓
theory of collinear gluons, light
quarks & b -quarks
(up to corrections of m_b^2/Q^2)
↓
theory of collinear gluons & light
quarks



$$\sigma^{\text{Resum}} = \sum_{i,j=g,q,\bar{q},b,\bar{b}} \mathcal{C}_{ij}(m_H, \mu_F) f_i^{[5]}(m_b, \mu_m, \mu_F) f_j^{[5]}(m_b, \mu_m, \mu_F)$$

$$f_i^{[5]}(m_b, \mu_m, \mu_F) = \sum_{\substack{k=g,q,\bar{q},b,\bar{b} \\ l,p=g,q,\bar{q}}} \mathcal{U}_{jk}^{[5]}(\mu_F, \mu_m) \mathcal{M}_{kl}(m_b, \mu_m) \mathcal{U}_{lp}^{[4]}(\mu_b, \mu_0) f_p^{[4]}(\mu_0)$$

↑
standard 5F-PDF set construction ($\mu_m = m_b$)

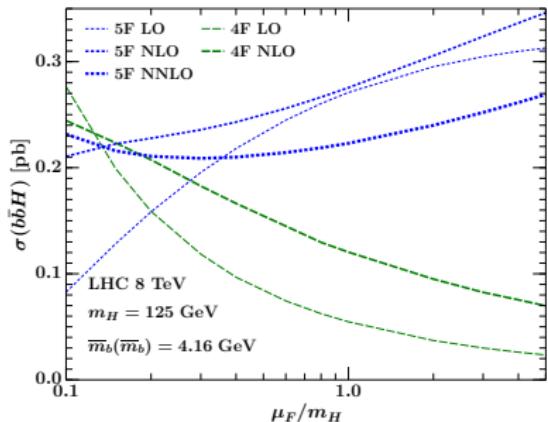
Perturbative counting

Standard counting

- ▶ counting in 4F and 5F schemes assigned only to $\textcolor{blue}{D}_i$ & $\textcolor{blue}{C}_{i,b}$
- ▶ in particular, 5FS counts $f_b^{[5]} \sim 1$

$$f_b^{[5]}(m_b, \mu_H) = \left[\textcolor{red}{U}_{bg}^{[5]}(\mu_H, \mu_m) + \textcolor{red}{U}_{bb}^{[5]}(\mu_H, \mu_m) \mathcal{M}_{bg}^{(1)}(m_b, \mu_m) + \dots \right] \textcolor{blue}{f}_g^{[4]}(\mu_m)$$
$$\sim 1 \qquad \qquad \qquad \sim \alpha_s \qquad \qquad \qquad (\text{naïvely})$$

At fixed order results can differ wildly



- ▶ LO: huge μ_F dependence
- ▶ NLO: not much better

Until recently, a theoretically consistent approach for combining virtues of 4F and 5F schemes for $b\bar{b}H$ was missing.

- ▶ large differences are due to different logarithmic structures present at fixed-order in each scheme
- ▶ ‘best’ prediction was a weighted average of 4F & 5F:
Santander Matching [Harlander,Krämer,Schumacher '11]

Perturbative counting

b -PDF is perturbative and counts as $\mathcal{O}(\alpha_s)$

- ▶ counting in 4F and 5F schemes assigned only to D_i & $C_{i,b}$
- ▶ in particular, 5FS counts $f_b^{[5]} \sim 1$

$$f_b^{[5]}(m_b, \mu_H) = \left[\textcolor{red}{U}_{bg}^{[5]}(\mu_H, \mu_m) + \textcolor{red}{U}_{bb}^{[5]}(\mu_H, \mu_m) \mathcal{M}_{bg}^{(1)}(m_b, \mu_m) + \dots \right] \textcolor{teal}{f}_g^{[4]}(\mu_m)$$
$$\sim 1 \qquad \qquad \qquad \sim \alpha_s \qquad \qquad \qquad (\text{naïvely})$$

- ▶ however, $\textcolor{red}{U}_{bg}^{[5]}$ is off-diagonal evolution factor: $\sim \alpha_s \log(\mu_H/\mu_m)$
- ▶ $\textcolor{red}{U}_{bg}^{[5]} \sim 1$ formally only for $\mu_H \gg \mu_m$, $\textcolor{red}{U}_{bg}^{[5]} \sim \alpha_s$ a more appropriate counting for $\mu_m \sim m_b$ and typical LHC hard scales
- ▶ for LHC pheno: $f_b^{[5]}(m_b, \mu_H)$ effectively counts as $\mathcal{O}(\alpha_s)$

Combining fixed-order and resummation

Add pure m_b^2/Q^2 corrections to resummed result

Want to combine the results $d\sigma^{\text{FO}}$ and $d\sigma^{\text{resum}}$ valid different parametric regions to obtain a result valid for any value of m_b/Q

- ▶ terms missing in $d\sigma^{\text{resum}}$ are the $\mathcal{O}\left(\frac{m_b^2}{Q^2}\right)$ terms in $d\sigma^{\text{FO}}$

Write full cross section as

$$d\sigma = d\sigma^{\text{resum}} + \underbrace{\left(d\sigma^{\text{FO}} - \overbrace{d\sigma^{\text{resum}}|_{\mu_m=\mu_H}}^{\text{d}\sigma^{\text{singular}}} \right)}_{\text{d}\sigma^{\text{nonsingular}}}$$

- ▶ taking perturbative counting we prescribe, then $d\sigma^{\text{resum}}|_{\mu_m=\mu_H}$ exactly reproduces all singular contributions (terms that do not vanish in $m_b \rightarrow 0$ limit) in $d\sigma^{\text{FO}}$
- ▶ note: $d\sigma \rightarrow d\sigma^{\text{FO}}$ in limit $\mu_m \rightarrow \mu_H$

[Bonvini, AP, Tackmann '15, '16]

NLO+NLL matched result

[Bonvini, AP, Tackmann '15, '16]

$$\begin{aligned}\sigma^{\text{FO+Resum}} = & \sum_{i,j=b,\bar{b}} \textcolor{blue}{C}_{ij}(\mathbf{m}_H, \boldsymbol{\mu}_F) \otimes f_i^{[5]}(m_b, \mu_F) \otimes f_j^{[5]}(m_b, \mu_F) \\ & + \sum_{\substack{i=b, \bar{b} \\ j=g, q, \bar{q}}} \left[\textcolor{blue}{C}_{ij}(\mathbf{m}_H, \boldsymbol{\mu}_F) \otimes f_i^{[5]}(m_b, \mu_F) \otimes f_j^{[5]}(m_b, \mu_F) + (i \leftrightarrow j) \right] \\ & + \sum_{i,j=g,q,\bar{q}} \bar{\textcolor{blue}{C}}_{ij}(\mathbf{m}_H, \mathbf{m}_b, \boldsymbol{\mu}_F) \otimes f_i^{[5]}(m_b, \mu_F) \otimes f_j^{[5]}(m_b, \mu_F),\end{aligned}$$

► fixed-order finite m_b -effects contained in $\bar{\textcolor{blue}{C}}_{ij}$ (up to NLO)

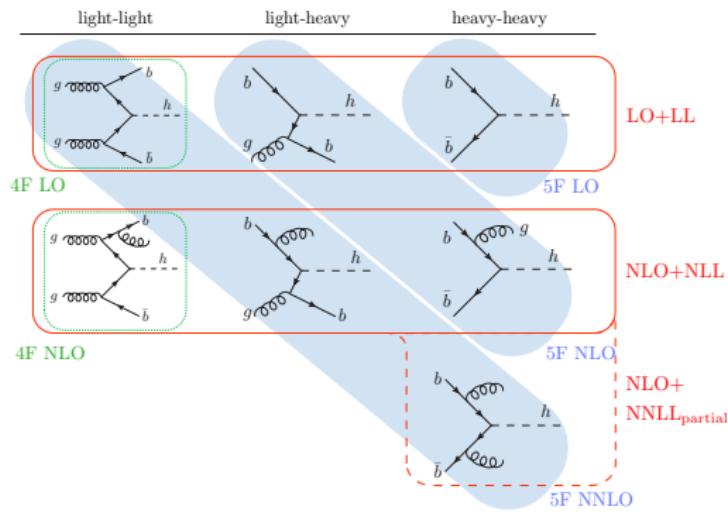
[similar construction to S-ACOT and FONLL coefficient functions]

Expanding (with perturbative counting as before):

$$\begin{array}{lll} \text{LO+LL} & \sigma = \alpha_s^2 \bar{\textcolor{blue}{C}}_{ij}^{(2)} f_i f_j + \alpha_s 4 \textcolor{blue}{C}_{bg}^{(1)} f_b f_g + 2 \textcolor{blue}{C}_{b\bar{b}}^{(0)} f_b f_b & \sim \alpha_s^2 \\ \text{NLO+NLL} & + \alpha_s^3 \bar{\textcolor{blue}{C}}_{ij}^{(3)} f_i f_j + \alpha_s^2 4 \textcolor{blue}{C}_{bk}^{(2)} f_b f_k + \alpha_s 2 \textcolor{blue}{C}_{b\bar{b}}^{(1)} f_b f_b & \sim \alpha_s^3 \end{array}$$

NLO+NLL matched result: ingredients

[Bonvini, AP, Tackmann '15, '16]



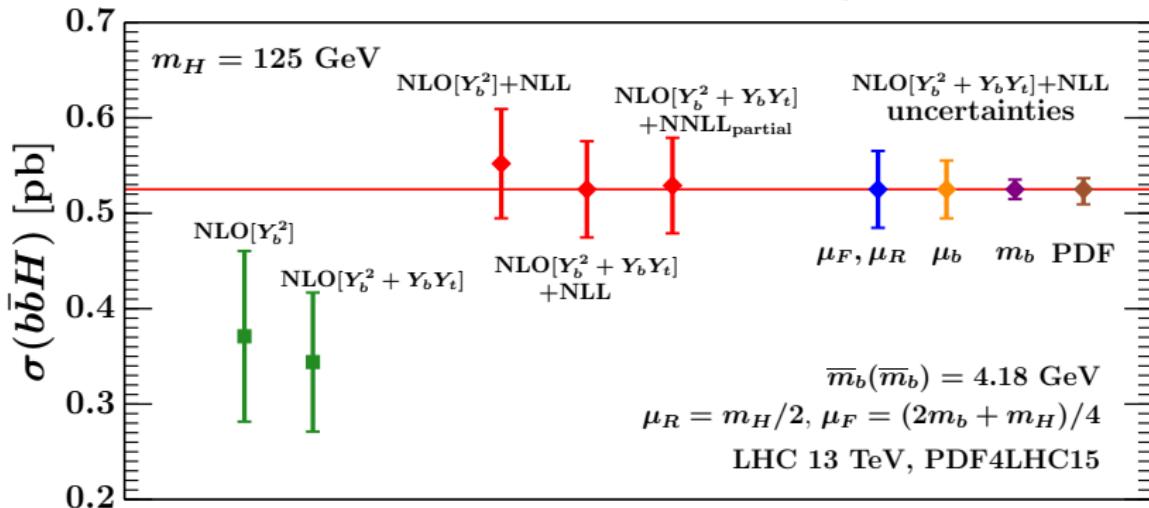
- we extract $D_i^{(3)}(m_H, m_b, \mu_H)$ using MG5_AMC@NLO [Alwall et al. '14]
- in-house implementation of $C_{b\bar{b}}^{(1,2)}$, $C_{bg}^{(1,2)}$, $C_{bq}^{(2)}$
- $\mathcal{M}_{ij}^{(2)}(m_b, \mu_m)$ known [Buza et al '96] : we implemented in APFEL [Bertone et al '13] for general μ_m
- construct $\bar{C}_{ij}^{(1,2)}$ from these known ingredients

$$\text{LO+LL} \quad \sigma = \alpha_s^2 \bar{C}_{ij}^{(2)} f_i f_j + \alpha_s 4 C_{bg}^{(1)} f_b f_g + 2 C_{b\bar{b}}^{(0)} f_b f_b \sim \alpha_s^2$$

$$\text{NLO+NLL} \quad + \alpha_s^3 \bar{C}_{ij}^{(3)} f_i f_j + \alpha_s^2 4 C_{bk}^{(2)} f_b f_k + \alpha_s 2 C_{b\bar{b}}^{(1)} f_b f_b \sim \alpha_s^3$$

$\sigma^{\text{NLO+NLL}}$ and its uncertainties (vary all scales)

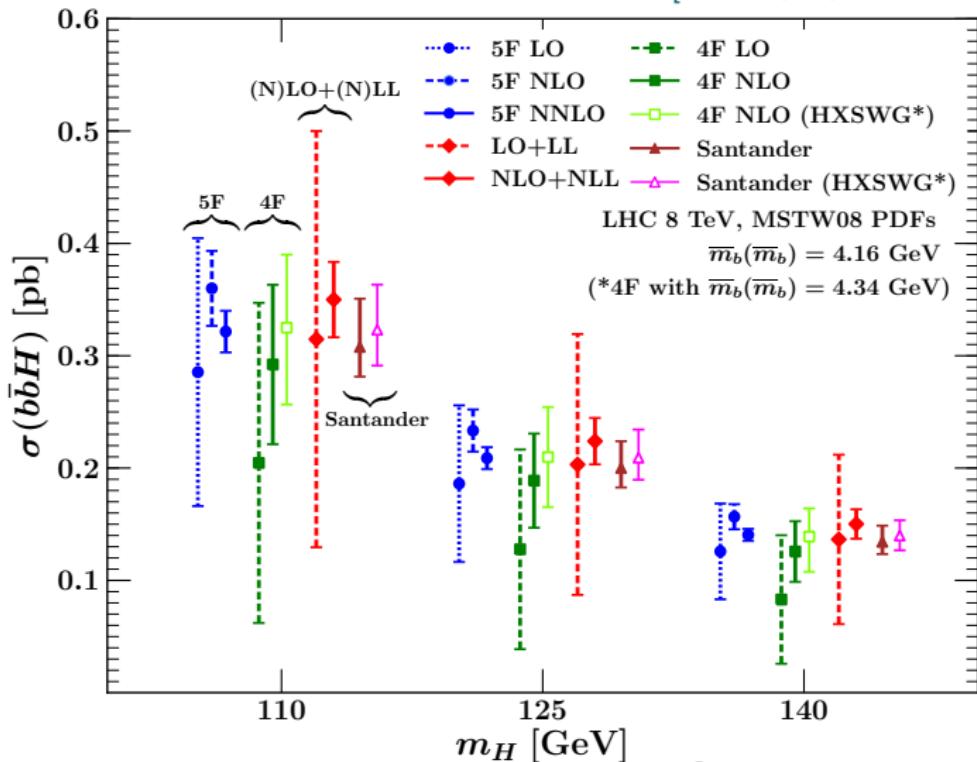
[Bonvini,AP,Tackmann '16]



- ▶ $Y_b Y_t$: interferences of Born-level diagrams with diagrams involving a top-quark loop (additional nonsingular terms)
- ▶ NLO+NNLL_{partial} contains pure 2-loop terms from 5F which are higher order in our approach

$\sigma^{\text{Santander}}$ systematically different from $\sigma^{\text{NLO+NLL}}$

[Bonvini, AP, Tackmann '15]

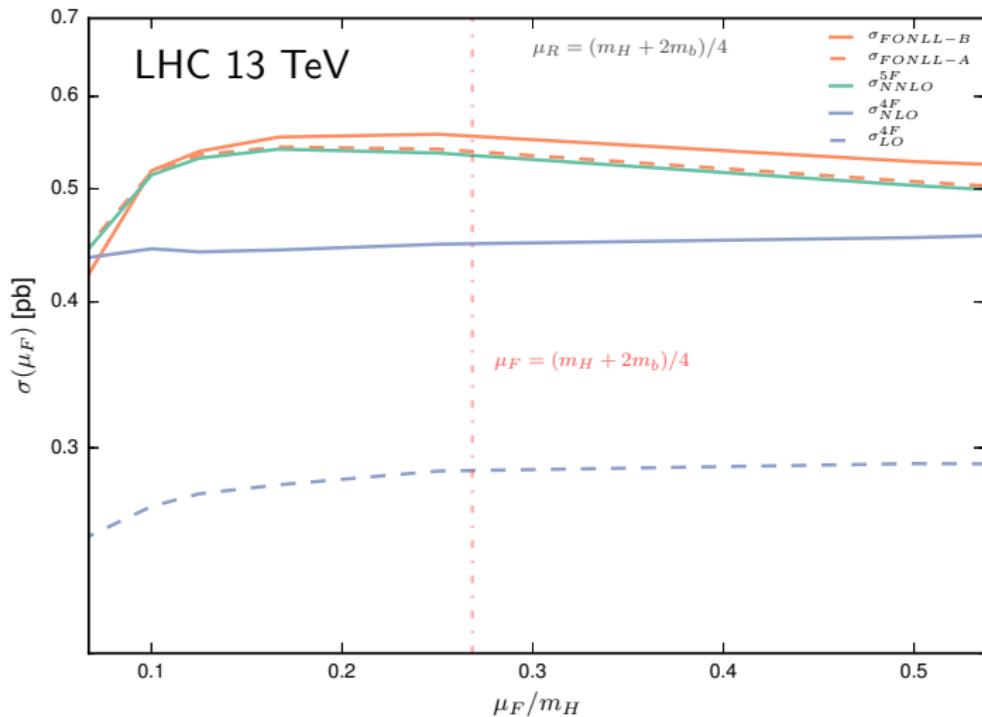


FONLL-A,B results

- ▶ in parallel, work done in FONLL approach by [Forte, Napoletano, Ubiali '15,'16]
- ▶ schematic result: $\sigma^{\text{FONLL}} = \sigma^{5\text{F}} + \sigma^{4\text{F}} - \sigma^{\text{double-counting}}$
- ▶ FONLL- A & B use $\sigma^{4\text{F}}$ at LO & NLO respectively (with appropriate subtractions to remove double-counting)
- ▶ construction of coefficient functions equivalent to that of NLO+NLL results
- ▶ results have been fully benchmarked against each other and agree when same perturbative ingredients are included and scales chosen to be the same (FONLL-B is equivalent to NLO+NNLL_{partial})

FONLL-A,B results

[Forte,Napoletano,Ubiali '16]



Choosing μ_F for $b\bar{b}H$

- ▶ standard factorization scale choice for both 4F and 5F is $\mu_F \sim m_H/4$
- ▶ this is smaller than what might be naively chosen, i.e. $\mu_F = m_H/2$ or $\mu_F = m_H$
- ▶ this is also a region where the 4F and 5F results are closest ...

To obtain a better understanding as to why this pattern holds, [Lim, Maltoni, Ridolfi, Ubiali '16] perform an analysis of the leading logarithms present in the 4F $b\bar{b}H$ cross section. These take the form $\sim \log \left(\frac{m_H}{m_b} \frac{(1-z)^2}{z} \right)$ [(z being the momentum fraction in one of the $g \rightarrow b\bar{b}$ splittings in the 4F LO diagram)]

Bottom line: to resum the leading collinear logarithms present in the 4F result, the choice $\mu_F \simeq m_H/3$ is more appropriate.

Differential – fixed order, resummation, parton showers

4F NLO parton-level, fully differential [Dittmaier, Kramer, Spira '03; Dawson et al '03]

5F NLO parton-level, fully differential [Dicus et al. '98;

Campbell et al. '02, Maltoni et al. '03 (MCFM)]

5F NNLO parton-level, fully differential [Bühler, Herzog, Lazopoulos, Müller '12]

5F NNLO+NNLL $p_T(H)$ distribution [Harlander, Tripathi, Wiesemann '14]

4F NLO+PS (MC@NLO) [Wiesemann, Frederix et al '14 (`mg5_aMC`)]

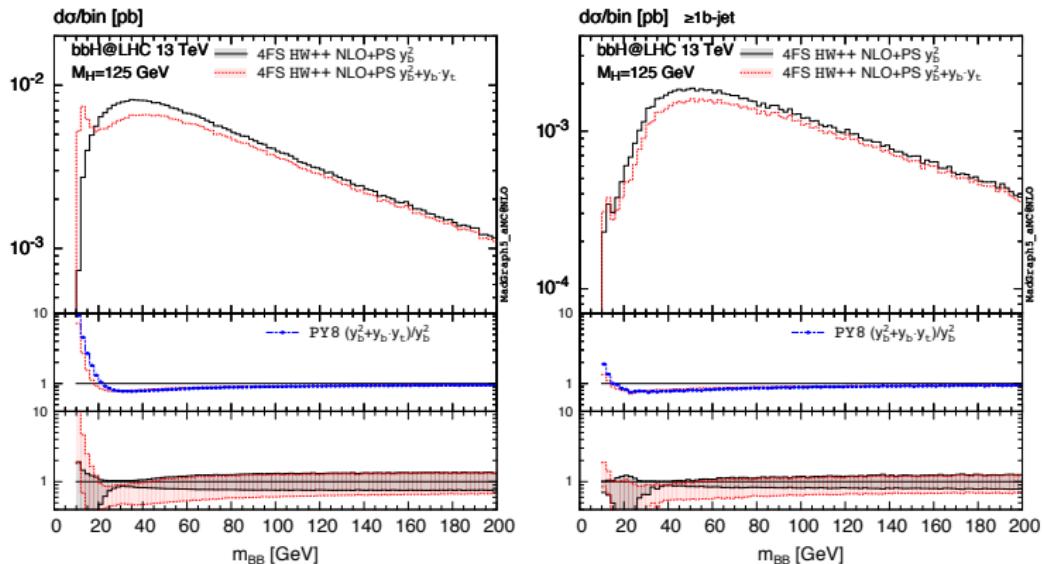
5F NLO+PS (MC@NLO) [Wiesemann, Frederix et al '14 (`mg5_aMC`)]

4F NLO+PS (POWHEG) [Jager, Reina, Wackerlo '15 (`POWHEG BOX`)]

} focus

4F NLO+PS (MC@NLO): $Y_b Y_t$ terms for m_{BB}

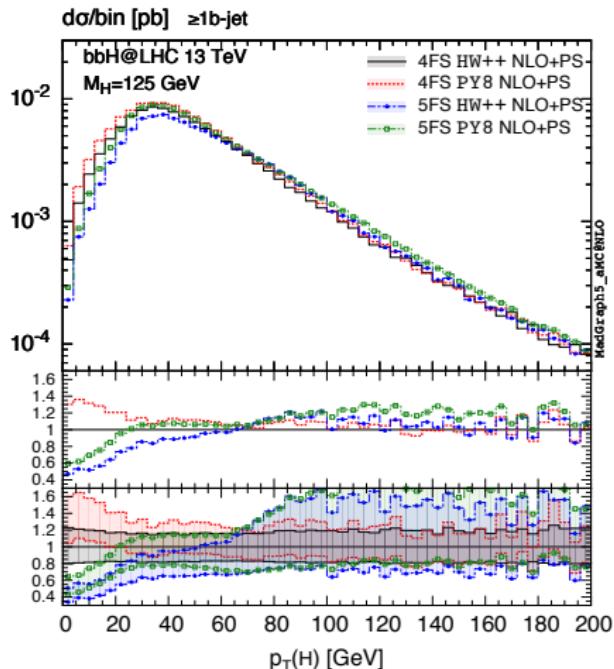
[Wiesemann, Frederix, Frixione, Hirschi, Maltoni, Torrielli '14]



- ▶ generically: $Y_b Y_t$ terms are flat across distributions (i.e. rescaling Y_b^2 distributions is relatively safe)
- ▶ however, striking peak structure of $Y_b Y_t$ terms for m_{BB} in inclusive H setup (**not** a shower effect)

4F vs 5F NLO+PS (MC@NLO): $p_T(H)$

[Wiesemann, Frederix, Frixione, Hirschi, Maltoni, Torrielli '14]

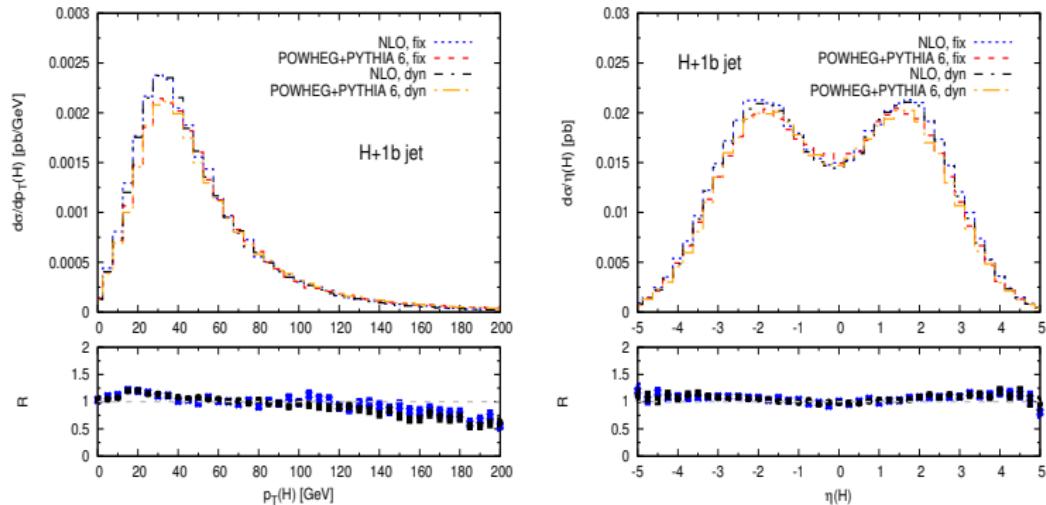


4F / 5F comparison:

- schemes agree within 20% of each other for $p_T(H) \gtrsim 50$ GeV
- schemes predict very different shapes at low $p_T(H)$
- differences when inclusive over 1 or 2 b -jets motivate studies on fixed-order/resummation matching for distributions
- also a better understanding of PS treatment of massless initial-state b -quarks ?

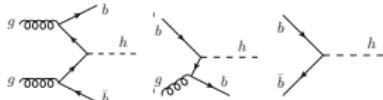
4F NLO+PS (POWHEG): $p_T(H)$ & $\eta(H)$

[Jäger, Reina, Wackerlo '15]



- ▶ shower effects generally small for many observables
- ▶ fixed order $p_T(H)$ distribution tilted by PS
- ▶ scale dependence generally large $\sim 25\%$ (not shown in plot)

Conclusions & Outlook



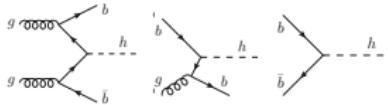
Inclusive cross section

- ▶ theoretically consistent combination of m_b^2/Q^2 -corrections with $\log(m_b/Q)$ -resummation for $b\bar{b}H$ has been made
- ▶ robust error-estimate through variation of all matching scales (fixed-order + resummation uncertainties)
- ▶ recommendation is to use these new consistently-matched predictions (grids are currently in preparation)

Differential and fully exclusive results

- ▶ NLO+PS codes available in MC@NLO (4F & 5F) and POWHEG (4F) matching schemes ($Y_b Y_t$ included)
- ▶ when inclusive over 1 or 2 b -jets still need to carefully study differences of 4F/5F results at the differential level
- ▶ many more details & discussion in YR4 chapter on $b\bar{b}H/bH$

Conclusions & Outlook



Thank you for your attention!

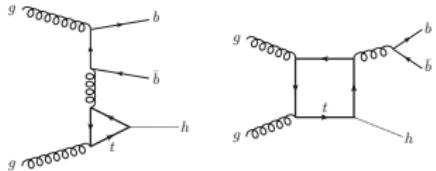
4F/5F: expansion of partonic cross sections

5F

$$\hat{\sigma}^{5F} = Y_b^2 (\underbrace{\sigma_{[Y_b^2]}^{(0)} + \alpha_s \sigma_{[Y_b^2]}^{(1)} + \alpha_s^2 \sigma_{[Y_b^2]}^{(2)} + \dots}_{\text{corrections to Born}})$$

4F

$$\hat{\sigma}^{4F} = Y_b^2 (\underbrace{\alpha_s^2 \sigma_{[Y_b^2]}^{(2)} + \alpha_s^3 \sigma_{[Y_b^2]}^{(3)} + \dots}_{\text{corrections to Born}}) + \underbrace{Y_b Y_t (\alpha_s^3 \sigma_{[Y_b Y_t]}^{(3)} + \dots)}_{\text{top-loop induced interferences}}$$



Comparing approaches

A practical way to combine 4F and 5F predictions is given by:

$$\sigma^{\text{Santander}} = \frac{\sigma^{\text{4FS,NLO}} + \omega \sigma^{\text{5FS,NNLO}}}{1+\omega}, \quad \text{where } \omega = \log\left(\frac{m_H}{m_b}\right) - 2,$$

and uncertainties obtained by applying formula to upper and lower 4F & 5F results.

Meaningful/fair comparison ('15 paper):

- ▶ PDFs: MSTW08 (NLO/NNLO and nf4/nf5 as appropriate)
- ▶ $m_b = 4.75$ GeV (as used in MSTW08)
- ▶ $\overline{m}_b(\overline{m}_b) = 4.18$ GeV, same RGE-evolution for all results
- ▶ central scale: $\mu_H = (m_H + 2m_b)/4$ ($\mu_H = \mu_F = \mu_R$)

Other approaches to initial-state HQs available (mainly DIS)

Alternative approaches to combining fixed-order mass effects with resummation of logarithms in DIS have been around for a while ...

- ▶ ACOT [Aivazis,Collins,Olness,Tung]
- ▶ TR [Thorne,Roberts]
- ▶ S-ACOT(χ) [Collins; Krämer,Olness,Soper; ...]
- ▶ FONLL [Cacciari,Greco,Nason; Forte,Laenen,Nason,Rojo]

These differ mainly in the way in which they choose to incorporate the m_b^2/Q^2 corrections. Compared to what we have outlined:

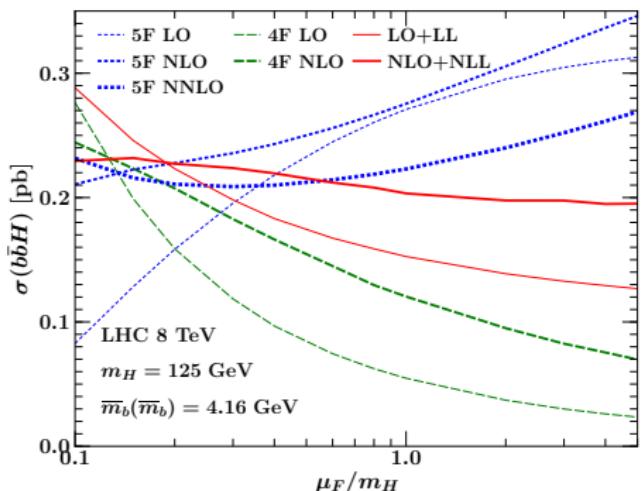
- ▶ construction of coefficient functions equivalent to S-ACOT and FONLL
- ▶ we consider $f_b(m_b, \mu_H)$ a perturbative $\mathcal{O}(\alpha_s)$ object (strictly expanded together with coefficient functions)
- ▶ we take the matching scale μ_m to be $\mathcal{O}(m_b)$, but not strictly equal to m_b

Perturbative stability

Consistent matching and counting stabilizes cross section

[Bonvini,AP,Tackmann '15]

(fixing $\mu_m = m_b$)

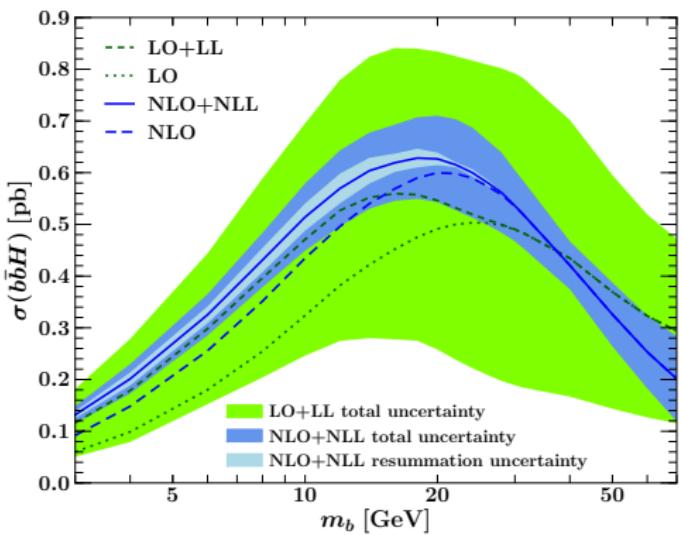


- ▶ scale dependence of **LO+LL** and **NLO+NLL** improved w.r.t LO and NLO **4F** and **5F**
- ▶ **4F stabilized:** resum large logs
- ▶ **5F stabilized:** put together channels that contribute at same perturbative order ($b\bar{b}$, bg , gg)

Proof of concept: cross section as a function of m_b

[Bonvini, AP, Tackmann '15]

- ▶ checks smooth matching to FO
- ▶ (\Rightarrow very stringent check of construction of coefficients \bar{C}_i and implementation of \mathcal{M}_{ij})
- ▶ check whether our uncertainty determination is useful



$$m_H = 125 \text{ GeV}, \mu_H = m_H/4$$

- ✓ $d\sigma^{(N)\text{LO}+(N)\text{LL}} \rightarrow \sigma^{(N)\text{LO}}$ in limit $\mu_m \rightarrow \mu_H$ (large m_b)
- ✓ NLO+NLL lies within LO+LL uncertainty band
- ✓ resummed result appears perturbatively more stable than FO result

$\sigma^{\text{Santander}}$ systematically lower than $\sigma^{\text{NLO+NLL}}$

[Bonvini, AP, Tackmann '15]

