

Associated production of vector bosons and jets in CMS

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on behalf of CMS collaboration

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Outline

➤ Standard Model Public Results from CMS

<http://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsSMP>

● Many experimental results on V+jets subjects, here only focus on:

➤ 8TeV ($L \leq 19.8 \text{ fb}^{-1}$, BX = 50 ns):

- Z boson + jets
- W boson + jets
- Photon (γ) + jets
- W/Z boson + heavy flavor quark jets
 - W boson + b quark jets
 - Z boson + c/b quark jets

◆ Data taken during 2012

➤ 13 TeV ($L = 2.5 \text{ fb}^{-1}$, BX = 25 ns):

- W boson + jets
- Z boson + jets

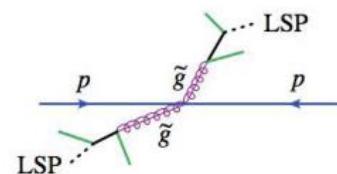
◆ Data taken during 2015

Motivations on V+jets physics

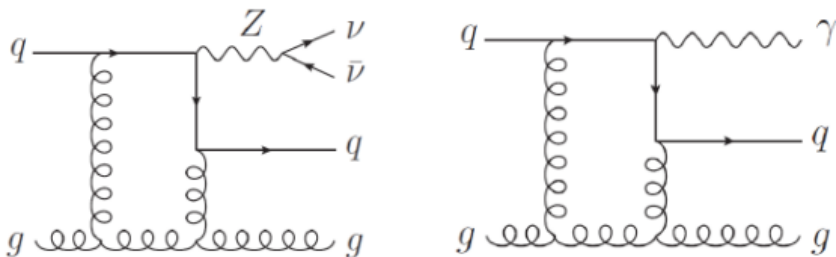
- One can probe different aspects of QCD calculations with V+jets processes
- Actual understanding and modeling of QCD interactions is crucial on the potentials for precision measurements
 - W/Z+jets is a dominant background for:
 - ◆ Top-quark measurements
 - ◆ Precision measurement of Higgs physics in VH(->bb) channel
 - It is significant for the modeling the production mechanism involved in new physics searches (e.g. Supersymmetry)

- Z+jets as background to new physics searches:
 - Z(->vv)+jets in SUSY (MET+jets) searches
 - Exploit NLO computations of W+jets/Z(->vv)+jets or γ +jets/Z(->vv)+jets ratios to calculate the **transfer functions** from W/ γ +jets to Z(->vv)+jets
 - ✓ important to constrain theory extrapolation with data

MET+ 4 jets



JHEP 10 (2012) 018
 PRD 90, 052008 (2014)
 arXiv:1405.7875



$$\sigma(pp \rightarrow Z(\rightarrow \nu\bar{\nu}) + \text{jets}) = \sigma(pp \rightarrow \gamma + \text{jets}) \times R_{Z/\gamma}$$

↑
↑
↑

irreducible background
measure this
theory input

The strategy of V+jets analyses

- W/Z bosons are reconstructed through their leptonic decays
 - ◆ Muons and electrons are reconstructed using Particle Flow algorithm based on the information from detector subsystems
 - ◆ Neutrinos are identified using missing transverse energy
- Jets are reconstructed using Particle Flow algorithm
 - ◆ Anti-Kt algorithm applied for jets with parameters $R=0.5$ (0.4) at 8 TeV (13 TeV)
 - ◆ Jets must be well isolated with the leptons from W/Z boson decays
 - ◆ Pileup subtraction is implemented in the isolation definition of the jets
 - ◆ Heavy flavor (c/b) jets are tagged for the dedicated analyses

- Restrict the phase space for an efficient selection and good acceptance
- Use Monte Carlo methodology to correct for detector effects on the signal extracted from data
- Compare the unfolded data (measurement) with different theoretical modellings on the cross sections at particle level, including the differential cross section as a function of various observables
 - Compared theoretical predictions can be in different accuracy levels

Data statistics for W/Z +jets measurement (8 TeV)

- ◆ Measured differential cross section as a function of several observables
- ◆ The presence of data goes up to large jet multiplicity region for both W/Z + jets

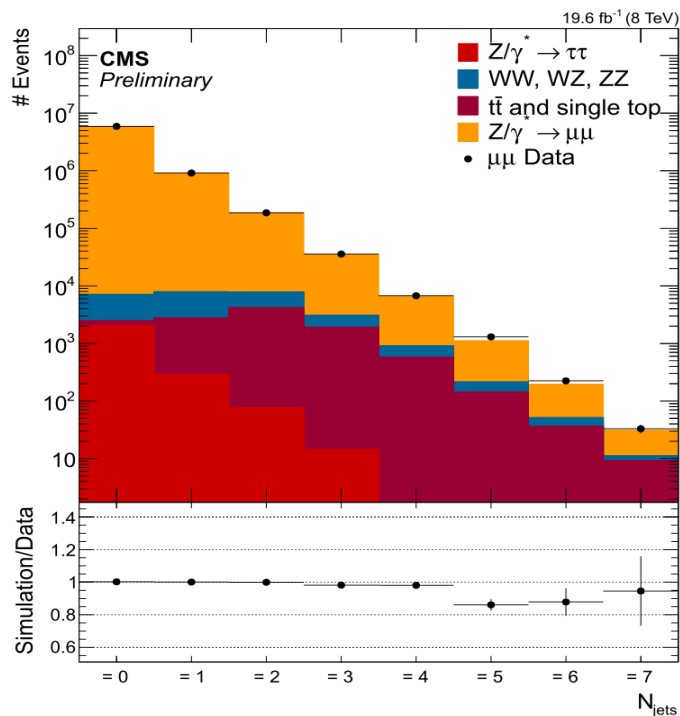
➤ Phase space for Z+jets:

● Lepton selection:

- $p_T(l) > 20 \text{ GeV}$
- $|\eta(l)| < 2.4$
- $71 < M(l\bar{l}) < 111 \text{ GeV}$

● Jet selection:

- $p_T(j) > 30 \text{ GeV}$
- $|\eta(j)| < 2.4$



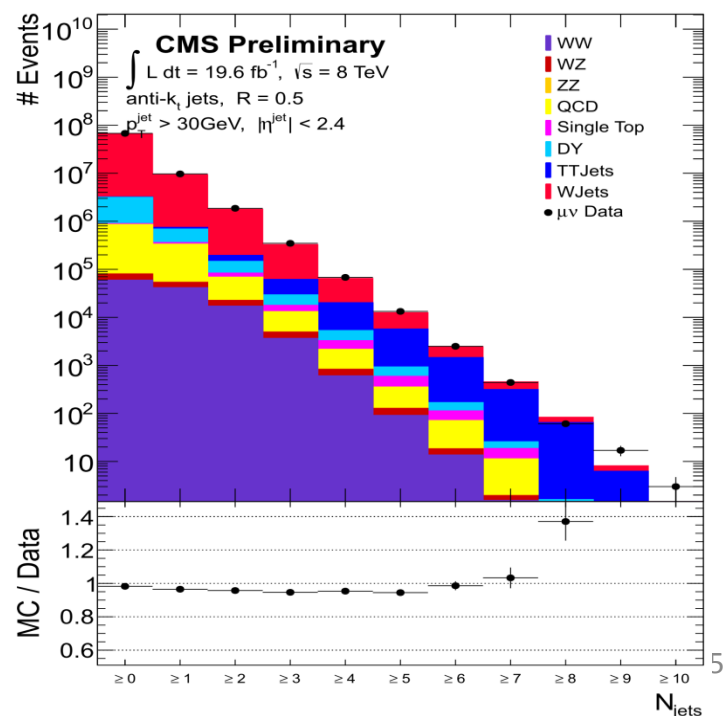
➤ Phase space for W+jets:

● Lepton selection:

- $p_T(l) > 25 \text{ GeV}$
- $|\eta(l)| < 2.1$
- $M_T > 50 \text{ GeV}$

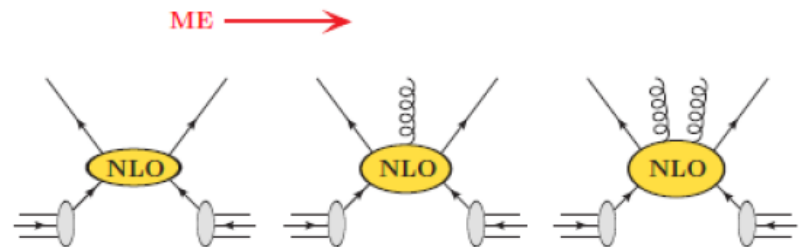
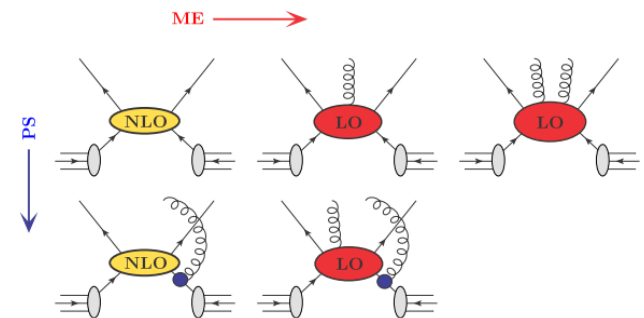
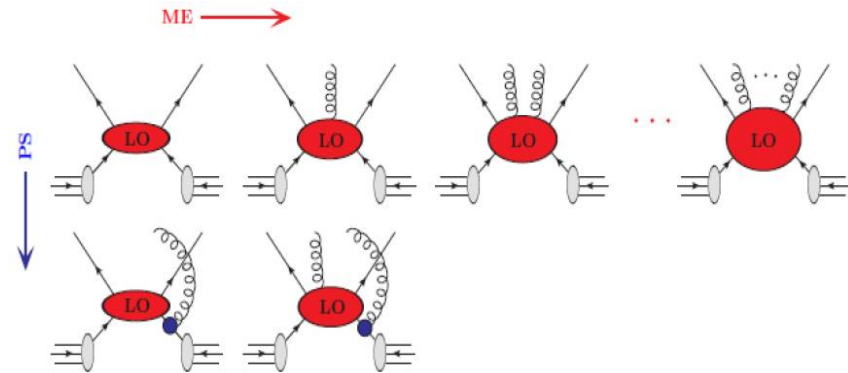
● Jet selection:

- $p_T(j) > 30 \text{ GeV}$
- $|\eta(j)| < 2.4$



Theoretical predictions for Z + jets cross section comparison at 8 TeV

- **Multileg LO with 0~4 final partons in matrix element:**
 - MADGRAPH5 + PYTHIA6: kt-MLM merging, CTEQ6L1 PDF
 - SHERPA1.4: CKKW merging, CT10 PDF
 - Hadronization and multiple parton interactions are implemented
- **Multileg with 0~2 partons at NLO, 3~4 partons at LO:**
 - SHERPA2 (+ BLACKHAT): MEPS@NLO merging, CT10 PDF
 - Hadronization and multiple parton interactions are implemented
- **Fixed order Z + N jets (NLO):**
 - BLACKHAT (+SHERPA): MSTW2008 NLO PDF
 - Correction for hadronization and multiple parton interactions computed with MG5+PYTHIA6



Theoretical predictions for W + jets cross section comparison at 8 TeV

- MADGRAPH5 + PYTHIA6
 - Leading order multileg matrix element up to 4 partons in final state
 - kt-MLM merging, CTEQ6L1 PDF

- MADGRAPH5_AMC@NLO + PYTHIA8
 - Next leading order multileg matrix element up to two partons in final state, LO accuracy for 3 partons
 - FxFx jet merging, NNPDF2.3 NLO PDF , CUETP8M1 PYTHIA8 tune

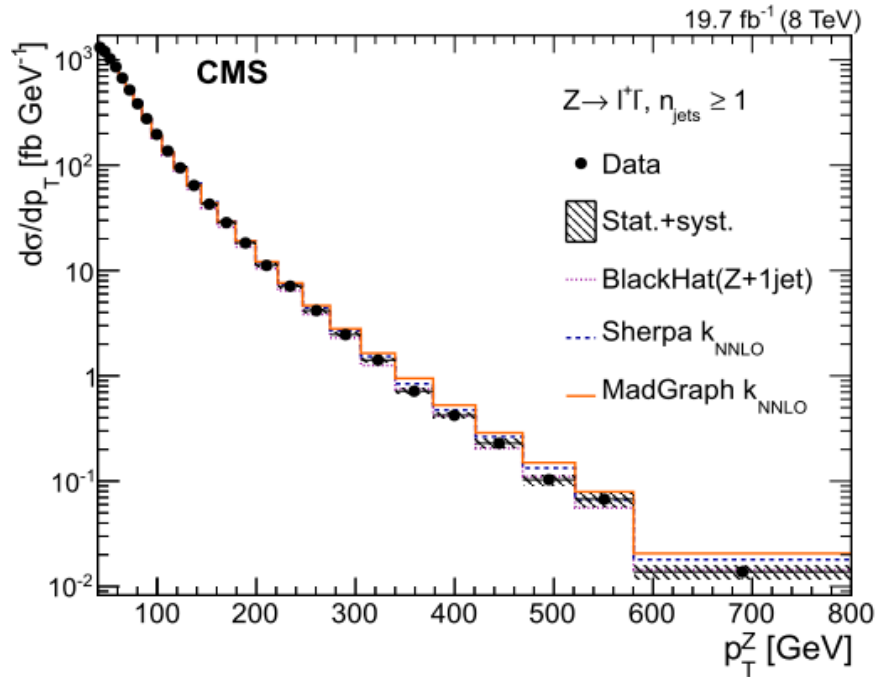
- SHERPA + BLACKHAT
 - Next leading order accuracy up to two partons, LO accuracy for 3~4 partons
 - MEPS@NLO merging, CT10 PDF

- BLACKHAT + SHERPA
 - NNLO W + njet with fixed order (n = 1~4)
 - CT10 PDF set
 - Correction for hadronization and multiple parton interactions computed with MG5+PYTHIA6

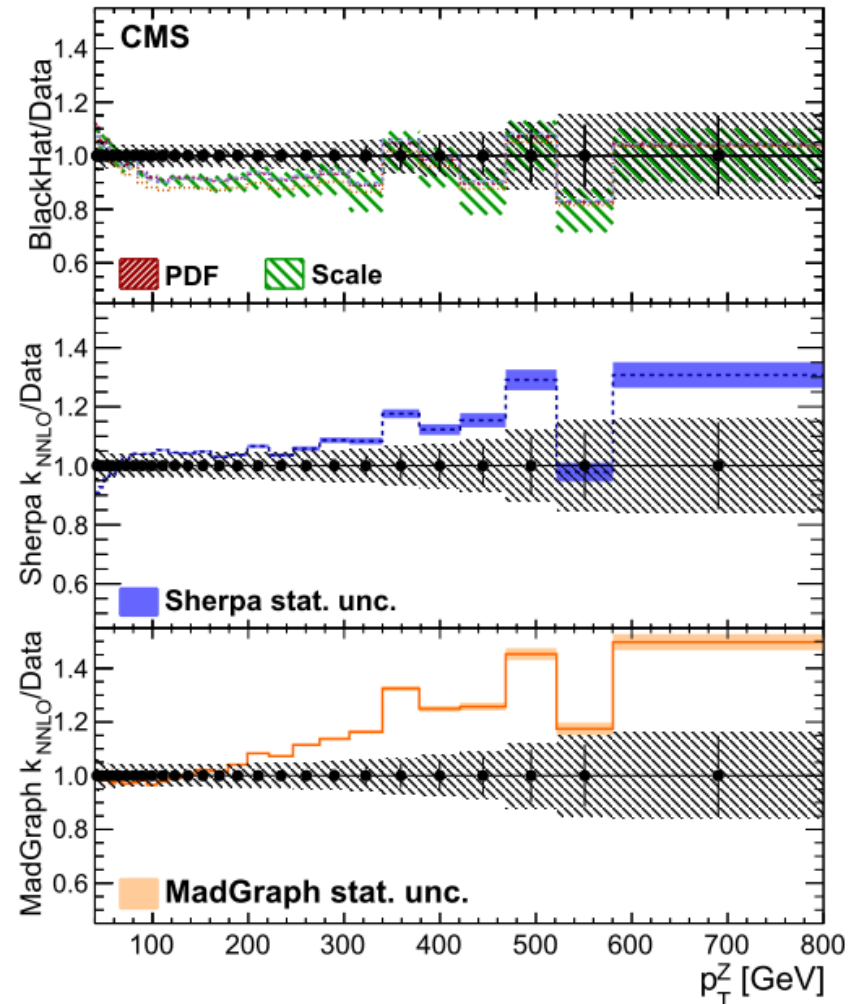
- NNLO (W+1jet fixed order)
 - Correction for hadronization and multiple parton interaction
 - CT14 PDF set
 - Reference: [PRL.115.062002](#); [arXiv:1602.08140](#); [arXiv:1512.01291](#); [arXiv:1602.06965](#)

$Z + \geq 1$ jet cross section as function of Z boson p_T

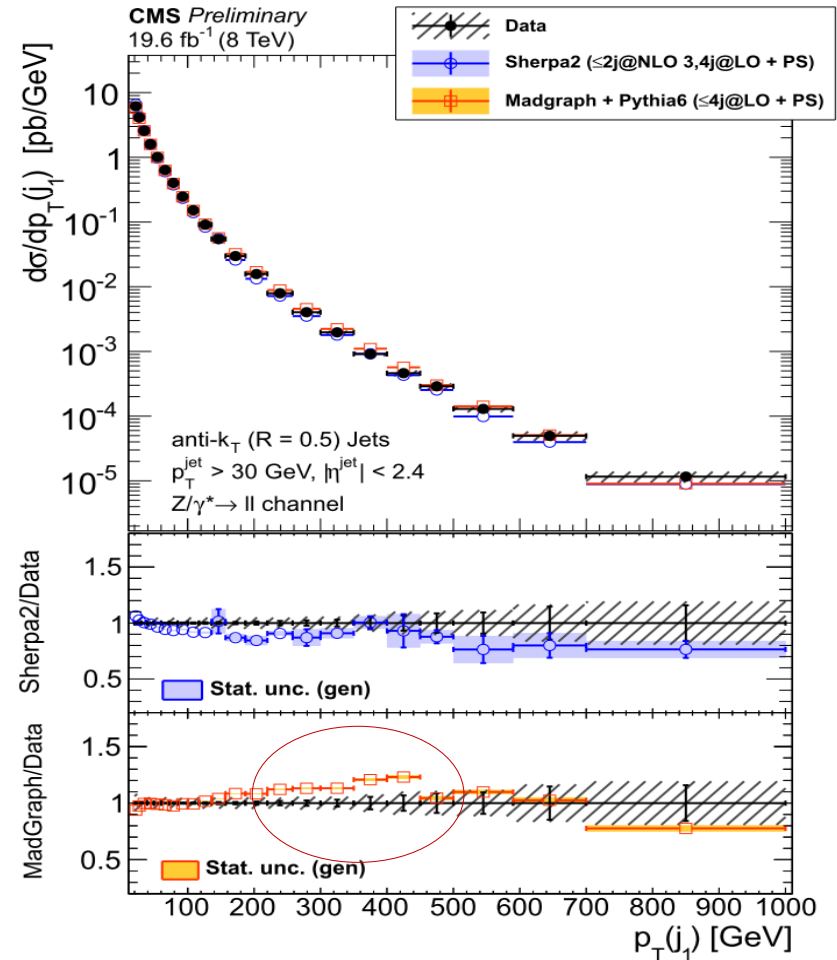
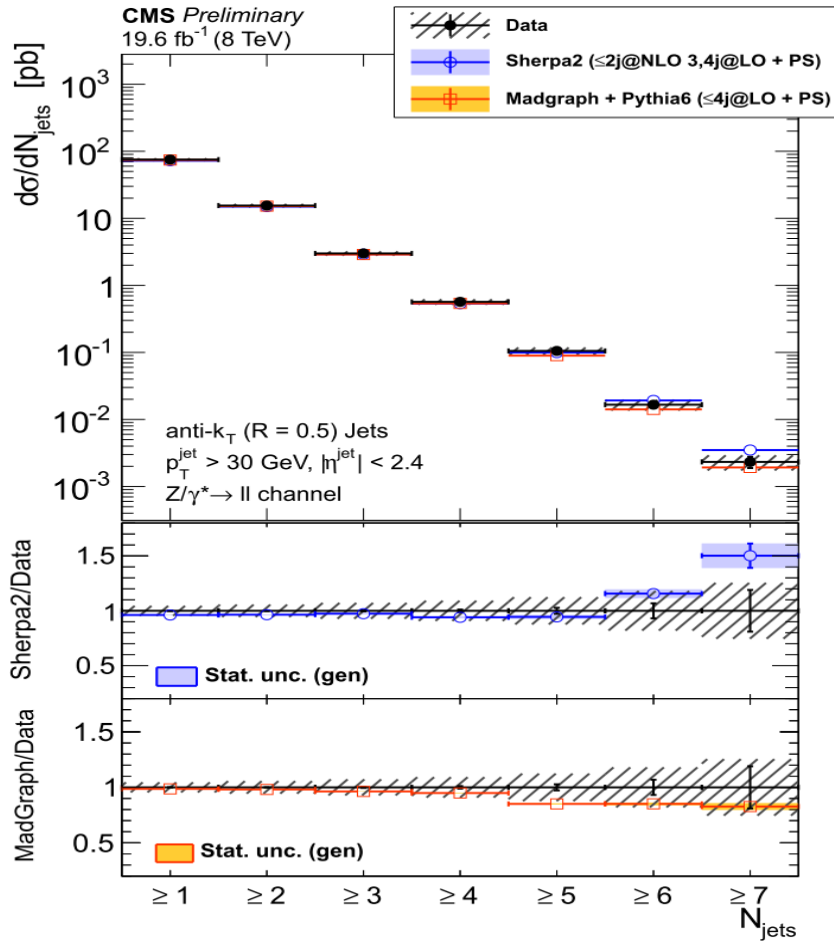
JHEP10(2015)128
Published



- K_{NNLO} : $Z + \geq 0$ jet cross section rescaled to NNLO value from FEWZ
- MG5+Pythia6 comparison is flat until around 200 GeV and up to about 30% discrepancy, which is same for $Z + \geq 2$ jets case
- BLACKHAT reproduces the shape of data in the same region



Z + jets differential cross section as function of jet observables

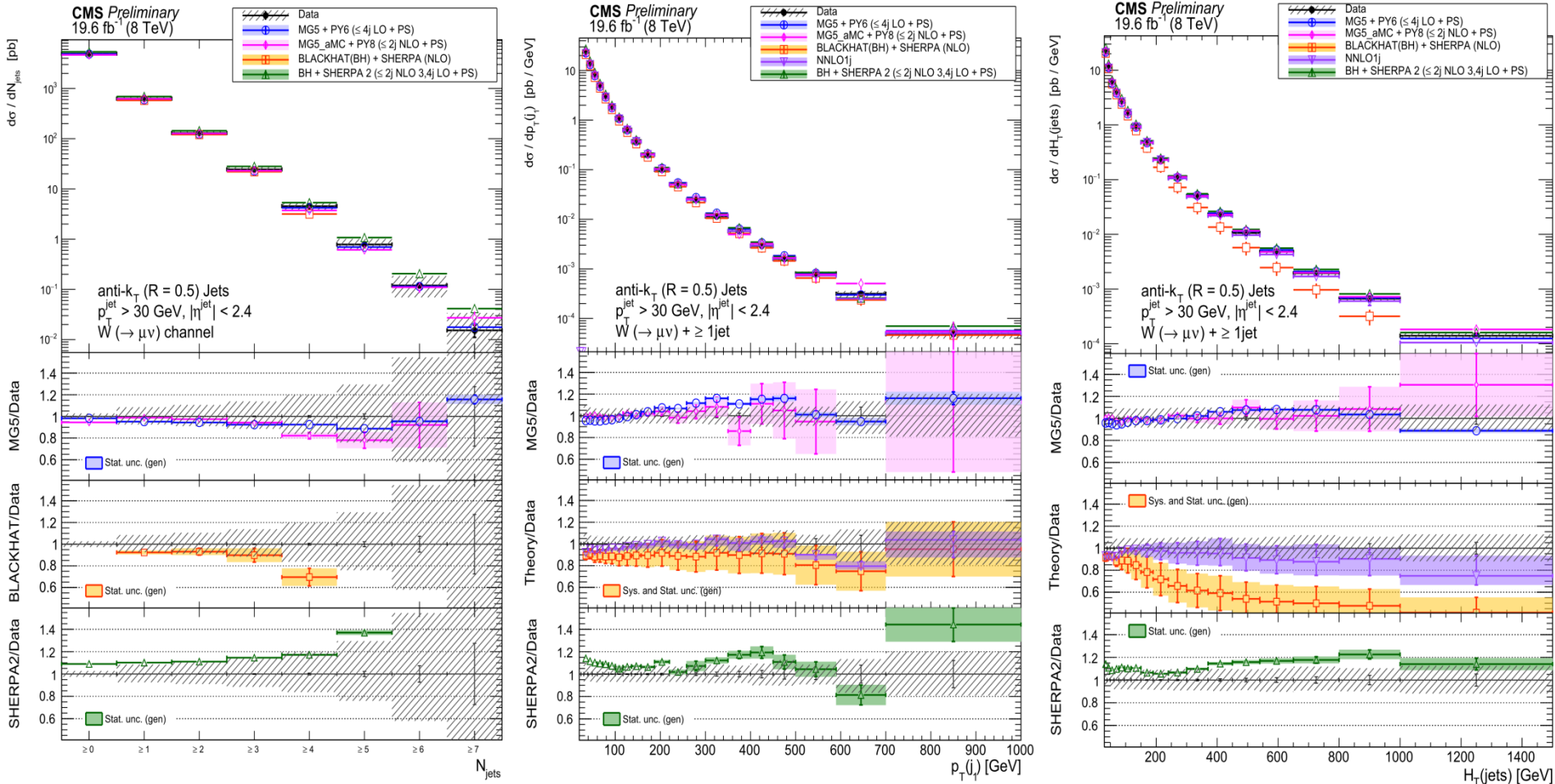


- Good agreement with SHERPA2 (NLO)
- Discrepancy with LO computation has disappeared with NLO accuracy

SMP-13-007

W + jets differential cross section as function of jet observables

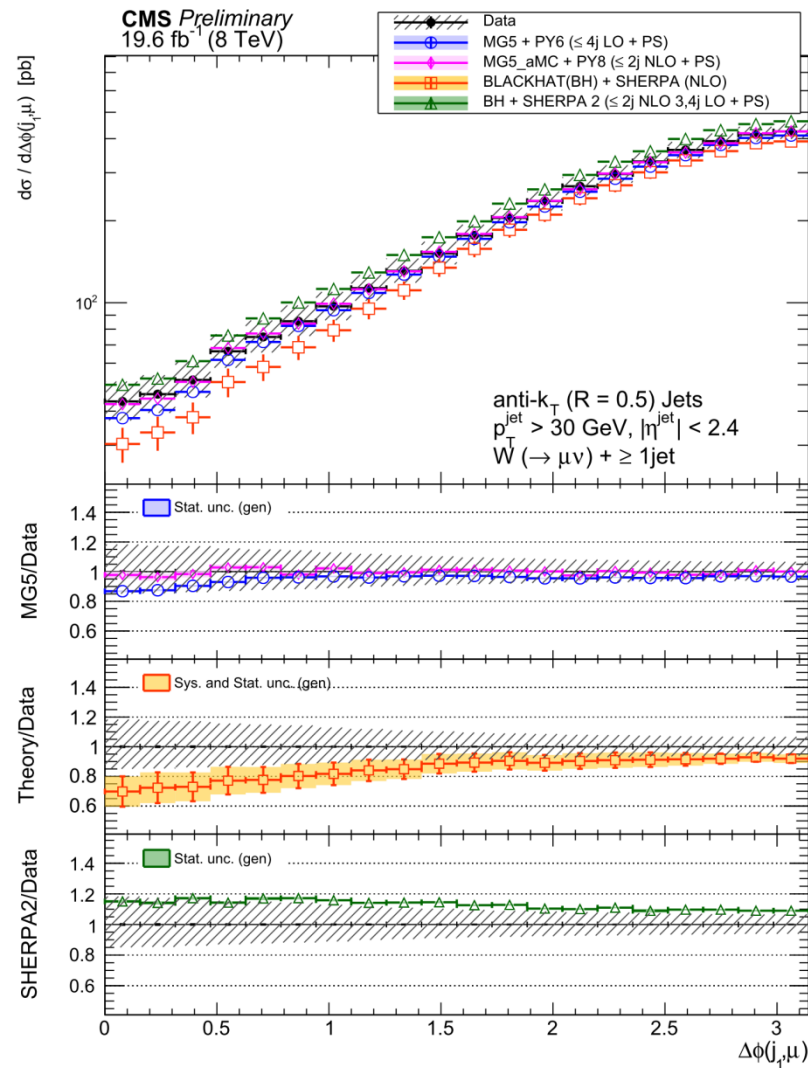
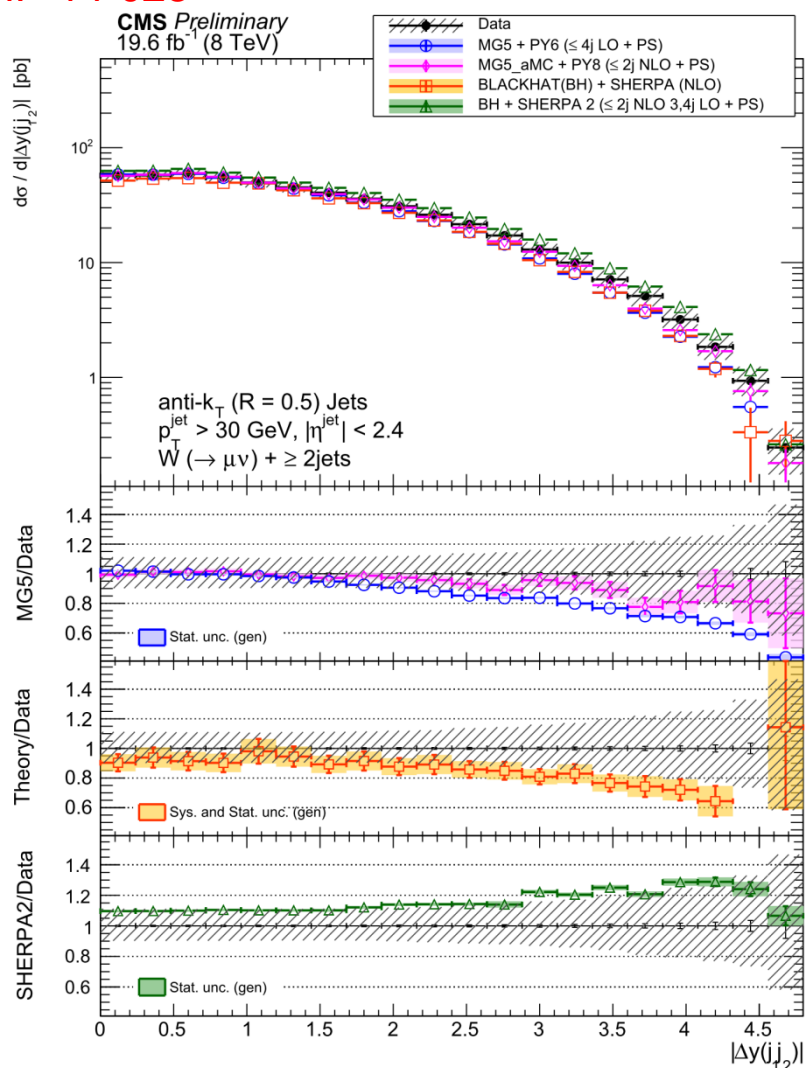
SMP-14-023



- Only the decay channel of $W(\rightarrow\mu\nu)$ + jets is measured
- Excellent agreement with AMC@NLO and NNLO W+1J predictions

Angular correlation measurement of W+jets

SMP-14-023



- Discrepancies in large rapidity difference between jets and in small azimuthal angle difference between muon and leading jet
- AMC@NLO describes the measured angular correlations best

Theoretical predictions for W/Z + jets cross section comparison at 13 TeV

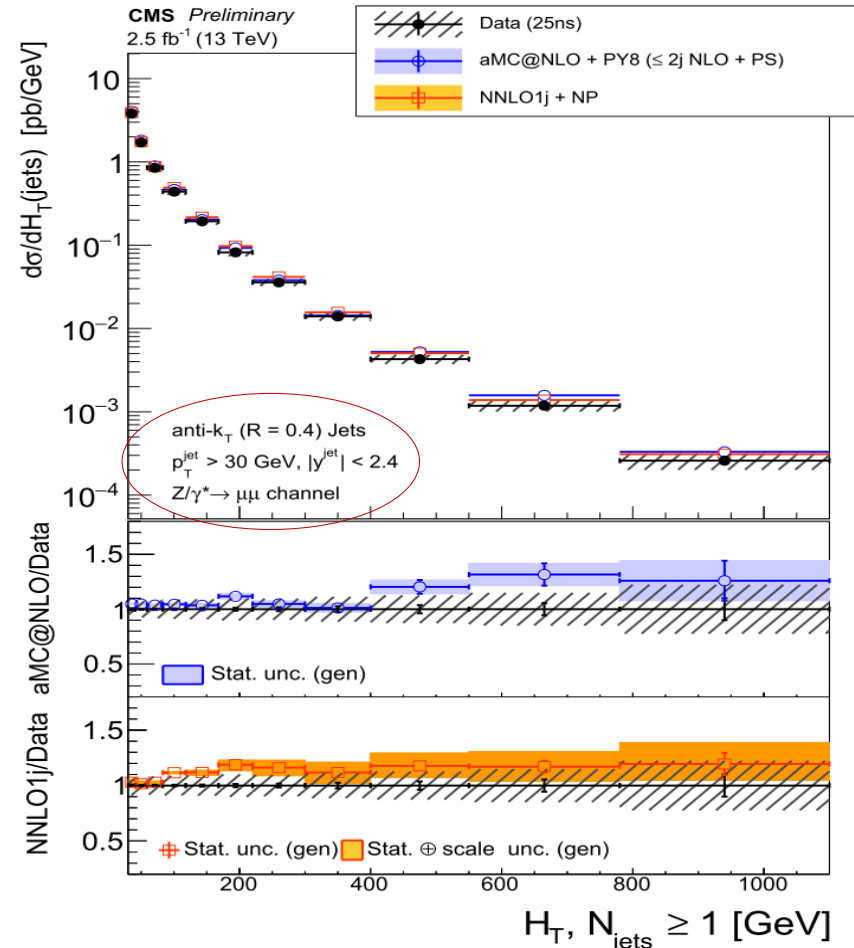
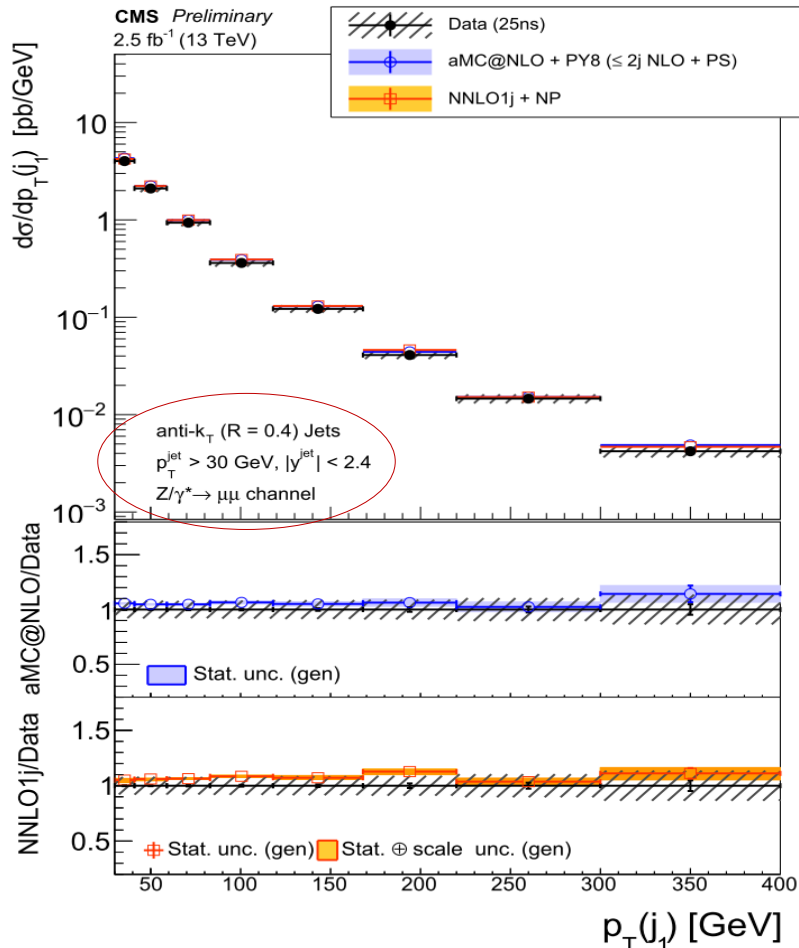
- MADGRAPH5_AMC@NLO + PYTHIA8
 - Next leading order multileg matrix element up to two partons in final state, LO accuracy for 3 partons
 - FxFx jet merging scheme
 - NNPDF3.0 NLO PDF set
 - CUETP8M1 PYTHIA8 tune

- NNLO
 - NNLO W/Z+1jet with fixed order
 - Correction for hadronization and multiple parton interaction computed with MADGRAPH5_AMC@NLO + PYTHIA8
 - CT14 PDF set
 - References:
 - [arXiv:1602.08140](#)
 - [arXiv:1512.01291](#)
 - [arXiv:1602.06965](#)
 - [Phys. Rev. Lett. 115, no.6, 062002 \(2015\)](#)

- MADGRAPH5 + PYTHIA (only for W+jets)
 - Leading order accuracy up to four partons
 - KT-MLM merging scheme
 - CTEQ6L1 PDF

Z + jets differential cross section at 13 TeV

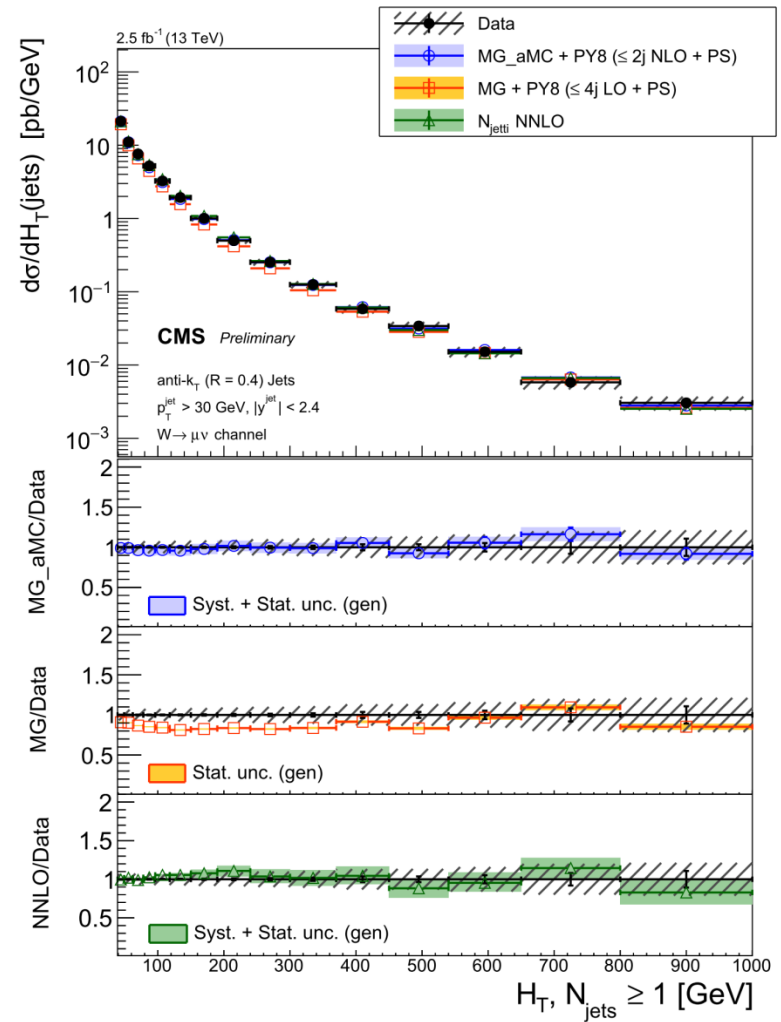
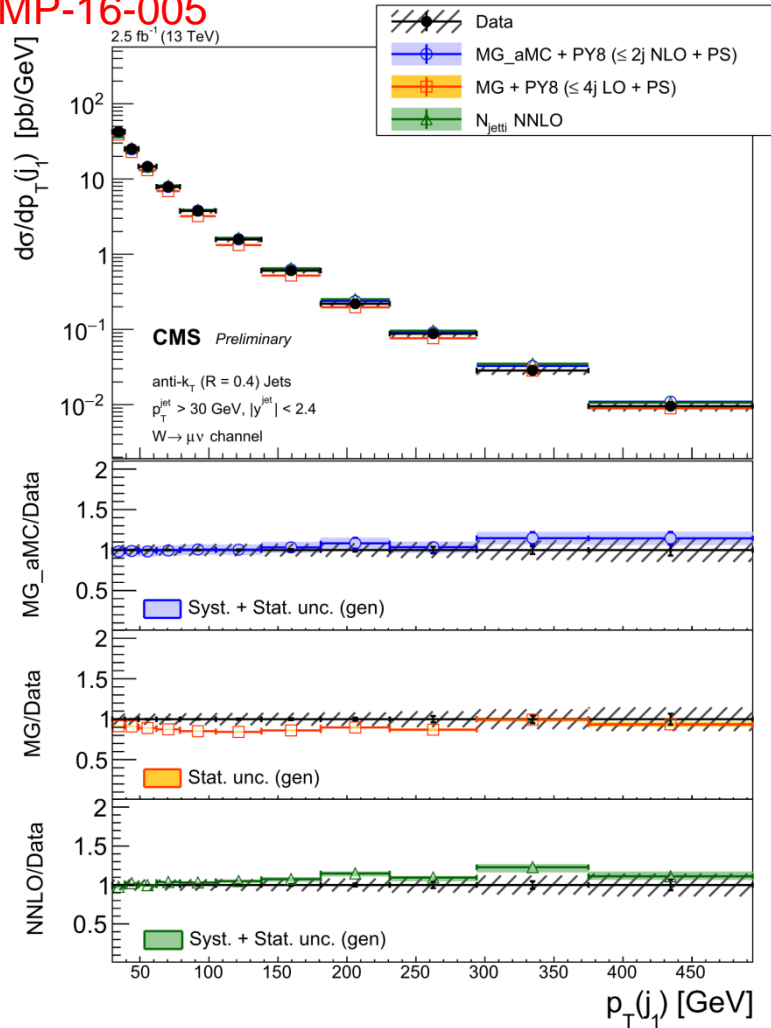
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- Good agreement with multileg NLO and NNLO calculations
- The p_T , η , H_T of jet for inclusive jet multiplicities up to 3 jets have also been measured
- H_T is the scalar sum of the p_T of jets

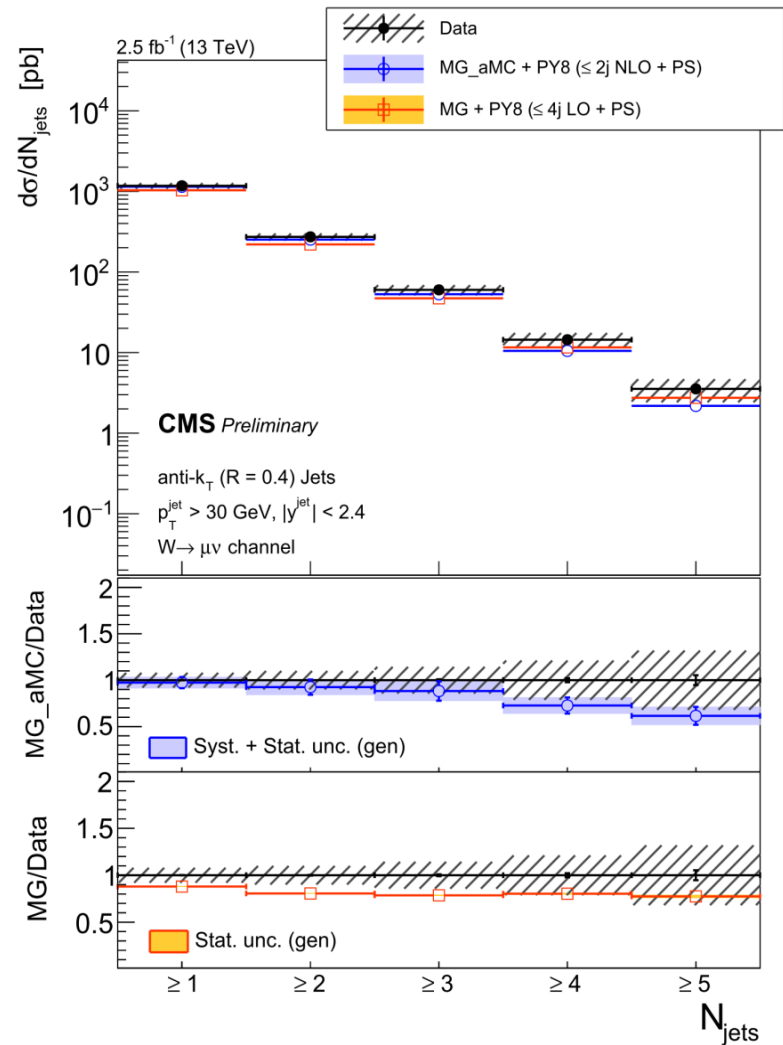
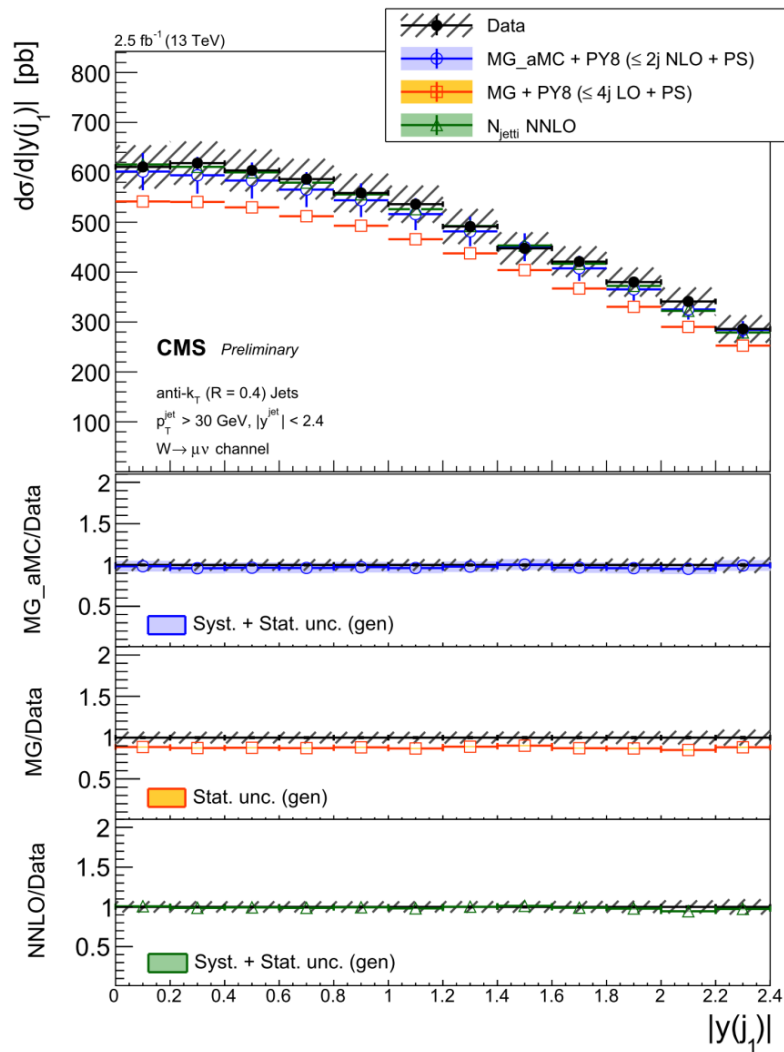
W + jets differential cross section at 13 TeV

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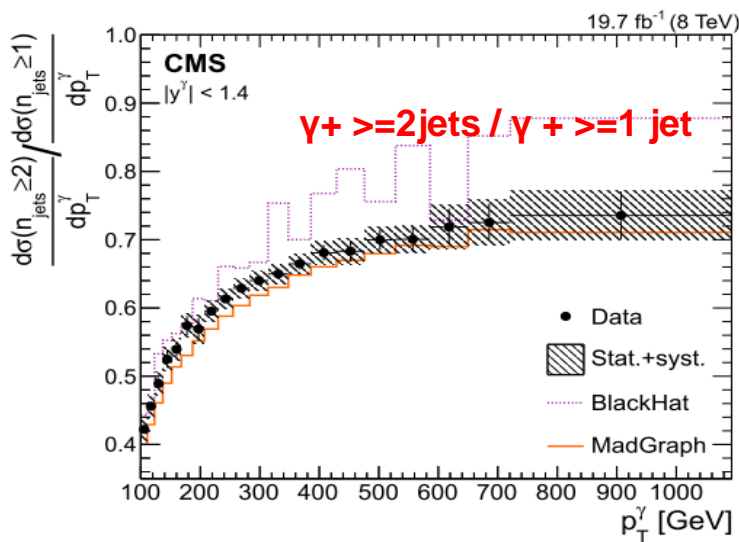
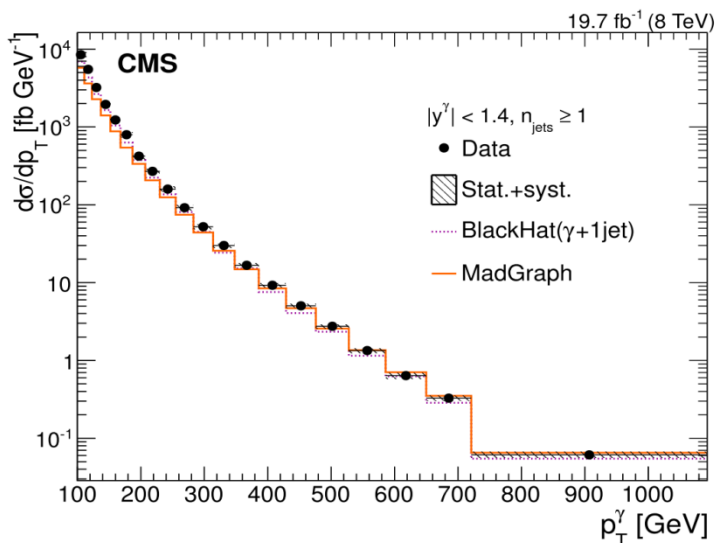
- Agreement is improved significantly with multileg NLO and NNLO calculations
- The p_T , η , H_T of jet for inclusive jet multiplicities up to 3 jets have also been measured
- H_T is the scalar sum of the p_T of jets

W + jets differential cross section at 13 TeV



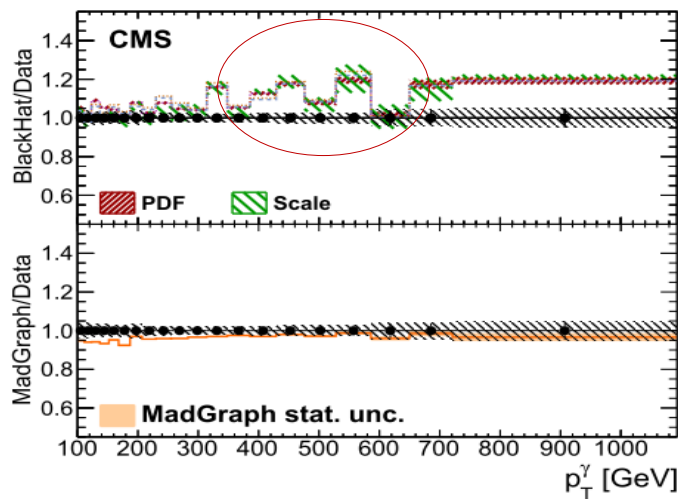
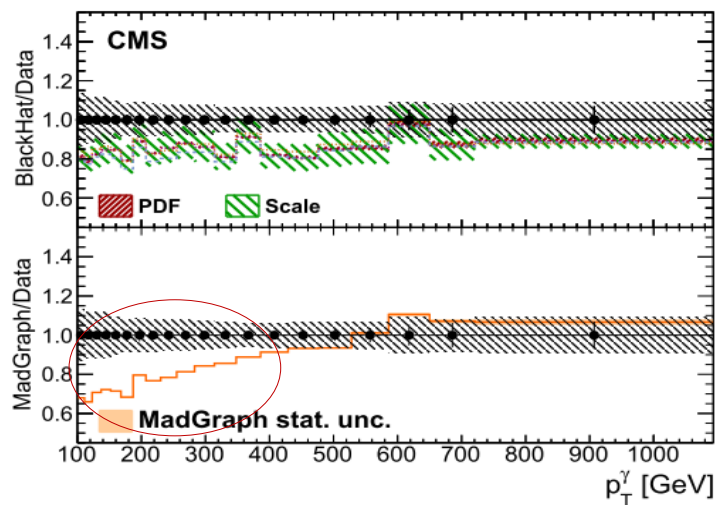
➤ Agreement is improved significantly with multileg NLO and NNLO calculations

Photon (γ) + jets cross section at 8 TeV



- **Lepton selection:**
 - $p_T(l) > 20 \text{ GeV}$
 - $|\eta(l)| < 2.4$
- **Photon selection:**
 - $p_T(\gamma) > 20 \text{ GeV}$
- **Jet selection:**
 - $p_T(j) > 30 \text{ GeV}$
 - $|\eta(j)| < 2.4$

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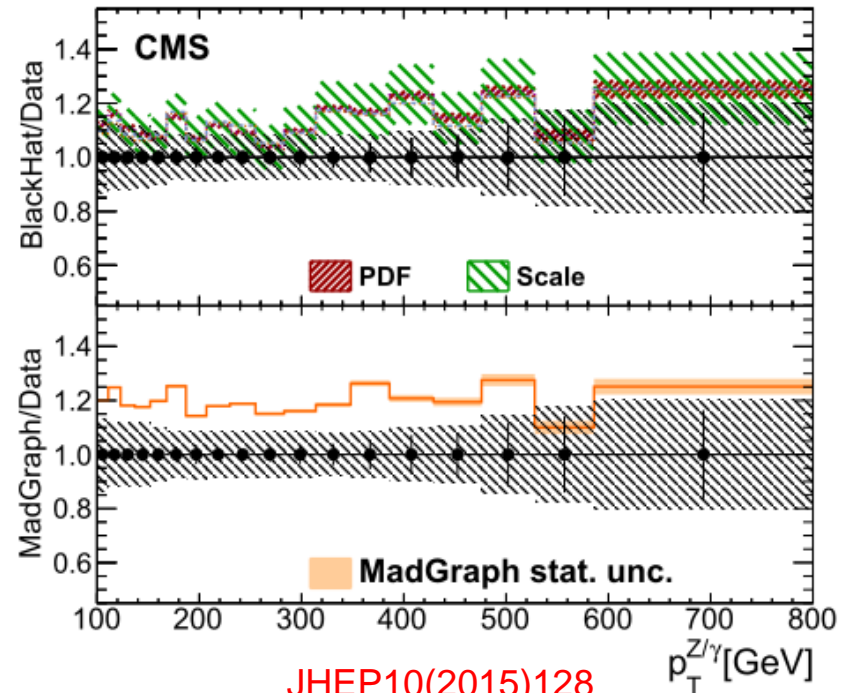
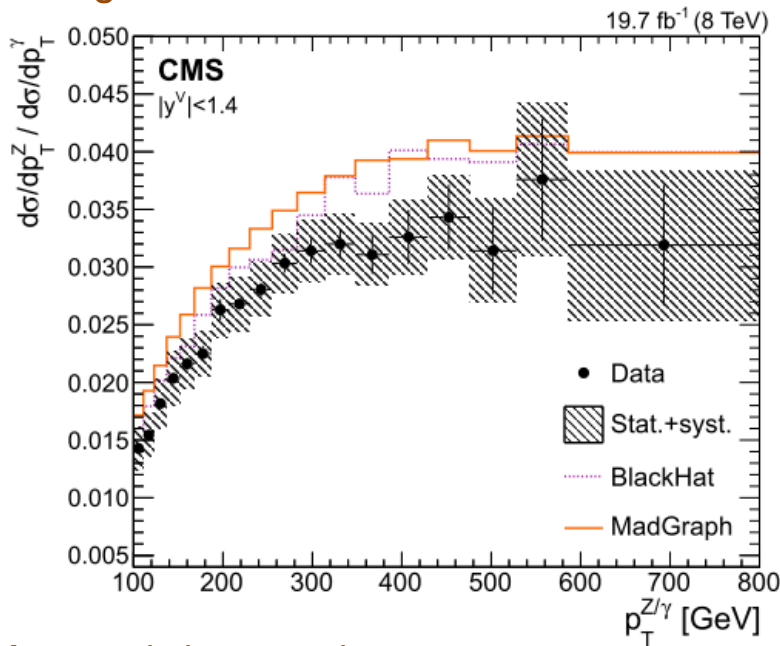


➤ BLACKHAT reproduces the shape of data better than MADGRAPH

- BLACKHAT overestimates the ratio by 10% for $p_T(\gamma) > 100 \text{ GeV}$
- LO prediction describes the data shape very well

Z + jets & γ +jets cross section ratio at 8 TeV

- Precise measurement of this ratio provides important information about the higher order effects of logarithmic corrections at higher transverse momentum
- It helps to reduce the systematical uncertainties corresponding to the Z(\rightarrow vv)+jets background estimation in SUSY searches



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- Measured plateau ratio:

$$R_{\text{dilep}} = \frac{\sigma_{Z \rightarrow \ell^+ \ell^-}(p_T^Z > 314 \text{ GeV})}{\sigma_{\gamma}(p_T^\gamma > 314 \text{ GeV})} = 0.0322 \pm 0.0008 (\text{stat}) \pm 0.0020 (\text{syst})$$

- The observed ratio increases gradually and saturates around 300~350 GeV of p_T :
 - The main distinctions between two processes are mass difference and different coupling
- Madgraph (Tree level) has already reproduced the shape of data well

W/Z + heavy flavor jets (b/\bar{b}) measurement at 8 TeV

- Theoretical uncertainties on W/Z + heavy flavor jets larger than for light jets case
 - ◆ Heavy quark content in the proton
 - ◆ Modeling of gluon splitting (initial state or final state)
 - ◆ Massive or massless b quark in computations
- Test of QCD predictions with various implementations (LO + Multipartons + parton shower, NLO, NLO+PS)
- Important process for backgrounds of Higgs measurements
- Descriptions of “b-quark initiated processes:”
 - ◆ 4 flavors number scheme (4FS): b quark generated with gluon splitting
 - ◆ 5 flavors number scheme (5FS): b quark (massless) generated in the initial state

W+b \bar{b} cross section at 8 TeV

➤ Lepton selection:

- ◆ $P_T > 30$ GeV
- ◆ $|\eta| < 2.1$

➤ B-tagged jet selection:

- ◆ $P_T > 25$ GeV
- ◆ $|\eta| < 2.4$

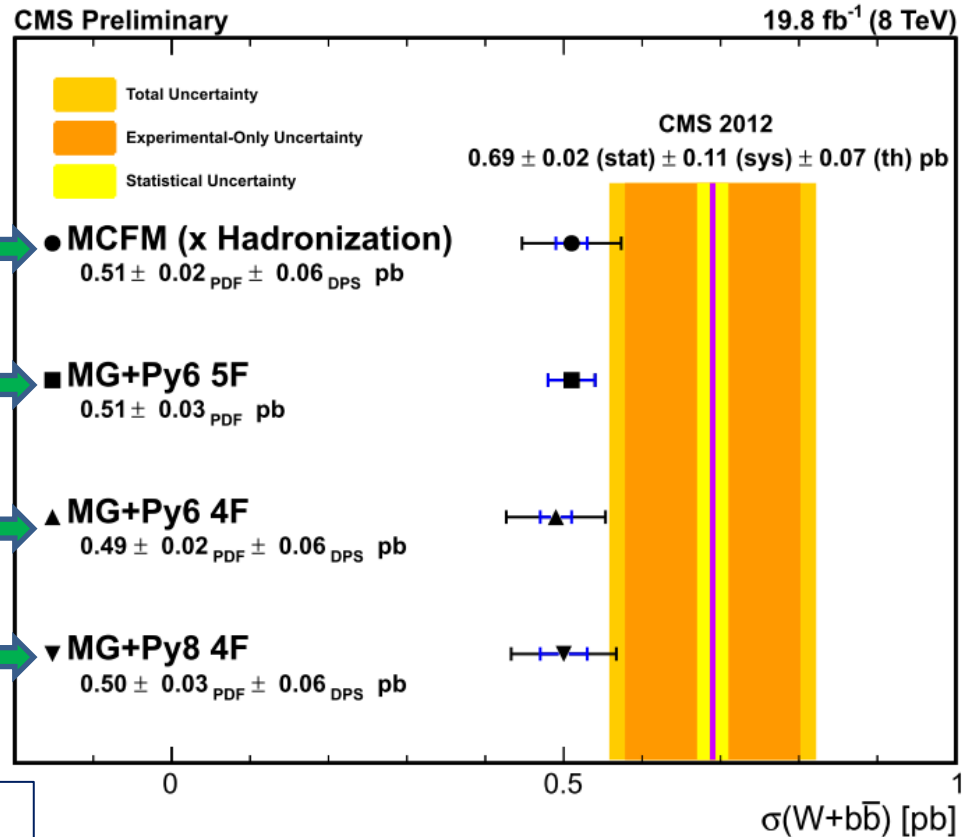
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NLO

LO

LO

LO

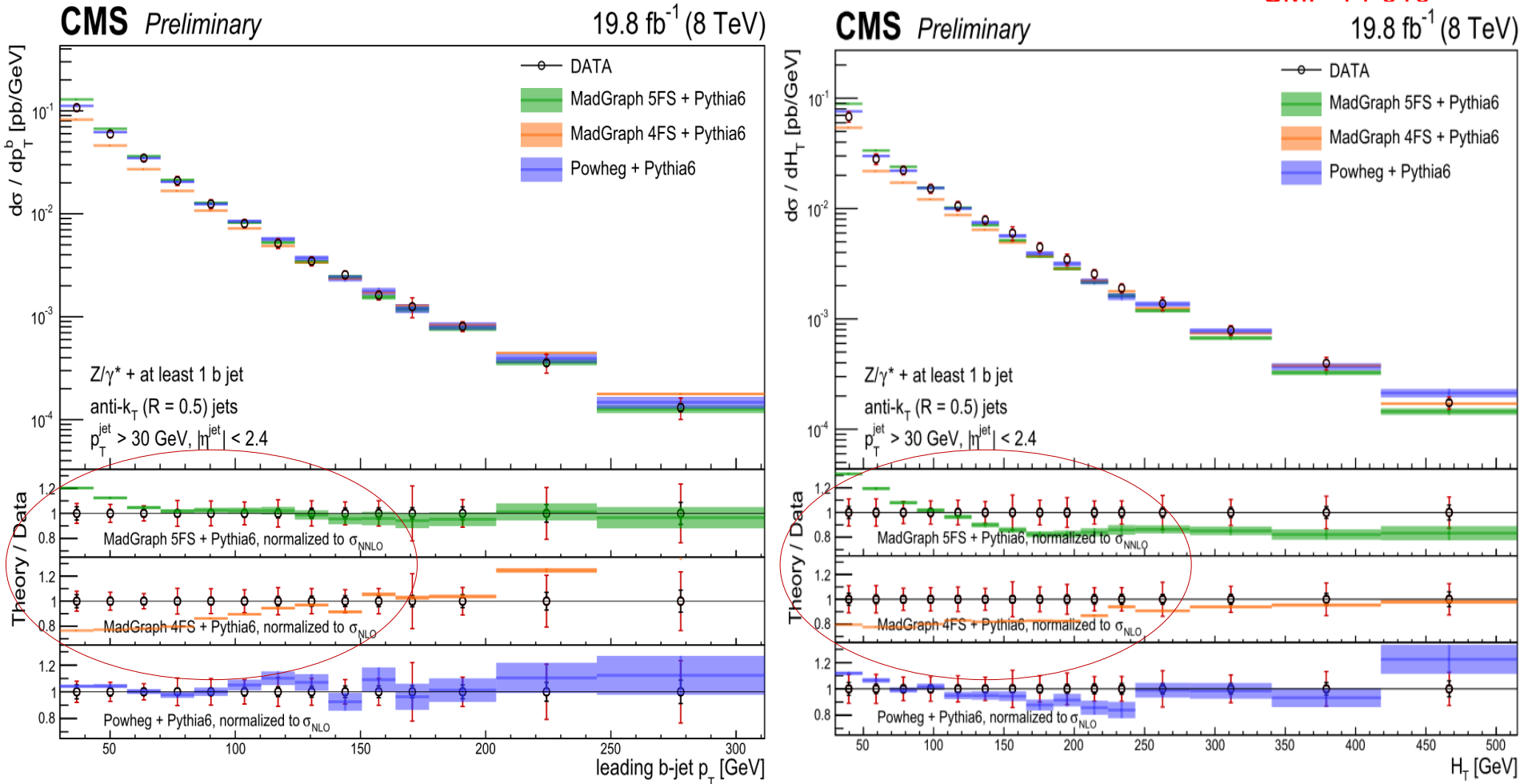


- The MCFM and MG+Py* with 4 flavor scheme do not account for the $b\bar{b}$ system coming from multiple parton interaction
- Hadronization correction is implemented for MCFM prediction which is at parton level

- All the predictions agree with measured cross section within one standard deviation

Z+b jets cross section at 8 TeV

SMP-14-010



- Discrepancy of about 20% observed for 4FNS-based LO prediction for Z + b jet case
- Overall agreement is observed between measurement and NLO prediction (Powheg), while the 5FNS-based prediction slightly overshoots the data at small transverse momentum of b jet (Z + b jet)

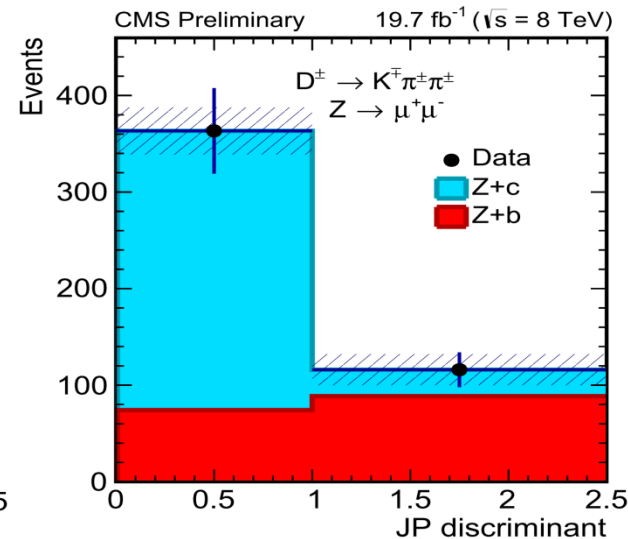
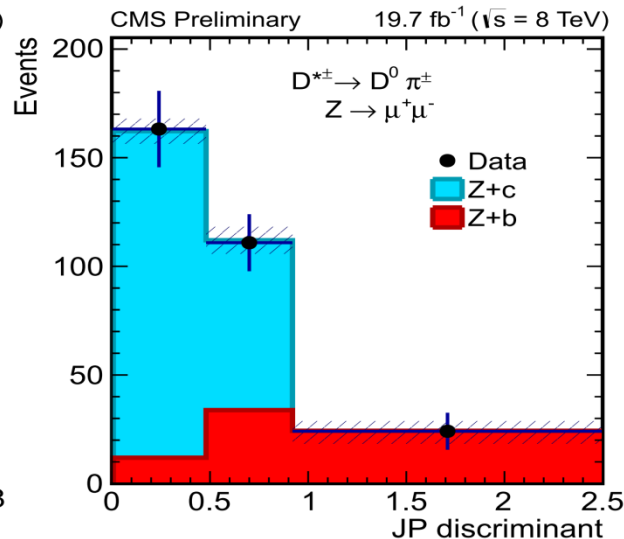
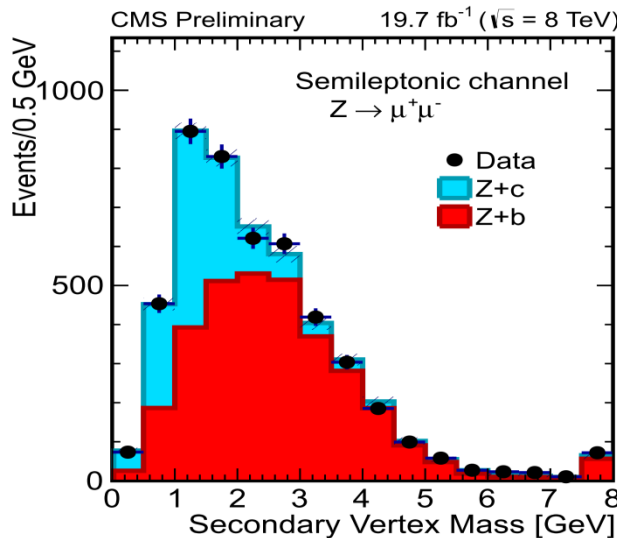
Z+c jets cross section at 8 TeV

➤ Three modes for identifying a jet from heavy flavor quarks:

- Semileptonic decay of a heavy flavor hadron:
 - A muon participating in a secondary vertex
- D^\pm mode: three tracks from a secondary vertex:
 - $D^\pm \rightarrow K^\mp \pi^\pm \pi^\pm$
- $D^{*\pm}$ mode: two tracks from a secondary vertex:
 - $D^{*+} \rightarrow D^0 \pi^+ \quad | \quad D^0 \rightarrow K^- \pi^+$
 - $D^{*-} \rightarrow \bar{D}^0 \pi^- \quad | \quad \bar{D}^0 \rightarrow K^+ \pi^-$

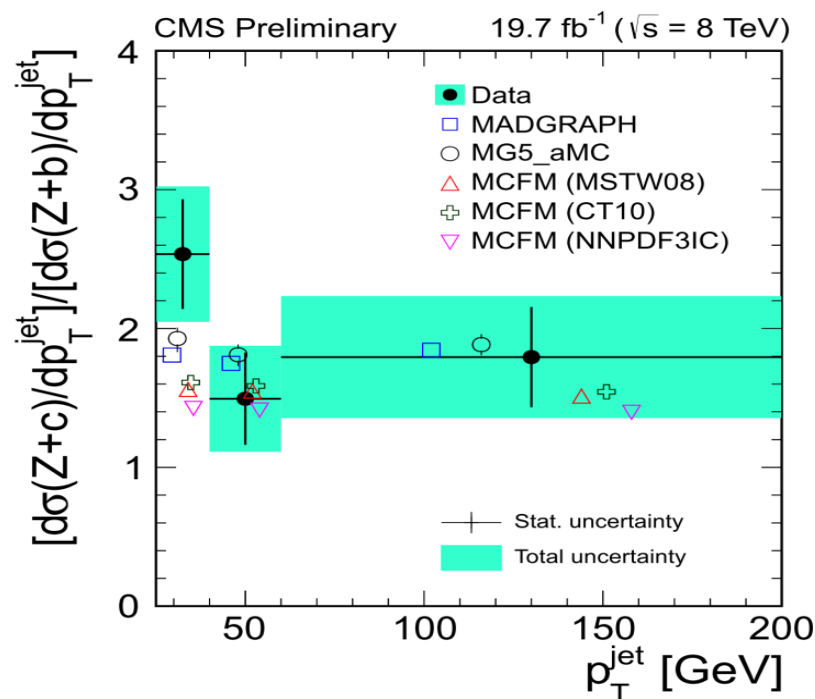
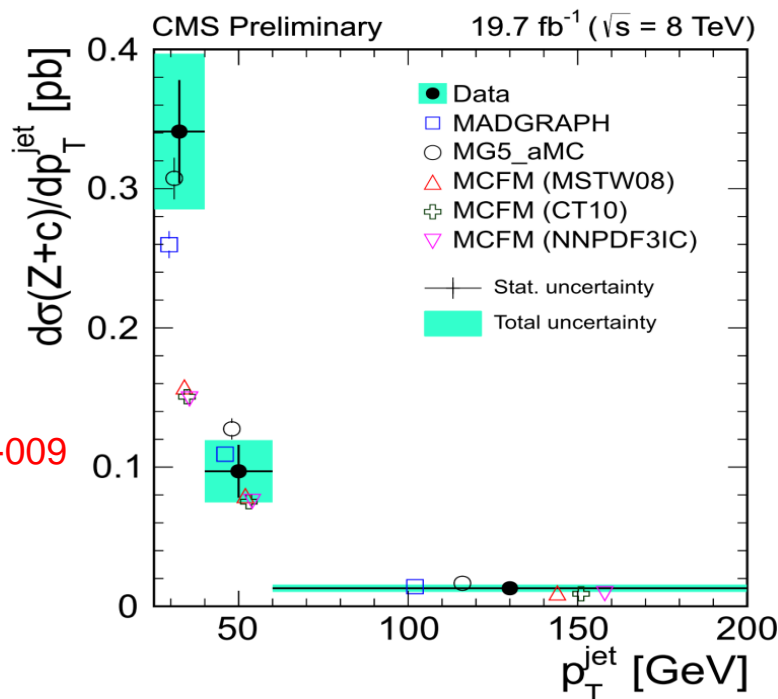
SMP-15-009

- Lepton selection:
 - ◆ $p_T > 20$ GeV
 - ◆ $|\eta| < 2.1$
 - ◆ $71 < M(\ell\ell) < 111$ GeV
- High flavor jet selection:
 - ◆ $p_T > 25$ GeV
 - ◆ $|\eta| < 2.5$



- A likelihood estimate of the probability that jet tracks comes from the primary vertex in the pure hadronic decay mode (JP discriminant)
- The invariant mass of all the charged particles (including a muon) constituting the secondary vertex in the semileptonic decay mode

Z+c jets cross section at 8 TeV



SMP-15-009

- The predictions with MADGRAPH LO and AMC@NLO predictions successfully reproduce the measurement in general
- The MCFM predictions are in good agreement for the p_T region above 40 GeV

- The measured Z + c-flavor-jet production cross section:
 8.6 ± 0.5 (stat.) ± 0.7 (syst.) pb
- The measured cross section ratio of Z+c to Z+b:
 2.0 ± 0.2 (stat.) ± 0.2 (syst.) pb

Electroweak W+2 jets cross section measurement at 8 TeV

SMP-13-012 arXiv:1607.06975

Submitted to JHEP recently

➤ Lepton selection:

◆ $p_T(\mu) > 25 \text{ GeV}$, $p_T(e) > 30 \text{ GeV}$

◆ $|\eta(\mu)| < 2.1$, $|\eta(e)| < 2.5$

◆ $E_T > 25 \text{ (30) GeV}$

◆ $M_T(W) > 30 \text{ GeV}$

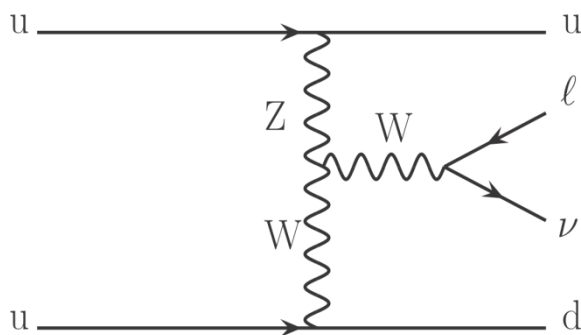
➤ Jet selection:

◆ $p_T(j1) > 60 \text{ GeV}$, $p_T(j2) > 50 \text{ GeV}$

◆ $|\eta(j)| < 4.7$

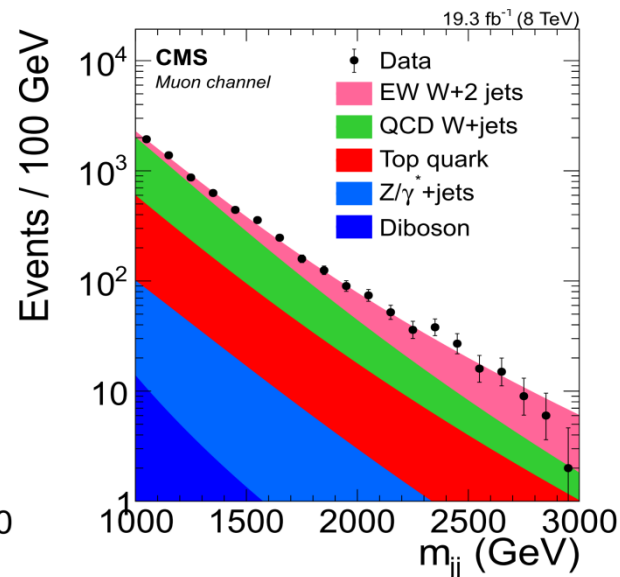
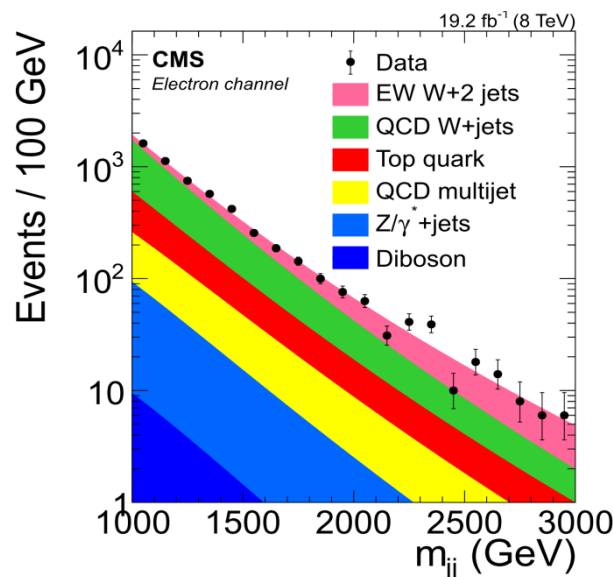
◆ $M(jj) > 1000 \text{ GeV}$

◆ $|y(W) - (y(j1) + y(j2))/2| < 1.2$



❑ The first time to measure the EW W+2jets (forward jets)

❑ It will benefit from more statistics at 13 TeV



- The signal process is predicted using MADGRAPH5 in LO accuracy (CTEQ6L1 PDF) and interfaced with PYTHIA6
- An unbinned maximum likelihood fit is used to extract the signal process yields

■ The measured EW W+2jets production cross section:
 $0.42 \pm 0.04 \text{ (stat.)} \pm 0.09 \text{ (syst.)} \pm 0.01 \text{ (lumi.) pb}$

■ The LO prediction using MADGRAPH5:
 $0.50 \pm 0.02 \text{ (scale)} \pm 0.02 \text{ (PDF) pb}$

Outlook

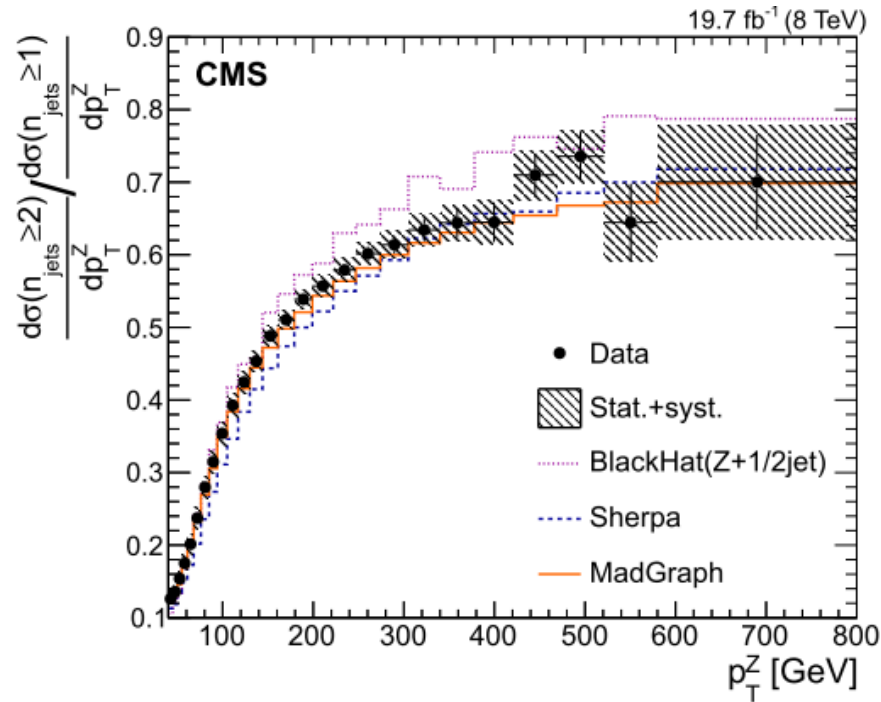
- The measurement on vector boson plus jets processes is quite important:
 - It deepens our understanding on QCD dynamics
 - It improves the modeling of background for Higgs measurement and new physics searches
- There have been significant improvement on theoretical predictions and experimental measurements:
 - Reach high precision of measurement
 - Better agreement with higher order (NLO) calculation than LO in general
 - ◆ Remained discrepancy and large uncertainty motivate the ongoing work to improve modeling and precision
- More results at both 8TeV and 13TeV will come out soon!

Backup

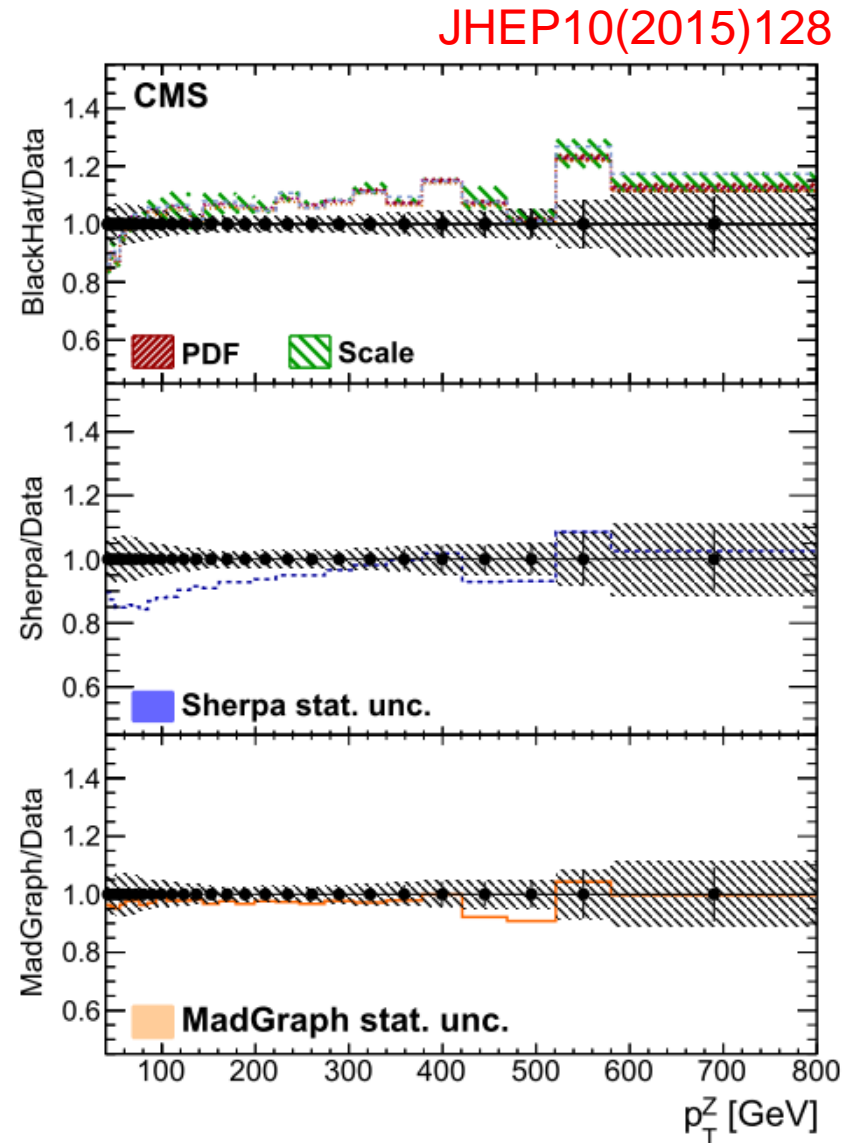
V+jets predictions on Monte Carlo evolution

- There are still theoretical uncertainties related to various sources which can be constrained by data:
 - ◆ Higher order QCD corrections (NNLO)
 - ◆ Electroweak corrections
 - ◆ Parton shower and its matched matrix element
 - ◆ Parton density functions
 - ◆ Underlying event modeling (multiple parton interactions)

$Z + \geq 1$ jet & $Z + \geq 2$ jets cross section ratio as function of Z p_T



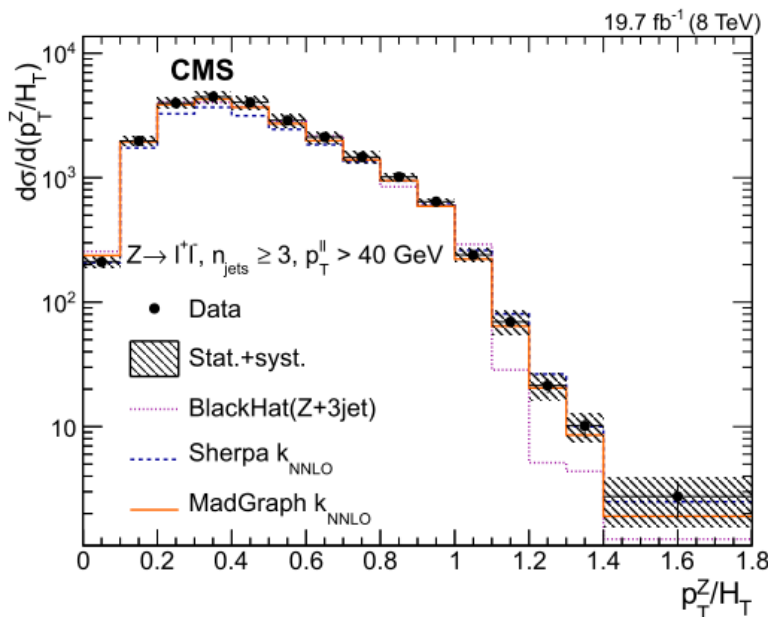
- The ratio increases until reaching around 350 GeV before a plateau
- SHERPA underestimates the relative rate of inclusive 2 jets case
- BLACKHAT overestimates the ratio after around 100 GeV



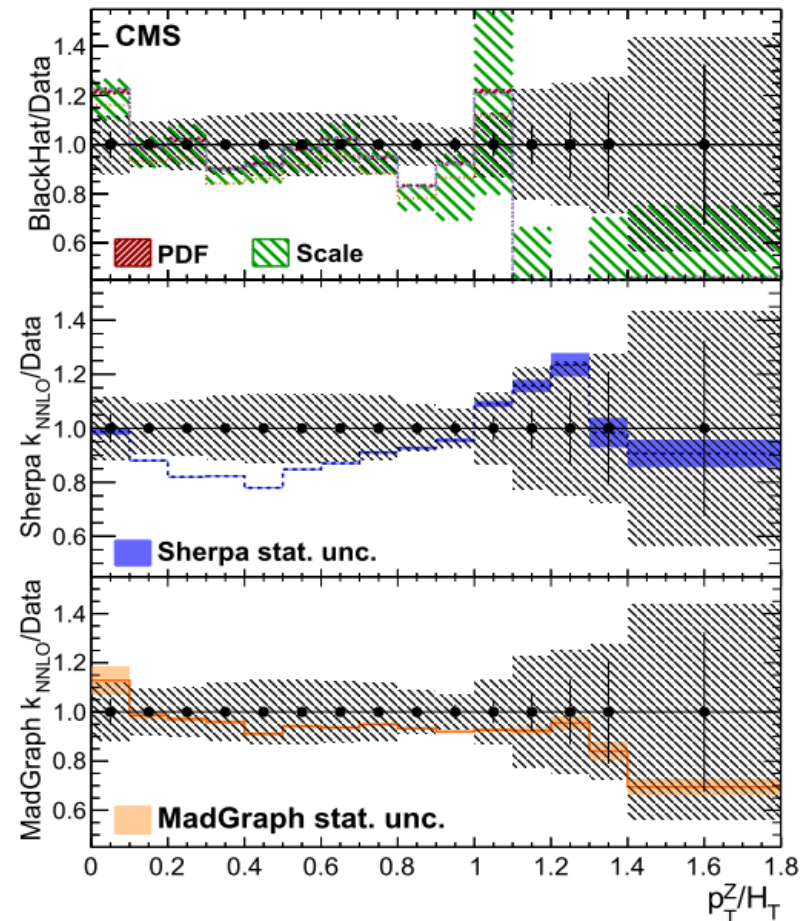
$Z + \geq 3$ jets cross section ratio as function of $Z p_T / H_T$

- ◆ This observable allows to test validity of NLO estimation, which might reach computational limit due to large logarithms or missing higher order effects may play a larger role
- ◆ Events in the high-end tail contribute to signatures with a high E_T/H_T ratio for hadronic new physics searches

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- MG5 generally predicts best on the shape
- SHERPA shows discrepancy on shape and rate
- BLACKHAT performs well on the bulk of distribution while failing to reproduce the tails



Data statistics for W/Z +jets measurement (13TeV)

➤ Data

- ◆ Measured differential cross section as a function of several observables
- ◆ The presence of data goes up to large jet multiplicity region for both W/Z + jets

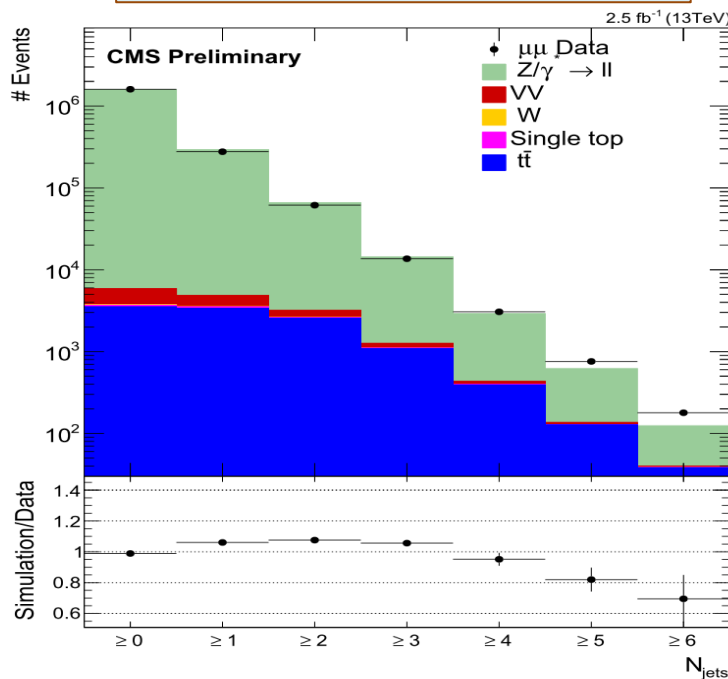
➤ Phase space for Z+jets:

● Lepton selection:

- $p_T(l) > 20 \text{ GeV}$
- $|\eta(l)| < 2.4$
- $71 < M(l\bar{l}) < 111 \text{ GeV}$

● Jet selection:

- $p_T(j) > 30 \text{ GeV}$
- $|\eta(j)| < 2.4$



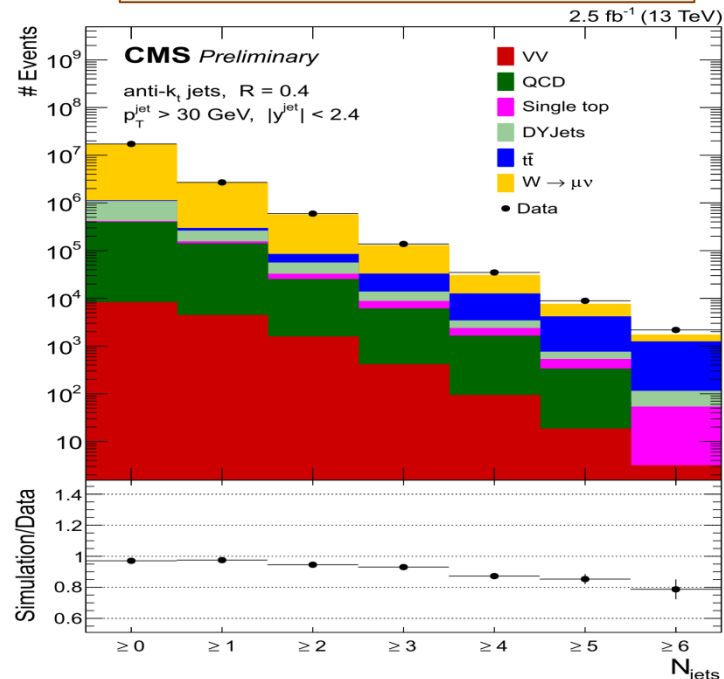
➤ Phase space for W+jets:

● Lepton selection:

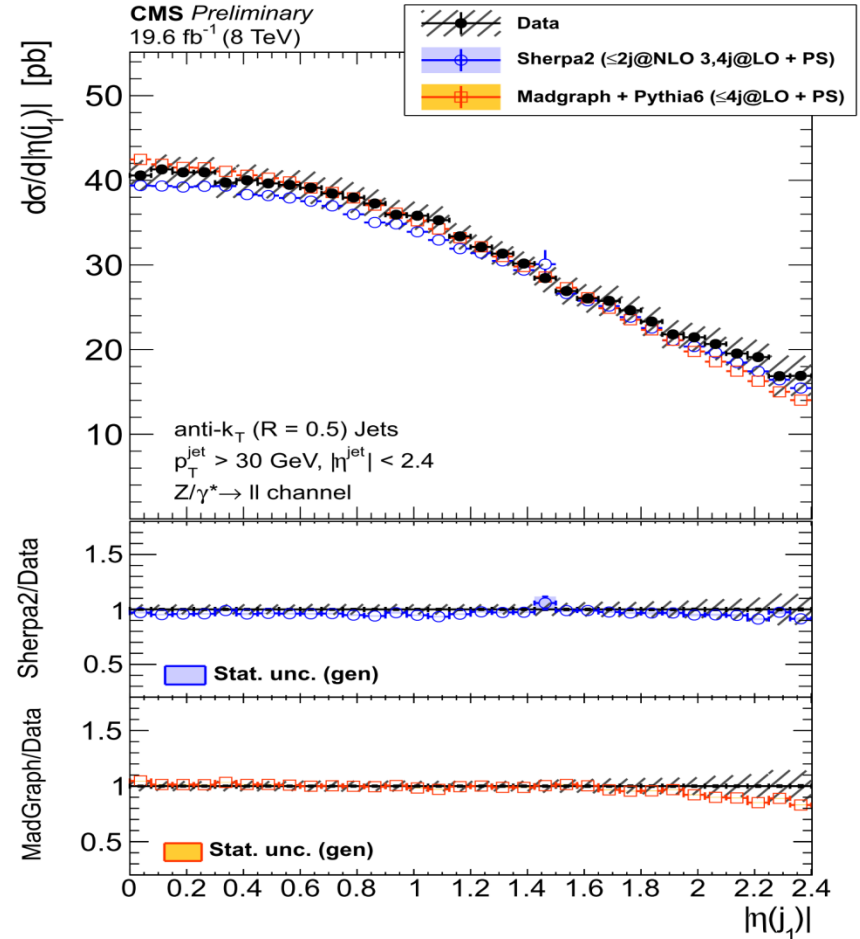
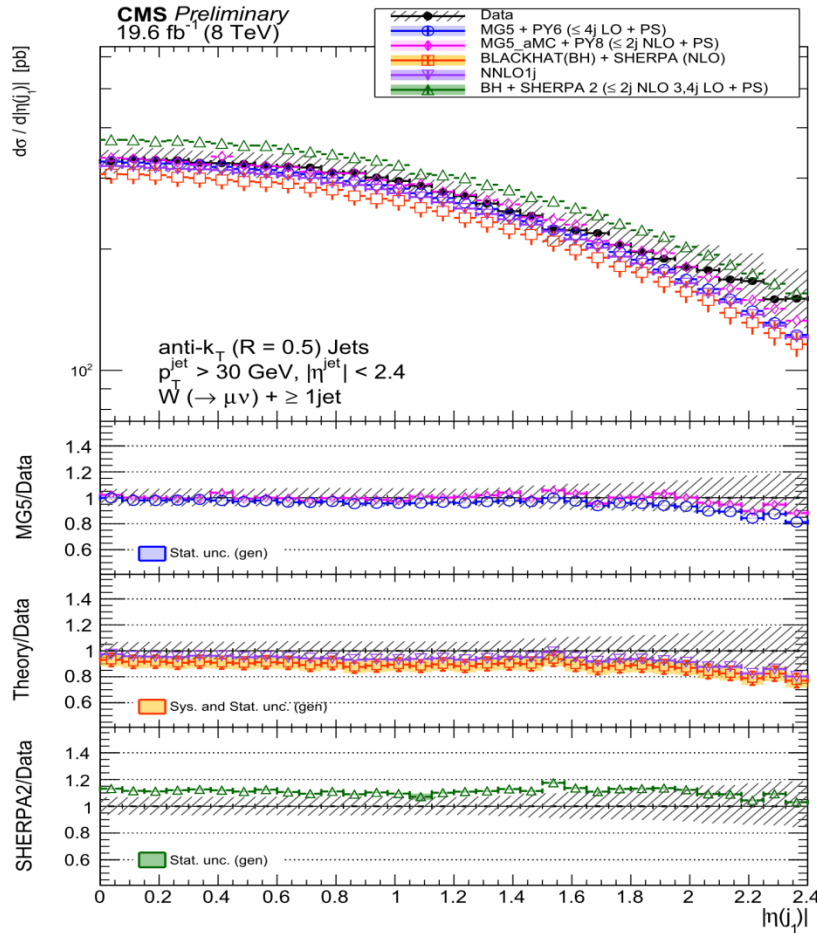
- $p_T(l) > 25 \text{ GeV}$
- $|\eta(l)| < 2.1$
- $M_T > 50 \text{ GeV}$

● Jet selection:

- $p_T(j) > 30 \text{ GeV}$
- $|\eta(j)| < 2.4$

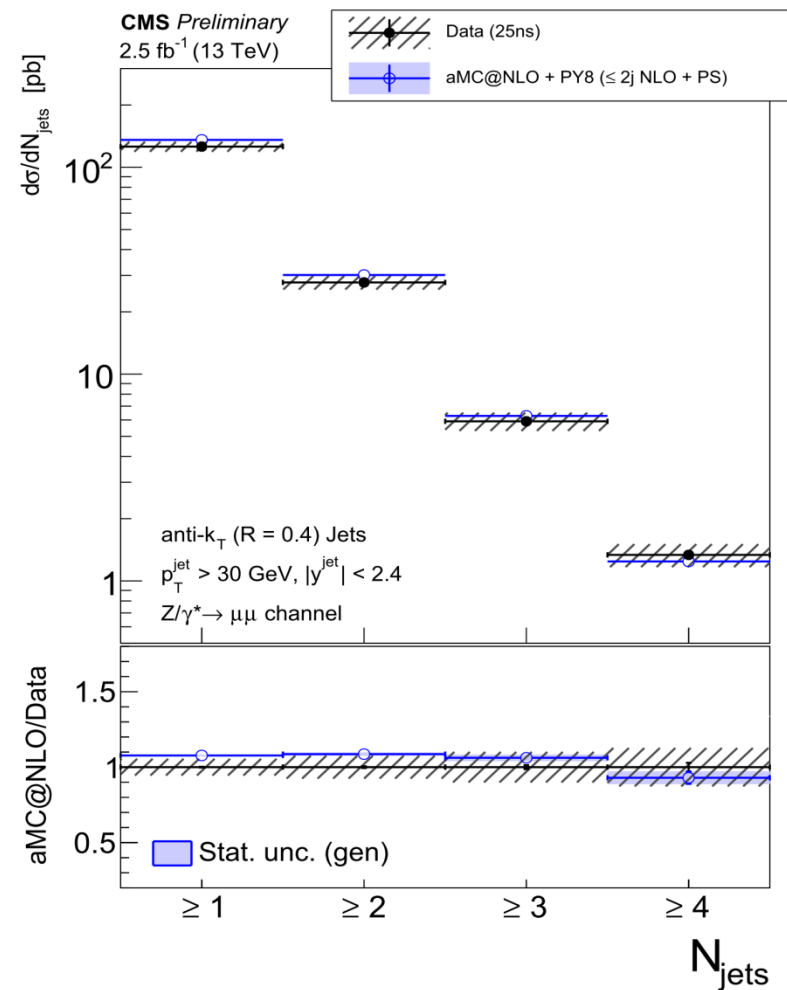
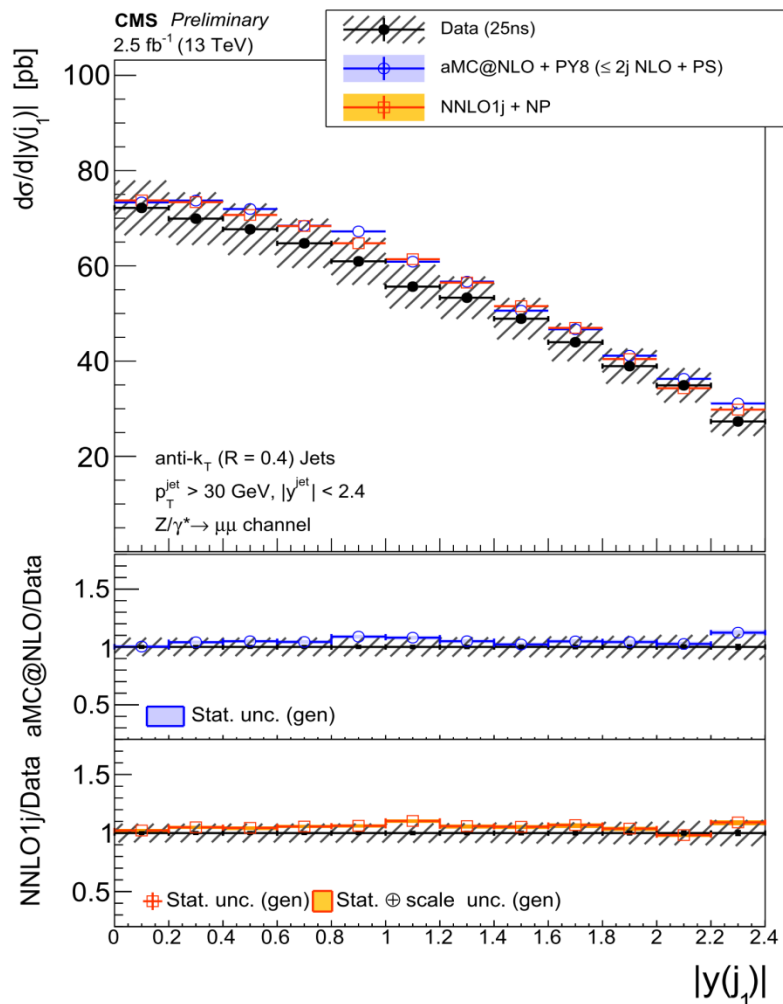


W/Z + jets differential cross section at 8 TeV



- Good agreement in general for all the predictions
- The p_T , η , H_T of jet for inclusive jet multiplicities up to 5 jets have also been measured
 - ▣ H_T is the scalar sum of the p_T of jets

Z + jets differential cross section at 13 TeV



- Good agreement with both multileg NLO and NNLO calculations
- The p_T , η , H_T of jet for inclusive jet multiplicities up to 3 jets have also been measured
- H_T is the scalar sum of the p_T of jets

$W+b\bar{b}$ measurement at 8 TeV

PAS: SMP-14-020

➤ Lepton selection:

◆ $P_T > 30$ GeV

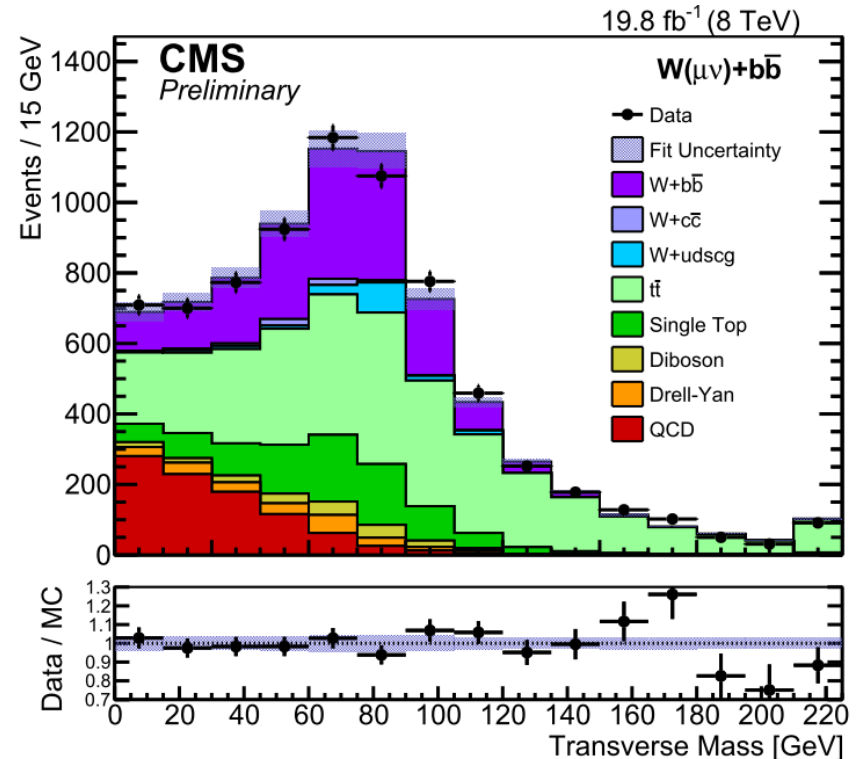
◆ $|\eta| < 2.1$

➤ B-tagged jet selection:

◆ $P_T > 25$ GeV

◆ $|\eta| < 2.4$

- The signal sample is generated at tree-level by MADGRAPH5 interfaced with PYTHIA6
- The $t\bar{t}$ is the dominant background, which is reweighted by data in the signal free region, in order to predict the transverse mass distribution in the signal region



Z+b jets measurement at 8 TeV

➤ Lepton selection:

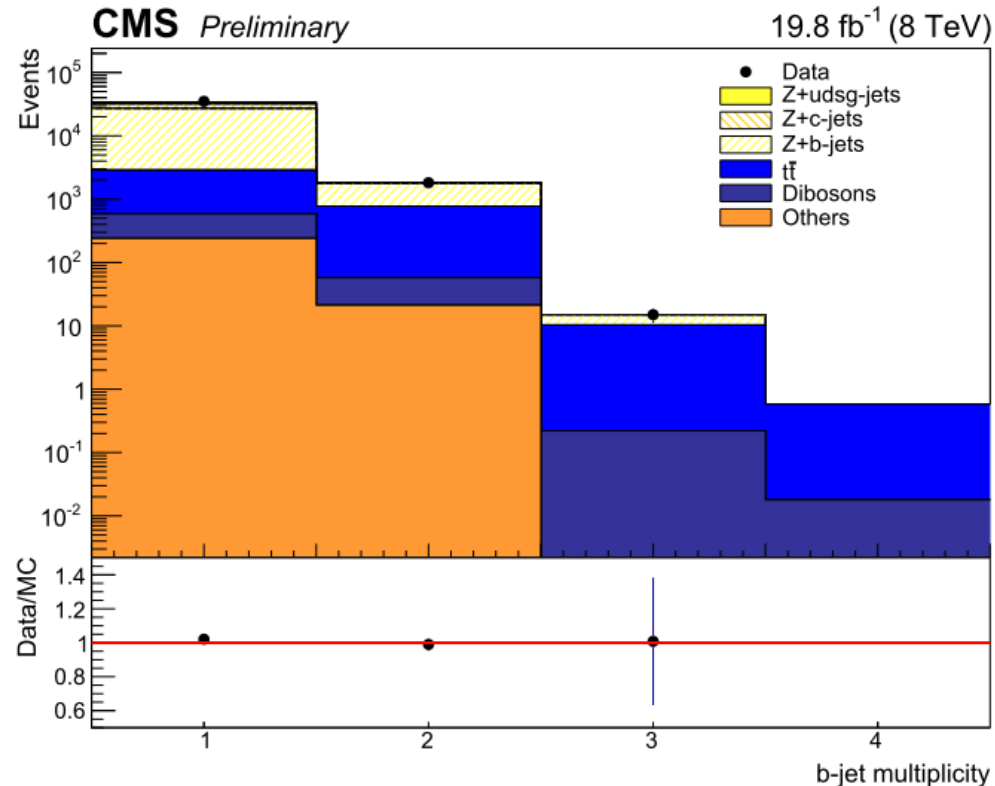
- ◆ $P_T > 20$ GeV
- ◆ $|\eta| < 2.4$
- ◆ $71 < M_{ll} < 111$ GeV

➤ B-tagged jet selection:

- ◆ $P_T > 30$ GeV
- ◆ $|\eta| < 2.4$

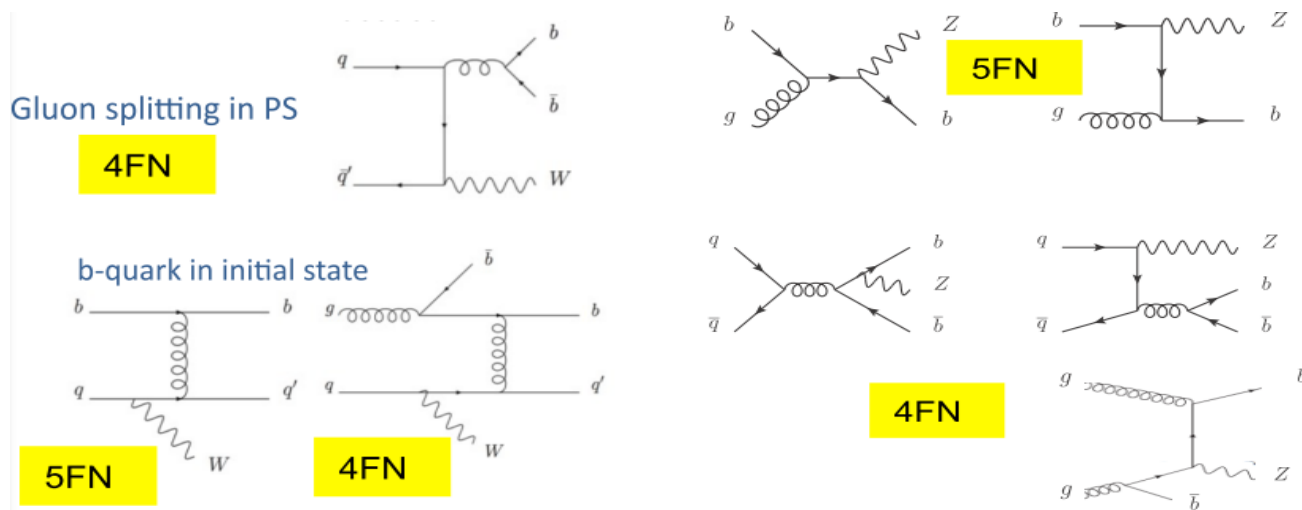
- The signal sample is generated with MADGRAPH5 implementing a matrix element up to 4 partons in the final state, interfaced with PYTHIA6
- The $t\bar{t}$ is the dominant background, which is also generated with MG5 implementing a matrix element up to 3 partons in the final state

PAS: SMP-14-010



W/Z + heavy flavor jets (b/\bar{b}) measurement at 8 TeV

- Theoretical uncertainties on W/Z + heavy flavor jets larger than for light jets case
 - ◆ Heavy quark content in the proton
 - ◆ Modeling of gluon splitting (initial state or final state)
 - ◆ Massive or massless b quark in computations
- Test of QCD predictions with various implementations (LO+Multipartons + parton shower, NLO, NLO+PS)
- Important process for backgrounds of Higgs searches



- Descriptions of “b-quark initiated processes:”
- 4 flavors number scheme (4FN): b quark generated with gluon splitting
- 5 flavors number scheme (5FN): b quark (massless) generated in the initial state

Z+c jets cross section at 8 TeV

SMP-15-009

- Lepton selection:
 - ◆ $p_T > 20$ GeV
 - ◆ $|\eta| < 2.1$
 - ◆ $71 < M(\ell\ell) < 111$ GeV
- High flavor jet selection:
 - ◆ $p_T > 25$ GeV
 - ◆ $|\eta| < 2.5$

Semileptonic mode

Channel	N_{Z+c}^{signal}	\mathcal{C}_{Z+c} (%)	$\sigma(Z+c)$ (pb)
$Z \rightarrow e^+e^-$	1066 ± 95	0.63 ± 0.03	$8.5 \pm 0.7 \pm 1.0$
$Z \rightarrow \mu^+\mu^-$	1449 ± 143	0.81 ± 0.03	$9.0 \pm 0.7 \pm 1.0$
Channel	N_{Z+b}^{signal}	\mathcal{C}_{Z+b} (%)	$\sigma(Z+c)/\sigma(Z+b)$
$Z \rightarrow e^+e^-$	2606 ± 114	2.90 ± 0.08	$1.9 \pm 0.2 \pm 0.2$
$Z \rightarrow \mu^+\mu^-$	3241 ± 147	3.93 ± 0.10	$2.2 \pm 0.3 \pm 0.2$

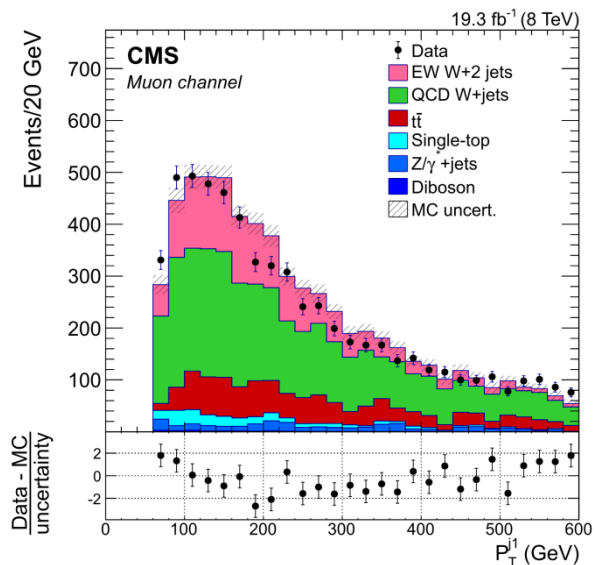
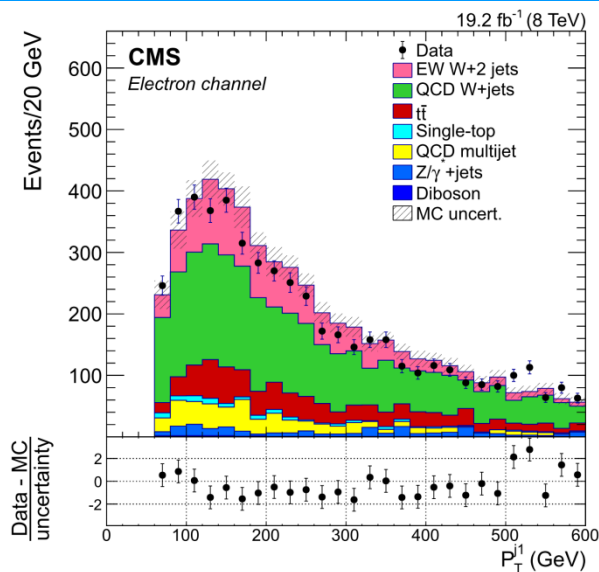
D^\pm mode

Channel	N_{Z+c}^{signal}	\mathcal{C}_{Z+c} (%)	$\sigma(Z+c)$ (pb)
$Z \rightarrow e^+e^-$	275 ± 55	0.13 ± 0.02	$11.0 \pm 2.1 \pm 0.9$
$Z \rightarrow \mu^+\mu^-$	315 ± 75	0.18 ± 0.02	$9.0 \pm 2.1 \pm 0.8$

$D^{*\pm}$ (2010) mode

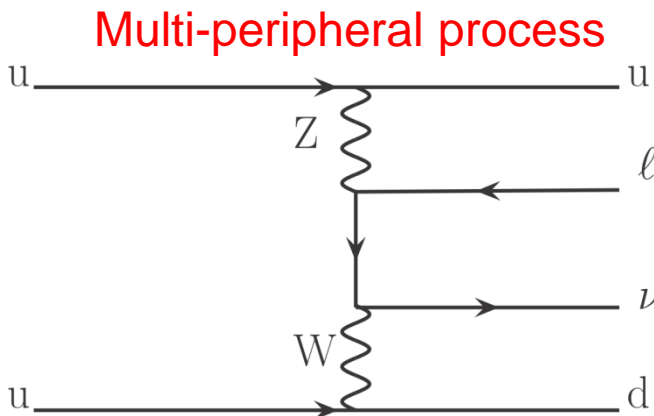
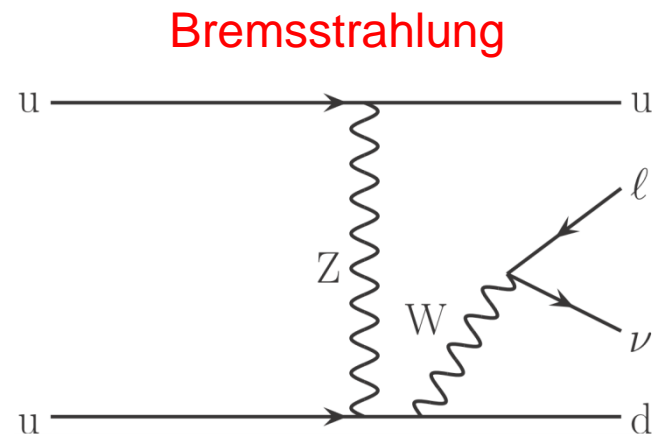
Channel	N_{Z+c}^{signal}	\mathcal{C}_{Z+c} (%)	$\sigma(Z+c)$ (pb)
$Z \rightarrow e^+e^-$	151 ± 31	0.11 ± 0.01	$7.3 \pm 1.5 \pm 0.6$
$Z \rightarrow \mu^+\mu^-$	228 ± 30	0.14 ± 0.01	$8.6 \pm 1.1 \pm 0.6$

Electroweak W+2 jets cross section measurement at 8 TeV



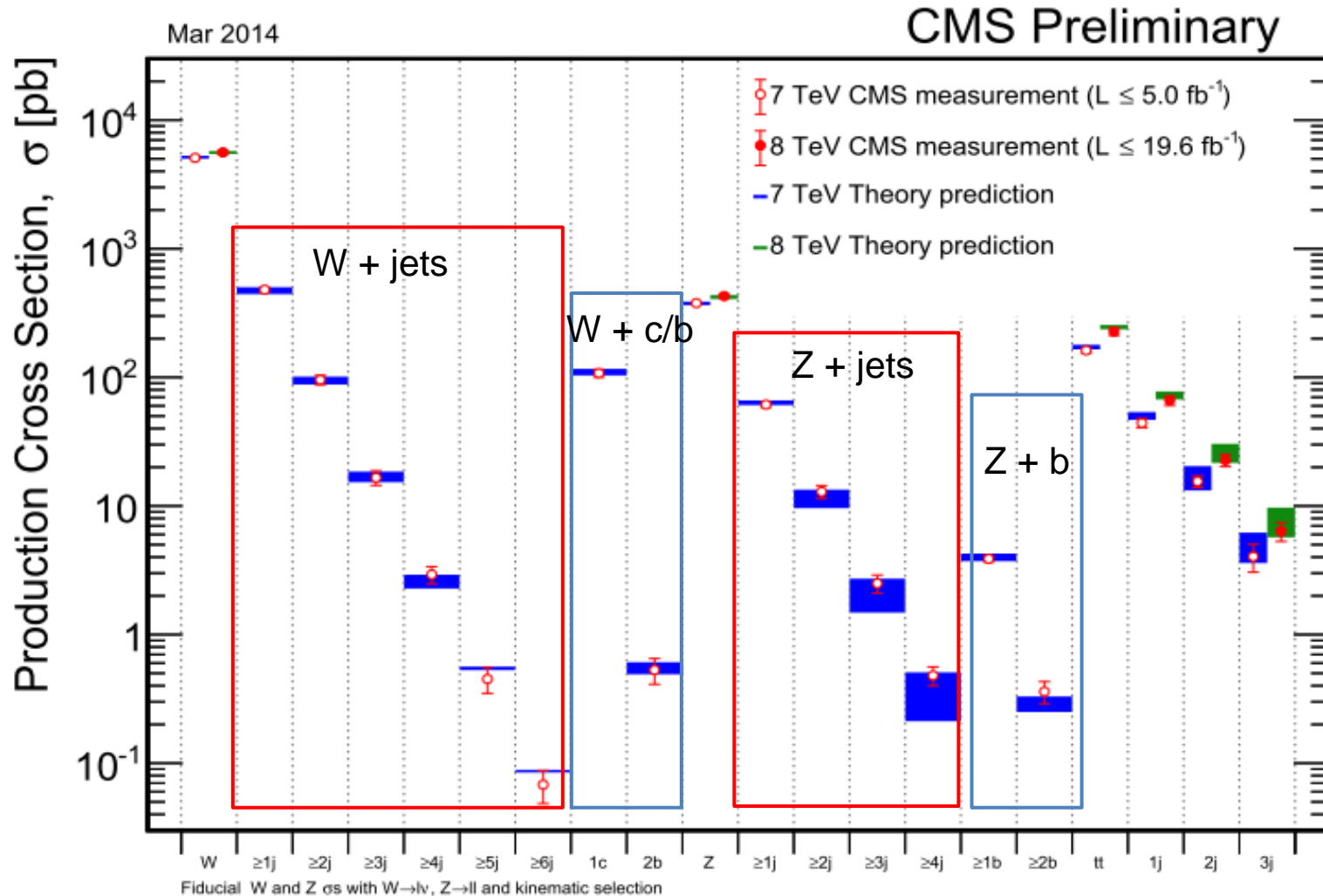
Process	Electrons		Muons	
Data	5481		6514	
	Pre-fit	Measured ratio	Pre-fit	Measured ratio
W+jets	3913	0.71 (fixed)	5084	0.70 (fixed)
Top quark	933	1.00 ± 0.07	1357	1.00 ± 0.07
QCD multijet	510	fixed to E_T^{miss} fit in data	—	—
DY+jets	236	1.00 (fixed)	256	1.00 (fixed)
Diboson	26	1.00 (fixed)	29	1.00 (fixed)
Total backgrounds	4488		5179	
Data – backgrounds	993		1335	
EW W+2-jets	1195	0.83 ± 0.08	1541	0.87 ± 0.08

Channel	Measured cross section
Electron	0.41 ± 0.04 (stat) ± 0.09 (syst) ± 0.01 (lumi) pb
Muon	0.43 ± 0.04 (stat) ± 0.10 (syst) ± 0.01 (lumi) pb
Combined	0.42 ± 0.04 (stat) ± 0.09 (syst) ± 0.01 (lumi) pb



■ **Measured signal strength:**
0.83 ± 0.08 (stat.) (electron channel)
0.87 ± 0.08 (stat.) (muon channel)

Legend of V+jets (Run I)



- Good agreement between experimental measurement and theory for Run I
- High precision up to very low cross section (high jet multiplicity)