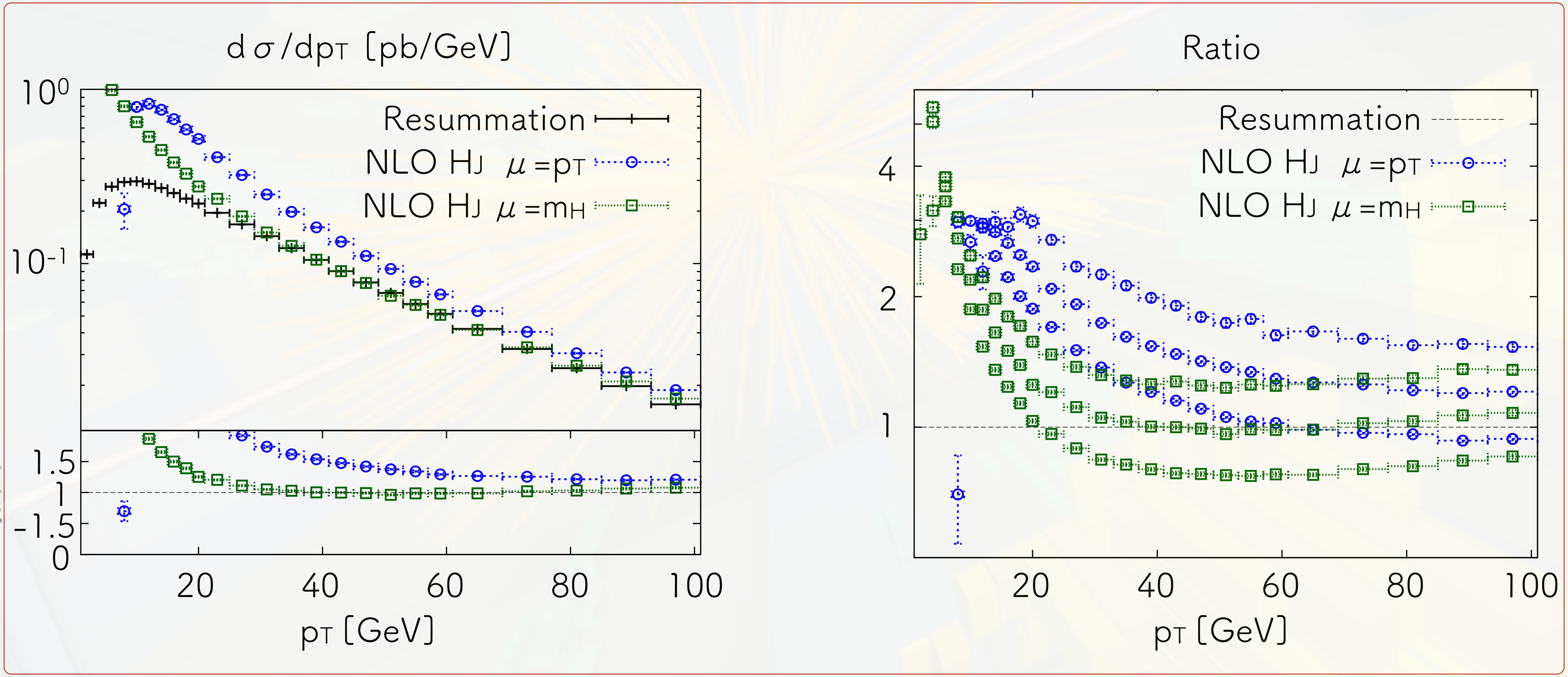
MiNLO

Outline

- MiNLO — Nason, Zanderighi, KH
- MiNLO′ — Nason, Oleari, Zanderighi, KH
- Extending the MiNLO method — Frederix, KH; Melia, Monni, Re, Zanderighi, KH

MiNLO introduction

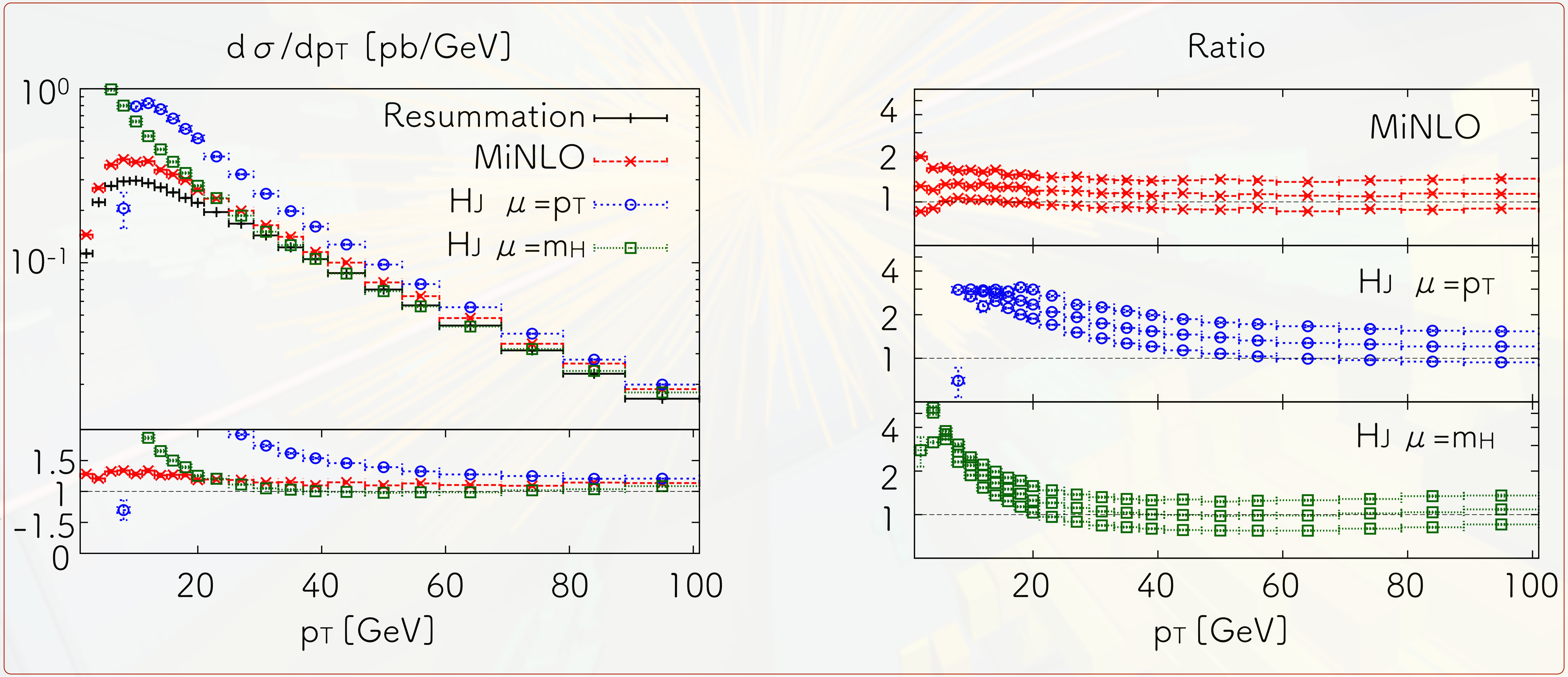
- Fixed order for jet prod^n procs fails if jets are close/low p_T
- Example: Higgs p_T from NLO H+jet calculation:



- Proper description at low p_T requires resummation

MiNLO introduction

- MiNLO matches fully differential NLO calc^{n}s to LL [nearly NLL$_\sigma$] Sudakov resummation
- MiNLO finite in all ph.space: no need of generation cuts



Question: what's MiNLO accuracy for inclusive quantities?

NLO H+jet radiation spectrum

- NLO H+jet calc^n : $$d\sigma = \boxed{d\sigma_{\mathcal{S}}} + \boxed{d\sigma_{\mathcal{SR}}} + \boxed{d\sigma_{\mathcal{F}}}$$

- Φ = H Born kinematics [here y_H], $L = \mathrm{Log}\,\frac{Q^2}{p_T^2}$

- $\boxed{d\sigma_{\mathcal{S}}}$: $p_T \to 0$ divergent terms

$$\boxed{\frac{d\sigma_{\mathcal{S}}}{d\Phi dL} = \frac{d\sigma_{\mathrm{LO}}}{d\Phi}\left[1+\bar{\alpha}_{\mathrm{S}}\left(\mu_R^2\right)\mathcal{H}_1\left(\mu_R^2\right)\right]\frac{d}{dL}\left[\Delta\left(Q,p_{\mathrm{T}}\right)\mathcal{L}\left(\{x_\ell\},\mu_F,p_{\mathrm{T}}\right)\right]\Big|_{\mathrm{NLO}}}$$

- $\boxed{d\sigma_{\mathcal{SR}}}$: $p_T \to 0$ divergent remainder as Sudakov only NLL$_\sigma$

$$\boxed{\frac{d\sigma_{\mathcal{SR}}}{d\Phi dL} = \frac{d\sigma_{\mathrm{LO}}}{d\Phi}\,\bar{\alpha}_{\mathrm{S}}^2\left(\mu_R\right)\left[L\,\widetilde{R}_{21}+\widetilde{R}_{20}\right]}$$

- $\boxed{d\sigma_{\mathcal{F}}}$: $p_T \to 0$ finite

MiNLO radiation spectrum

- NLO H+jet calc^n : $d\sigma = \boxed{d\sigma_{\mathcal{S}}} + \boxed{d\sigma_{\mathcal{SR}}} + \boxed{d\sigma_{\mathcal{F}}}$

MiNLO

- MiNLO H+jet calc^n : $d\sigma_{\mathcal{M}} = \boxed{d\sigma_{\mathcal{R}}} + \boxed{d\sigma_{\mathcal{MR}}} + \boxed{d\sigma_{\mathcal{F}}}$

- $\boxed{d\sigma_{\mathcal{R}}}$: $p_T \to 0$ divergent terms, $d\sigma_{\mathcal{S}}$, after MiNLO

$$\boxed{\frac{d\sigma_{\mathcal{R}}}{d\Phi dL} = \frac{d\sigma_{\mathrm{LO}}}{d\Phi} \left[1 + \bar{\alpha}_{\mathrm{S}}\left(\mu_R^2\right) \mathcal{H}_1\left(\mu_R^2\right)\right] \frac{d}{dL}\left[\Delta\left(Q, p_{\mathrm{T}}\right) \mathcal{L}\left(\left\{x_\ell\right\}, \mu_F, p_{\mathrm{T}}\right)\right]}$$

- $\boxed{d\sigma_{\mathcal{MR}}}$: $p_T \to 0$ divergent remainder, $d\sigma_{\mathcal{SR}}$, after MiNLO

$$\boxed{\frac{d\sigma_{\mathcal{MR}}}{d\Phi dL} = \frac{d\sigma_{\mathrm{LO}}}{d\Phi} \Delta\left(Q, p_{\mathrm{T}}\right) \prod_{\ell=1}^{n_i} \frac{q^{(\ell)}\left(x_\ell, p_{\mathrm{T}}\right)}{q^{(\ell)}\left(x_\ell, \mu_F\right)} \left[\bar{\alpha}_{\mathrm{S}}^2\left(p_{\mathrm{T}}\right)\left[\widetilde{R}_{21} L + \widetilde{R}_{20}\right] - \bar{\alpha}_{\mathrm{S}}^3\left(p_{\mathrm{T}}\right) L^2\, \Sigma_\ell C_\ell\, \bar{\beta}_0 \mathcal{H}_1\right]}$$

- $\boxed{d\sigma_{\mathcal{F}}}$: $p_T \to 0$ finite

MiNLO integrated

- Integrate out all the radiation [including jet in the Born]:

$$\frac{d\sigma_{\mathcal{M}}}{d\Phi} = \boxed{\int_L \frac{d\sigma_{\mathcal{R}}}{d\Phi dL} + \int_L \frac{d\sigma_{\mathcal{F}}}{d\Phi dL}} + \boxed{\int_L \frac{d\sigma_{\mathcal{MR}}}{d\Phi dL}}$$

$$= \boxed{\frac{d\sigma_{\text{NLO}}}{d\Phi}} + \boxed{\int_L \frac{d\sigma_{\mathcal{MR}}}{d\Phi dL}}$$

- $\boxed{d\sigma_{\mathcal{MR}}}$: $p_T \to 0$ divergent remainder, $d\sigma_{\mathcal{SR}}$, after MiNLO

- $\boxed{\widetilde{R}_{21} \neq 0 \ : \quad \int_L \frac{d\sigma_{\mathcal{MR}}}{d\Phi dL} = -\frac{d\sigma_{\text{LO}}}{d\Phi}\,\bar{\alpha}_{\text{S}}\,\frac{\widetilde{R}_{21}}{\Sigma_\ell C_\ell}}$

- $\boxed{\widetilde{R}_{21} = 0 \ : \quad \int_L \frac{d\sigma_{\mathcal{MR}}}{d\Phi dL} = -\frac{d\sigma_{\text{LO}}}{d\Phi}\,\bar{\alpha}_{\text{S}}^{3/2}\,\sqrt{\frac{\pi}{2}}\,\frac{\widetilde{R}_{20} - \bar{\beta}_0\mathcal{H}_1}{\sqrt{\Sigma_\ell C_\ell}}}$

MiNLO′

- But H+1-jet spectrum known analytically to high accuracy
- Analytic control allows to formulate Sudakov s.t. $d\sigma'_{\mathcal{MR}} = 0$

MiNLO → MiNLO′

$$\Delta\left(Q,p_{\mathrm{T}}\right) \rightarrow \Delta'\left(Q,p_{\mathrm{T}}\right) = \Delta\left(Q,p_{\mathrm{T}}\right)\,\delta\Delta\left(Q,p_{\mathrm{T}}\right)$$

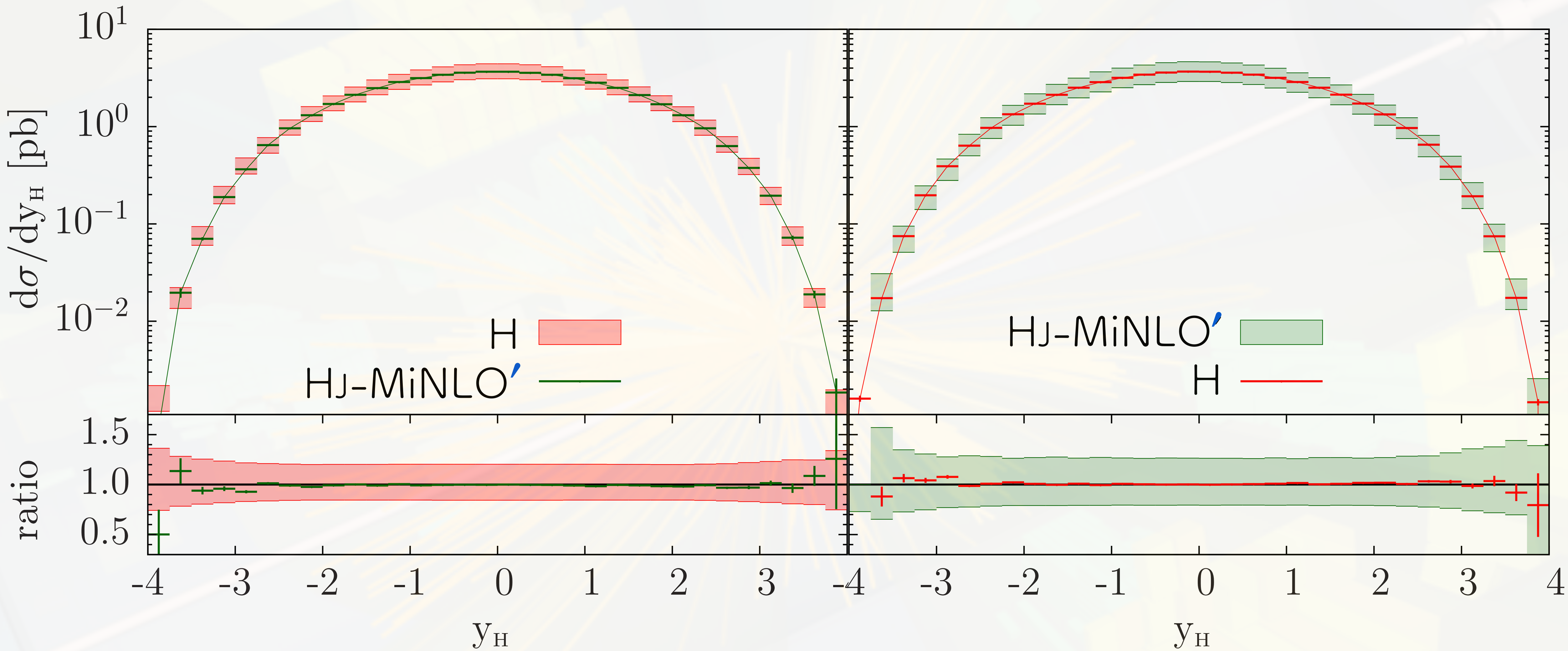
$$\delta\Delta\left(Q,p_{\mathrm{T}}\right) = \exp\left[\int_0^L dL'\,\bar{\alpha}_{\mathrm{S}}^2\left(p'_{\mathrm{T}}\right)\left[\widetilde{R}_{21}\,L' + \widetilde{R}_{20} - \bar{\beta}_0\mathcal{H}_1\right]\right]$$

$$\frac{d\sigma'_{\mathcal{M}}}{d\Phi} = \int_L \frac{d\sigma'_{\mathcal{R}}}{d\Phi dL} + \int_L \frac{d\sigma_{\mathcal{F}}}{d\Phi dL} = \frac{d\sigma_{\mathrm{NLO}}}{d\Phi}$$

- MiNLO′ simultaneously NLO for H & H+1-jet prod$^{\mathrm{n}}$

MiNLO′ H+1-jet

- Higgs rapidity



- Conventional NLO H prod^n: red
- MiNLO′ H+1-jet+parton shower: green
- Agree with each other ~ to within the line thickness

Extending the MiNLO method

- MiNLO′ needed rad^n spectrum at $\mathrm{N^3LL}_\sigma$ to NLO
- For MiNLO → MiNLO′ H+1-jet we put in $\mathrm{N^3LL}_\sigma$ term ~ `B_2′
- Known for p_T of Boson/y_{01} jet rate in W/Z/H/HW/HZ prod^n
- Generally we don′t know `p_T′ → 0 divergent terms to NLO
- Then what?

Extending the MiNLO method

- Earlier we computed the difference w.r.t. conventional inclusive NLO that unknown/uncontrolled terms give rise to:

$$\frac{d\sigma_{\rm NLO}}{d\Phi} - \frac{d\sigma_{\mathcal{M}}}{d\Phi} = \frac{d\sigma_{\rm LO}}{d\Phi}\,\bar{\alpha}_{\rm S}\,\frac{\widetilde{R}_{21}}{\Sigma_\ell C_\ell}\,(1+\mathcal{O}(\sqrt{\bar{\alpha}_{\rm S}}))$$

- Can manipulate this to get approx Sudakov coefficients:

$$\frac{\dfrac{d\sigma_{\rm NLO}}{d\Phi} - \dfrac{d\sigma_{\mathcal{M}}}{d\Phi}}{\displaystyle\int_L \bar{\alpha}_{\rm S}^2 L^2\,\frac{d\sigma_{\mathcal{M}}}{dL\,d\Phi}} = \frac{1}{2}\widetilde{R}_{21}\left(1+\mathcal{O}\left(\sqrt{\bar{\alpha}_{\rm S}}\right)\right)$$

Extending the MiNLO method

- If MiNLO Sudakov had all NLL$_\sigma$ terms, neglecting N^{3}LL$_\sigma$, can write MiNLO$'$ Sudakov equivalently as

$$\Delta'(Q,\,p_\mathrm{T}) \;=\; \Delta(Q,\,p_\mathrm{T})\,\exp\left[\int_0^L dL'\,\bar{\alpha}_\mathrm{S}(p'_\mathrm{T})\left[\widetilde{R}_{21}L'+\widetilde{R}_{20}-\bar{\beta}_0\mathcal{H}_1\right]\right]$$

$$\to\; \Delta(Q,\,p_\mathrm{T})\left[1+\bar{\alpha}_\mathrm{S}^2L^2\,\frac{\dfrac{d\sigma_\mathrm{NLO}}{d\Phi}-\dfrac{d\sigma_\mathcal{M}}{d\Phi}}{\displaystyle\int_L\bar{\alpha}_\mathrm{S}^2L^2\,\frac{d\sigma_\mathcal{M}}{dL\,d\Phi}}\right]$$

- We implement this as a reweighting of the MiNLO xsec$^\mathrm{n}$

$$d\sigma'_\mathcal{M} \;=\; d\sigma_\mathcal{M}\left[1+\bar{\alpha}_\mathrm{S}^2L^2\,\frac{\dfrac{d\sigma_\mathrm{NLO}}{d\Phi}-\dfrac{d\sigma_\mathcal{M}}{d\Phi}}{\displaystyle\int_L\bar{\alpha}_\mathrm{S}^2L^2\,\frac{d\sigma_\mathcal{M}}{dL\,d\Phi}}\right] \;\to\; \frac{d\sigma'_\mathcal{M}}{d\Phi}\equiv\frac{d\sigma_\mathrm{NLO}}{d\Phi}$$

Extending the MiNLO method

- We arrived at a numerical recipe for MiNLO $\rightarrow$ MiNLO$'$

- Traded problem of computing $\mathrm{N}^{2/3}\mathrm{LL}_{\sigma}$ MiNLO$'$

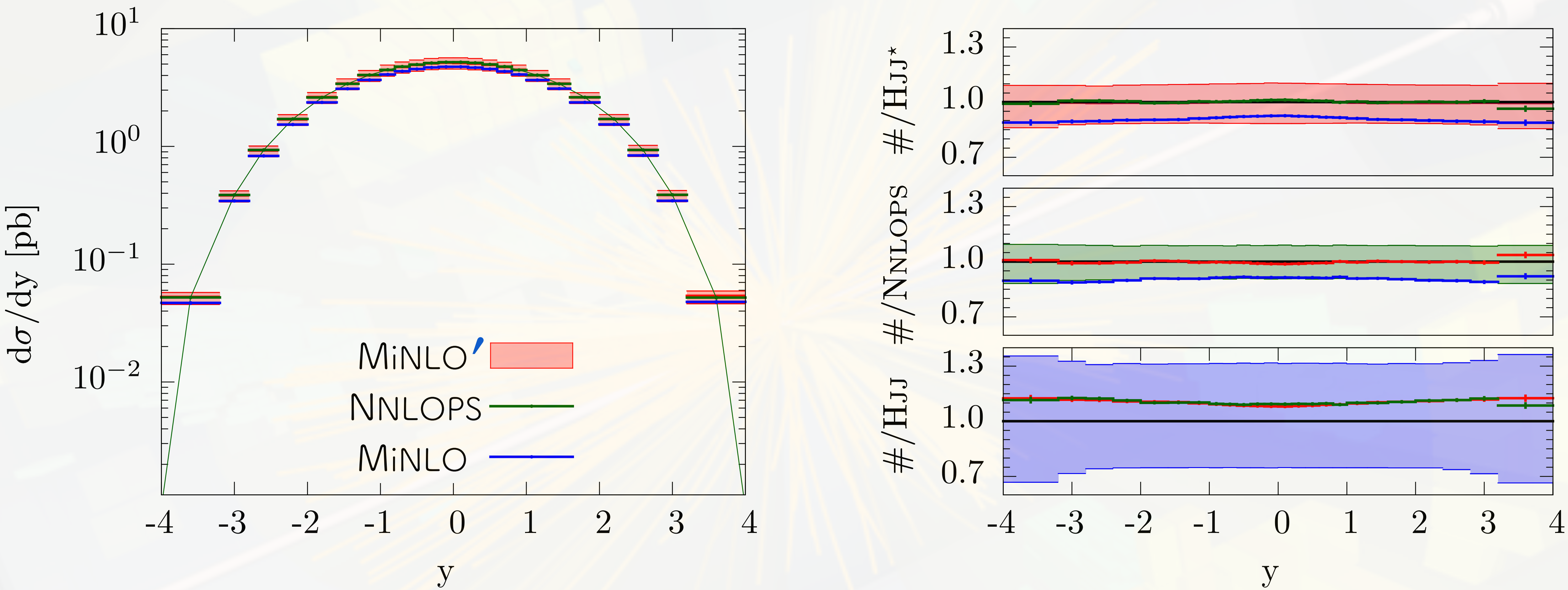
  Sudakov terms for that of computing NLO xsec^{n} for

  the process with one less jet [differential in Born vars]

- Minimal requirement for Sudakov in initial MiNLO is NLL_{σ}

Extending the MiNLO method applied to H+2-jets

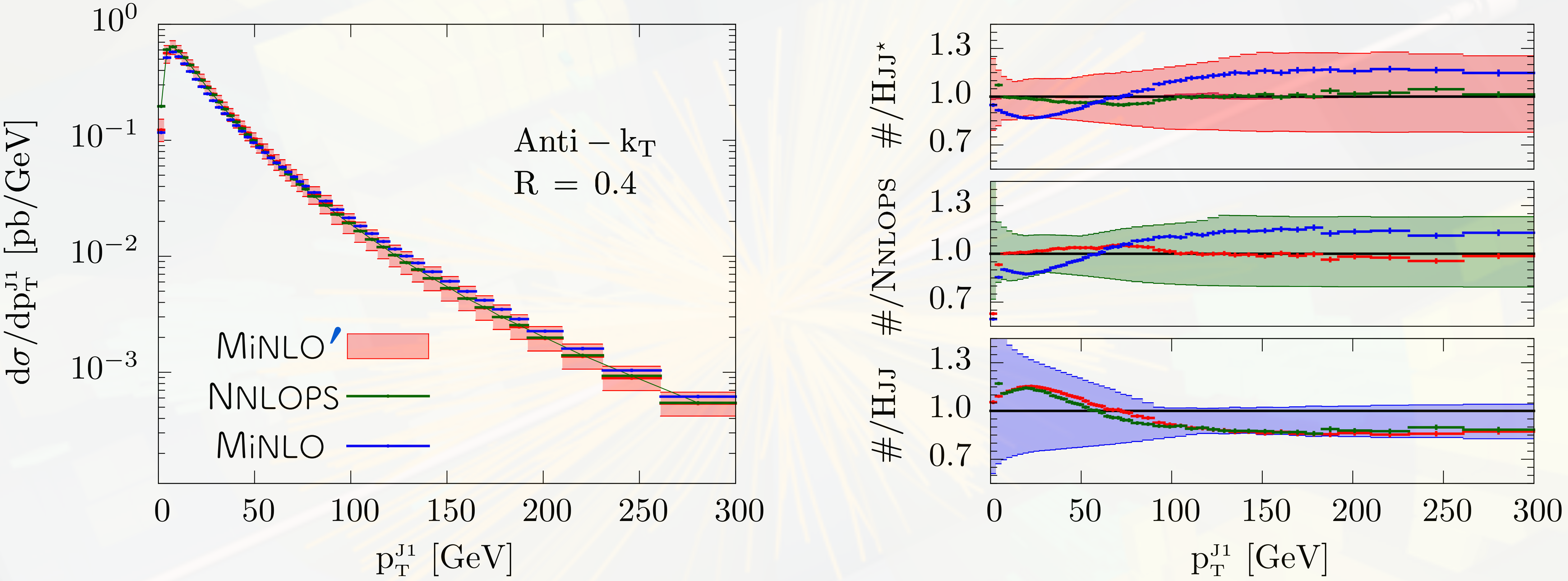
Higgs rapidity



- MiNLO′ H+2-jets: red, formally NNLO
- HNNLOPS: green, formally NNLO
- MiNLO H+2-jets: blue, formally not quite LO

Extending the MiNLO method applied to H+2-jets

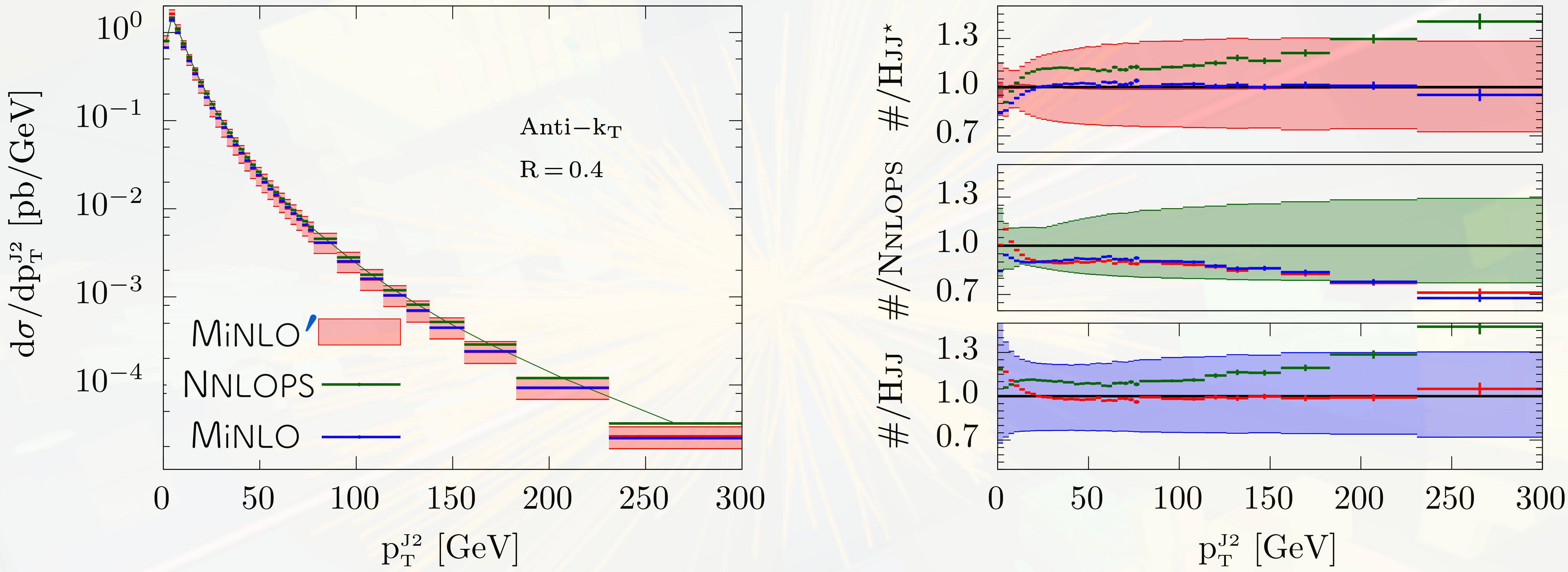
Leading jet transverse momentum



- MiNLO′ H+2-jets: red, formally NLO
- HNNLOPS: green, formally NLO
- MiNLO H+2-jets: blue, formally not quite LO

Extending the MiNLO method applied to H+2-jets

Second jet transverse momentum



- MiNLO′ H+2-jets: red, formally NLO
- HNNLOPS: green, formally LO
- MiNLO H+2-jets: blue, formally NLO

Summary

- MiNLO — Nason, Zanderighi, KH
- MiNLO′ — Nason, Oleari, Zanderighi, KH
- Extending the MiNLO method — Frederix, KH; Melia, Monni, Re, Zanderighi, KH

Cuts

MiNLO introduction

- MiNLO matches fully differential B+n-jets NLO calc^{n}s to LL [nearly NLL$_\sigma$] Sudakov resummation
- Example: NLO H+jet calculation:

$$\frac{d\sigma_{\mathcal{M}}}{dp_{\mathrm{T}}d\Phi} = \Delta\left(Q,p_{\mathrm{T}}\right)\,\frac{\alpha_{\mathrm{S}}\left(p_{\mathrm{T}}\right)}{\alpha_{\mathrm{S}}\left(Q\right)}\left[\frac{d\sigma}{dp_{\mathrm{T}}d\Phi} - \left.\frac{d\sigma}{dp_{\mathrm{T}}d\Phi}\right|_{\mathrm{LO}}\left[\Delta\left(Q,p_{\mathrm{T}}\right)\,\frac{\alpha_{\mathrm{S}}\left(p_{\mathrm{T}}\right)}{\alpha_{\mathrm{S}}\left(Q\right)}\right]_{\alpha_{\mathrm{S}}}\right]$$

Sudakov form factor

Resum β0 logs

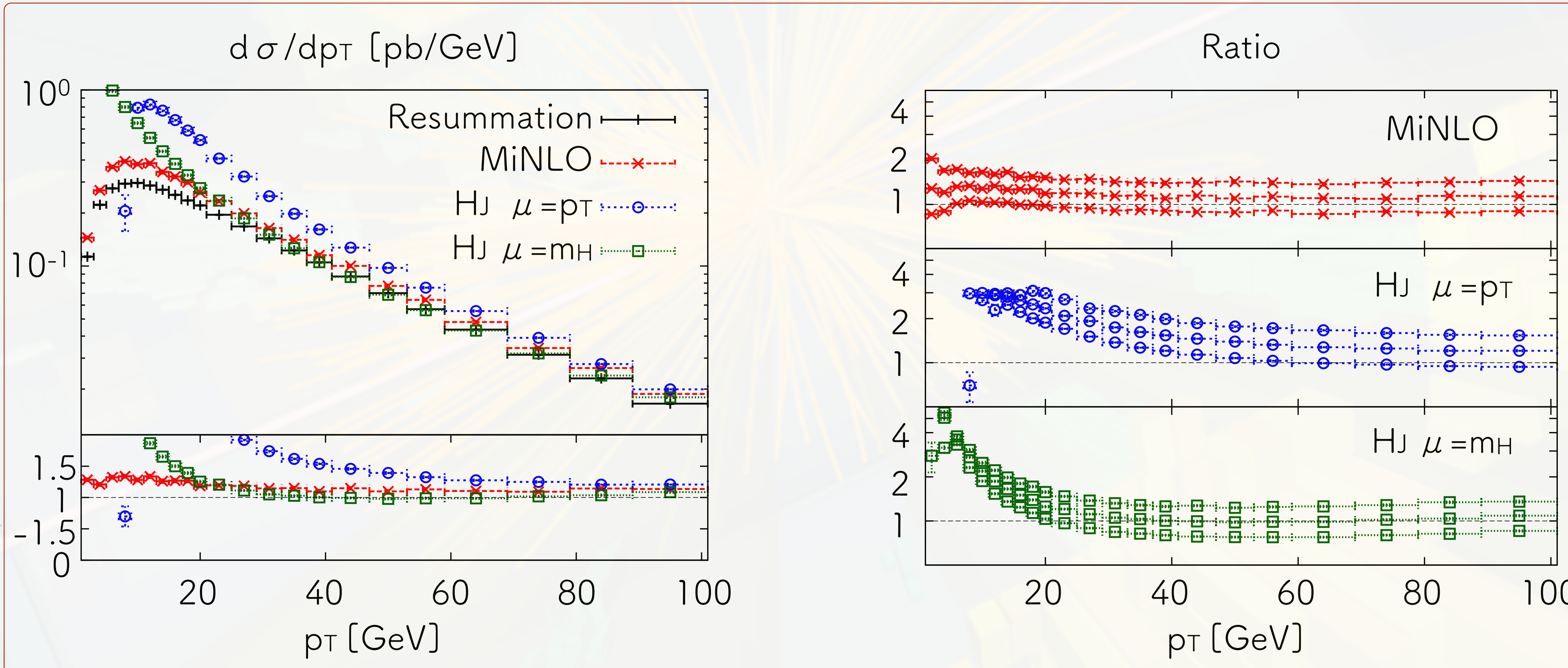
NLO xsec
μ_R = Q
μ_F = pT

Maintain fixed order expansion to NLO

$$\Delta\left(Q,p_{\mathrm{T}}\right) = \exp\left[-\int_{p_{\mathrm{T}}^2}^{Q^2}\frac{d\mu^2}{\mu^2}\left[A\left(\alpha_{\mathrm{S}}\left(\mu\right)\right)\log\frac{Q^2}{\mu^2} + B\left(\alpha_{\mathrm{S}}\left(\mu\right)\right)\right]\right]$$

MiNLO introduction

- MiNLO transitions to the resummed calc^n at low p_T
- MiNLO finite in all ph.space: no need of generation cuts



Question: what's MiNLO accuracy for inclusive quantities?

MiNLO′ integrated

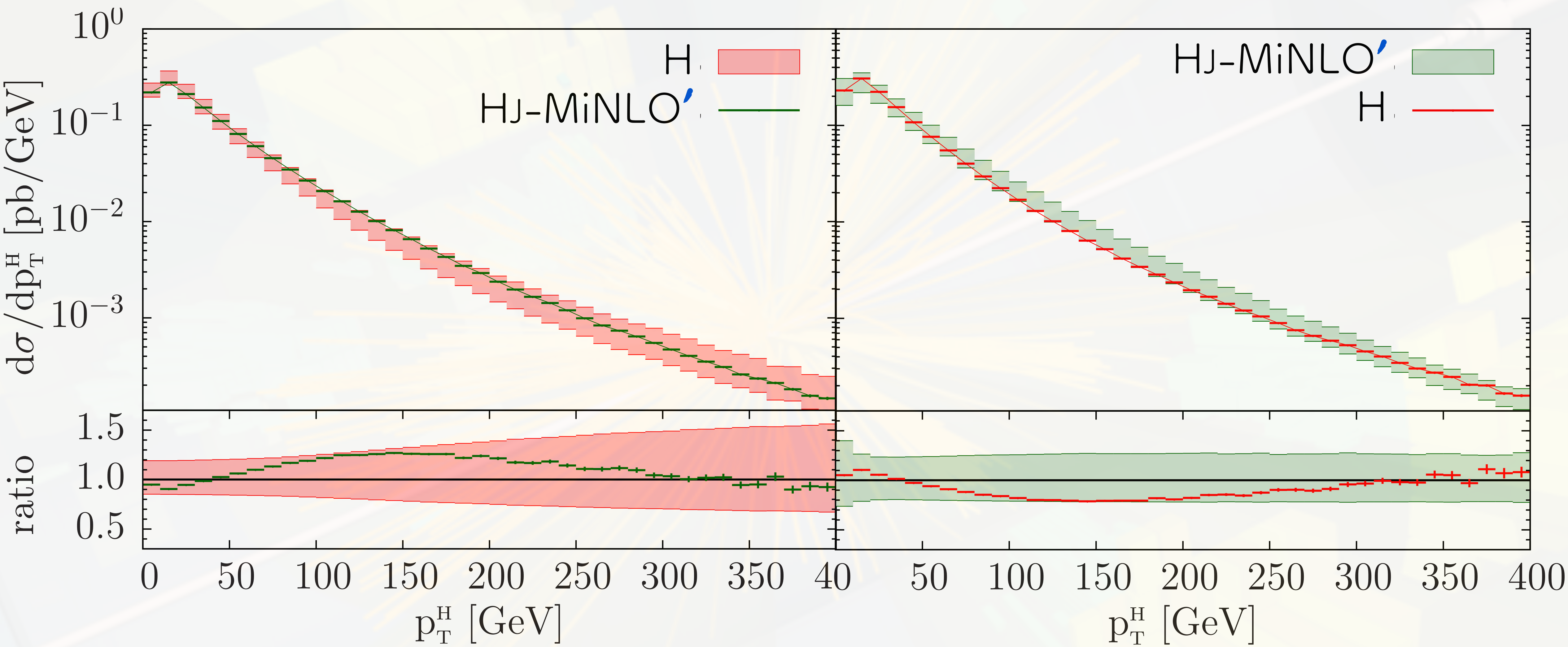
- Integrate out all the radiation [including jet in the Born]:

$$\boxed{\frac{d\sigma'_{\mathcal{M}}}{d\Phi} = \int_L \frac{d\sigma'_{\mathcal{R}}}{d\Phi dL} + \int_L \frac{d\sigma_{\mathcal{F}}}{d\Phi dL} = \frac{d\sigma_{\rm NLO}}{d\Phi}}$$

- MiNLO′ also NLO when Born jet in H+1-jet integrated out!
- MiNLO′ simultaneously NLO for H & H+1-jet prod^n
- This is NLO merging but without actually merging anything
- Relies on **analytic** control of $p_T \to 0$ divergent terms to NLO
- Trivially replicated for H/W/Z/HW/HZ prod^n [successfully]

MiNLO′ H+1-jet

- Higgs p_T



- NLO+parton shower H prod^n: red
- MiNLO′ H+1-jet+parton shower: green
- NLO+parton shower H prod^n LO here, MiNLO′ NLO here

Extending the MiNLO method

- Can MiNLO′ H+2-jets return NLO accuracy for H inclusive as well as H+1-jet?
- By construction the method makes MiNLO′ H+2-jets identical to the target when 2nd jet integrated out [y12]

$$d\sigma'_{\mathcal{M}} = d\sigma_{\mathcal{M}} \left[1 + \bar{\alpha}_{\mathrm{S}}^2 L^2 \underbrace{\boxed{\frac{\int_L d\sigma_{\mathrm{NLO}} - \int_L d\sigma_{\mathcal{M}}}{\int_L \bar{\alpha}_{\mathrm{S}}^2 L^2 \, d\sigma_{\mathcal{M}}}}}_{\text{coefficient}} \right]$$

target

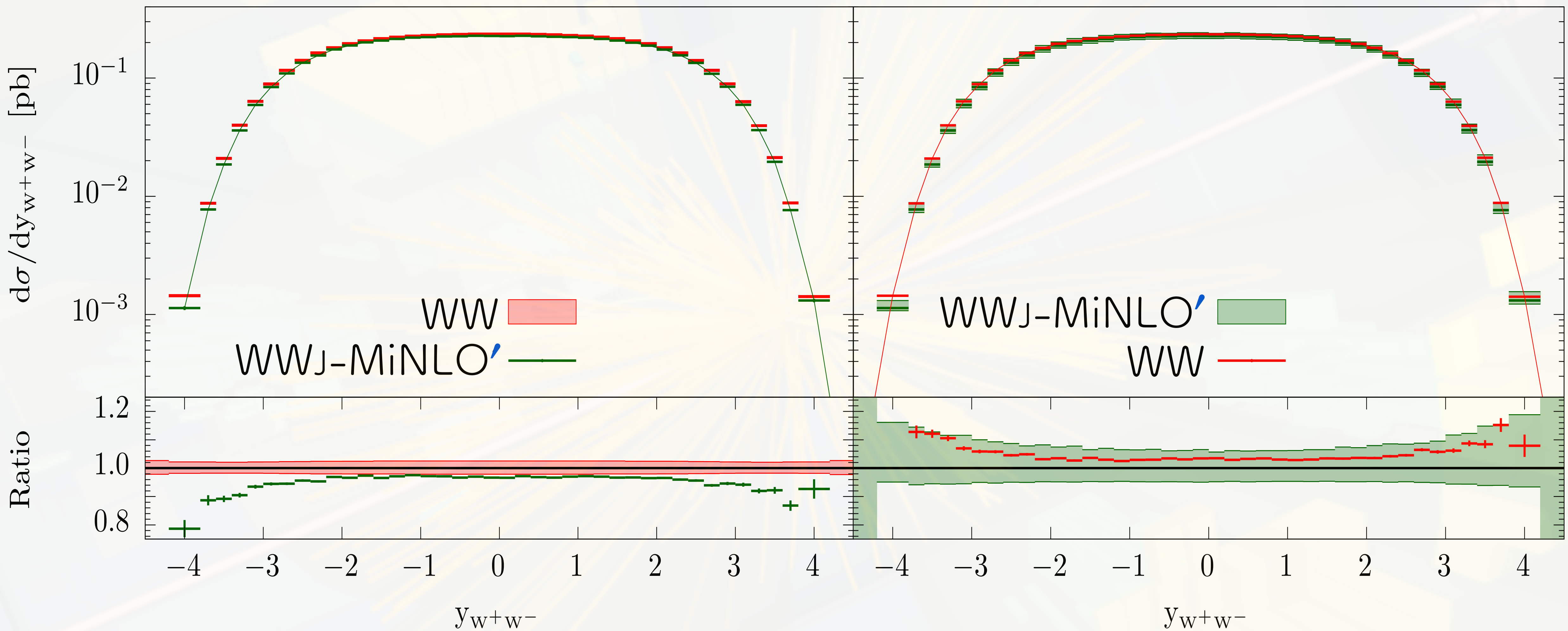
$$\rightarrow \quad \int_L d\sigma'_{\mathcal{M}} \equiv d\sigma_{\mathrm{NLO}}$$

- So if you `target' HNNLOPS instead of conventional NLO H+1-jet you'll get NNLO H inclusive **and** NLO H+1-jet
- Additional y_{01} Sudakov keeps coeff O(1) also for $y_{01} \rightarrow 0$

MiNLO′ W^+W^-+1-jet

[Melia, Monni, Re, Zanderighi, KH]

- MiNLO′ recently extended to procs w. non-trivial virtuals

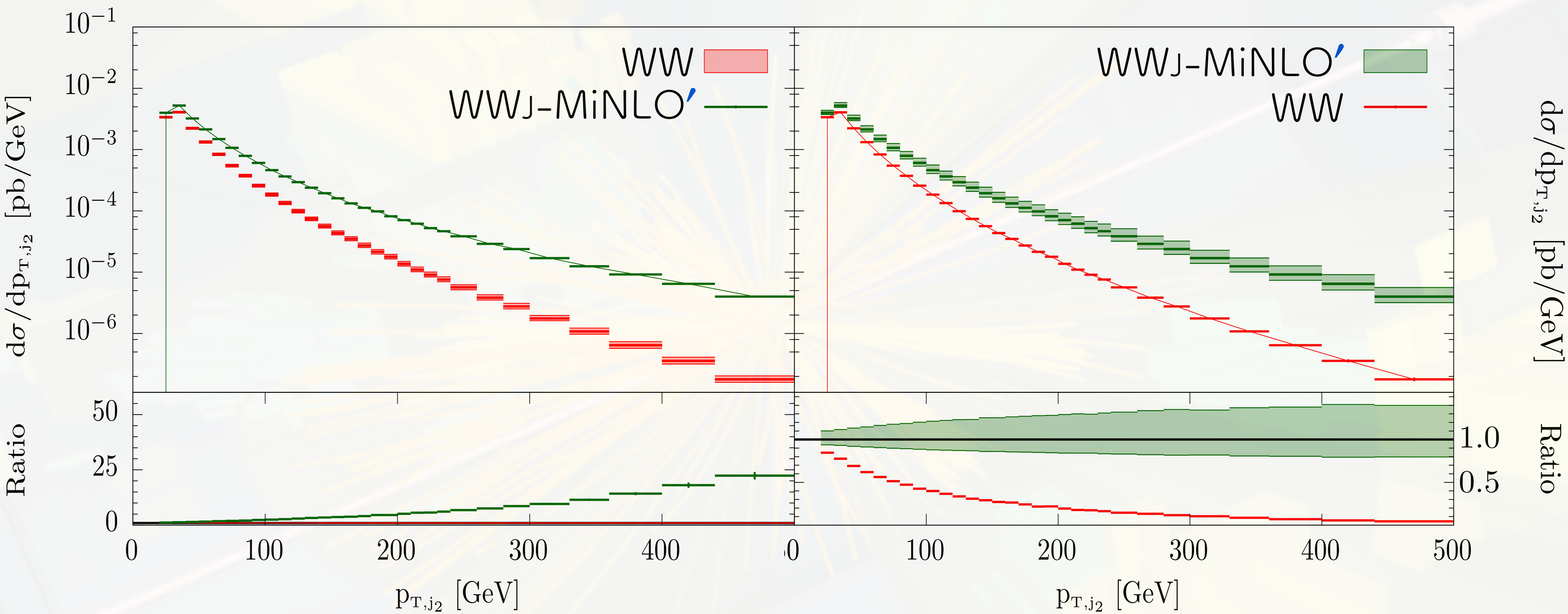


- Conventional NLO: red
- MiNLO′ W^+W^- +1-jet+parton shower: green
- MiNLO′ W^+W^- +1-jet

MiNLO′ W^+W^-+1-jet

[Melia, Monni, Re, Zanderighi, KH]

- MiNLO′ recently extended to procs w. non-trivial virtuals



- NLO+parton shower W^+W^-: red
- MiNLO′ W^+W^- +1-jet+parton shower: green
- MiNLO′ W^+W^- +1-jet

NNLOPS

H+1-jet NLO

- 0-jet: unphysical
- 1-jet: NLO
- 2-jet: LO
- All else: no predictions

H+1-jet MiNLO′

- 0-jet: NLO
- 1-jet: NLO
- 2-jet: LO
- All else: no predictions

H+1-jet MiNLO′ w. PS

- 0-jet: NLO
- 1-jet: NLO
- 2-jet: LO
- All else: PS

NNLOPS

- 0-jet: NNLO
- 1-jet: NLO
- 2-jet: LO
- All else: PS

NNLOPS very quickly [vanilla]

- In its most basic form:

$$d\sigma_{\text{NNLOPS}} = d\sigma_{\text{MiNLO}} \times W(\Phi) \quad \text{with} \quad W(\Phi) = \frac{\frac{d\sigma_{\text{NNLO}}}{d\Phi}}{\frac{d\sigma_{\text{MiNLO}}}{d\Phi}}$$

- $\frac{d\sigma_{\text{MiNLO}}}{d\Phi} = \frac{d\sigma_{\text{NNLO}}}{d\Phi}$ to NLO $\cdots\cdots\rightarrow$ $W(\Phi) = 1 + O(\alpha_S^2)$

- Multiplying $d\sigma_{\text{MiNLO}}$ by $W(\Phi)$ to get NNLO accuracy doesn't spoil NLO already in $d\sigma_{\text{MiNLO}}$ for ≥ 1 jet obs

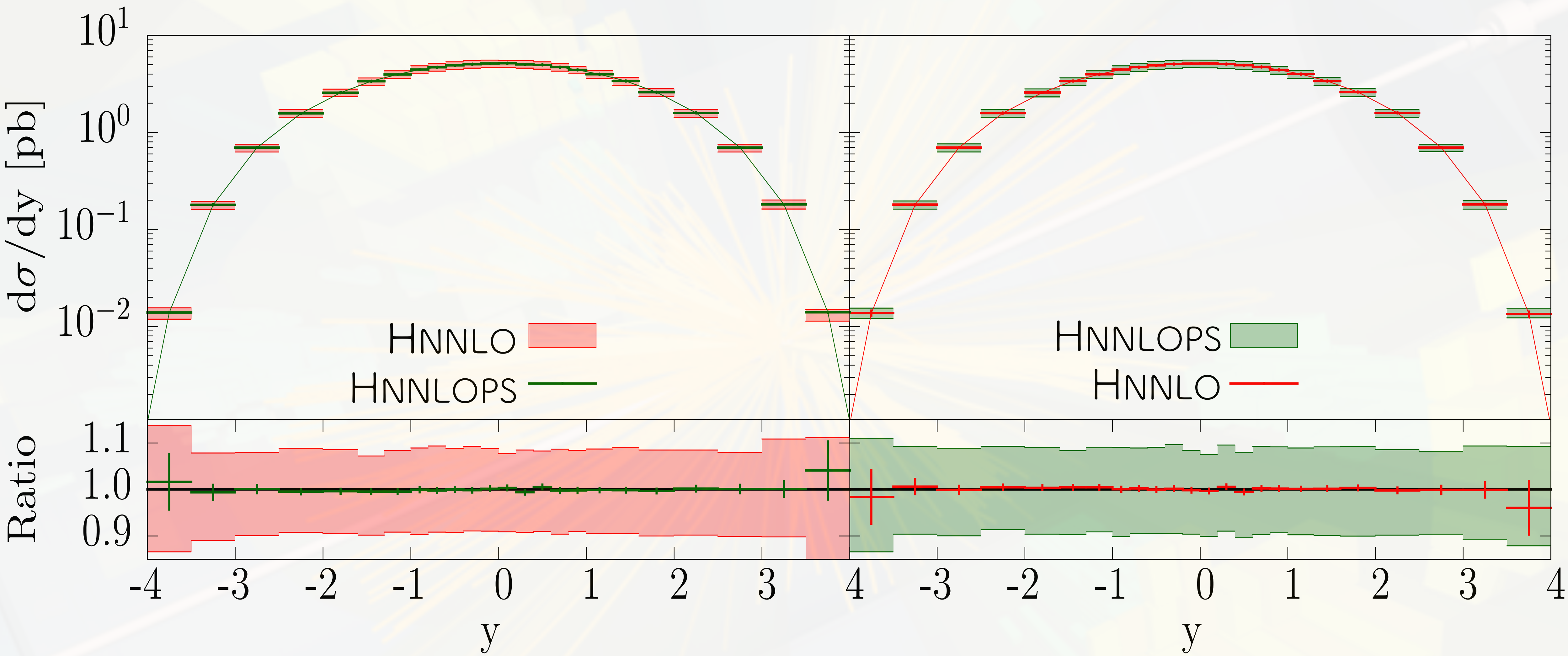
- If $\frac{d\sigma_{\text{MiNLO}}}{d\Phi} \neq \frac{d\sigma_{\text{NNLO}}}{d\Phi}$ $W(\Phi)$ spoils NLO $d\sigma_{\text{MiNLO}}$ for ≥ 1 jet

- Bottleneck for NNLOPS is making a NLO x NLO MiNLO′

Of course, the NNLO calc^{n} is the really hard part! We fully depend on NNLO friends for this.

NNLOPS results

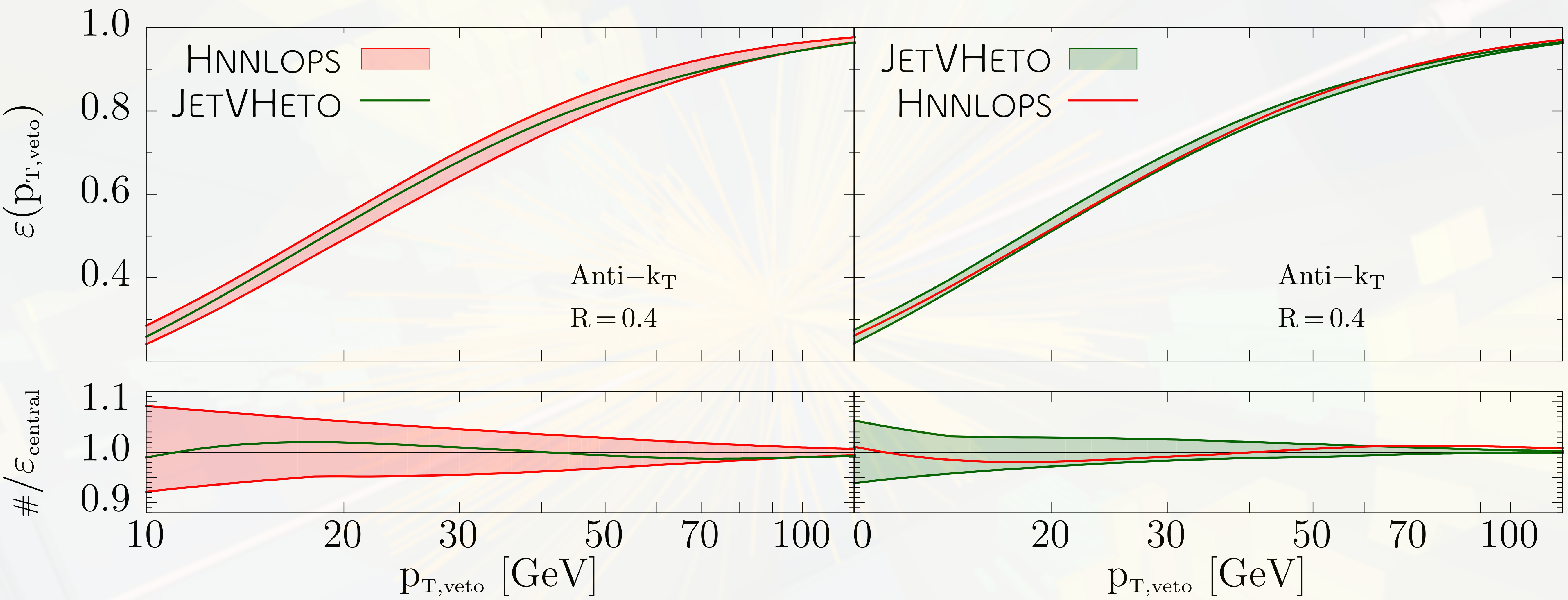
- Higgs rapidity



- Conventional NNLO H prod^n: red [Catani, Grazzini, Sargsyan]
- HNNLOPS: green
- HNNLOPS = Conventional NNLO H prod^n

NNLOPS results

- $\varepsilon = \sigma\,(0\text{-jet}) / \sigma\,(\text{total})$ xsec^n as f^n of jet p$_T$ threshold



- JETVHETO NNLO+NNLL: green [Banfi, Monni, Salam, Zanderighi]
- HNNLOPS tuned to NNLO+NNLL Higgs p$_T$ spectrum: red

Extending the MiNLO method

- In the beginning we saw the MiNLO matching is constructed s.t. NLO accuracy is preserved

- Extending MiNLO → MiNLO′ must also respect this, i.e. can only affect MiNLO by relative α_S^2 terms

- I.E. coeff of $\alpha_S^2 L^2$ must be O(1) and not e.g. $O(1/\sqrt{\alpha_S})$ here:

$$d\sigma'_{\mathcal{M}} \quad = \quad d\sigma_{\mathcal{M}} \left[1 + \bar{\alpha}_{\mathrm{S}}^2 L^2 \, \frac{\int_L \, d\sigma_{\mathrm{NLO}} - \int_L \, d\sigma_{\mathcal{M}}}{\int_L \bar{\alpha}_{\mathrm{S}}^2 L^2 \, d\sigma_{\mathcal{M}}} \right]$$

- Since denominator is guaranteed to be O(α) numerator must therefore be as well: $\int_L \, d\sigma_{\mathrm{NLO}} - \int_L \, d\sigma_{\mathcal{M}} \sim \mathcal{O}\left(\bar{\alpha}_{\mathrm{S}}\right)$

- **Demands** MiNLO Sudakov in $d\sigma_{\mathcal{M}}$ be already NLL_{σ} correct

Extending the MiNLO method

- Test: MiNLO → MiNLO′ for H/W/Z+2-jets
- Derived CAESAR N^2LL_σ resummation of y_{01} and y_{12} k_T jet rates in H+jets production
- This req^d analytic N^2LL_σ multiple emission correction in Sudakov form factor [`F_2´] for k_T jet rates
- Following CAESAR formalism we derived general expression

$$\mathcal{F}_2 = -\frac{\pi^2}{16}\frac{\sum_{\ell=1}^{n} C_\ell^2 - \sum_{\ell=1}^{n_i} C_\ell^2}{\left(\sum_{\ell=1}^{n} C_\ell\right)^2}$$

- Checked it numerically agrees with automatized CAESAR program to 4 s.f. for all channels in jet prod^n and Z+jets

Extending the MiNLO method

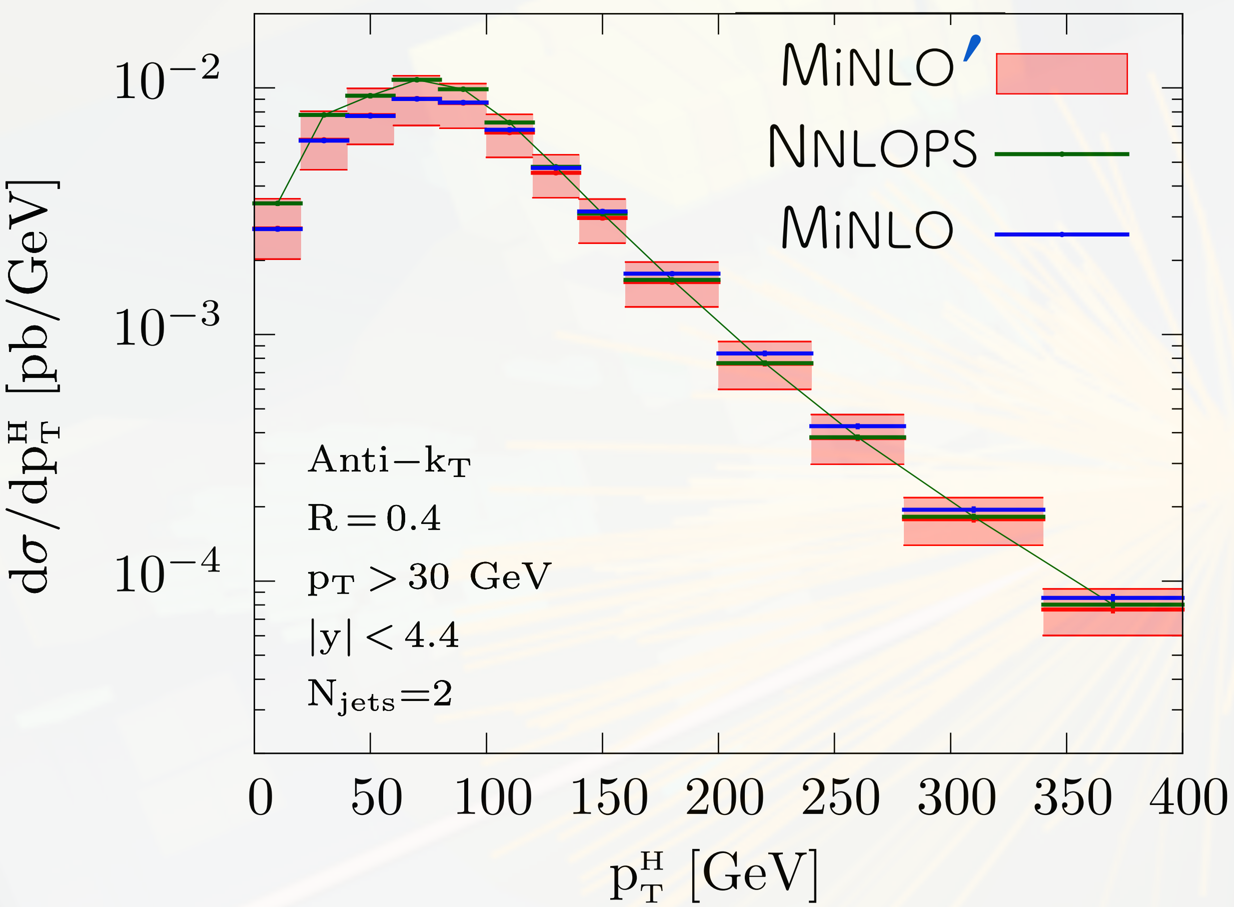
- Is coefficient O(1) ?
- In moderate-high y_{01} region there's no issue HNNLOPS equals NLO H+1-jet up to unenhanced H.O. terms
- At low y_{01} you need to worry about large y_{01} logs
- If H+2-jets MiNLO′ has no joint y_{01} resummation the coefficient is out of control in the small y_{01} region
- We implement a y_{01} Sudakov together with the y_{12} Sudakov according to the coherent parton branching formalism [as in the first MiNLO paper]

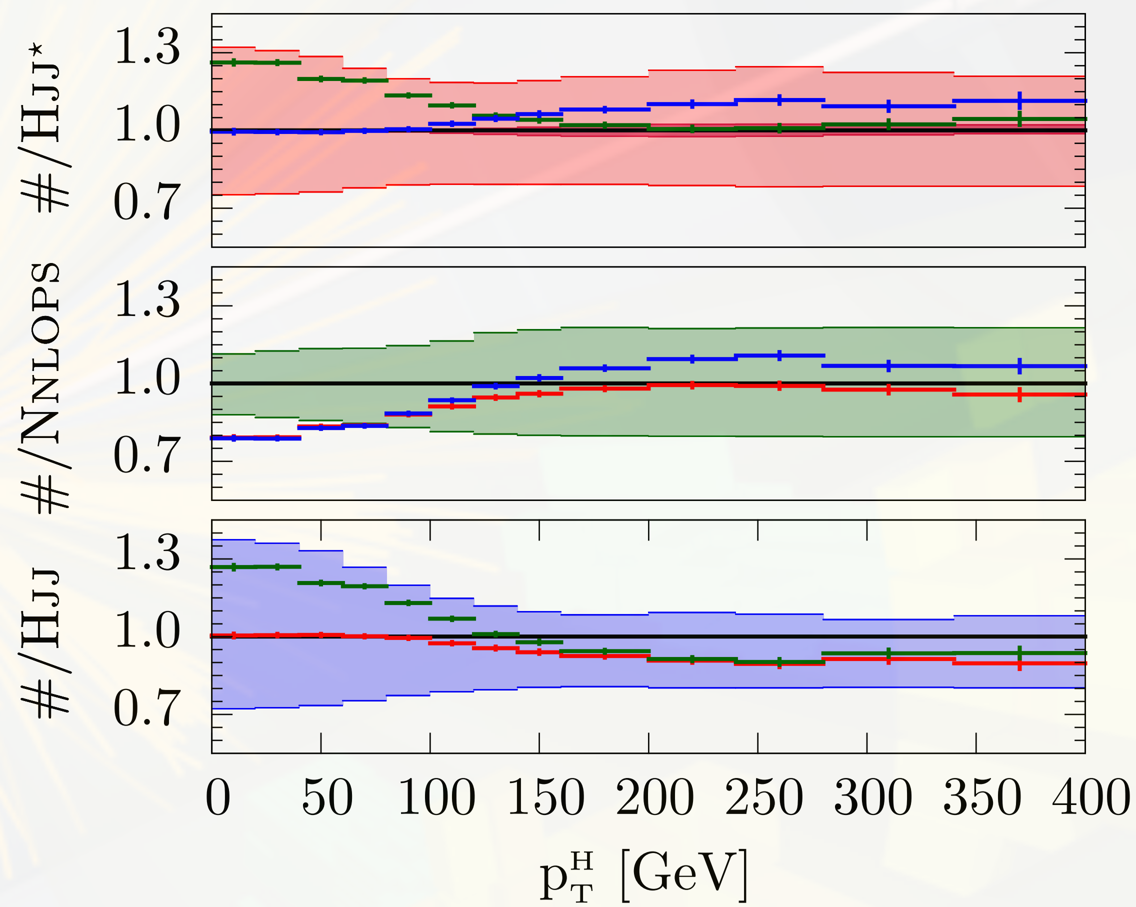
Extending the MiNLO method

- We argue coefficient is then O(1) throughout ph. space
- Except in the region where $\alpha_S L^2 >> 1$, but control in this region is generally limited anyway for all kinds of calc^{n}s
- The **conjecture** is largely based on detailed comparison of `nested' CAESAR jet rate resummations vs. CKKW
- They are identical, up to a subleading soft wide angle term missing in CKKW [beyond CKKW's remit]
- Even if it's wrong and MiNLO is **only** LL_σ for $y_{01} \to 0$, that's enough to have HJJ stay LO [in F.O. domain], which is enough to have NNLO 0-jet & NLO 1-jet [coeff goes like ~ $1/\sqrt{\alpha_S}$]!

Extending the MiNLO method applied to H+2-jets

Higgs transverse momentum exclusively in 2-jet events

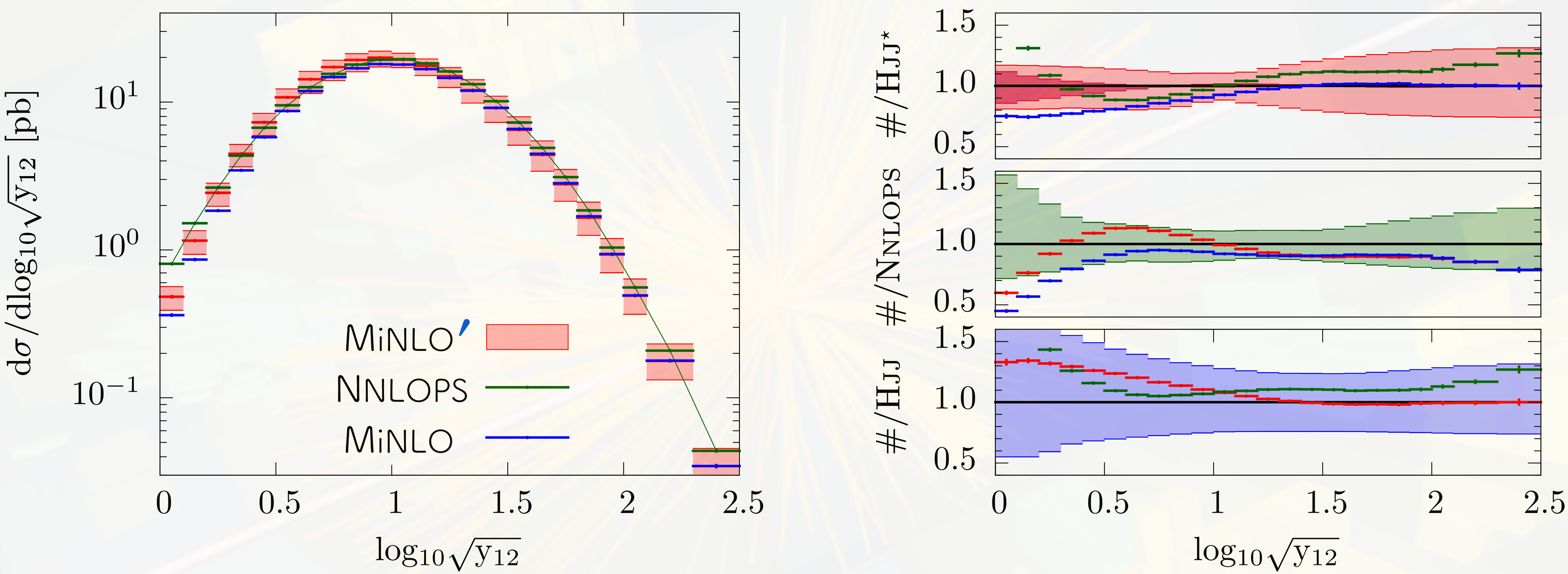




- MiNLO′ H+2-jets: red, NLO at low p_T, NLO at high p_T
- HNNLOPS: green, LO at low p_T, NLO at high p_T
- MiNLO H+2-jets: blue, NLO at low p_T, LO at high p_T

Extending the MiNLO method applied to H+2-jets

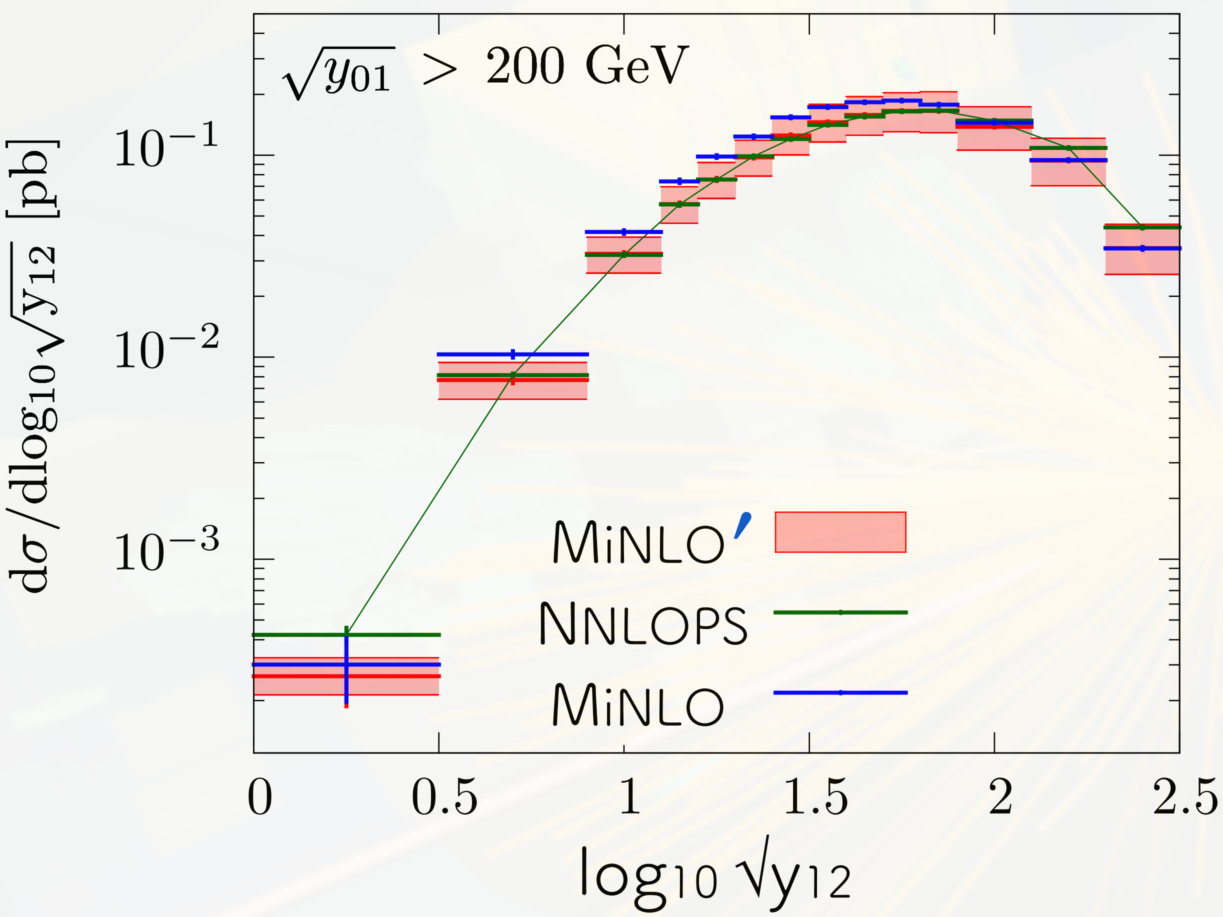
jet rate $\log_{10}\sqrt{y_{12}}$

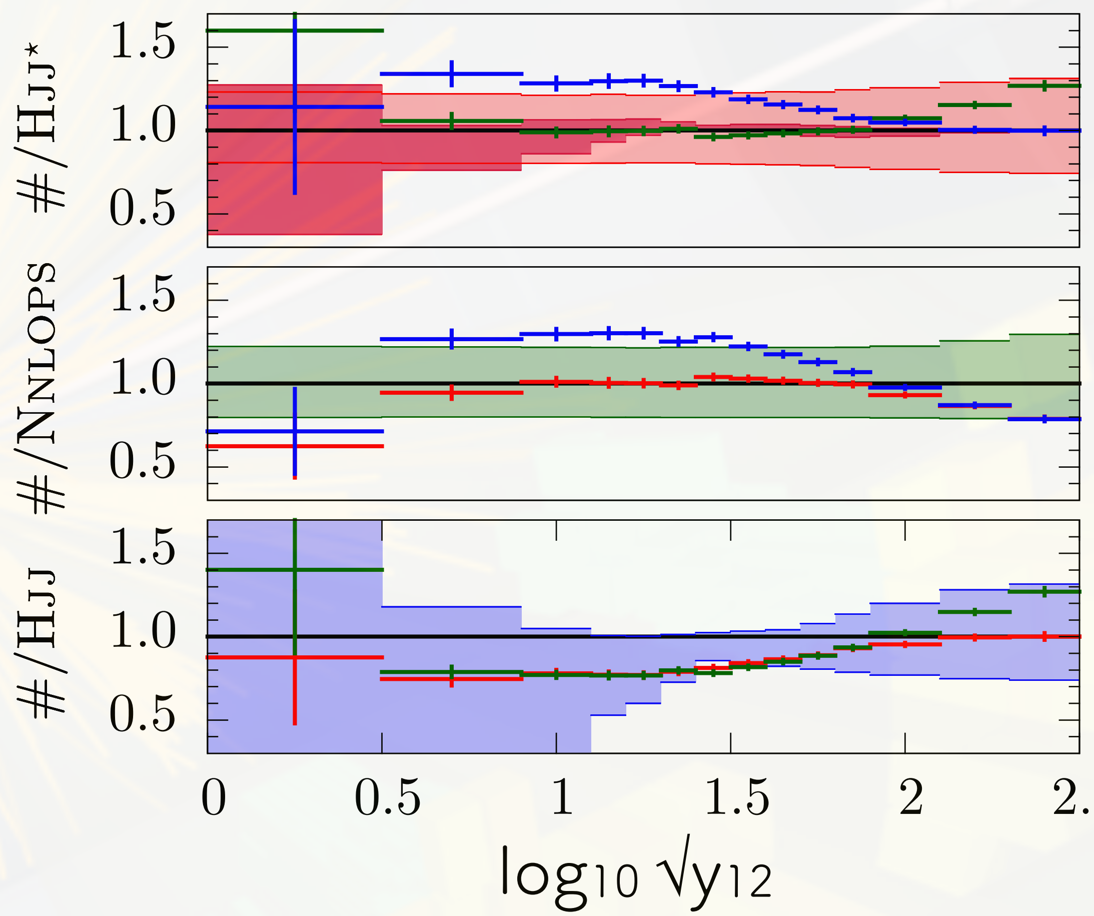


- MiNLO′ H+2-jets: red
- HNNLOPS: green
- MiNLO H+2-jets: blue

Extending the MiNLO method applied to H+2-jets

jet rate $\log_{10}\sqrt{y_{12}}$ in events with $\sqrt{y_{01}}$ > 200 GeV





- MiNLO′ H+2-jets: red
- HNNLOPS: green
- MiNLO H+2-jets: blue