Threshold logarithms in a parton shower

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DESY

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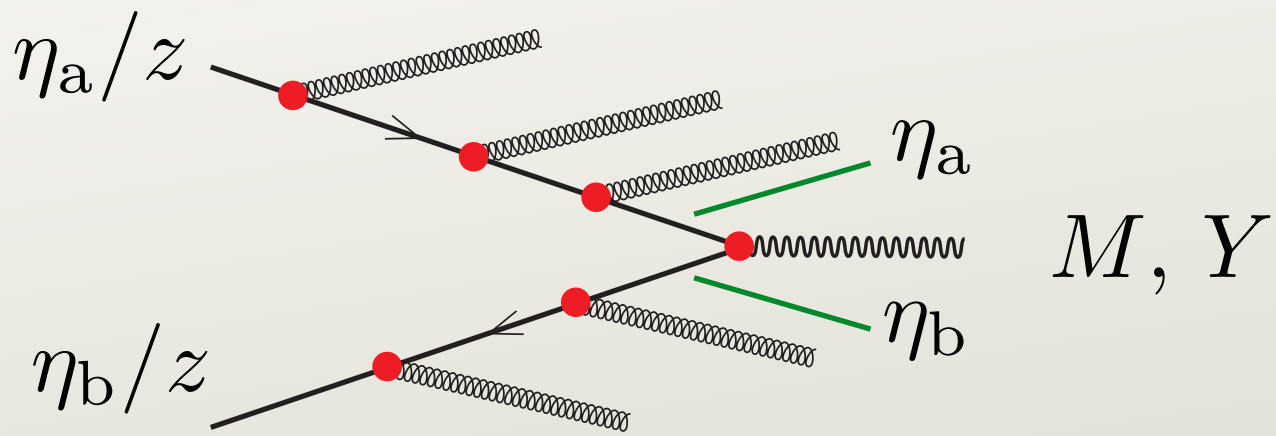
- Consider the Drell-Yan process with dimuon rapidity Y and mass M.



- There are logarithms of $(1-z)$:

$$\int_0^1 dz\ f_{a/A}(\eta_{\rm a}/z,\mu_{\rm F}^2)\left\{\delta(1-z)+C\alpha_s\left[\frac{\log(1-z)}{1-z}\right]_+ +\cdots\right\}$$

- There is a large literature on summing these logarithms starting with Sterman (1987).

Deductor

- http://pages.uoregon.edu/soper/deductor/

- Dipole shower.

- In principle, uses quantum density matrix in color & spin.

- LC+ approximation for color.

- Z. Nagy and D. E. Soper,
  "Summing threshold logs in a parton shower,"
  arXiv:1605.05845 [hep-ph]

Coming in Deductor

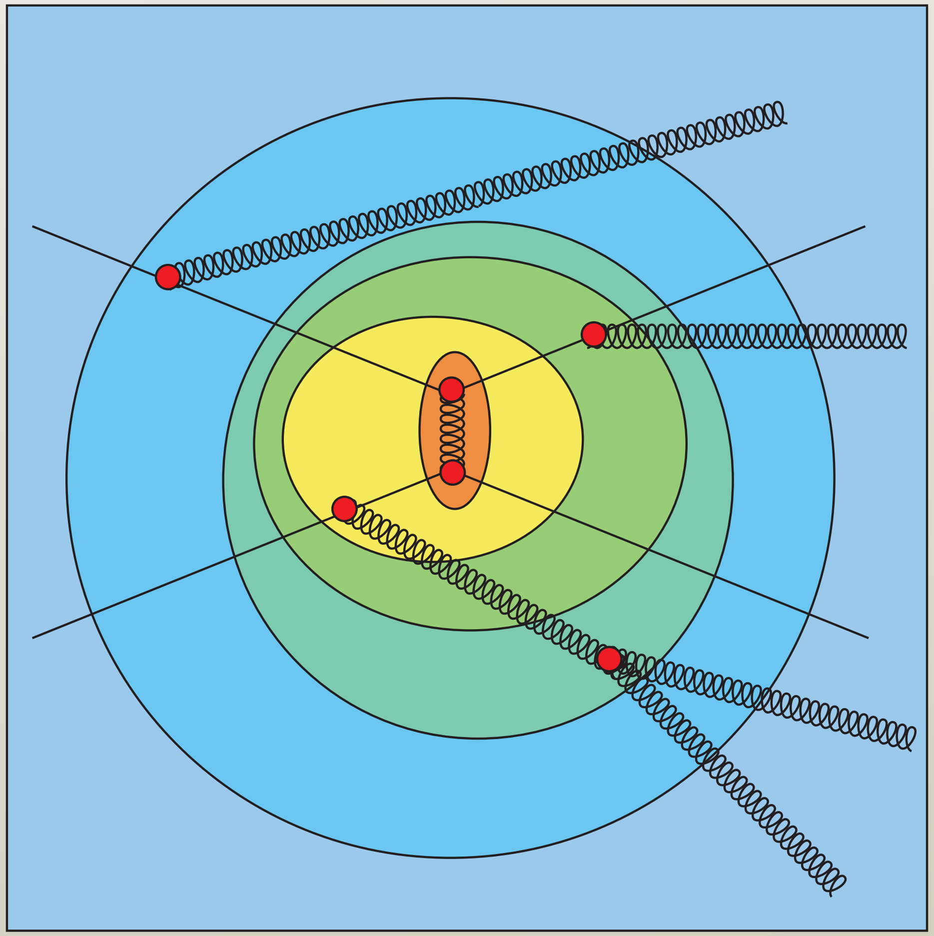
- Perturbative improvement to LC+ approximation.
- Quantum spin.
- Interface to hadronization model.

Now in Deductor

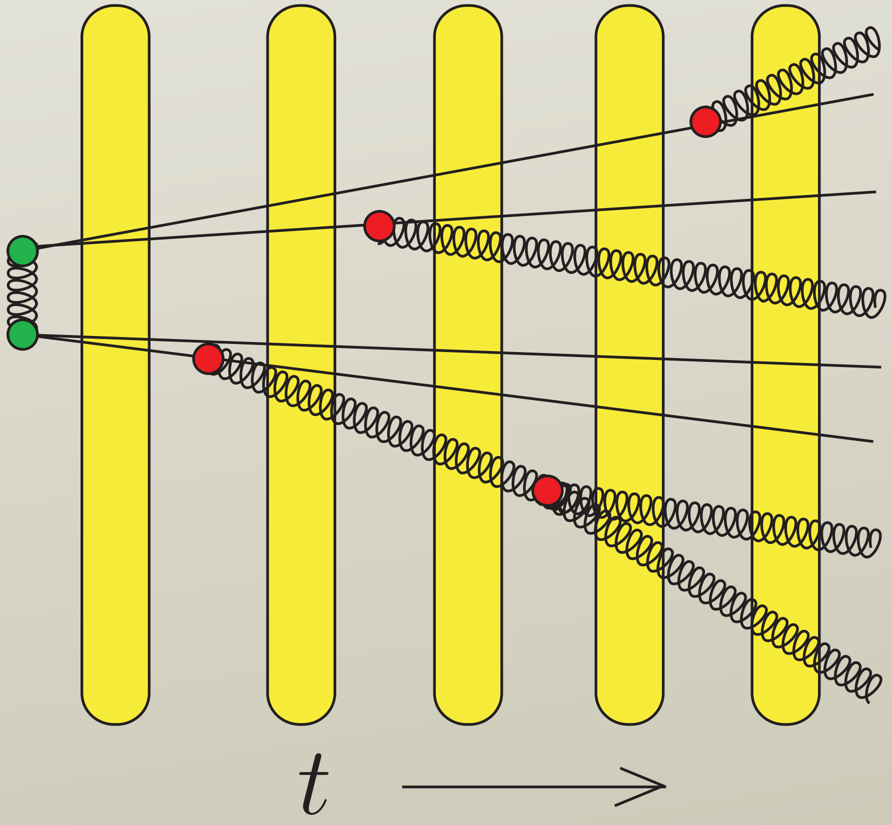
- Threshold logs (this talk).
- Choices, e.g. for definition of ordering variable.

Shower evolution

- Showers develop in “shower time.”
- Hardest interactions first.



Real time picture



Shower time picture

Shower ordering variable

- Originally, PYTHIA used virtuality to order splittings.
- Now, PYTHIA and SHERPA use "k_{T}."
- DEDUCTOR uses Λ,

$$\Lambda^2 = \frac{(\hat{p}_l + \hat{p}_{m+1})^2}{2p_l \cdot Q_0}\, Q_0^2 \qquad \text{final state,}$$

$$\Lambda^2 = -\frac{(\hat{p}_{\mathrm{a}} - \hat{p}_{m+1})^2}{2p_{\mathrm{a}} \cdot Q_0}\, Q_0^2 \qquad \text{initial state}$$

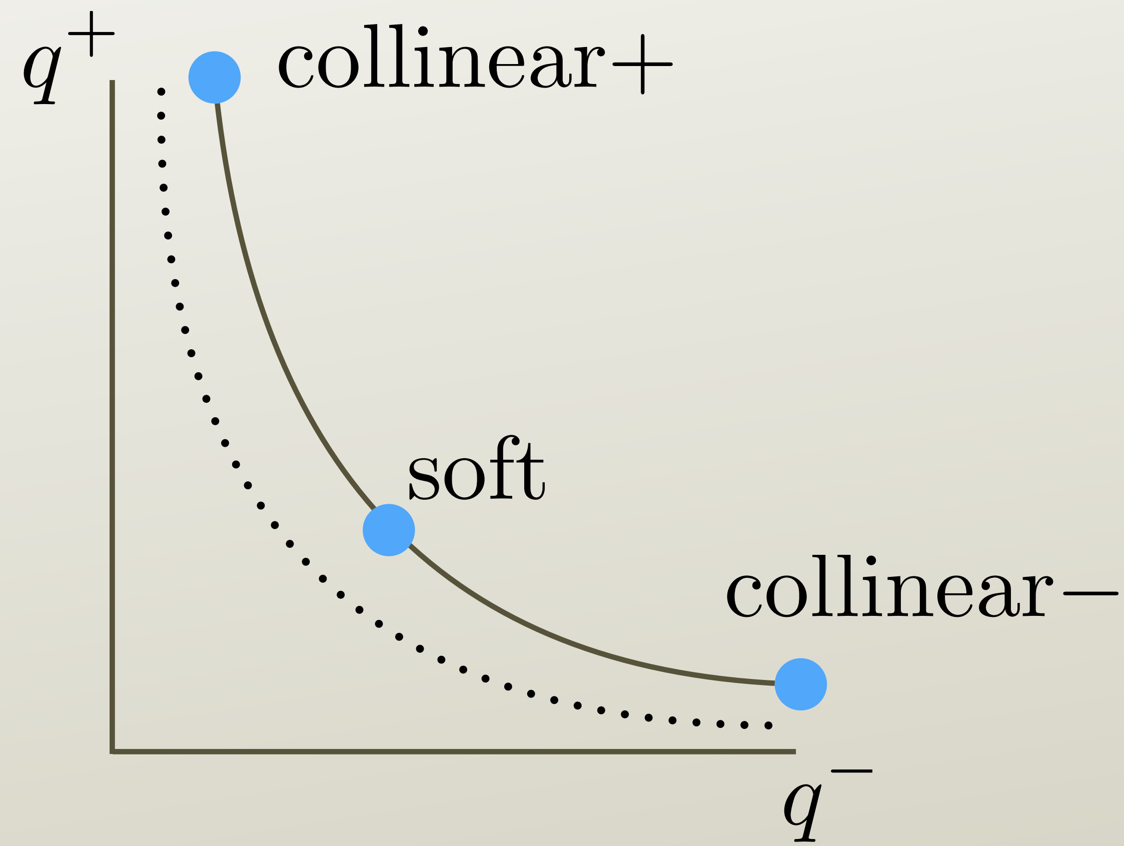
- Here Q_0 is a fixed timelike vector.

Contrast with SCET

- SCET distinguishes multiple regions.

$$(q^+, q^-, q_\perp) = (1, \lambda^2, \lambda)Q \quad \text{collinear}+$$
$$(q^+, q^-, q_\perp) = (\lambda^2, 1, \lambda)Q \quad \text{collinear}-$$
$$(q^+, q^-, q_\perp) = (\lambda, \lambda, \lambda)Q \quad \text{soft}$$

$$\lambda \ll 1$$



- A parton shower has just larger λ and smaller λ.
- Shower evolution from

$$\frac{d}{d\lambda}$$

The shower state

- Here I am ignoring spin.

- To describe the state at shower time t based on an ensemble of runs of the program, use the density operator in color space

$$\rho(\{p,f\}_m,t) = \sum_{\{c\}_m,\{c'\}_m} \rho(\{p,f,c',c\}_m,t)\,\big|\{c\}_m\big\rangle\big\langle\{c'\}_m\big|$$

- Here $\rho(\{p,f,c',c\}_m,t)$ is the probability to find the system with momenta and flavors $\{p,f\}_m$ in this color state.

- Denote this function by $\big|\rho(t)\big)$.

Evolution equation

$$\big|\rho(t)\big) = \mathcal{U}_{\mathcal{S}}(t,t_0)\big|\rho(t_0)\big)$$

$\mathcal{H}_{\mathrm{I}}(t)$ = splitting operator

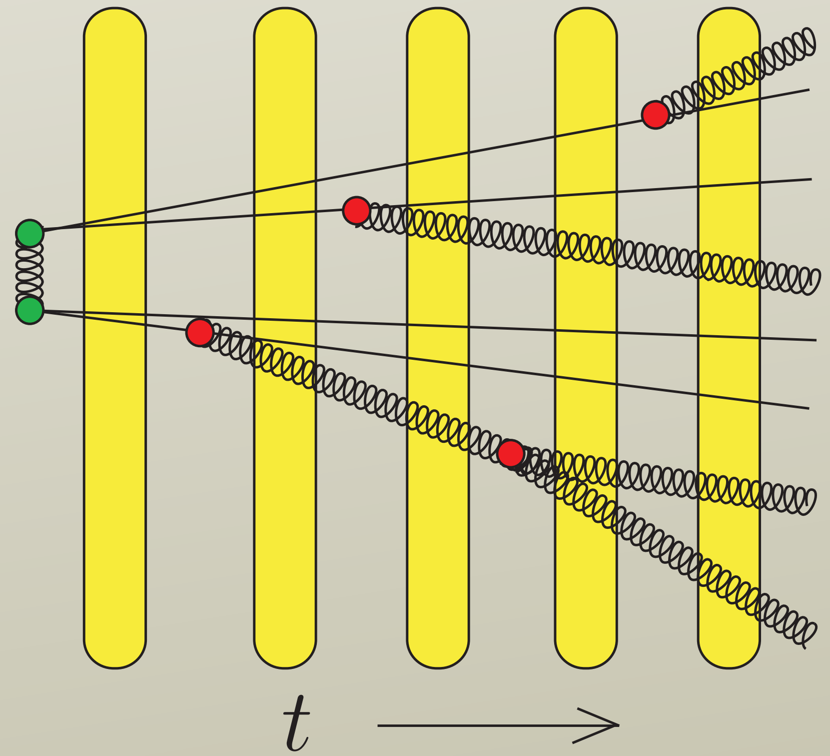
$$\frac{d}{dt}\,\mathcal{U}_{\mathcal{S}}(t,t') = [\mathcal{H}_I(t) - \mathcal{S}(t)]\,\mathcal{U}_{\mathcal{S}}(t,t')$$

$\mathcal{S}(t)$ = no-splitting operator

$$\mathcal{U}_{\mathcal{S}}(t,t') = \mathcal{N}_{\mathcal{S}}(t,t') + \int_{t'}^{t} d\tau\; \mathcal{U}_{\mathcal{S}}(t,\tau)\mathcal{H}_I(\tau)\mathcal{N}_{\mathcal{S}}(\tau,t')$$

where

$$\mathcal{N}_{\mathcal{S}}(\tau,t') = \mathbb{T}\exp\left[-\int_{t'}^{\tau} d\tau'\;\mathcal{S}(\tau')\right]$$



Role of parton distributions

- $\rho(t)$ contains a factor of parton distributions,

$$\big|\rho(t)\big) = \mathcal{F}(t)\big|\rho_{\rm pert}(t)\big)$$

- Here $\big|\rho_{\rm pert}(t)\big)$ is $\big|M\big\rangle\big\langle M\big|$ from Feynman diagrams.

- We include parton distribution functions at the current scale.

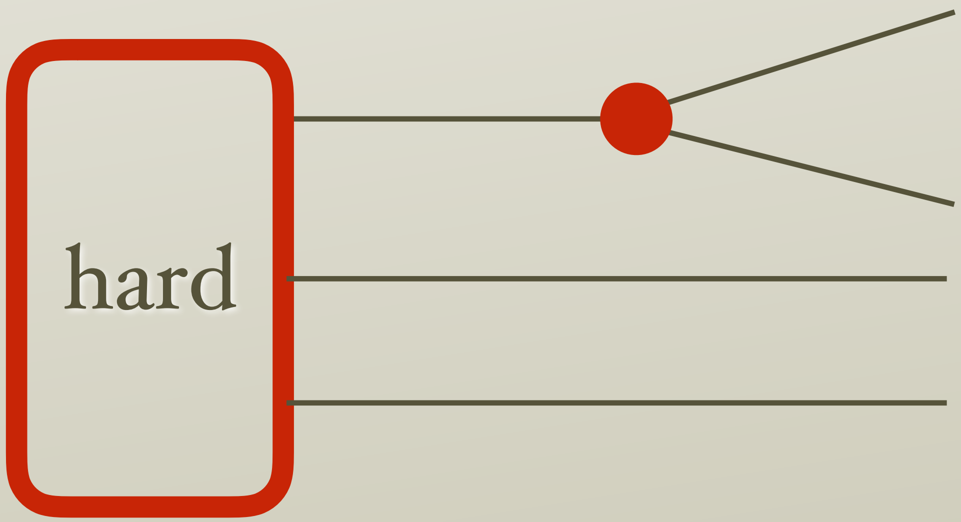
$$\begin{aligned}
&\mathcal{F}(t)\big|\{p,f,c',c\}_m\big) \\
&\qquad = \frac{f_{a/A}(\eta_{\rm a},\mu^2_{\rm a}(t))\,f_{b/B}(\eta_{\rm b},\mu^2_{\rm b}(t))}{4n_{\rm c}(a)n_{\rm c}(b)\,4\eta_{\rm a}\eta_{\rm b}p_{\rm B}\!\cdot\! p_{\rm B}}\;\big|\{p,f,c',c\}_m\big)
\end{aligned}$$

Evolution for the perturbative state

$$\big|\rho(t)\big) = \mathcal{F}(t)\big|\rho_{\rm pert}(t)\big)$$

$$\frac{d}{dt}\big|\rho_{\rm pert}(t)\big) = [\mathcal{H}_I^{\rm pert}(t) - \mathcal{S}^{\rm pert}(t)]\big|\rho_{\rm pert}(t)\big)$$

- Calcuate $\mathcal{H}_I^{\rm pert}(t)$ from Feynman diagrams.



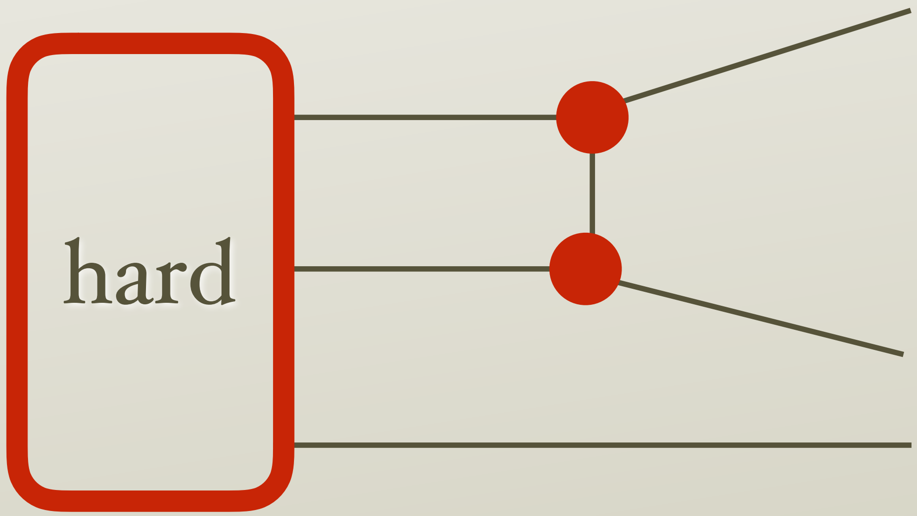
- Then use

$$\mathcal{H}_I(t) = \mathcal{F}(t)\mathcal{H}_I^{\rm pert}(t)\mathcal{F}(t)^{-1}$$

$$\big|\rho(t)\big) = \mathcal{F}(t)\big|\rho_{\rm pert}(t)\big)$$

$$\frac{d}{dt}\big|\rho_{\rm pert}(t)\big) = [\mathcal{H}_I^{\rm pert}(t) - \mathcal{S}^{\rm pert}(t)]\big|\rho_{\rm pert}(t)\big)$$

- Calculate $\mathcal{S}^{\rm pert}$ from Feynman diagrams.



- Then use

$$\mathcal{S}(t) = \mathcal{S}^{\rm pert}(t) - \mathcal{F}(t)^{-1}\left[\frac{d}{dt}\mathcal{F}(t)\right]$$

Including threshold logs

- A parton shower can sum logarithms if you let it.

- I can show you the main idea.

What not to do

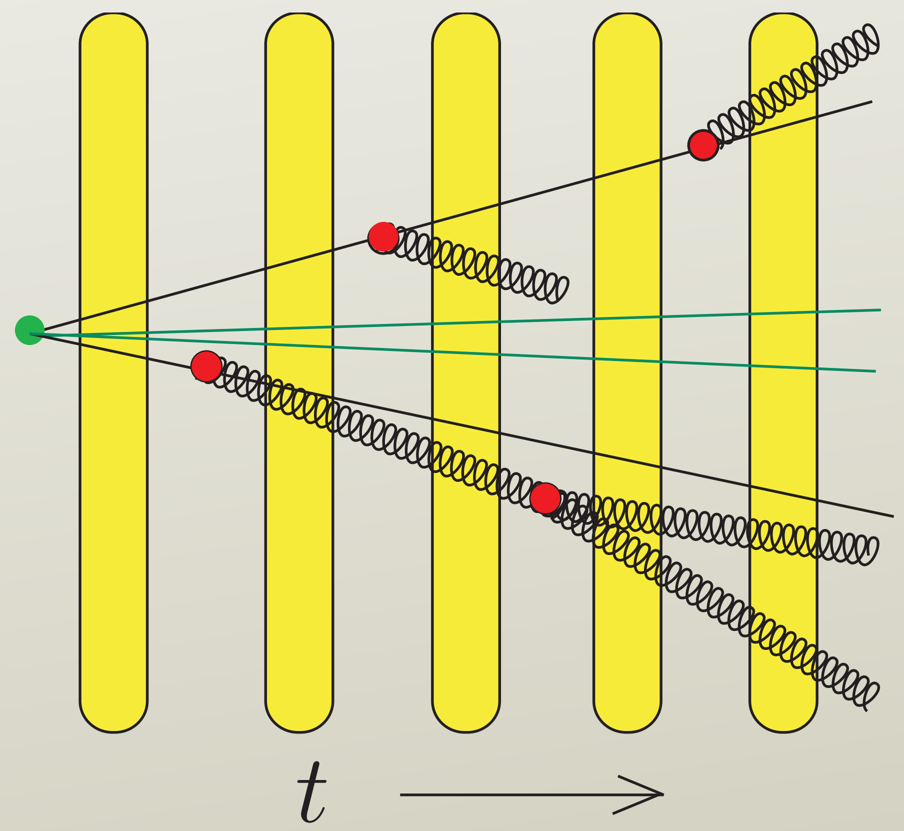
- Suppose that the shower state evolves according to

$$\big|\rho(t)\big) = \mathcal{U}_{\mathcal{V}}(t,t')\big|\rho(t')\big)$$

$$\frac{d}{dt}\mathcal{U}_{\mathcal{V}}(t,t') = [\mathcal{H}_{\mathrm{I}}(t) - \mathcal{V}(t)]\mathcal{U}_{\mathcal{V}}(t,t')$$

$$\mathcal{H}_{\mathrm{I}}(t) = \text{splitting operator}$$

$$\mathcal{V}(t) = \text{no-splitting operator}$$



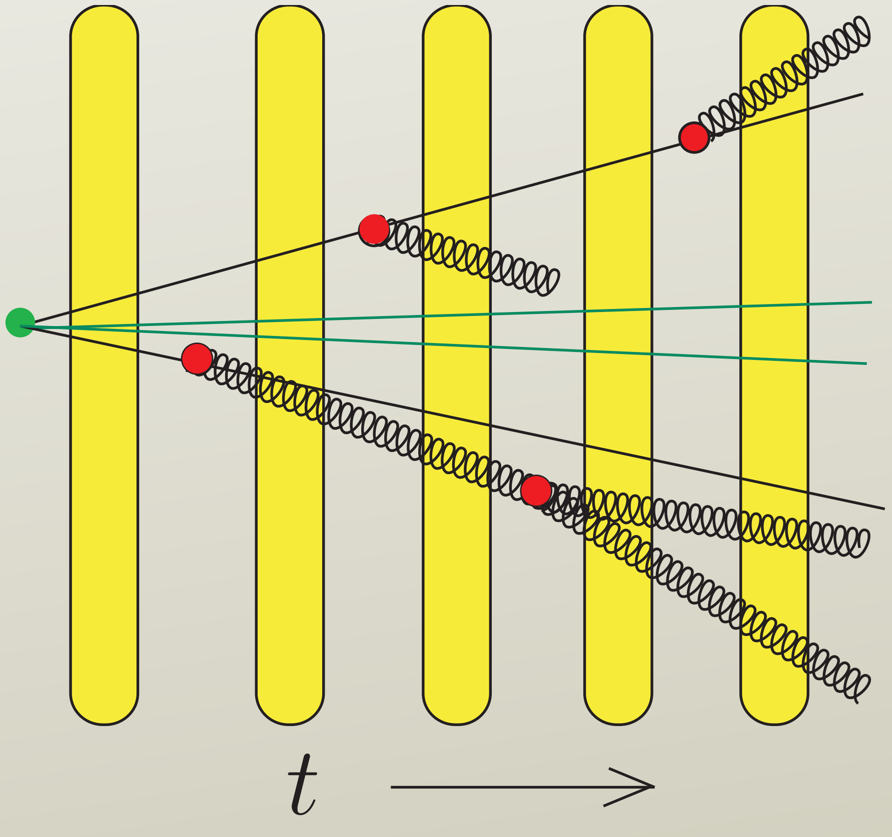
- We calculate $\mathcal{V}(t)$ from $\mathcal{H}_I(t)$ so that the inclusive cross section does not change during the shower.

What to do

- The shower state evolves in shower time.

$$\big|\rho(t)\big) = \mathcal{U}_{\mathcal{S}}(t,t')\big|\rho(t')\big)$$

$$\frac{d}{dt}\,\mathcal{U}_{\mathcal{S}}(t,t') = [\mathcal{H}_{\rm I}(t) - \mathcal{S}(t)]\,\mathcal{U}_{\mathcal{S}}(t,t')$$



$\mathcal{H}_{\rm I}(t)$ = splitting operator

$\mathcal{S}(t)$ = virtual splitting and parton evolution

- We simply calculate $\mathcal{S}(t)$ from one loop virtual graphs plus parton evolution.

What happens

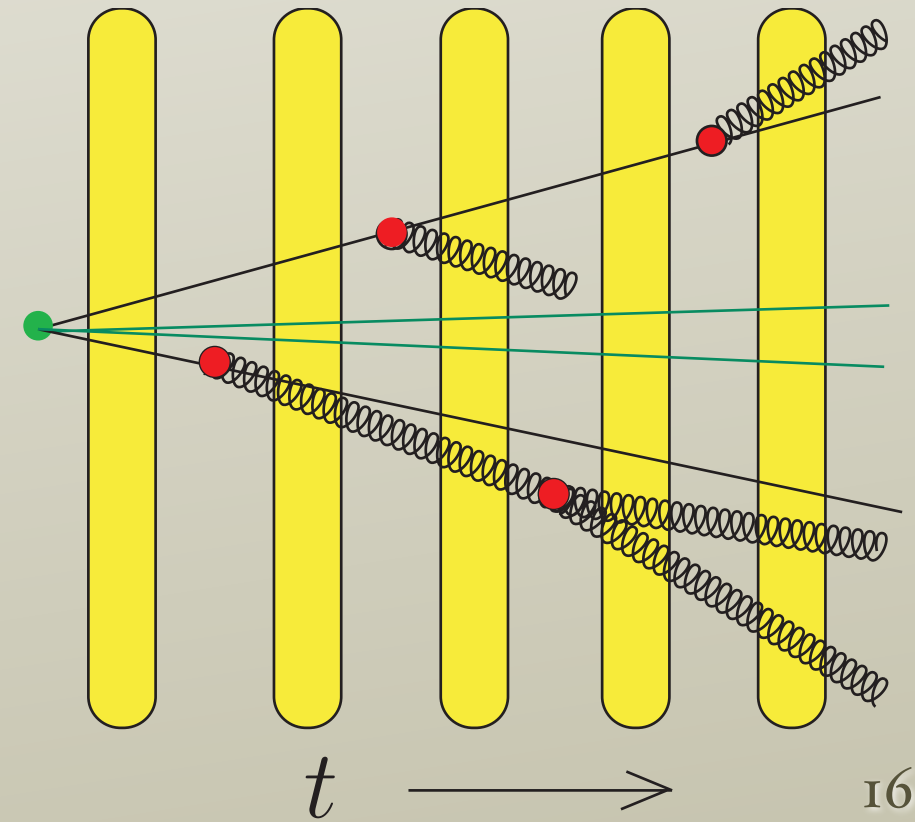
$$\mathcal{U}_{\mathcal{S}}(t,t_0) = \mathcal{N}_{\mathcal{S}}(t,t_0) + \int_{t_0}^{t} d\tau\, \mathcal{U}_{\mathcal{S}}(t,\tau)\mathcal{H}_I(\tau)\mathcal{N}_{\mathcal{S}}(\tau,t_0)$$

$$\mathcal{N}_{\mathcal{S}}(t_2,t_1) = \mathbb{T}\exp\left[\int_{t_1}^{t_2} d\tau\, [-\mathcal{V}(\tau) + (\mathcal{V}(\tau) - \mathcal{S}(\tau))]\right]$$

- Within the LC+ approximation, the operators commute.
- There is an extra factor

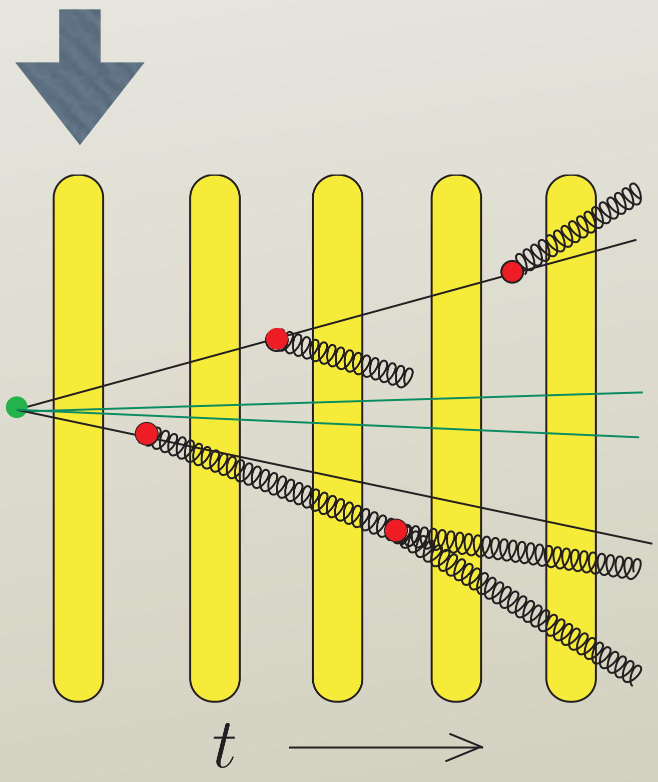
$$\exp\left[\int_{t_1}^{t_2} d\tau\, (\mathcal{V}(\tau) - \mathcal{S}(\tau))\right]$$

that changes the cross section.



The most important term

- Look at the Drell-Yan process.
- Look at the factor for line "a" just after the hard interaction.
- Assume that no real gluons have been emitted yet.



- Use y = dimensionless virtuality variable (with $y \ll 1$) and z = momentum fraction.

- Result: almost everything cancels.

- Two terms do not quite cancel.

$$
\begin{aligned}
&[\mathcal{V}_{\rm a}(t) - \mathcal{S}_{\rm a}(t)]\big|\{p,f,c',c\}_m) = \\
&\Bigg\{ \int_0^{1/(1+y)} \frac{dz}{z} \sum_{\hat a} \frac{\alpha_s}{2\pi} \left(\frac{f_{\hat a/A}(\eta_{\rm a}/z, Q^2 y)}{f_{a/A}(\eta_{\rm a}, Q^2 y)} P_{a\hat a}(z) - \delta_{a\hat a} \frac{2C_a z}{1-z} \right) [1 \otimes 1] \\
&\quad - \int_0^1 \frac{dz}{z} \sum_{\hat a} \frac{\alpha_s}{2\pi} \left(\frac{f_{\hat a/A}(\eta_{\rm a}/z, Q^2 y)}{f_{a/A}(\eta_{\rm a}, Q^2 y)} P_{a\hat a}(z) - \delta_{a\hat a} \frac{2C_a z}{1-z} \right) [1 \otimes 1] \\
&\quad + \cdots \Bigg\} \big|\{p,f,c',c\}_m)
\end{aligned}
$$

- $z < 1/(1+y)$ comes from splitting kinematics.

- $z < 1$ comes from parton evolution.

- This gives

$$[\mathcal{V}_{\rm a}(t) - \mathcal{S}_{\rm a}(t)]\big|\{p,f,c',c\}_m\big) =$$

$$\left\{ \int_{1/(1+y)}^{1} dz\; \frac{\alpha_s}{2\pi}\, \frac{2C_a}{1-z} \left(1 - \frac{f_{a/A}(\eta_{\rm a}/z, Q^2 y)}{f_{a/A}(\eta_{\rm a}, Q^2 y)}\right) [1 \otimes 1] \right.$$

$$\left. + \cdots \right\}\big|\{p,f,c',c\}_m\big)$$

- The $1/(1-z)$ factor creates the "threshold log."

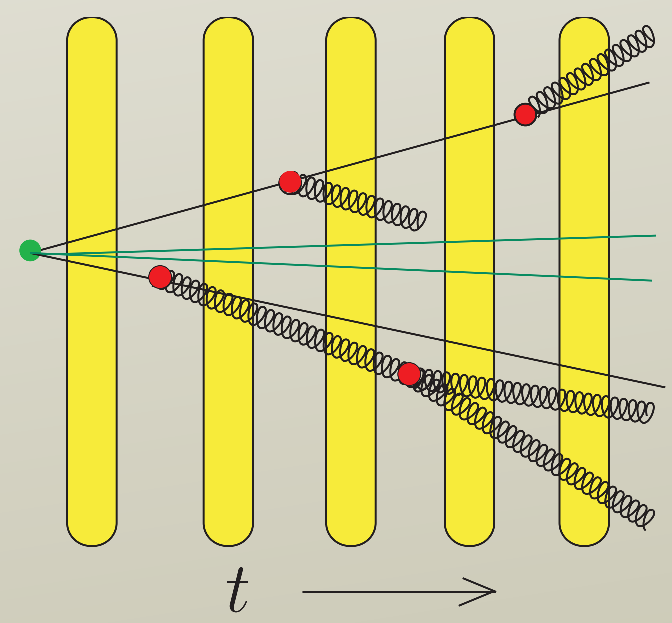
- But the parton factor contains a factor $(1-z)$ so there is no actual log.

- For $y \ll 1$, this contribution is suppressed by a factor of y.

- But, the parton factor can be large, so we keep this.

Conclusion on threshold logs

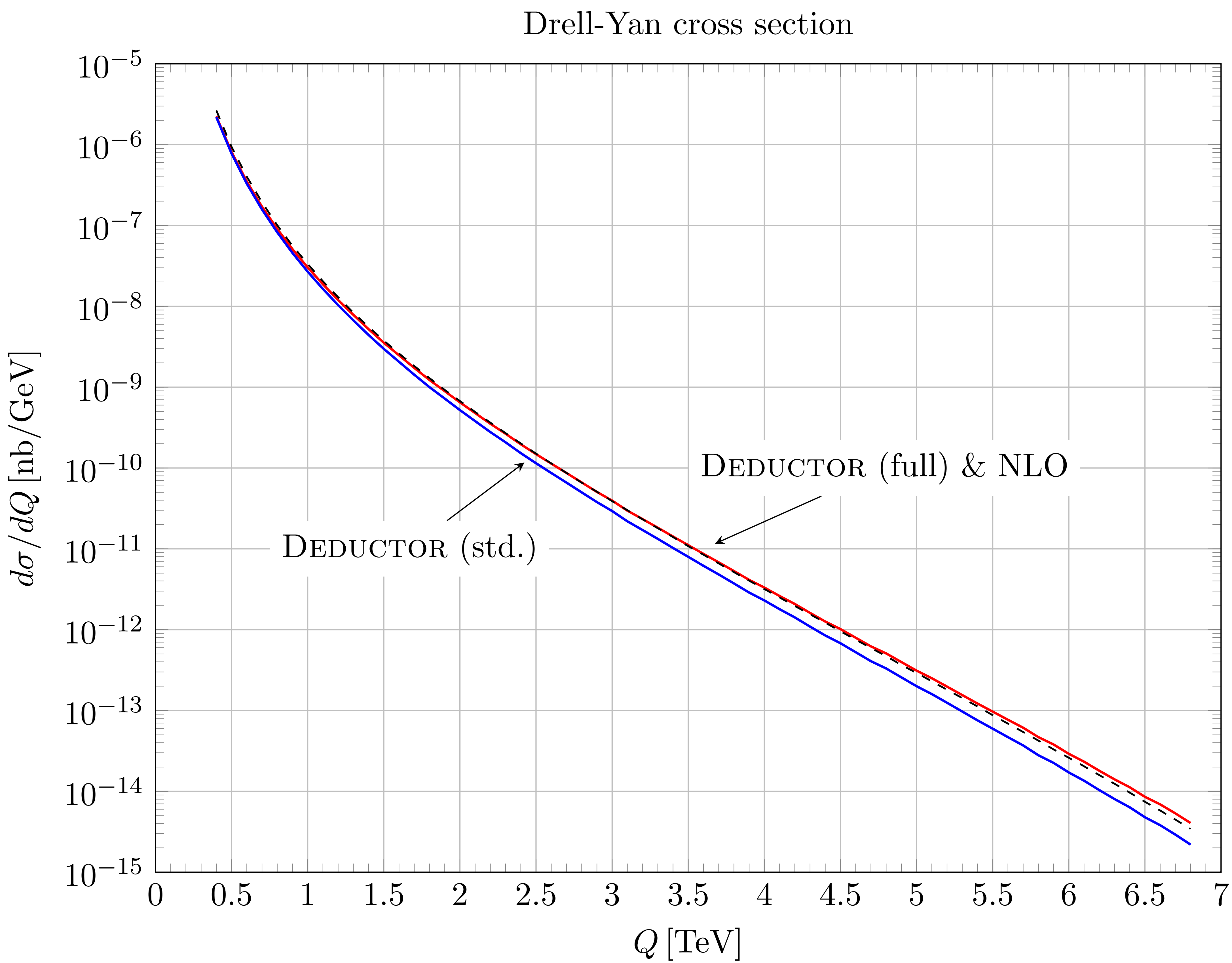
- We find simple and intuitive leading order formulas.
- This is in the context of a leading order parton shower not "NLO," "NLL" or "NNLL."
- This is implemented as part of DEDUCTOR.
- The summation applies to all hard processes.
- The shower sums the threshold logs jointly with other large logs.



Some details

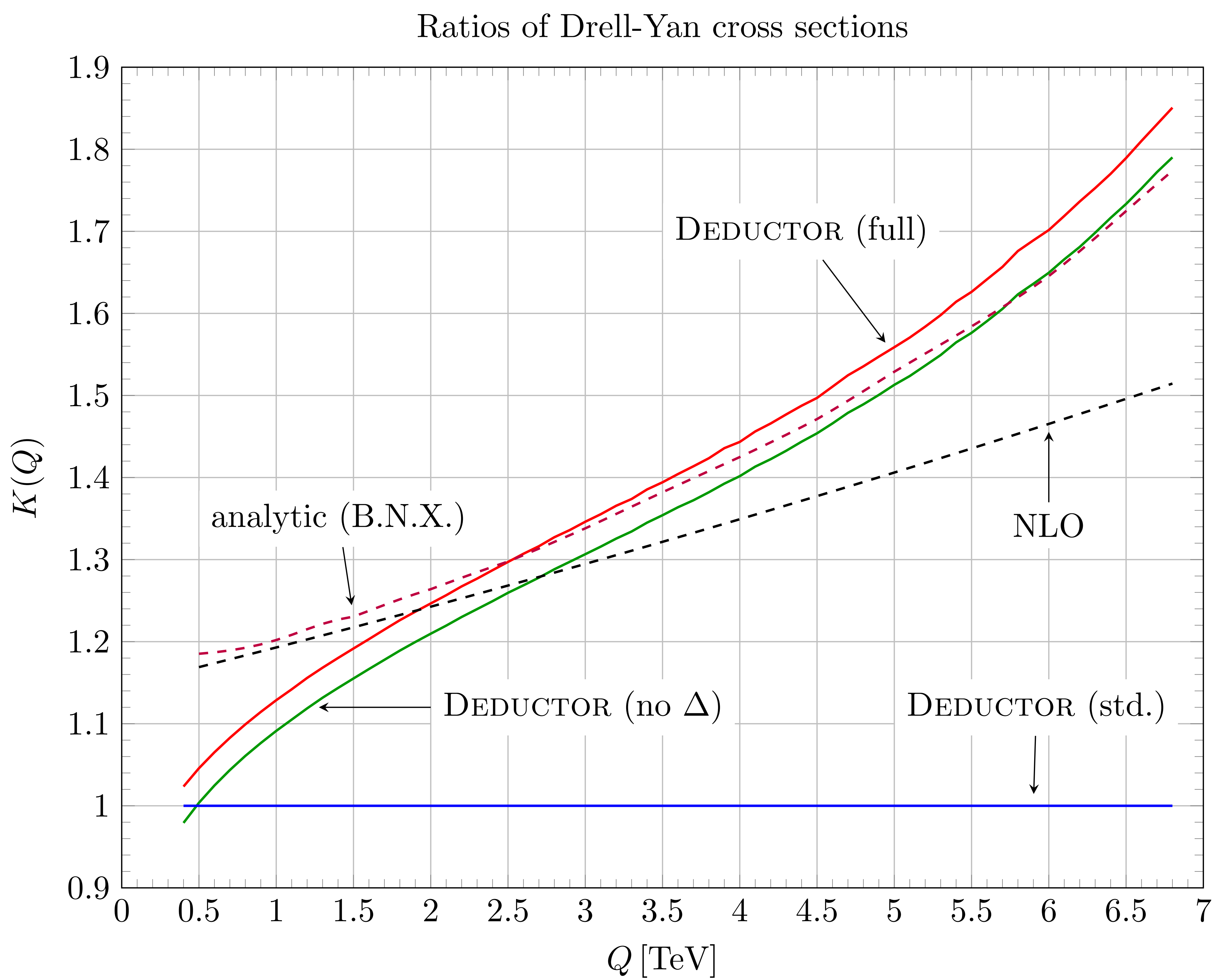
- There are more terms, some with non-trivial color.
- We need to account for switch to parton distributions based on virtuality instead of transverse momentum as the measure of hardness.

Drell-Yan

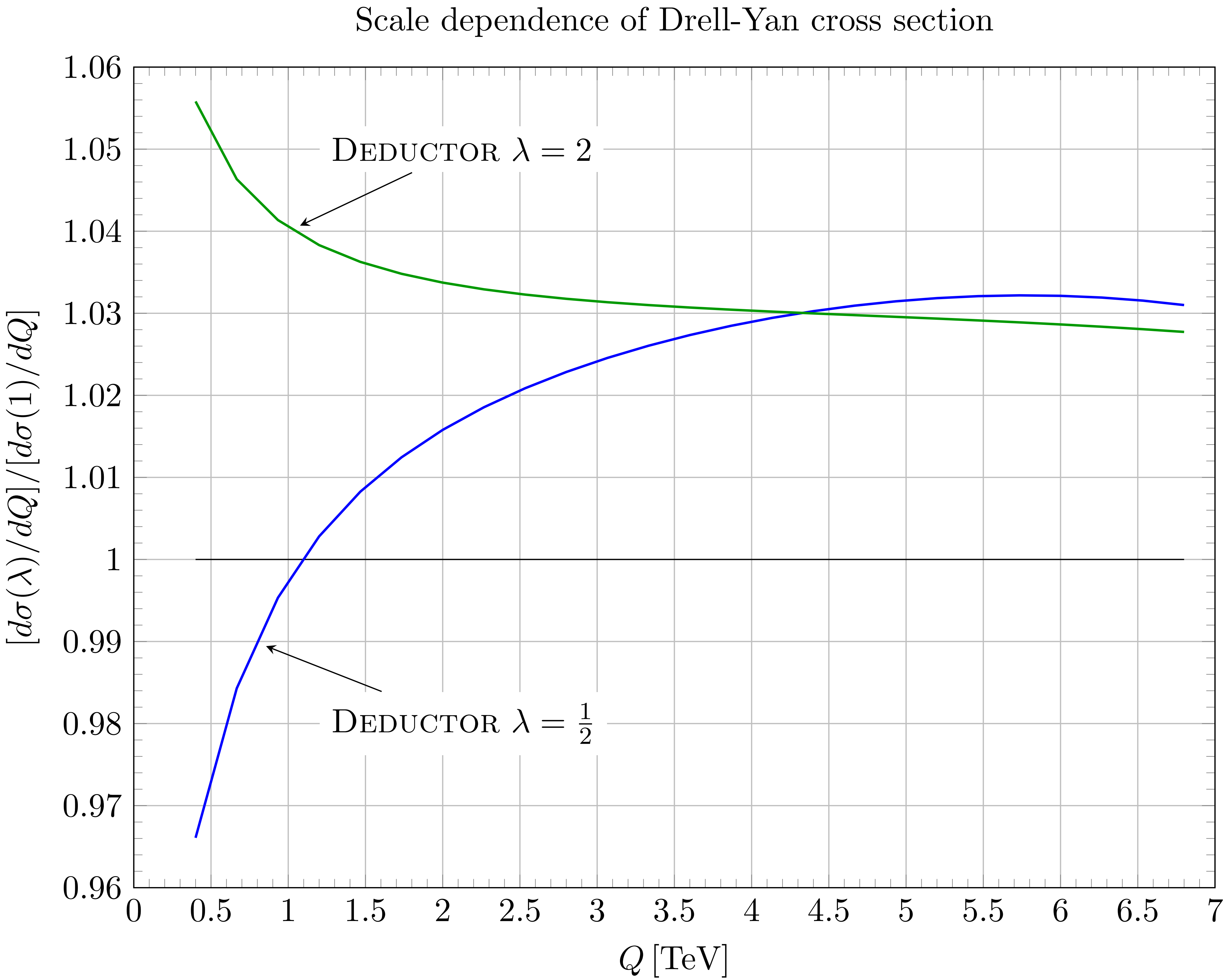


Drell-Yan cross section
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{-15}
$d\sigma/dQ\,[\mathrm{nb/GeV}]$
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
$Q\,[\mathrm{TeV}]$
DEDUCTOR (full) & NLO
DEDUCTOR (std.)

Drell-Yan

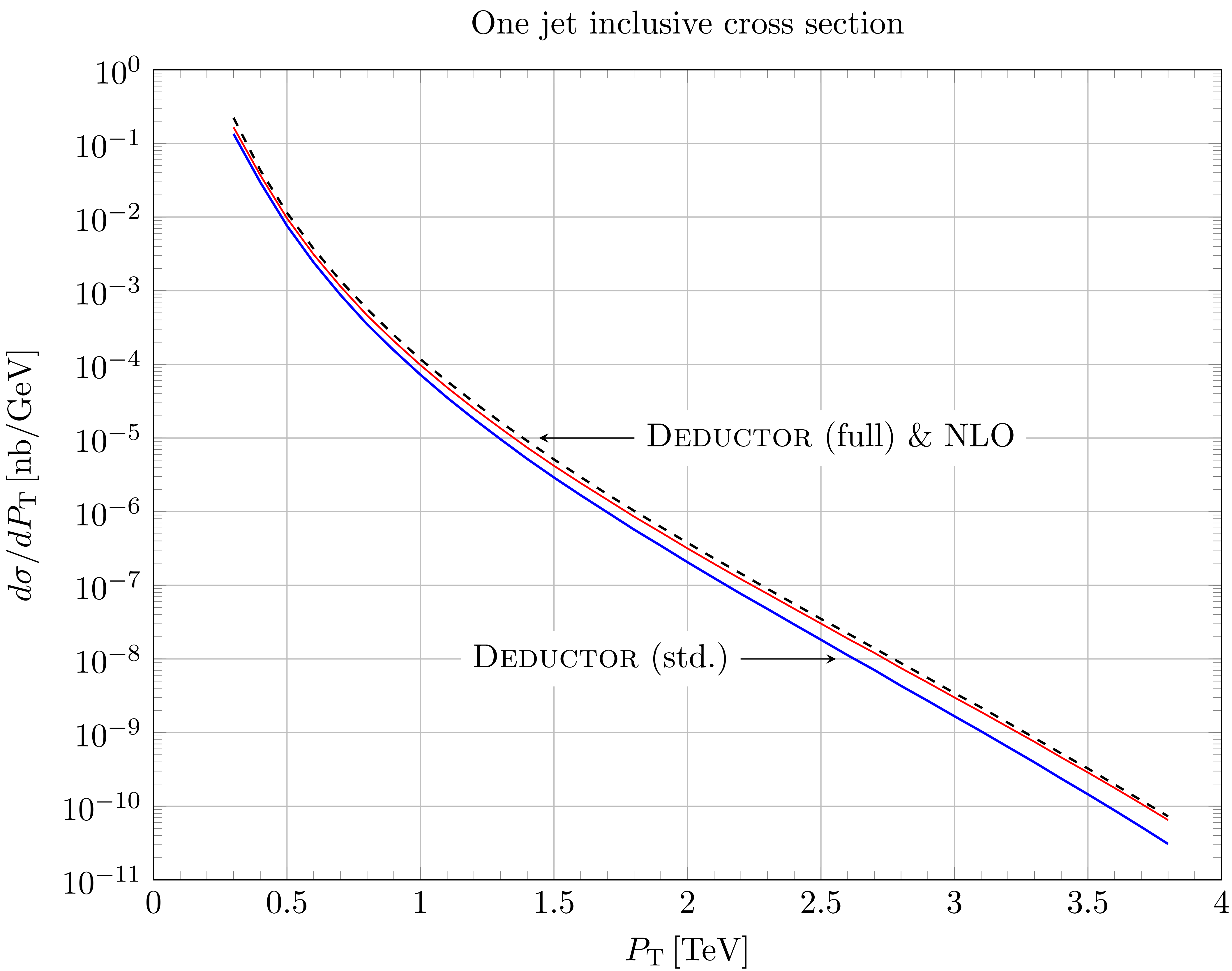

Ratios of Drell-Yan cross sections
K(Q)
Q [TeV]
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1
0.9
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
Deductor (full)
analytic (B.N.X.)
NLO
Deductor (no Δ)
Deductor (std.)

Drell-Yan

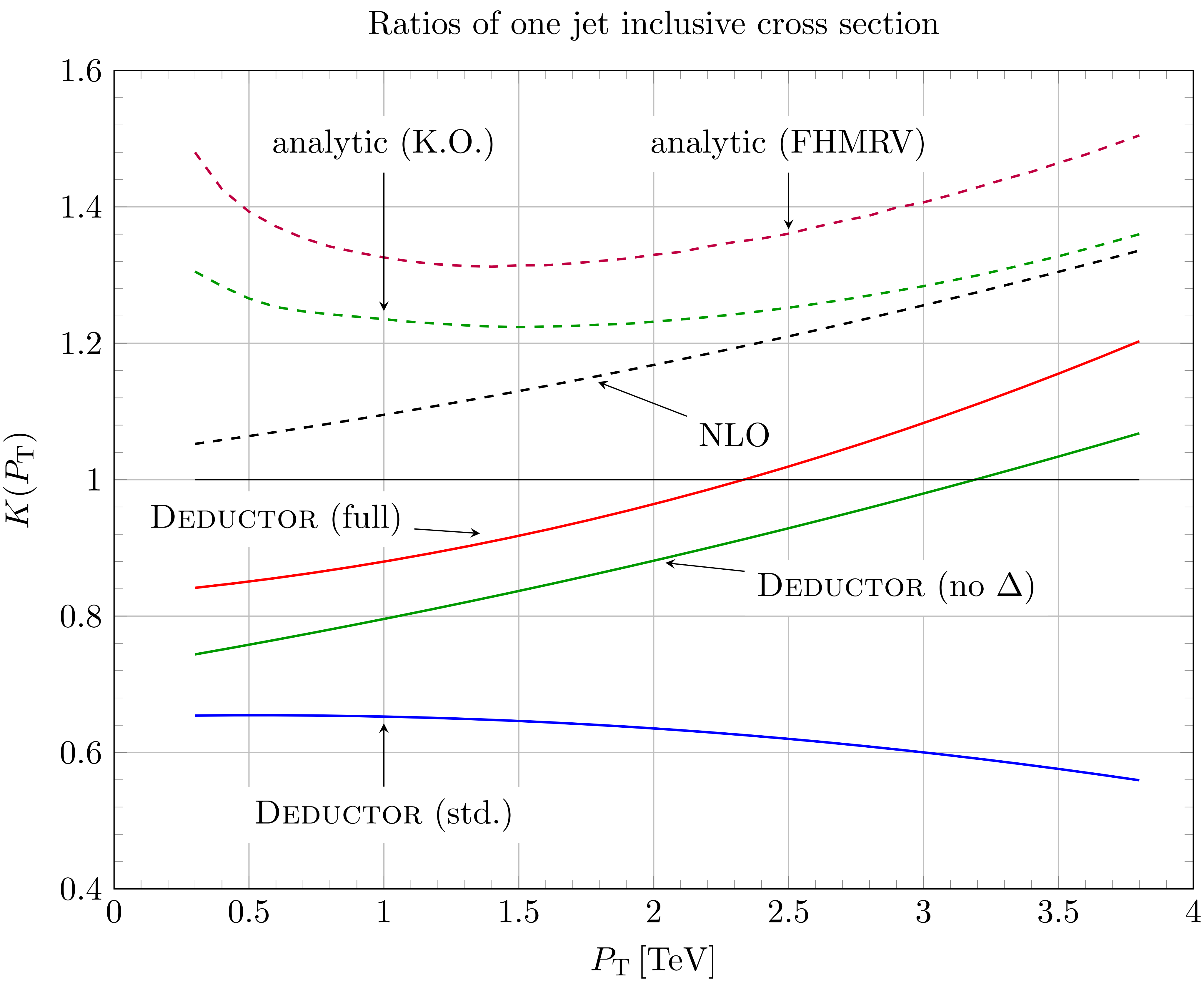


Scale dependence of Drell-Yan cross section
1.06
1.05
1.04
1.03
1.02
1.01
1
0.99
0.98
0.97
0.96
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
$[d\sigma(\lambda)/dQ]/[d\sigma(1)/dQ]$
$Q\,[\mathrm{TeV}]$
DEDUCTOR $\lambda = 2$
DEDUCTOR $\lambda = \frac{1}{2}$

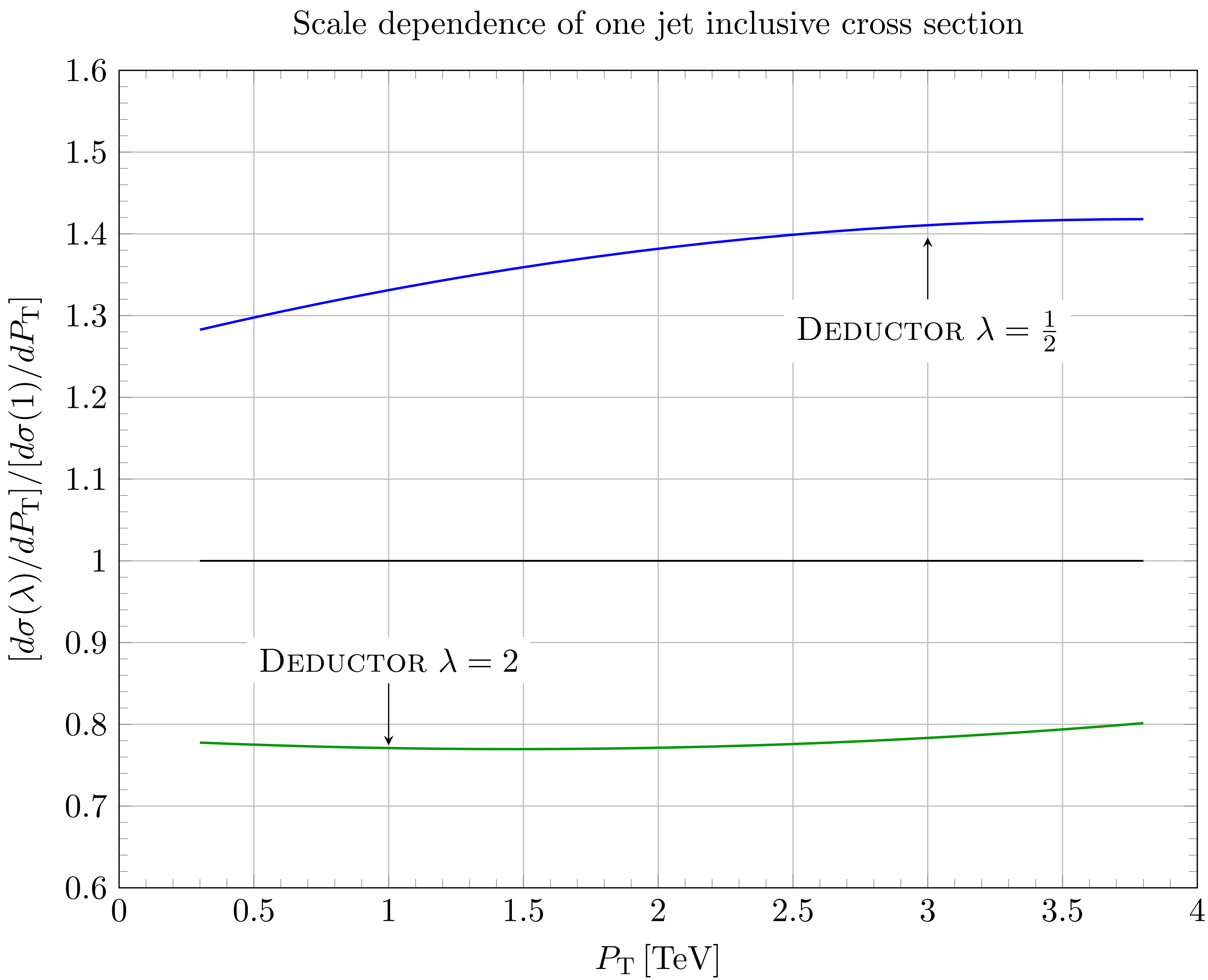
Jet Production



Jet Production



Jet Production



General conclusion

- Parton shower event generators can sum logarithms.
- They are leading order, so not as precise as SCET.
- But they are useful because they are more general.
- Summing threshold logs with a parton shower is possible.