Matching NLO QCD with parton shower in Monte Carlo scheme - the KrkNLO method

Andrzej Siódmok

in collaboration with:

S. Jadach, G. Nail, W. Płaczek, S. Sapeta and M. Skrzypek

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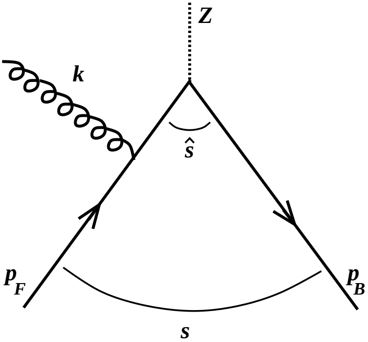
Outline

- Motivation/notation.
- Our approach to NLO+PS matching (example: Drell-Yan) [JHEP 1510 (2015) 052]
- PDF in MC factorization scheme - full definition [arXiv:1606.00355]
- KrkNLO for the Higgs boson production [arXiv:1607.06799]
- Final remarks and outlook

Motivation

- **Why would you like another method of NLO+PS matching?**
  - The method is extremely simple (can be applied on event record).
  - No negative weight events.
  - In angular ordered PS - no need for a truncated shower.
  - Simple at NLO $\Rightarrow$ you may hope that pushing the method to NNLO+NLO PS should be possible.

Drell-Yan process



$$s = (p_F + p_B)^2$$

$$z = \frac{\hat{s}}{s}$$

Sudakov variables:

$$\alpha = \frac{2k \cdot p_B}{\sqrt{s}} = \frac{2k^+}{\sqrt{s}}$$

$$\beta = \frac{2k \cdot p_F}{\sqrt{s}} = \frac{2k^-}{\sqrt{s}}$$

$$z = 1 - \alpha - \beta$$

$$k_T^2 = s\alpha\beta$$

$$y = \frac{1}{2} \ln \frac{\alpha}{\beta}$$

Basic idea of the MC scheme

DY cross section at NLO in collinear $\overline{\text{MS}}$ factorization for the $q\bar{q}$ channel:

$$\sigma^1_{\text{DY}} - \sigma^B_{\text{DY}} \quad = \quad \sigma^B_{\text{DY}}\, D_1^{\overline{\text{MS}}}(x_1,\mu^2) \otimes \frac{\alpha_s}{2\pi} C_q^{\overline{\text{MS}}}(z) \otimes D_2^{\overline{\text{MS}}}(x_2,\mu^2)\,,$$

where

$$C_q^{\overline{\text{MS}}}(z) = C_F\left[4\,(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ - 2\frac{1+z^2}{1-z}\ln z + \delta(1-z)\left(\frac{2}{3}\pi^2 - 8\right)\right].$$

All solutions for NLO + PS matching which use $\overline{\text{MS}}$ PDFs, need to implement collinear remnant term of the type $4\,(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+$ that are technical artefacts of $\overline{\text{MS}}$ scheme.

The implementation is not easy since those terms correspond to the collinear limit but Monte Carlo lives in 4 dimensions and not in the phase space restricted by $\delta(k_T^2)$.
The idea behind the MC scheme is to absorb those terms to PDF.

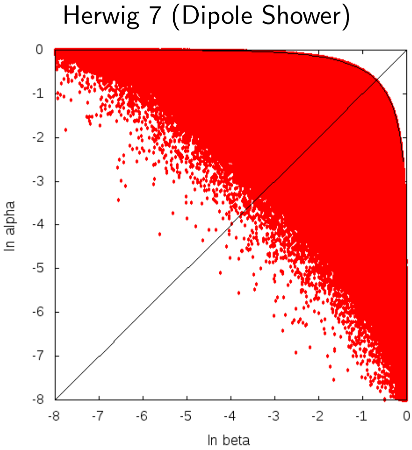
KRK method [Jadach, Kusina, Płaczek, Skrzypek & Sławińska '13]

1. Take a parton shower that covers the (α, β) phase space completely (no gaps, no overlaps) and produces emissions according to approx. real matrix element K.
2. Upgrade the real emissions to exact ME R by reweighting the PS events by $W_R = R/K$.
3. We define the coefficion function $C^R(z) = \int (R - K)$. To avoid unphysical artifacts of $\overline{\mathrm{MS}}$.
4. Transform PDF for $\overline{\text{MS}}$ scheme to this new physical MC factorization scheme.
5. As a result the virtual+soft correction, Δ_{S+V}, is just a constant, without x-depended collinear remnant terms now. Multiply the whole result by $1 + \Delta_{S+V}$ to achieve complete NLO accuracy.

This has been shown to reproduce exactly the NLO result of fixed order collinear factorization, for the case of simplistic PS by means of analytical integration.
[S. Jadach at al. Phys.Rev. D87]

Could we implement the method in a popular, general purpose MC?

1. Take a PS that covers the (α, β) phase space



The evolution variable:
$$q^2 = k_T^2 = \alpha\,\beta\,s.$$

2. Upgrade the real emissions to exact ME by reweighting.

The hardest real emission is upgraded to ME by reweighting:

$$W_R = R/K$$

Where the kernel K is just a CS dipole written in terms of shower's internal variables multiplied by the ratio of PDFs due to backward evolution. The "Sudakov" form factor for he CS shower

$$S(Q^2, \Lambda^2, x) = \int_{\Lambda^2}^{Q^2} \frac{dq^2}{q^2} \int_{z_{\min}(q^2)}^{z_{\max}(q^2)} dz \;\; K(q^2, z, x)\,,$$

Real part:

$$\begin{aligned} W_R^{q\bar{q}}(\alpha, \beta) &= 1 - \frac{2\alpha\beta}{1 + (1-\alpha-\beta)^2} \\ W_R^{qg}(\alpha, \beta) &= 1 + \frac{\alpha(2-\alpha-2\beta)}{1 + 2\,(1-\alpha-\beta)(\alpha+\beta)} \end{aligned}$$

<u>Note:</u>

Very simple weight dependent only on the kinematics α, β. One can compute it on the fly, inside an MC, or outside, using information from event record.

3. The coefficient function $C(z)$

$\hookrightarrow$ It turns out that coefficient functions of the CS shower equal to those from the MC scheme of Jadach et al. arXiv:1103.5015. Hence, CS $\equiv$ MC.

The $C(z)$ function:

$$C^{\mathrm{MC}}(z)\Big|_{\mathrm{real}} = \int (R - K)$$

- For the $q\bar{q}$ channel:

$$C_q^{\mathrm{MC}}(z)\Big|_{\mathrm{real}} = \frac{\alpha_s}{2\pi} C_F \left[-2(1-z)\right]$$

- For the qg channel:

$$C_g^{\mathrm{MC}}(z)\Big|_{\mathrm{real}} = \frac{\alpha_s}{2\pi}\, T_R\, \frac{1}{2}(1-z)(1+3z)$$

Simple form of the coefficient functions with no singular terms!

- Quark and anti-quark PDFs are redefined by:
  - subtracting $C_q^{\mathrm{MC}}(z)$ and $C_g^{\mathrm{MC}}(z)$ from $\overline{\mathrm{MS}}$ PDFs
  - absorbing all z-dependent terms from $\overline{\mathrm{MS}}$ coefficient functions

4. Redefine PDFs: MC PDF

Recipe: Make convolution of the LO PDF in $\overline{\mathrm{MS}}$ (q and $\bar{q}$) with the difference of coefficient functions in $\overline{\mathrm{MS}}$ and MC schemes:

$$f_{q(\bar{q})}^{\mathrm{MC}}(x,Q^2) = f_{q(\bar{q})}^{\overline{\mathrm{MS}}}(x,Q^2) + \int_x^1 \frac{dz}{z}\, f_{q(\bar{q})}^{\overline{\mathrm{MS}}}\left(\frac{x}{z},Q^2\right)\Delta C_q(z) + \int_x^1 \frac{dz}{z}\, f_g^{\overline{\mathrm{MS}}}\left(\frac{x}{z},Q^2\right)\Delta C_g(z)$$

where

$$\Delta C_q(z) = \frac{1}{2}[C_q^{\overline{\mathrm{MS}}}(z) - C_q^{\mathrm{MC}}(z)] = \frac{\alpha_s}{2\pi} C_F \left[\frac{1+z^2}{1-z}\ln\frac{(1-z)^2}{z} + 1 - z\right]_+$$

$$\Delta C_g(z) = C_g^{\overline{\mathrm{MS}}}(z) - C_g^{\mathrm{MC}}(z) = \frac{\alpha_s}{2\pi} T_R \left\{\left[z^2 + (1-z)^2\right]\ln\frac{(1-z)^2}{z} + 2z(1-z\right.$$

The formula is valid for any process up to $\mathcal{O}(\alpha_s^2)$.

The gluon PDF for DY: $f_g^{\mathrm{MC}}(x,Q^2) = f_g^{\overline{\mathrm{MS}}}(x,Q^2)$

Notes:

- The MC scheme has been validated by reproducing the scheme-independent relations between DY and DIS. [S. Jadach at al. Phys.Rev. D87]
- LHAPDF grid (easy to use by all PS MC) for the MC PDF. (As a source we used MSTW2008lo, other $\overline{\mathrm{MS}}$ PDF possible).

5. Virtual+soft correction, Δ_{S+V}

Virtual + soft:

$$
\begin{aligned}
W_{V+S}^{q\bar{q}} &= \frac{\alpha_s}{2\pi} C_F \left[\frac{4}{3}\pi^2 - \frac{5}{2} \right] \\
W_{V+S}^{qg} &= 0
\end{aligned}
$$

Notes:

- Simple, kinematics independent!
- No need to generate strictly collinear contributions (like $d\Sigma^{c\pm}$ terms in MC@NLO).

Upgrading to NLO: the hardest emission

Steps:

1. Run LO PS[1] (Herwig/Sherpa) using MC PDF (via LHAPDF interface)
2. Get and an event record (for example in the HepMC format).

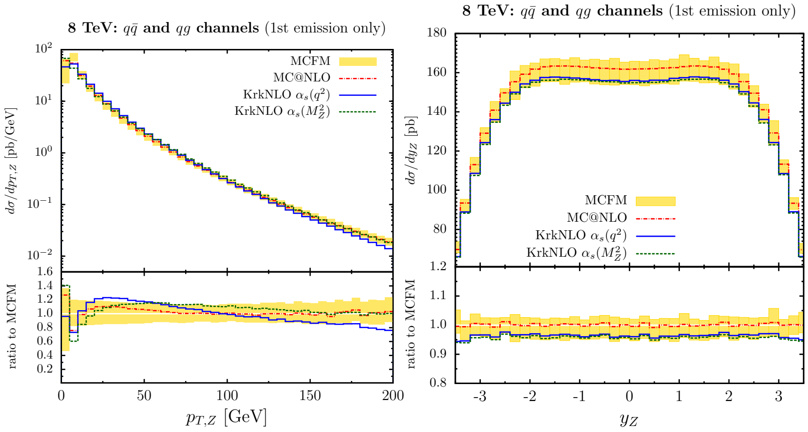
```
GenEvent: #8 ID=0 SignalProcessGenVertex Barcode: 0
 Momenutm units:     GEV     Position units:      MM
 Cross Section: 697.653 +/- 206.627
 Entries this event: 1 vertices, 5 particles.
 Beam Particles are not defined.
 RndmState(0)=
 Wgts(9)=(0,3023.17) (1,0.17886) (2,3023.17) (3,9) (4,0) (5,1.14371) (6,0) (7,1) (8,1)
 EventScale -1 [energy]         alphaQCD=0.139387       alphaQED=-1
                                   GenParticle Legend
        Barcode   PDG ID      ( Px,       Py,       Pz,     E ) Stat  DecayVtx
________________________________________________________________________________
GenVertex:       -1 ID:    0 (X,cT):0
 I: 2     10001        1 +0.00e+00,+0.00e+00,+6.26e+02,+6.26e+02   2        -1
          10002       21 +0.00e+00,+0.00e+00,-1.84e+01,+1.84e+01   2        -1
 O: 3     10003        1 -1.82e+00,+5.68e-01,-1.50e+01,+1.51e+01   1
          10004       11 +2.58e+01,+9.16e+00,+5.71e+02,+5.71e+02   1
          10005      -11 -2.40e+01,-9.73e+00,+5.17e+01,+5.78e+01   1
```

3. Book histograms (for example using Rivet) with MC weight calculated from the event record (and information on α_s).

It is almost as fast as LO+PS calculation!

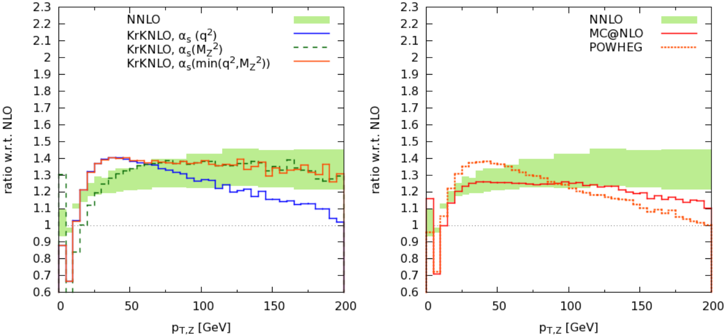
[1] Cover full Phase Space.

Matched results: DY botch channels, 1st emission



- MCFM band is an uncertainty estimate obtained by independent variation of μ_F and μ_R by a factor $1/2$ and 2
- Moderate differences between KrkNLO $\alpha_s(q^2)$ and MC@NLO in the region below M_Z and between KrkNLO $\alpha_s(M_Z^2)$ and MC@NLO in the region above M_Z

DY comparison with fixed order NNLO results (DYNNLO)



- DYNNLO green band is an uncertainty estimate obtained by independent variation of μ_F and μ_R by a factor $1/2$ and 2
- KrkNLO $\alpha_s(min(q^2, M_Z))$ and NNLO results show the same trends (left).
- Similar comparisons for POWHEG and MCatNLO are also shown (right).

Full (including gluon) PDFs in the MC scheme

[S. Jadach, W. Płaczek, S. Sapeta, AS and M. Skrzypek, arXiv:1606.00355]

Reminder: The gluon PDF for DY: $f_g^{\rm MC}(x, Q^2) = f_g^{\overline{\rm MS}}(x, Q^2)$

The entire transformation rule takes the form

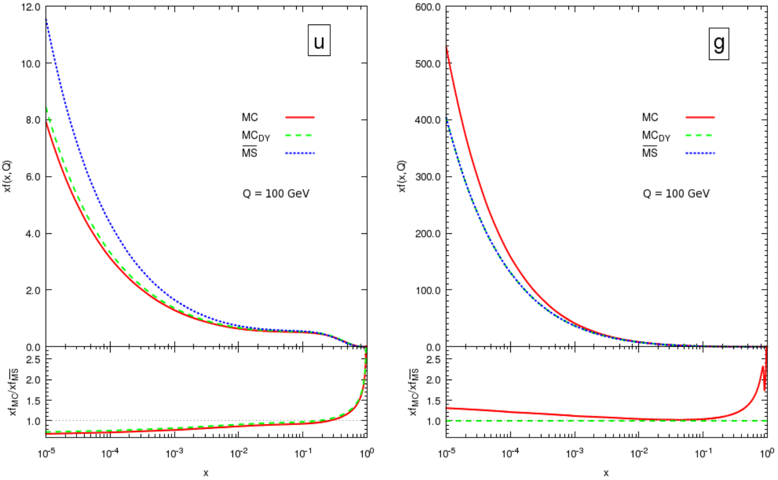
$$\begin{bmatrix} q(x,Q^2) \\ \bar{q}(x,Q^2) \\ g(x,Q^2) \end{bmatrix}_{\mathsf{MC}} = \begin{bmatrix} q \\ \bar{q} \\ g \end{bmatrix}_{\overline{\mathsf{MS}}} + \int dzdy \begin{bmatrix} K_{qq}^{\mathsf{MC}}(z) & 0 & K_{qg}^{\mathsf{MC}}(z) \\ 0 & K_{\bar{q}\bar{q}}^{\mathsf{MC}}(z) & K_{\bar{q}g}^{\mathsf{MC}}(z) \\ K_{gq}^{\mathsf{MC}}(z) & K_{g\bar{q}}^{\mathsf{MC}}(z) & K_{gg}^{\mathsf{MC}}(z) \end{bmatrix} \begin{bmatrix} q(y,Q^2) \\ \bar{q}(y,Q^2) \\ g(y,Q^2) \end{bmatrix}_{\overline{\mathsf{MS}}} \delta(x-yz)$$

see backup slides for K's.

- All virtual parts $\sim \delta(1-z)$ adjusted using momentum sum rules.
- We provided all information (MC-scheme coeff. functions, Q^2 evolution governed by LO kernels) needed for direct fitting of PDFs!

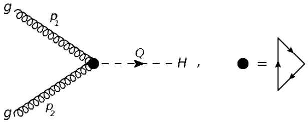
MC factorization scheme

Numerical examples of PDFs in the MC scheme



- Change with respect to $\overline{\text{MS}}$ PDFs is noticeable.
- Version labeled MC is complete MC scheme.
- Version MC$_{DY}$ neglects certain $\mathcal{O}(\alpha_s^2)$ terms, limited to DY process.

KrkNLO for Higgs-boson production in gluon–gluon fusion



As expected we get **simple** weights:

- Real part:
  1. $g + g \longrightarrow H + g$:

$$W_R^{gg}(\alpha, \beta) = \frac{1 + z^4 + \alpha^4 + \beta^4}{1 + z^4 + (1 - z)^4} \tag{2}$$

  2. $g + q \longrightarrow H + q$:

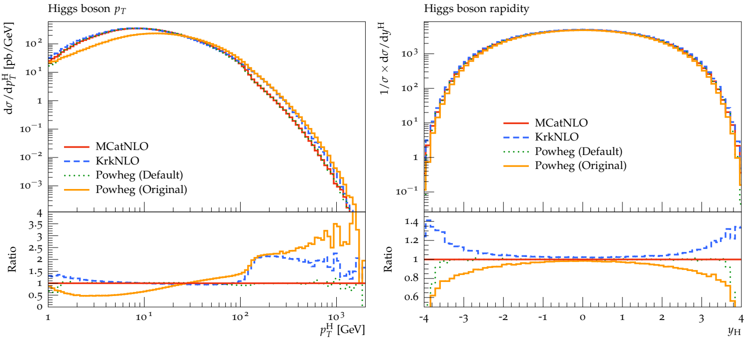
$$W_R^{gq}(\alpha, \beta) = \frac{1 + \beta^2}{1 + (1 - z)^2} \tag{3}$$

  also $W_R^{qg}(\alpha, \beta) = W_R^{gq}(\beta, \alpha)$.

- VS part $W_{VS} = 1 + \Delta_{VS}$:

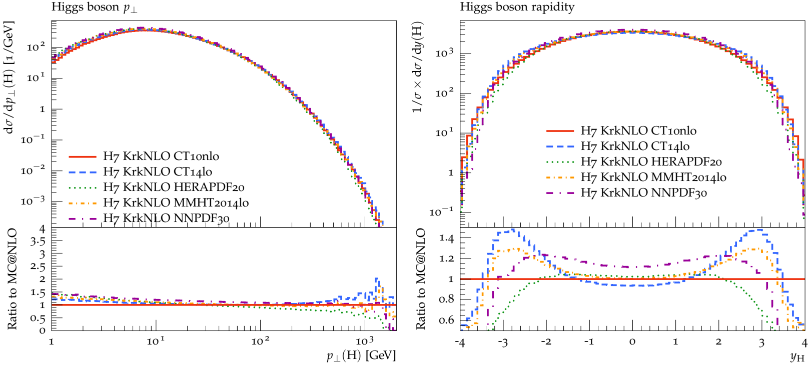
$$\Delta_{VS}^{gg} = \frac{\alpha_s}{2\pi} C_A \left(\frac{4\pi^2}{3} + \frac{473}{36} - \frac{59}{18} \frac{T_f}{C_A} \right), \qquad \Delta_{VS}^{gq} = 0.$$

KrkNLO for Higgs-boson production — comparison with other methods



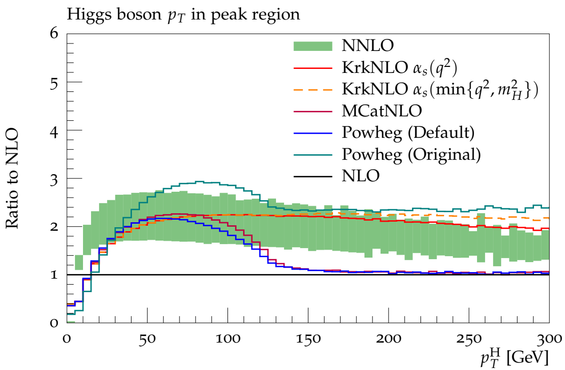
Comparisons of the Higgs-boson transverse momentum and rapidity distributions from the KrkNLO, MCatNLO and Powheg methods implemented in H7

KrkNLO for Higgs-boson production — different PDF sets



KrkNLO method using different PDF sets in the MC factorization scheme for the Higgs-boson production in gluon–gluon fusion at the LHC . Easy to get from LHAPDF6 unified format.

Comparison with NNLO



- Higgs-boson transverse-momentum distributions from KrkNLO, MCatNLO and Powheg compared with the fixed-order NNLO result from the HNNLO. All distributions are divided by the NLO results.
- Similar observation as for DY, KrkNLO pt spectra similar to NNLO fixed order

Conclusions

- I have discussed the KrkNLO method of NLO+PS matching:
  - Real emissions are corrected by simple reweighting.
  - No troublesome "collinear remnant terms" - artifacts of the MS-bar scheme. They are absorbed in PDFs by changing the factorization scheme from MS-bar to MC.
  - Virtual correction is just a constant and does not depend on Born-like kinematics.
- The method has been implemented on top of Catani-Seymour shower in H7 both for Drell-Yan and Higgs production processes.
- It has been validated against fixed order NLO and compared to MC@NLO and POWHEG.
- Pt spectra from KrkNLO and NNLO show the same trends.

Near future: Public version implemented in Herwig 7 (krknlo.hepforge.org), diboson production, correction of n emissions.
Next: work on extension of the method to NNLO+NLO PS.

Thank you for the attention!

krknlo.hepforge.org

$$
\begin{aligned}
K_{gq}^{\mathrm{MC}}(z) &= \frac{\alpha_s}{2\pi}\, C_F \left\{ \frac{1+(1-z)^2}{z} \ln\frac{(1-z)^2}{z} + z \right\}, \\
K_{gg}^{\mathrm{MC}}(z) &= \frac{\alpha_s}{2\pi}\, C_A \left\{ 4\left[\frac{\ln(1-z)}{1-z}\right]_+ + 2\left[\frac{1}{z} - 2 + z(1-z)\right] \ln\frac{(1-z)^2}{z} \right. \\
&\qquad\qquad \left. - 2\frac{\ln z}{1-z} - \delta(1-z)\left(\frac{\pi^2}{3} + \frac{341}{72} - \frac{59}{36}\frac{T_f}{C_A}\right) \right\}, \\
K_{qq}^{\mathrm{MC}}(z) &= \frac{\alpha_s}{2\pi}\, C_F \left\{ 4\left[\frac{\ln(1-z)}{1-z}\right]_+ - (1+z)\ln\frac{(1-z)^2}{z} - 2\frac{\ln z}{1-z} + 1 - z \right. \\
&\qquad\qquad \left. - \delta(1-z)\left(\frac{\pi^2}{3} + \frac{17}{4}\right) \right\}, \\
K_{qg}^{\mathrm{MC}}(z) &= \frac{\alpha_s}{2\pi}\, T_R \left\{ \left[z^2 + (1-z)^2\right] \ln\frac{(1-z)^2}{z} + 2z(1-z) \right\}, \\
K_{g\bar{q}}^{\mathrm{MC}}(z) &= K_{gq}^{\mathrm{MC}}(z), \qquad K_{\bar{q}g}^{\mathrm{MC}}(z) = K_{qg}^{\mathrm{MC}}(z).
\end{aligned}
\tag{4}
$$