The LPM effect in sequential gluon bremsstrahlung

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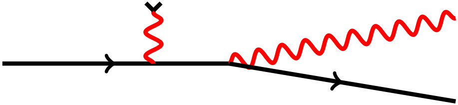
Outline

1. Introduction
2. Overview of the calculation
3. Results

- At high energies, energy loss is dominated by nearly collinear bremsstrahlung and pair production.
- Naively, the bremsstrahlung rate will be roughly,

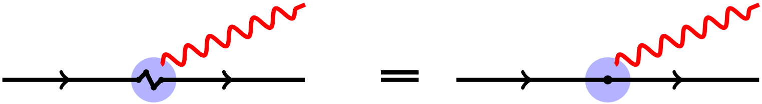


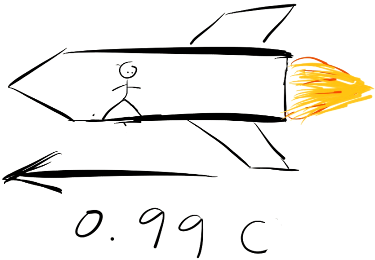
$$\Rightarrow \Gamma = n\sigma v$$

where n is the density of particles in the medium; σ is the cross section for bremsstrahlung arising from a single collision; and v is the relative velocity.

Complication: The Landau-Pomeranchuk-Migdal effect

- When the formation time of the radiated particle becomes larger than the mean free path in the medium, leading to a significant reduction in the bremsstrahlung rate.
- Consider an *electron* scattering multiple times from the medium and radiating a *photon*.
- The photon cannot resolve details that are smaller than its wavelength. This will create a region of fuzziness, depicted as the blue shaded region.

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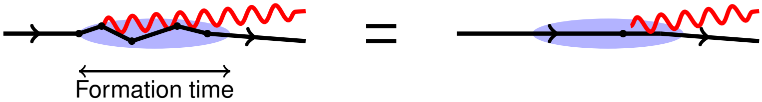
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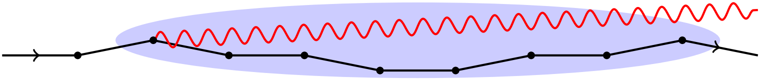
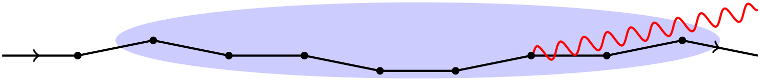
Formation time





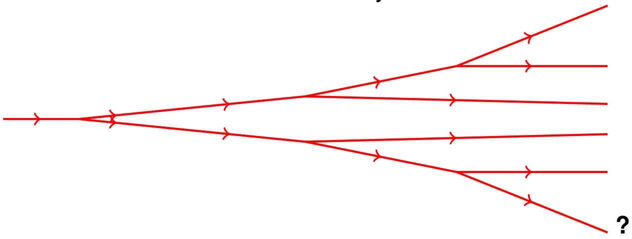
Two possibilities interfere quantum mechanically ⇒ A reduction in the bremsstrahlung rate from the naive expectation.

A qualitative difference between QED and QCD formation times

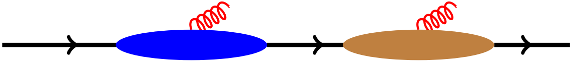
- QED: Softer photon, bigger wavelength ⇒ larger formation time.
- QCD: Softer gluon , easier to deflect ⇒ shorter formation time.

- LPM effect for QED was figured out in 1950's.
- QCD generalization in 1990's.

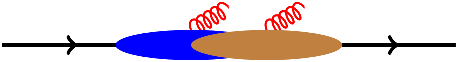
So does that solve the problem? Can we just use the LPM result for the rate into a Monte-Carlo-like calculation to study



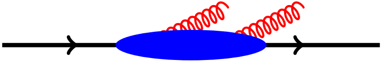
Yes, when splittings are well separated.



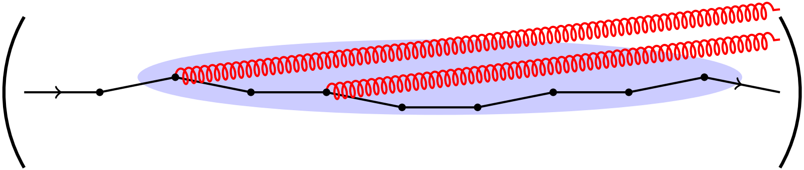
But not when the formation times overlap.

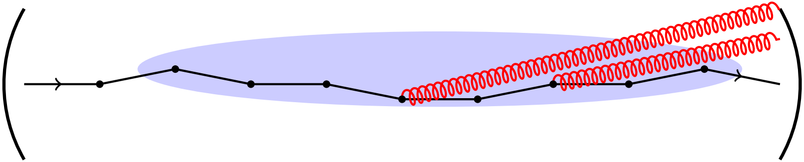


Alternative picture: When we have two splittings within a formation time of each other.

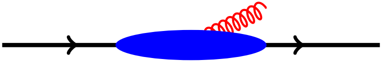


We would need to calculate interferences of the type

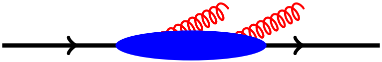




How important could this correction be?



Probability $\sim \alpha$



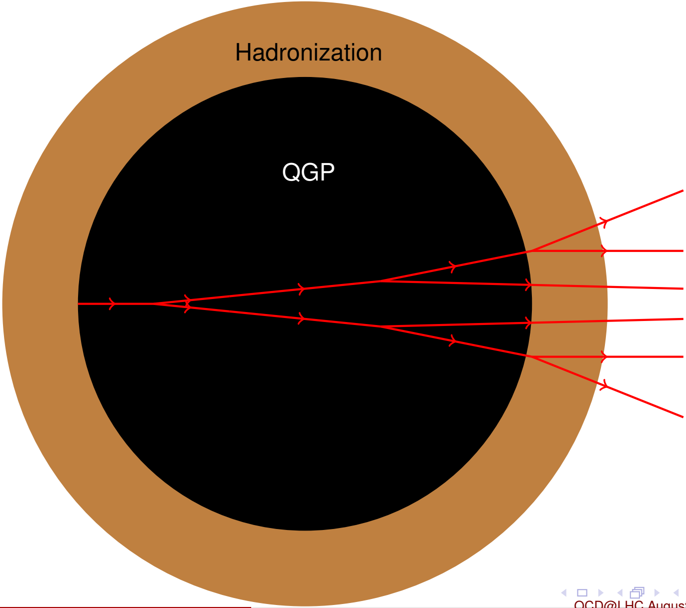
Probability $\sim \alpha^2$

α is only moderately small at energy scales reached typically in heavy-ion collisions.

Conclusion: Interesting to calculate the effects of overlapping formation times.

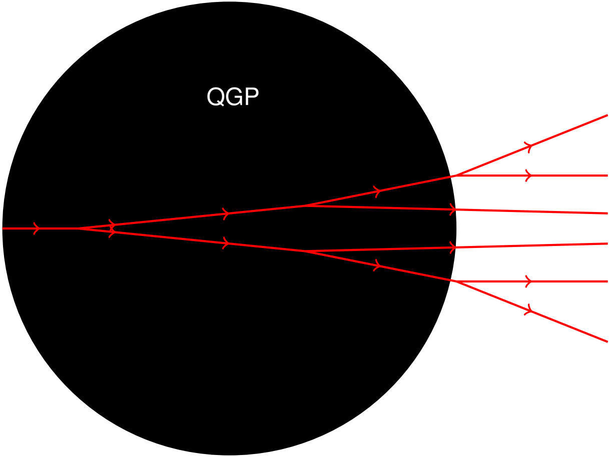
- Previous authors have analyzed this problem in the limiting cases where the radiated gluons have very small energies [Wu, Mehtar-Tani, Blaizot].
- Our calculation goes beyond that approximation and will be valid for general energies of radiated gluons.

What we do (and what we don't)



What we do (and what we don't)

- We will focus on the in-medium part of the evolution.

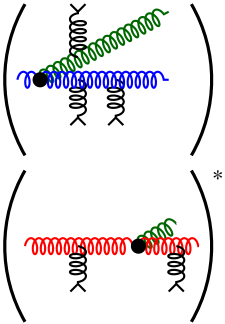


What we do (and what we don't)

- We will assume the medium to be static, homogeneous and thick, i.e. wider than a typical formation length.
- To simplify the color algebra, we will work in the large N_C limit and will focus on the all-gluon case.
- We have calculated the **correction** introduced because of overlapping formation times for general energies of the radiated gluons.

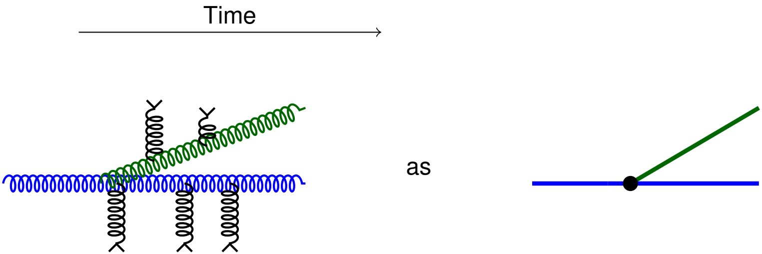
LPM effect in QCD

In the terms of Feynman diagrams, the LPM effect represents interferences between splittings happening before and after a sequence of elastic scatterings with medium.



Notation

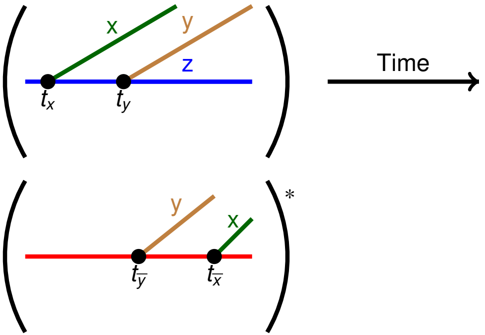
But for diagrammatic convenience, we will draw



with interactions with the medium being implicit.

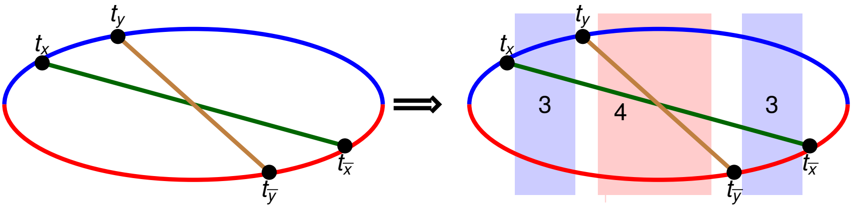
Double Gluon bremsstrahlung:Formalism

- Using this notation, consider one such interference diagram contributing to the double bremsstrahlung rate,



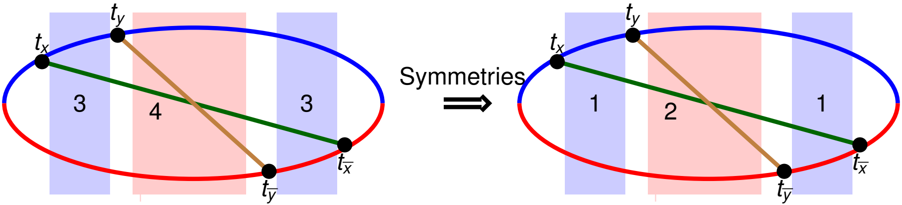
- We will call this the $xy\overline{yx}$ diagram. So the name indicates the order of splittings.

- For calculations like these, it is helpful to think of these terms as being a single ”process”.
- In time between the splittings, the number of high energy particles doesn’t change.

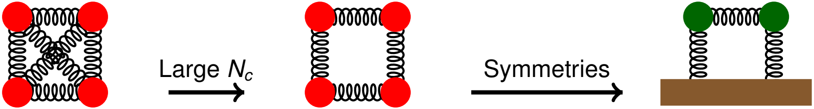


- So the problem reduces to sewing QCD splitting matrix elements with the evolution in these three regions.

Complications: 4-particle evolution



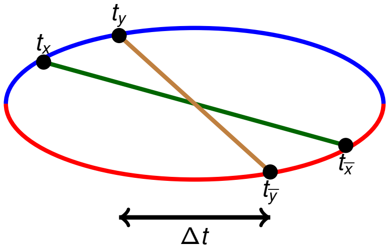
$\hat{q}$ Approximation $\Rightarrow$ Harmonic Oscillators with non-Hermitian Springs!



Need to calculate Eigenfrequencies and then, QM HO propagators.

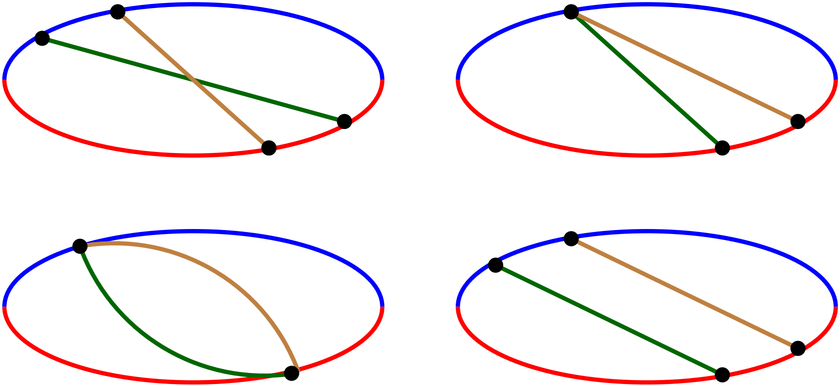
More Complications and the result

- **Non-Trivial Helicity contractions:** Must use helicity dependent DGLAP splitting functions.
- **Divergences:** Diagrams involving two 3-gluon vertices diverge as $\Delta t \to 0$. The divergences cancel when contribution from different diagrams are added together, but we still need to carefully calculate the non-trivial contributions from the poles.



More Complications and the result

- A lot of diagrams to calculate!.
- Many different permutations and time orderings.



$$\textbf{Result} = \int \Delta t(\text{Complicated expression})$$

- Final Δt integral easy to do numerically.

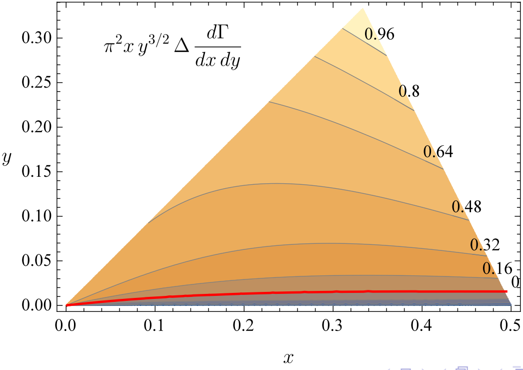
What we have done

- Completed the calculation of the correction introduced by the considering overlapping formation times.
- P. Arnold and Shahin Iqbal, ”The LPM effect in sequential bremsstrahlung”,JHEP 04(2015)070
- P. Arnold, H.C. Chang and Shahin Iqbal, ”The LPM effect in sequential bremsstrahlung 2: factorization” arXiv:1605.07624 [hep-ph].
- P. Arnold, H.C. Chang and Shahin Iqbal, ”The LPM effect in sequential bremsstrahlung: dimensional regularization” arXiv:1606.08853 [hep-ph].
- P. Arnold, H.C. Chang and Shahin Iqbal, ”The LPM effect in sequential bremsstrahlung: 4-gluon vertices” arXiv:1608.05718 [hep-ph].
- Can reproduce the results of previous authors in the relevant limiting cases.

What we find

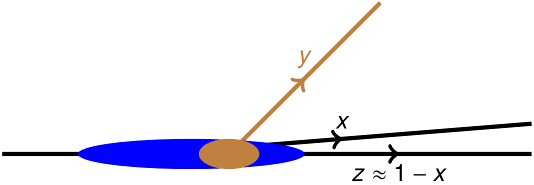
- The correction is negative only when one of the three daughters is much softer than the the other two.
- In in the limiting case $y \ll x \ll 1$, the correction introduced by considering the effects of overlapping formation times is given by

$$\Delta \frac{d\Gamma}{dxdy} \approx -\frac{C_A^2 \alpha^2}{\pi^2 x y^{3/2}} \sqrt{\frac{\hat{q}_A}{E}}$$

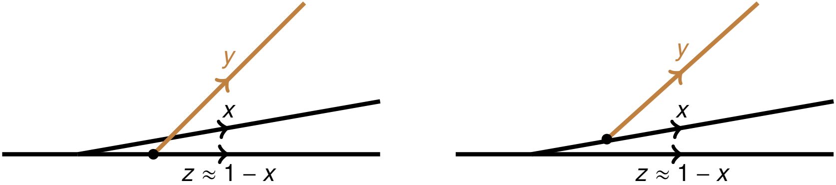


Why is the correction negative?

For $y \ll x \ll 1$, x and z are so close to each other that y sees them as if they were a *single* adjoint color particle.



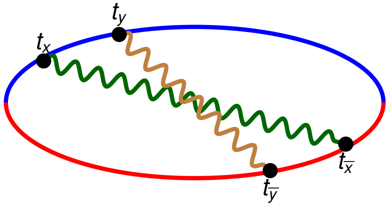
A naive Monte-Carlo calculation treating these splittings as independent would count both possibilities and therefore, double count the chance of y emission.



Thank You

Work in progress

- Calculating the effects of overlapping formation times in QED.

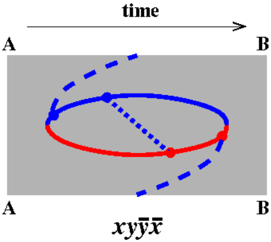


References

1- L.D. Landau and I. Pomeranchuk, Limits of applicability of the theory of bremsstrahlung electrons and pair production at high-energies, Dokl. Akad. Nauk Ser. Fiz. 92 (1953) 535 [INSPIRE].
2- A.B. Migdal, Bremsstrahlung and pair production in condensed media at high-energies, Phys. Rev. 103 (1956) 1811 [INSPIRE].
B.G. Zakharov, Fully quantum treatment of the Landau-Pomeranchuk-Migdal effect in QED and QCD, JETP Lett. 63 (1996) 952 [hep-ph/9607440] [INSPIRE].
3- R. Baier, Y.L. Dokshitzer, A.H. Mueller, S. Peigne and D. Schiff, Radiative energy loss and pT broadening of high-energy partons in nuclei, Nucl. Phys. B 484 (1997) 265 [hep-ph/9608322] [INSPIRE].
4- J.P. Blaizot and Y. Mehtar-Tani, Renormalization of the jet-quenching parameter, Nucl. Phys. A 929 (2014) 202 [arXiv:1403.2323] [INSPIRE].
5- B. Wu, Radiative energy loss and radiative pŁ-broadening of high-energy partons in QCD matter, JHEP 12 (2014) 081 [arXiv:1408.5459] [INSPIRE].

Large N limit

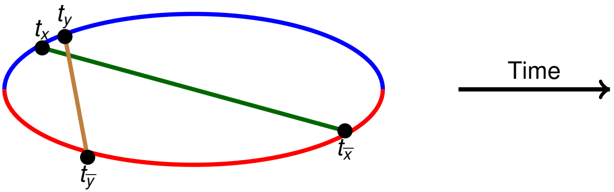
- In the 3-particle region, the constraint $T_1 + T_2 + T_3 = 0$ determines all of the $T_i.T_j$.
- In the 4-particle region, the constraint $T_1 + T_2 + T_3 + T_4 = 0$ cannot determine all $T_i.T_j$.
- Things become much more simplified in the large N limit.



(a) (b)

A Problem

- Integrand $\propto \frac{\text{stuff}}{\Delta t^2} + \frac{\text{stuff}}{\Delta t}$ as $\Delta t \to 0$.
- $\frac{1}{\Delta t^2}$ divergence is canceled by subtracting the vacuum contribution.
- We find that the $\frac{1}{\Delta t}$ divergence was arising in the limit when three of the 4 times t_i approached each other simultaneously.



- All $\frac{1}{\Delta t}$ divergences get canceled when all six time orderings are added.

A subtlety!

- Even though the divergence cancels, one needs to be careful about the $i\epsilon$ prescriptions.
- For example,

$$\int_{-\infty}^{+\infty} dt(\frac{1}{t} - \frac{1}{t}) = 0$$

but

$$\int_{-\infty}^{+\infty} dt(\frac{1}{t - i\epsilon} - \frac{1}{t + i\epsilon}) \neq 0$$

- The different diagrams turn out to get different $i\epsilon$ prescriptions and therefore,we also had to calculate the contributions from these poles.

LPM effect in QED vs QCD