NNLL soft and Coulomb threshold resummation for squark and gluino production at the LHC

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M. Beneke, P. Falgari, J. Piclum, C. Schwinn, CW [arXiv: 1312.0837]
M. Beneke, J. Piclum, C. Schwinn, CW [arXiv: 1607.07574]

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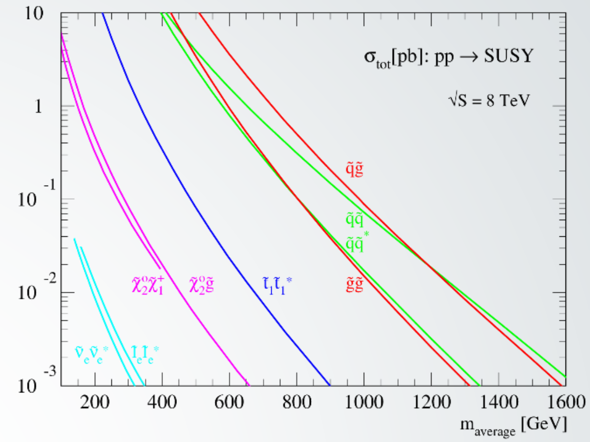
Motivation

- SUSY searches important at LHC
- In MSSM SUSY particles are pair produced
- Main production: squark and gluino pairs
- Strong exclusion bounds on masses



[Plehn, Prospino 2.1]

- First 13 TeV limits: $$m_{\tilde{g}} \geq 1.75\text{TeV}, m_{\tilde{q}} \geq 1.26\text{TeV}$$

- Hadronic processes: $$PP \to \tilde{s}\tilde{s}'X \qquad \tilde{s}, \tilde{s}' = \text{squarks, gluinos}$$

- Partonic processes:
$$\begin{aligned} gg,\, q_i\bar{q}_j &\to \tilde{q}\bar{\tilde{q}} \\ q_i q_j &\to \tilde{q}\tilde{q} \qquad \bar{q}_i\bar{q}_j \to \bar{\tilde{q}}\bar{\tilde{q}} \\ g q_i &\to \tilde{g}\tilde{q} \qquad g\bar{q}_i \to \tilde{g}\bar{\tilde{q}} \\ gg,\, q_i\bar{q}_i &\to \tilde{g}\tilde{g} \end{aligned}$$

- We only consider degenerate squark masses and also not stops

LO cross sections

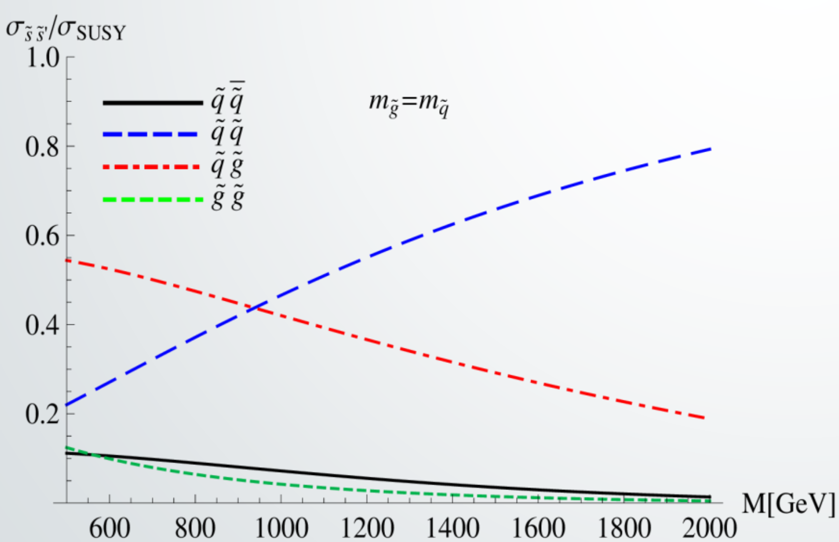
- Squark and gluino pair production processes:

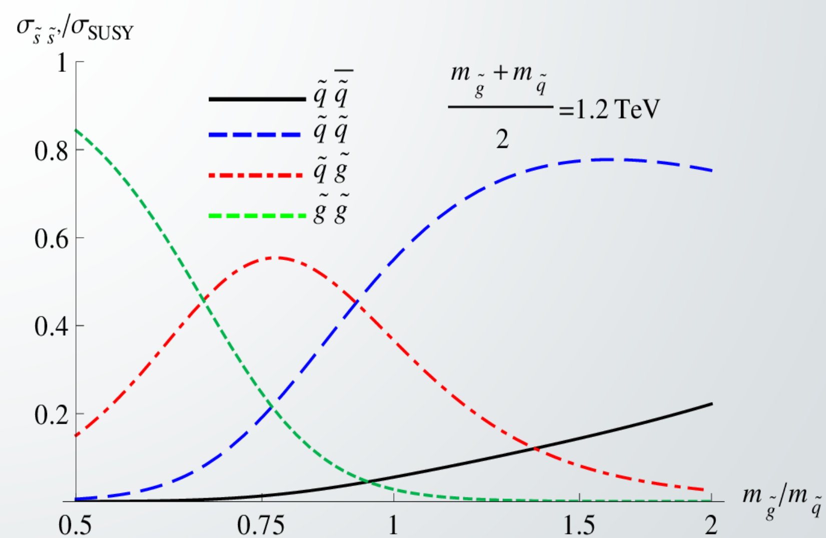
$$gg,\, q\bar{q} \to \tilde{q}\bar{\tilde{q}} \qquad qq \to \tilde{q}\tilde{q} \qquad gq \to \tilde{g}\tilde{q} \qquad gg,\, q\bar{q} \to \tilde{g}\tilde{g}$$

Analytic LO calculations [Kane, Leveille '82; Harisson, Smith '83; Dawson, Eichten, Quigg '85]

- Relative contributions to LO cross sections

$$M = \frac{m_{\tilde{s}} + m_{\tilde{s}'}}{2}$$

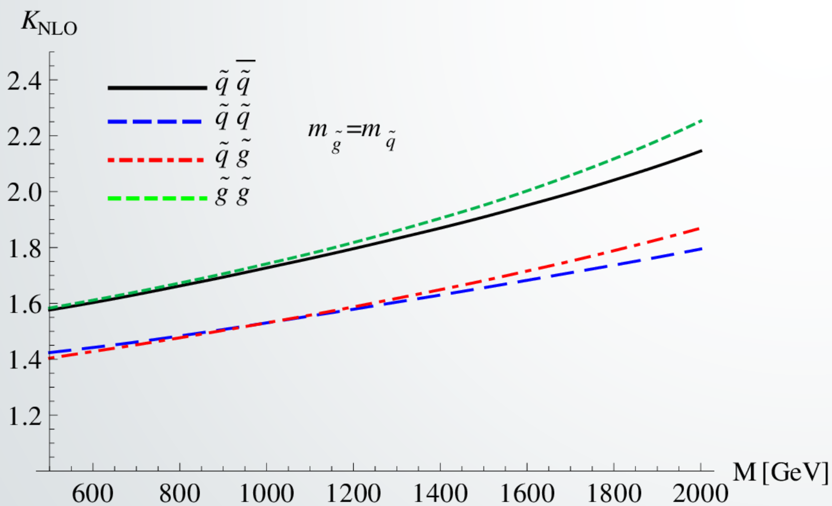


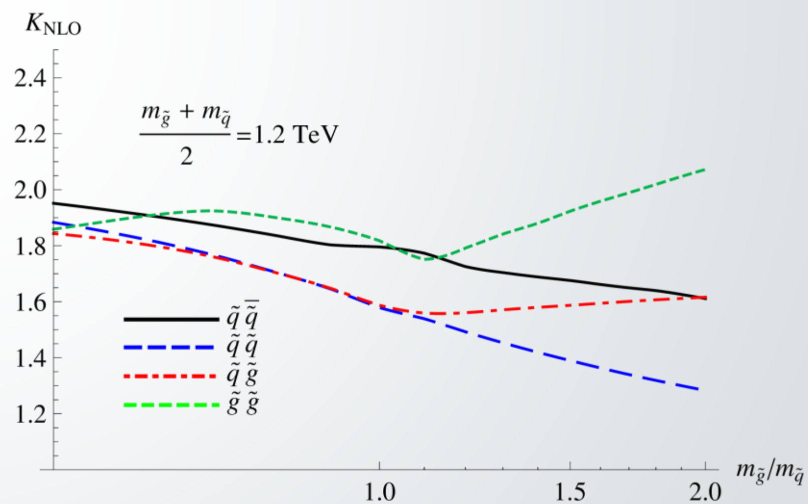


NLO cross sections

- NLO SUSY QCD: [Beenakker et al. ’97, PROSPINO]
  including PS matching: [Gavin et al. ’13, Degrande et al. ’16]
- NLO squark-squark production and decay: [Hollik et al.’13, Gavin et al. ’14]
- EW corrections: [Bornhauser et al. ’07, Germer, Hollik et al. ’08-’11, Hollik et al. ’15]

$gg,\, q\bar{q} \to \tilde{q}\bar{\tilde{q}}$ $\quad qq \to \tilde{q}\tilde{q}$ $\quad gq \to \tilde{g}\tilde{q}$ $\quad gg,\, q\bar{q} \to \tilde{g}\tilde{g}$

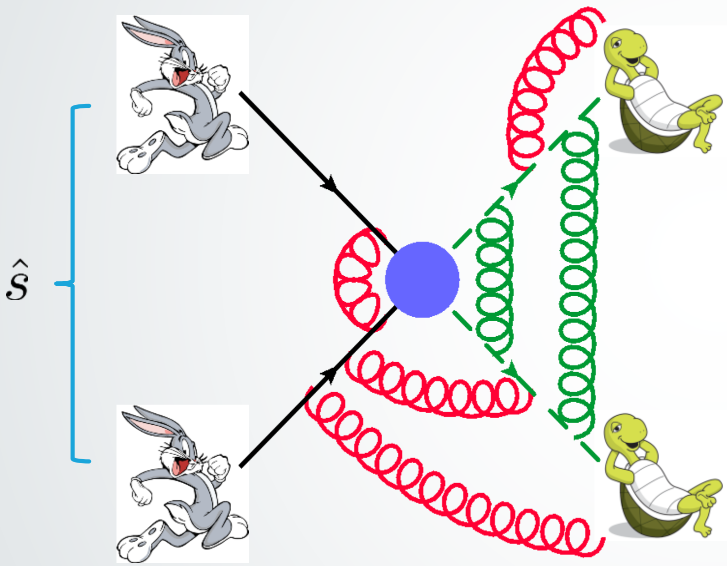




- Increasingly large corrections with increasing average mass *M*
- Heavy pairs $s \gtrsim 2M := m_{\tilde{s}} + m_{\tilde{s}'}$ $\longrightarrow$ close to threshold

Threshold

- Partonic processes: $pp' \to \tilde{s}\tilde{s}' X$ $\qquad p, p' = \text{partons}$, $\tilde{s}, \tilde{s}' = \text{squarks,gluinos}$
- Threshold region: $\beta := \sqrt{1 - \frac{(2M)^2}{\hat{s}}} \ll 1, \quad M := \frac{m_{\tilde{s}} + m_{\tilde{s}'}}{2}, \quad \hat{s} = \tau s = \text{partonic cm energy}$



Relevant modes at threshold:

Collinear: $k_- \sim M, k_+ \sim M\beta^2, k_\perp \sim M\beta$

Hard: $k \sim M$

Soft gluons: $k_0 \sim |k| \sim M\beta^2 \longrightarrow \alpha_s^n \ln^m \beta$

Potential (gluons): $k_0 \sim M\beta^2, |k| \sim M\beta \longrightarrow (\alpha_s/\beta)^n$

[Catani et al. ’96; Becher, Neubert ’06; Kulesza, Motyka ’08; …]

[Fadin, Khoze ’87-’89; Fadin et al. ’90; Kulesza, Motyka ’09; …]

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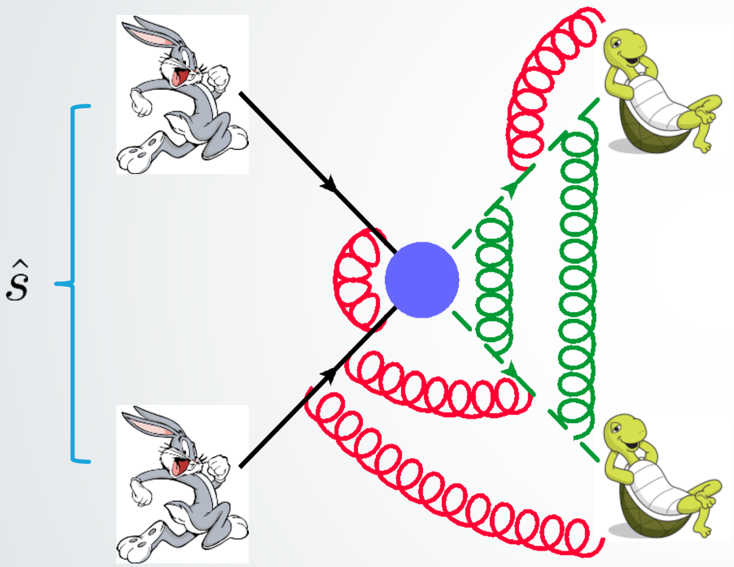

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- Partonic cs enhanced near threshold by soft and coulomb corrections $\longrightarrow$ need to resum
- Threshold enhanced terms also approximate well away from threshold

$$\alpha_s \ln \beta, \left(\frac{\alpha_s}{\beta}\right) \sim 1 \longrightarrow \hat{\sigma}_{pp'}(\hat{s}) \sim \hat{\sigma}^{(0)}_{pp'} \sum_{k=0} \left(\frac{\alpha_s}{\beta}\right)^k \exp[\underbrace{\ln \beta g_0(\alpha_s \ln \beta)}_{\text{(LL)}} + \underbrace{g_1(\alpha_s \ln \beta)}_{\text{(NLL)}} + \underbrace{\alpha_s g_2(\alpha_s \ln \beta)}_{\text{(NNLL)}} + \ldots]$$
$$\times \{1(\text{LL,NLL}); \alpha_s, \beta(\text{NNLL}); \alpha_s^2, \alpha_s\beta, \beta^2(\text{NNNLL}); \ldots\}$$

Some history

$$\hat{\sigma}_{pp'}(\hat{s}) \sim \hat{\sigma}^{(0)}_{pp'} \sum_{k=0} \left(\frac{\alpha_s}{\beta}\right)^k \exp[\underbrace{\ln\beta g_0(\alpha_s \ln\beta)}_{\text{(LL)}} + \underbrace{g_1(\alpha_s \ln\beta)}_{\text{(NLL)}} + \underbrace{\alpha_s g_2(\alpha_s \ln\beta)}_{\text{(NNLL)}} + \ldots]$$

$$\times \{1(\text{LL,NLL}); \alpha_s, \beta(\text{NNLL}); \alpha_s^2, \alpha_s\beta, \beta^2(\text{NNNLL}); \ldots\}$$

- (N)NLL soft resummation [Beenakker et al. '09-'14, NLLFast; Broggio et al. '13]
- Coulomb resummation [Hagiwara/Yokoya '09, Kauth et al. '11]
- Combined soft and Coulomb resummation [Beneke, Falgari, Schwinn '09-'10]
  - ttbar production at NNLL [Beneke, Falgari, Klein, Schwinn '11]
  - Squark and gluino pair production:

    NLL [Falgari, Schwinn, W '12]

    NNLL *with annihilation contribution* [Beneke, Piclum, Schwinn, W '16]

    NNLL in Mellin-space [Beenakker et al. '16]

Factorization using EFT

[Beneke, Falgari, Schwinn '10]

- Hierarchy in scales: $M \gg M\beta \gg M\beta^2$ $\longrightarrow$ use EFT
- Effective lagrangian: $\mathcal{L}_{EFT} = \underbrace{\mathcal{L}_{SCET}}_{\text{Collinear-soft}} + \underbrace{\mathcal{L}_{PNRQCD}}_{\text{Potential-soft}}$
- Field redefinitions: soft gluons decouple from collinear and potential modes at LO in β $\longrightarrow$

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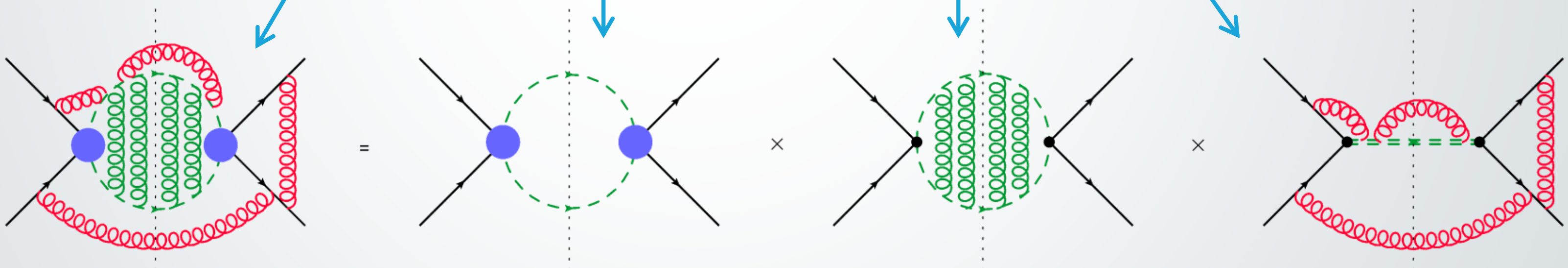
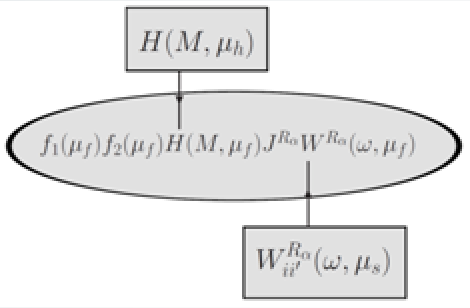
$$\hat{\sigma}_{pp'}(\hat{s},\mu_f) = \sum_{R_\alpha} H^{R_\alpha}_{pp'}(m_{\tilde{q}}, m_{\tilde{g}}, \mu_f) \int d\omega\, J_{R_\alpha}(E - \frac{\omega}{2})\, W^{R_\alpha}(\omega, \mu_f)$$



Partonic cross section | Hard contributions | Coulomb contributions (potential gluons exchange) | Soft contributions (soft gluons exchange)

- Coulomb contributions also contain bound-state effects below threshold
- Factorization valid up to NNLL for S-wave processes and for P-wave processes [Falgari et al. '12]
- H and W satisfy evolution equations $\longrightarrow$ choose scales to minimize higher order corrections

Resummation using RG flow

$\sigma(s) \sim$ 

$\mu_h = 2 k_h M$

- RG flow resums logs $\ln(\frac{\mu_f}{\mu_h}), \ln(\frac{\mu_f}{\mu_s})$

$\mu_f = \mu_R = k_f M, \quad \mu_C = k_C \mathrm{Max}\{2\alpha_s(\mu_C) m_r |D_{R_\alpha}|, 2\sqrt{2 m_r M}\beta\}$

$\mu_s = k_s \frac{2 M e^{-\gamma_E}}{s^{(1)}_{pp'}}, \quad s^{(1)}_{pp'} = -\frac{\mathrm{d}\ln(L_{pp'}(\tau,\mu))}{\mathrm{d}\ln(\tau)}|_{\tau = 4M^2/s}$

Similar results if $\mu_s \sim M\beta^2$

$k_f, k_h, k_C, k_s \sim 1$

- One-loop SUSY hard matching coefficients computed by [Kulesza et al. ’11-’13, Kauth et al. ’11, Langenfeld ’12] $\longrightarrow$ NNLL ingredients known

Resummation using RG flow

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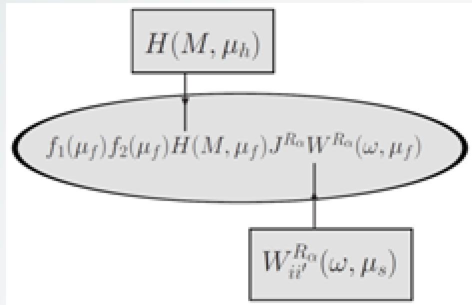

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- One-loop SUSY hard matching coefficients computed by [Kulesza et al. '11-'13, Kauth et al. '11, Langenfeld '12] $\longrightarrow$ NNLL ingredients known
- Use PDF4LHC15 NNLO_30 PDF's and match the c.s. to an approximation of the NNLO result:

$$\mathrm{NNLL}^{\mathrm{matched}} = [\mathrm{NNLL} - \mathrm{NNLL}(\alpha_s^{0,1,2})] + \mathrm{NNLO}_{\mathrm{app}}$$

$$\mathrm{NNLO}_{\mathrm{app}} = \mathrm{NLO}_{\mathrm{Prospino}} + \mathrm{NNLL}(\alpha_s^2)_{\mathrm{soft+Coulomb\ terms}}$$

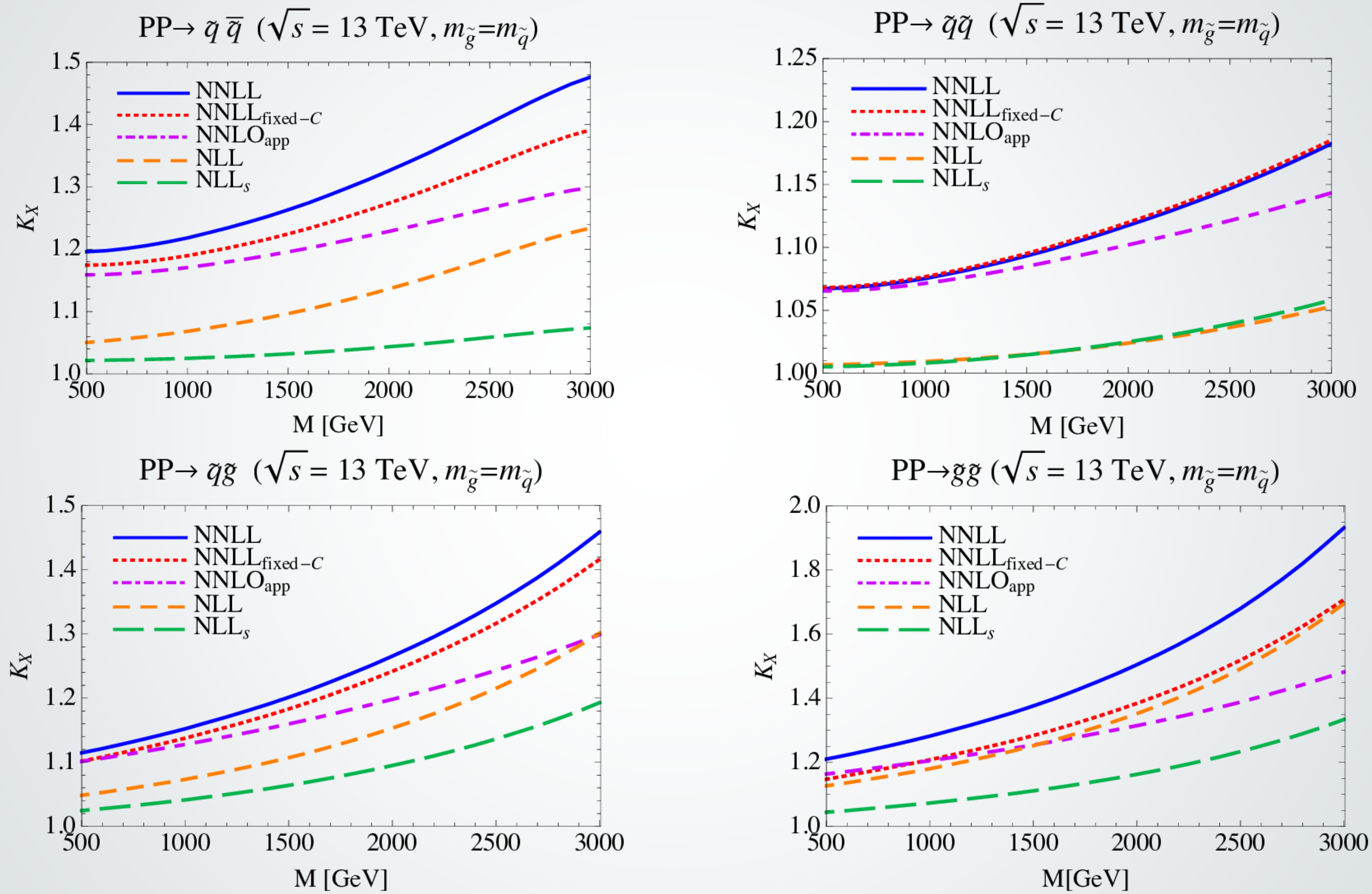
- Theoretical errors:

1) Scale variations: $\frac{1}{2} \le k_f, k_h, k_C, k_s \le 2$
2) Parameterization errors: $E = \sqrt{\hat{s}} - 2M, \quad M\beta^2$
3) NNLO approximation uncertainty
4) PDF + α_s uncertainty

Next we present our results resumming both soft and Coulomb terms at NNLL

$$K_{\mathrm{NNLL}} = \frac{\sigma^{\mathrm{matched}}}{\sigma^{\mathrm{NLO}}}$$

[Beneke, Piclum, Schwinn, W '16]

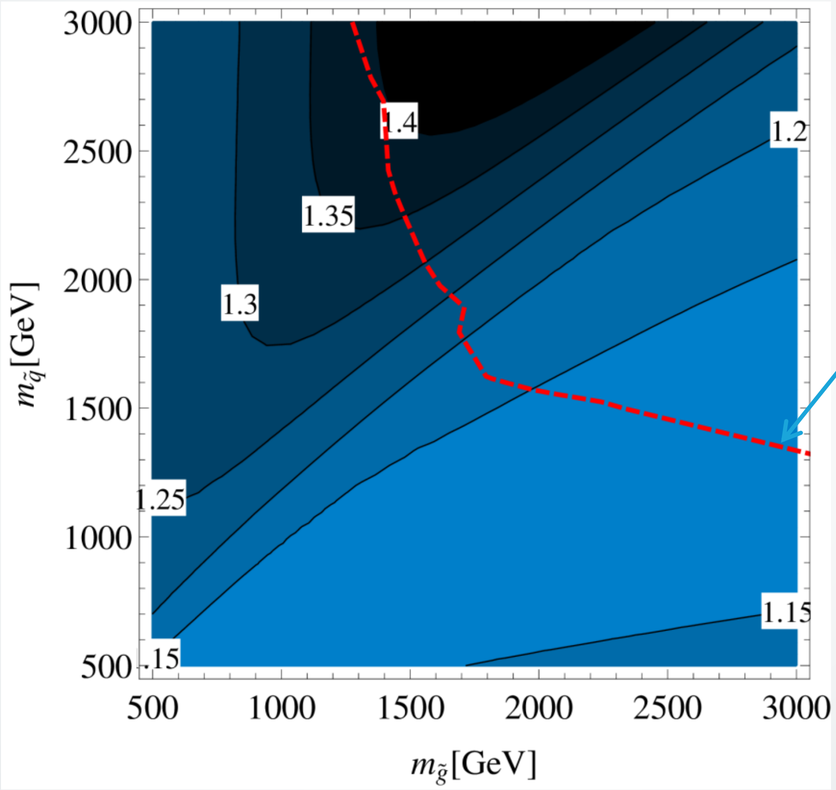


- Large NNLL corrections: 10-90% of NLO
- Corrections on top of NLL: 10-20%
- Corrections beyond NNLO: 0-50%

Contour plot K_{NNLL}

[Beneke, Piclum, Schwinn, W '16]

$$PP \to \tilde{q}\bar{\tilde{q}} + \tilde{q}\tilde{q} + \tilde{q}\tilde{g} + \tilde{g}\tilde{g} \; (\sqrt{s} = 13 \text{ TeV})$$



ATLAS 8 TeV exclusion bound in massless neutralino model

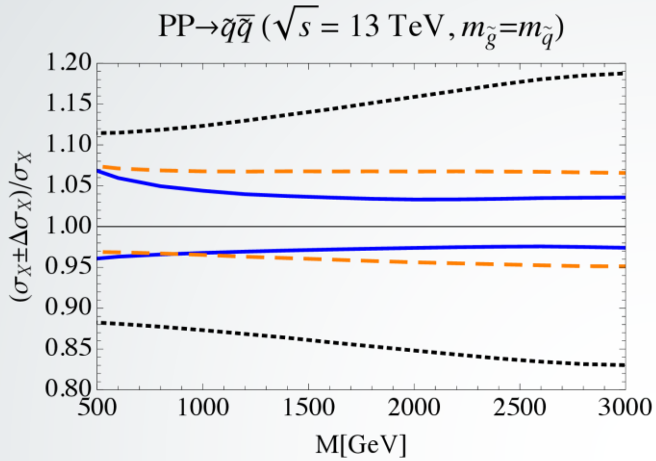
- Public NNLL grids:

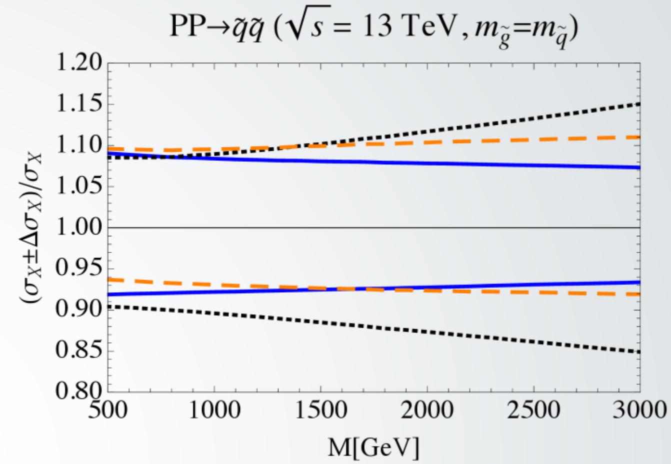
[http://users.ph.tum.de/t31software/SUSYNNLL/]

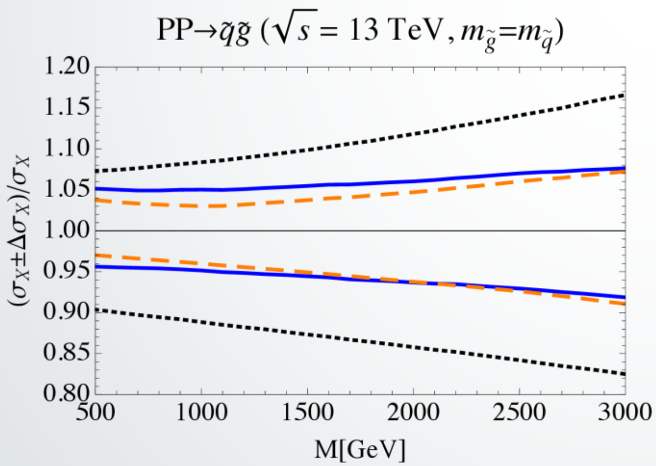
- Corrections can become as large as 40%, if squark mass is larger than gluino mass
- Exclusion bound goes through large K_{NNLL} regions

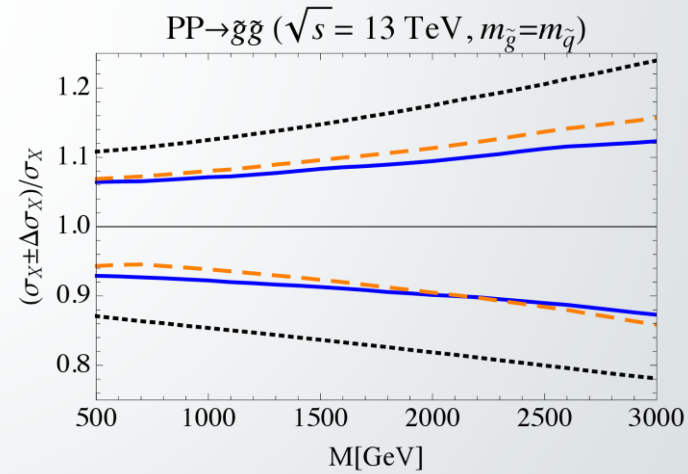
Uncertainties

[Beneke, Piclum, Schwinn, W '16]









Scale and parameterization errors of:

- Blue: NNLL resummation
- Orange: NLL resummation
- Black: NLO fixed order calculation to α_s^3

- NLL corrections reduce NLO errors to $\pm$5-16%
- NNLL corrections reduce further to $\pm$4-12%
- Public code available to reproduce NNLL results

Summary

- NNLL corrections for the SUSY processes can be as large as 10-90%
- From NLL to NNLL: errors reduced from $\pm$5-16% to $\pm$4-12%
- Coulomb corrections can be as large as soft corrections $\longrightarrow$ need to resum them
- Corrections need to be taken into account for setting more accurate squark-gluino mass (bounds) [Kulesza et al. '11]
- Public squark and gluino NNLL grids:

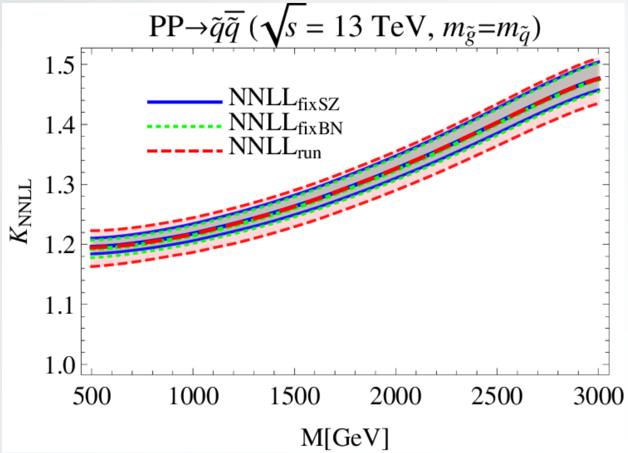
http://users.ph.tum.de/t31software/SUSYNNLL/

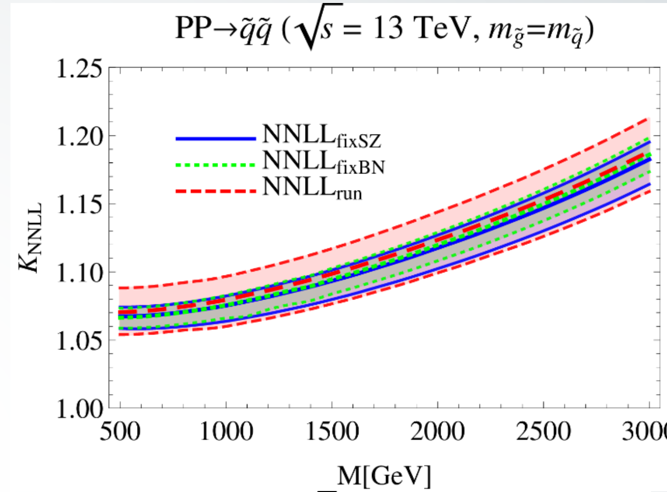
Outlook

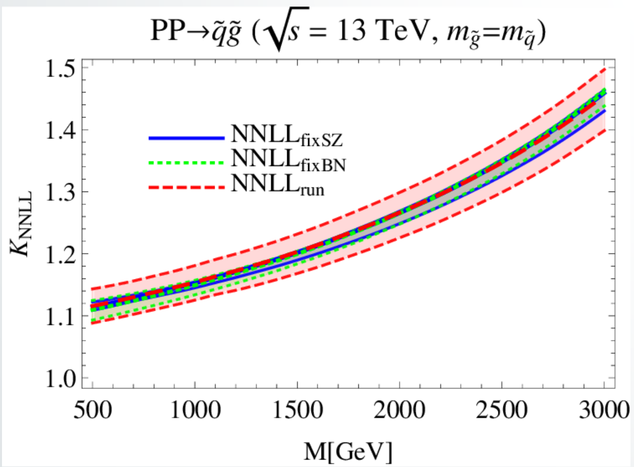
- Detailed comparison with Mellin results [Beenakker et al. '16]
- Extend results to non-degenerate squark masses

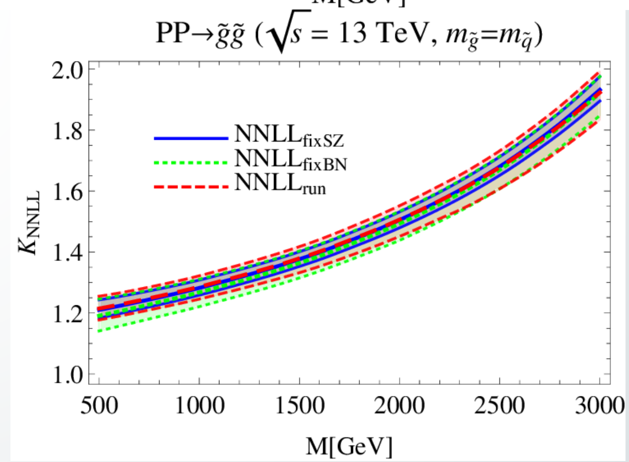
Backup slides

Soft scale implementation



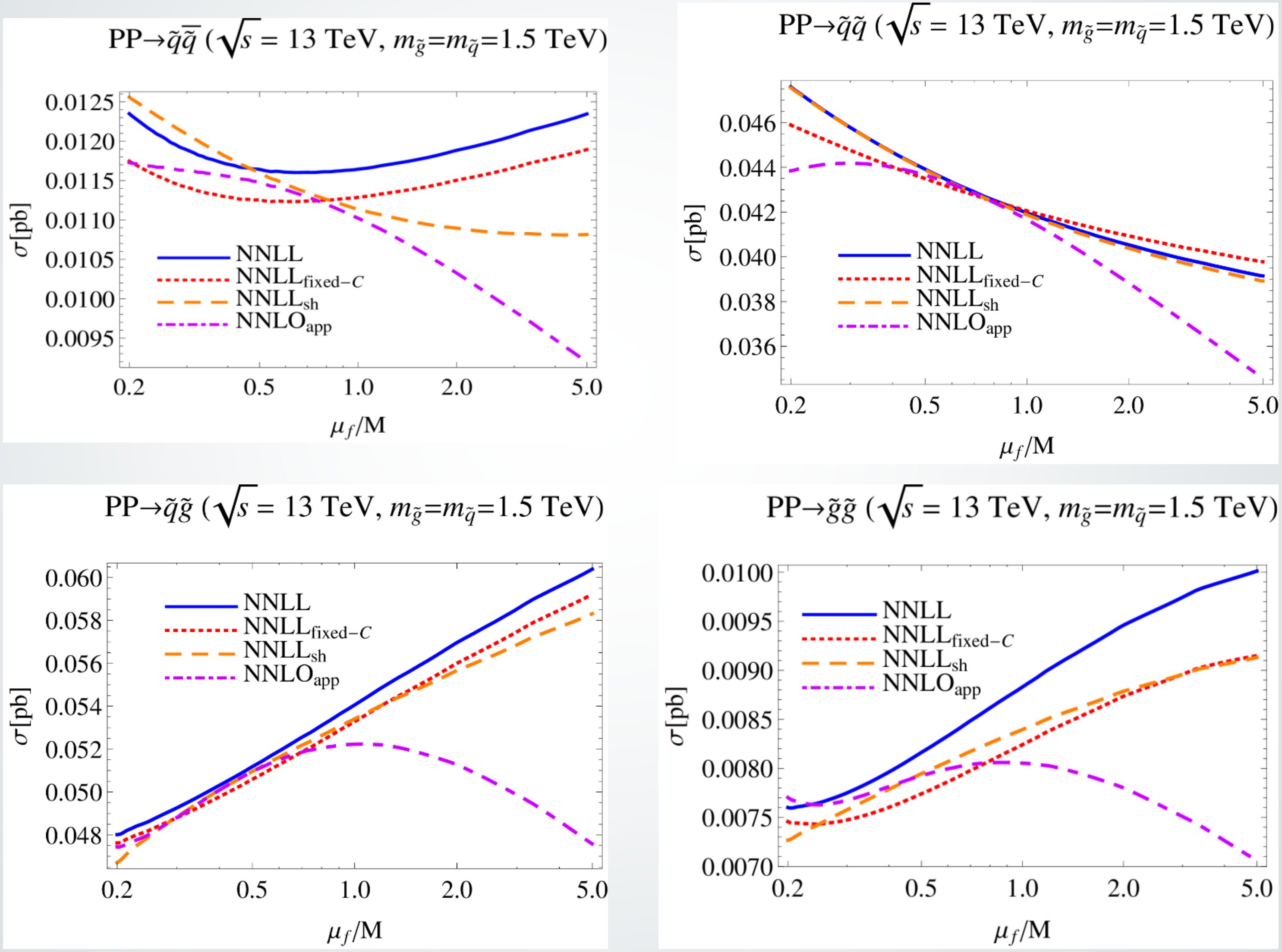






- Overlapping error bands
- Central values fall within error bands of each implementation
- NNLL vs. NLL: difference between soft scale implementation less relevant at NNLL

Factorization scale dependence



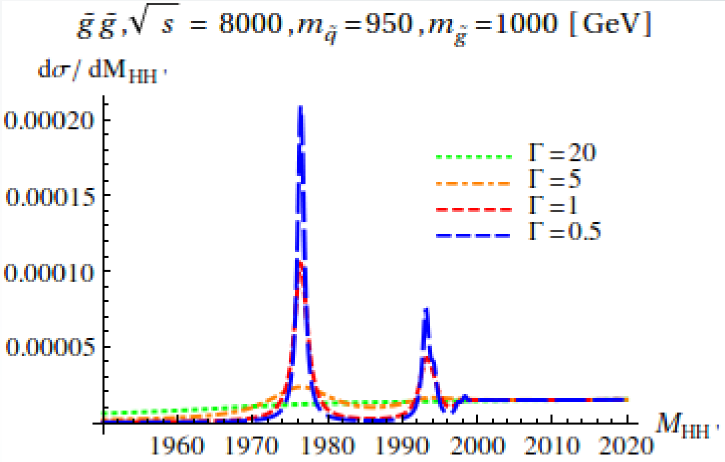
Finite width

[Falgari, Schwinn, W '12]

- Squarks and gluinos decay

- Finite width taken into account by:

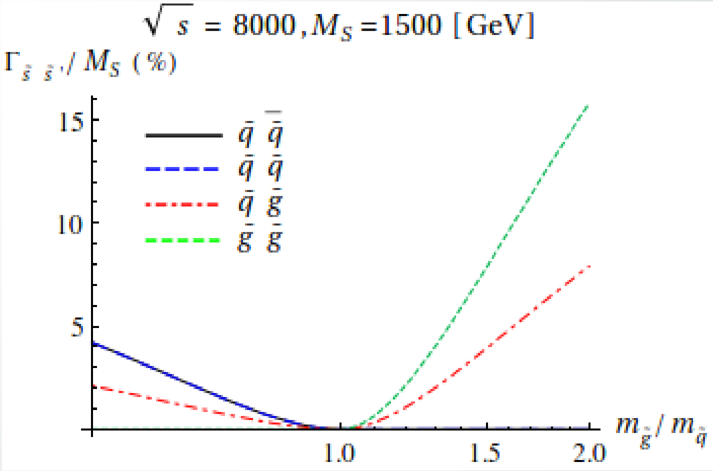
$$E \to E + i\Gamma_{\tilde{s}\tilde{s}'}$$
$$\Gamma_{\tilde{s}\tilde{s}'} = (\Gamma_{\tilde{s}} + \Gamma_{\tilde{s}'})/2$$



Bound state peaks smeared out

Soft logs: $\alpha_s^n \ln^m \beta \to \alpha_s^n \ln^m (\beta^4 + (\Gamma/M)^2)^{1/4}$

Coulomb: $(\alpha_s/\beta)^n \to (\alpha_s/(\beta^4 + (\Gamma/M)^2)^{1/4})^n$



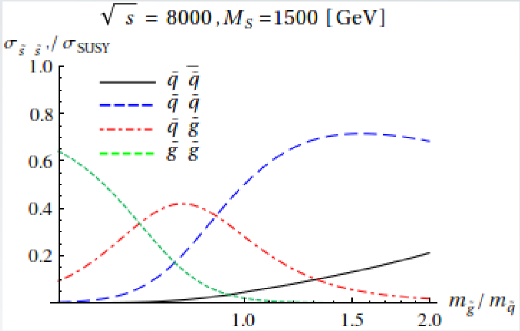
- SQCD widths are most often dominant

- Q: How much does the width effect our previous results?

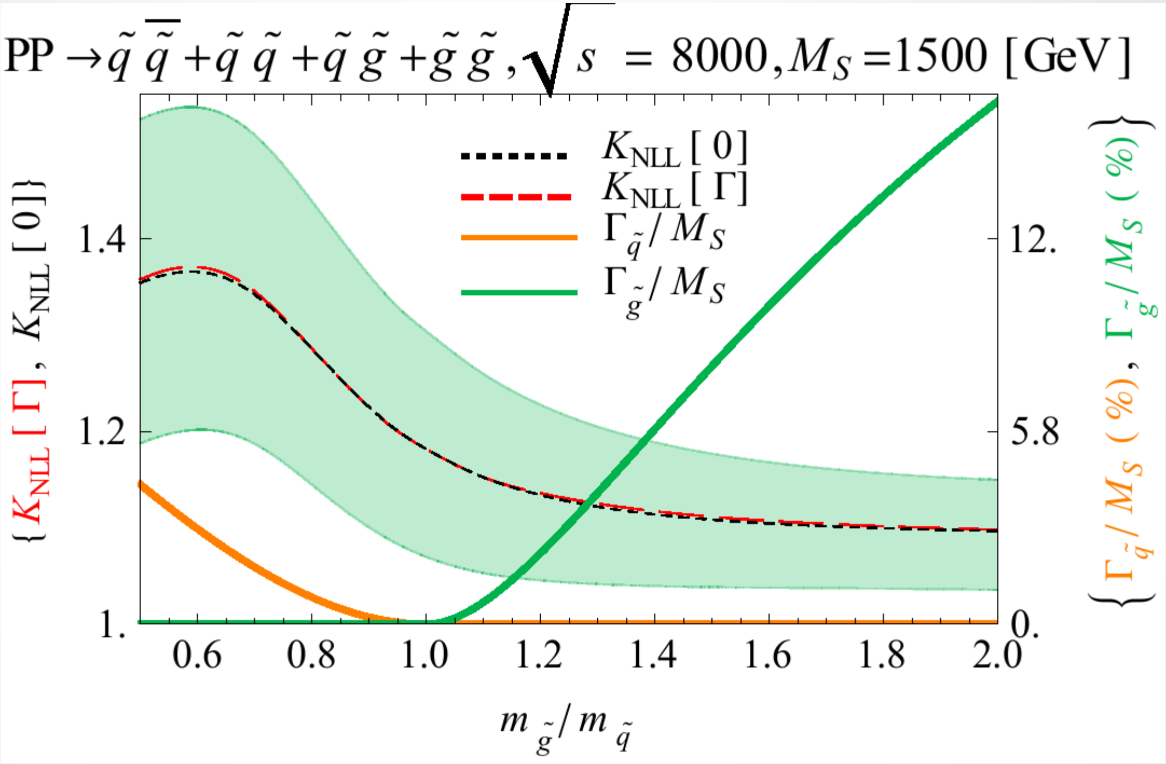
Backup

15

$$K_{\mathrm{NLL}}[\Gamma] = \frac{\sigma^{\mathrm{matched}}[\Gamma]}{\sigma^{\mathrm{NLO}}[\Gamma]}$$



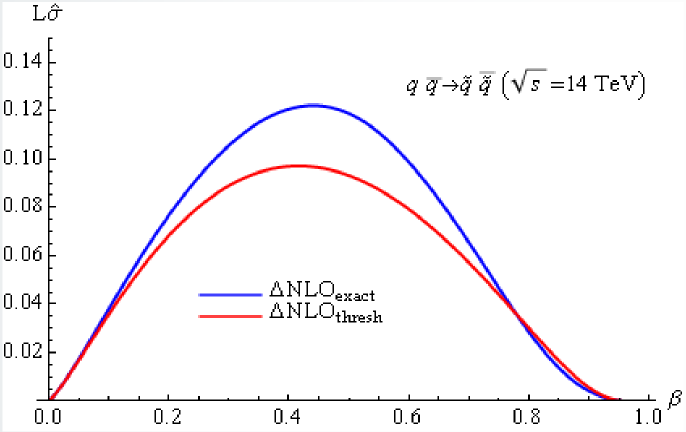
- Experimentally relevant total SUSY production:

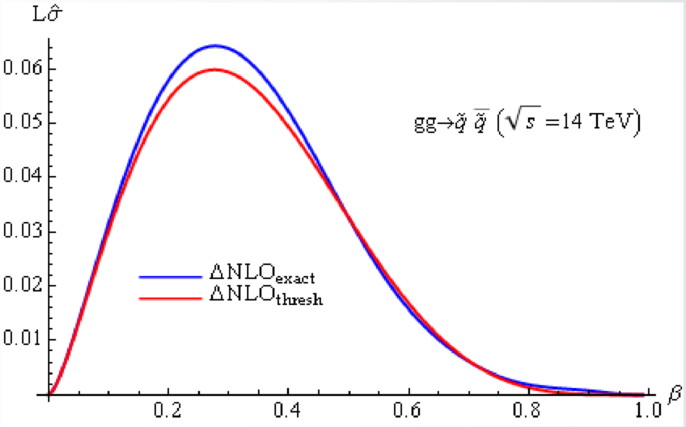


- Green band: total error for zero width
- Negligible difference → BS contributions correctly included by delta peaks
- Width can be neglected for the total SUSY process

Need for resummation

$$\sigma_{PP\to\tilde{s}\tilde{s}'X}(s) = \int_0^{\beta_1} d\beta \sum_{p,p'=q,\bar{q},g} \boxed{\left(\frac{\partial\tau}{\partial\beta}\right) L_{pp'}(\tau,\mu_f)\hat{\sigma}_{pp'\to\tilde{s}\tilde{s}'X}(\tau s,\mu_f)}$$





- Sizeable contribution from small β region $\longrightarrow$ need to resum at threshold
- Threshold enhanced terms also approximate well away from threshold

• Soft logarithms resummation [Catani et al. ’96; Becher, Neubert ’06; Kulesza, Motyka ’08; Langenfeld, Moch ’09; Beenakker et al. ’09]
• Coulomb resummation [Fadin, Khoze ’87-’89; Fadin, Khoze, Sjostrand ’90; Kulesza, Motyka ’09]
• Simultaneous soft and Coulomb resummation for squark-antisquark at NLL [Beneke, Falgari, Schwinn ’10] and top-quark pairs at NNLL [Beneke et al. ’11]

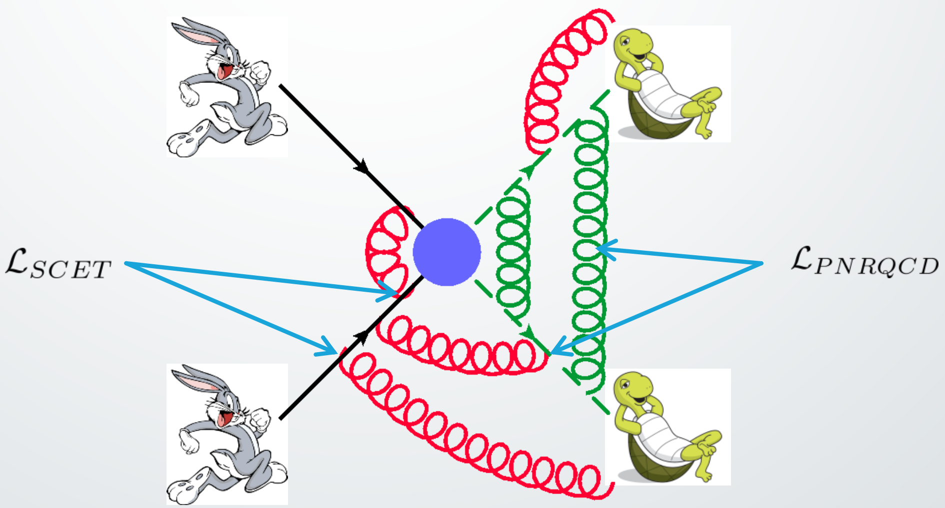
Effective lagrangian

- Effective lagrangian: $\mathcal{L}_{EFT} = \mathcal{L}_{SCET} + \mathcal{L}_{PNRQCD}$

Collinear-soft: $$\mathcal{L}_{SCET} = \bar{\xi}_c\left(in.D + i\not{D}_{\perp c}\frac{1}{i\bar{n}D_c}i\not{D}_{\perp c}\right)\frac{\not{\bar{n}}}{2}\xi_c - \frac{1}{2}tr\left(F_c^{\mu\nu}F^c_{\mu\nu}\right)$$

LO Potential-soft: $$\mathcal{L}_{PNRQCD} = \psi^\dagger\left(iD_s^0 + \frac{\vec{\partial}^2}{2m_{\tilde{s}}} + \frac{i\Gamma_{\tilde{s}}}{2}\right)\psi + \psi'^\dagger\left(iD_s^0 + \frac{\vec{\partial}^2}{2m_{\tilde{s}'}} + \frac{i\Gamma_{\tilde{s}'}}{2}\right)\psi' + \int d^3\vec{r}\,[\psi^\dagger \mathbf{T}^{(R)a}\psi](\vec{r})\left(\frac{\alpha_s}{r}\right)[\psi'^\dagger \mathbf{T}^{(R')a}\psi'](0)$$

$$pp' \to \tilde{s}\tilde{s}' X$$



LO Potential function

- The potential function sums the Coulomb terms: $(\alpha_s/\beta)^n$

  The potential function equals twice the imaginary part of the LO Coulomb Green's function:

$$G_C^{R_\alpha(0)}(0,0;E) = -\frac{(2m_r)^2}{4\pi}\Big\{\sqrt{-\frac{E}{2m_r}} + (-D_{R_\alpha})\alpha_s\Big[\frac{1}{2}\ln(-\frac{8m_rE}{\mu^2}) - \frac{1}{2} + \gamma_E + \psi\Big(1 - \frac{(-D_{R_\alpha})\alpha_s}{2\sqrt{-E/(2m_r)}}\Big)\Big]\Big\}$$

- Potential function J:

$$J_{R_\alpha}(E) = \frac{(2m_r)^2\pi D_{R_\alpha}\alpha_s}{2\pi}\left(e^{\pi D_{R_\alpha}\alpha_s\sqrt{\frac{2m_r}{E}}} - 1\right)^{-1} \qquad E > 0$$

$$J_{R_\alpha}^{\text{bound}}(E) = 2\sum_{n=1}^{\infty}\delta(E - \Big(-\frac{2m_\text{r}\alpha_s^2 D_{R_\alpha}^2}{4n^2}\Big))\left(\frac{2m_\text{r}(-D_{R_\alpha})\alpha_s}{2n}\right)^3 \qquad E < 0$$

  It depends on the Casimir coefficients: $D_{R_\alpha} = \frac{1}{2}(C_{R_\alpha} - C_p - C_{p'})$

NLL resummation formula

- NLL partonic cross section is a sum over the total color representations of final state:

$$\hat{\sigma}^{\rm NLL}_{pp'}(\hat{s},\mu_f) = \sum_{R_\alpha} H^{(0),R_\alpha}_{pp'}(\mu_h)\, U_i(M,\mu_h,\mu_s,\mu_f)\,\frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)} \int_0^\infty d\omega\, \frac{J_{R_\alpha}(M\beta^2-\frac{\omega}{2})}{\omega}\left(\frac{\omega}{2M}\right)^{2\eta}$$

[Beneke, Falgari, Schwinn'10]

Hard function H is determined by Born cross section at threshold: $\hat{\sigma}^{(0),R_\alpha}_{pp'}(\hat{s}) \underset{\hat{s}\to 4M^2}{\approx} \frac{(2m_r)^2}{2\pi}\sqrt{\frac{E}{2m_r}}H^{(0),R_\alpha}_{pp'}$

- The function Ui follows from the evolution equations of H and W:

$$U_i(M,\mu_h,\mu_f,\mu_s) = \left(\frac{4M^2}{\mu_h^2}\right)^{-2a_\Gamma(\mu_h,\mu_s)}\left(\frac{\mu_h^2}{\mu_s^2}\right)^{\eta} \times \exp\Big[4(S(\mu_h,\mu_f)-S(\mu_s,\mu_f))$$
$$-2a_i^V(\mu_h,\mu_s)+2a^{\phi,p}(\mu_s,\mu_f)+2a^{\phi,p'}(\mu_s,\mu_f)\Big]$$

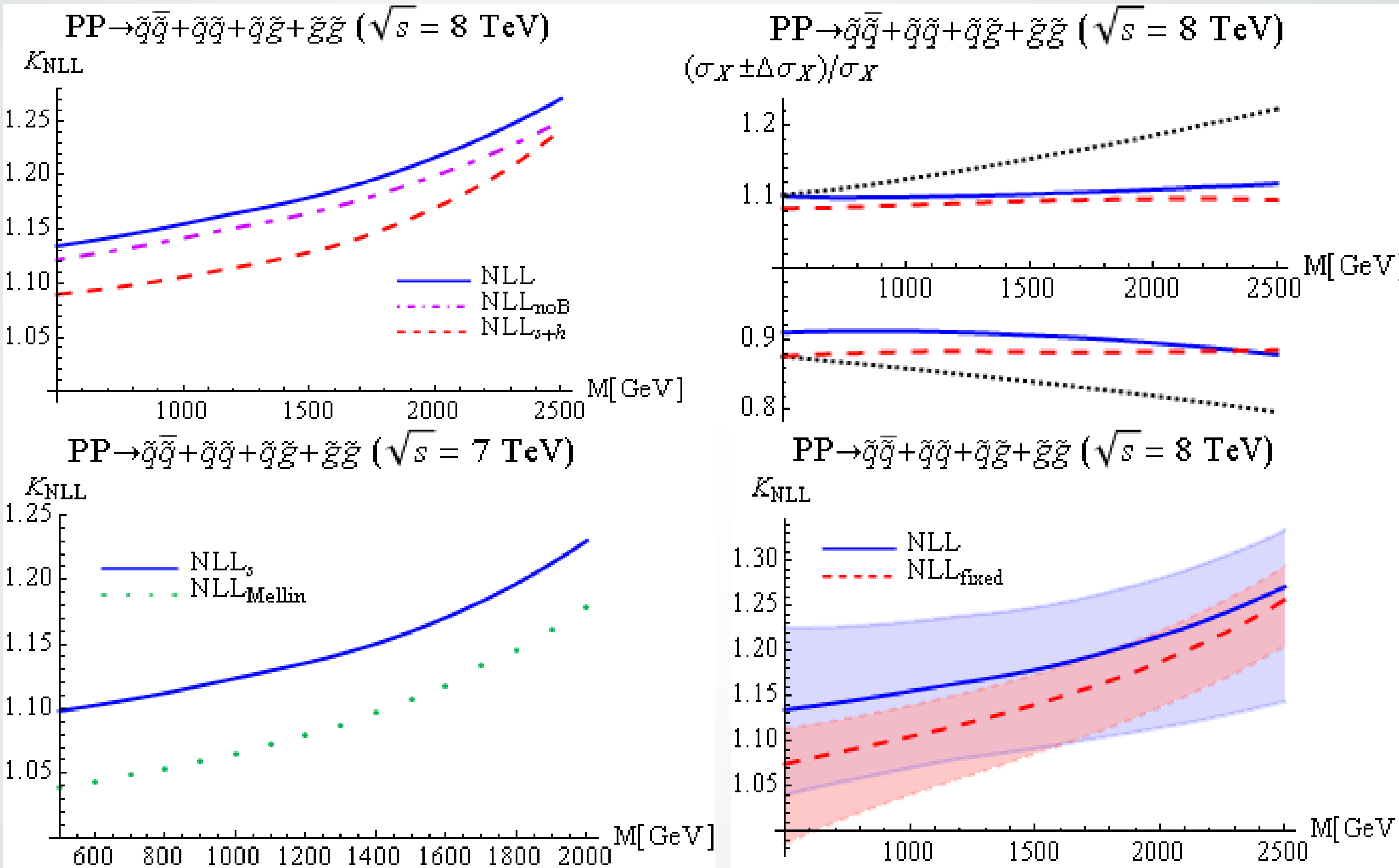
$$S(\mu_i,\mu_j) = \frac{C_p+C_{p'}}{2\beta_0^2}\left[\frac{4\pi}{\alpha_s(\mu_i)}\left(1-\frac{1}{r}-\ln r\right)+\left(2K-\frac{\beta_1}{\beta_0}\right)(1-r+\ln r)+\frac{\beta_1}{2\beta_0}\ln^2 r\right]$$

$$a_\Gamma(\mu_i,\mu_j) = \frac{C_p+C_{p'}}{\beta_0}\ln r\,,\quad a_i^V(\mu_i,\mu_j) = \frac{\gamma_i^{(0),V}}{2\beta_0}\ln r,\qquad a^{\phi,p}(\mu_i,\mu_j) = \frac{\gamma^{(0)\phi,p}}{2\beta_0}\ln r$$

γ's are the one-loop anomalous-dimension coefficients, β's the beta coefficients and C's are the Casimir invariants, while other constants are:

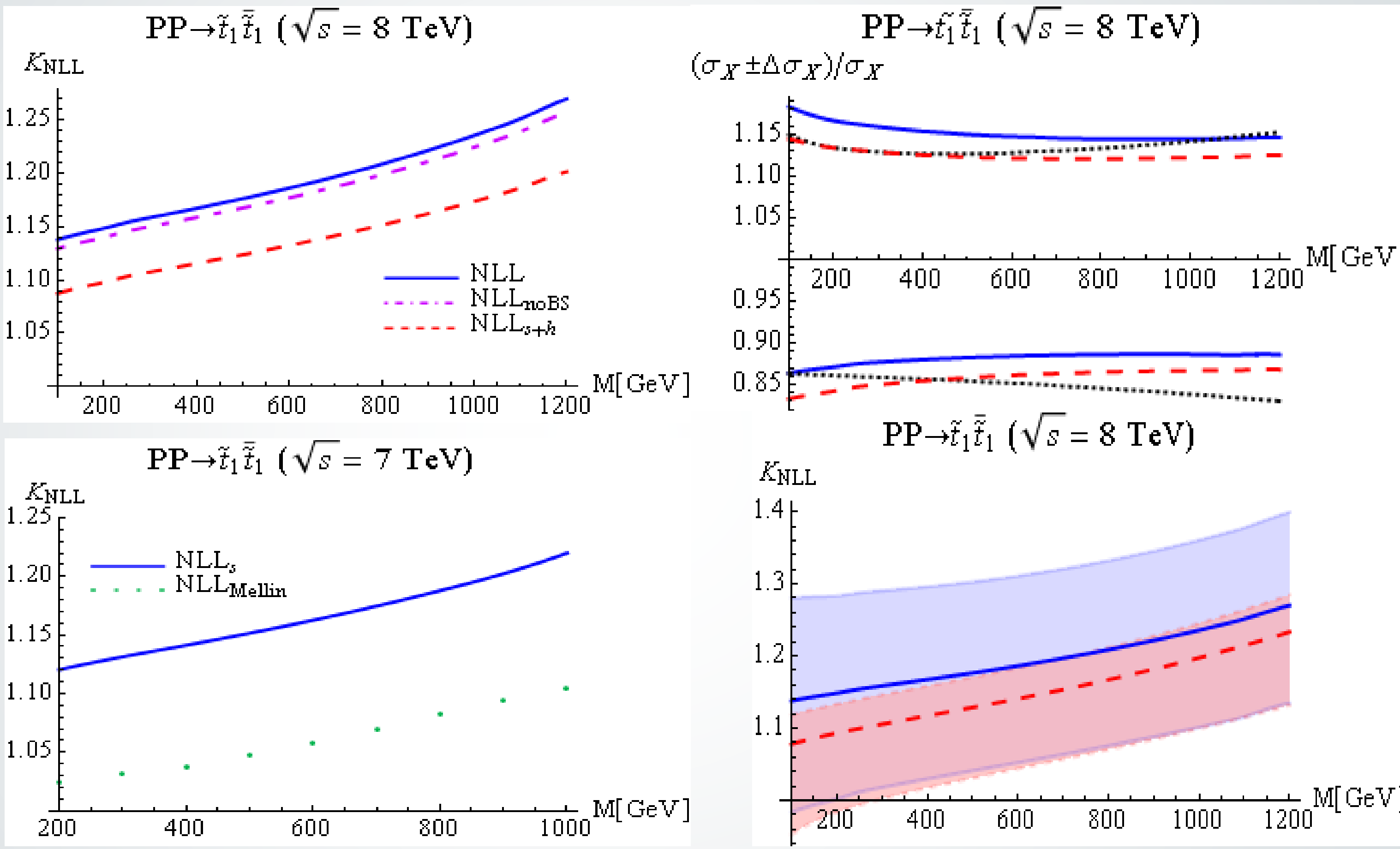
$$\eta = 2a_\Gamma(\mu_s,\mu_f),\quad r = \alpha_s(\mu_j)/\alpha_s(\mu_i),\quad K = \left(\frac{67}{18}-\frac{\pi^2}{6}\right)C_A - \frac{10}{9}T_F n_f$$

Total SUSY cross section



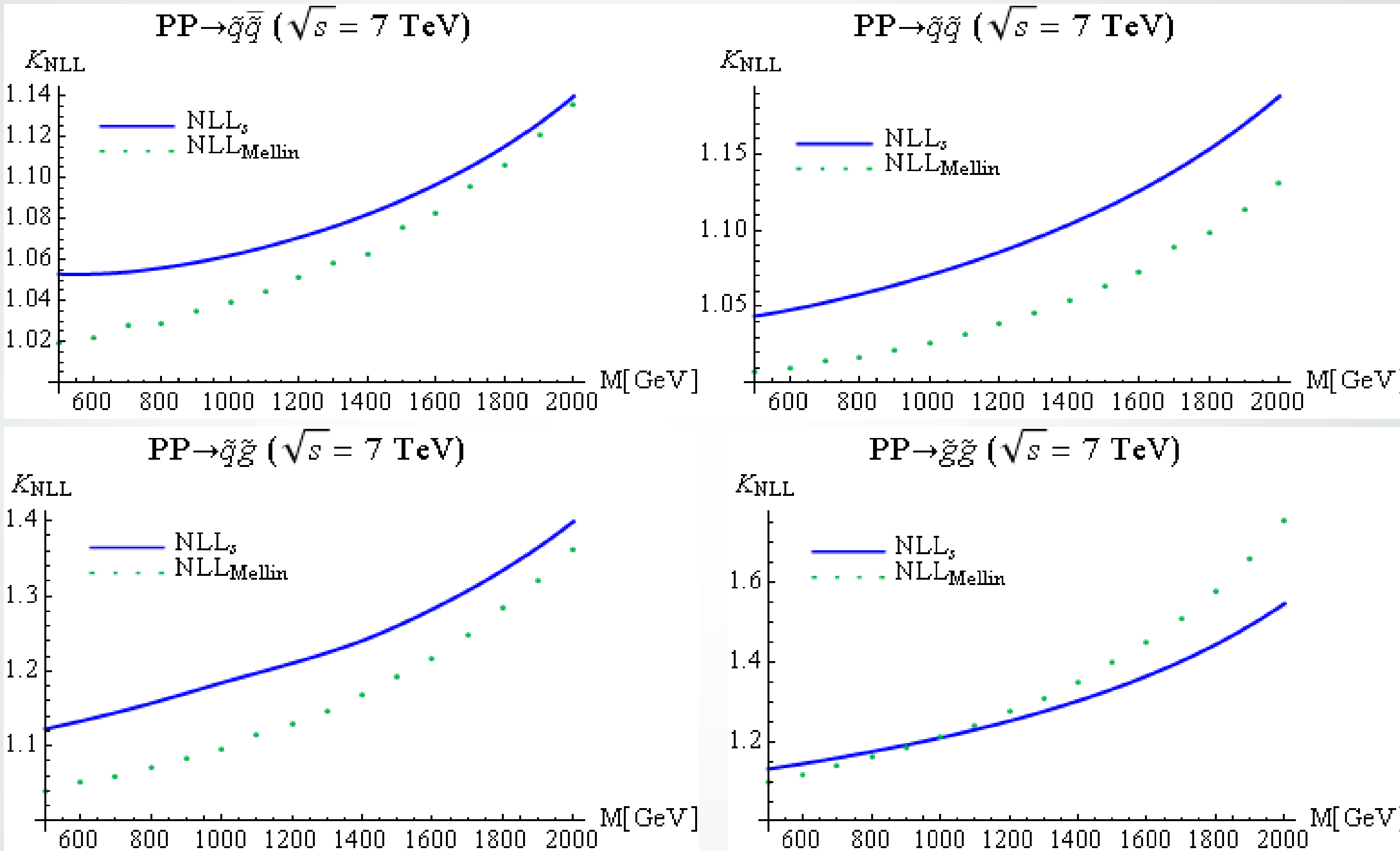
- The corrections are about 15-30%
- Errors reduced to $\pm$10%

Stops



- Q-qbar fusion P-wave suppressed compared to gluon fusion
- Relatively larger soft corrections from gluon fusion than squark-antisquark

NLL Mellin space comparison



- Mellin transf.: $f(x) \to \tilde{f}(N) = \int_0^\infty dx\, f(x) x^{N-1}$

  $N \sim \frac{1}{\beta^2}$ $\longrightarrow$ Threshold: $N \gg 1$
- Resum and transform back to momentum space

- Only soft resummation: $\mathrm{NLL_s}$
- Compare with $\mathrm{NLL_{Mellin}}$ [Kulesza et al. '09, NLLFast]
- Agreement is within error bars