

Photonic crystals, graphene, and new effects in Čerenkov radiation

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Marie Curie IOF
project BSICS

Čerenkov Radiation – Shock Wave of Light

$$v_{source} > v_{sound\ wave}$$

$$v_{particle} > v_{photon}$$

$$\beta c > \frac{c}{n}$$

The Čerenkov threshold: $\beta > \frac{1}{n}$

Radiation cone

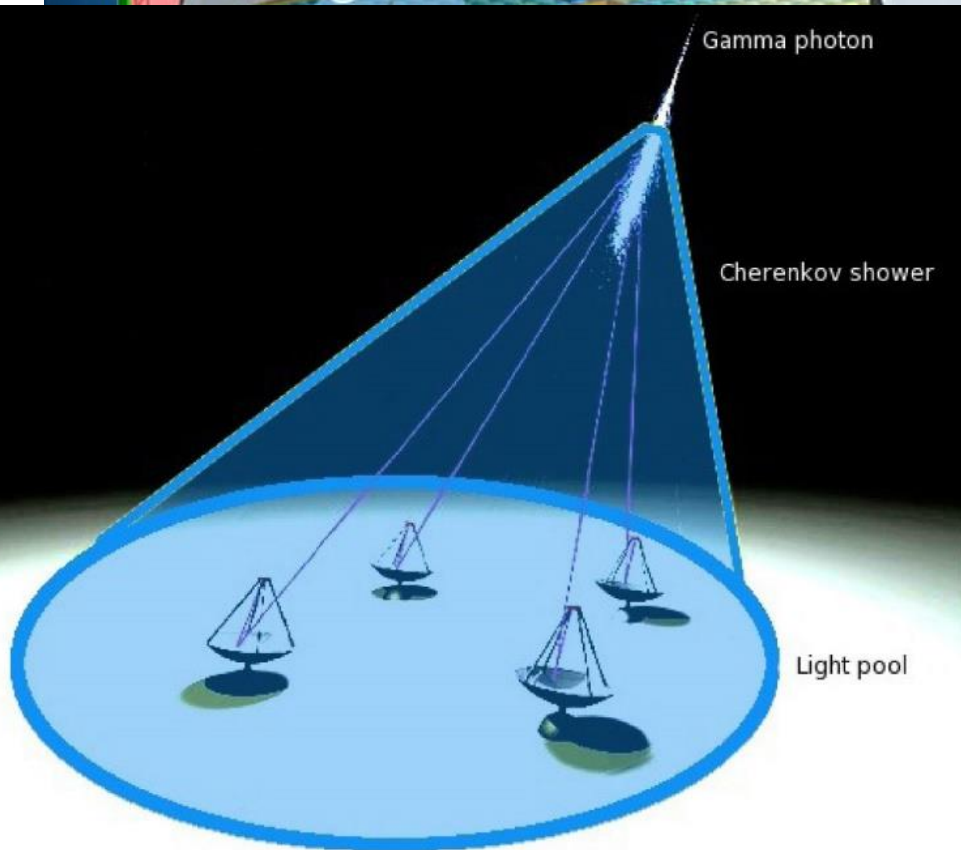
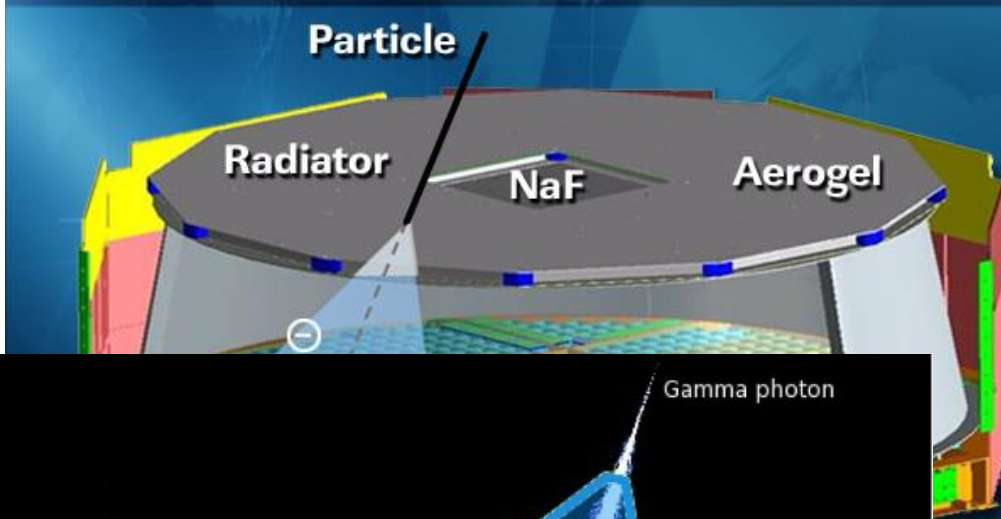
Charged particle

Čerenkov, **Dokl. Akad. Nauk SSSR** 2, 451 (1934)

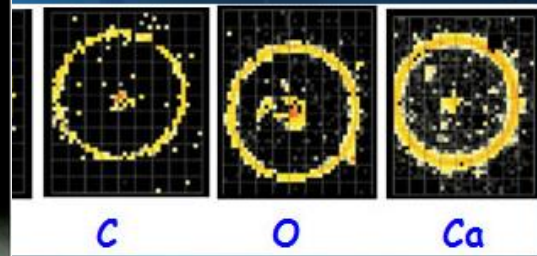
Nobel Prize in Physics 1958



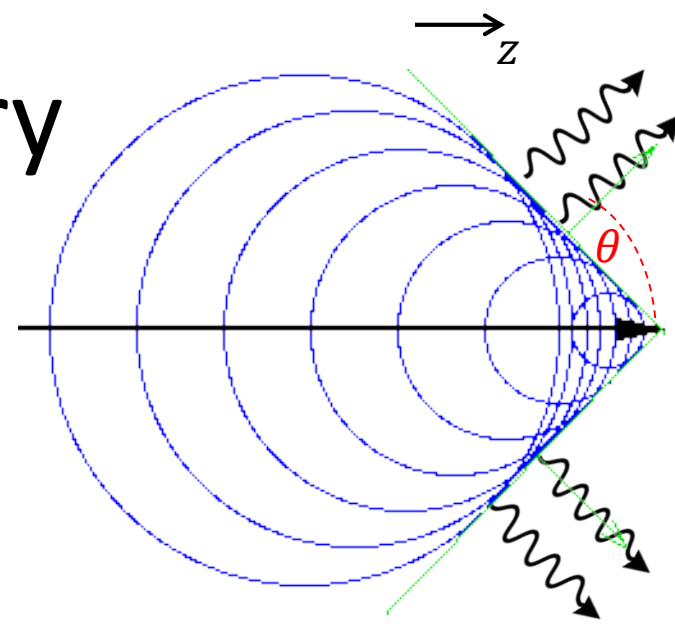
Ring Imaging Cherenkov (RICH)



Images from the Test beam $E=158 \text{ GeV/n}$



The Conventional Theory



$$\nabla^2 \vec{A} - \epsilon \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2} = \mu_0 \vec{J} = \mu_0 q \vec{v} \delta(z - vt) \delta(x) \delta(y)$$

$$k_{\perp}^2 + k_z^2 = \epsilon \frac{\omega^2}{c^2}$$

$$k_z = \frac{\omega}{v} \leftarrow \mu_0 q \vec{v} e^{i\omega \frac{z}{v}} \delta(x) \delta(y)$$

$$k_{\perp} = \pm \sqrt{\epsilon \frac{\omega^2}{c^2} - \frac{\omega^2}{v^2}} = \pm \frac{\omega}{c} \sqrt{n^2 - \frac{1}{\beta^2}}$$

The Čerenkov threshold: $\beta > \frac{1}{n}$

Tamm&Frank, **Dokl. Akad. Nauk SSSR** 14, 109 (1937)

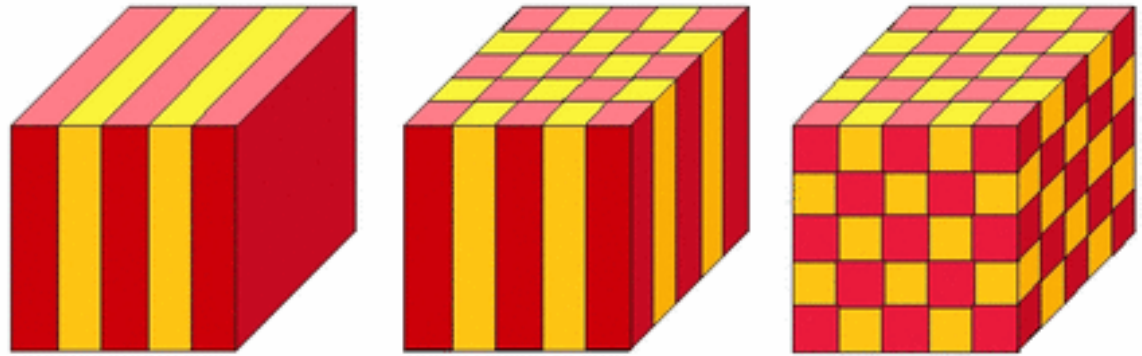
Nobel Prize in Physics 1958

$$\text{The Čerenkov angle: } \cos(\theta) = \frac{1}{\beta n}$$

New materials and new types of matter

Photonic crystals

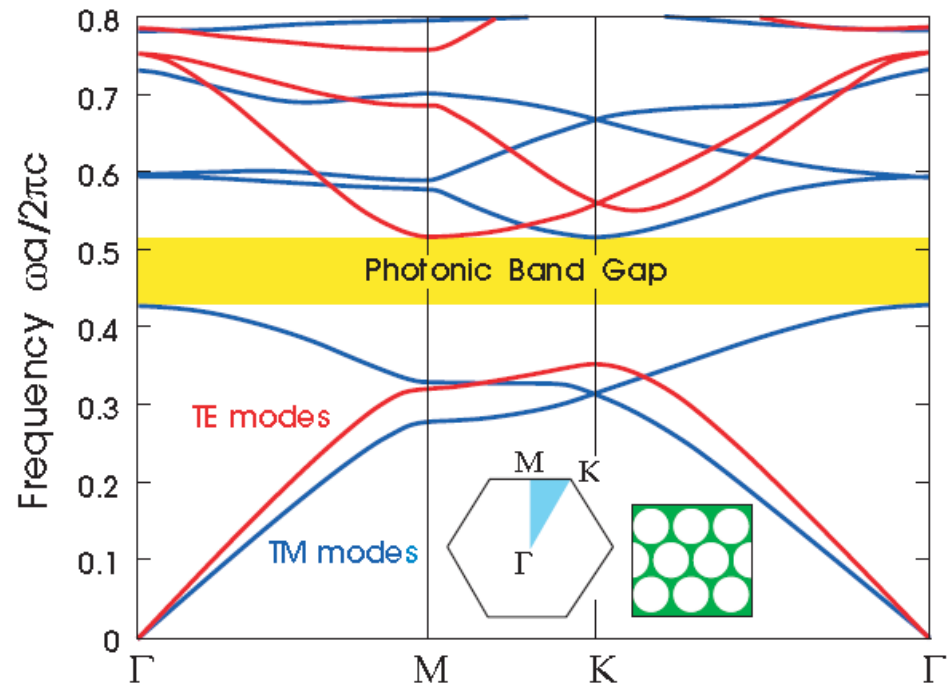
Knapitsch and Lecoq, "Review on photonic crystal coatings for scintillators." **Int. J. Mod. Phys. A** 29, 1430070 (2014).



$$\cancel{k_x^2 + k_z^2 = \epsilon \frac{\omega^2}{c^2}}$$

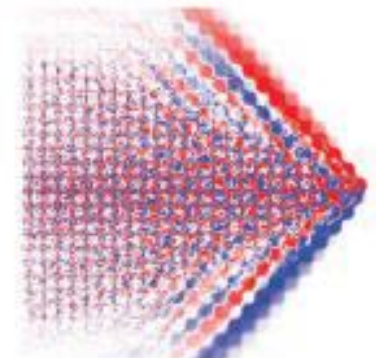
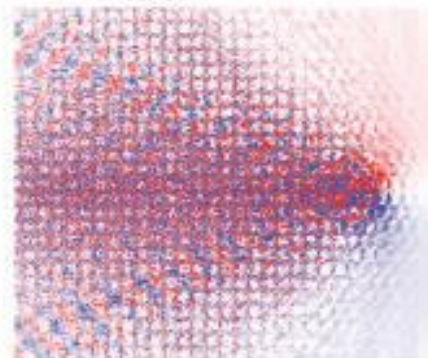
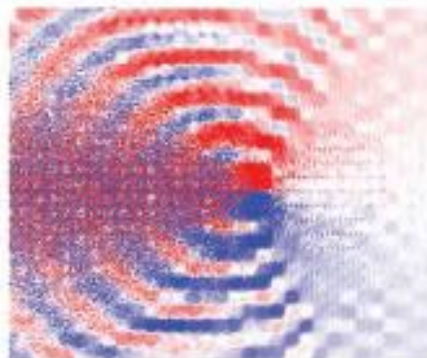
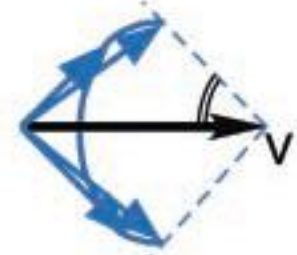
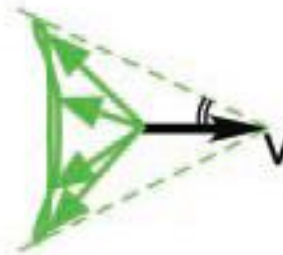
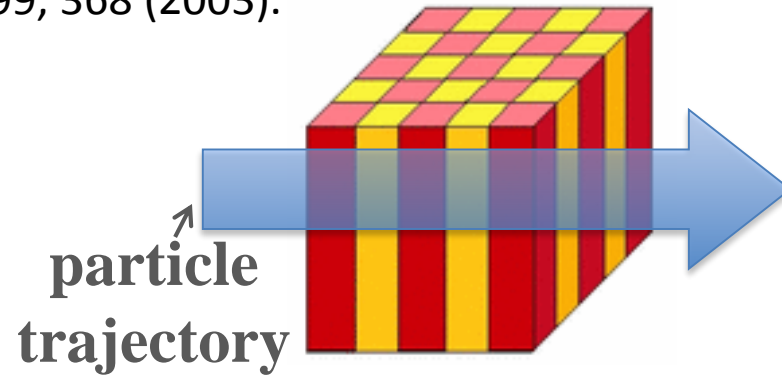
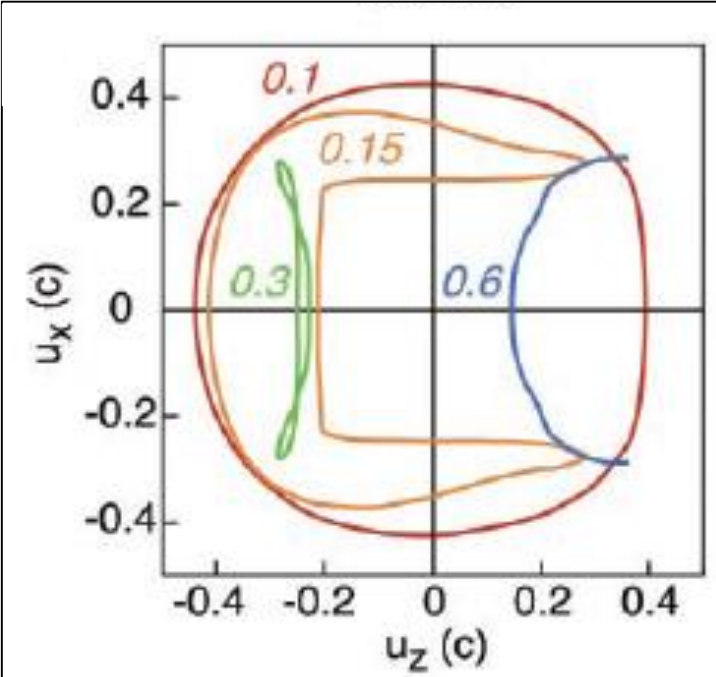
Metamaterials

Graphene



Čerenkov radiation in photonic crystal

Luo, Ibanescu, Johnson, and Joannopoulos, **Science** 299, 368 (2003).



Phase matching condition $\omega = \vec{k} \cdot \vec{v}$

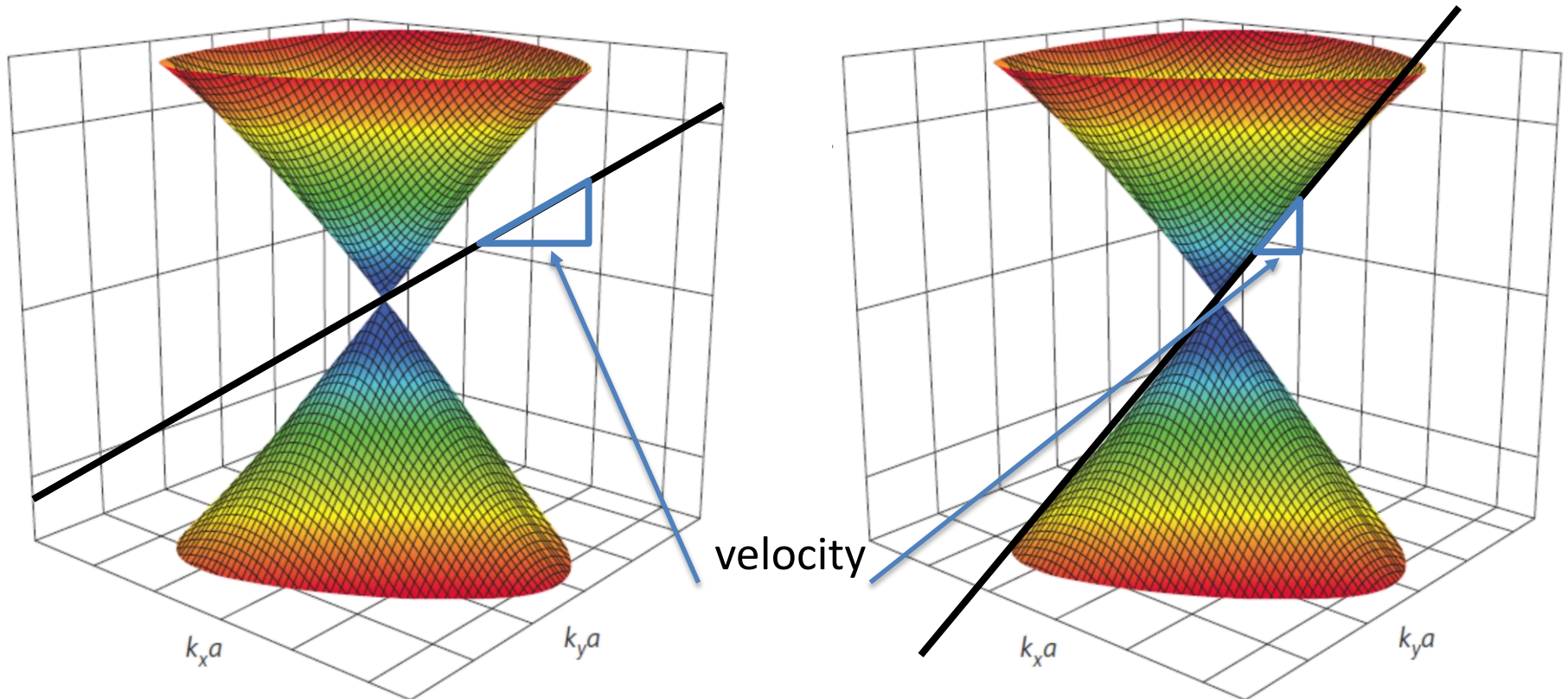
$$k_z = \frac{\omega}{v} \leftarrow \mu_0 q \vec{v} e^{i\omega \frac{z}{v}} \delta(x) \delta(y)$$

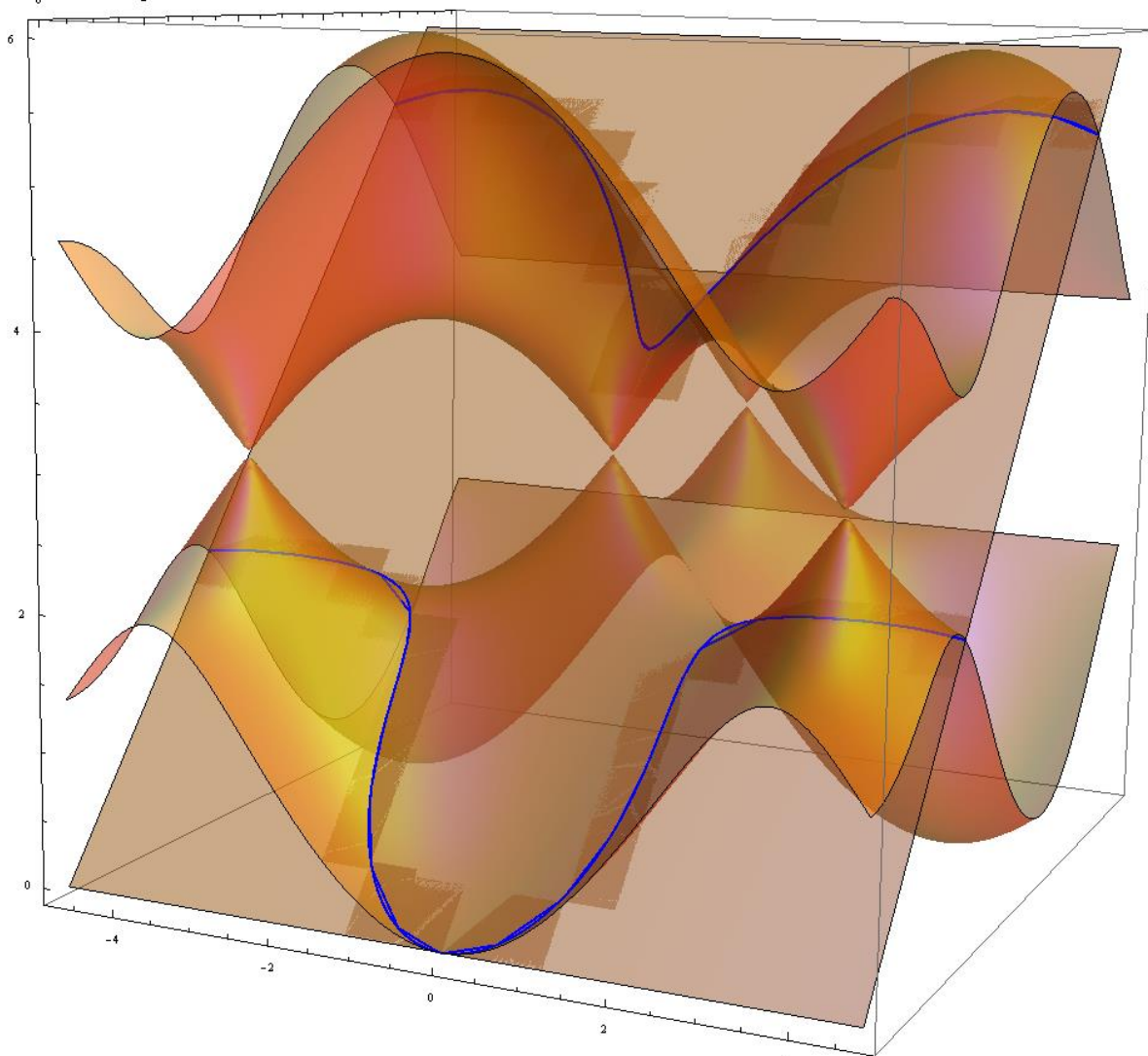
$\omega(\vec{k}) = \text{photonic bandstructure}$

Can be found numerically by MPB (MIT Photonic Bands)

Johnson and Joannopoulos, "Block-iterative frequency-domain methods for Maxwell's equations in a planewave basis," **Optics Express** 8, 173 (2001)

The case of a homogeneous medium - conventional Čerenkov effect:

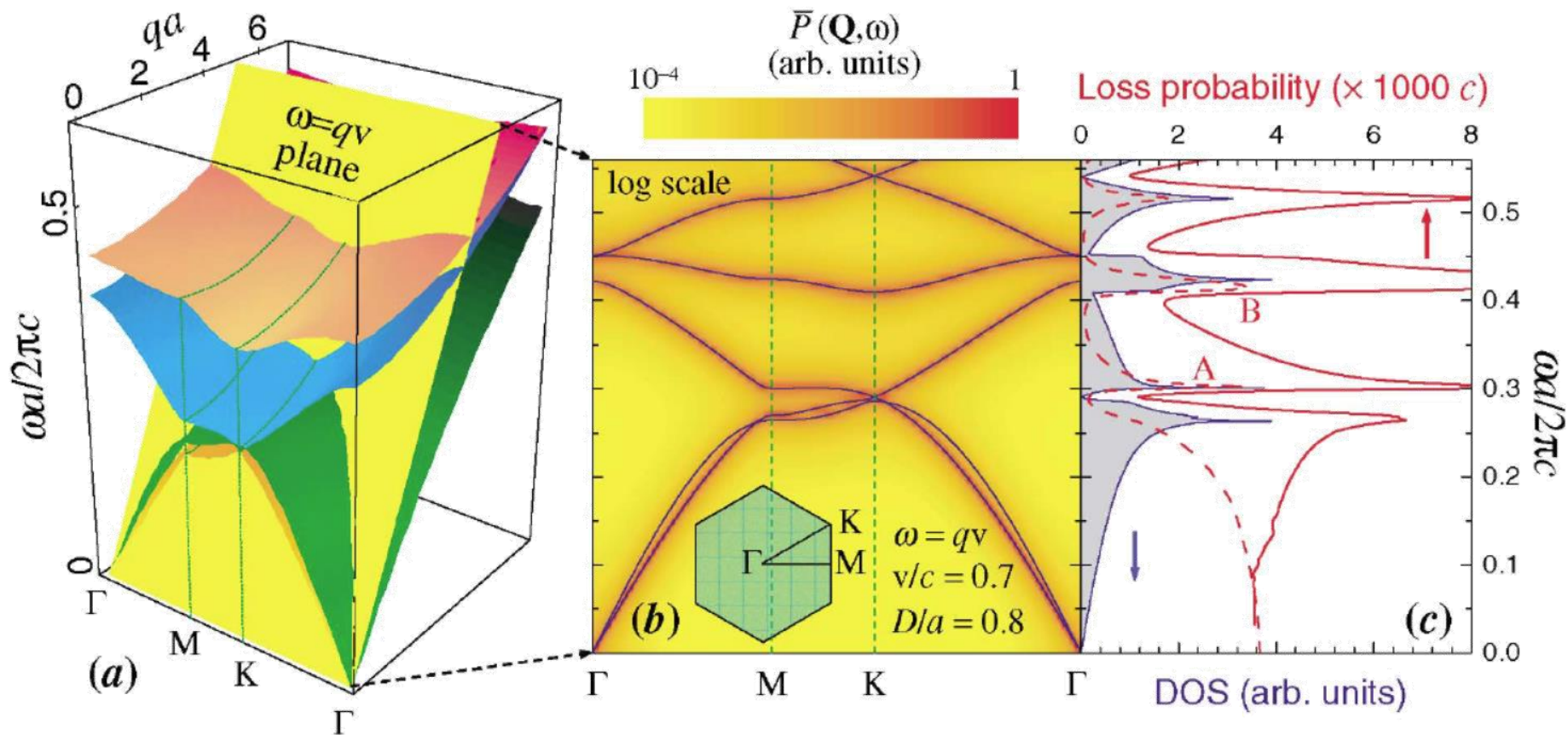




Different frequencies are generally emitted to different directions.
No longer a single Čerenkov angle

Enhancements from the properties of photonic crystals

Also seen through energy loss spectroscopy



the phase matching condition:
intersection between a plane
and the bands

photonic bandstructure

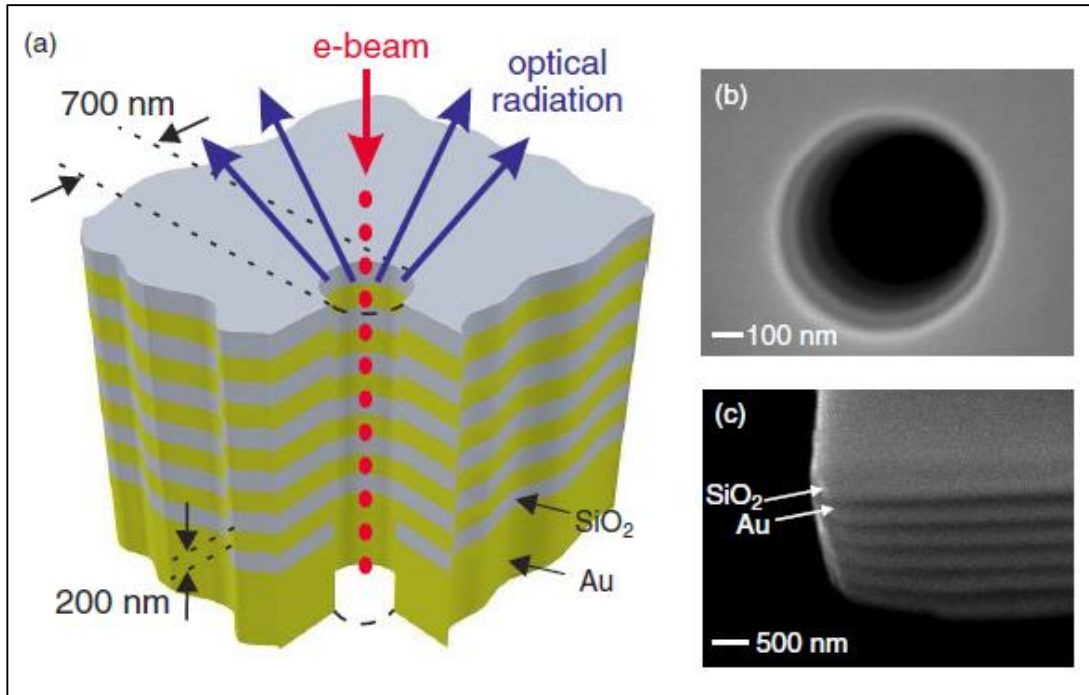
photonic bandstructures

(2D photonic crystal slab)

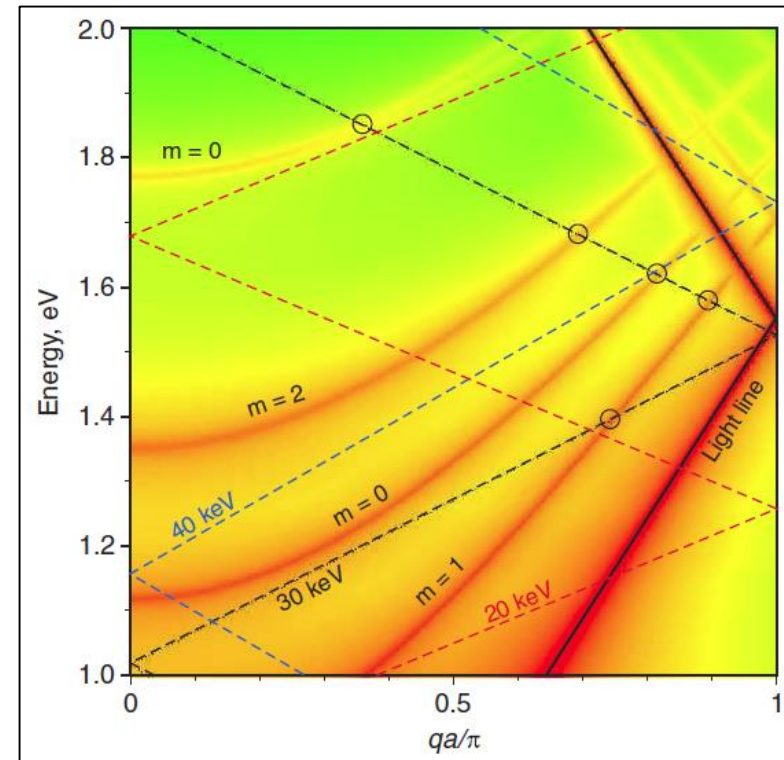
de Abajo, et al., "Cherenkov effect as a probe of
photonic nanostructures." **PRL** 91, 143902 (2003)

A different representation

(yet it all drills down to the photonic bandstructure)



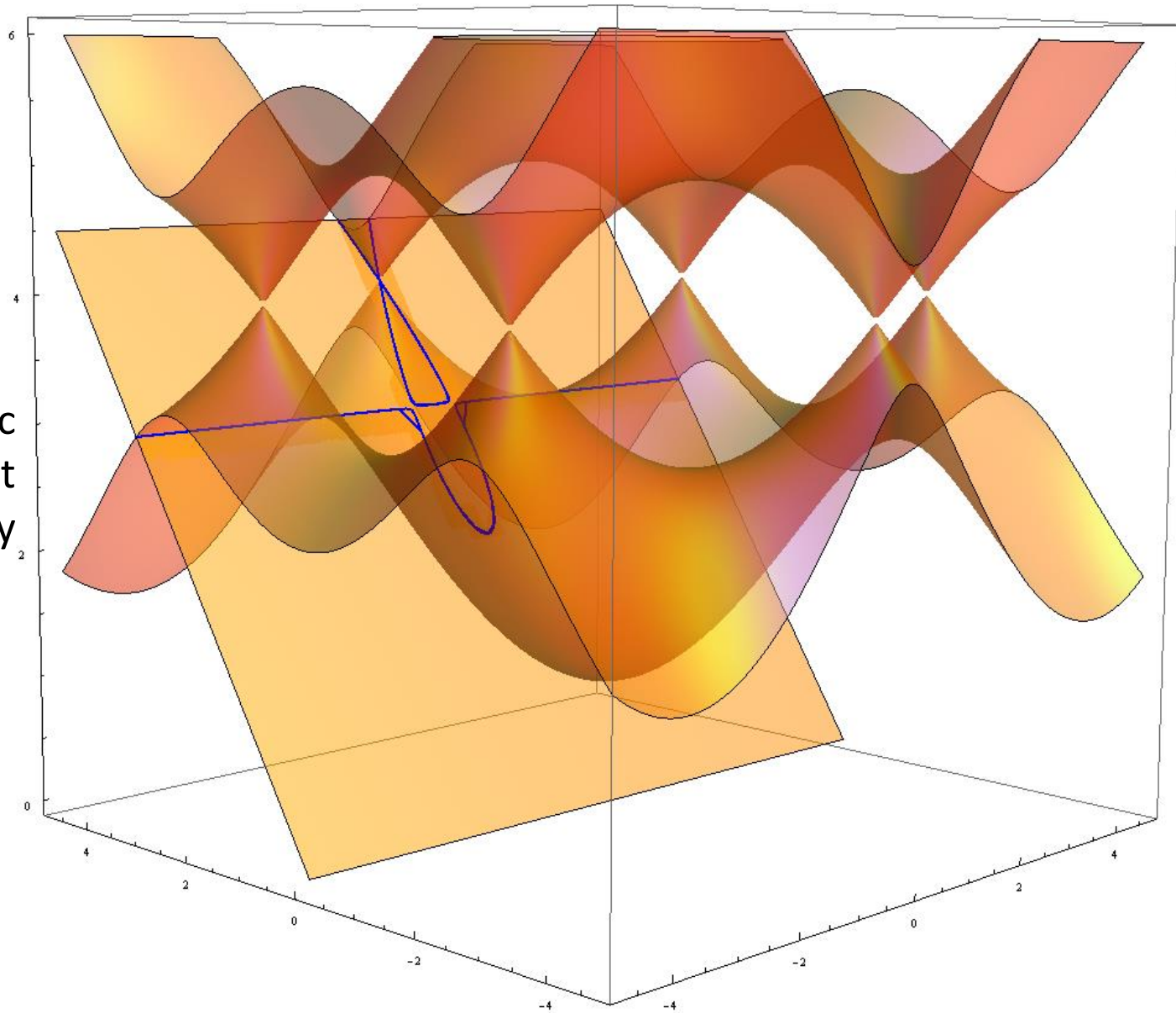
"Light well"



Electron beam passed through a metal-dielectric 1D photonic crystal/metamaterial

Adamo, et al. "Light well: a tunable free- light source on a chip." **PRL** 103, 113901 (2009).

A supercollimation effect in the Čerenkov radiation: directional monochromatic radiation

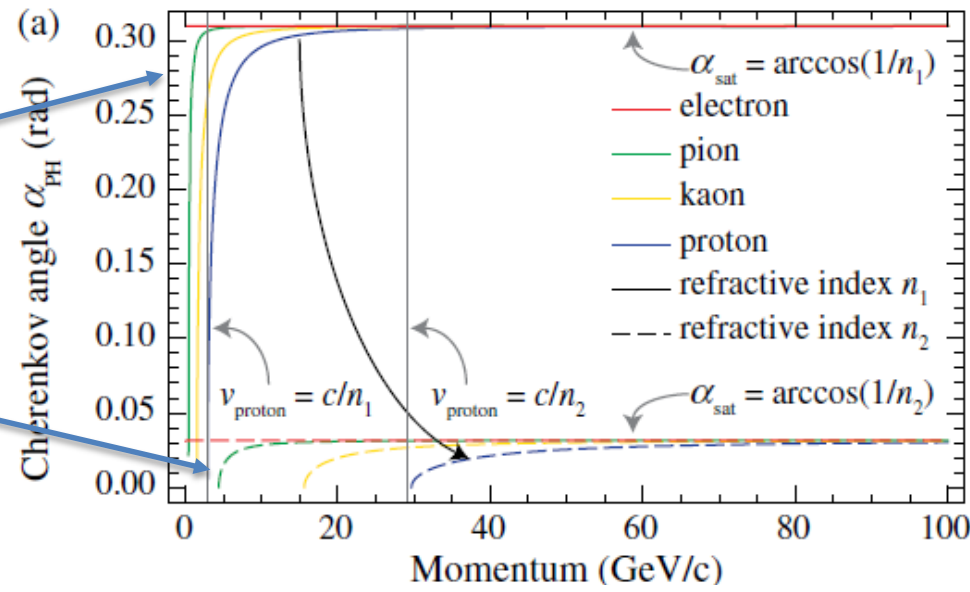
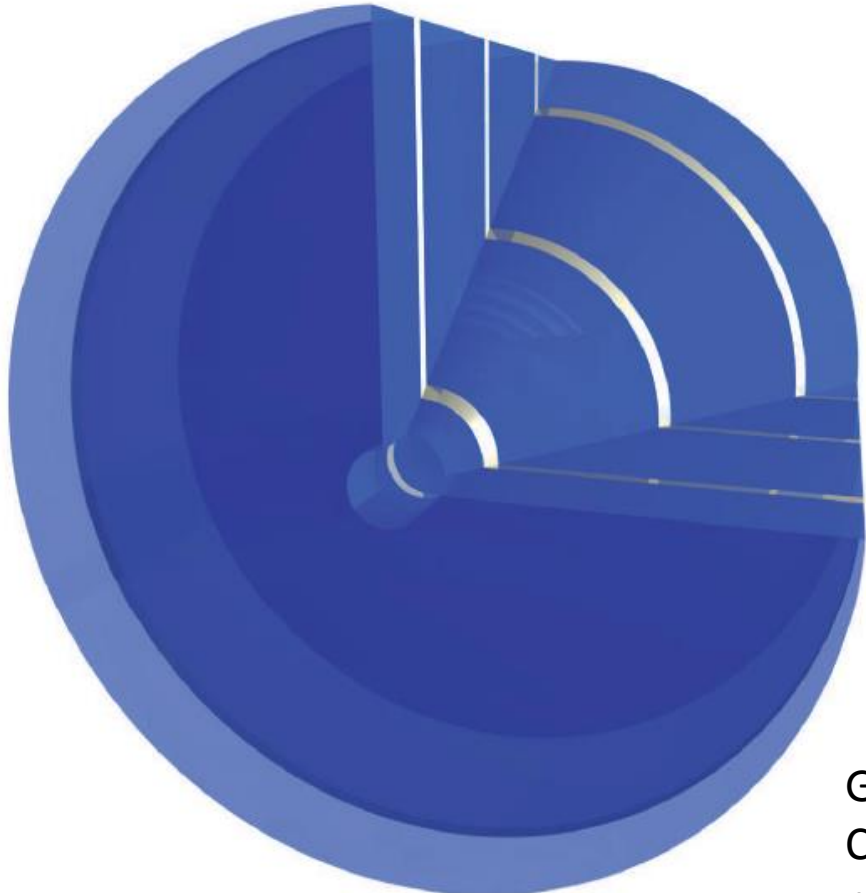


Extremely enhanced
coupling of a charged
particle at one specific
velocity to radiation at
one specific frequency

In a strongly anisotropic medium
(designed with transformation optics)

large opening angles of a silica
aerogel radiator ($n=1.05$)

resolution of a CF_4 radiator ($n=1.0005$)

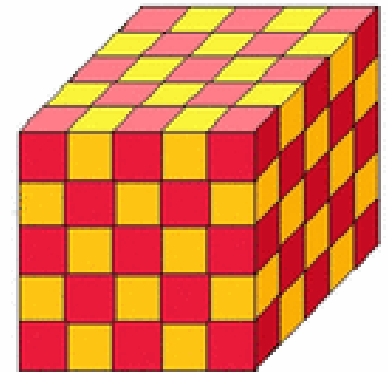
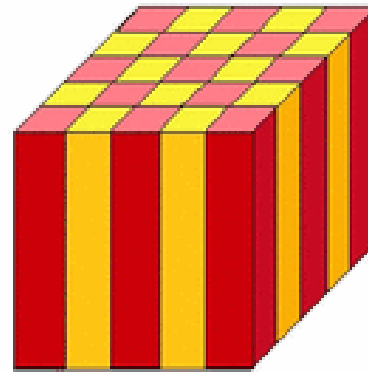
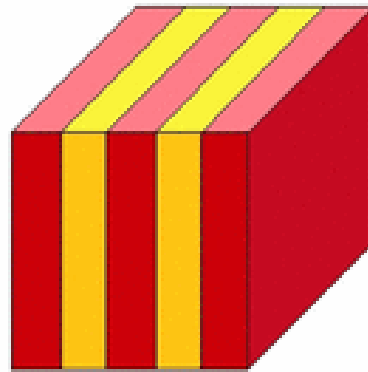
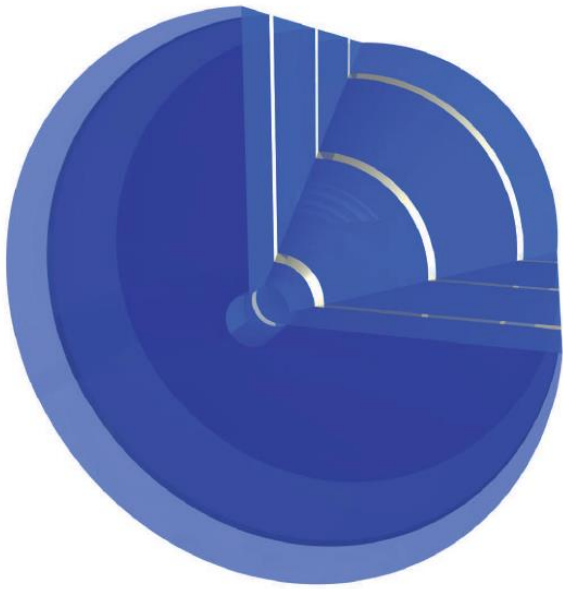
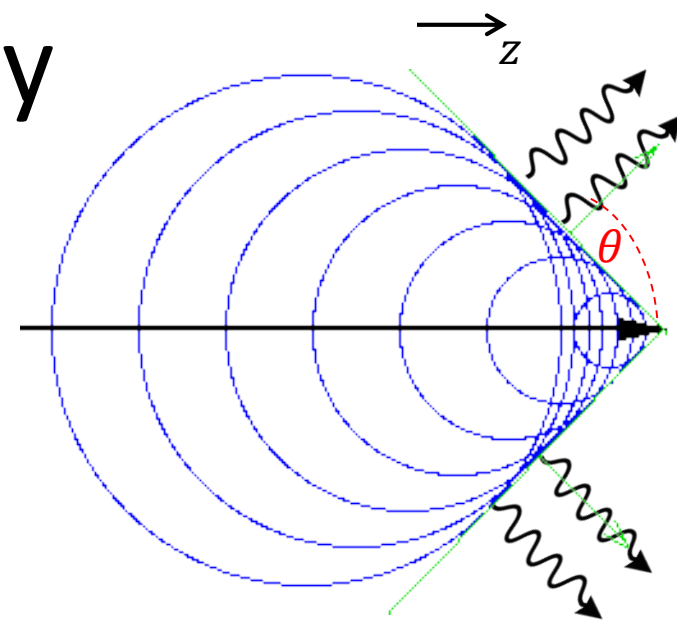


Ginis, Danckaert, Veretennicoff, & Tassin, "Controlling Cherenkov radiation with transformation-optical metamaterials." **PRL** 113, 167402 (2014).

The Conventional Theory

$$v_{source} > v_{sound\ wave}$$

**What about the repulsion?
Where is the conservation of energy?**



$$= \pm \frac{\omega}{c} \sqrt{n^2 - \frac{1}{\beta^2}}$$

The Čerenkov threshold: $\beta > \frac{1}{n}$

Tamm&Frank, **Dokl. Akad. Nauk SSSR** 14, 109 (1937)

Nobel Prize in Physics 1958

The Čerenkov angle: $\cos(\theta) = \frac{1}{\beta n}$

This may be written

$$\cos \theta = 1/n\beta + (\Lambda/\lambda)(n^2 - 1)/2n^2,$$

where $\beta = u/c$ and Λ is the de Broglie wave-length of the electron before the emission of the photon.

The first term on the right is the expression for $\cos \theta$ derived classically by Frank and Tamm.

The second, in which h is a factor, illustrates the rule that a quantum equation becomes identical with the corresponding classical one when h is replaced by zero. Or, to say much the same thing in another way, this term is a natural one to find as the difference between a quantum calculation and a classical one, since the classical calculation of course ignores any effect associated with the wave-length of the electron.

This term will be very small since, as was mentioned earlier, the wave-length of the electron is very much smaller than that of the photon. Hence the classical approximation is a very good one, and the direction of emission of the light does not appreciably depend upon its wave-length except as the wave-length determines the refractive index.

Cox, *Phys. Rev.* 66, 106 (1944)

Quantum Corrections

$$\cos(\theta_{\check{C}R}) = \frac{1}{\beta n} + \frac{\hbar\omega}{\beta\gamma mc^2} \frac{n^2 - 1}{2n}$$

$$\cos(\theta) = \frac{1}{\beta n} \quad \text{Conventional Čerenkov angle}$$

Later papers, e.g,

Jauch&Watson, *Phys. Rev.* 74, 1485 (1948)

Neamtan, *Phys. Rev.* 92, 1362 (1953)

Derived the Čerenkov Effect

from the Dirac Hamiltonian, **always reconfirming the conventional result**

$$\omega_{cutoff} = \frac{2mc^2}{\hbar} \frac{\beta n - 1}{(n^2 - 1)\sqrt{1 - \beta^2}}$$

$\omega_{Compton}$

No Čerenkov radiation for $\omega > \omega_{cutoff}$

The quantum condition for radiation (5) is different from the classical condition

Quantum Corrections

In all previous quantum derivations the charged particle was a plane wave

The wavepacket nature of the particle brings additional effects to the ČR process

Physics – Uspekhi **39** (10) 973–982 (1996)

©1996 Uspekhi Fizicheskikh Nauk, Russian Academy of Sciences

LECTURE OF A LAUREATE OF THE M V LOMONOSOV GREAT GOLD MEDAL

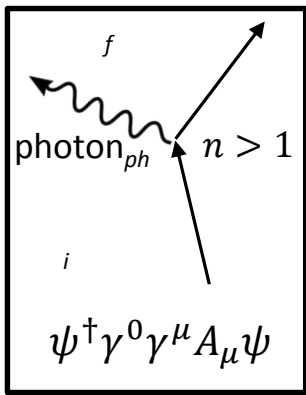
PACS numbers: 41.60.-m, 41.60.Bq, 41.90.+e

Radiation by uniformly moving sources (Vavilov – Cherenkov effect, transition radiation, and other phenomena)†

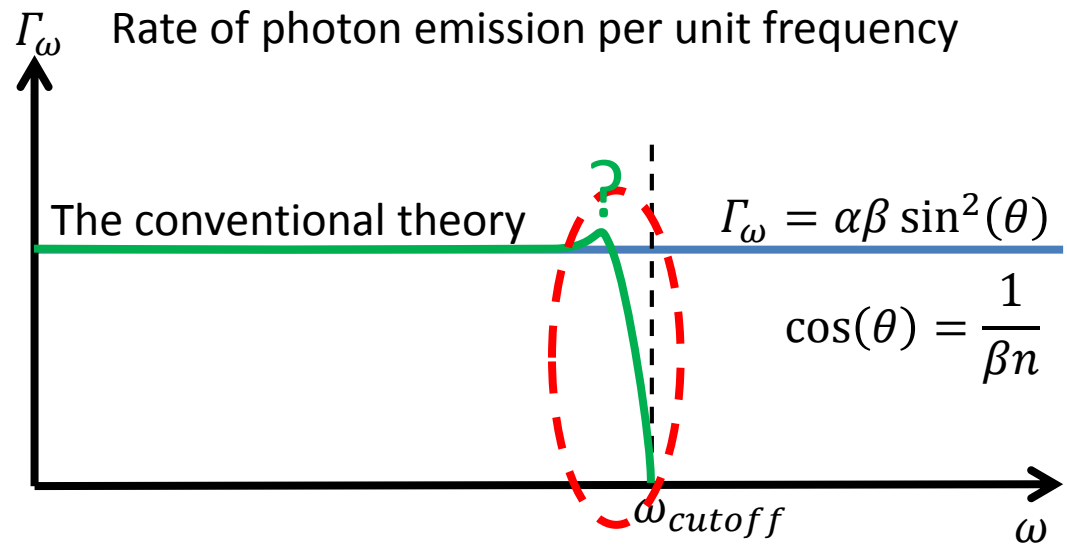
V L Ginzburg

In the optical region, the only one where applications of the VC effect are normally feasible, the ratio $\hbar\omega/mc^2 \sim 10^{-5}$ even for electrons, i.e. quantum corrections are immaterial. In 1940, L D Landau told about my work [36] stated that it was of no interest (see Ref. [20] p. 380). It follows from the above, that he was fully justified in drawing this conclusion, and his comment hit the mark as was usual with his criticism.

Ginzburg, V. L. *Phys. Usp.* **39**, 973 (1996)



Does not occur in vacuum...



The only possible first-order interaction, hence it is the dominant effect

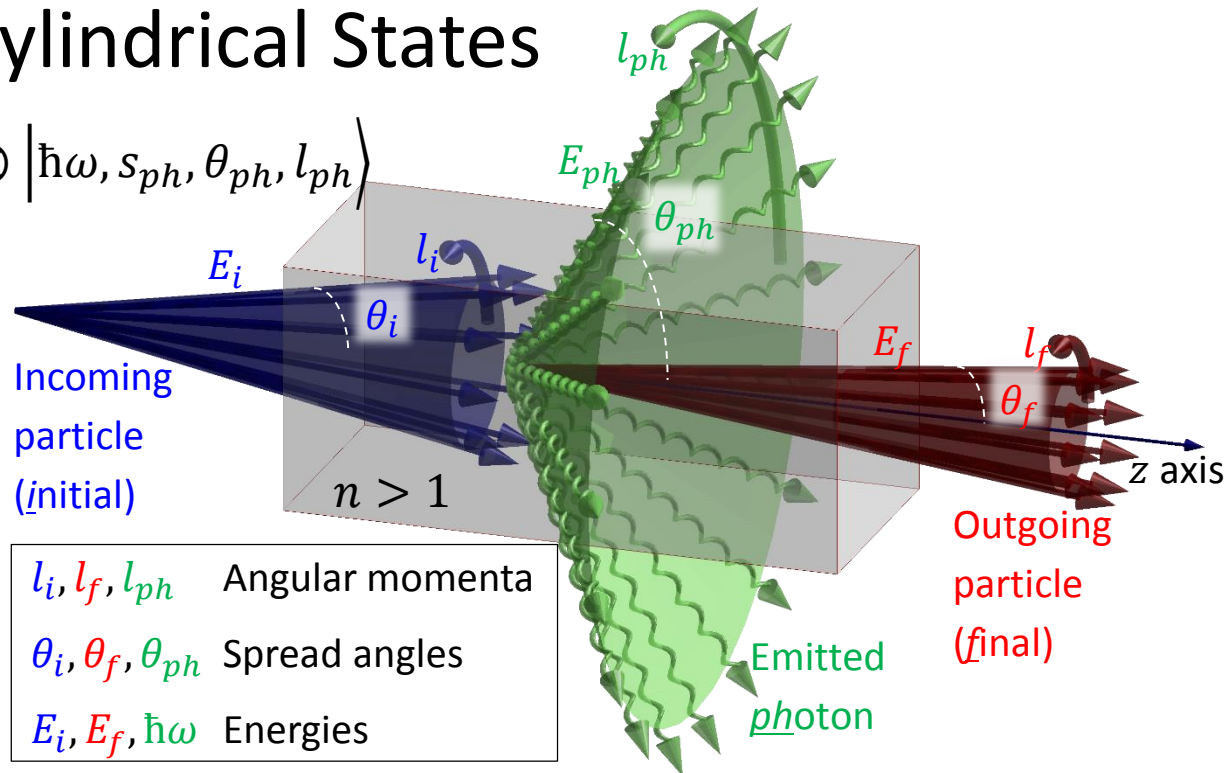
Cylindrical States

$$\begin{aligned} |p_i^{cyl}\rangle &= |E_i, s_i, \theta_i, l_i\rangle \\ |p_f^{cyl}\rangle \otimes |k^{cyl}\rangle &= |E_f, s_f, \theta_f, l_f\rangle \otimes |\hbar\omega, s_{ph}, \theta_{ph}, l_{ph}\rangle \end{aligned}$$

Bliokh, Dennis, Nori,
PRL 107, 174802 (2011)

Derived the cylindrical beams of the Dirac equation for the first time

Kaminer et al., "Quantum Čerenkov Radiation: Spectral Cutoffs and the Role of Spin and Orbital Angular Momentum." **PRX 6, 011006 (2016).**



$$M_{p_i^{cyl} \rightarrow p_f^{cyl} + k^{cyl}}^{density} (t, r, \varphi, z) = \left\langle p_f^{cyl} \text{ particle}, k^{cyl} \text{ photon} \left| \underbrace{\psi^\dagger \gamma^0 \gamma^\mu \psi}_{\square j^\mu(t, r, \varphi, z)} q A_\mu (t, r, \varphi, z) \right| p_i^{cyl} \text{ particle}, 0 \right\rangle$$

$$\int_0^\infty J_{l_i} (p_{ir} r / \hbar) J_{l_f} (p_{fr} r / \hbar) J_{l_{ph}=l_i-l_f} (k_r r) r dr = \frac{\cos(l_i \alpha_f - l_f \alpha_i)}{2\pi S_\Delta (\frac{1}{\hbar} p_{ir}, \frac{1}{\hbar} p_{fr}, k_r)}$$

Gervois&Navelet, *J. Math. Phys.* 25, 3350 (1984)

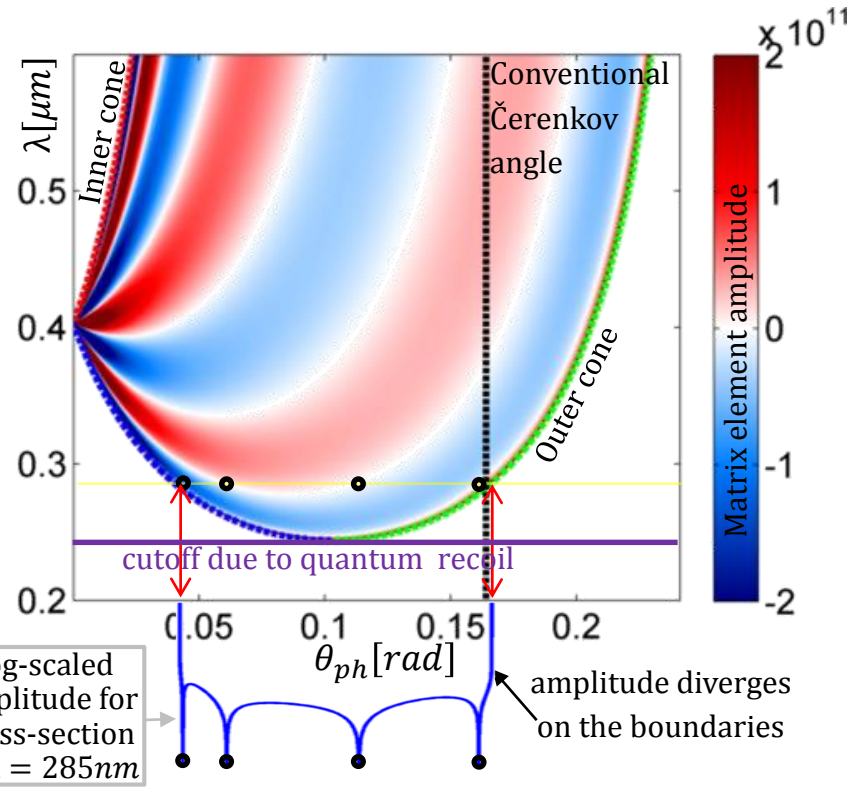
The Čerenkov angle splits in two!

- $\theta_{ph} = \theta_i - \theta_{\check{C}R}$
- $\theta_{ph} = \theta_{\check{C}R} - \theta_i$
- $\theta_{ph} = \theta_i + \theta_{\check{C}R}$

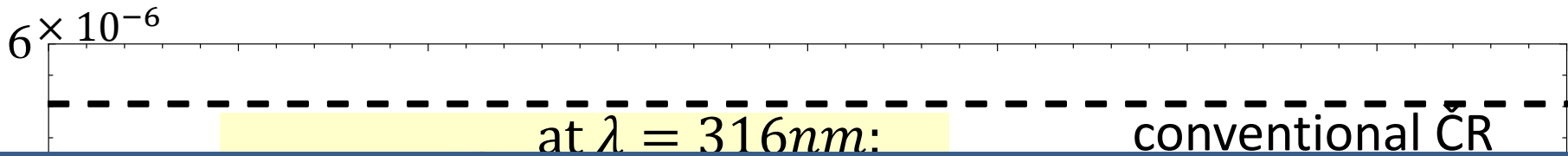
Preferred emission angles controlled by OAM, spin, and polarization

Creates coupling between the charged particle and the emitted photon

Kaminer et al., "Quantum Čerenkov Radiation: Spectral Cutoffs and the Role of Spin and Orbital Angular Momentum." *PRX* 6, 011006 (2016).

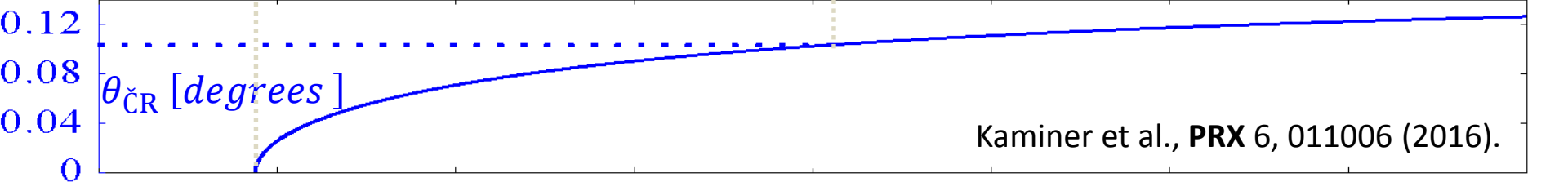
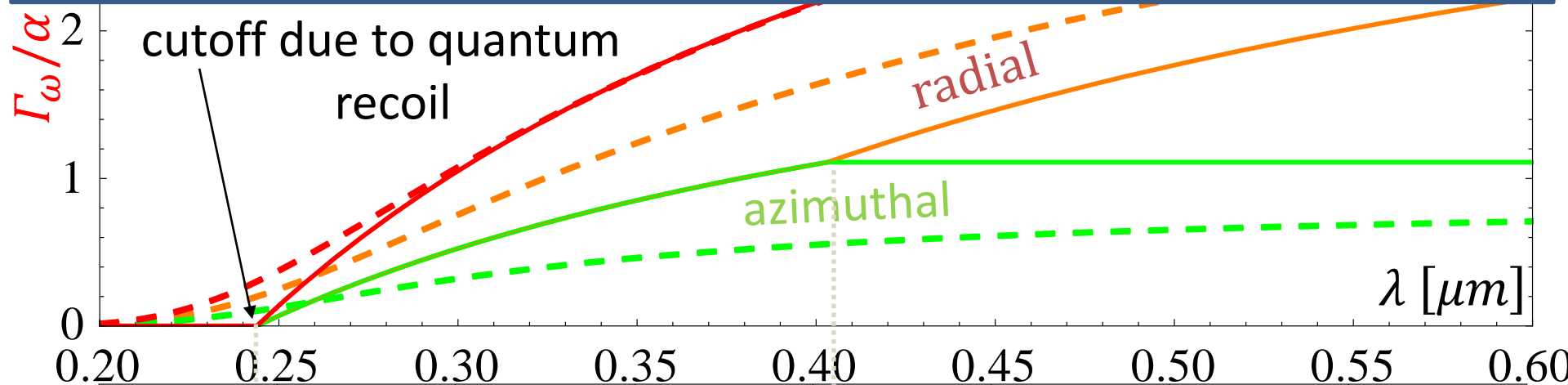


$\theta_i = 0.1^\circ$ $\beta = 0.685$ $n = 1.45986$ $l_{ph} = 8$



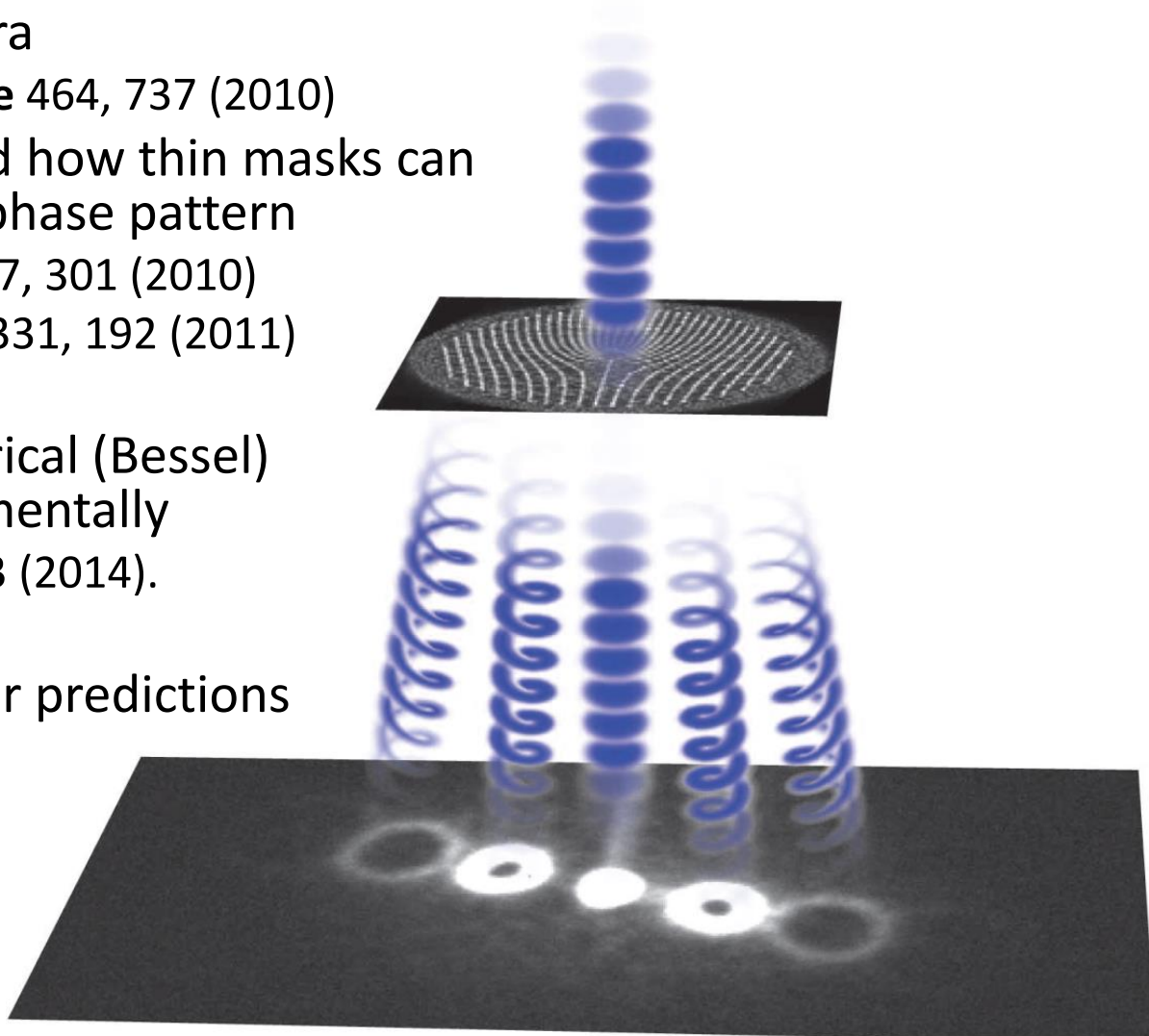
The importance of this result is in the fact that the quantum deviation remains in Gaussian particles

observable even when there is a variance in the particle energy (here $\Delta E \approx 0.5\text{eV}$)



Vortex Electron Beams

- Predicted by Bliokh et al
 - Bliokh, Bliokh, Savel'ev, Nori, **PRL** 99, 190404 (2007)
- First observed by Tonomura
 - Uchida&Tonomura, **Nature** 464, 737 (2010)
- Then other groups showed how thin masks can imprint the beam with a phase pattern
 - Verbeeck et al., **Nature** 467, 301 (2010)
 - McMorran et al., **Science** 331, 192 (2011)
- Recently the actual cylindrical (Bessel) beam was created experimentally
 - Grillo *et al.*, **PRX** 4, 011013 (2014).
- This means that part of our predictions can already be observed

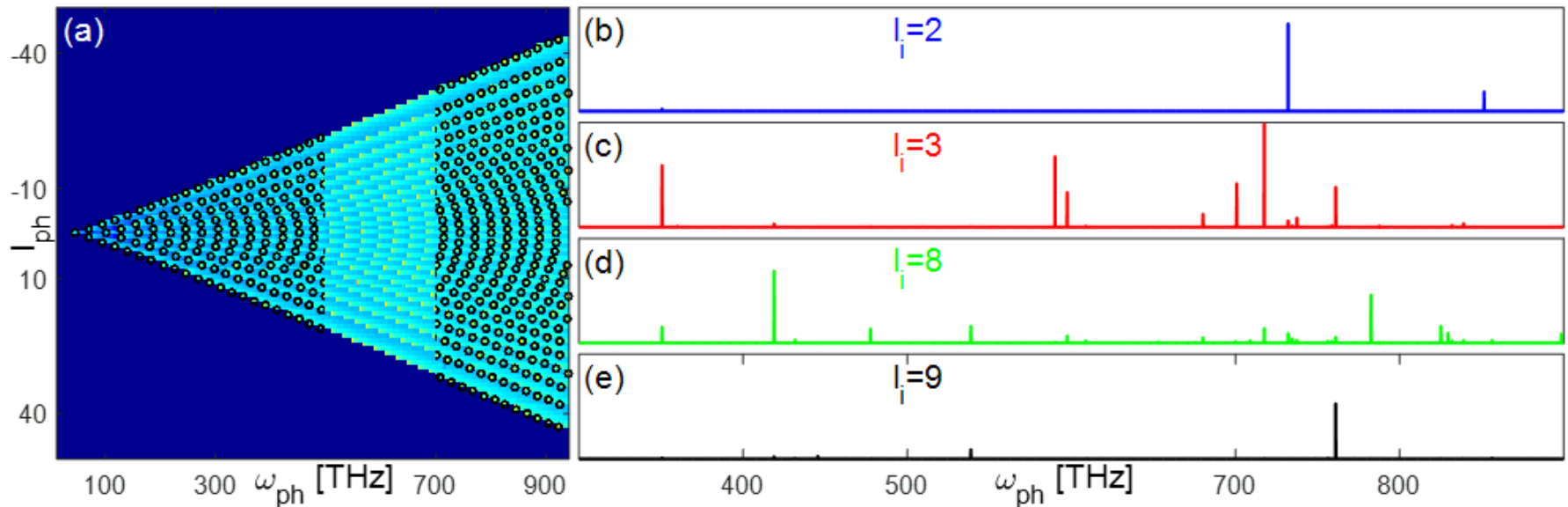


OAM beyond electrons

pions, kaons, protons

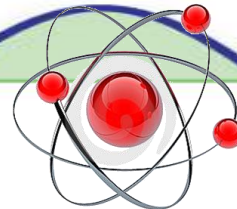
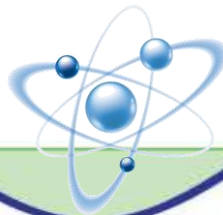
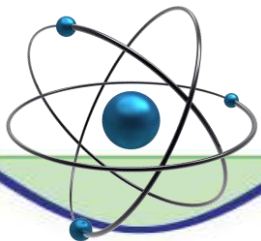
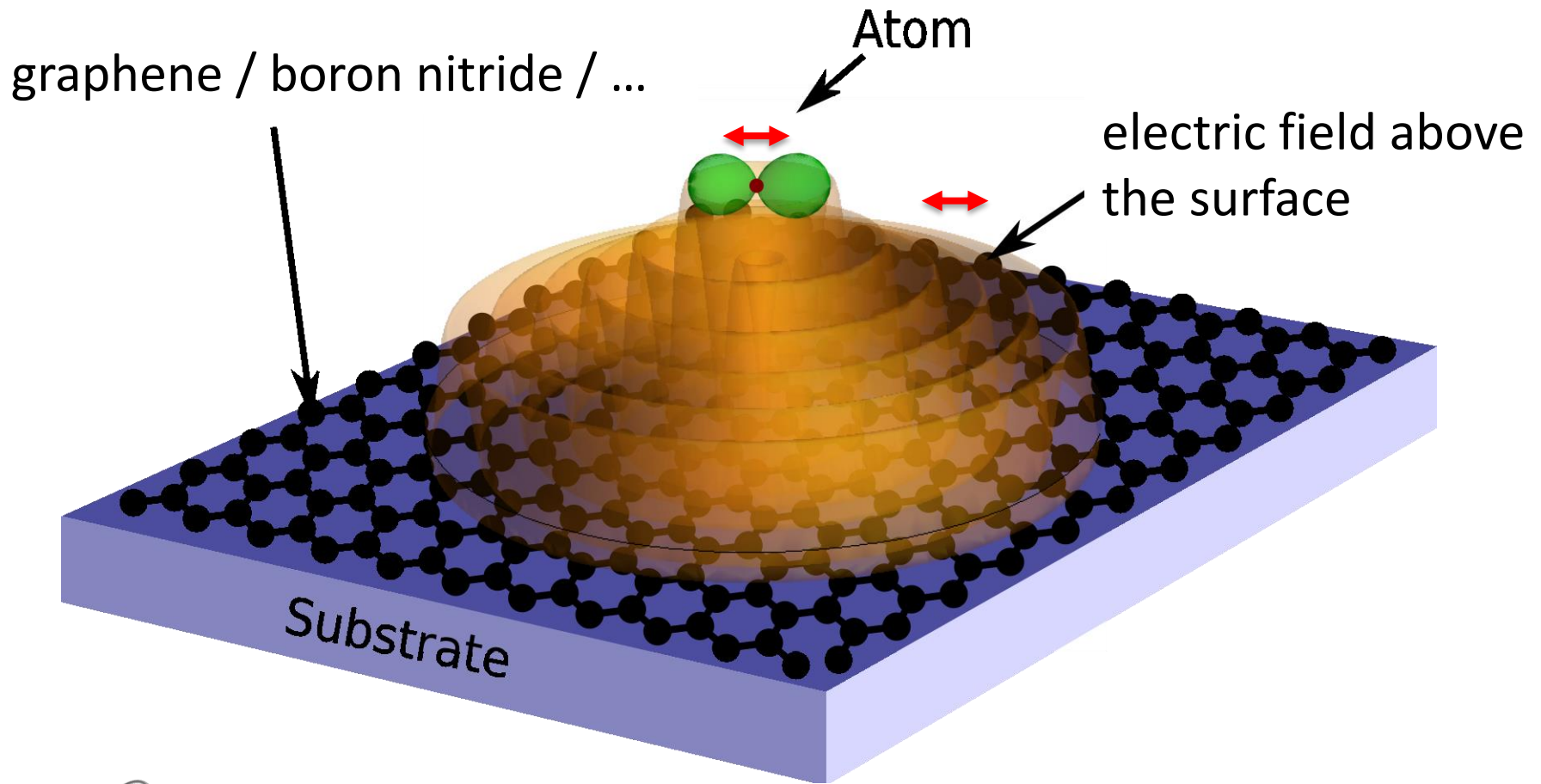
Fundamental particles emerging from collisions might carry orbital angular momentum.

If we could measure it, what would it tell us about the collision process?

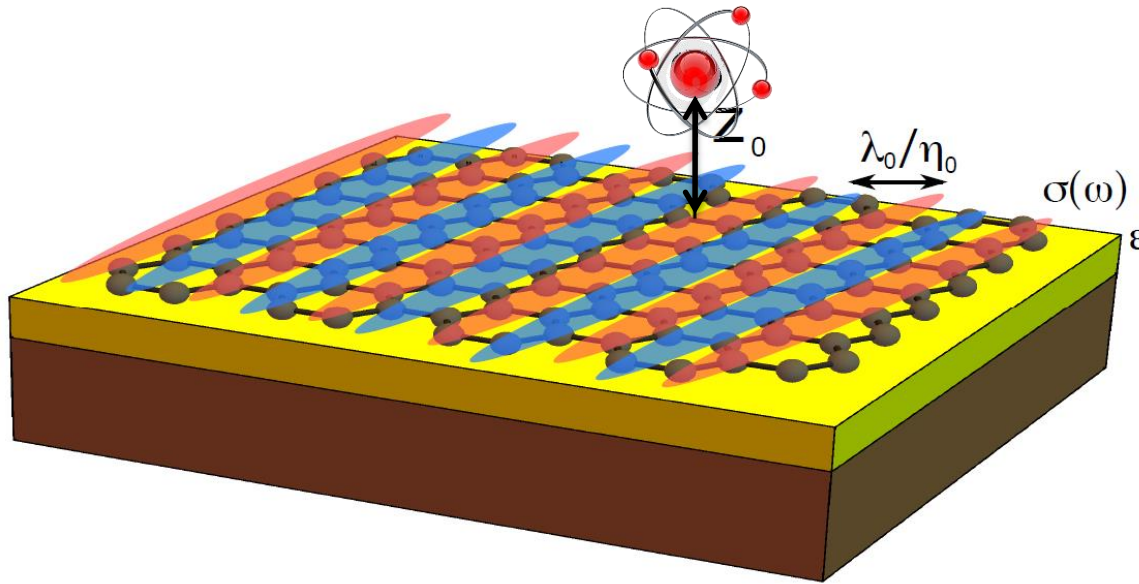


Shapira, Mutzafi, Harari, Kaminer, Alon and Segev, "Čerenkov Radiation from Particles Carrying Orbital Angular Momentum in a Cylindrical Waveguide." *in preparation*

Motivation - shrinking light



Light-matter interaction with composite photons of extreme confinement



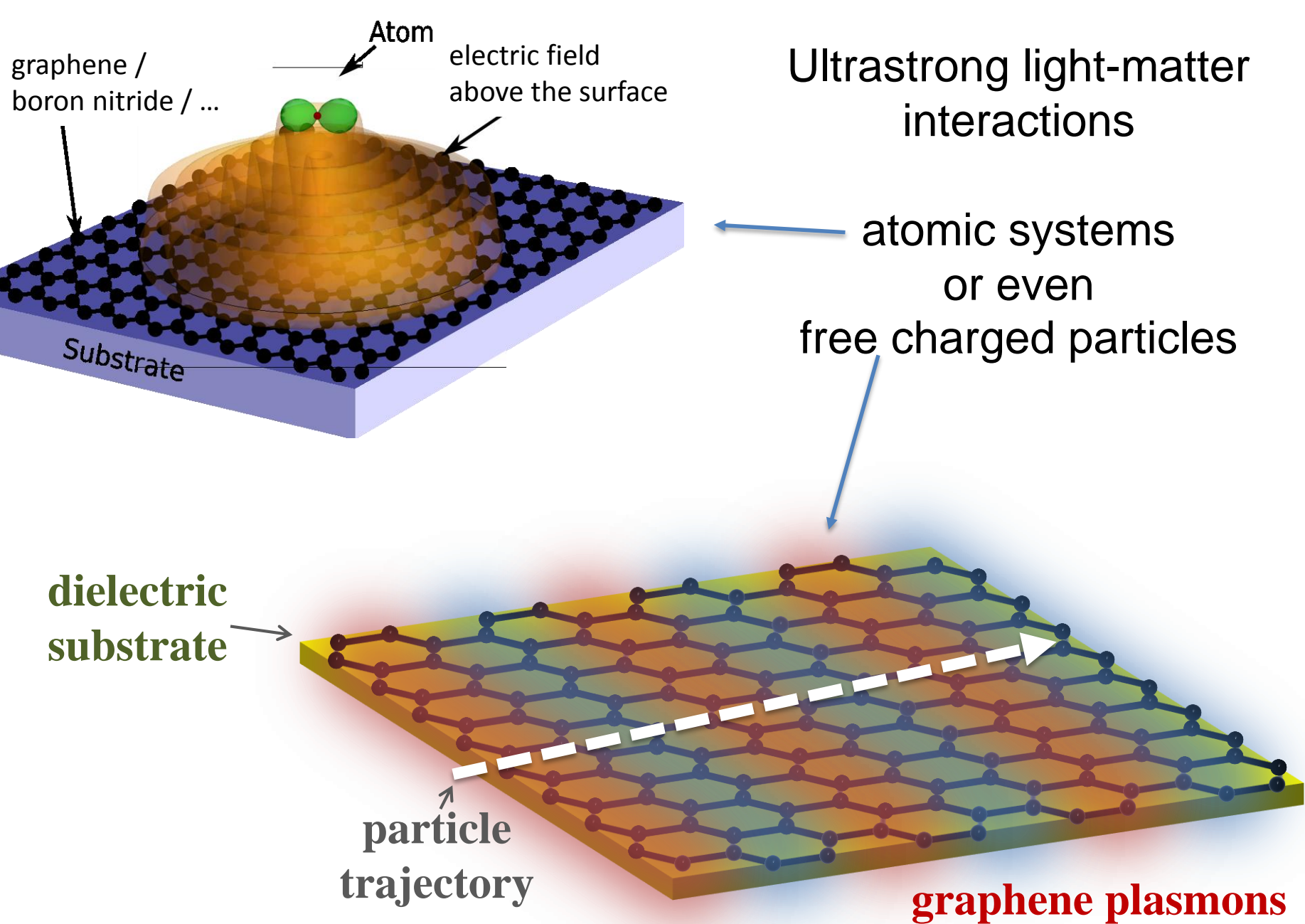
η_0 confinement factor
150-250 for graphene

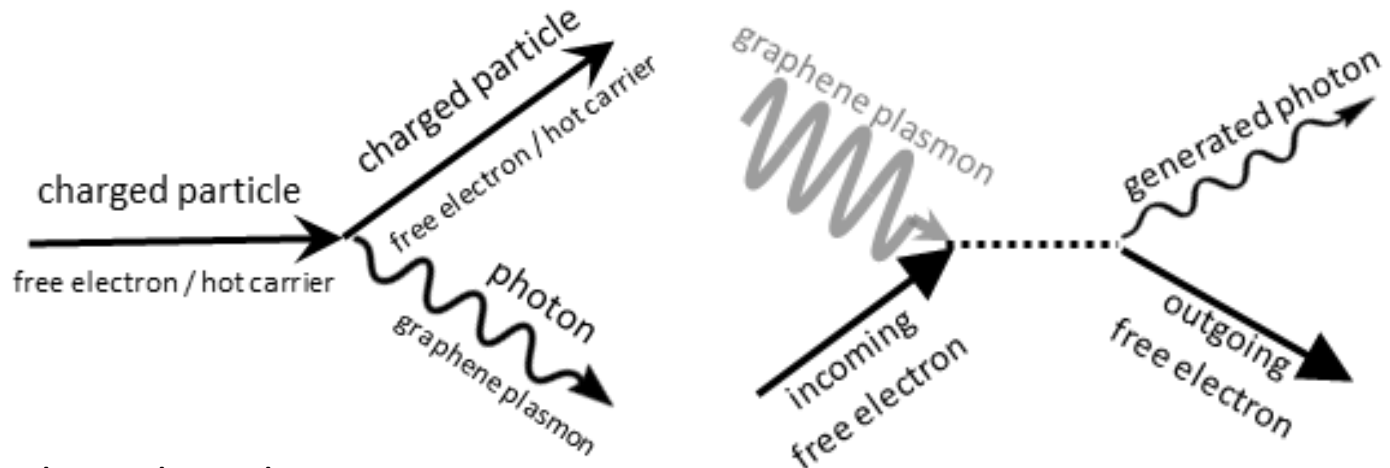
$\lambda_{plasmon}$ can be as small as **$\sim 10\text{nm}$** in graphene,
and much smaller for other 2D conductors (e.g., 2D silver)
or phononic materials (e.g., boron nitride, silicon carbide)

→ Almost at the atomic scale!

New platforms for spectroscopy, sensing, and broadband light generation,
as well as a new source of entangled photons

Rivera*, Kaminer*, Zhen, Joannopoulos, Soljacic ([arXiv:1512.04598](https://arxiv.org/abs/1512.04598)), under review in *Science*





Kaminer, Katan, Buljan, Shen, Ilic,
 López, Wong, Joannopoulos, Soljačić
 (under review *Nature Comm.*,
 arXiv:1510.00883)

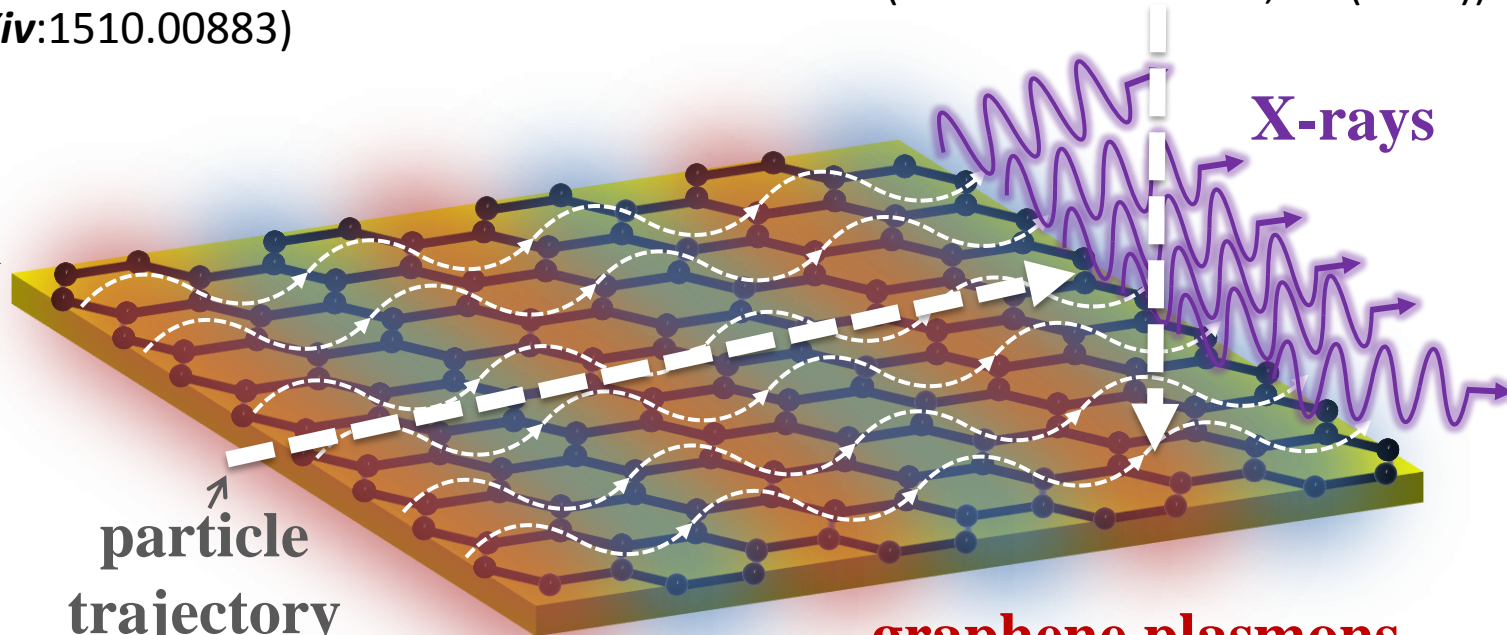
Wong*, Kaminer*, Ilic,
 Joannopoulos, Soljačić
 (**Nature Photon.** 10, 46 (2016))

dielectric
 substrate

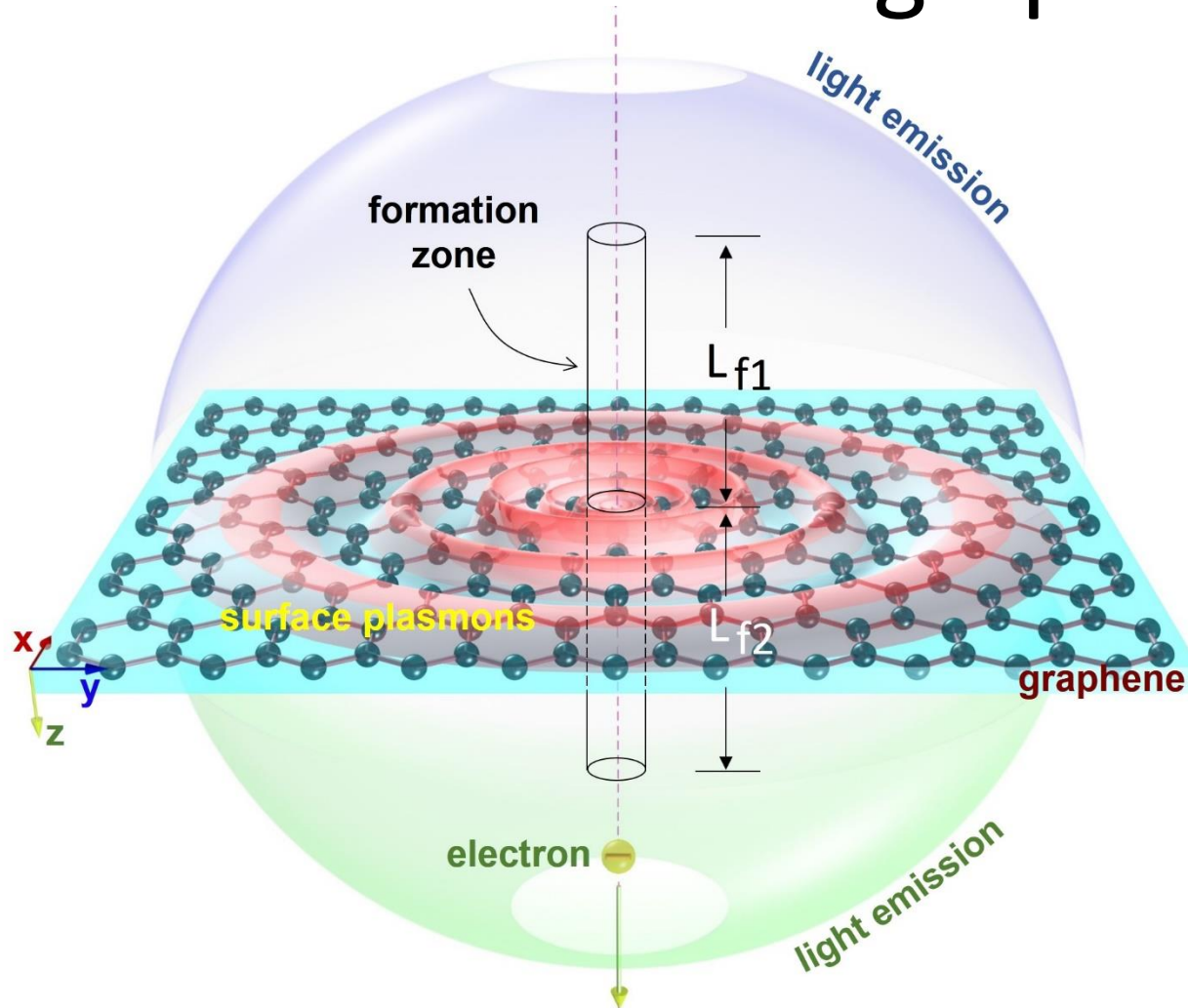
particle
 trajectory

graphene plasmons

X-rays



Transition radiation - graphene



Lin, Shi, Gao, Kaminer, Yang, Gao, Buljan, Joannopoulos, Soljačić, Chen, Zhang,
"Dynamical Mechanism of Two-Dimensional Plasmon Launching by Swift Electrons."
under review in PRL ([arXiv:1507.08369](https://arxiv.org/abs/1507.08369)).

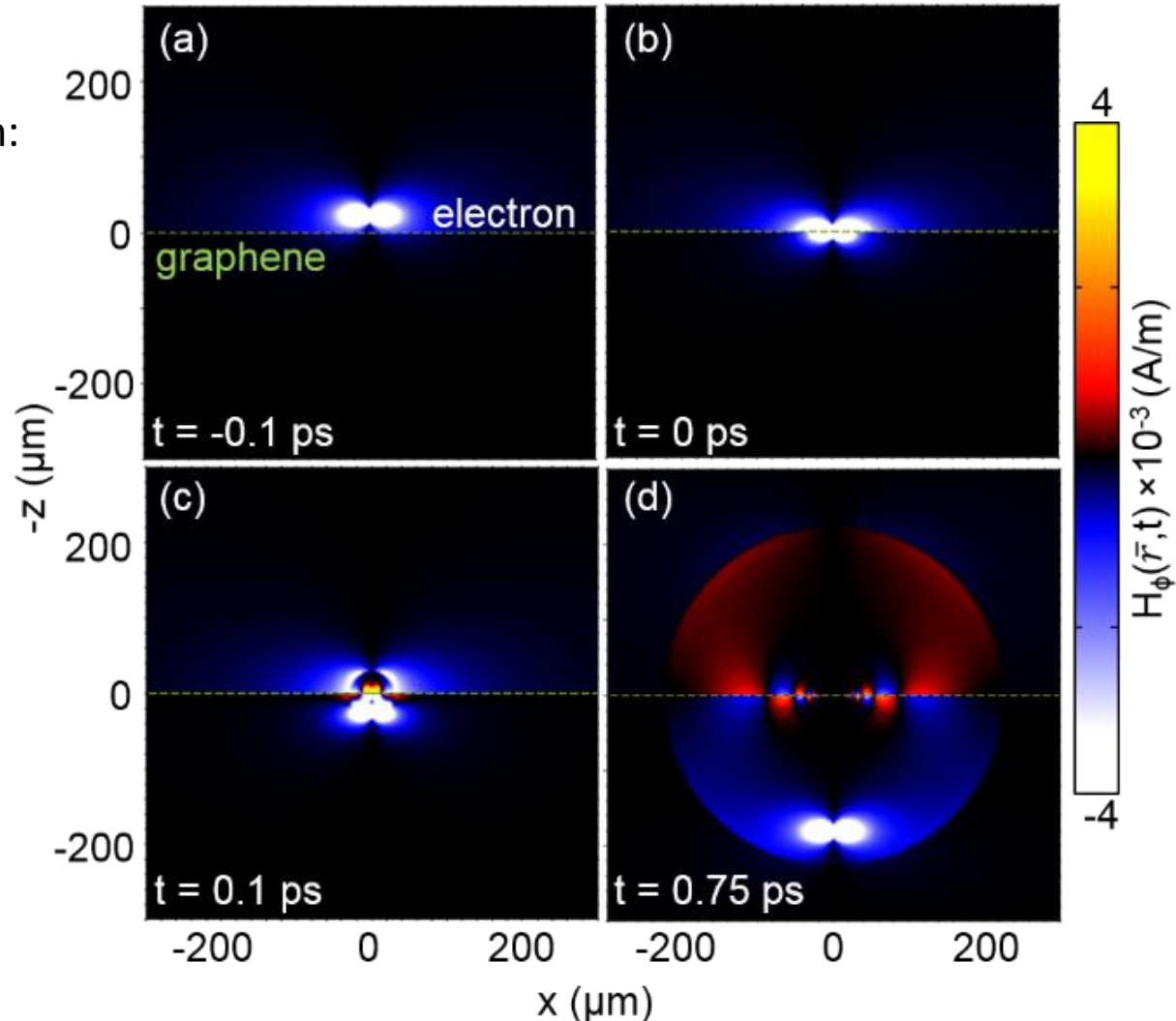
Emission into plasmons and into photons

Conventional transition radiation:
formation zone of a finite length

Graphene transition radiation:
only a single atomic layer

Especially efficient for low
velocities $v \sim c/100$

Most emission into plasmons



Lin, Shi, Gao, Kaminer, Yang, Gao, Buljan, Joannopoulos, Soljačić, Chen, Zhang,
“Dynamical Mechanism of Two-Dimensional Plasmon Launching by Swift Electrons.”
under review in PRL ([arXiv:1507.08369](https://arxiv.org/abs/1507.08369)).

Čerenkov Radiation

Čerenkov (*Dokl. Akad. Nauk SSSR* 2, 451, 1934)
Nobel Prize in Physics 1958

$$v_{particle} > v_{photon}$$

$$\beta c > \frac{c}{n}$$

The Čerenkov threshold: $\beta > \frac{1}{n}$

$$\cos(\theta) = \frac{1}{\beta n} \quad \text{Conventional Čerenkov angle}$$

Most of optics: $n \sim 1.5 - 2$
→ particle has to be relativistic

Taking this concept to graphene:

- Charge particles with non-relativistic velocities can emit Čerenkov radiation
- The quantum correction becomes significant [Kaminer, et al. **PRX**, (2016)]
– lowering the velocity threshold in graphene
- Even the Fermi velocity can cross the threshold
– Čerenkov effect from charge particles flowing inside graphene (hot carriers)

electron's (Fermi) velocity: $c/300$

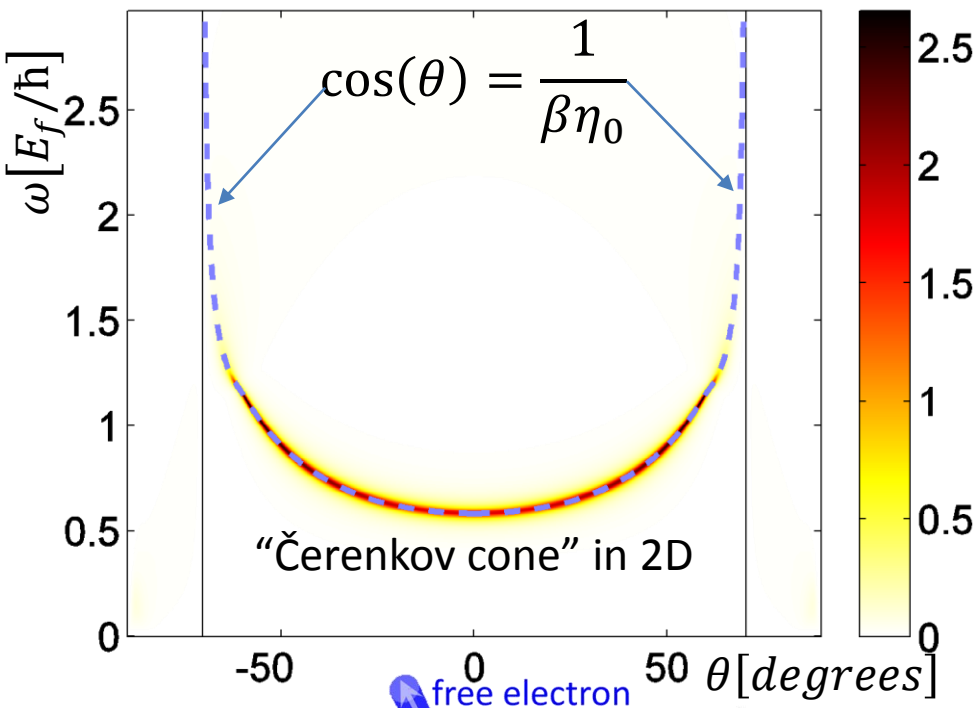
~

plasmons (phase) velocity: $\sim c/300$

Electron beam physics *on-chip*

confinement factor

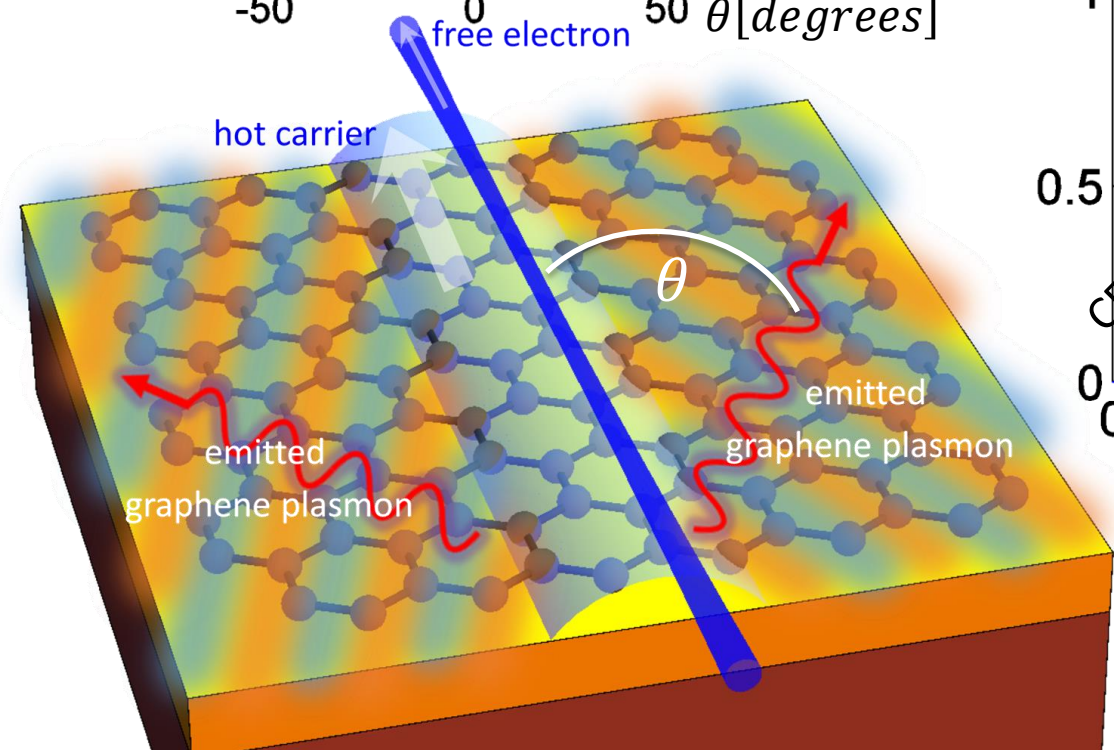
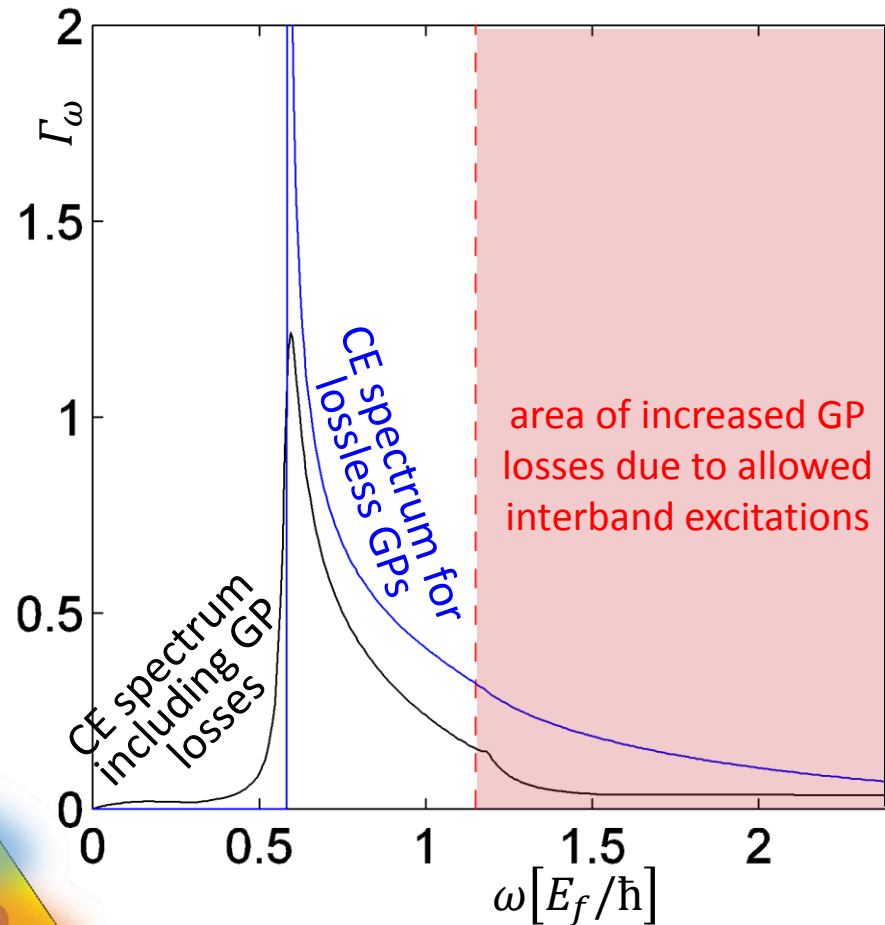
Kaminer, Katan, Buljan, Shen, Ilic, López, Wong, Joannopoulos, Soljačić
(*under review Nature Comm.*, [arXiv:1510.00883](https://arxiv.org/abs/1510.00883))



$$\beta c = v = 3v_f$$

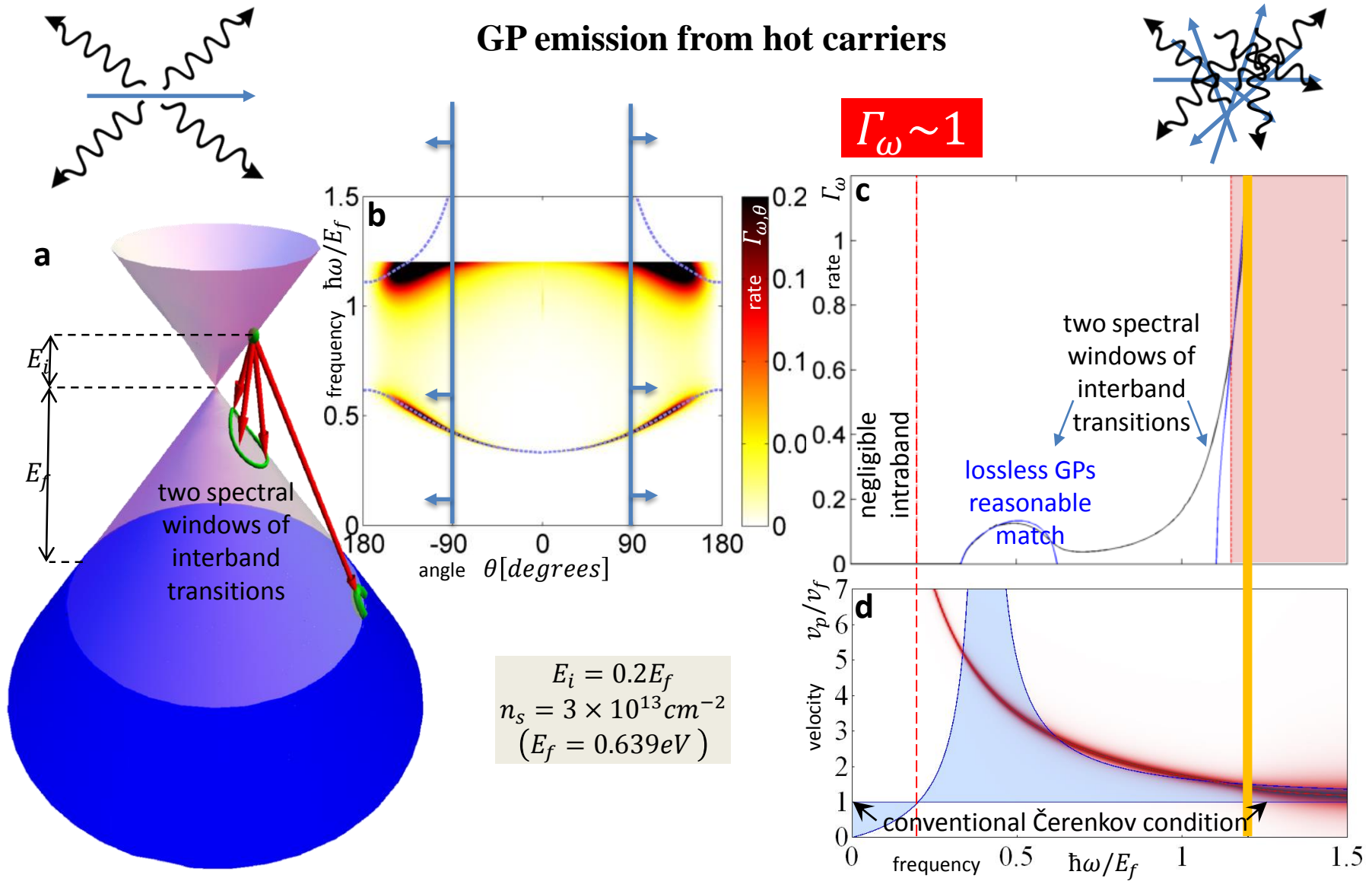
$$n_s = 3 \times 10^{13} \text{ cm}^{-2} (E_f = 0.639 \text{ eV})$$

$$\Gamma_\omega \sim 1$$



Kaminer, Katan, Buljan, Shen, Ilic,
 López, Wong, Joannopoulos, Soljačić
 (under review *Nature Comm.*,
arXiv:1510.00883)

GP emission from hot carriers



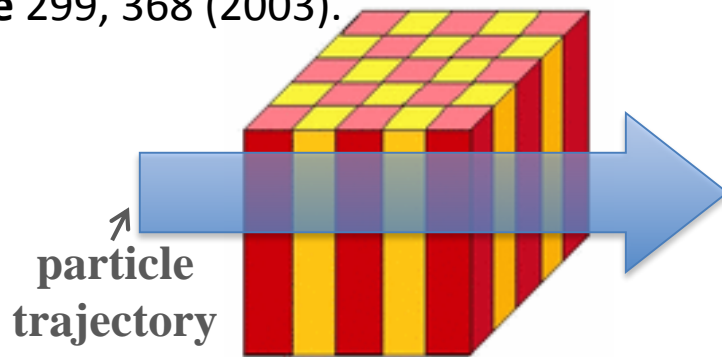
Most of the emission goes backward! (conventional Čerenkov radiation never goes backward)

I. Kaminer, Y. T. Katan, H. Buljan, Y. Shen, O. Ilic, J. J. López, L. J. Wong,
 J. D. Joannopoulos, M. Soljačić (under review *Nature Comm.*, [arXiv:1510.00883](https://arxiv.org/abs/1510.00883))

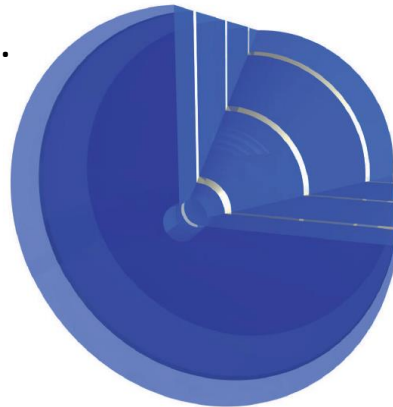
Summary

Čerenkov Radiation in Photonic Crystals

Luo, Ibanescu, Johnson, and Joannopoulos, *Science* 299, 368 (2003).

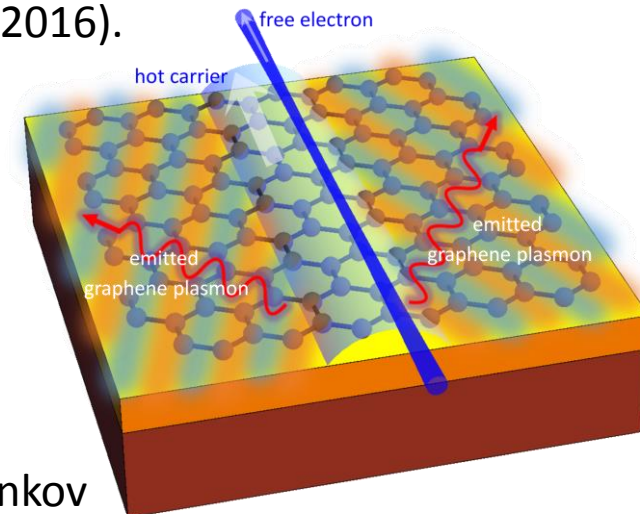
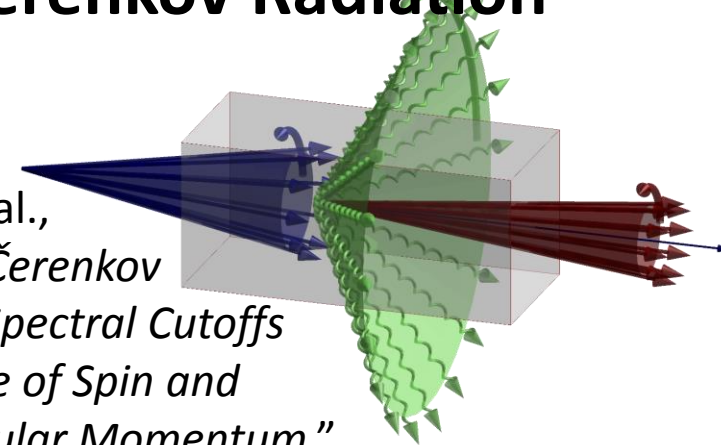


Ginis, Danckaert, Veretennicoff, & Tassin, "Controlling Čerenkov radiation with transformation-optical metamaterials." *PRL* 113, 167402 (2014).



New Effects in Čerenkov Radiation

Kaminer et al., "Quantum Čerenkov Radiation: Spectral Cutoffs and the Role of Spin and Orbital Angular Momentum." *PRX* 6, 011006 (2016).



Kaminer, et al., "Quantum Čerenkov Effect from Hot Carriers in Graphene: An Efficient Plasmonic Source." *in review*, ([arXiv:1510.00883](https://arxiv.org/abs/1510.00883))