More precise predictions for hard processes at the Tevatron and the LHC

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Based on work done together with John Campbell, Walter Giele, Zoltan Kunszt, Kirill Melnikov and Giulia Zanderighi

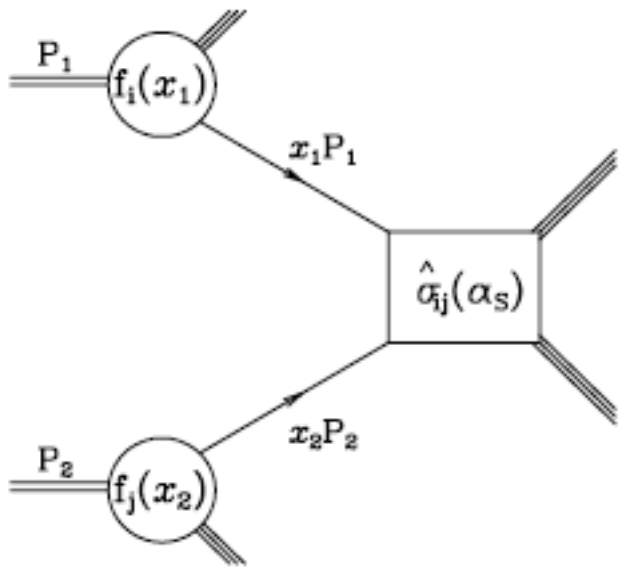
I review the status of next-to-leading order QCD as applied to hard processes at the Tevatron and the LHC. Next-to-leading order QCD generally leads to an improvement of the precision of theoretical calculations. I will briefly describe the theoretical advances that lead to optimism that all hard processes can be calculated at NLO. Recent applications of the NLO parton level generator MCFM will be presented.

Menu

- Theoretical setup
- Status of alphas and parton distributions
- MCFM and comparison with Tevatron data
- Theoretical advances in the calculation of one loop diagrams.
- Recent phenomenological results.

QCD improved parton model

Hard QCD cross section is represented as the convolution of a short distance cross-section and non-perturbative parton distribution functions.
Physical cross section is formally independent of μ_F and μ_R



Physical cross section

Parton distribution function

Renormalization scale μ_R

$$\sigma(P_1, P_2) = \sum_{i,j} \int dx_1 dx_2 \, f_i(x_1, \mu_F) f_j(x_2, \mu_F) \, \hat{\sigma}_{ij}(p_1, p_2, \alpha_S(\mu_R), Q^2, \mu_R, \mu_F).$$

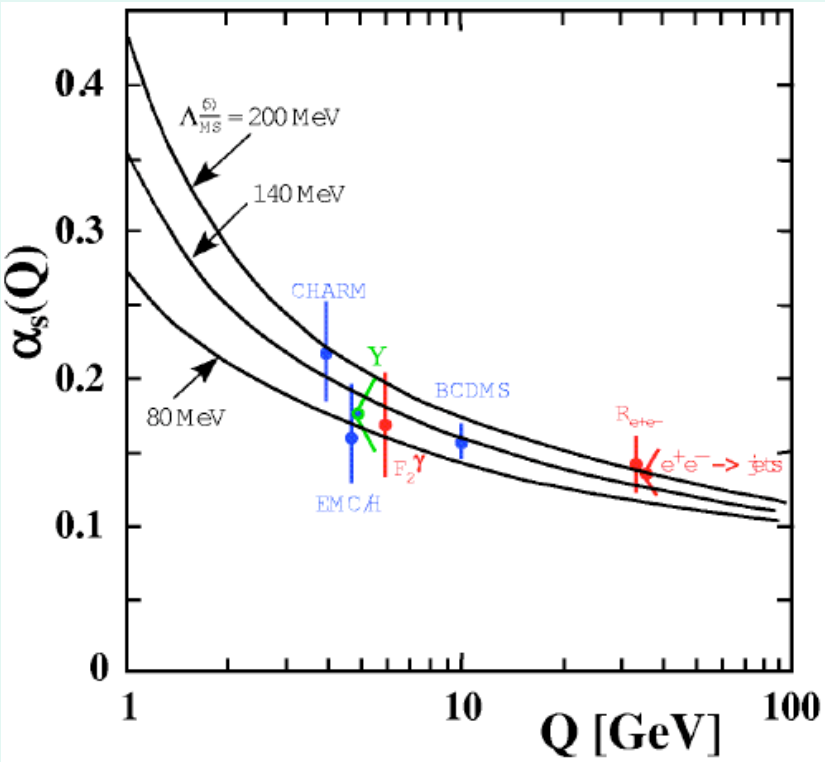
Factorization scale μ_F

Short distance cross section, calculated as a perturbation series in α_S

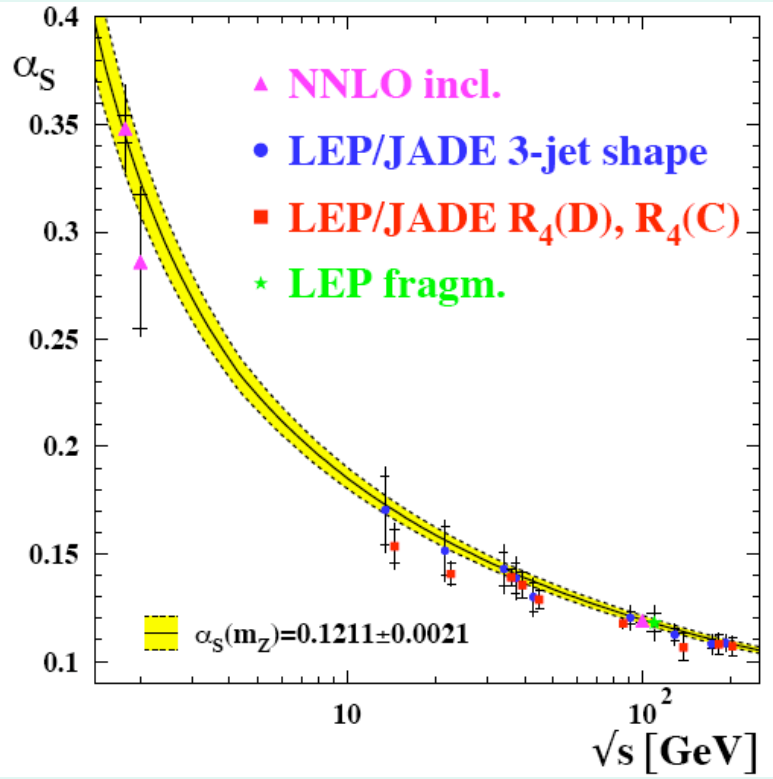
α_S

α_S is small(ish) at high energies because of the property of asymptotic freedom.

The role of LEP in determining the size of α_S has been crucial

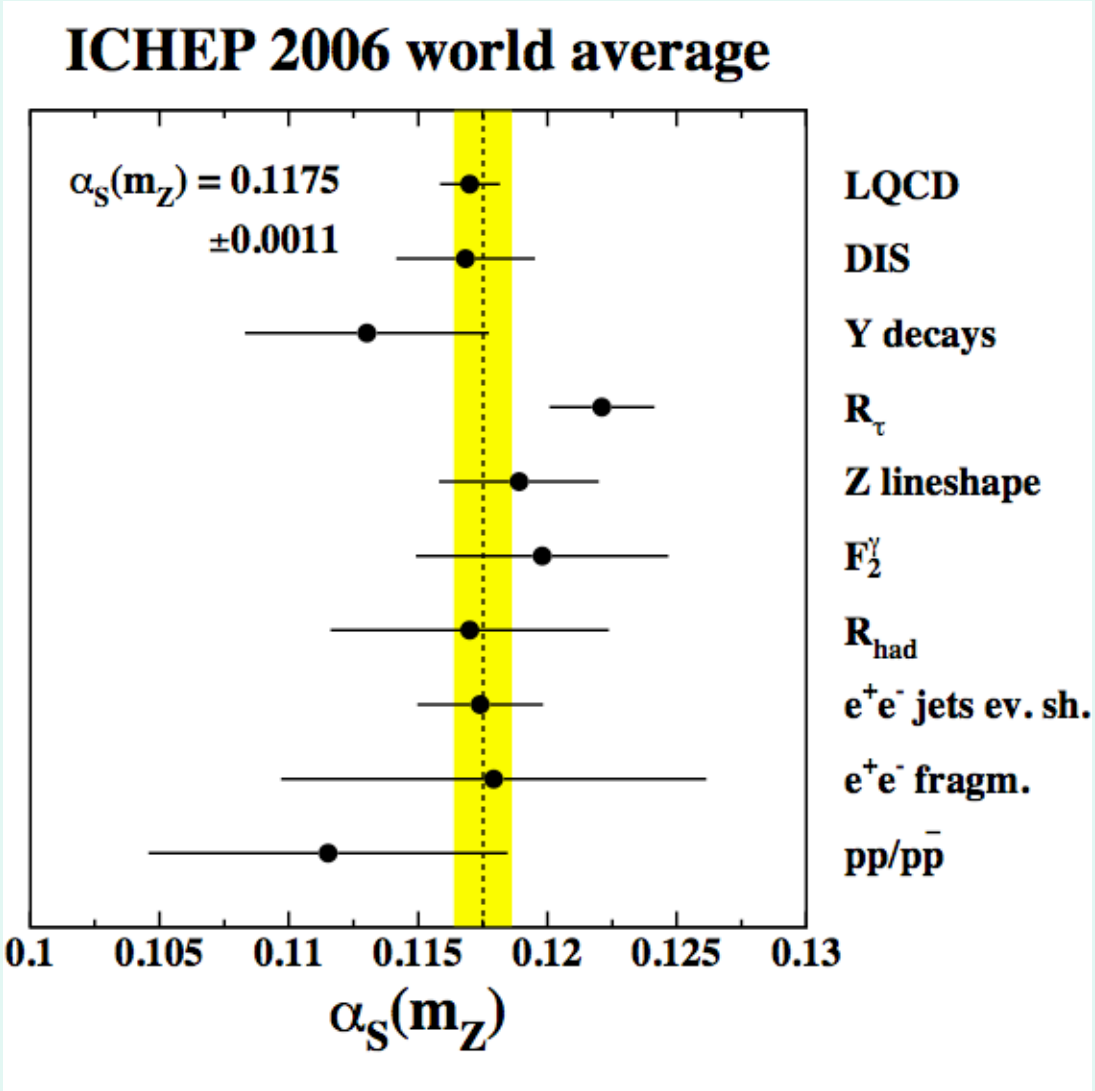


G. Altarelli 1989



S. Kluth EPS, 2007

S Kluth, hep-ex/0609020



2006 World average $\alpha_s(M_Z) = 0.1175 \pm 0.0011$

Parton distribution functions

Measurement of the non-perturbative parton distributions at lower energies allow extrapolations to higher values of μ and lower values of x using the DGLAP equation

$$\frac{\partial}{\partial \ln \mu^2} f_i(x,t) = \frac{\alpha_S(\mu)}{2\pi} \int_x^1 \frac{d\xi}{\xi} P_{ij}\left(\frac{x}{\xi}, \alpha_S(\mu)\right) f_j(\xi,\mu)$$

The evolution kernel is calculable as a perturbation series in α_s

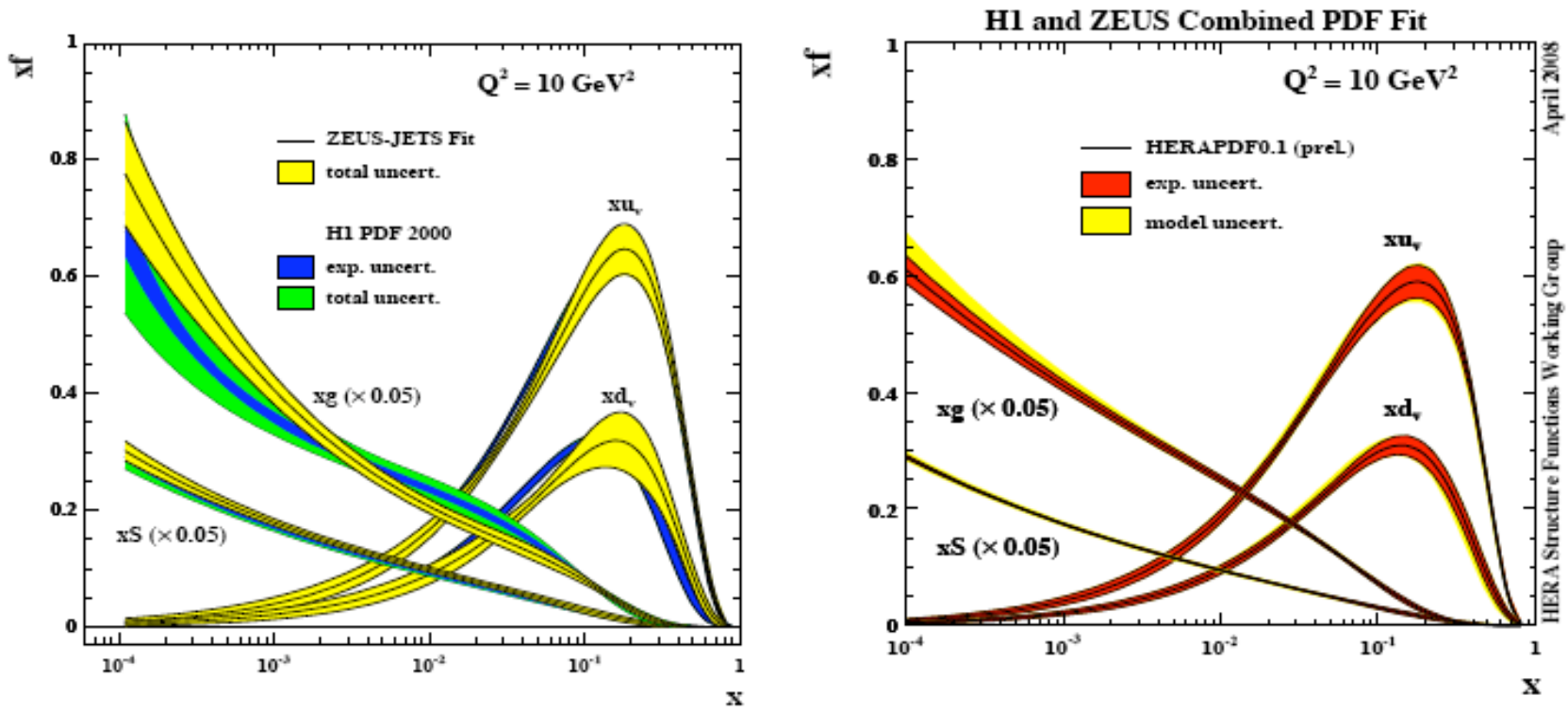
$$P_{ij}(z,\alpha_S) = P_{ij}^{(0)}(z) + \frac{\alpha_S}{2\pi} P_{ij}^{(1)}(z) + \left(\frac{\alpha_S}{2\pi}\right)^2 P_{ij}^{(2)}(z) \ldots$$

LO NLO NNLO

NNLO is known completely. (Moch et al, hep-ph/0403192)

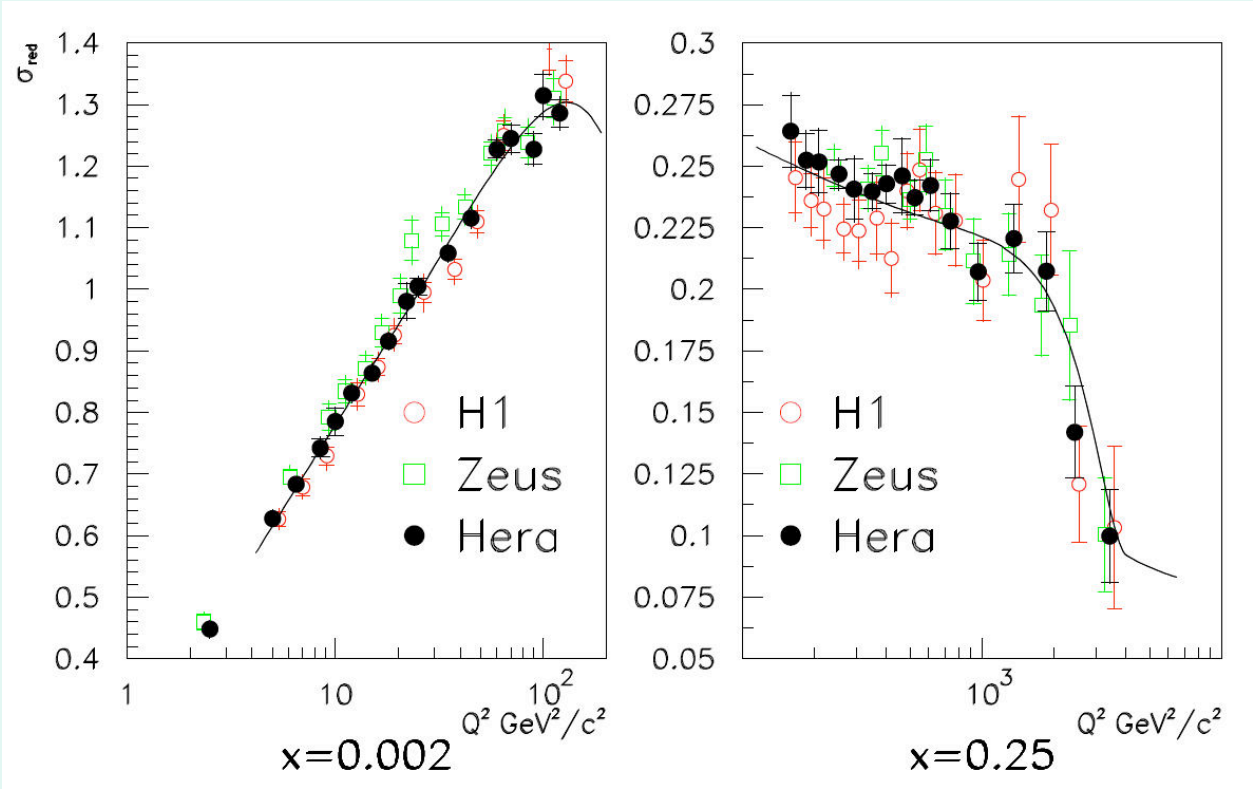
arXiv:0903.3861

Comparison of H1 and Zeus



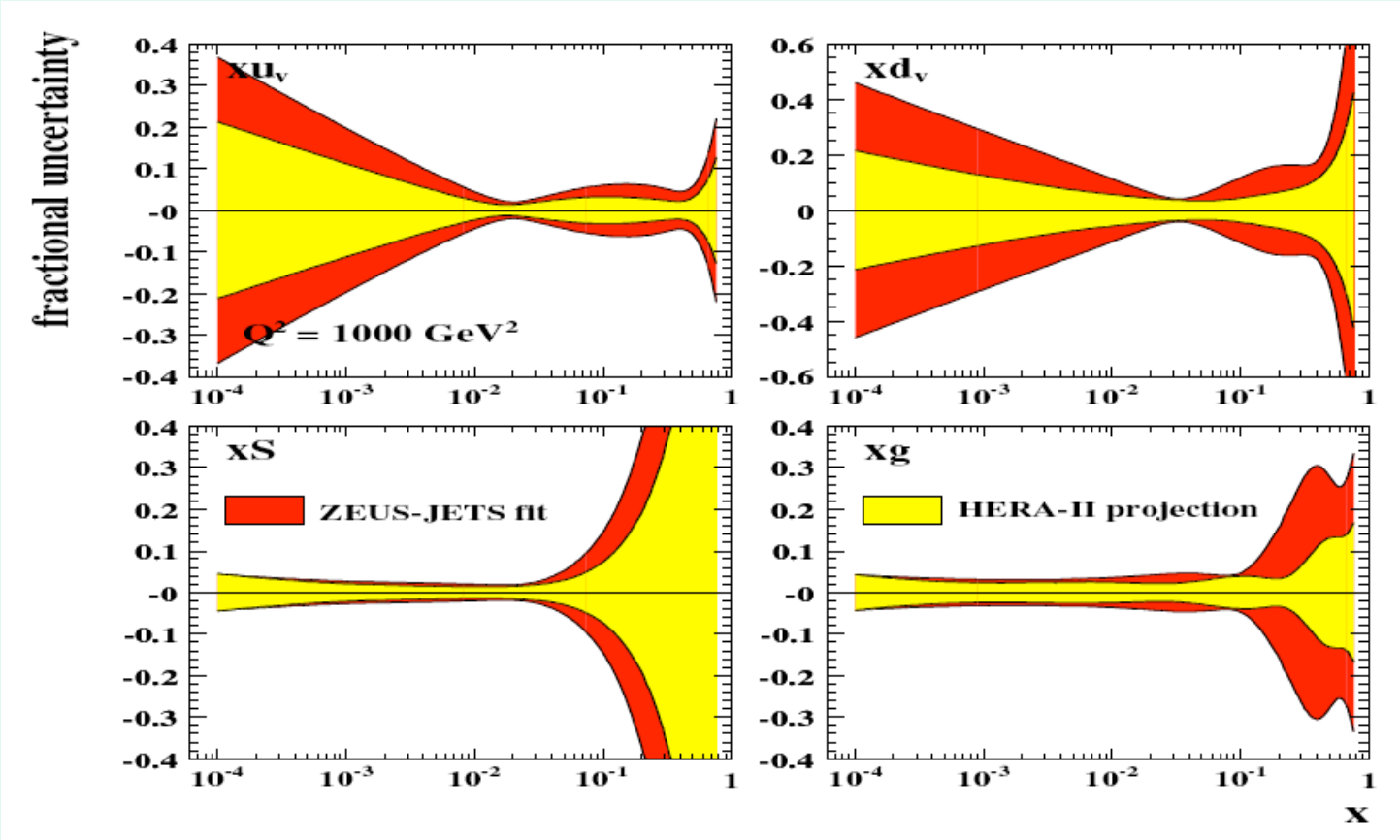
Some of the differences are understood (inclusion of BCDMS at large x (ZEUS) ; inclusion of jet data for mid x gluon (H1))

Combination of ZEUS and H1 data (taken from Rik Yoshida, Fermilab JETP seminar, 11/30/2007)



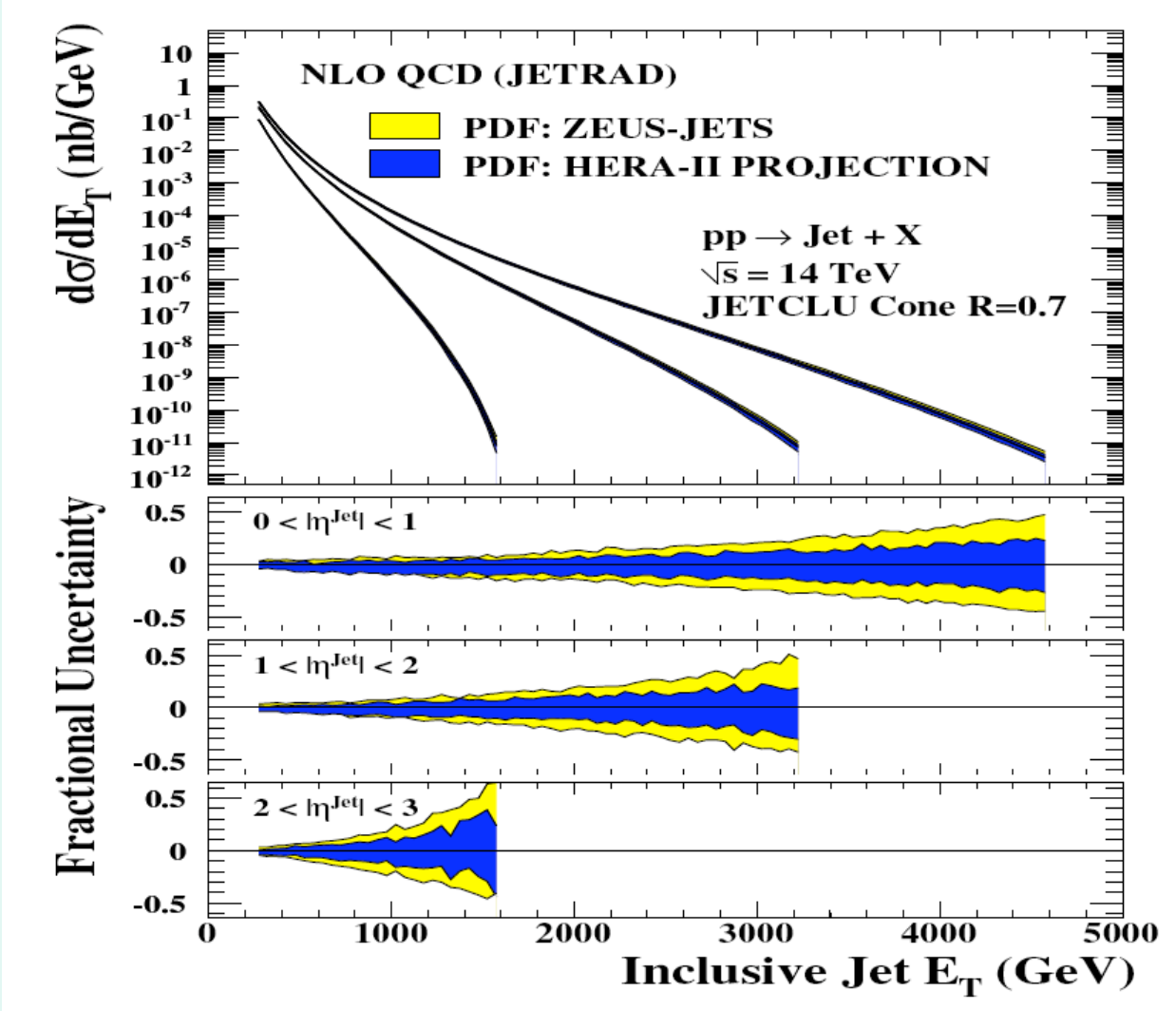
The improvement is (much) better than $\sqrt{2}$!
How is this possible?
"Systematics correlated across the kinematic plane, but uncorrelated between experiments cancel." (Yoshida)

Projected parton model uncertainties after HERAII

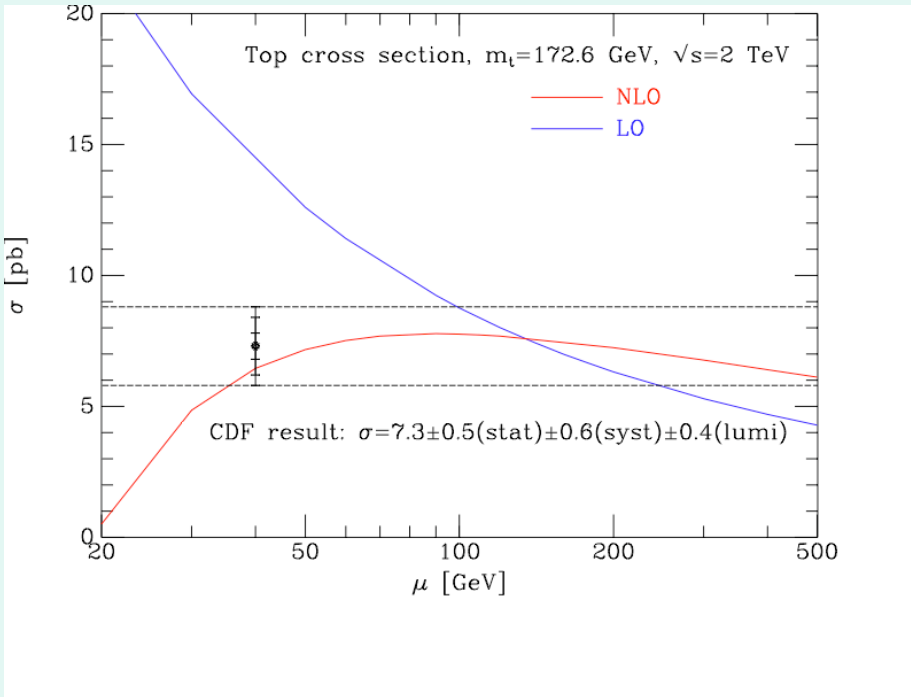


HERA for LHC workshop,hep-ph/0601012

…and consequent improvement on uncertainty of jet cross section



Why NLO?

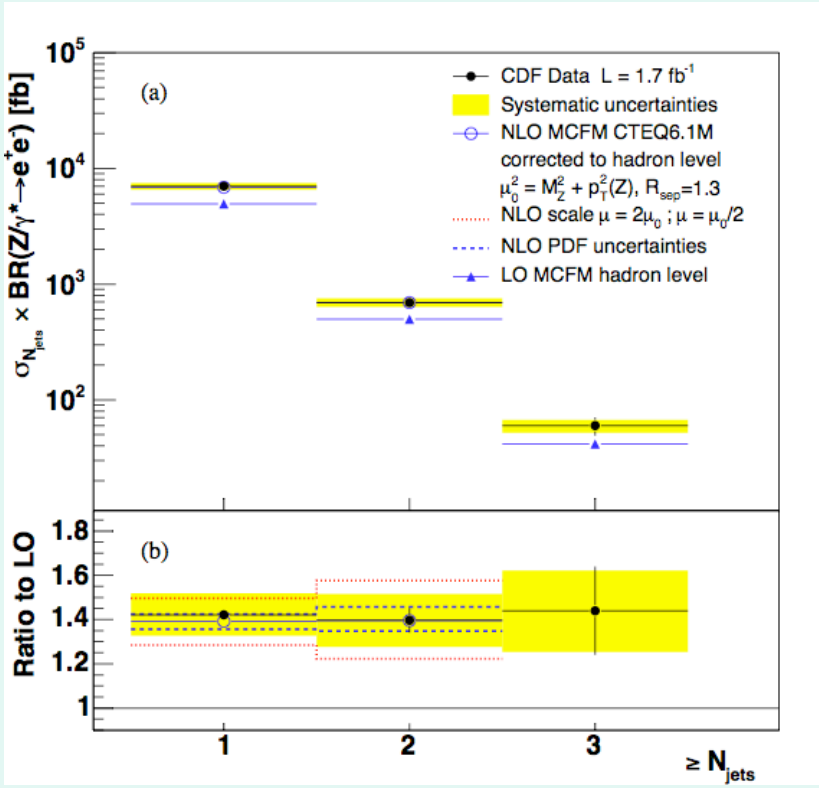


In order to get ~10% accuracy we need to include NLO.

- Less sensitivity to unphysical input scales, (eg. renormalization and factorization scales)
- NLO first approximation in QCD which gives an idea of suitable choice for μ.
- NLO has more physics, parton merging to give structure in jets, initial state radiation, more species of incoming partons enter at NLO.
- A necessary prerequisite for more sophisticated techniques which match NLO with parton showering.

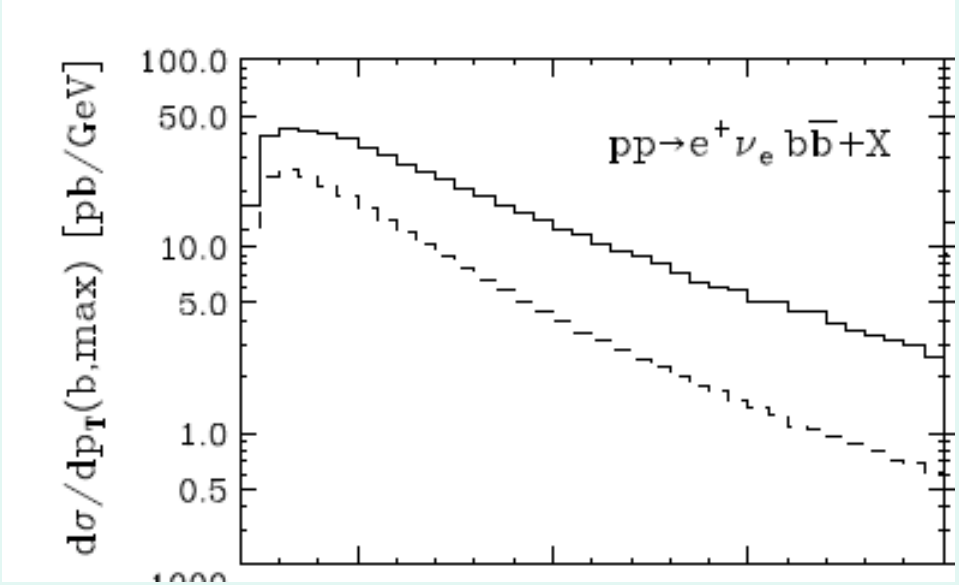
Isn't it just an overall K-factor?

Sometimes…..



Z+jet production at the Tevatron

but not always



NLO-solid, LO-dashed

Wbb production at the LHC

MCFM
A NLO parton level generator

- pp → W/Z
- pp → W+Z, WW, ZZ
- pp → W/Z + 1 jet
- pp → W/Z +2 jets
- pp → t W
- pp → tX (s&t channel)
- pp → tt
- pp → W/Z+H
- pp (gg) → H
- pp→(gg) → H + 1 jet
- pp→(gg) → H + 2 jets
- pp(VV) → H +2 jets
- pp → W/Z +b , W+c
- pp → W/Z +bb

Processes calculated at NLO, but no automatic procedure for including new processes.

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Code available at http://mcfm.fnal.gov

Current version 5.4 (March 11 2009)

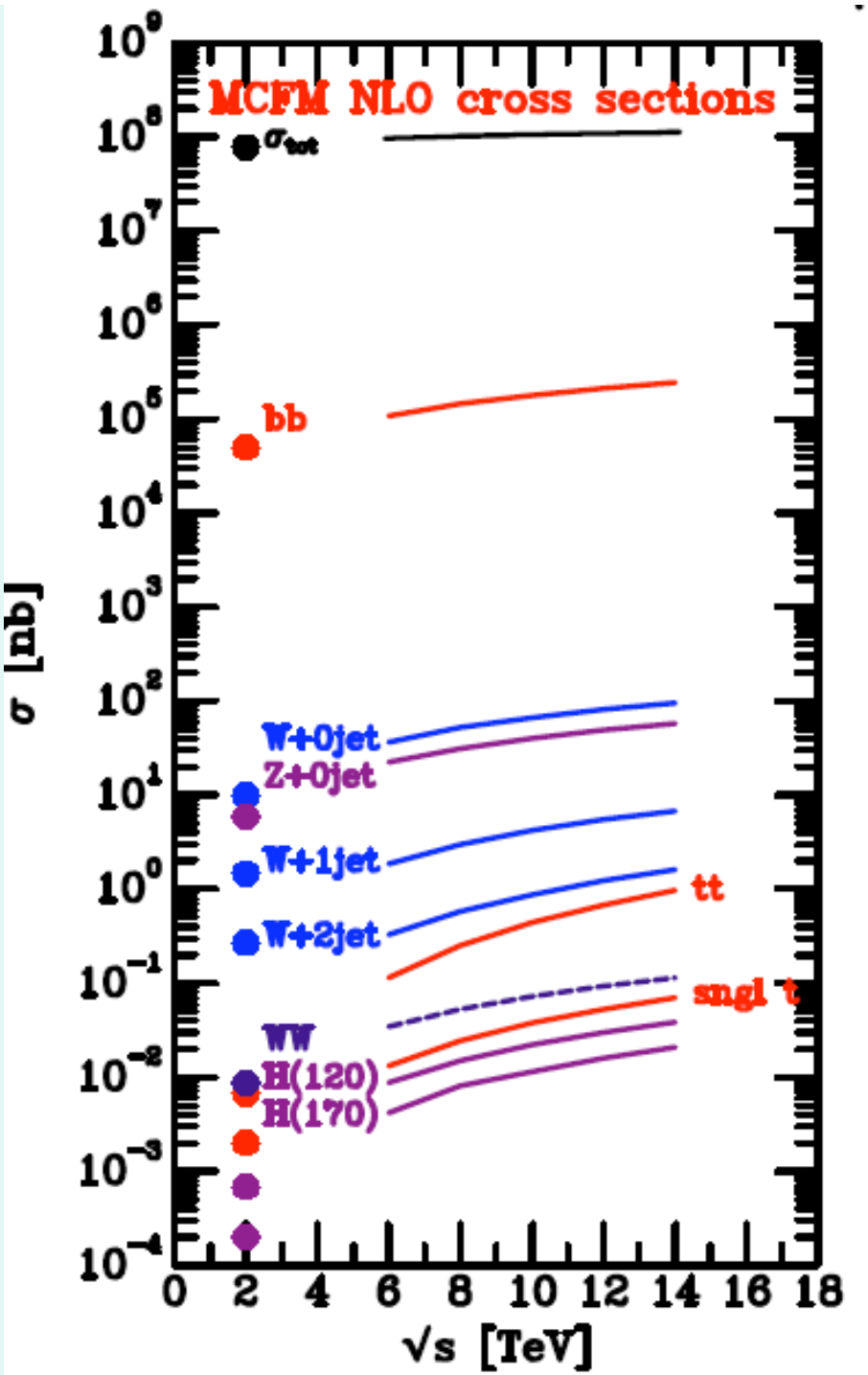
An experimenter's wishlist

Run II Monte Carlo Workshop

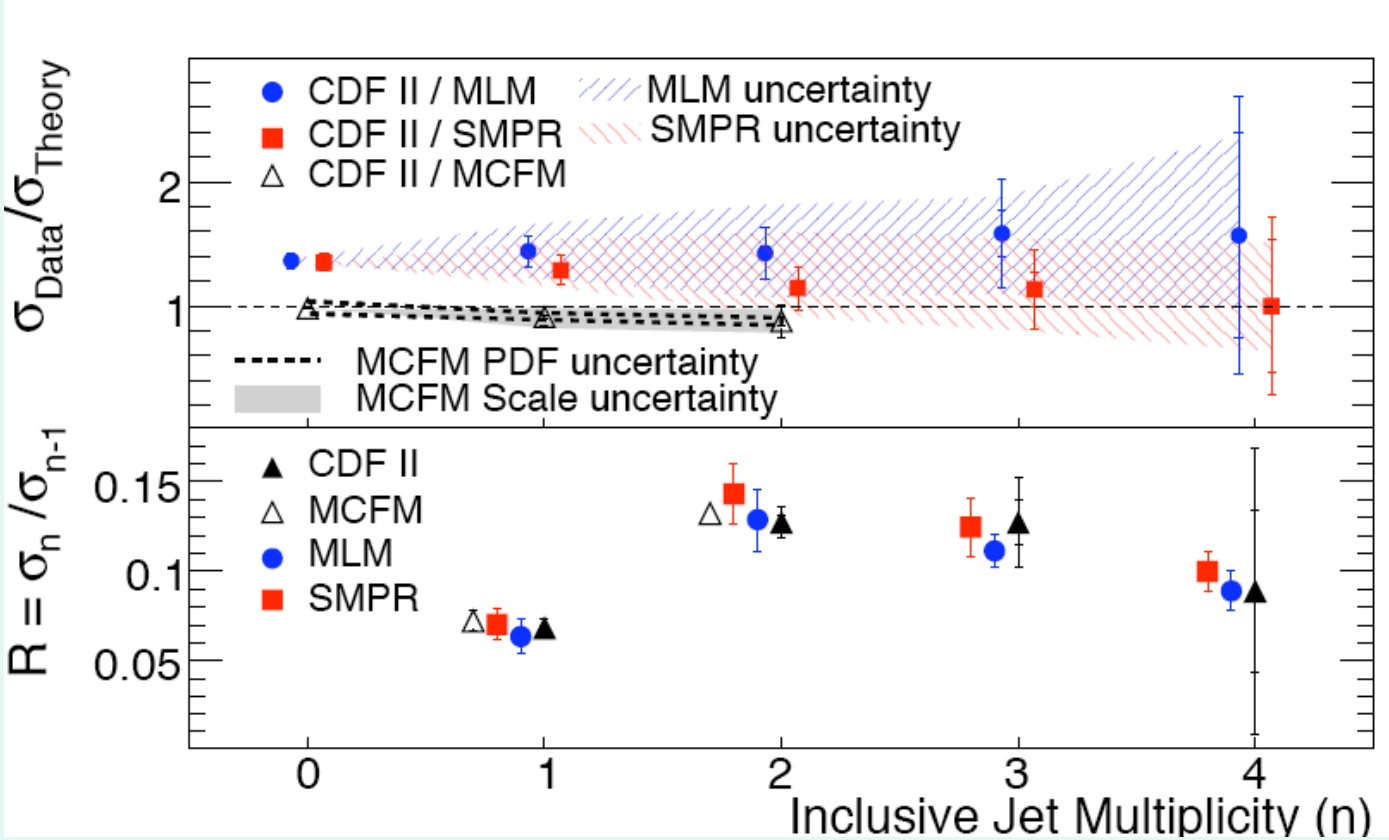
Single Boson	Diboson	Triboson	Heavy Flavour
$W+ \leq 5j$	$WW+ \leq 5j$	$WWW+ \leq 3j$	$t\bar{t}+ \leq 3j$
$W + b\bar{b} \leq 3j$	$W + b\bar{b}+ \leq 3j$	$WWW + b\bar{b}+ \leq 3j$	$t\bar{t} + \gamma+ \leq 2j$
$W + c\bar{c} \leq 3j$	$W + c\bar{c}+ \leq 3j$	$WWW + \gamma\gamma+ \leq 3j$	$t\bar{t} + W+ \leq 2j$
$Z+ \leq 5j$	$ZZ+ \leq 5j$	$Z\gamma\gamma+ \leq 3j$	$t\bar{t} + Z+ \leq 2j$
$Z + b\bar{b}+ \leq 3j$	$Z + b\bar{b}+ \leq 3j$	$ZZZ+ \leq 3j$	$t\bar{t} + H+ \leq 2j$
$Z + c\bar{c}+ \leq 3j$	$ZZ + c\bar{c}+ \leq 3j$	$WZZ+ \leq 3j$	$t\bar{b} \leq 2j$
$\gamma+ \leq 5j$	$\gamma\gamma+ \leq 5j$	$ZZZ+ \leq 3j$	$b\bar{b}+ \leq 3j$
$\gamma + b\bar{b} \leq 3j$	$\gamma\gamma + b\bar{b} \leq 3j$		single top
$\gamma + c\bar{c} \leq 3j$	$\gamma\gamma + c\bar{c} \leq 3j$		
	$WZ+ \leq 5j$		
	$WZ + b\bar{b} \leq 3j$		
	$WZ + c\bar{c} \leq 3j$		
	$W\gamma+ \leq 3j$		
	$Z\gamma+ \leq 3j$		

The big picture

- MCFM contains the best predictions for many processes of relevance for Tevatron and LHC
- LHC will be a great machine because of the increase of both energy and luminosity wrt to the Tevatron
- Dramatic growth with energy of gluon-induced processes (eg tt)



W+n jet rates from CDF



Both uncertainty on rates and deviation of Data/Theory from 1 are smaller than other calculations. The ratio R agrees well for all theory calculations but only available with from MCFM with small error for n≤2.

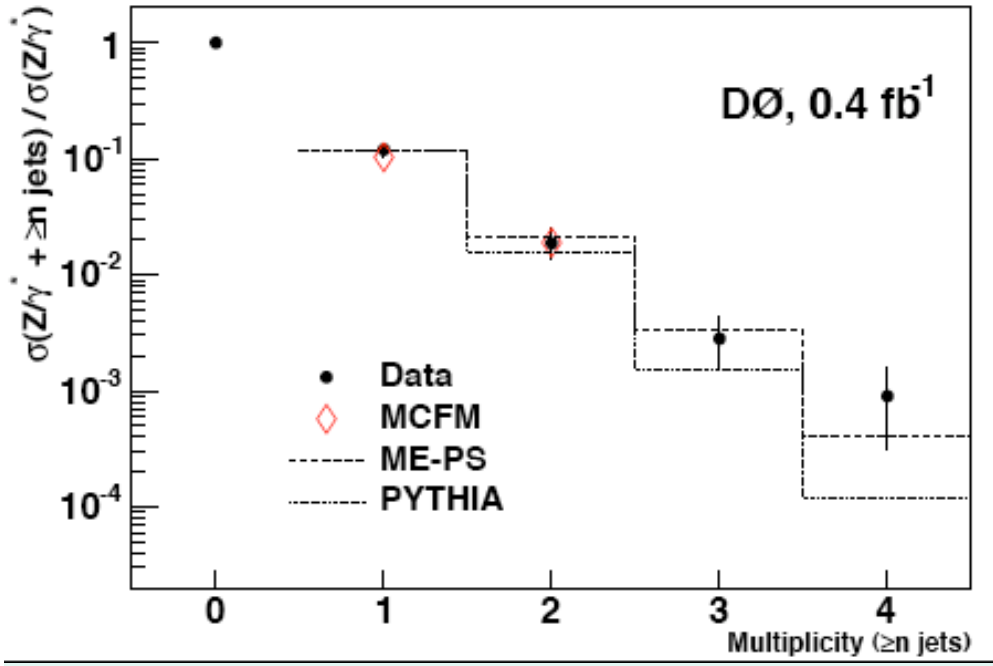
MCFM RKE, Campbell

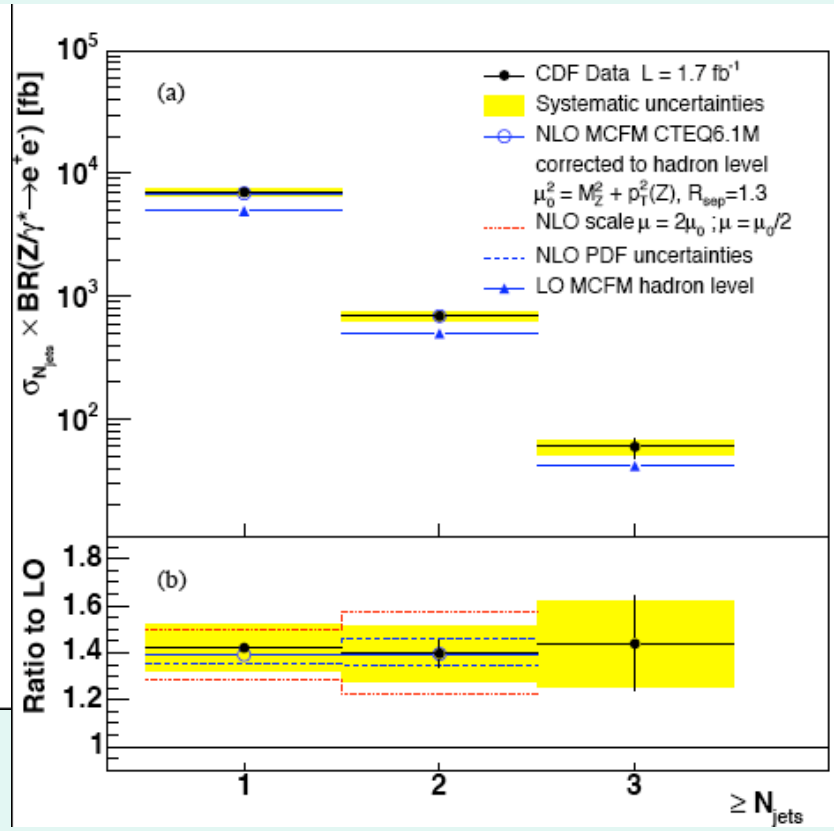
Z + jets

NLO QCD (which for Z + jets currently only available for n ≤ 2) does a good job of describing the inclusive cross sections.

D0 arXiv:hep/ex 0608052v3

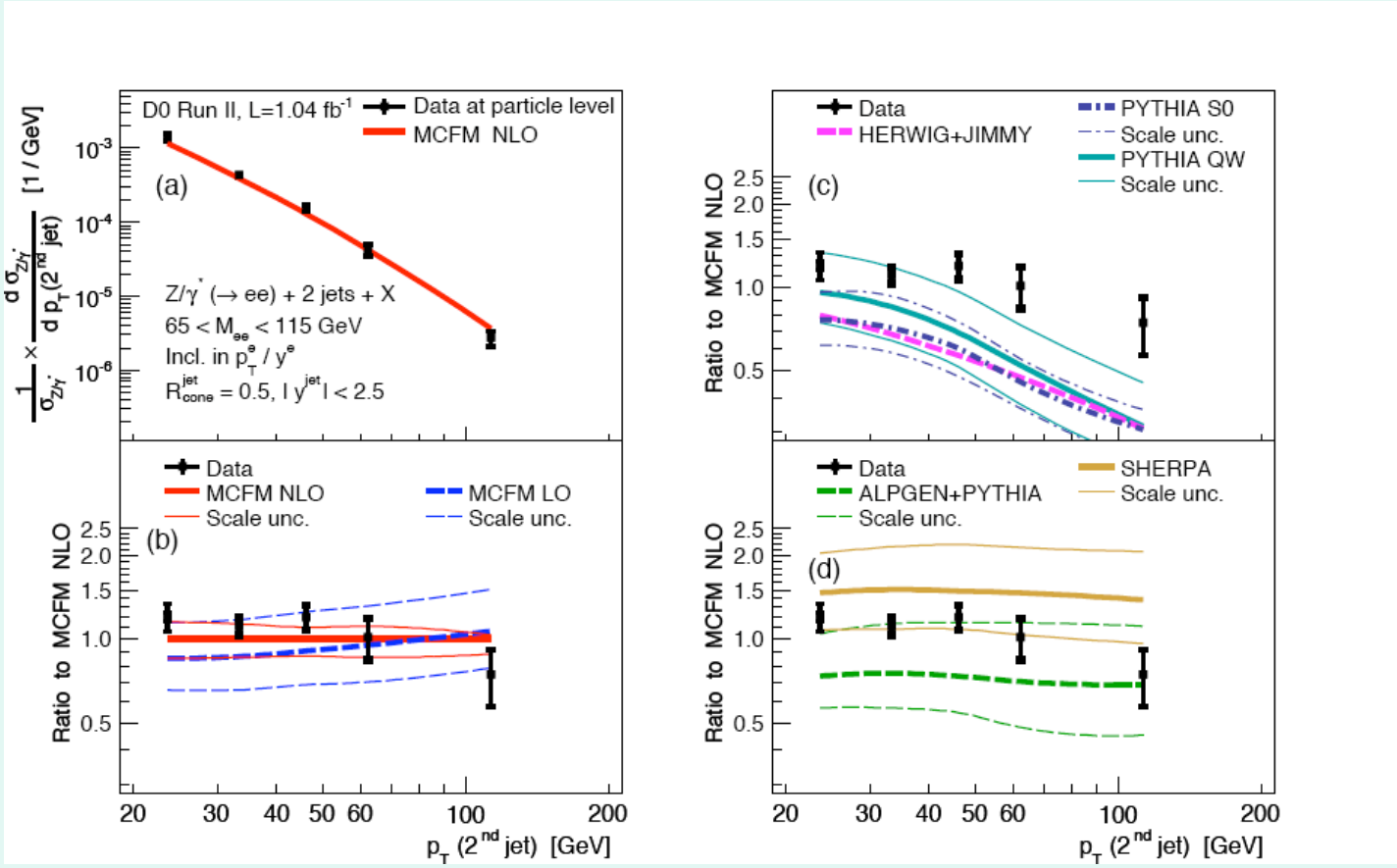
CDF arXiv:0711.3717v1





D0 arXiv:0903.1748

New Z + jets results from D0

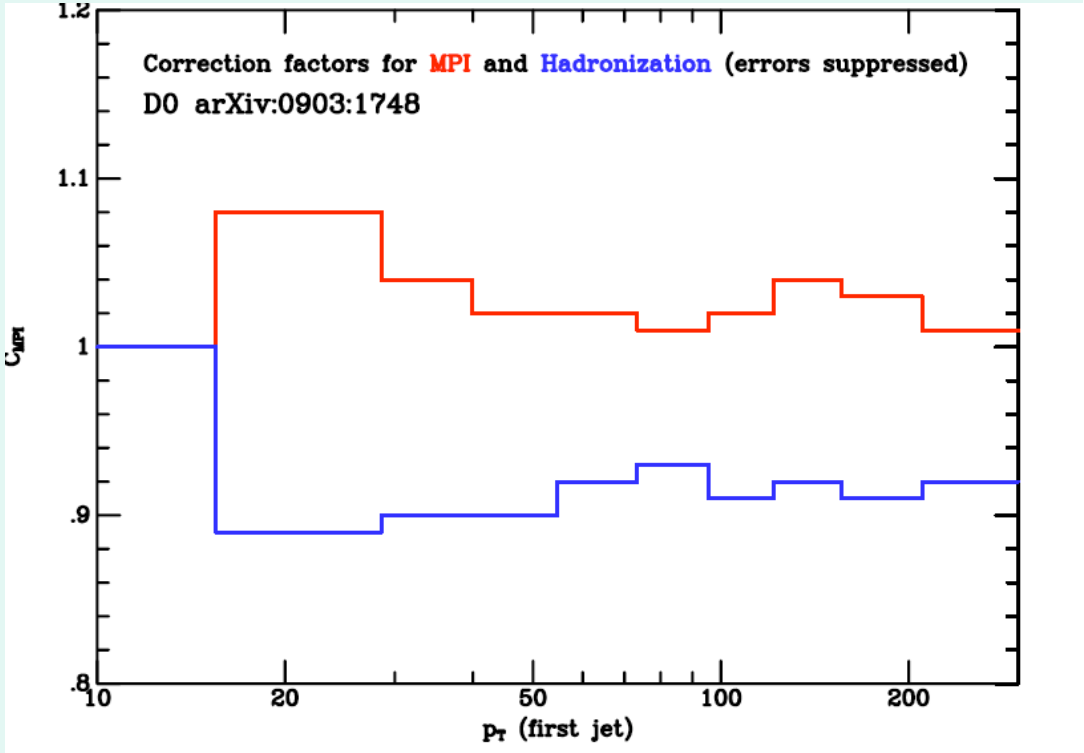


- MCFM, LO and NLO agrees with data;
- shower-based generators show significant differences with data;
- matrix element + parton shower models agree in shape, but with larger normalization uncertainties.

Comparison of parton level results with real data

Correction factors are required for multiple parton interactions and for hadronization.

The general pattern at the Tevatron is that these corrections are small, O(10%), opposite in sign and decreasing functions of P_T



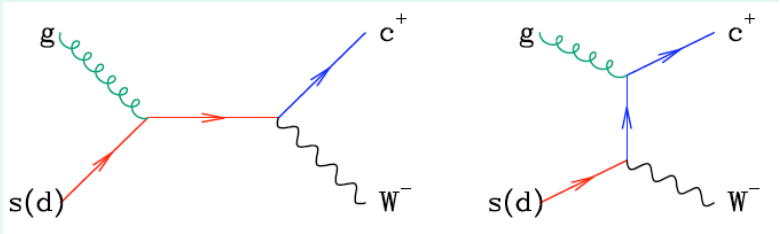
Vector boson + jets containing heavy flavors

- The data is much more limited
- The stakes are higher (single top, WH, ZH, …)
- The theory predictions can depend on poorly known heavy flavor distributions in the proton.
- The theory predictions can be performed with either a fixed or variable flavor schemes.

D0:PLB666(2008)23,CDF:PRL100(2008)091803

W + charm

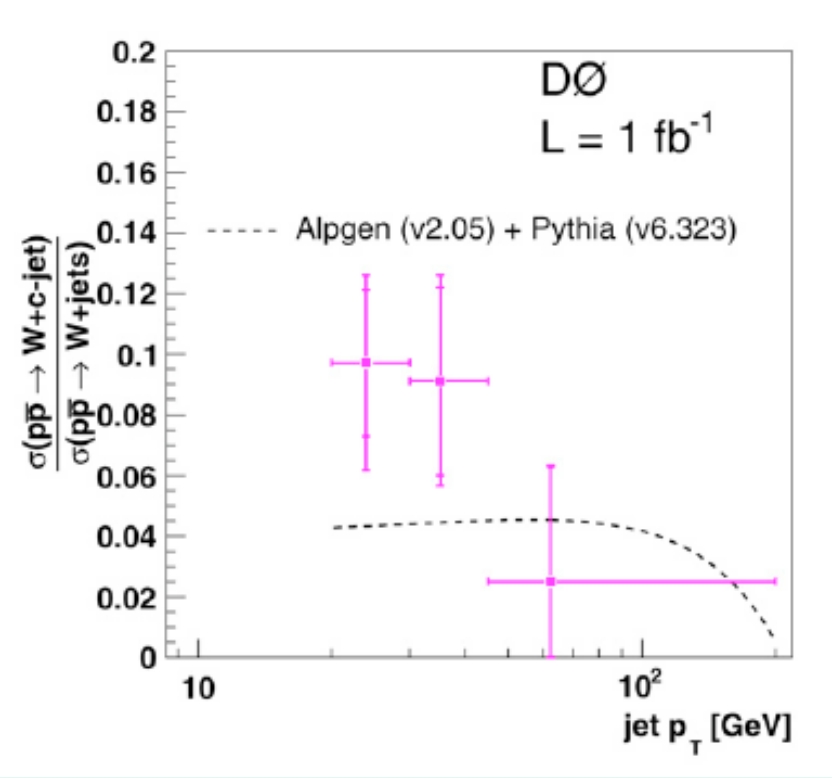
- W+charm samples are isolated by exploiting the opposite charge of the W and charm.
- Measurements from CDF and D0



$$\frac{\sigma[W+c\text{−jet}]}{\sigma[W+\text{jets}]} = 0.074 \pm 0.019(\text{stat})^{+0.012}_{-0.014}(\text{syst})$$

$$\frac{\sigma[W+c\text{−jet}]}{\sigma[W+\text{jets}]} = 0.044\ (\text{Alpgen})$$

$$\frac{\sigma[W+c\text{−jet}]}{\sigma[W+\text{jets}]} = 0.045\ (\text{MCFM})$$



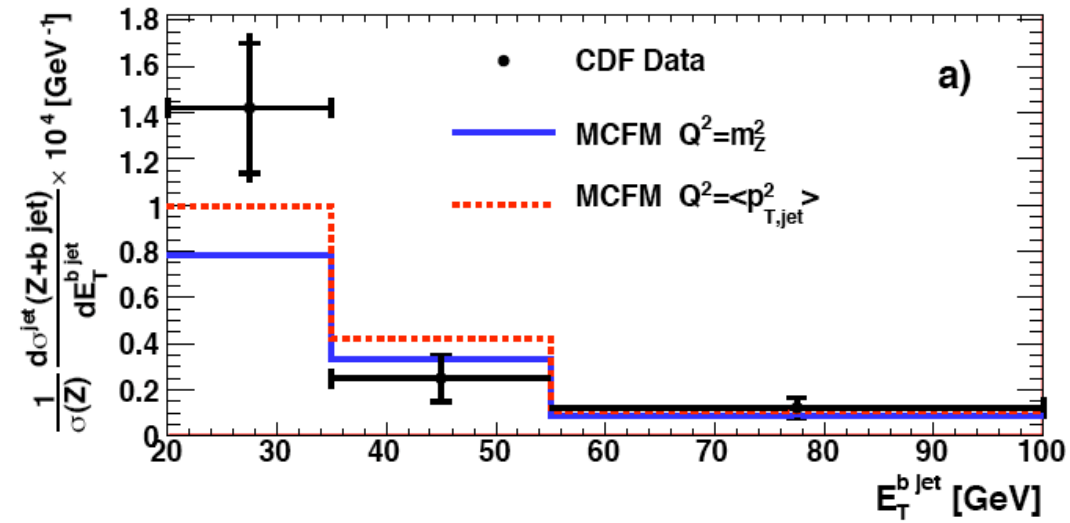
- Agreement within the large errors, (theory errors ~ 10%)
- It will be important to have the more precise kinematic distributions.

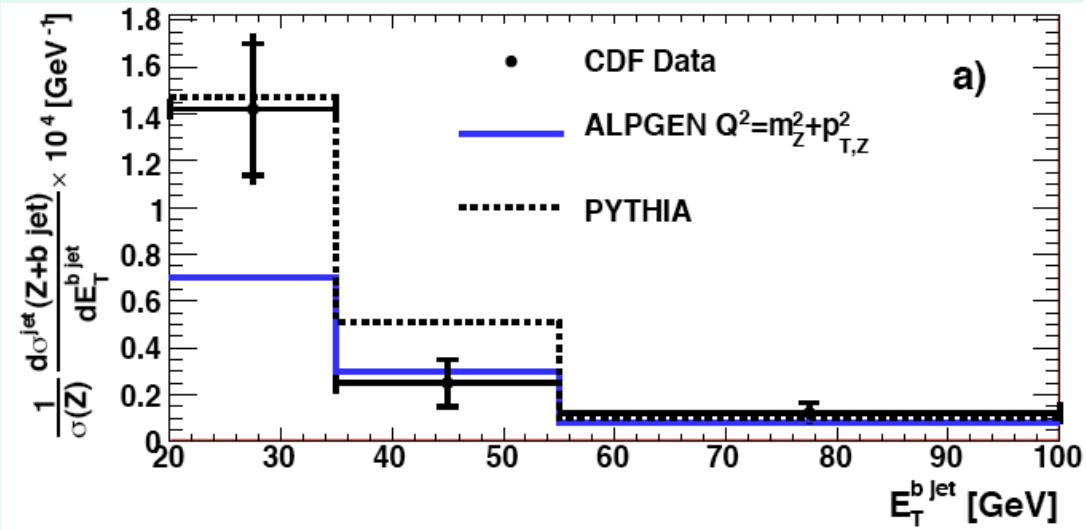
CDF arXiv:0812.4458

Z + bottom

- Data still quite limited
- Low scale preferred to try and explain data
- All of the models seem to have difficulty to describe the shape of the data.
- CDF result is:-

$$E_T > 20 \text{ GeV}, \quad |\eta| < 1.5$$
$$\sigma(Z + b\text{jet})/\sigma(Z) =$$
$$(3.32 \pm 0.53(\text{stat}) \pm 0.42(\text{syst})) \times 10^{-3}$$



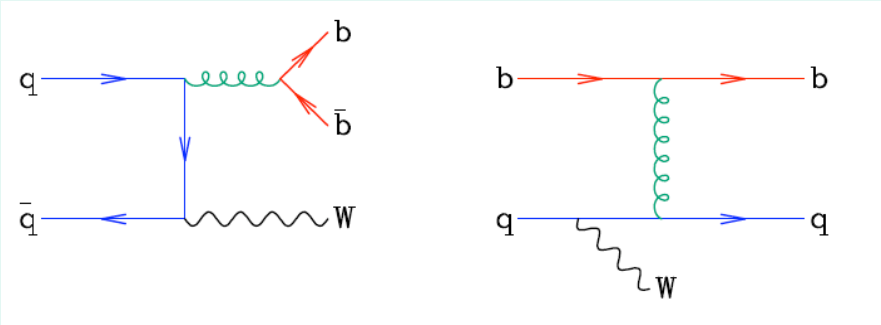


W + bottom

- W + 1 or 2 jets, either or both of which may be b-tagged.
- Important for single top, WH, etc …
- CDF measurements, b's identified by secondary vertex tag, (1 or 2 E_T>20GeV,|η|<2 jets (CDF Note 9321)

$$\sigma_{b-\text{jets}}(W + b-\text{jets}) \times \text{BR}(W \to \ell\nu) = 2.74 \pm 0.27(\text{stat}) \pm 0.42(\text{syst})\text{pb}$$
$$\sigma_{b-\text{jets}}(W + b-\text{jets}) \times \text{BR}(W \to \ell\nu)\text{Alpgen} = 0.78\text{pb}$$

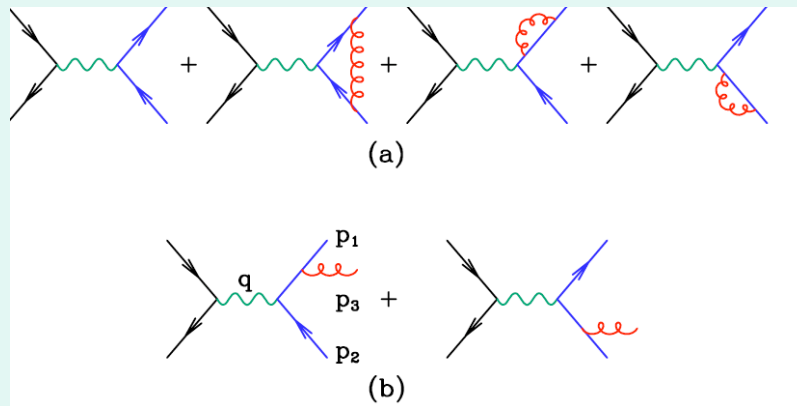
- Ongoing work to combine two sources of W+b events at NLO, (but still hard to explain a factor of 3.5)



Ingredients for a NLO calculation

- Born process LO
- Interference of one-loop with LO
- Real radiation (also contributes to the two jet rate in the region of soft or collinear emission).

Example $e^+e^- \Rightarrow$ 2 jets



Virtual:

$$\sigma^{q\bar{q}(g)}(\epsilon) = \sigma_0\, 3 \sum_q Q_q^2 \frac{C_F \alpha_s}{2\pi} H(\epsilon) \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2 + O(\epsilon) \right]$$

Real:

$$\sigma^{q\bar{q}g}(\epsilon) = \sigma_0\, 3 \sum_q Q_q^2 \frac{C_F \alpha_s}{2\pi} H(\epsilon) \left[\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2 + O(\epsilon) \right]$$

Sum:

$$R = 3 \sum_q Q_q^2 \left\{ 1 + \frac{\alpha_s}{\pi} + O(\alpha_s^2) \right\}$$

General NLO parton integrator

- We want to compute a cross section $\sigma = \sigma^{LO} + \sigma^{NLO}$.
- Born approximation involves m partons in the final state $\sigma^{LO} = \int_m d\sigma^B$
- At NLO we have real correction with m+1 final state partons, and a virtual correction with m partons

$$\sigma^{NLO} \equiv \int d\sigma^{NLO} = \int_{m+1} d\sigma^R + \int_m d\sigma^V$$

- Two integrals are separately divergent in four dimensions, although their sum is finite.

Finiteness

- The general idea of the subtraction method is to use the identity $d\sigma^{NLO} = \left[d\sigma^R - d\sigma^A\right] + d\sigma^A + d\sigma^V$
- $d\sigma^A$ is and approximation to $d\sigma^R$ which has the same singular behavior point-by-point as $d\sigma^R$
- Final results consists of two terms which are separately finite.

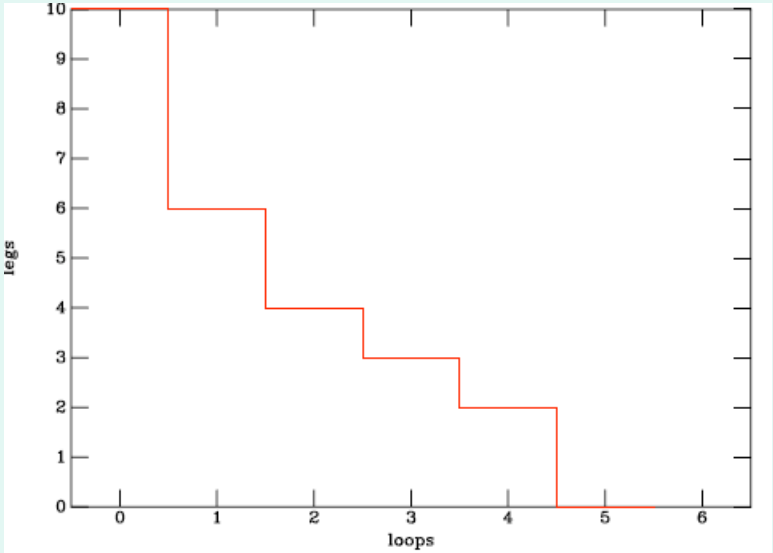
$$\sigma^{NLO} = \int_{m+1} \left[d\sigma^R - d\sigma^A\right] + \int_{m+1} d\sigma^A + \int_m d\sigma^V$$

Short-distance cross section

For a hard process the short distance cross section can be calculated in various approximations.

Leading order (LO) tree graphs	α_S^n
Next-to-leading order (NLO)	α_S^{n+1}
Next-to-next_to_leading order (NNLO)	α_S^{n+2}

Current state of the art can calculate large number of loops and small number of legs or a smaller number of legs and a larger number of loops.



Extension to multi-leg processes

- At the LHC we are interested in processes with many jets; these have standard model backgrounds involving many legs.
- The NLO calculation of multileg processes is pressing because the dependence on the unphysical scales is so strong.
- We need both efficient methods to calculate tree diagrams and efficient methods to calculate loops.

Tree graphs

The calculation of any tree graph is essentially solved.

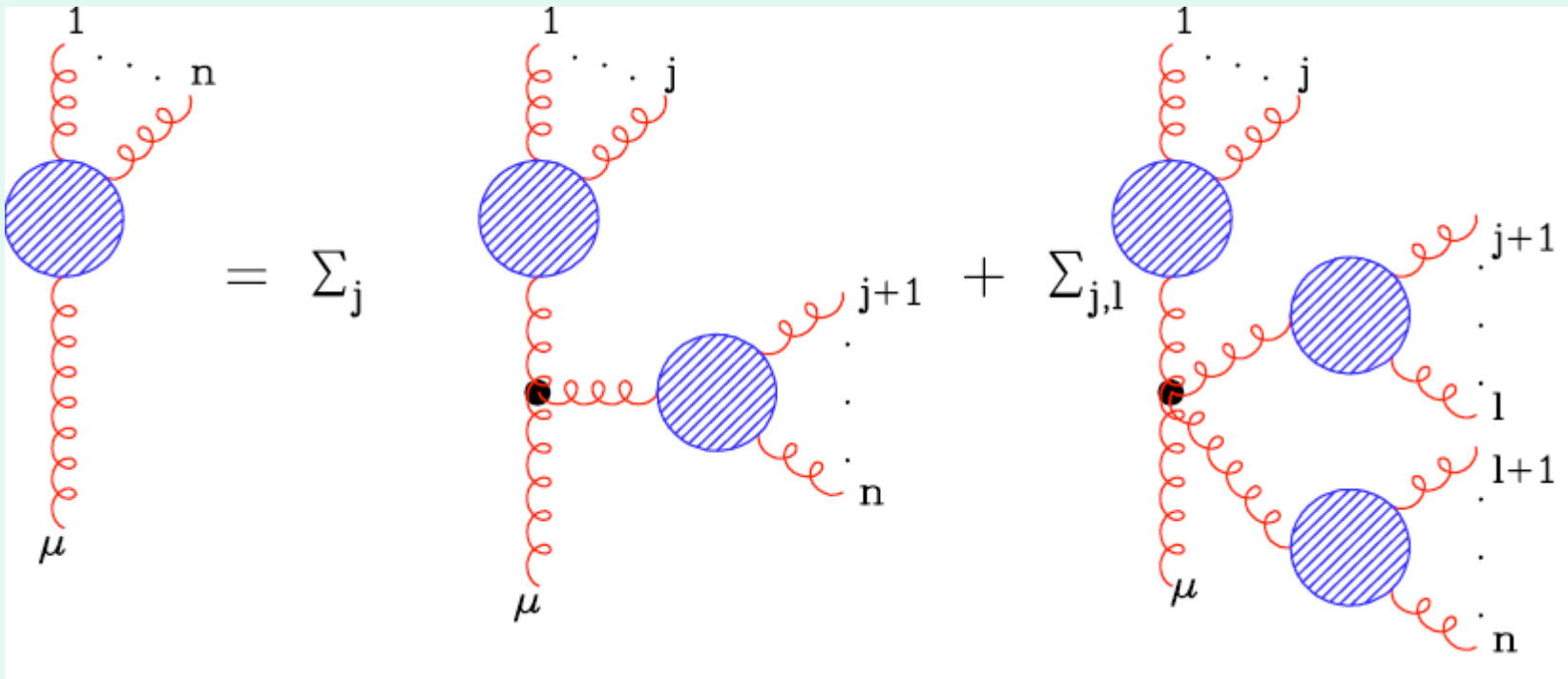
- Berends-Giele recursion } Off-shell recursion
- Madgraph/Helas Feynman diagrams
- MHV based recursion
- BCFW on-shell recursion

} On-shell recursion techniques

- Comparison of methods

Berends-Giele recursion

Building blocks are non-gauge invariant color-ordered off-shell currents. Off-shell currents with n legs are related to off-shell currents with fewer legs (shown here for the pure gluon case).



Despite the fact that it is constructing the complete set of Feynman diagrams, BG recursion is a very economical scheme.

Berends Giele recursion

- Time required for calculation of gluonic amplitudes grows as N^4 where N is the number of legs. Kleiss, Kuijf, Nucl. Phys.B312:616,1989
- Recursive scheme, so simple to program
- Extension to complex momenta and integer dimensions different than four is straightforward.

Dinsdale, Ternick, Weinzierl, cf. Duhr, Hoche & Maltoni

Comparison of speed for numerical evaluation

Tree level amplitude with n external gluons may be written as

$$\mathcal{A}_n(k_1^{\lambda_1},\ldots,k_n^{\lambda_n}) = g^{n-2} \sum_{\sigma\in S_n/Z_n} 2\,\mathrm{Tr}\left(T^{a_{\sigma(1)}}\ldots T^{a_{\sigma(n)}}\right) A_n\left(k_{\sigma(1)}^{\lambda_{\sigma(1)}},\ldots,k_{\sigma(n)}^{\lambda_{\sigma(n)}}\right)$$

Leading color matrix element squared is given by

$$\mathcal{M}_n = \sum_{\lambda_1,\ldots,\lambda_n} \left|A_n\left(k_1^{\lambda_1},\ldots,k_n^{\lambda_n}\right)\right|^2$$

CPU time in seconds to calculate $\mathcal{M}_n$ using the various methods

n	4	5	6	7	8	9	10	11	12
Berends-Giele	0.00005	0.00023	0.0009	0.003	0.011	0.030	0.09	0.27	0.7
Scalar	0.00008	0.00046	0.0018	0.006	0.019	0.057	0.16	0.4	1
MHV	0.00001	0.00040	0.0042	0.033	0.24	1.77	13	81	—
BCF	0.00001	0.00007	0.0003	0.001	0.006	0.037	0.19	0.97	5.5

Conclusion on calculation of tree graphs

- For calculation of six jet rate, BCFW shows a modest advantage in computer speed over BG (for leading color amplitudes).
- Once full color information is included even this advantage is removed. (Duhr, Hoche, Maltoni)
- Berends-Giele off-shell recursion is universal, fast enough and simple to program.

Theoretical digression on the calculation of one loop amplitudes

- The classical paradigm for the calculation of one-loop diagrams was established in 1979.
- Complete calculation of one-loop scalar integrals
- Reduction of tensors one-loop integrals to scalars.

Nuclear Physics B153 (1979) 365–401
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SCALAR ONE-LOOP INTEGRALS

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Received 16 January 1979

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ONE-LOOP CORRECTIONS FOR e^+e^- ANNIHILATION INTO $\mu^+\mu^-$ IN THE WEINBERG MODEL

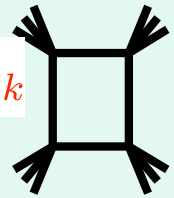
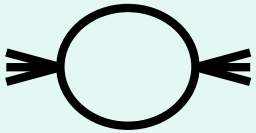
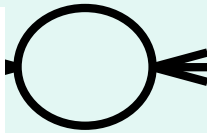
G. PASSARINO* and M. VELTMAN
Institute for Theoretical Physics, University of Utrecht, Utrecht, The Netherlands

Received 22 March 1979

Neither will be adequate for present-day purposes.

Basis set of scalar integrals

Any one-loop amplitude can be written as a linear sum of boxes, triangles, bubbles and tadpoles

$\mathcal{A}_N(\{p_i\}) = \sum d_{ijk}$  $+ \sum c_{ij}$  $+ \sum b_i$  $+ \sum_i a_i$ 

In addition, in the context of NLO calculations, scalar higher point functions, can always be expressed as sums of box integrals. Passarino, Veltman - Melrose ('65)

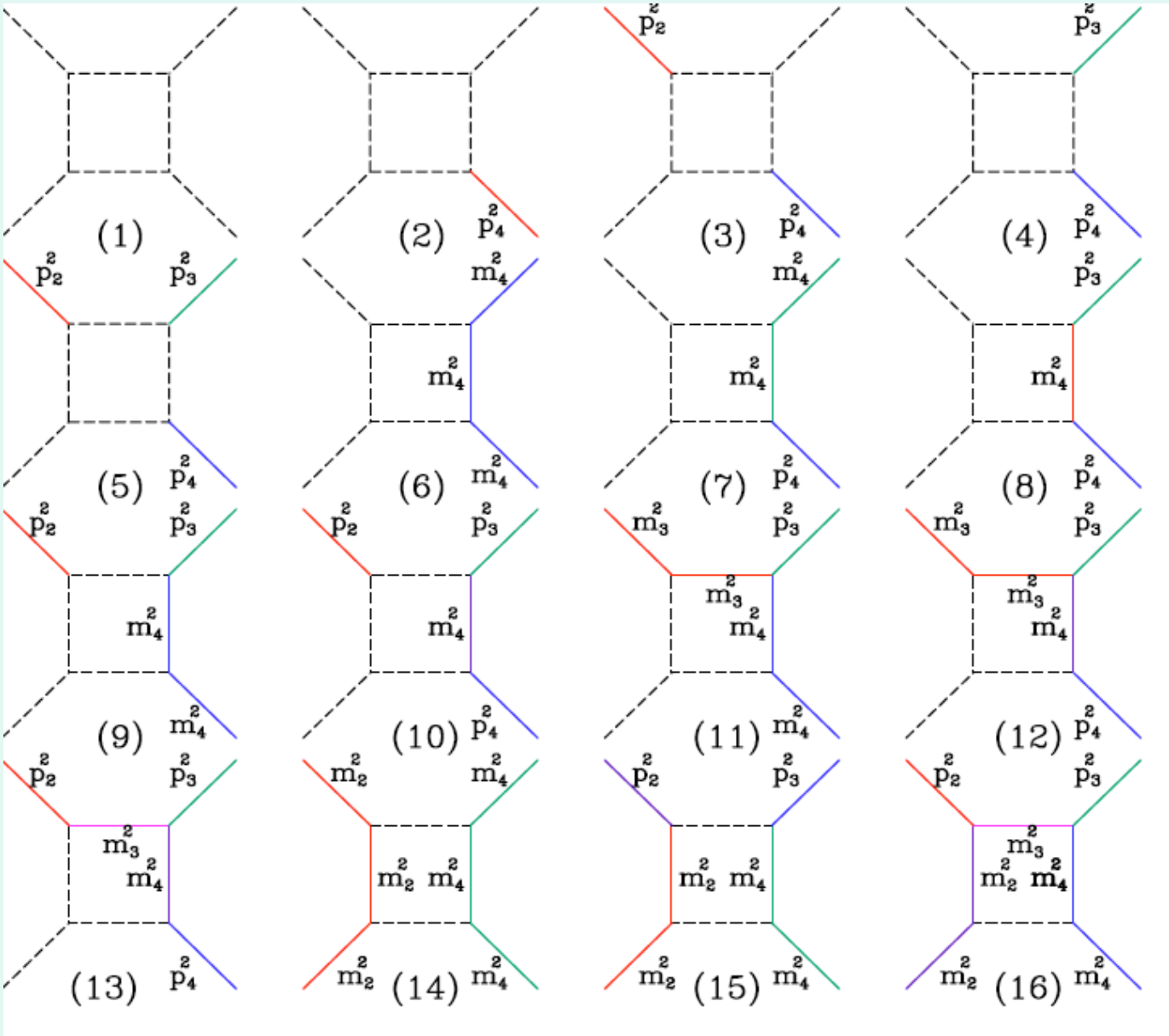
- Scalar hexagon can be written as a sum of six pentagons.
- For the purposes of NLO calculations, the scalar pentagon can be written as a sum of five boxes.
- In addition to the 'tH-V integrals we need integrals containing infrared and collinear divergences.

Scalar one-loop integrals

- ‘t Hooft and Veltman’s integrals contain internal masses; however in QCD many lines are (approximately) massless. The consequent soft and collinear divergences are regulated by dimensional regularization.
- So we need general expressions for boxes, triangles, bubbles and tadpoles, including the cases with one or more vanishing internal masses.

Basis set of sixteen divergent box integrals

RKE, Zanderighi

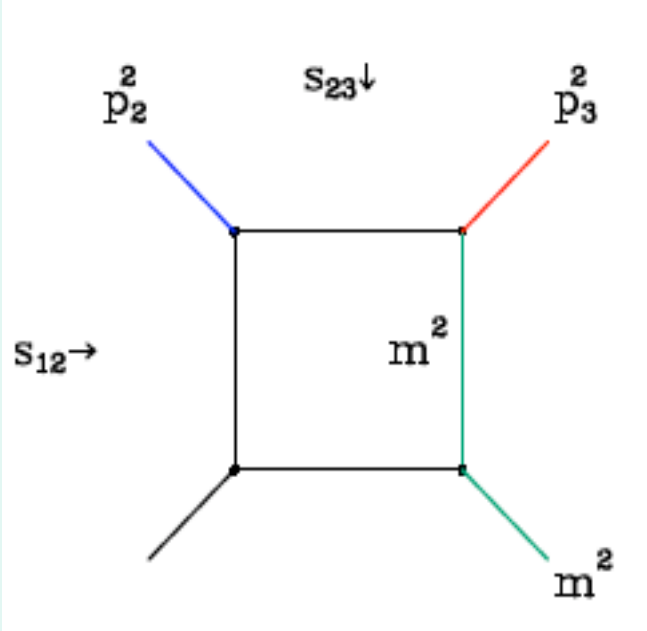


QCDLoop

- Analytic results are given for the complete set of divergent box integrals at http://qcdloop.fnal.gov
- Fortran 77 code is provided which calculates an arbitrary scalar box, triangle, bubble or tadpole integral.
- Finite integrals are calculated using the ff library of Van Oldenborgh. (Used also by Looptools)
- For divergent integrals the code returns the coefficients of the Laurent series $1/\varepsilon^2$, $1/\varepsilon$ and finite.
- Problem of scalar integrals for one-loop calculations is completely solved numerically and analytically!

Example of box integral from qcdloop.fnal.gov

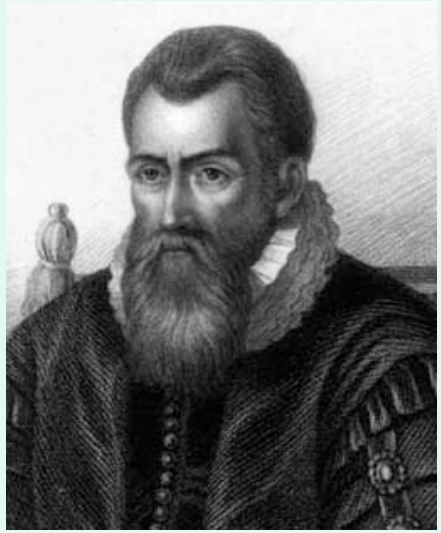
Basis set of 16 basis integrals allows the calculation of any divergent box diagram.
Result given in the spacelike region.
Analytic continuation as usual by
$s_{ij} \Rightarrow s_{ij} + i\,\varepsilon$



$$I_4^{\{D=4-2\epsilon\}}(0,p_2^2,p_3^2,m^2;s_{12},s_{23};0,0,0,m^2) = \frac{1}{s_{12}(s_{23}-m^2)}\left[\frac{1}{2\epsilon^2} - \frac{1}{\epsilon}\ln\left(\frac{s_{12}}{p_2^2}\frac{(m^2-s_{23})}{\mu m}\right)\right.$$
$$\left. + \;\mathrm{Li}_2\left(1+\frac{(m^2-p_3^2)(m^2-s_{23})}{m^2p_2^2}\right) + 2\,\mathrm{Li}_2\left(1-\frac{s_{12}}{p_2^2}\right) + \frac{\pi^2}{12} + \ln^2\left(\frac{s_{12}}{p_2^2}\frac{(m^2-s_{23})}{\mu m}\right)\right] + \mathcal{O}(\epsilon)\,.$$

Limit $p_3^2 = 0$ can be obtained from this result, (limit $p_2^2 = 0$ cannot)

Scottish functions.



- We have expressed the one-loop integrals entirely in Scottish functions, logarithms (Napier) and dilogarithms (Spence).
- Shown are John Napier, 1550-1617, laird of Merchiston, inventor of the Napierian logarithm and William Spence of Greenock (1777-1815) author of “Essay on logarithmic transcendents”

Determination of coefficients of scalar integrals

Feynman diagrams + Passarino-Veltman reduction cannot be the answer as the number of legs increases. There are too many diagrams with cancellations between them.

Process	Amplitude	# of diagrams at 1 loop
$t\bar{t}$	$t\bar{t}gg$	30
$t\bar{t}$+1 jet	$t\bar{t}ggg$	341
$t\bar{t}$+2 jets	$t\bar{t}gggg$	4341
$t\bar{t}$+3 jets	$t\bar{t}ggggg$	63800

Semi-numerical methods based on unitarity are the answer. "Semi-numerical" because the integral containing the divergences is determined analytically, but its coefficient is determined numerically.

Unitarity for one-loop diagrams

- Important steps include:-
- First modern use of the idea Bern, Dixon,Kosower
- Cuts w.r.t. to loop momenta give (box) coefficients directly Cachazo, Britto, Feng
- OPP tensor reduction scheme, Ossola Pittau Papadopoulos
- The OPP procedure meshes well with unitarity Ellis Giele, Kunszt
- D-dimensional unitarity Giele, Kunszt, Melnikov

Generalized Unitarity

- Any one loop amplitude can be written as

$$A^{\text{1-loop}}(p) = \int \frac{d^D l}{(2\pi)^D} \frac{\text{Num}_D(l,p)}{\prod d_i} = \sum c_j I_j$$

- Ossola, Pittau and Papadopoulos showed us how to compute the coefficients c_j by considering (complex) loop momenta for which the denominators vanish.
- Set of propagators vanishing, corresponds to putting lines on-shell -- This is unitarity

Decomposition of a one-loop amplitude in terms of ■▲●…

$$
\begin{aligned}
\mathcal{A}_N(p_1, p_2, \ldots, p_N) \;=\; & \sum_{1 \le i_1 < i_2 < i_3 < i_4 \le N} d_{i_1 i_2 i_3 i_4}(p_1, p_2, \ldots, p_N) I_{i_1 i_2 i_3 i_4} \\
& + \sum_{1 \le i_1 < i_2 < i_3 \le N} c_{i_1 i_2 i_3}(p_1, p_2, \ldots, p_N) I_{i_1 i_2 i_3} \\
& + \sum_{1 \le i_1 < i_2 \le N} b_{i_1 i_2}(p_1, p_2, \ldots, p_N) I_{i_1 i_2} \\
& + \sum_{1 \le i_1 \le N} a_{i_1}(p_1, p_2, \ldots, p_N) I_{i_1} \;,
\end{aligned}
$$

$$
I_{i_1 \cdots i_M} = \int [d\,l]\, \frac{1}{d_{i_1} \cdots d_{i_M}}
$$

- Without the integral sign, the identification of the coefficients is straightforward.
- Determine the coefficients of a multipole rational function.

Residues of poles and unitarity cuts

Define residue function

$$\mathrm{Res}_{ij\ldots k}\left[F(l)\right] \equiv \left(d_i(l)d_j(l)\cdots d_k(l)F\left(l\right)\right)\Big|_{l=l_{ij\ldots k}}$$

We can determine the d-coefficients, then the c-coefficients and so on

$$\begin{aligned}
\overline{d}_{ijkl}(l) &= \mathrm{Res}_{ijkl}\Big(\mathcal{A}_N(l)\Big) \\
\overline{c}_{ijk}(l) &= \mathrm{Res}_{ijk}\left(\mathcal{A}_N(l) - \sum_{l\neq i,j,k}\frac{\overline{d}_{ijkl}(l)}{d_i d_j d_k d_l}\right) \\
\overline{b}_{ij}(l) &= \mathrm{Res}_{ij}\left(\mathcal{A}_N(l) - \sum_{k\neq i,j}\frac{\overline{c}_{ijk}(l)}{d_i d_j d_k} - \frac{1}{2!}\sum_{k,l\neq i,j}\frac{\overline{d}_{ijkl}(l)}{d_i d_j d_k d_l}\right) \\
\overline{a}_i(l) &= \mathrm{Res}_i\left(\mathcal{A}_N(l) - \sum_{j\neq i}\frac{\overline{b}_{ij}(l)}{d_i d_j} - \frac{1}{2!}\sum_{j,k\neq i}\frac{\overline{c}_{ijk}(l)}{d_i d_j d_k} - \frac{1}{3!}\sum_{j,k,l\neq i}\frac{\overline{d}_{ijkl}(l)}{d_i d_j d_k d_l}\right)
\end{aligned}$$

Giele, Kunszt,Melnikov, arXiv 0801.2237

Unitarity in D-dimensions

- The theory contains divergences which we regulate dimensionally. Divergences appear as poles as $\varepsilon = (4-D)/2 \to 0$
- We calculate the unitarity cuts numerically in integer dimensions D>4. Internal degrees of freedom are taken to be D_s dimensional.

$$\mathcal{N}^{D_s}(l) = \mathcal{N}_0(l) + (D_s - 4)\mathcal{N}_1(l)$$

- Dependence on D_s is linear so we calculating in a two different integer dimensions and extrapolate to $\varepsilon=0$
- Only the length of the loop momentum in the extra dimension is relevant so we can treat

Giele, Kunszt,Melnikov, arXiv 0801.2237

Extension to full amplitude

- Keep dimensions of virtual unobserved particles integer and perform calculations in more than one dimension.
- Arrive at non-integer values D=4-2ε by polynomial interpolation.
- Results for six-gluon amplitudes agree with original Feynman diagram calculation of RKE, Giele, Zanderighi.

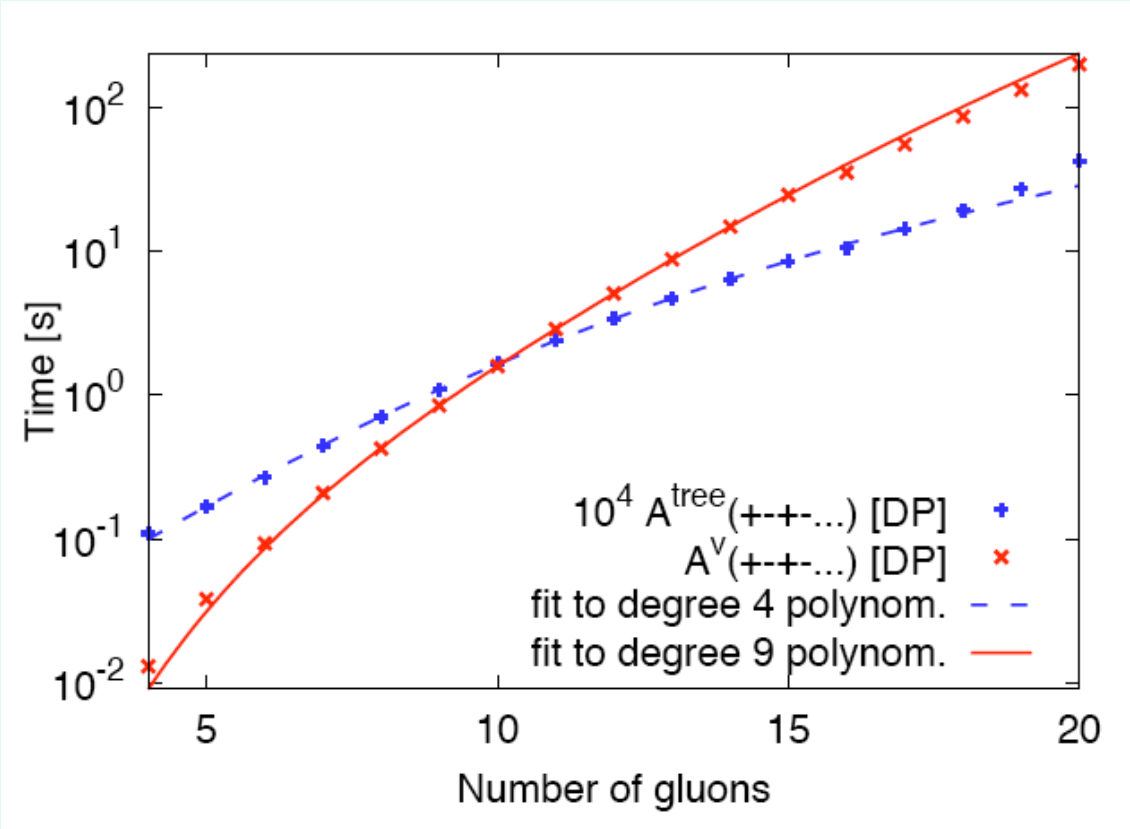
$\lambda_1, \lambda_2, \ldots, \lambda_6$	Δ^{cut}	Δ^{rat}	Δ
- - + + + +	-19.481065+78.147162 i	28.508591-74.507275 i	9.027526+3.639887 i
- + - + + +	-241.10930+27.176200 i	250.27357-25.695269 i	9.164272+1.480930 i
- + + - + +	5.4801516-12.433657 i	0.19703574+0.25452928 i	5.677187-12.179127 i
- - - + + +	15.478408-2.7380153 i	2.2486654+1.0766607 i	17.727073-1.661354 i
- - + - + +	-339.15056-328.58047 i	348.65907+336.44983 i	9.508509+7.869351 i
- + - + - +	31.947346+507.44665 i	-17.430910-510.42171 i	14.516436-2.975062 i

Giele, Zanderighi arXiv:0805.2152

One loop calculation of pure gluon amplitudes

Time to calculate one-loop amplitude scales as N^9 as expected.

For small numbers of legs N=4,5,6 the times are of the order of 10's of milliseconds

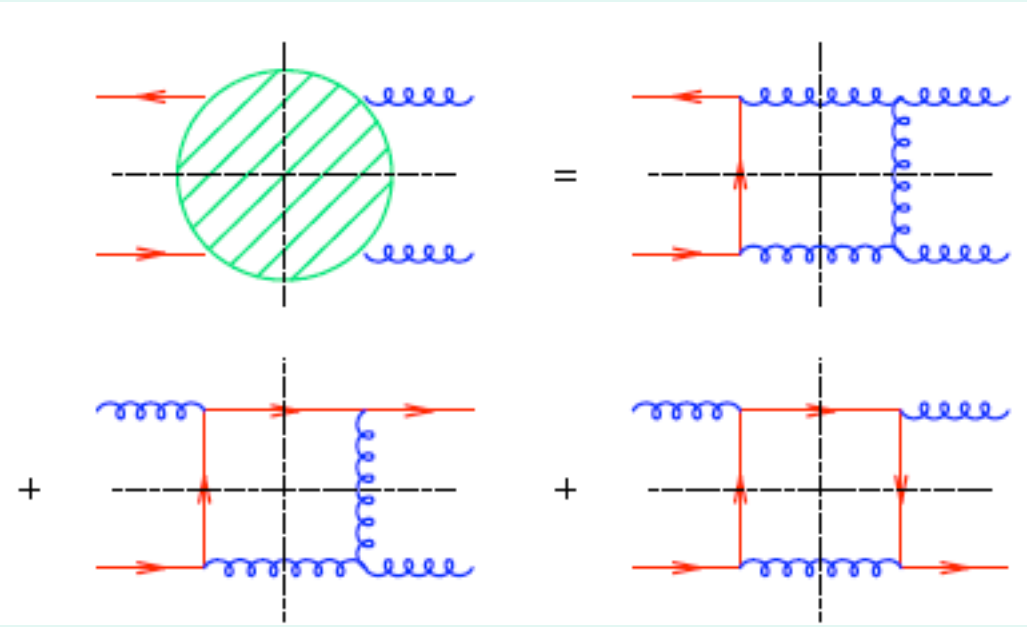


Ellis, Giele, Kunszt Melnikov:arXiv:0806.3467

Generalized unitarity and massive fermions

We have calculated the one-loop amplitudes for ttgg and ttggg as a proof of principle that the method can be extended to massive particles. Calculation times are longer than pure gluon amplitudes

Thus ttgg ~ 10 ms (cf 1ms for gggg) and ttggg ~ 40 ms.



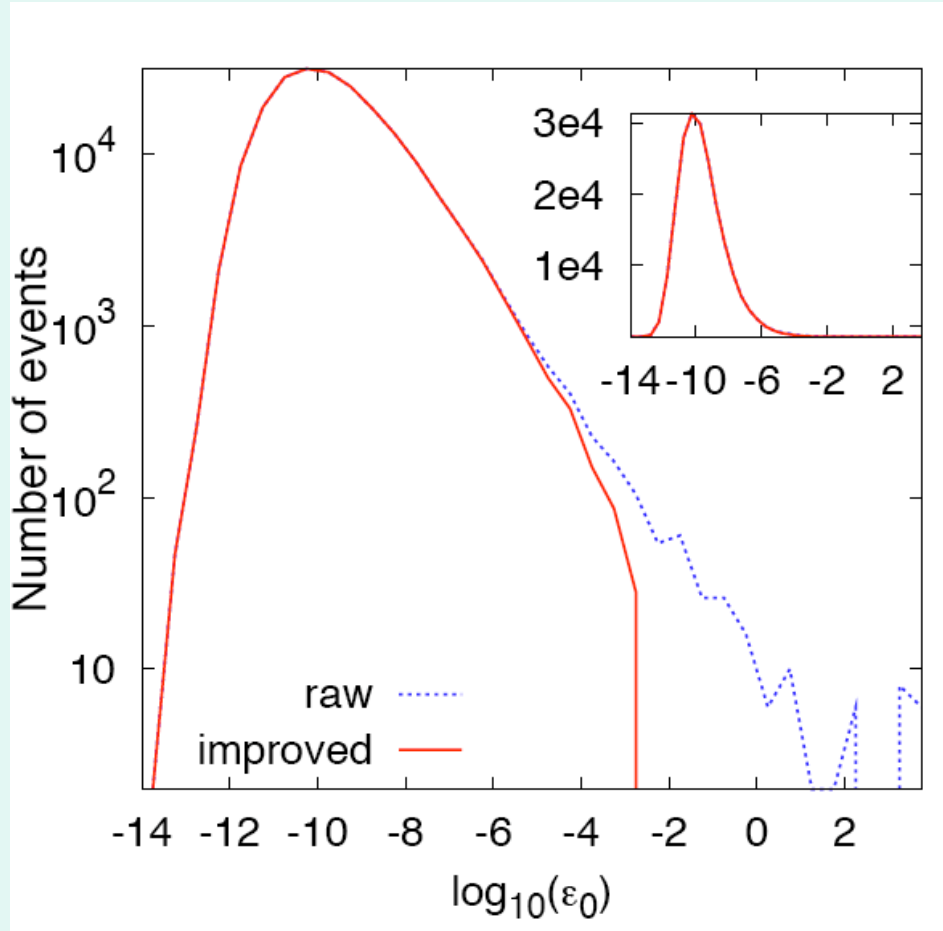
EGKMZ, arXiv:0810.2762

W+qqggg amplitudes

Numerical stability assured by computation, (where necessary) in quadruple precision.

$\varepsilon_0 = |(DP-QP)/QP|$

Evaluation times are 45-50 msec per leading color primitive on 2.33 GHz pentium Xeon machine.



One-loop amplitude summary

- There are a number of groups which use unitarity and OPP ideas to perform one-loop calculations (Berger et al, OPP, Lazopoulos, Giele & Winter).
- The F90 Rocket program (Ellis, Melnikov, Zanderighi) can compute results at one loop for:-
- N gluon scattering amplitudes
- two quarks (massless and massive) + N gluons,
- W-boson + two quarks + N gluons,
- W-boson + four quarks + 1 gluon
- tt+N gluons, ttqq+ N gluons (EGKM +Schulze)
- Note that extension to arbitrary number of gluons (using Berends-Giele recursion), and the proven ability to deal with massive fermions.

RKE, Melnikov,Zanderighi arXiv:0901.4101

W + 3 jets

- Here I report on recent calculations of W+3-jet rate at hadron colliders
- The calculation represents a proof of principle for the unitarity-based methods and is challenging with traditional methods, (1480 1-loop diagrams)
- W+3 jets is phenomenologically relevant because of Tevatron measurements, single top, SUSY searches, …
- More generally the rates for vector boson + jets production at the LHC will give you the keys to the BSM kingdom

RKE, Melnikov, Zanderighi, arXiv:0901.4101

W+3 jets: First NLO QCD results

- We simplify the problem by working at large N_c (N_c is the number of colors) and by keeping only the two quark channels qqW+ggg
- These are 10-30 percent approximations, so the phenomenology is rather preliminary.
- Virtual corrections are computed using a grid determined from the leading order computation
- Dipole subtraction is used for the real emission corrections.

Berger,Bern,Dixon,Febres-Cordero, Forde, Gleisberg,Ita,Kosower and Maitre, arXiv:0902.2760

A second calculation of W + 3 jets

Calculation also performed in the large N_c approximation, but with the addition of 4 quark processes which are formally subleading in color.

Important to have cross checks of these complicated calculations.

Detailed comparison will have to await publication of details and/or codes.

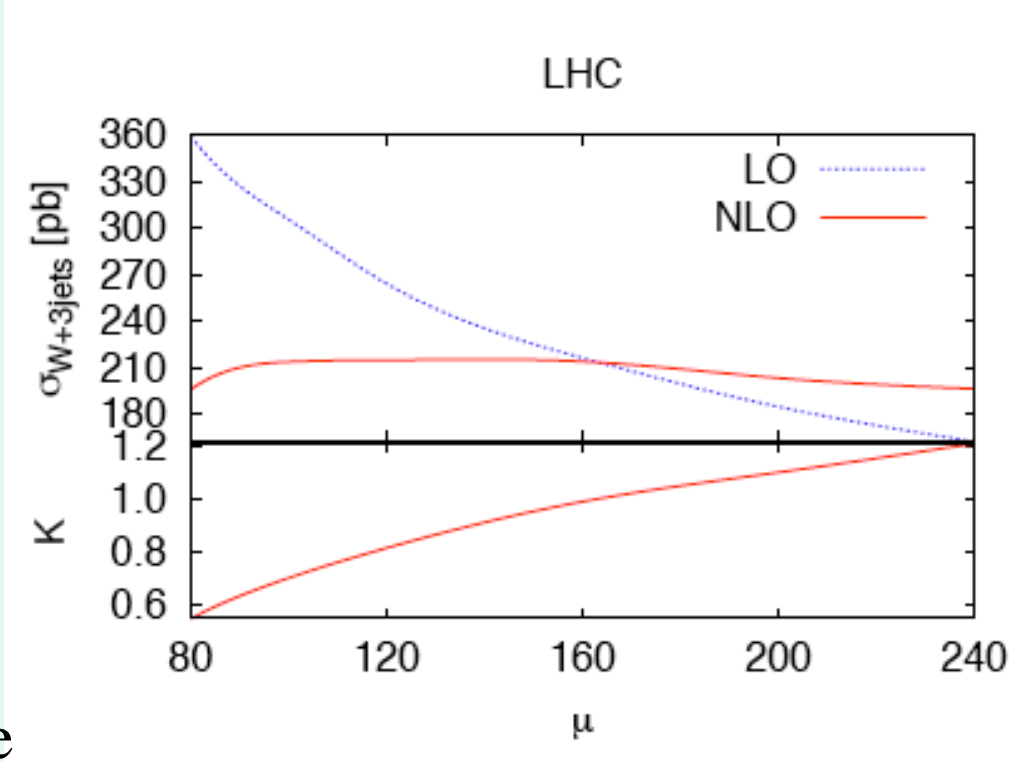
RKE, Melnikov,Zanderighi arXiv:0901.4101

W+3 jets: First NLO QCD results for LHC

Inclusive W+3jets + K factor
p_T>50GeV, |η| <3, R=0.7,
√S=14 TeV

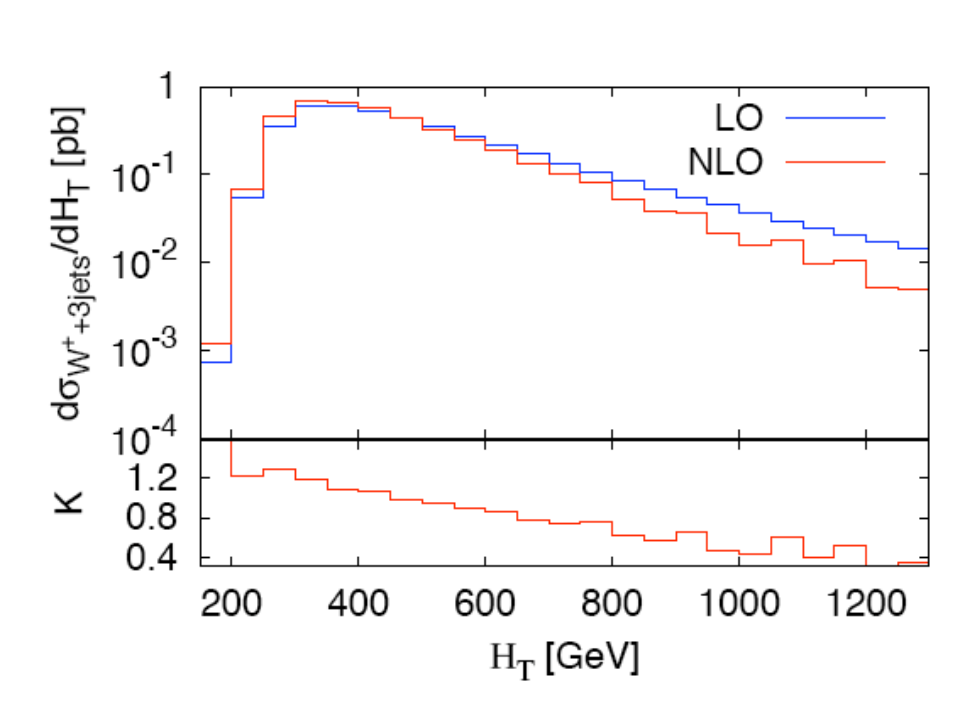
This calculation displays the standard improvement of scale dependence.

Detailed phenomenology at the 10% level will have to await the inclusion of all processes.



W+3 jets: First NLO QCD results for LHC

- Distribution of the transverse energy shows a softening at LHC, at least wrt to a lowest order result calculated at fixed scale μ=160 GeV



$$H_T = \sum_j E_{\perp,j} + E_\perp^{\text{miss}} + E_\perp^e$$

Summary

- $\alpha_S(M_Z)$ is known to < 2% and parton distributions are known well enough to predict most cross sections to 20%, (0.005<x<0.3)
- At high p_T, parton level integrators, such as MCFM, can do an adequate job of describing data with smaller theoretical errors than other methods.
- Calculation of tree graphs is a solved problem, for all practical purposes. Berends-Giele recursion is numerically the best method.

Summary (continued)

- Open theoretical problem has been the calculation of one-loop amplitudes
- All one-loop integrals for QCD are known.
- Unitarity based methods have achieved important results for one-loop amplitudes, these methods are now being tested in real physical calculations.
- I have presented first results on W+3 jet production.
- The hope is to have several semi-automatic methods of calculating one-loop amplitudes.