

$B \rightarrow \pi\pi$ form factors from light cone sum rules

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QCD sum rules approach

The **QCD sum rule approach** is based on a twofold way of treating process-dependent **correlation functions** of quark-antiquark operators: (I) a correlation function is calculated in QCD operator product expansion, yielding a combined perturbative and power expansion, which contains universal input parameters encoding the nonperturbative quark-gluon interactions; (II) the same correlation function is related to the **hadronic matrix elements** via a dispersion relation. Mutually connected versions of QCD sum rules for different types of hadronic matrix elements include the **light-cone sum rules (LCSRs)** for hadronic form factors and **two-point sum rules** for the decay constants of hadrons. One employs two different types of correlation functions with, respectively, **distribution amplitudes (DAs) of hadrons** and QCD **vacuum condensates** as nonperturbative inputs.

$B \rightarrow \pi\pi$ form factors in LCSRs

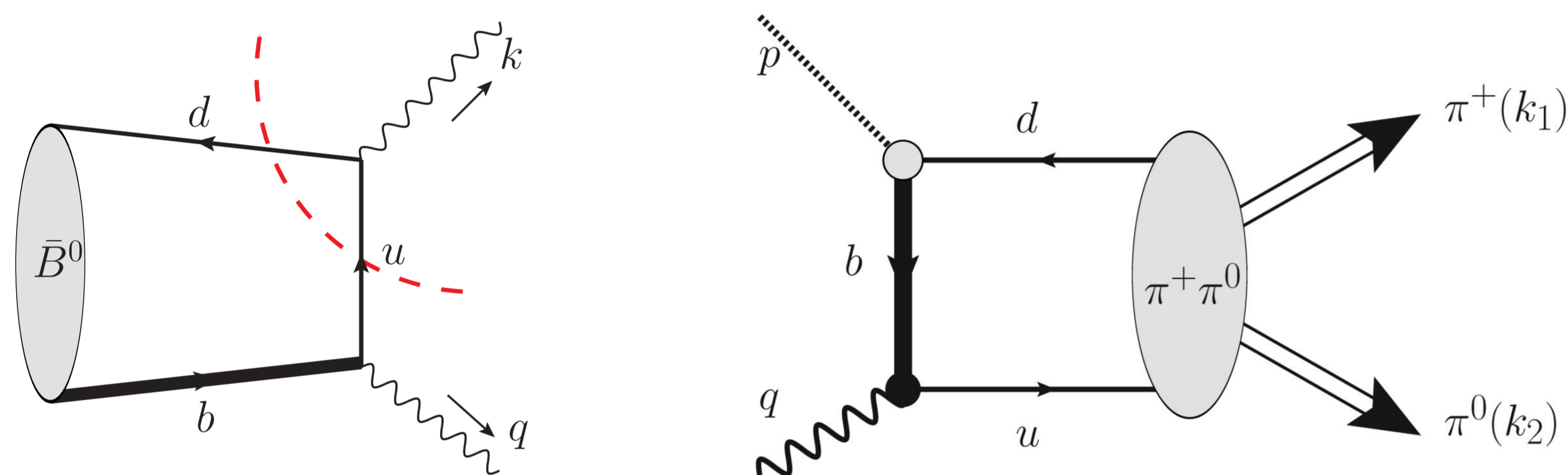


Illustration: Leading order diagram for the correlation functions used to derive QCD LCSRs for the form factors in $B \rightarrow \pi\pi\ell\bar{\nu}$ with the B (left) and dipion (right) DAs. See [1, 2, 3] for details.

With B meson distribution amplitude [1]

→ **Correlation function:** $\bar{d}\gamma_\mu u$ interpolation current with the $b \rightarrow u$ weak current,

$$F_{\mu\nu}(k, q) = i \int d^4x e^{ikx} \langle 0 | T \{ \bar{d}\gamma_\mu u(x), \bar{u}\gamma_\nu(1 - \gamma_5)b(0) \} | \bar{B}^0 \rangle.$$

→ **QCD calculation:** $q^2 \ll m_B^2, |k^2| \gg \Lambda_{\text{QCD}}^2$

long distant matrix element is defined by B meson light cone DAs;
short distant contribution is carried by internal free propagator.

→ **Hadron dispersion relation:** $\pi^-\pi^0$ interpolation, (unitarity relation)

$$\langle \pi^-(k_1)\pi^0(k_2) | \bar{u}\gamma_\mu d | 0 \rangle = -\sqrt{2}(k_1 - k_2)_\mu F_\pi(k^2),$$

⇒ convolution of F_π and $F_{B \rightarrow \pi\pi}$ in invariant mass s , ⇒ impossible to solve analytically.

→ **Quark-hadron duality and Borel transfer**

→ **Resonance model for $B \rightarrow \pi^-\pi^0$ form factor** to fit the form factor.

an independent and apposite channel to study $B \rightarrow \rho, \rho'$ from factors in LCSRs;

how large of ρ contribution to $B \rightarrow \pi^-\pi^0$ form factor ?

With dipion distribution amplitude [2, 3]

→ **Correlation function:** pseudoscalar heavy-light current j_5^B and weak current $j_{b \rightarrow u}^{(m)}$

$$\Pi^{(m)}(q, k_1, k_2) = i \int d^4x e^{iqx} \langle \pi^-(k_1)\pi^0(k_2) | T \{ j_{b \rightarrow u}^{(m)}(x), j_5^B(0) \} | 0 \rangle.$$

→ **Analytical solution in terms of isospin-vector dipion DAs**

→ $m = V - A$: limit knowledge of dipion DAs ⇒ only $F_{\perp,||}(q^2, s)$ is available at $s = 4m_\pi^2$;

kinematic singularity ⇒ $F_{t,0}(q^2, s)$ should be considered separately.

→ $m = P$: for **timelike-helicity form factor $F_{t,0}$**

$$B_{01}^|| (s) \simeq F_\pi(s) \text{ with B factory data, go to a large prediction } s \in [4m_\pi^2, 1.5 \text{ GeV}^2].$$

→ **Resonance model to estimate the ρ contribution**

Numerics

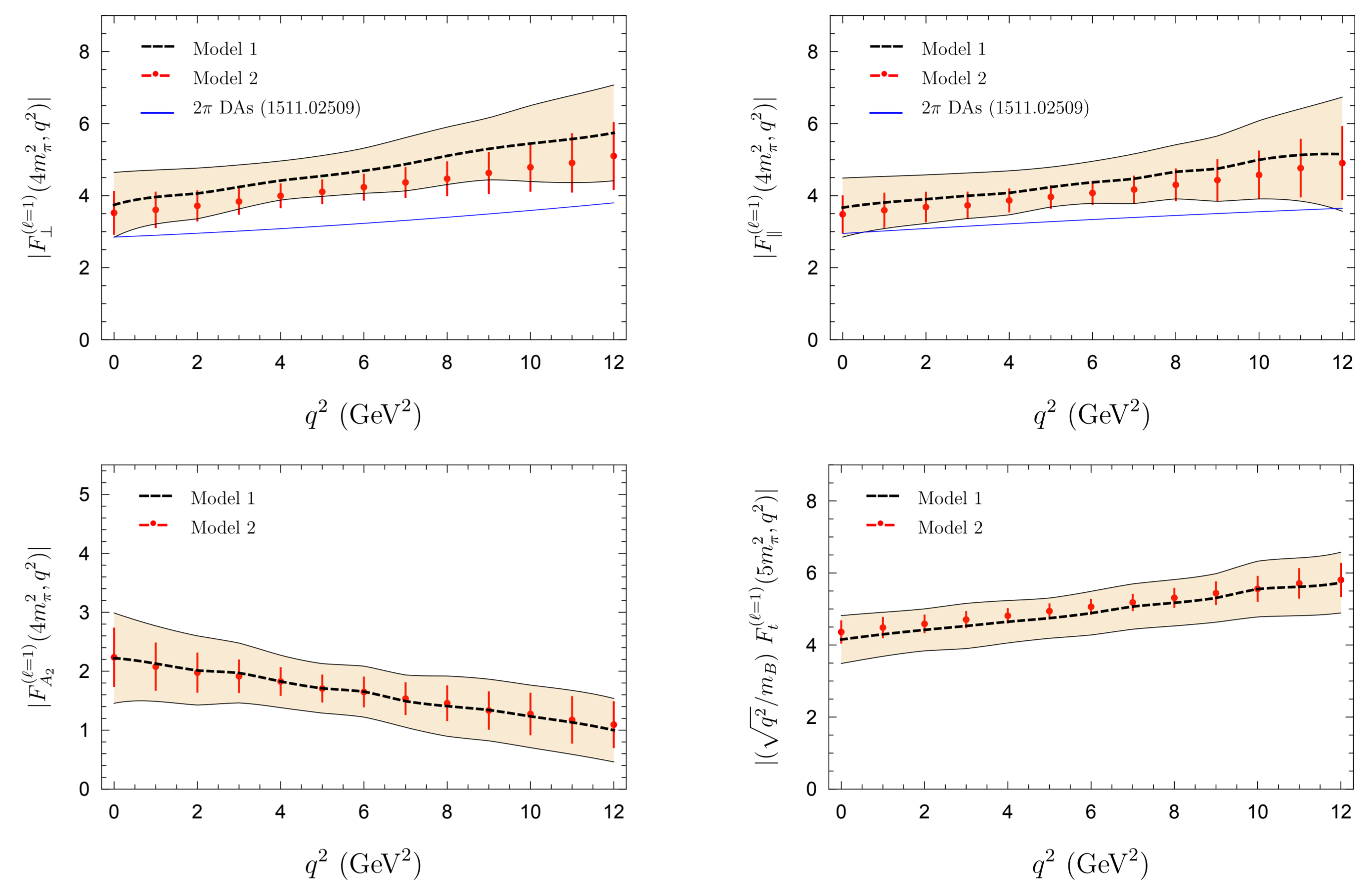


Figure.1: $B \rightarrow \pi^-\pi^0$ transition form factors obtained with B meson LCDAs.

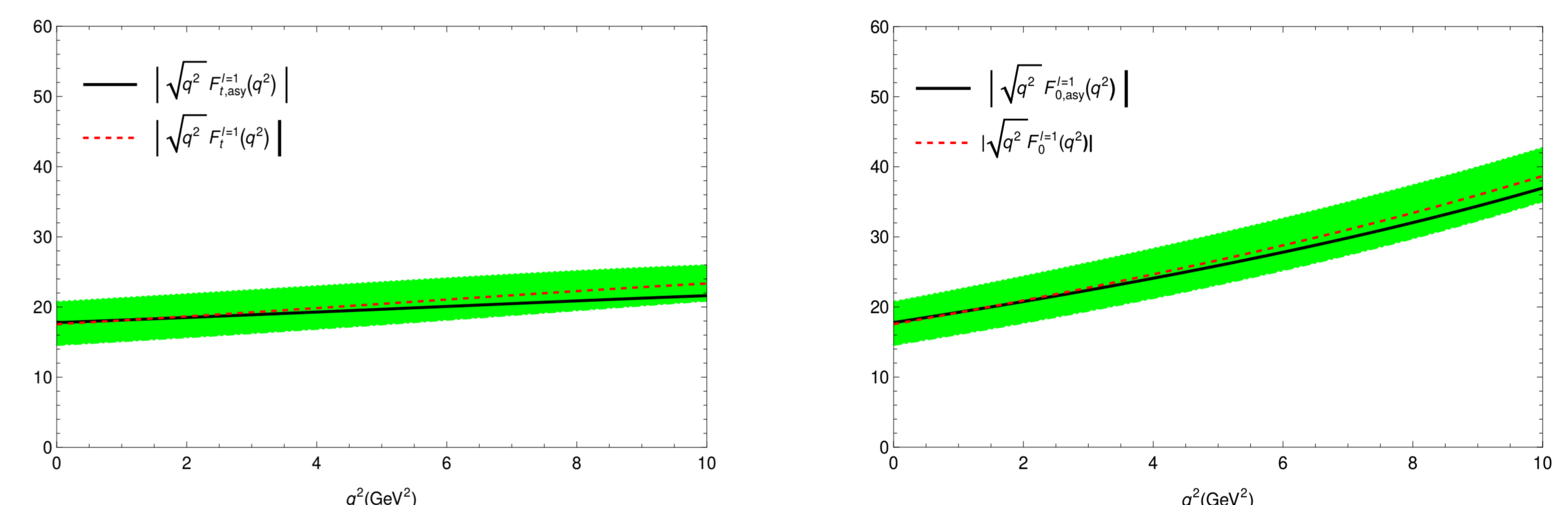


Figure.2: Timelike-helicity $B \rightarrow \pi^-\pi^0$ transition form factors $F_{t,0}$ obtained with diipion LCDAs.

Conclusion

- **Model-I in Fig.1:** two resonances model, with using the $B \rightarrow \rho$ form factor obtained from ρ meson LCSRs;
- **Model-II in Fig.1:** three resonances model, with their relative size being the same as in pion vector form factor parameterized to describe data;
- **Fig.2:** Timelike-helicity form factors $F_{t,0}$ are calculated separately to $F_{\perp,||}$ with diipion DAs, lowest expansion ($l = 1, n = 0$) gives almost the whole contribution;
- **P-wave contribution:** Only P-wave contribution with B meson DAs \ll pion vector form factor; absolute dominate with diipion DAs ($> 95\%$);
- **ρ contribution:** $80\% - 90\% \Rightarrow$ the residual deficit can be regarded as the finite-width effect in $B \rightarrow \rho$ form factors, rethink the single pole assuming;
- **k^2 -dependent evolution:** all four form factors with B meson DAs, only timlike-helicity form factors $F_{t,0}(k^2)$ with diipion DAs.

Outlooks

- Accuracy of OPE calculation: NLO correction & input parameters in B meson;
- light cone DAs of isospin-scalar diipion state and $K\pi$ state;
- $B \rightarrow \pi^0\pi^0$ to study B decay form factor with scalar final states;
- $B \rightarrow K\pi$ with SU(3) flavour violation final states $\Rightarrow B \rightarrow K^*$ form factors.

References

- [1] S. Cheng, A. Khodjamirian and J. Virto, " $B \rightarrow \pi\pi$ Form Factors from Light-Cone Sum Rules with B -meson Distribution Amplitudes", arXiv:1701.01633 [hep-ph].
- [2] C. Hambrock and A. Khodjamirian, "Form factors in $\bar{B}^0 \rightarrow \pi\pi\ell\bar{\nu}_\ell$ from QCD light-cone sum rules", Nucl. Phys. B **905**, 373 (2016), arXiv:1511.02509 [hep-ph].
- [3] S. Cheng, A. Khodjamirian and J. Virto, Timelike-helicity " $B \rightarrow \pi\pi$ form factor from light-cone sum rules with dipion Distribution Amplitudes", arXiv:1706.xxxxx [hep-ph].