

# Standard Model Extended by a Heavy Singlet: Linear vs. Nonlinear EFT

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## Introduction

The discovery of the Higgs boson together with the absence of new-physics has triggered a renewed interest in effective field theories (EFTs) at the electroweak scale.

**Building a *bottom-up* EFT for the electroweak sector requires that the nature of the underlying dynamics is specified.**

- If new-physics appears at a high scale  $\Lambda \gg v$  and decouples [1]:

### Standard Model Effective Field Theory or linear EFT

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{C_i}{\Lambda^{n_i-4}} \mathcal{O}_i,$$

with  $n_i$  the canonical dimension of  $\mathcal{O}_i$ . The leading order (LO) corresponds to the renormalisable SM. New physics effects are encoded in higher dimensional operators that enter suppressed by the appropriate power of  $\Lambda$ . The EFT expansion is organized by canonical dimensions.

- If the underlying dynamics responsible for electroweak symmetry breaking is strongly coupled (around the scale  $f \gtrsim v$ ) and new-physics effects do not decouple:

### Electroweak Chiral Lagrangian (EWChL) or nonlinear EFT

The LO EWChL is non-renormalisable and is given by [2]

$$\begin{aligned} \mathcal{L}_{\text{LO}} = & -\frac{1}{2} \langle G_{\mu\nu} G^{\mu\nu} \rangle - \frac{1}{2} \langle W_{\mu\nu} W^{\mu\nu} \rangle - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \bar{q} i \not{D} q + \bar{\ell} i \not{D} \ell + \sum_{f=u,d,e} \bar{f} i \not{D} f \\ & + \frac{v^2}{4} (D_\mu U^\dagger D^\mu U) (1 + F_U(h)) + \frac{1}{2} \partial_\mu h \partial^\mu h - V(h) + \\ & - v [\bar{q} Y_u(h) U u + \bar{q} Y_d(h) U d + \bar{\ell} Y_e(h) U e + \text{h.c.}], \end{aligned}$$

where  $U = \exp(2i\varphi^a T^a/v)$  is the Goldstone-boson matrix,  $h$  is the physical Higgs, and

$$V(h) = v^4 \sum_{n=2}^{\infty} f_{V,n} \left(\frac{h}{v}\right)^n, \quad F_U(h) = \sum_{n=1}^{\infty} f_{U,n} \left(\frac{h}{v}\right)^n, \quad Y_f(h) = \left( Y_f + \sum_{n=1}^{\infty} Y_f^{(n)} \right).$$

The EFT is organized as a loop expansion, or equivalently in the chiral dimension of fields and couplings. The assignment of chiral dimensions is 0 for bosons, and 1 for each derivative, weak coupling or fermion bilinear. The LO has chiral dimension 2. NLO corrections are encoded in operators of chiral dimension 4 and enter suppressed by two powers of the cut-off scale  $\Lambda \sim 4\pi f$ .

### Linear vs. Nonlinear EFT, a bottom-up view

In the linear EFT, we expect deviations from the SM in the Higgs and electroweak gauge sectors at the level of  $v^2/\Lambda^2$ . In the nonlinear EFT, deviations from the SM in the Higgs sector are expected at the level of  $v^2/f^2$  while the gauge sector is SM-like up to corrections of order  $v^2/(16\pi^2 f^2)$ .

Deviations from the SM in the Higgs couplings are parametrically larger than those in the electroweak gauge interactions within the nonlinear EFT, making it suitable for LHC studies of the Higgs properties [3].

A simple UV model can be used to illustrate the systematics of these two EFTs.

## Model

Consider the SM extended with a real scalar singlet

$$\mathcal{L} = (D^\mu \Phi)^\dagger (D_\mu \Phi) + \partial^\mu S \partial_\mu S - V(\Phi, S) + \mathcal{L}_{\text{Yukawa}},$$

with

$$V(\Phi, S) = -\frac{\mu_1^2}{2} \Phi^\dagger \Phi - \frac{\mu_2^2}{2} S^2 + \frac{\lambda_1}{4} (\Phi^\dagger \Phi)^2 + \frac{\lambda_2}{4} S^4 + \frac{\lambda_3}{2} \Phi^\dagger \Phi S^2.$$

We impose a  $Z_2$  symmetry under which  $S \rightarrow -S$ . The scalar fields develop vacuum expectation values (vevs),

$$\Phi = \frac{v + h_1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad S = \frac{v_s + h_2}{\sqrt{2}}.$$

Here we write  $\Phi$  in polar coordinates. The physical states are given by

$$\begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} \cos \chi & -\sin \chi \\ \sin \chi & \cos \chi \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}, \quad \tan(2\chi) = \frac{2\lambda_3 v v_s}{\lambda_2 v_s^2 - \lambda_1 v^2},$$

with  $M_h \equiv m < M_H \equiv M$  by convention. Fixing  $m = 125$  GeV, the model depends only on three combinations of parameters

$$r \equiv \frac{m^2}{M^2}, \quad \xi \equiv \frac{v^2}{f^2}, \quad \omega \equiv \sin^2 \chi, \quad \text{with } f \equiv \sqrt{v^2 + v_s^2}.$$

We assume an approximate  $SO(5)$  symmetry in the scalar sector. In the strict  $SO(5)$  symmetric limit, we have  $\lambda_1 = \lambda_2 = \lambda_3 \equiv \lambda = 2M^2/f^2$ ,  $r = 0$  and  $\omega = \xi$ .

We consider different limits of the parameter space and analyze the resulting low-energy EFT after integrating out the heavy state:

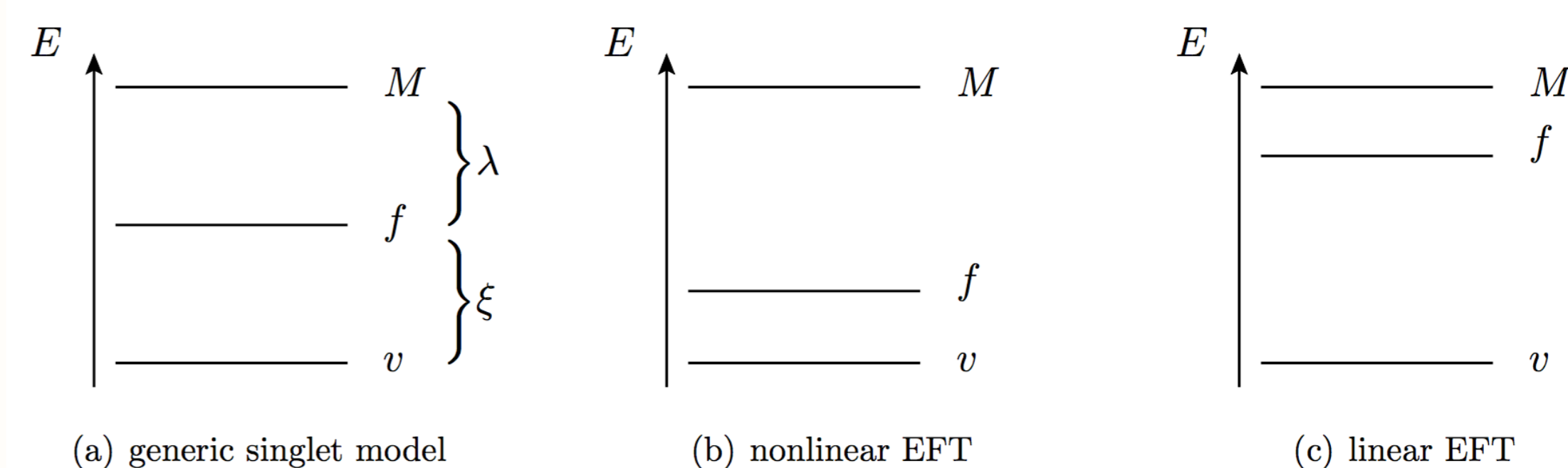
I) strongly-coupled regime (nonlinear EFT)

$$|\lambda_i| \lesssim 32\pi^2, \quad m \sim v \sim f \ll M \quad \Rightarrow \quad \xi, \omega = \mathcal{O}(1)$$

II) weakly-coupled regime (linear EFT)

$$\lambda_i = \mathcal{O}(1), \quad m \sim v \ll f \sim M \quad \Rightarrow \quad \xi, \omega \ll 1$$

illustrated here



## Linear vs. Nonlinear EFT, a top-down view

The Lagrangian of the model contains terms that depend on the heavy field  $H$  with the form

$$\mathcal{L} \supset \frac{1}{2} H(-\partial^2 - M^2)H + J_1 H + J_2 H^2 + J_3 H^3 + J_4 H^4,$$

where  $J_i \equiv M^2 \bar{J}_i^0 + \bar{J}_i$  making the  $M$  dependence explicit. The heavy field is integrated out at tree level by solving its equation of motion

$$(-\partial^2 - M^2 + 2J_2)H + J_1 + 3J_3 H^2 + 4J_4 H^3 = 0,$$

via an expansion in powers of  $1/M^2$ . In the strongly-coupled regime the resulting EFT takes the form of the nonlinear EFT with

$$F_U(h) = 2\sqrt{1-\xi} \left(\frac{h}{v}\right) + (1-2\xi) \left(\frac{h}{v}\right)^2 - \frac{4}{3} \xi \sqrt{1-\xi} \left(\frac{h}{v}\right)^3 + \mathcal{O}(h^4),$$

$$\begin{aligned} V(h) = & m^2 v^2 \left[ \frac{1}{2} \left(\frac{h}{v}\right)^2 + \frac{1-2\xi}{2\sqrt{1-\xi}} \left(\frac{h}{v}\right)^3 + \frac{1}{1-\xi} \left( \frac{1}{8} - \frac{7}{6}\xi + \frac{7}{6}\xi^2 \right) \left(\frac{h}{v}\right)^4 \right. \\ & \left. - \frac{\xi(1-2\xi)}{2\sqrt{1-\xi}} \left(\frac{h}{v}\right)^5 + \mathcal{O}(h^6) \right], \end{aligned}$$

and

$$Y_f + \sum_{n=1}^{\infty} Y_f^{(n)} \left(\frac{h}{v}\right)^n = Y_f \left[ 1 + \sqrt{1-\xi} \left(\frac{h}{v}\right) - \frac{\xi}{2} \left(\frac{h}{v}\right)^2 - \frac{1}{6} \xi \sqrt{1-\xi} \left(\frac{h}{v}\right)^3 + \mathcal{O}(h^4) \right].$$

In the weakly-coupled limit, integrating out the heavy scalar gives rise to an EFT organized by canonical dimensions of the fields. The only dimension six operator generated is  $(\Phi^\dagger \Phi) \square (\Phi^\dagger \Phi)$ . The resulting linear EFT can also be derived by expanding the leading order nonlinear effective Lagrangian through  $\mathcal{O}(\omega, \xi)$ .

## Summary and conclusions

A simple model has been used to illustrate the systematics of the linear and nonlinear EFTs. These two EFTs possess the same degrees of freedom and symmetries, yet they are organized by different power counting principles. The toy model allows to understand the different organization principles based on the character of the underlying dynamics. In the non-decoupling regime the low-energy EFT approaches the nonlinear EFT formulation organized as a loop expansion. In the decoupling regime, the low-energy EFT falls into the linear EFT paradigm and is organized via canonical dimensions.

## References

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