Gauge boson masses from fermion masses in BSM

Petr Beneš

IEAP CTU Prague

Žilina, June 2, 2016

Outline

1. Introduction: Particle masses and symmetries (a reminder of the basic logic)
2. Masses $W^{\pm}$ and Z from fermion masses (in models BSM)
3. A byproduct: Correction to the Pagels–Stokar formula of QCD

1. Introduction

Particle masses

The basic *experimental* facts:

- The gauge bosons $W^{\pm}$, Z are massive
- The fermions are massive

The basic *theoretical* facts:

- The gauge boson masses are protected by the electroweak symmetry $\mathbb{SU}(2)_{\mathrm{L}} \times \mathbb{U}(1)_{\mathrm{Y}}$
- The fermion masses are protected by chiral symmetries (i.e., those treating differently left- and right-handed chiral fields)
  (right-handed neutrinos are another story ...)

$\Rightarrow$ Two requirements for any theory of Nature:

- Electroweak symmetry breaking (EWSB)
  - $\mathbb{SU}(2)_{\mathrm{L}} \times \mathbb{U}(1)_{\mathrm{Y}}$ must be spontaneously broken down to $\mathbb{U}(1)_{\mathrm{em}}$
  - $\rightarrow W^{\pm}$, Z masses
- Chiral symmetry breaking (χSB)
  - The chiral symmetries must be broken
  - $\rightarrow$ fermion masses

EWSB $\not\Rightarrow$ χSB

Key fact: The electroweak symmetry is a subgroup of the chiral symmetries

$\Rightarrow$

- EWBS doesn't break enough chiral symmetries to allow for fermion masses
- Massiveness of $W^{\pm}$, Z doesn't necessarily imply massiveness of fermions
- $\Rightarrow$ Having only EWSB, an independent mechanism has to be devised for χSB

Examples:

- SM, 2HDM, MSSM, . . .
  - EWSB: condensing scalar(s)
  - χSB: Yukawa interactions
- (E)TC
  - EWSB: Technicolor
  - χSB: Extended Technicolor

Alternative

Starting point:

- Sure, EWSB does not neccessarily imply χSB
- But the opposite is true: χSB implies EWSB

$\Rightarrow$ The idea: turn the logic upside down:

- Better think about getting χSB, and EWSB will come *automatically*
- In other words: Invent a way how to generate fermion masses, and the gauge boson masses will pop up for free
- (Actually the old idea of the top-quark condensate model)

$\Rightarrow$ Three steps:

1. Invent a model that dynamically generates the fermions masses (χSB)
2. Calculate the $W^{\pm}$, Z masses from the fermion masses (χSB $\Rightarrow$ EWSB)
3. Ensure that the low energy effective theory is just the SM Higgs sector

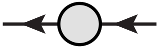
The step **2.** is model independent $\longrightarrow$ topic of this talk

2. $W^{\pm}$, Z masses from fermion masses

Dynamical fermion mass generation

- Basic assumption: Some dynamics generates the fermion self-energy $\mathbf{\Sigma}(p^2)$

$$\langle \psi \bar{\psi} \rangle_{\mathrm{1PI}} \;=$$



$$= \quad -\mathrm{i}\mathbf{\Sigma}(p^2)$$

- Fermion masses are given by the poles of the full propagator $\left[\not{p} - \mathbf{\Sigma}(p^2)\right]^{-1}$:

$$\det\left[m^2 - \mathbf{\Sigma}^\dagger(m^2)\mathbf{\Sigma}(m^2)\right] \quad = \quad 0$$

(N.B.: A self-energy $\mathbf{\Sigma}(p^2)$ is in general a matrix.)

- Another assumption: The self-energies break $\mathbb{SU}(2)_{\mathrm{L}} \times \mathbb{U}(1)_{\mathrm{Y}} \longrightarrow \mathbb{U}(1)_{\mathrm{em}}$

$$\left[\mathbf{\Sigma}(p^2), T_a\right] \quad \neq \quad 0 \qquad \text{for broken generators}$$

Gauge boson masses

- The self-energies break $\mathbb{SU}(2)_{\mathrm{L}} \times \mathbb{U}(1)_{\mathrm{Y}}$ down to $\mathbb{U}(1)_{\mathrm{em}}$
- $\Rightarrow W^{\pm}$ and Z must obtain masses
- These masses are given by poles of the full gauge boson propagator
- Specifically, the gauge boson mass matrix M^2_{ab} is given by the leading term of the polarization tensor

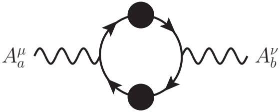
$$\Pi^{\mu\nu}_{ab}(q) = \langle A^{\mu}_{a} A^{\nu}_{b} \rangle_{\mathrm{1PI}} =$$

$$= \left(g^{\mu\nu} - \frac{q^{\mu} q^{\nu}}{q^2} \right) \left[M^2_{ab} + \mathcal{O}(q^2) \right]$$

Polarization tensor

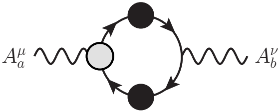
How to calculate the polarization tensor $\Pi^{\mu\nu}_{ab}(q)$ using the fermion self-energies?

- What about this:

$\Pi^{\mu\nu}_{ab}(q) \quad = $ 

  Wrong! $\Pi^{\mu\nu}_{ab}(q)$ is not transversal!

- But:

$\Pi^{\mu\nu}_{ab}(q) \quad = $ 

  Correct!

- Provided the dressed vertex $\Gamma^\mu_a(p', p)$ satisfies the Ward–Takahashi identity

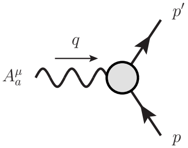
$$(p' - p)_\mu \Gamma^\mu_a(p', p) \quad = \quad G^{-1}(p')\, T_a - T_a\, G^{-1}(p)$$

where $G^{-1}(p) = \not{p} - \mathbf{\Sigma}(p^2)$

How to find the vertex

⇒ The key quantity we need to know is the vertex function $\Gamma_a^\mu(p', p)$:

$$\Gamma_a^\mu(p', p) \quad = $$



⇒ We must somehow find it (calculate/construct/approximate/guess/. . .)

Our approach:
We construct an Ansatz for $\Gamma_a^\mu(p', p)$ by requiring the following conditions:

- The Ward–Takahashi identity
- Correct transformation properties under continuous and discrete ($\mathcal{C}$, $\mathcal{P}$, $\mathcal{T}$) symmetries
- Hermiticity
- Correct analytic structure (Nambu–Goldstone pole)
- Linearity in $\mathbf{\Sigma}(p'^2)$, $\mathbf{\Sigma}(p^2)$ and T_a

Resulting vertex

- One can straightforwardly find:

$$\Gamma^\mu_a(p',p) = \underbrace{\gamma^\mu T_a - \frac{1}{2}\frac{q^\mu}{q^2}\Big[\boldsymbol{\Sigma}(p'^2)+\boldsymbol{\Sigma}(p^2),T_a\Big] - \frac{1}{2}q'^\mu\Big\{\frac{\boldsymbol{\Sigma}(p'^2)-\boldsymbol{\Sigma}(p^2)}{p'^2-p^2},T_a\Big\}}_{\text{saturates the WT identity (Ball, Chiu, 1980)}}$$

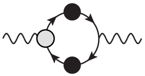
$$+\underbrace{x\left(\frac{q^\mu}{q^2}[\not q,\not q'] - [\gamma^\mu,\not q']\right)\Big[\frac{\boldsymbol{\Sigma}(p'^2)-\boldsymbol{\Sigma}(p^2)}{p'^2-p^2},T_a\Big]}_{\text{generically new (transversal) term}}$$

where $q' = p' + p$

- Notice the parameter x: in principle arbitrary real number, undetermined by the WT identity
  $\Rightarrow$ the vertex is ambiguous

Symmetricity of the mass matrix

- Having $\Gamma_a^\mu(p',p)$, we can calculate the gauge boson mass matrix M_{ab}^2
- The resulting M_{ab}^2 has two flaws:
  - It depends on some (in principle arbitrary) parameter x: $M_{ab}^2 = M_{ab}^2(x)$
  - It is, in general, not symmetric: $M_{ab}^2 \neq M_{ba}^2$
- However, these two flaws can be fruitfully combined to cancel each other:
  - M_{ab}^2 is symmetric if and only if $x = -1/12$
  - This holds even for larger class of theories than just the EW interactions
  - (Notice that in the older literature it was always $x = 0$)
- The mass matrix M_{ab}^2 of the photon, W and Z then has the correct form



$$= \underbrace{\begin{pmatrix} g^2F_W^2 & 0 & 0 & 0 \\ 0 & g^2F_W^2 & 0 & 0 \\ 0 & 0 & g^2F_Z^2 & -gg'F_Z^2 \\ 0 & 0 & -gg'F_Z^2 & g'^2F_Z^2 \end{pmatrix}}_{M_{ab}^2} \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) + \dots$$

with only two independent form factors F_Z, F_W

Spectrum

- The resulting spectrum:

$$\begin{aligned} M_Z &= \sqrt{g^2 + g'^2} F_Z \\ M_W &= g F_W \end{aligned}$$

- The form factors:

$$\begin{aligned} F_Z^2 &= -\mathrm{i}\frac{N_c}{2} \sum_{f=u,d} \mathrm{Tr} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \Big[\mathbf{\Sigma}_f \mathbf{\Sigma}_f^\dagger - \frac{p^2}{2} (\mathbf{\Sigma}_f \mathbf{\Sigma}_f^\dagger)' \Big] D_f^2 \\ &\quad + \text{contribution of leptons} \\ F_W^2 &= -\mathrm{i}\frac{N_c}{2} \mathrm{Tr} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \Big\{ \Big[\mathbf{\Sigma}_u \mathbf{\Sigma}_u^\dagger - \frac{p^2}{2} (\mathbf{\Sigma}_u \mathbf{\Sigma}_u^\dagger)' \Big] D_d D_u \\ &\qquad - \frac{p^2}{2} (\mathbf{\Sigma}_u \mathbf{\Sigma}_u^\dagger) \Big[D_d D_u' - D_d' D_u \Big] \Big\} \\ &\quad + (u \leftrightarrow d) + \text{contribution of leptons} \end{aligned}$$

where $D_f \equiv (p^2 - \mathbf{\Sigma}_f \mathbf{\Sigma}_f^\dagger)^{-1}$.

- The lepton contributions are more complicated due to the presence of (possibly Majorana) massive neutrinos

3. Chiral symmetry of QCD

Generalization

The previous analysis can be generalized:

- Instead of a spontaneously broken local symmetry (e.g., the EW symmetry) one can consider a global one.
  - I.e., instead of $\langle A_a^\mu A_b^\nu \rangle$ one can consider $\langle j_a^\mu j_b^\nu \rangle$
- It is natural to assume that the functional form of $\Gamma_a^\mu(p', p)$ should be the same in all theories.
  - I.e., including $x = -1/12$ even when symmetricity of $\langle A_a^\mu A_b^\nu \rangle$ or $\langle j_a^\mu j_b^\nu \rangle$ is not an issue

Example:
Spontaneous breakdown of the (global) chiral symmetry $\mathbb{SU}(N_f)_A$ in QCD:

- The relevant quantity is

$$\langle j_a^\mu j_b^\nu \rangle \quad \sim \quad F_\pi^2 \delta_{ab}$$

  where F_π is the pion decay constant

Pagels–Stokar formula

⇒ The Pagels–Stokar formula (1979) for estimating F_π:

- Original PS formula (corresponding to $x = 0$):

$$F_\pi^2 = -2\mathrm{i}N_f N_c \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{\boldsymbol{\Sigma}^2 - \frac{1}{4}p^2 \frac{\mathrm{d}}{\mathrm{d}p^2}\boldsymbol{\Sigma}^2}{(p^2 - \boldsymbol{\Sigma}^2)^2}$$

- Our suggested correction of the PS formula ($x = -1/12$):

$$F_\pi^2 = -2\mathrm{i}N_f N_c \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{\boldsymbol{\Sigma}^2 - \frac{1}{2}p^2 \frac{\mathrm{d}}{\mathrm{d}p^2}\boldsymbol{\Sigma}^2}{(p^2 - \boldsymbol{\Sigma}^2)^2}$$

For $\boldsymbol{\Sigma}(p^2) = 4m^3/p^2$ with constituent quark mass $m = 244\,\mathrm{MeV}$ and $N_f = 2$ (Ansatz originally used by Pagels and Stokar) we have

- Original PS formula:

$$F_\pi = 83\,\mathrm{MeV}$$

- Corrected PS formula:

$$F_\pi = 96\,\mathrm{MeV}$$

N.B.: The experimental value is $F_\pi = 93\,\mathrm{MeV}$

Summary

- Most of the models utilize more or less independent mechanisms for generating gauge boson masses and fermion masses
- I explored a simpler way: Assuming that the fermion masses are already somehow generated, the gauge boson masses follow automatically
- I provided model-independent explicit formulas for $W^{\pm}$ and Z masses in terms of model-dependent fermion masses/self-energies
- Unlike similar previous analyses in the literature and thanks to a generically new term in the gauge boson-fermion vertex, the resulting gauge boson mass matrix is consistent (symmetric) even for a general form of the fermion self-energies (e.g., matrix-like)
- Generalizing the analysis slightly, I provided a correction to the Pagels–Stokar formula for F_{π}, yielding better results

- Reference: Mod.Phys.Lett. A30 (2015) no.19, 1550094 [arXiv:1402.5055]

Thank you for your attention!